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ESSAYS ON LUXEMBOURG'S RESIDENTIAL ENERGY TRANSITION AND THE RARE EARTH ELEMENTS COMPETITION

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Abstract

The present dissertation focuses on two subjects: the efficiency of energy policies in the Luxembourgish residential building sector and the strategic interactions between the U.S. and China in the rare earth elements supply market. I investigated these topics in the following works by means of computational and game-theoretical models.

Energy Policies for Eco-Friendly Households in Luxembourg *(single author)*. In the Grand Duchy of Luxembourg, the residential building sector is a major energy consumer and greenhouse gases emitter that plays a key role in achieving the country's environmental objectives. The purpose of this work is to assess the effectiveness of the most important policy instruments in decreasing the final energy consumption and direct CO_2 emissions of Luxembourgish households. To this end, we developed the LuxHEI model, which is an enhanced and upgraded version of the well-known French simulation model Res-IRF. This variant has also been adjusted to the particular problems of a small country with growing economy and a quickly increasing population. The LuxHEI model goes beyond standard energy-economy models by incorporating global warming as a decision-making factor. The model outcomes reveal that in 2060, and compared to the no-policy baseline scenario, the most aspirational policy mix enables energy savings of 42% and emission reductions of 60%. However, in none of the projections, the residential building sector meets the national energy and climate targets on time. From the results we can draw the following policy implications: for a significant improvement of the sector's energy efficiency and sufficiency, the implementation of a remediation duty for existing buildings and the tightening of the performance standards for new constructions, together with the application of a national carbon tax, are crucial.

The U.S.–China Market for Rare Earth Elements: A Dynamic Game View *(joint with Luisito Bertinelli and Benteng Zou)*. Rare earth elements govern today's high-tech world and are deemed to be essential for the attainment of sustainable development goals. Since the 1990s, these elements have been predominantly supplied by one single actor, China. However, due to the increasing relevance of their availability, the United States, who imports 80% of its rare earths from China, recently announced its plan to (re-)launch national rare earths production. This work analyzes the two countries' competitive and cooperative interactions in open-loop strategy spaces. Particular emphasis is devoted to (1) the optimal timing for the U.S. to start the rare earth elements rivalry, (2) the effects of China's response to the U.S.'s market entry under competitive behaviors, and (3) the discrepancies between non-cooperative and cooperative supply approaches. By setting up a continuous-time differential game model, we show that in the absence of arbitrage opportunities, (1) the U.S. optimally enters the competition when its rare earths reserves coincide with those of China, (2) China's rivalry turns out best when the U.S.'s extraction is withheld by a conservative monopolistic behavior and a continuous entry-supply, and (3) compared to non-cooperative environments, cooperation does not lead to a Pareto improvement.

Dynamic Rare Earth Elements Game under Markovian Feedback Strategies *(joint with Benteng Zou)*. We study the very particular U.S.-China supply competition for rare earth elements (REEs) under Markovian strategies. With a view to determining the REEs competitors' optimal strategy space, this work includes a comparison between the present models' Markovian outcomes and the open-loop strategies of our earlier work. The findings yield that when competitors have the Markovian flexibility to update supply actions throughout the entire game, this at first triggers a relatively aggressive but beneficial extraction behavior. At a later stage of the game, this trend reverses and the initially more conservative open-loop behavior prevails over the Markovian one. The accompanying gradual switch in instantaneous profitability between the two strategy spaces then eventually makes open-loop strategies the optimal choice for both competitors; provided that China keeps its supply continuous at the U.S.'s market entry.

Contents

Chapter 1

General Introduction

With a view to universally fighting the devastating effects of anthropogenic climate change, almost all members of the United Nations Framework Convention on Climate Change (2015), emitting together the quasi-totality of global greenhouse gases (GHG), have presently ratified the Paris Agreement. By this means, parties officially declare their willingness to ambitiously and progressively contribute to the Agreement's long-term goal of keeping the current century's average worldwide temperature rise significantly below 2°C, preferably 1.5°C, above the levels of the late 19^{th} century. The relevance of such global climate-protection efforts becomes apparent when considering that the fossil-fuel-based industrialization greatly contributed to concentrations of heat-trapping gases last seen millennia ago (Pachauri et al., 2014). Consequently, under business as usual, further growth in world population and economic development enhances the greenhouse effect, which in turn provokes irreversible extreme weather conditions and higher sea levels that both heavily threaten modern civilization (Watts et al., 2018). To reduce instead the dangerous climatic risks, substantial technological and behavioral changes are required as they allow for a more efficient and sustainable performance of the GHG emitting sectors. Yet, just as countries are unevenly exposed to the damaging effects of global warming, they possess unequal adaptation and mitigation capabilities: developing countries are typically affected most but have low climate change combating power (Huq and Ayers, 2007; Islam and Winkel, 2017). For this reason, all Agreement parties can determine individualized emissions reduction goals in accordance with their national particularities.

On this basis, the world's third largest polluter, the European Union, subscribed to challenging climate and energy targets, among them a GHG emissions reduction goal of 20%

and at least 40% by 2020 and 2030, respectively (Pöttering, H.-G. and Necas. P., 2009; European Council, 2014). For these overall EU-wide targets to be achieved, each Member State must deliver sufficient contributions, necessitating, on the one side, ambitious but realistic national objectives and, on the other side, meticulously designed action plans to guarantee implementation. With even cost-effective climate measures not being fully applied by emitters (Jaffe and Stavins, 1994; Gillingham and Palmer, 2014), appropriate incentives are needed and thus the inclusion of target-oriented energy policies is not to be neglected in the countries' environmental approaches (Gillingham et al., 2018). In default of a one-size-fits-all policy mix (Köppel and Ürge-Vorsatz, 2007), countries are counseled to determine their own optimal instrument bundle. To this end, computational models can be of great benefit as they yield theoretical projections of real-world situations and hence enable us to approximate policies' future impacts (Carley, 2009).

In light of the above, the first part of the present dissertation aims at contributing to the literature on energy policy assessment. Even though this topic has been dealt with in other studies,[1] only scarce contributions provide computerized evaluations of policy tools' environmental and economic effectiveness in boosting the eco-friendliness of residential buildings.

Chapter 2 aspires to contribute to changing this state of affairs by examining how the final energy consumption and direct CO_2 emissions react to policy tools of financial and regulatory nature, in the Luxembourgish residential building sector. While Luxembourg was selected for both its particular economic and population growth anticipations (Haas and Peltier, 2017) and its ambitious climate targets, the choice of the sector is based on its comparatively large and lucrative energy and emission saving potential (Schulz and Mavroyiannis, 2012; Levine et al., 2007). For the analysis to be conducted, we set up a considerably developed Luxembourgish version LuxHEI (Luxembourgish Households' Energy Indicators model) of the French hybrid energy-economy model Res- IRF (Residential module of IMACLIM-R) (Giraudet et al., 2012). Starting from the national housing stock of 2013, the LuxHEI model computes, until 2060, the annual quantity and quality of retrofits and new constructions. What generates these outcomes is a decision-making progress that influences households' behavior not solely by means of micro- and macroeconomic variables, as existing models typically do, but also via the consideration of climate change

[1] For example: Weitzman (1974); Pizer (2002); Lee and Yik (2004); Bovenberg et al. (2005); Boonekamp (2006); Geller et al. (2006); Böhringer et al. (2008); Fankhauser et al. (2010); Boeters and Koornneef (2011); Flues et al. (2014); Knobloch et al. (2019); Bye et al. (2018).

effects. Beyond this novelty, decision-makers can now as well choose highly efficient energy classes, renewable carriers and sustainable heating systems—and this in a fashion that ensures their choices' technical feasibility. Further extension examples in the LuxHEI model are the inclusion of the green-value concept, the energy-class and energy-carrier dependency of the adjustment factor, the endogenous encoding of new constructions' building-type evolution and the modified integration of discount rates. Finally, to fully account for the country's particularities, parameter settings, determination of retrofitting, carrier-switching and construction costs as well as conventional unit final energy needs, just like calibration executions and energy policy modeling, are all based on empirical analysis of national data. Although the model projections obtained yield that none of the country's climate targets is reached on time, energy performance standards for new buildings and a remediation duty for existing buildings, the two with progressive threshold strengthening, emerge to hold the highest cost- and environmental effectiveness. What ranks next is the application of a CO_2 price and then, far behind, financial incentives, that is, in our case, subsidies and loans at reduced/zero interest rate. Despite the low mitigation potential of the latter aids, our conclusions are in essence consistent with those of other policy-related studies (Schaefer et al., 2000; Köppel and Ürge-Vorsatz, 2007; Giraudet et al., 2011; Weiss et al., 2012).

With the building sector's energy transition being, similarly to that of any other polluting sector, promoted presently on a global scale, primarily through the growing utilization of energy-saving and eco-friendly technologies that require advanced digitization levels for optimal usage (Kagermann, 2015), the question of resource availability arises—again (De Boer and Lammertsma, 2013; Bazilian, 2018). Actually, unlike in previous industrial revolutions, where the scarcity problem stemmed from the non-renewable and noxious energy sources they were essentially powered by, the current revolution is characterized by an ongoing shift towards renewable and clean energies. What instead causes the latest availability issue are the new production and consumption processes that largely hinge on exhaustible resources like, most notably, the increasingly-demanded rare earth elements (REEs) (Dutta et al., 2016; Balaram, 2019). These elements' suggested rarity, however, does not refer to their geological occurrence but rather to their stable market supply: over the past three decades, REEs have nearly exclusively been supplied by China. With this quasi-monopolistic supply having been observed repeatedly not to provide satisfying reliability levels for importers (let us only recall China's REEs export ban to

Japan in 2010), quite a number of affected countries have decided to improve national rare earths availability in future. In this context, the most concrete plan was recently announced by the U.S., who envisage, in spite of their significantly smaller reserves (U.S. Geological Survey, 2020), to (re-)enter the global REEs market as a serious supplier in addition to China (U.S. Department of Commerce, 2019).

Against this backdrop, the second part of this thesis aspires to extend the literature on oligopolistic non-renewable resource games by investigating the strategic interactions between two potential REEs market actors, China and the U.S. The specific idiosyncrasies that come along with rare earths are, on the one hand, the absence of a backstop technology for these indispensable elements (which we assume to be depleted in infinite time) and, on the other hand, the long-term supply dominance of a powerful incumbent that, in our case, one newcomer with initially smaller reserves, tries to put an end to, by becoming a worthy supplier.

