SOS Sudoku

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The game Sudoku consists in filling a 9×9 grid (thus, 81 cells) with numbers from 1 to 9 such that the numbers in each row and in each column are distinct. There is another rule, namely consider the grid to be composed of nine 3×3 grids, called boxes: the numbers in each box must be distinct. One does not start with an empty grid but some numbers are already given so that there is only one possible way of completing the grid. With logical reasonings one progressively writes down the correct numbers in the cells until the grid is complete. We are going to describe some basic strategies which may help you if you are stuck with your Sudoku.

The core method is analyzing one box (or a row, or a column) and look at which numbers are possible in each of the cells, discarding the choices which would contradict one of the rules. One can temporarily write down in the cells all the still possible values for the numbers because this conveys useful information.

Only choice: It often happens that one finds a cell where only one number is possible, which means that we can write down the correct number in that cell.

An example of the above elimination method leading to "only choice" is the following.
• Consider the top center box. The number 9 already appears in the second and the third row, so the 9 in that box must be in the only free place on the first row.
• Consider the top right box. The number 1 already appears in the first and the second row, so the 1 in that box must be in one of the two free places on the third row. We may discard the left place because there is a 1 in that column, so the 1 must be in the bottom right cell of that box.

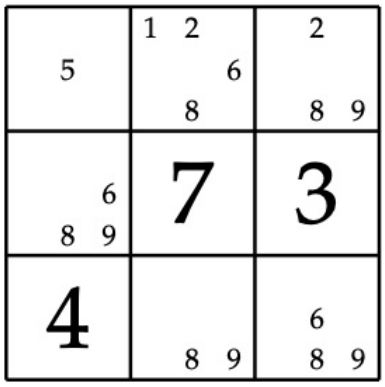
• Consider the center left box. In the top center cell there cannot be the numbers $8, 7, 3, 4$ (already present in that box) nor the numbers $9, 5$ (already present in that column) nor the numbers $1, 6$ (already present in that row). So the only possible number for that cell is the number 2.

			1		4			
		1				9		
	9		7		3		6	
8		7				1		6
3		4				5		9
	5		4		2		3	
		8				6		
			8		6			

Hidden Single: It may happen that in a box (or a row, or a column) one number can be put in only one position. In this case we may write the number in that cell.

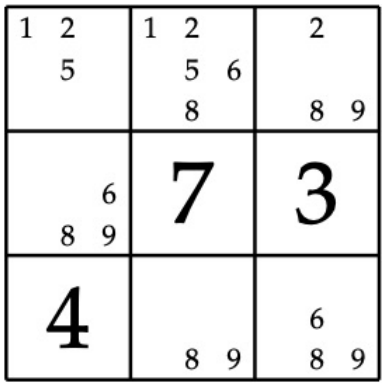
Subset: It may happen that in a box (or a row, or a column) there are n cells whose possible numbers are exactly n distinct numbers. Then you are sure that the n numbers fit those cells, and hence you may discard them from other cells.

In the figure we illustrate the above techniques for a box. The numbers $4, 7, 3$ are already in the correct position, and suppose that with the elimination technique the possibilities for the numbers in the other cells are what we have written down. Then because of the "only choice" we know that the number 5 must be in the top left cell. By "hidden single" we know that the number 1 must be in the top center cell. Moreover, in the three free cells in the center and bottom row there must be the numbers $6, 8, 9$ (three cells with only three possible distinct numbers). Then by "subset" we deduce that the numbers 8 and 9 are not in the top row. This implies that in the top right cell there must be the number 2.



Hidden numbers: This is a generalization of hidden single to more numbers. It may happen that in a box (or a row, or a column) there are n numbers that can only be placed in n cells. Then we can remove the other numbers from those cells.

Let us modify the previous example to explain this last method "hidden numbers" for a box. We see that the three numbers $1, 2, 5$ can occupy at most the three cells in the top row, so they must fill the top row. We deduce that $6, 8, 9$ cannot be in the top row. Consequently, the number 2 must be in the top right cell while the numbers $1, 5$ occupy the top left and top center cell (we do not know their exact position).



Locked candidate in a box: Suppose that in a given box a number can only appear in one row (or one column). Then we may discard that number from the remaining cells of that row (or column).

Locked candidate in a row or column: Suppose that a number in a given row (or column) must belong to some box. Then we may discard that number from the other rows (or columns) of that box.

The intersecting value: Suppose that for a given row a number must be inside a certain box, and also that for a given column it must be in the same box. Then that number must reside at the intersection of the row and the column (which of course resides in the considered box).

The X method: Suppose that there are two rows (respectively, columns) such that a given number can occupy only two positions in each of them. If these cells happen to be on the same two columns (respectively, rows) then we can discard all other possibilities for that number on those two columns (respectively, rows).

We illustrate the advanced X method with an example. Consider the number 4 in the following grid. We have marked all possible places for the missing numbers 4. Consider the third and seventh row of the diagram: there are exactly two possibilities for the number 4, which correspond to the same columns (first and sixth column). We may then discard the following possibilities for the number 4: the first, second, and eighth entry in the first column; the second entry in the sixth column.

4?		*4?*	*4?*		7	*4?*	*4?*	6
4?		*4?*	2		*4?*	5	*4?*	
4?		6	1	3	*4?*	7	9	
6	3	1	9	4			5	7
2	9		7	6		*4?*	8	3
7	4		3	2	5	1	6	9
4?		2	6	1	*4?*			8
4?		3	*4?*	7	9	6		5
9		7	8			3		4

There are many advanced techniques for solving hard Sudoku puzzles without resolving to the method of trial and error, and there is a vast liter-

ature on the subject. You are invited to became a Sudoku master!

References: Some of the examples in this article were inspired from `https://www.conceptispuzzles.com/de/index.aspx?uri=puzzle/sudoku/techniques` and `https://www.sudokudragon.com/sudokustrategy.htm`.

Question for the reader: In the last example, can you justify why the numbers 4 can only be in the places that we marked in the grid?

Solution to the question for the reader: We removed the remaining possibilities in the top boxes because 4 cannot appear twice in those columns. Moreover, there is no further 4 in the center left and the central box, as well as the bottom right box. In the center right box, the 4 cannot be in the first row, in the bottom left box the 4 cannot be in the second column, and in the bottom center box the 4 cannot be in the third row (because there is already a number 4 in these rows).