In Chapter 3, we begin our investigation by setting up a competitive continuous-time model that starts off with a first finite period, where China holds a monopolistic market position, and switches, whenever the U.S. enters the game, to a second infinite period, in which China and the U.S. share the market. With the intention of determining the U.S.'s optimal entry condition, China's optimal entry behavior and the players' optimal monopolistic and duopolistic supply commitments, we solve these games for time-dependent open-loop (Nash equilibrium) strategies. In fact, applying the backward induction principle, we first solve the duopolistic differential game and then use these findings as transversality conditions in the optimal control monopolistic game to conduct the solving procedure. The model outcomes achieved yield that the U.S.'s market entry should optimally occur when China's monopolistic supply has decreased its initial REEs reserves to a level that makes them coincide with those of the U.S. By implication, it is the speed of China's first-period extraction that determines the time of the U.S.'s production launch. We furthermore find that it is economically more beneficial for China, unlike for the U.S., to postpone this launch through a conservative monopolistic supply behavior, followed by supply-continuity at the entry itself and a moderate second-period extraction, rather than to combine an more aggressive supply in the monopolistic period with a supply drop at the entry and a conservative behavior in the duopolistic period. We complete the study by analyzing whether or not a Pareto improvement emerges when the U.S. enters the market right at the outset via a cooperation deal that splits the total supply and fixes the two countries' profits proportionally to their initial reserves. For this purpose, we set up a continuous-time cooperative-game model, with only one (infinite) period, in which China and the U.S. both act as market suppliers, and search again for open-loop strategic Nash equilibria. In this way, we find that while the cooperation agreement would make the U.S. financially better off, the opposite holds true for China.

Chapter 4 pursues the REEs investigation by means of a competitive continuous-time model that we solve for time- and reserve-dependent Markovian (Nash equilibrium) strategies. Even though this new strategy space generally allows for a more accurate modeling of extraction behaviors (Clemhout and Wan, 1991; Dockner and Sorger, 1996), it received so far almost no attention in non-renewable resource games. This observation could be related to the fact that providing analytical solutions of Markovian-based problems necessitates "a good deal of experience and mathematical creativity" (Dockner et al., 2000). Moreover, due to the before-mentioned special REEs characteristics, the solving techniques that are applied in the few existing related works prove themselves to be of no help for our study. Our alternative method consists in guessing the form of the Bellman value function. We thereby manage to explicitly solve both, the first- and second-period games. Since the findings of dynamic games, and most particularly those of differential games, are commonly affected by the choice of the strategy space (Reinganum and Stokey, 1985; Dockner et al., 2000), we round off our analysis by comparing the present solutions with those of Chapter 3. We were able to confirm this theory, as our results of optimal control problems with uniform initial and terminal conditions remain unchanged under heterogeneous strategies, whereas those of differential games do not. Actually, while the duopolistic Markovian extraction starts off more aggressively and lucratively than the open-loop one, the situation reverses as the competition continues. Based on reserve data of the U.S. Geological Survey (2020), this phenomenon eventually implies that the application of laborious Markovian decision rules turns out to be sub-optimal for either of the two players. Only in the event that China is incapable of keeping its supply continuous at the U.S.'s market entry, its aggregated open-loop payoffs drop below the Markovian ones. In all other scenarios, competitors are thus advised not to update continuously their supply strategies, but rather to increase market stability and consumer confidence by committing to a supply path right at the outset of each game.

Chapter 2

Energy Policies for Eco-Friendly Households in Luxembourg

2.1 Introduction

Despite the fact that residential building sector has been known for a long time to hold a large and cost-effective energy- and emission-saving potential (European Commission, 2010; Pachauri et al., 2014), only a fraction of it is currently exploited (Jaffe and Stavins, 1994). Yet, with today's efforts to mitigate the devastating consequences of human-made climate change (Lindner et al., 2010) being largely insufficient to meet the vital goals of the Paris Agreement (Olhoff and Christensen, 2018), the further exhaustion of this key GHG-emitting sector's potential is now more important than ever. In order for this to be realized, the application of energy policies that aim at increasing energy efficiency and sufficiency is considered to be crucial (Gillingham et al., 2018).

In this light, the current study aims at evaluating the effectiveness of the most important policy instruments in making Luxembourgish households more eco-friendly, that is, reduce final space heating energy consumption and direct CO_2 emissions. More precisely, we analyze: (1) the ranking of the policy instruments in terms of environmental and economic effectiveness when applied individually; (2) the ways in which the instruments generate savings; (3) how the instruments' effectiveness is affected when applied concurrently; and (4) whether or not the national energy and climate objectives are achievable in the country's residential building sector.

Although there exists a rich literature on the assessment of various energy policy instruments,[1] the one that specifically analyses their environmental and economic effectiveness, when being applied (individually or in combination) to promote energy efficiency in the residential building sector, is relatively scarce. Additionally, the latter analysis has, to our best knowledge, so far not been performed for Luxembourg; yet the impacts of policy tools can strongly differ among countries (Köppel and Ürge-Vorsatz, 2007). To fill this gap, we built on the work of Giraudet et al. (2011, 2012, 2015) and design a significantly enhanced Luxembourgish version LuxHEI (Luxembourgish Households' Energy Indicators model) of the French hybrid energy-economy model Res-IRF (Residential module of IMACLIM-R). What makes this project all the more important for Luxembourg's political decision-makers is the fact that the country has committed itself to meet ambitious energy and climate targets over the next decades, while at the same time being expected to face both good economic development and the largest population growth rate (the population is projected to double until 2060) among all EU Member States (Haas and Peltier, 2017).

The LuxHEI model is basically designed as a bottom-up model: technologically powerful but microeconomically rather limited (Hourcade et al., 2006). Since the model's microeconomic weakness is, however, compensated by incorporating several "barriers" to energy efficiency, it is considered a hybrid energy-economy model (Hourcade et al., 2006). Indeed, engineering bottom-up models typically tend to follow the assumption of neoclassical economics, that is, consumers behave efficiently when making energy conservation investments. As this hypothesis requires a correct modeling of the costs and the decision-making behavior, Giraudet et al. (2012) modeled the impacts of "market barriers" like hidden-costs and consumer heterogeneity. Besides that, the real world decision-making has been found to not always coincide with the neoclassical standpoint, that is, not all capital expenditures with positive net present value are realized. This phenomenon is often referred to as the energy efficiency gap or paradox, for which several explanations exist in the literature: Jaffe and Stavins (1994); Weber (1997); Sorrell et al. (2000); Rohdin and Thollander (2006); Schleich and Gruber (2008); Thollander and Ottosson (2008); Fleiter et al. (2011); Trianni and Cagno (2012). For neoclassical economists, on the one hand, such suboptimal decisions result from an imperfect market structure; in a perfect market consumers would still act rationally (Gillingham and Palmer, 2014). To include this "market failure", the model

[1] Just to mention a few: Weitzman (1974); Pizer (2002); Lee and Yik (2004); Bovenberg et al. (2005); Boonekamp (2006); Geller et al. (2006); Böhringer et al. (2008); Fankhauser et al. (2010); Boeters and Koornneef (2011); Flues et al. (2014); Knobloch et al. (2019); Bye et al. (2018).

takes into consideration asymmetric information, learning-by-using and the principal-agent problem. For behavioral economists, on the other hand, "behavioral failures" are to be blamed for suboptimal consumer investments (Gillingham and Palmer, 2014). This explains why the model also relies on restricted consumer awareness. Finally, in order to reduce overestimations of the sector's energy saving potential, the rebound effect is considered in the simulation.

The most important innovative feature of the LuxHEI model is probably the fact that it encodes climate change as a decision-making influence factor. More specifically, we assume that the significant consequences of global warming imply, firstly, that the percentage of the existing building stock that is renovated annually increases over time (from 1% to 3%) and, secondly, that the market becomes more heterogeneous. This allows us to go beyond standard models, which are often based on financial considerations only. Yet, models that do not take sufficient account of the effects of climate change have reduced informative value and misinform policy-makers. The LuxHEI model also changes various other modeling methods of the Res-IRF model or adapts them to the available national data and the peculiarities of a small country with growing economy and quickly increasing population. We encoded, for example, the special national situation in all calibration procedures, parameterizations and evaluated policy instruments.[2] Beyond that, we included more sustainable energy efficiency classes (Zero Energy Buildings (ZEB) and Positive Energy Buildings (PEB)), energy carriers (pellets and solar), and heating systems (heat pumps). As to the carriers considered in the LuxHEI model, some of them are now authorized only in higher energy efficiency classes and some carrier switches became prohibited. The discrepancy that exists between the Luxembourgish households' conventional and effective energy needs for heating was incorporated through an adjustment factor, which we determined empirically—for each energy efficiency class and each carrier. Furthermore, when an owner retrofits its dwelling, the tenant or potential buyer profits from reduced heating energy costs. The LuxHEI model encodes the corresponding green value, that is, the percentage of the energy costs savings that the owner can expect recovering through the monthly rent or the sales price. In addition, we encoded a dynamic evolution of the new constructions' building types and changed the inclusion of discount-rates.

Regarding our main findings, we get that: (1) the highest environmental and economic

[2] We wish to emphasize that the LuxHEI model allows to perform the present study for any other country; as long as there exists sufficient data to complete the necessary calibration and parameterization procedures.

effectiveness is achieved by the building codes, followed by the national carbon tax and the remediation duty; (2) while subsidy schemes and regulatory policies have a stronger impact on energy efficiency than on energy sufficiency, it is the other way round for taxes; (3) when the policies are applied concurrently, their individual effects are summed, so that the greatest savings are realized by the policy package with the largest number of instruments; and (4) even in the projection with the highest environmental effectiveness, the residential building stock can only meet Luxembourg's energy and climate targets with delay.

The chapter is structured as follows. Section 2 describes how the final energy consumption of Luxembourgish households is encoded in the LuxHEI model. In Section 3, we outline the modeling of the existing building stock's transformation. Section 4 completes the explanation of the model by elucidating the dynamics of the new building stock. In Section 5, we depict the policy instruments that are analyzed and explain how they are modeled. Section 6 presents and discusses the results of our simulations. In a final section, we draw conclusions and make policy recommendations.

2.2 The LuxHEI model (I): Energy consumption

Our objective is to study, between 2014 and 2060, the impact of various policy tools on the space heating final energy consumption E_{fin} of Luxembourgish households.[3]

2.2.1 Final energy consumption

The final energy consumption $E_{\text{fin}}(t)$ in the year t (in kWh) is given by

$$E_{\text{fin}}(t) = S(t)\frac{E_{\text{con}}(t)}{S(t)}\frac{E_{\text{fin}}(t)}{E_{\text{con}}(t)}, \tag{2.1}$$

where $S(t)$ denotes the total residential building stock (in m^2), $\frac{E_{\text{con}}(t)}{S(t)}$ is the theoretically/conventionally needed final energy (in kWh/m^2) and where $\frac{E_{\text{fin}}(t)}{E_{\text{con}}(t)}$ is the quotient of the effective and the conventional needs (dimensionless).

We attribute an energy efficiency class to each dwelling. For existing dwellings we use

[3] Starting from the situation on 31 December of our initial year $t = 2013$, we compute the situation on 31 December of the year $t + 1 = 2014$, from this, the situation in 2015, and so on, up to 2060.

classes $i \in \mathcal{I} = \{I, \ldots, B, A\}$, where A is the most efficient and I the least efficient of the 9 classes in $\mathcal{I}$. For new buildings we consider only 4 classes $j \in \mathcal{J} = \{B, A, \mathrm{ZEB}, \mathrm{PEB}\}$. Besides that, we introduce 32 additional categories: (1) we distinguish between owner-occupied individual houses and flats and tenant-occupied individual houses and flats, which defines the 4 categories $D \in \mathcal{D} = \{$O-H, O-F, T-H, T-F$\}$; and (2) we consider the 8 energy carriers heating oil, gas, electricity, pellets, oil combined with a solar thermal system, gas with solar, electricity with solar and pellets with solar, which gives the 8 categories $e \in \mathcal{E} = \{$F, G, E, P, F+s, G+s, E+s, P+s$\}$. Altogether, we thus obtain for each $k \in \mathcal{I} \cup \mathcal{J}$ a 4×8 matrix of categories D, e. Note that here each class k is defined by an overall primary energy demand that we can transform for each type e of carrier into a conventional final energy $\rho_{k,e}$ needed for heating per square meter (and year). Subsection 2.2.2 describes this conversion process in more detail.

Consequently, when denoting the residential building stock in k, D, e by $S_{k,D,e}(t)$ and the factor $\frac{E_{\mathrm{fin},k,e}(t)}{E_{\mathrm{con},k,e}(t)}$ in k, e by $F_{k,e}(t)$, we get that the final space heating energy of Equation (2.1) is actually computed by

$$E_{\mathrm{fin}}(t) = \sum_{\substack{k \in \mathcal{I} \cup \mathcal{J} \\ D \in \mathcal{D} \\ e \in \mathcal{E}}} S_{k,D,e}(t)\, \rho_{k,e}\, F_{k,e}(t), \tag{2.2}$$

where the dimensionless factor $F_{k,e}(t)$ is the adjustment factor, which we present in Subsection 2.2.3.

2.2.2 Conventional unit space heating energies

The Luxembourgish "Energy Performance Certificate" (EPC) of a dwelling contains inter alia the energy efficiency class k, which depends on the insulation, the heating system characteristics and the carrier. Since each class k is characterized by its "overall primary energy demand" $Q_{\mathrm{pri}}(k)$, consisting of the sum of the overall primary energy demand for heating $Q_{\mathrm{pri},h}(k)$, hot water $Q_{\mathrm{pri},hw}(k)$ and auxiliary usages $Q_{\mathrm{pri},a}(k)$, we have

$$Q_{\mathrm{pri},h}(k) = \kappa_k Q_{\mathrm{pri}}(k),$$

where we empirically determined the percentage κ_k by means of the data from the Ministry of the Economy of Luxembourg (2017). Furthermore, while $Q_{\mathrm{pri},h}(k)$ is the conventional unit primary energy needed for heating in the class k, the model actually uses the conventional unit final energy needed for heating

$$\rho_{k,e} = \varepsilon_e^{-1} Q_{\text{pri},h}(k), \tag{2.3}$$

where ε_e depends exclusively on the carrier e and is defined by the Ministry of Luxembourg (2007). Concerning the conventional unit (per square meter and per year) energies $\rho_{\text{ZEB},e}$ for a "Zero Energy Building" and $\rho_{\text{PEB},e}$ for a "Positive Energy Building", both with carrier e, they cannot be computed by Equation (2.3) as the EPC does not yet include these energy efficiency classes. Instead, we rely on the average energy production of existing ZEBs and PEBs and set

$$\rho_{\text{ZEB,E+s}} = 0 \text{ and } \rho_{\text{PEB,E+s}} = -27,$$

where $e = \text{E} + \text{s}$ was found the unique significant carrier. Moreover, for $k \in \{\text{ZEB}, \text{PEB}\}$ and $e = \text{E} + \text{s}$, the conventional energies $\rho_{k,e}$ are the sums of the buildings' theoretical energy consumption $\rho_{k,e}^{\text{con}}$ and the opposite $\rho_{k,e}^{\text{pro}} < 0$ of their theoretical energy production. In this case, Equation (2.2) must hence be rewritten as

$$E_{\text{fin}}(t) = \sum_{\substack{k \in \mathcal{I} \cup \mathcal{J} \\ D \in \mathcal{D} \\ e \in \mathcal{E}}} S_{k,D,e}(t) \left(\rho_{k,e}^{\text{con}} F_{k,e}(t) + \rho_{k,e}^{\text{pro}} \right). \tag{2.4}$$

The ensuing subsection clarifies the idea behind this modeling choice.

2.2.3 Adjustment factor

With a view to taking account of the discrepancy that exists between the effective and the conventional energy needs, the model computes the space heating final energy consumption by Equation (2.2) and not by the approximation

$$E_{\text{fin}}(t) = \sum_{k,D,e} S_{k,D,e}(t) \, \rho_{k,e}.$$

This is because the national EPC, which allowed us to compute the conventional unit energies $\rho_{k,e}$, was initially designed to compare the energy efficiency of buildings and therefore, the calculations in the energy passports are simply based on standard usage parameters. Yet, there is evidence that heterogeneous user behavior can entail significant deviations in the energy consumption of structurally identical buildings (Schuler et al., 2000; Braun, 2010; Cayla et al., 2011; Wei et al., 2014; Hörner et al., 2016).[4]

Various ways exist to model the adjustment factor. In Lichtmeß (2013), for example, the author determines the Luxembourgish factor empirically using a function of the type

$$F_{k,e} = a + \frac{b}{1 + \frac{\rho_{k,e}}{c}} \ .$$

We follow Cayre et al. (2011) and Giraudet et al. (2012) and model this factor as a logistic function

$$F_{k,e}(t) = a + \frac{b}{1 + \exp(c\,\rho_{k,e} P_e(t) - d)} \qquad (a, b, c, d \text{ constant}), \tag{2.5}$$

of $\rho_{k,e} P_e(t)$, where the price $P_e(t)$ of the carrier e in the year t (expressed in € per kWh) is defined by means of the energy price projections in Capros et al. (2016) and Birol et al. (2010). This way, we capture the impact that energy efficiency measures and energy price variations have on energy sufficiency; known as prebound or rebound effect (Sunikka-Blank and Galvin, 2012). The concept behind this effect is that while households in dwellings with a low energy performance (high $\rho_{k,e}$) tend to consume less energy than the conventional energy $\rho_{k,e}$ (prebound effect, $F_{k,e}(t) < 1$), the exact opposite occurs in buildings with a high energy performance (low $\rho_{k,e}$): the measured energy consumption of these households is close to or even exceeds $\rho_{k,e}$ (rebound effect, $F_{k,e}(t) > 1$). The user behavior is similar when $P_e(t)$ passes from high values to low ones. The best modeling choice for $F_{k,e}(t)$ is therefore a decreasing logistic function ($c > 0$) in $\rho_{k,e}\, P_e(t)$.

The formula used by Lichtmeß (2013) [Giraudet et al. (2012)] to compute F in Luxembourg [France] associates the "lowest" possible $\rho = 0$ with $F = 110\%$ [$F = 120\%$] and the "highest" possible $\rho = 500$ [$\rho P = 100$] with $F = 45\%$ [$F = 40\%$]. The increased value 45% is natural when compared with the Luxembourgish disposable income. In this sense, as projections estimate that the Luxembourgish households' disposable income will have almost doubled in 2060 (Haas and Peltier, 2017), we adopt the values 120% and 50% and set

$$\lim_{\rho_{k,e} P_e(t) \to -\infty} F_{k,e}(t) = 150\% = a + b \quad \text{and} \quad \lim_{\rho_{k,e} P_e(t) \to +\infty} F_{k,e}(t) = 50\% = a,$$

[4]Standardized values are also used in the EPC to describe technical systems, construction materials or climate conditions. The consideration of these values in the passports' computation base further explains the observed discrepancy between the effective and conventional needs.

where the first limit means that even if ρP becomes negative, the average population behaves responsibly. Although using prices $P_e(t)$ that vary over time, we keep the values of a, b, c and d in Equation (2.5) constant. Based on the data of the Ministry of the Economy of Luxembourg (2017), a linear regression over all categories k, e (with very good R-squared, standard error and p-value) then yields $c = 0.175$ and $d = 0.875$. when substituting these values into Equation (2.5), we get

$$F_{k,e}(t) = 0.5 + \frac{1}{1 + \exp(0.175\,\rho_{k,e} P_e(t) - 0.875)},$$

and find the desired values 50% and 120% for $\rho P = 100$ and $\rho P = 0$, respectively.[5] Finally, we wish to highlight that the adjustment factor depends on

$$\rho_{k,e}\, P_e(t) = \left(\rho^{\text{con}}_{k,e} + \rho^{\text{pro}}_{k,e}\right) P_e(t),$$

and not on $\rho^{\text{con}}_{k,e}\, P_e(t)$ alone. This is because the households are assumed to adjust their behavior to the net amount of money they earn and not to the money that they spend for heating. Furthermore, using the latter sum of money would mean that the factor $F_{k,e}(t)$ is the same in the classes A, ZEB and PEB, since these classes have the same insulation and thus the same theoretical energy consumption. Our hypothesis rather implies that the adjustment factor increases when passing from A to ZEB and from there to PEB.

2.3 The LuxHEI model (II): Existing building stock dynamics

We separately study the building stock that existed at the end of 2013 (EBS) and the building stock that was newly constructed as of 2014 (NBS).

Regarding the EBS, for each $i \in \mathcal{I}$, $D \in \mathcal{D}$ and $e_i \in \mathcal{E}$ (subscript i added to avoid possible subsequent ambiguity), we must compute the existing building stock $S_{i,D,e_i}(\tau)$ in $\tau = t + 1$ from the known entries $S_{\iota,\Delta,\varepsilon_\iota}(t)$ of a $9 \times 4 \times 8$ matrix. For this purpose, we use

$$S_{i,D,e_i}(\tau) = (1 - \gamma_{i,D,e_i}(t))S_{i,D,e_i}(t) - \sum_{f>i} \text{TRANS}_{i,f\,;\,D,e_i}(\tau) + \sum_{\varphi<i} \text{TRANS}_{\varphi,i\,;\,D,e_i}(\tau), \quad (2.6)$$

[5]We conduct at present a separate study on the dependence of the adjustment factor on socio-economical variables like income or occupancy status.

where $\gamma_{i,D,e_i}(t)$ is the demolition rate of the stock $S_{i,D,e_i}(t)$. The second and third terms are the renovations/transitions in τ from class i to a higher efficiency class f and from a lower class φ to the class i, respectively. For example, to get the existing stock in class $i = F$ in 2017, we start from the existing stock in class F in 2016 that was not destroyed. From this stock we deduct the buildings in the energy class F that were upgraded to any higher energy class in 2017 and add the buildings that were upgraded to the energy class F in 2017.

The next equation explains the computation of the second term of Equation (2.6) (the third term is calculated analogously):

$$\mathrm{TRANS}_{i,f\,;\,D,e_i}(\tau) = (1 - \gamma_{i,D,e_i}(t))S_{i,D,e_i}(t)\ \mathrm{X}_{i,D,e_i}(\tau)\ \mathrm{PR}_{i,f\,;\,D,e_i}(\tau). \tag{2.7}$$

To find the transitions/retrofits from the initial class i to any higher final class f, the model thus computes the fraction $\mathrm{X}_{i,D,e_i}(\tau)$ of the undamaged stock in i that is retrofitted in τ (proportion of retrofits in class i) before the fraction $\mathrm{PR}_{i,f\,;\,D,e_i}(\tau)$ of the latter that is retrofitted to f in τ (proportion of retrofits to class f). In Section 2.3.5 we explain how the demolition rate $\gamma_{i,D,e_i}(t)$ is computed from the time-invariant average demolition rate γ in the whole stock $S(t)$.

2.3.1 Distribution of retrofits

Without climate change

The distribution $\mathrm{PR}_{i,f\,;\,D,e_i}(\tau)$ in the year τ of decided retrofits in a class i over all higher classes f is given by

$$\mathrm{PR}_{i,f\,;\,D,e_i}(\tau) = \frac{\mathrm{LCC}_{i,f,D,e_i}(\tau)^{-\nu}}{\sum\limits_{h>i}\mathrm{LCC}_{i,h,D,e_i}(\tau)^{-\nu}}. \tag{2.8}$$

Indeed, when a retrofit from i was decided, the number of retrofits from i to f is roughly proportional to the inverse of the life cycle costs $\mathrm{LCC}_{i,f,D,e_i}(\tau)$ of such a renovation. Hence, the percentage $\mathrm{PR}_{i,f\,;\,D,e_i}(\tau)$ is obtained by Equation (2.8) with $\nu = 1$. In this case, the observed percentages in the initial year, however, do not correspond well with the computed ones. While the accordance becomes better for higher values of ν, the best one is obtained for $\nu = 7$. This technique was first introduced by Jaccard and Dennis (2006) to model consumer heterogeneity, which corresponds to one of the above-mentioned market barriers. More specifically, values of ν close to 1 reflect preference heterogeneity: the choice of different investment options is relatively even. In contrast, higher values of ν, such as $\nu = 7$, reflect a more homogeneous investment behavior: the retrofitting option with the lowest life cycle costs $\mathrm{LCC}_{i,h,D,e_i}(\tau)$ is selected by most consumers.

Let us clarify that whenever ν increases, the price elasticity of demand (for us the elasticity of the number of retrofits) increases (in absolute value). This implies that if the life cycle costs P of a retrofit increase by 100%, then the number Q of retrofits decreases by $(2^{-\nu} - 1) \times 100\%$. The price elasticity at the initial price and initial number is thus

$$\frac{\Delta Q/Q}{\Delta P/P} = \frac{(2^{-\nu} - 1) \times 100\%}{100\%}. \tag{2.9}$$

With climate change

Up to here the model is based on typical price-demand relationships and ignores possible shocks that could suspend this rule. Yet, in view of current climate trends (Lindner et al., 2010), it is likely that over the next decades, the effects of climate change will steadily become more perceptible for society. Additionally, not only will the Luxembourgish population's educational level keep raising (Schofer and Meyer, 2005) but also is the country projected to face both economic growth and increasing disposable household incomes (Haas and Peltier, 2017).

Consequently, as environmental awareness is an increasing function of the experience of global warming impacts (Reynolds et al., 2010) and the educational level (Palmer et al., 1999; Aminrad et al., 2011), the latter can be expected to increase over the modeling period. Moreover, we know from Huang et al. (2006) that the inhabitants of a territory with economic growth, an above-average income per capita and bad environmental quality, have great willingness to invest in environmental improvement measures. This tendency is further strengthened by self-serving reasons (Huang et al., 2006), for instance, an improvement of the insulation of a dwelling to decrease suffering from heat rather than to protect the climate.

Against this backdrop, we suppose that Luxembourgish households will progressively accept spending more money for a retrofitting to a low energy class and a nonfossil carrier—even if this decision is not optimal from a financial viewpoint. In this case, the market will become more heterogeneous, that is, the parameter ν decreases over time.

We model ν as a decreasing logistic function of time with asymptotic values 7.5 and 1:

$$\nu(t) = 1 + \frac{6.5}{1 + \exp(ct - d)} \qquad (c > 0).$$

While climate summit meetings target zero emissions around 2050 and a limitation of global warming to less than 2 degrees Celsius by the end of this century (Falkner, 2016; Schleussner et al., 2016), the effects of climate change are expected to be seriously perceptible around 2030 (Pachauri et al., 2014). For this reason, we set the inflection point of the sigmoid curve at the year 2040 ($t = 27$), which connotes that $27c = d + \ln 0.5$. This condition and the information $\nu(0) = 7$ yield $d = 2.48$ and $c = 0.066$, so that

$$\nu(t) = 1 + \frac{6.5}{1 + \exp(0.066\, t - 2.48)}. \tag{2.10}$$

In light of Equation (2.10), the value of ν in 2060 is (a bit higher than) 3.25, that is, approximately 10% of the population maintain their renovation choice even when the costs double (see Equation (2.9)).

Life cycle costs

Coming back to the life cycle costs $\mathrm{LCC}_{i,f,D,e_i}(\tau)$ in Equation (2.8), they are the sum of the investment/retrofitting costs $\mathrm{INVC}_{i,f}(\tau)$, the energy operating costs $\mathrm{ENERC}_{i,f,D,e_i}(\tau)$ and the intangible costs $\mathrm{IC}_{i,f}(\tau)$:

$$\mathrm{LCC}_{i,f,D,e_i}(\tau) = \mathrm{INVC}_{i,f}(\tau) + \mathrm{ENERC}_{i,f,D,e_i}(\tau) + \mathrm{IC}_{i,f}(\tau). \tag{2.11}$$

The model assumes that first the decision to renovate from i to f is made and that only then the decision to switch from the initial carrier e_i to a final one is taken. Therefore the energy operating costs $\mathrm{ENERC}_{i,f,D,e_i}(\tau)$ are based on the initial carrier e_i; below we explain their dependence on $i \in \mathcal{I}$ and $D \in \mathcal{D}$. The remainder of this subsection more precisely depicts the three terms in Equation (2.11).

(1) ***Investment costs***

The evolution of the investment costs $\mathrm{INVC}_{i,f}(\tau)$ is modeled by

$$\mathrm{INVC}_{i,f}(\tau) = \mathrm{INVC}_{i,f}(0) \left(\alpha + (1-\alpha)(1-l)^{\log_2 \frac{C_f(\tau)}{C_f(0)}} \right). \tag{2.12}$$

A large spectrum of measures can be taken to retrofit a building from an energy class i to a higher class f. The initial retrofitting costs $\mathrm{INVC}_{i,f}(0)$ of Table 2.1 are hence average costs, which include costs ranging from small improvements of the envelope and the heating system to significant ones.[6]

Table 2.1 – Initial retrofitting costs $\mathrm{INVC}_{i,f}(0)$ (in € $/\mathrm{m}^2$)

	H	G	F	E	D	C	B	A
I	100	250	475	775	1150	1600	2125	2725
H		190	415	715	1090	1540	2065	2665
G			280	580	955	1405	1930	2530
F				370	745	1195	1720	2320
E					460	910	1435	2035
D						550	1075	1675
C							640	1240
B								730

The idea of Equation (2.12) is that the retrofitting costs $\mathrm{INVC}_{i,f}(0)$ in the year 0 have decreased in the year τ due to the experience $C_f(\tau)$ accumulated in τ through realized retrofits to the class f. The term $\mathrm{INVC}_{i,f}(0)\,\alpha$ is the percentage α of the initial retrofitting costs that cannot be decreased by experience (see Table 2.4 for the precise value of α). The reduction of the remaining costs $\mathrm{INVC}_{i,f}(0)(1-\alpha)$ is modeled through the multiplication by the exponential function $(1-l)^{\log_2 \frac{C_f(\tau)}{C_f(0)}}$. Here the constant l (see Table 2.4) is the learning-by-doing rate and the accumulated experience $C_f(\tau)$ is calculated from $C_f(t)$ by

$$C_f(\tau) = C_f(t) + \sum_{i<f} \sum_{D,e_i} \mathrm{TRANS}_{i,f\,;\,D,e_i}(t), \tag{2.13}$$

and

$$C_f(0) = 15 \times 1\% \times S_f(0), \tag{2.14}$$

where the experience $C_f(0)$ in 2013 was accumulated through retrofits between 1998

[6]The matrix of initial investment costs respects similar rules to those used in Giraudet et al. (2012) and was determined after concertation with experts from renovation companies.

and 2012. For $C_f(\tau) = 2^n C_f(0)$, we find that

$$(1-l)^{\log_2 \frac{C_f(\tau)}{C_f(0)}} = (1-l)^n, \tag{2.15}$$

which means that for each doubling of the experience, the price $\mathrm{INVC}_{i,f}(0)(1-\alpha)$ is multiplied by $1-l$, that is, it decreases by l.

(2) ***Energy operating costs***

Step 1: *Approximate energy costs*

The energy operating costs (in € per m^2) over the average lifetime N (see Table 2.4) of a retrofit are the sums

$$\mathrm{ENERC}_{f,e_i}(\tau) = \sum_{\mathfrak{t}=1}^{N} P_{e_i}(\tau + \mathfrak{t})\, \rho_{f,e_i}, \tag{2.16}$$

where the energy price $P_e(t)$ of carrier e (in € per kWh) again follows the projections of Capros et al. (2016) and Birol et al. (2010).

The terms of the sums in Equation (2.16) are costs, denoted by $C_{\mathfrak{t}}$, that are paid over the N years of the lifetime. To cover the retrofitting cost, the decision maker may use money from an interest-bearing investment with interest rate r_D and therefore bases her decision on the net present value of the periodic cash flows $C_{\mathfrak{t}}$. Besides that, the model considers the prices $P_{e_i}(\tau + \mathfrak{t})$ as constant over lifetime and replaces them by $P_{e_i}(\tau)$. The reason for this is a market failure: similarly to the findings in Simon (1955), we assume that uncertainty about the energy price evolution leads people to drop a part of the information at disposal when making decisions about energy conservation investments. By implication, the model calculates the energy operating costs as follows:

$$\mathrm{ENERC}_{f,D,e_i}(\tau) = P_{e_i}(\tau)\ \rho_{f,e_i} \sum_{\mathfrak{t}=1}^{N} (1 + r_D)^{-\mathfrak{t}}. \tag{2.17}$$

This modeling allows to account for the Landlord-Tenant dilemma (or principal-agent problem), which constitutes an important market failure to energy renovation in the residential sector of the European Union (Ástmarsson et al., 2013). Actually, this dilemma occurs if tenants and landlords have split incentives, for example, if tenants wish to reduce their energy bill through energy efficiency measures but owners are reticent to come up for the costs (as they have no direct return on the investment) (Gillingham et al., 2012; Charlier, 2015). As a result, when it comes to energy efficiency investments, non-occupying homeowners require a higher profitability than occupying homeowners. In order to model the lower [higher] number of renovations in the categories T-H and T-F [O-H and O-F], we assign different interest rates to these four decision situations D:

$$r_{\text{T-H}} = 0.10,\ r_{\text{T-F}} = 0.07,\ r_{\text{O-H}} = 0.30 \text{ and } r_{\text{O-F}} = 0.25.$$ [7]

The lower [higher] rates for tenant-occupied [owner-occupied] dwellings produce higher [lower] net present values or energy operating costs. The model therefore yields lower [higher] numbers of renovations in the categories T-H and T-F [O-H and O-F].

Step 2: *Energy costs with green value*

In Luxembourg, owners sell their dwellings after an average period T of 9 years. When an owner retrofits (we assume that he renovates right after he bought the habitation), the potential tenant and the future buyer have the advantage of reduced energy costs. On that account, we include the green value $\mathcal{G}$ (see Table 2.4), which corresponds to the percentage of the energy cost savings that the owner recovers through monthly rents or an increased sales price. For occupying owners D, we therefore replace the approximate energy costs of Equation (2.17) by the energy costs

$$\text{ENERC}_{i,f,D,e_i}(\tau) = P_{e_i}(\tau)\,\rho_{f,e_i}\sum_{\text{t}=1}^{T}(1+r_D)^{-\text{t}} - \left(P_{e_i}(\tau)\,(\rho_{i,e_i}-\rho_{f,e_i})\sum_{\text{t}=T+1}^{N}(1+r_D)^{-\text{t}}\right)\mathcal{G}, \quad (2.18)$$

where the last term is the percentage of the energy cost savings of the new owner, which D recovers when selling her dwelling. If D is a non-occupying owner, she can furthermore recover the same percentage through the rents that the tenant pays

[7] The weighted mean of the four percentages is 0.24, which agrees with the observation of Hausman (1979) and Train (1985) that the range of this mean is 0.20–0.25.

during the first T years:

$$\mathrm{ENERC}_{i,f,D,e_i}(\tau) =$$
$$P_{e_i}(\tau)\,(\rho_{f,e_i} - (\rho_{i,e_i} - \rho_{f,e_i})\mathcal{G}) \sum_{\mathrm{t}=1}^{T} (1+r_D)^{-\mathrm{t}} - \left(P_{e_i}(\tau)\,(\rho_{i,e_i} - \rho_{f,e_i}) \sum_{\mathrm{t}=T+1}^{N} (1+r_{D'})^{-\mathrm{t}} \right) \mathcal{G}, \tag{2.19}$$

where $r_{D'}$ is the owner interest rate that corresponds to the tenant interest rate r_D (for example, if $D =$ T-H, then $D' =$ O-H, since the buyer tries to reduce the increase of the sales price).

(3) ***Intangible costs***

When the calculation of the proportions $\mathrm{PR}_{i,f\,;\,D,e_i}(\tau)$ is based only on the two former costs, that is, $\mathrm{INVC}_{i,f}(\tau)$ and $\mathrm{ENERC}_{i,f,D,e_i}(\tau)$, the computed proportions in the year 0 do not coincide with the observed proportions. To counter this gap, Giraudet et al. (2012) use intangible costs $\mathrm{IC}_{i,f}(\tau)$, split into hidden intangible costs $\mathrm{HIC}_{i,f}(\tau)$ (market barrier) and intangible costs $\mathrm{IIC}_{i,f}(\tau)$ due to imperfect information.[8] Given that hidden costs can, on the one side, hardly be changed, they are calculated as a constant percentage β (see Table 2.4) of the initial intangible costs: $\mathrm{HIC}_{i,f}(\tau) = \mathrm{IC}_{i,f}(0)\beta$. On the other side, imperfect information gets smaller with growing accumulated experience $C_f(\tau)$ (Jaffe and Stavins, 1994; Jaffe et al., 2004), so that the costs $\mathrm{IIC}_{i,f}(\tau)$ decrease and eventually tend to disappear completely. We model the evolution of these costs by

$$\mathrm{IIC}_{i,f}(\tau) = \mathrm{IC}_{i,f}(0) \frac{1}{1 + c\exp\left(d\frac{C_f(\tau)}{C_f(0)}\right)} \qquad (c, d > 0),$$

where the decreasing logistic function of the relative accumulated experience $\frac{C_f(\tau)}{C_f(0)}$ takes the value $1-\beta$ for $\tau = 0$. Equation

$$\mathrm{IC}_{i,f}(\tau) = \mathrm{HIC}_{i,f}(\tau) + \mathrm{IIC}_{i,f}(\tau) = \mathrm{IC}_{i,f}(0)\left(\beta + \frac{1}{1 + c\exp\left(d\frac{C_f(\tau)}{C_f(0)}\right)}\right) \tag{2.20}$$

[8] The idea of using intangible costs to ameliorate the modeling of life cycle costs stems from the energy-economy model CIMS (Jaccard and Dennis, 2006; Rivers and Jaccard, 2005).

is therefore consistent in the year 0.

Additionally, when the initial accumulated experience[9] doubles, the initial value $1-\beta$ is multiplied by a factor $1-\mu$. The percentage μ can be compared with the learning-by-doing rate l: recall that when the initial accumulated experience doubles, the initial value $1-\alpha$ is multiplied by $1-l$ (see Equation (2.15)). In this sense, the percentage μ (see Table 2.4) can actually be interpreted as the information acceleration rate, which is related to the asymmetric information that causes the market failure; the learning-by-doing rate can be interpreted analogously. We obtain that way the system of equations

$$\frac{1}{1+c\exp(d)} = 1-\beta \quad \text{and} \quad \frac{1}{1+c\exp(2d)} = (1-\beta)(1-\mu),$$

where

$$c = \frac{(1-\mu)\beta^2}{(\mu+(1-\mu)\beta)(1-\beta)} > 0 \quad \text{and} \quad d = \ln\left(\frac{\mu}{(1-\mu)\beta}+1\right) > 0. \tag{2.21}$$

Here the constant c determines the proportion $\frac{1}{1+c}$ that corresponds to $C_f(\tau)=0$ and the constant d is responsible for the steepness of the sigmoid curve. Equation (2.21) shows that if the information acceleration rate μ increases, the values of $\frac{1}{1+c}$ and d increase; just the way it should be.

Equation (2.20) can be used once the initial intangible costs are known. However, because they are intangible, the initial costs $\mathrm{IC}_{i,f}(0)$ cannot be observed but must be calculated. In order for this calculation to be realized, we consider for any $i<B$ the system

$$\mathrm{PR}_{i,f}(0) = F_f\left(\mathrm{INVC}_{i,h>i}(0),\mathrm{ENERC}_{h>i}(0),\mathrm{IC}_{i,h>i}(0)\right) \qquad (i<f\leq A), \tag{2.22}$$

which is obtained from Equations (2.8) and (2.11).

We derive the proportions $\mathrm{PR}_{i,f}(0)$ from the analysis of 402 retrofitting operations undertaken in the Luxembourgish residential sector. As this sample does not allow for the proportions $\mathrm{PR}_{i,f\,;\,D,e_i}(0)$ to be observed, the initial energy costs must be

[9]The accumulated experience is calculated as before by Equations (2.13) and (2.14).

independent of D and e_i. The sample is split into two building types: individual houses and flats. In order to eliminate D from the energy costs $\mathrm{ENERC}_{h,D,e_i}(0)$, we use a weighted mean r of the average discount rates of the building types. To eliminate the carrier, we calculate the proportions $\mathrm{PR}_{e_i}(0)$ from the available data and compute the energy costs in each efficiency class as weighted mean:

$$\mathrm{ENERC}_h(0) = \sum_{e_i} \mathrm{PR}_{e_i}(0) P_{e_i}(0) \rho_{h,e_i} \sum_{\mathrm{t}=1}^{N} (1+r)^{-\mathrm{t}}.$$

Notice that the sum of the proportions on the left hand side of (2.22) is equal to 1, just as the sum of the functions on the right hand side (see Equation (2.8)). For this reason, the system in (2.22) reduces to the same system but with $f > i$ and $f < A$. As this entails that the new system consists of $8 - i$ equations (see possible values of f) and of $9 - i$ unknown intangible costs (see possible values of h), an additional equation must be added. To this end, we base on the fact that the percentage λ of the average $\mathrm{LCC}_{i,h>i}(0)$ that consists of the average $\mathrm{IC}_{i,h>i}(0)$ can be defined by

$$\sum_{h>i} \mathrm{PR}_{i,h}(0) \mathrm{IC}_{i,h}(0) = \lambda \sum_{h>i} \mathrm{PR}_{i,h}(0) \mathrm{LCC}_{i,h}(0),$$

in the same unknown intangible costs $\mathrm{IC}_{i,h>i}(0)$. This constitutes the required additional equation, where the parameter λ should of course have a low value. On this account, for any $i < B$, we search for the lowest value of λ that solves the total system (new system and additional equation). This finishes the calibration of the initial intangible costs.

2.3.2 Further description of the transformations of the existing building stock

We now rewrite Equation (2.7) by incorporating the carrier switch that we mentioned below Equation (2.11), that is, we must compute the transitions from i to f and e_i to e_f. Therefore we calculate the total proportions $\mathrm{PRT}_{i,f\,;\,e_i,e_f\,;\,D}$, which correspond to the product of the proportion $\mathrm{PR}_{i,f\,;\,D,e_i}$ of retrofits from i to f and the conditional proportion $\mathrm{PRS}_{e_i,e_f\,;\,D}\,|_{i,f}$ of switches from e_i to e_f, presented in the next Subsection 2.3.3.

We thus determine the transitions

$$\mathrm{TRANS}_{i,f\,;\,e_i,e_f\,;\,D}(\tau) = \\ (1-\gamma_{i,D,e_i}(t))S_{i,D,e_i}(t)\ \mathrm{X}_{i,D,e_i}(\tau)\ \mathrm{PR}_{i,f\,;\,D,e_i}(\tau)\ \mathrm{PRS}_{e_i,e_f\,;\,D}\,|_{i,f}(\tau), \quad (2.23)$$

where $f > i$, and the transitions

$$\mathrm{TRANS}_{\varphi,i\,;\,e_\varphi,e_i\,;\,D}(\tau) = \\ (1-\gamma_{\varphi,D,e_\varphi}(t))S_{\varphi,D,e_\varphi}(t)\ \mathrm{X}_{\varphi,D,e_\varphi}(\tau)\ \mathrm{PR}_{\varphi,i\,;\,D,e_\varphi}(\tau)\ \mathrm{PRS}_{e_\varphi,e_i\,;\,D}\,|_{\varphi,i}(\tau), \quad (2.24)$$

where $\varphi < i$. To obtain the number of transitions (or the corresponding number of square meters) that is needed in Equation (2.6), we sum the transitions in (2.23) over all $f > i$ and all e_f, and we sum the transitions in (2.24) over all $\varphi < i$ and all e_φ. While the first sum is

$$\sum_{f>i}\mathrm{TRANS}_{i,f\,;\,D,e_i}(\tau) = \\ (1-\gamma_{i,D,e_i}(t))S_{i,D,e_i}(t)\ \mathrm{X}_{i,D,e_i}(\tau)\ \sum_{f>i}\mathrm{PR}_{i,f\,;\,D,e_i}(\tau)\ \sum_{e_f}\mathrm{PRS}_{e_i,e_f\,;\,D}\,|_{i,f}(\tau)\ = \\ (1-\gamma_{i,D,e_i}(t))S_{i,D,e_i}(t)\ \mathrm{X}_{i,D,e_i}(\tau), \quad (2.25)$$

the second sum is equal to

$$\sum_{\varphi<i}\mathrm{TRANS}_{\varphi,i\,;\,D,e_i}(\tau) = \\ \sum_{\varphi<i}\sum_{e_\varphi}(1-\gamma_{\varphi,D,e_\varphi}(t))S_{\varphi,D,e_\varphi}(t)\ \mathrm{X}_{\varphi,D,e_\varphi}(\tau)\ \mathrm{PR}_{\varphi,i\,;\,D,e_\varphi}(\tau)\ \mathrm{PRS}_{e_\varphi,e_i\,;\,D}\,|_{\varphi,i}(\tau), \quad (2.26)$$

and really depends on the proportions PR and PRS.

2.3.3 Distributions of carrier switches

Homogeneous market

We calculate the (conditional) proportions $\mathrm{PRS}_{e_i,e_f\,;\,D}\,|_{i,f}(\tau)$ of switches from e_i to e_f analogously to the proportions $\mathrm{PR}_{i,f\,;\,D,e_i}(\tau)$ of retrofits from i to f:

$$\mathrm{PRS}_{e_i,e_f\,;\,D}\,|_{i,f}(\tau) = \frac{\mathrm{LCCS}^{-\nu(\tau)}_{f,D,e_i,e_f}(\tau)}{\sum\limits_{e_h} \mathrm{LCCS}^{-\nu(\tau)}_{f,D,e_i,e_h}(\tau)}. \tag{2.27}$$

Here $\nu(\tau)$ is the dynamic heterogeneity parameter of Equation (2.10) and

$$\mathrm{LCCS}_{f,D,e_i,e_f}(\tau) = \mathrm{SWIC}_{e_i,e_f} + P_{e_f}(\tau)\ \rho_{f,e_f}\ \sum_{\mathrm{t}=1}^{M} (1+r_D)^{-\mathrm{t}}. \tag{2.28}$$

The life cycle costs of a switch from e_i to e_f in (2.28) are similar to the life cycle costs of a retrofit from i to f in (2.11). While the switching costs[10] SWIC_{e_i,e_f} of Equation (2.28) include costs arising, for example, from oil tank removal, drilling for geothermal probes or laying a gas pipe as well as services provided by electricians or masons, the analogous investment costs $\mathrm{INVC}_{i,f}(\tau)$ in Equation (2.11) include the heater and heater-installation costs. The second term of (2.28) can be compared with the term $\mathrm{ENERC}_{i,f,D,e_i}$ of (2.11), except that in the present situation the final carrier is known and we can thus compute the energy costs using this carrier (which is more natural). Unlike the lifetime of a retrofit, which is N years, the lifetime of a carrier switch is M years (see Table 2.4). Moreover, we do not use a green value in (2.28) because the carrier is switched in a fixed efficiency class. As opposed to (2.11), (2.28) does also not contain intangible costs because in Luxembourg, the observations needed for the calibration of the initial intangible costs are unavailable. Lastly, the switching costs are considered as constant, that is, no learning effect is included; also due to infeasibility.

As illustrated in Table 2.2, the final carriers “pellets” (P), “pellets combined with a solar thermal system” (P + s), “electricity” (E) and “electricity with solar” (E + s) can be chosen only in higher energy efficiency classes. Firstly, we mentioned earlier in Subsection 2.2.1 that each energy efficiency class is initially defined in primary energy Q_{pri} and then transformed in the model for each type of carrier into final energy Q_{fin}. Based on the data of the Ministry of the Economy of Luxembourg (2017), we find that the final energy of almost all Luxembourgish dwellings is lower than 643 $\mathrm{kWh}\,/\,\mathrm{m}^2\,/\,\mathrm{year}$. This means that if a person who renovates chooses the final carrier P or P+s, the primary energy $Q_{\text{pri}} = 0.07\,Q_{\text{fin}}$ is lower than 45, which, however, means that the dwelling has the energy efficiency class A.

[10] These costs were determined after concertation with experts from renovation companies.

In other words, a person who renovates to the final class $f = C$ or $f = B$ cannot choose the carriers P and P + s, otherwise it almost always misses its goal to renovate to f. On this account, our model allows the choice of the carriers P and P + s only if the chosen final class is $f = A$ (see Table 2.2). Secondly, given the bad overall efficiency of electric heaters and the resulting environmental disadvantages, the Luxembourgish government wants to push back these heating systems and promotes the use of heat pumps instead. Hence, in our model, if e_f = E or e_f = E + s, the heating system used is a heat pump. Yet, for technical reasons, heat pumps are solely adapted for space heating in the energy classes B, A, ZEB and PEB (Myenergy Luxemgourg, 2018). This is why carrier switches to $e_f \in \{\text{E}, \text{E} + \text{s}\}$ are only permitted if $f > C$ (see Table 2.2).

Table 2.2 – Switching costs SWIC_{e_i,e_f} (in € / m^2)

	Oil	Gas	Electricity (if k<B)	Electricity (if k>=B)	Pellets (if k<A)	Pellets (if k=A)	Oil + Solar	Gas + Solar	Electricity + Solar (if k<B)	Electricity + Solar (if k>=B)	Pellets + Solar (if k<A)	Pellets + Solar (if k=A)
Oil	0	64		50		82	27	91		77		109
Gas	61	0		50		82	88	27		77		109
Electricity	183	186	0	172		204	210	213		199		231
Pellets	61	64		50	0	0	88	91		77	27	27
Oil+Solar							0	64		50		82
Gas+Solar							61	0		50		82
Electricity+Solar							61	64	0	0		82
Pellets+Solar							61	64		50	0	0

In contrast to the above, the heating system of an initial carrier $e_i \in \{\text{E}, \text{E} + \text{s}\}$ is an electric heater. As these systems consist mostly of direct-heating electric radiators and not of central heating systems (as do all other carriers in the model), switching from such an e_i to any other carrier is very expensive. Finally, because carrier switches are related to retrofits to higher energy classes, households who already used "solar" do usually not switch to a carrier without "solar". For this reason, carrier switches from $e_i \in \{\text{F}+\text{s}, \text{G}+\text{s}, \text{E}+\text{s}, \text{P}+\text{s}\}$ to $e_f \in \{\text{F}, \text{G}, \text{E}, \text{P}\}$ are not allowed (see Table 2.2).

Heterogeneous marked

If we calculate the percentages $\text{PRS}_{e_i,e_f\,;\,D\,|_{i,A}}(\tau)$ using the homogeneous market behavior defined by the heterogeneity parameter $\nu(\tau)$, then the numbers of houses using E, E + s, P or P + s are rather low in 2060. This insufficiency of the model comes from the values of $\nu(\tau)$, which range from 7 to 3.25, that is, when the life cycle costs double the percentage of switching decisions deceases from 100% to a percentage between approximately 1% and 10%. The values of $\nu(\tau)$ can of course decrease in specific subpopulations, for example, switches in the class A to one of the carriers $\mathrm{E}, \mathrm{E+s}, \mathrm{P}$ or $\mathrm{P+s}$ reflect a very good environmental consciousness, which in turn decreases the effect of costs on the switching decision. In order to remedy for the mentioned insufficiency of the model, we decrease $\nu(\tau)$ in the calculation of the proportions $\mathrm{PRS}_{e_i,e_f\,;\,D\,|_{i,f}}(\tau)$.

We justified above that: (1) if $f \leq C$ only the carriers $\mathrm{F}, \mathrm{F+s}, \mathrm{G}$ and $\mathrm{G+s}$ are possible; (2) for $f = B$ the decider can choose $e_f \in \{\mathrm{F}, \mathrm{F+s}, \mathrm{G}, \mathrm{G+s}, \mathrm{E}, \mathrm{E+s}\}$; and (3) for $f = A$ all eight carriers are possible final carriers. In the first case the parameter $\nu(\tau)$ used in the calculation of the proportions PRS is given by Equation (2.10). Yet, in the final class A $[B]$ we choose $\nu(\tau) - 1$ $[\nu(\tau) - 0.5]$ and further reduce this parameter in a way that depends on the chosen carrier.

With a view to specifying a coherent way to further reduce ν, we record numerically, on a scale from 0 to 5, the environmental awareness α of the deciders who switch in A $[B]$ to the carrier

$$e_f = \mathrm{F}\ (\mathrm{F+s}, \mathrm{G}, \mathrm{G+s}, \mathrm{E}, \mathrm{E+s}, \mathrm{P} \text{ or } \mathrm{P+s})$$
$$\Big[e_f = \mathrm{F}\ (\mathrm{F+s}, \mathrm{G}, \mathrm{G+s}, \mathrm{E} \text{ or } \mathrm{E+s})\Big].$$

Respectively we set

$$\alpha = 0.0\ (0.4, 0.4, 0.8, 3.2, 3.6, 4.0 \text{ and } 4.4)$$
$$\Big[\alpha = 0.0\ (0.2, 0.2, 0.4, 1.6 \text{ and } 1.8)\Big].$$

In the calculation of the proportions $\mathrm{PRS}_{e_i,e_f\,;\,D\,|_{i,A}}(\tau)$ we thus replace $\nu(\tau)$ by

$$\nu_A(\alpha, \tau) = \nu(\tau) - 1 - \pi\,\alpha,$$

where the coefficient π is determined by the request that for the maximal awareness 5 the heterogeneity parameter is $\nu_A(5, 47) = 1$ in the year 47 (that is, in 2060). From this we obtain

$$\nu_A(\alpha, \tau) = \nu(\tau) - 1 - 0.25\,\alpha,$$

and for B we get

$$\nu_B(\alpha, \tau) = \nu(\tau) - 0.5 - 0.35\,\alpha.$$

Conclusively, we use Equation (2.27) to find the proportions PRS in the classes $f \leq C$. In $f = A$ and $f = B$, we use the same equation but replace $\nu(\tau)$ by $\nu_A(\alpha, \tau)$ and $\nu_B(\alpha, \tau)$ and choose the value α that corresponds in A and B to the final carriers e_f and e_h.[11]

2.3.4 Fraction of retrofitted buildings

Without climate change

The proportion $X_{i,D,e_i}(\tau)$ of retrofits of class i dwellings is correlated to the profitability of the corresponding investment. The net present value $\mathrm{NPV}_{i,D,e_i}(\tau)$ of such a retrofit is the difference between the lifetime energy costs in the class i (when no retrofit is made) and the weighted average lifetime costs of a retrofit from i to any higher class f:

$$\mathrm{NPV}_{i,D,e_i}(\tau) = \mathrm{ENERC}_{i,D,e_i}(\tau) - \sum_{f>i} \mathrm{PR}_{i,f\,;\,D,e_i}(\tau)\ \ \mathrm{LCC}_{i,f,D,e_i}(\tau).$$

The precise relation between $\mathrm{NPV}_{i,D,e_i}(\tau)$ and $X_{i,D,e_i}(\tau)$ is defined by a logistic function:

$$X_{i,D,e_i}(\tau) = \frac{1}{1 + a \exp(-b\ \mathrm{NPV}_{i,D,e_i}(\tau))} \qquad (a, b > 0).^{12} \tag{2.29}$$

This models that if the net present value begins to increase, it is not yet really attractive and the proportion of retrofits increases only slowly and that, on the contrary, if the profit of a retrofit becomes more and more attractive the proportion increases quicker.

Equation

$$\sum_{i,D,e_i} \frac{S_{i,D,e_i}(0)}{1 + a \exp(-b\ \mathrm{NPV}_{i,D,e_i}(1))} = 0.01\,S(0)$$

asks for the retrofitted surface in the first year to be 1% of the surface of the existing building stock in the initial year. The constants a and b are the positive solutions of this equation for which the percentage $\frac{1}{1+a}$ of retrofits for zero-profitability is minimal. Given that this calibration problem is an optimization problem under constraint, we solve it numerically using Lagrange multipliers.

[11] This alternative modeling approach produces good (in particular not at all excessive) results.
[12] The asymptotic values are 0% and 100%.

With climate change

The percentage of the existing stock $S(t)$ that is renovated in the next year will increase over time.[13] This percentage $p(\tau)$ is modeled as an increasing logistic function,[14] and by applying the same procedure as for $\nu(\tau)$ in Subsection 2.3.1, we get

$$p(\tau) = 0.01\left(0.85 + \frac{2.30}{1+\exp(-0.124\,\tau + 2.66)}\right).$$

In this modeling alternative, a and b in Equation (2.29) depend on τ: $a = a_\tau$ and $b = b_\tau$. Their calculation uses the time-dependent constraint

$$\sum_{i,D,e_i} \frac{S_{i,D,e_i}(t)}{1 + a_\tau \exp(-b_\tau \ \mathrm{NPV}_{i,D,e_i}(\tau))} = (p(\tau) - 0.01)\, S(t) + \sum_{i,D,e_i} \frac{S_{i,D,e_i}(t)}{1 + a \exp(-b \ \mathrm{NPV}_{i,D,e_i}(\tau))}.$$

In this formula the sum at the left hand side is the total surface that is renovated in the year τ after consideration of the economic and the climatic issues encoded in the LuxHEI model. The percentage $p(\tau)$ in the right hand side increases from the current 1% to 3% due to (essentially) climatic reasons. Since we subtract in the term $(p(\tau) - 0.01)S(t)$ the approximate total surface $0.01\,S(t)$ that is renovated in τ for economic reasons, this term represents the total surface renovated in τ for climatic reasons. Adding the last term of the right hand side means replacing the approximate ($0.01\,S(t)$) by the true total surface renovated in τ for economic reasons.

2.3.5 Demolition rates

The demolition rate $\gamma_{i,D,e_i}(t)$ in the stock $S_{i,D,e_i}(t)$ remains to be calculated. We regard the demolition rate γ in the whole stock $S(t)$ as time-independent: γ is equal to the demolition rate 0.35%, which we observed for $S(0)$. Furthermore, the calculation of $\gamma_{i,D,e_i}(t)$ is based on the suggestion of Sartori et al. (2009) to first demolish the low energy classes.

The total destruction in t in the category D, e is

$$\mathrm{Tot}_{D,e}(t) = 0.0035 \times S_{D,e}(t) = 0.0035 \times \sum_i S_{i,D,e}(t).$$

[13]For the reasons already set out in the paragraph "Climate change" of Subsection 2.3.1.

[14]The asymptotic values are 0.85% (in the year 0 the value of p was 1% in Luxembourg) and 3.15% (newer versions of Giraudet et al. (2012) use the value $p = 3\%$ constantly, from the initial to the final year).

The inclusion of the suggestion to begin demolishing this surface in the worst energy class is modeled as follows: if in the category D, e the percentage $\frac{S_{I,D,e}(t)}{S_{I,D,e}(0)}$ of class I dwellings in the year 0 that do still exist in the year t, is still high [already low], we demolish much [we do not demolish much] of the total destruction $\mathrm{Tot}_{D,e}(t)$ in the class I. Here the demolition in I is taken to be

$$\mathrm{D}_{I,D,e}(t) = \mathrm{Tot}_{D,e}(t) \times \frac{S_{I,D,e}(t)}{S_{I,D,e}(0)}. \tag{2.30}$$ [15]

The remainder of the total destruction is demolished in the next class:

$$\mathrm{D}_{H,D,e}(t) = \mathrm{Tot}_{D,e}(t) - \mathrm{D}_{I,D,e}(t). \tag{2.31}$$

If in some year the class I has been completely destroyed we destroy first in H, then in G, and so on. Equation (2.30) and (2.31) can thus be written, respectively, as

$$\mathrm{D}_{I,D,e}(t) = \frac{\mathrm{Tot}_{D,e}(t)}{S_{I,D,e}(0)} S_{I,D,e}(t) = \gamma_{I,D,e}(t) S_{I,D,e}(t), \tag{2.32}$$

and

$$\mathrm{D}_{H,D,e}(t) = \frac{\mathrm{Tot}_{D,e}(t) - \mathrm{D}_{I,D,e}(t)}{S_{H,D,e}(t)} S_{H,D,e}(t) = \gamma_{H,D,e}(t) S_{H,D,e}(t), \tag{2.33}$$

which allows to calculate $\gamma_{I,D,e}(t)$ and $\gamma_{H,D,e}(t)$.

2.4 The LuxHEI model (III): New building stock dynamics

Section 2.3 dealt with the building stock that existed in 2013 (EBS), its transformation and the associated demolitions: we calculated the evolution over time of the surface S_{i,D,e_i} of the EBS in all $9 \times 4 \times 8$ categories i, D, e_i.

Hereinafter, we study the building stock growth or new building stock (NBS). Therefore we will calculate for all $4 \times 4 \times 8$ categories j, D, e_j the temporal development of the surface $\mathcal{S}_{j,D,e_j}$ (or the number $\mathcal{H}_{j,D,e_j}$) of new houses constructed in 2014 or later (in the case of new buildings $j \in \{B, A, \mathrm{ZEB}, \mathrm{PEB}\}$).

The total housing needs

$$H = \frac{L}{\mathrm{LPH}}$$

[15] As we cannot demolish more in the class I than available, the real destruction in I is the minimum of $\mathrm{D}_{I,D,e}(t)$ and $S_{I,D,e}(t)$.

are the quotient of the "population" and the "average population per house" (for example, if $L = 500,000$ and $\mathrm{LPH} = 4$ the total housing needs are $125,000$). The evolution over time of L is obtained exogenously using the findings of Haas and Peltier (2017). The data of STATEC (2011) suggest that the number LPH of people per house decreases over time. In the model, the decrease of LPH is bounded by a minimal number[16] and is calculated endogenously.

We denote the number of new constructions in 2014, 2015, etc., up to τ ($\tau = t+1$, $t \geq 0$) by $\mathcal{H}(\tau)$. The difference $(\Delta\mathcal{H})(\tau) = \mathcal{H}(\tau) - \mathcal{H}(t)$ is thus the number of new constructions in the year τ. Yet, this number is also the difference

$$(\Delta\mathcal{H})(\tau) = H(\tau) - \left(\frac{1}{\mathrm{SPH}} \sum_{i,D,e_i} S_{i,D,e_i}(\tau) + \mathcal{H}(t)\right) \tag{2.34}$$

between the housing needs $H(\tau)$ in τ and the sum of the number of dwellings from 2013 and earlier that still exist in τ and the number of new dwellings constructed in 2014, 2015, up to t. The fact that Equation (2.34) is expressed in (number of) houses and the existing stock $\sum_{i,D,e_i} S_{i,D,e_i}(\tau)$ is expressed in m^2, explains why the latter must be divided by the average surface SPH of a house that existed in 2013.

The surface $(\Delta\mathcal{S})(\tau)$ of the new constructions $(\Delta\mathcal{H})(\tau)$ naturally depends on the surface per house:

$$(\Delta\mathcal{S})(\tau) = \mathrm{SPH}(Y(\tau))(\Delta\mathcal{H})(\tau),$$

where the surface $\mathrm{SPH}(Y(\tau))$ is an increasing function of the disposable income per capita, with the value of $Y(\tau)$ for the years τ up to 2060 coming from the projections of Haas and Peltier (2017). Actually, the surface SPH is modeled by incorporating a maximal surface per house and by assuming that the annual increase of SPH shrinks as the surface gets closer to this limit; the modeling of the evolution over time of LPH is very similar. Since the data of STATEC (2017) yields that the surface $\Sigma = \mathrm{SPH}$ increases by 20% if the income doubles,[17] we have

$$\Sigma_\tau = \Sigma_t \left(1 + \frac{\Delta Y}{Y} \times 20\%\right). \tag{2.35}$$

Although Σ is in fact bounded by a limit or maximal surface $\Sigma_{\max}$, Equation (2.35) produces an increasingly higher surface over time. To rather model that Σ increases by lower

[16] Which is set equal to 2.

[17] The percentage 20% is only valid in the categories O-H and T-H; for O-F and T-F it is only 1%.

percentages than 20% when it comes closer to $\Sigma_{\max}$, the quotient $\frac{\Sigma_{\max}-\Sigma_t}{\Sigma_{\max}-\Sigma_0}$ is included into (2.35):

$$\mathrm{SPH}(Y(\tau)) = \Sigma_\tau = \Sigma_t \left(1 + \frac{\Delta Y}{Y} \times \frac{\Sigma_{\max} - \Sigma_t}{\Sigma_{\max} - \Sigma_0} \times 20\%\right).$$

Similarly to $S(\tau)$ in the EBS, we distribute $\mathcal{S}(\tau)$ in the NBS among the categories j, D, e_j. More specifically, the surface of new constructions in the category j, e_j is given by

$$(\Delta\mathcal{S})_{j,e_j}(\tau) = \mathrm{PRN}_{j,e_j}(\tau)(\Delta\mathcal{S})(\tau),$$

and the proportion $\mathrm{PRN}_{j,e_j}(\tau)$ of new constructions in j, e_j is calculated exactly as the proportions in Equations (2.8) and (2.27):

$$\mathrm{PRN}_{j,e_j}(\tau) = \frac{\mathrm{LCCN}_{j,e_j}^{-\nu(\tau)}(\tau)}{\sum\limits_{k,e_k} \mathrm{LCCN}_{k,e_k}^{-\nu(\tau)}(\tau)},$$

where

$$\mathrm{LCCN}_{j,e_j}(\tau) = \mathrm{INVCN}_{j,e_j}(\tau) + \mathrm{ENERCN}_{j,e_j}(\tau) + \mathrm{ICN}_{j,e_j}(\tau).$$ [18]

As depicted in Table 2.3, the carrier e_j is $\mathrm{E+s}$ for $j \in \{\mathrm{ZEB}, \mathrm{PEB}\}$. Indeed, a ZEB [PEB] is a house with a neutral [positive] annual energy balance, that is, it produces as much [more] energy as the household consumes over a year. With this being achieved by perfect insulation and an efficient heating system, we suppose households living in such buildings to have high environmental awareness: they desire sustainable heating and want to maximize the energy production from renewable energies. Therefore the model only allows solar thermal heating combined with a heat pump that works mainly with electricity from the in-house photovoltaic system.

[18] The initial construction costs $\mathrm{INVCN}_{j,e_j}(0)$ were again determined after concertation with experts from renovation companies and do only contain direct building costs, that is, no land costs are included.

Table 2.3 – Initial construction costs $\mathrm{INVCN}_{j,e_j}(0)$ (in € /m^2)

	B	A	ZEB	PEB
Oil	2527	2716		
Gas	2504	2692		
Electricity	2582	2776		
Pellets	2588	2782		
Oil+Solar	2626	2823		
Gas+Solar	2603	2799		
Electricity+Solar	2682	2883	3084	3285
Pellets+Solar	2688	2889		

However, unlike the proportions in Equations (2.8) and (2.27), the share $\mathrm{PRN}_{j,e_j}(\tau)$ does not dependent on the category D. The reason for this is that while the dependence on D in (2.8) and (2.27) comes from the different discount rates used for the different categories D, these discount rates are not needed in the case of new dwellings. On this account, the actually searched surface $(\Delta\mathcal{S})_{j,D,e_j}(\tau)$ is thus simply given by

$$(\Delta\mathcal{S})_{j,D,e_j}(\tau) = \mathrm{PR}_D(\tau)\mathrm{PRN}_{j,e_j}(\tau)(\Delta\mathcal{S})(\tau),$$

where $\mathrm{PR}_D(\tau)$ is the proportion of D-dwellings (for example, owner-occupied houses when $D =$ O-H) in the new constructions in the year τ. As $D = \mathcal{P} \cap \mathcal{T}$, with $\mathcal{P} \in \{\mathrm{O}, \mathrm{T}\}$ and $\mathcal{T} \in \{\mathrm{H}, \mathrm{F}\}$, we have

$$\mathrm{PR}_D(\tau) = \mathrm{PR}_{\mathcal{T}}(\tau)\ \mathrm{PR}_{\mathcal{P}|\mathcal{T}}(\tau),$$

where the percentage $\mathrm{PR}_{\mathcal{P}|\mathcal{T}}(\tau)$ was observed in the year $\tau = 1$ and we use that value. Concerning the two shares $\mathrm{PR}_{\mathcal{T}}(\tau)$, they are known once we found the percentage $\mathrm{PR}_{\mathrm{F}}(\tau)$ of flats in the new constructions in τ. Notice that we consider the fact the latter percentage increases over time. The data of STATEC (2017) suggests that the percentage $\mathrm{PR}_{\mathrm{F}}(\tau)$ of flats in the new constructions is an increasing logistic function of the relative growth

$$G(\tau) = \frac{H(\tau) - H(1960)}{H(1960)}$$

of the total building stock with respect to 1960:

$$\mathrm{PR}_{\mathrm{F}}(\tau) = a + \frac{b}{1 + \exp(-c\,G(\tau) + d)} \qquad (a, b, c, d > 0).$$

We have

$$\lim_{G\to+\infty} \mathrm{PR}_{\mathrm{F}} = a + b = 1,$$

and set

$$\lim_{G\to-\infty} \mathrm{PR}_{\mathrm{F}} = a = 0.$$

This choice is justified as the linear regression that gives $c = 0.54$ and $d = 0.42$ is of good quality and the law

$$\mathrm{PR}_{\mathrm{F}}(\tau) = \frac{1}{1 + \exp(-0.54\, G(\tau) + 0.42)},$$

leads to a good approximation of the observed value $\mathrm{PR}_{\mathrm{F}}(1960)$.

2.5 Description of the energy policy tools

As the building sector is accountable for about half of the EU's energy needs and greenhouse gas (GHG) emissions (Lechtenböhmer and Schüring, 2011), it is considered essential to meet large energy and climate objectives (for example, the EU's 20-20-20 strategy or the Paris Agreement) and so is the residential building sector (Itard, 2008). For this reason, Luxembourg's policy makers also devote particular importance to this sector: since the 1990s, the government promotes energy conservation in the building stock through the application of various policy instruments. More precisely, energy policy tools of direct nature (regulatory instruments) and indirect nature (communication or financial instruments) were implemented to address the barriers that hinder the full exploitation of the sector's significant energy conservation potential; often referred to as the energy efficiency gap or paradox. In this light, the LuxHEI model aims at evaluating the effects of currently applied and possible future financial and regulatory instruments. A detailed synopsis of the considered instruments and their modeling is provided in this section.

2.5.1 Existing instruments: initial and extended form

This subsection presents the energy policies that are currently applied in Luxembourg.

2.5.1.1 Capital grants

Since 2013, the Luxembourgish state offers PRIMe-House capital grants to promote household investments in insulation measures on existing dwellings, green building and sustainable heating systems.[19]

Here the subsidy ($\mathcal{S}_1$) granted for insulation measures in an existing dwelling increases with both the quantity of insulation material used (including windows) and its quality. As information about material properties is not captured by the model, we use available data of Myenergy Luxemgourg (2018)[20] and compute that for a retrofit from i to f, 15% of the average capital expenditures for insulation measures are covered by the subsidy (up to 2026).[21]

Between 2013 and 2016, state aids were also granted for all new energy class B and A constructions (for example, the maximal grant for a new single-family house of class A was 24,000 € in 2014). Yet, with the introduction in 2016 of the Luxembourgish environmental certification system LENOZ, the regulations of this policy changed. In fact, the new scheme determines a building's sustainability no longer exclusively by its energy class but also through its geographical location and factors of economic and social nature. To benefit from the grant, new constructions must now obtain a certain amount of points in the LENOZ evaluation. Since such specifications are not tangible for the model, we build on the assertions of consultants from Myenergy and assume that between the beginning of 2017 and the end of 2020, 35% of new energy class A dwellings, 50% of new ZEB, and 65% of new PEB remain eligible for this second type of PRIMe-House grant ($\mathcal{S}_2$).

Besides that, until the end of 2024, the Luxembourgish state also offers grants for solar thermal plants, pellet heating systems and heat pumps (varying between 2,500 € and 8,000 €). An additional subsidy (of 1,000 €) is accorded whenever the two latter systems are combined with a solar thermal plant. As the overall costs of a heating system replacement are split into system and installation costs (included in the investment costs $\mathrm{INVC}_{i,f}(\tau)$) and ancillary costs induced by the carrier swap (included in the switching costs SWIC_{e_i,e_f}),

[19] Instruments that were already applied in 2013 are considered in the calibrations of the model.

[20] Myenergy is the main national structure to promote the transition to sustainable energy.

[21] Note that this modeling choice reflects reality as the level of subsidies increases, as it should, with the quality of the final energy class f, that is, the number or grade of the undertaken actions.

this last type of grant ($\mathcal{S}_3$) is split ($\mathcal{S}_3 = \mathcal{S}_3^a + \mathcal{S}_3^b$, $\mathcal{S}_3^a$ and $\mathcal{S}_3^b$ are fixed percentages of $\mathcal{S}_3$):

$$\mathrm{INVC}_{i,f}(\tau)' = \mathrm{INVC}_{i,f}(\tau) - (\mathcal{S}_1(\tau) + \mathcal{S}_3^a(\tau)),$$

$$\mathrm{SWIC}_{e_i,e_f}(\tau)' = \mathrm{SWIC}_{e_i,e_f} - \mathcal{S}_3^b(\tau),$$

$$\mathrm{INVCN}_{j,e_j}(\tau)' = \mathrm{INVCN}_{j,e_j}(\tau) - (\mathcal{S}_2(\tau) + \mathcal{S}_3(\tau)).$$

Based on the findings of ADEME (2013), we take into account that probably not all eligible households actually use the capital grants; mainly because of imperfect information. Due to the substantial efforts of the Luxembourgish government to promote the PRIMe-House grants, we, however, suppose the utilization rates to be slightly higher than those in France (ADEME, 2013): in Luxembourg, on average, 75% of retrofits and 90% of new constructions apply the instrument.

Furthermore, an extended version of the policy is modeled. This means that at the end of the instruments' initial application period, the model prolongs the grants for 15 additional years and considers their application as mandatory: as in Giraudet et al. (2011, 2012) all eligible households apply the instrument.

2.5.1.2 Subsidized loans

With the launch of the Luxembourgish climate bank in 2017, households became eligible for a retrofitting credit at reduced interest or even zero-interest rate. Under the interest-free loan, recipients can take out a credit of up to 50,000 € , repayable (without interests) within 15 years, and further get a capital grant of 10% of the loan. On the contrary, under the loan at reduced interest rate, the credit is limited to a maximum of 100,000 € , repayable within 15 years, and the state grants a 1.5% subsidy on the interest rate of the bank.

To encode both retrofitting credits, we again refer to ADEME (2013) and consider that not all households borrow money to pay a retrofit. Similarly to the situation observed in France, we assume the proportion $\mathcal{P}$ of retrofitting households taking out a loan to be 30%, whether or not the policy tool is applied. For this proportion $\mathcal{P}$, the investment costs $\mathrm{INVC}_{i,f}(\tau)$ are then increased by the accrued interests under an averaged fixed interest rate[22]. Only if the government applies the interest-free or reduced interest loan, the increased investment

[22]This rate corresponds to the mean of the fixed rates that Luxembourgish banks charged on mortgage loans between 2009 and 2017.

costs $\mathrm{INVC}_{i,f}(\tau)$ of the proportion $\mathcal{P}$ are decreased by the saved interests. Furthermore, the instruments' effectiveness is improved by limiting their duration of application (Köppel and Ürge-Vorsatz, 2007): as for capital grants, the subsidized loans are available until 2026.

Comparable with the extended version of the *PRIMe-House* grants, we encode a complementary scenario where both instruments' period of application is prolonged until 2041 (+15 years) and where for the reduced interest loan the 1.5% subsidy is increased to 2.0%.

2.5.1.3 Energy tax

Since the European Commission implemented the EU Energy Taxation Directive (ETD) (Matteoli, 2003) in 2003, minimum tax rates are imposed on energy products in the Member States. Within this framework, the Luxembourgish government taxes the use of electricity and fuels like oil, gas and coal (if they are not used to produce electricity). Here the tax rates depend on the energy source (the carrier), the sector of application and the volume of the annual consumption. Based on the data of the Ministry of the Economy of Luxembourg (2017), we find that carriers used for space heating in the residential sector are taxed between $1.5\,€/\mathrm{t\,CO_2}$ and $5\,€/\mathrm{t\,CO_2}$. In this first form of the policy tool we consider the energy tax as time-independent and encode the instrument by adding the amount of the tax (converted into € per kWh) to the energy price $P_e(t)$ of the corresponding carrier e.

An enhanced energy tax is also included in our model: following the objective of the Ministry of the Economy of Luxembourg et al. (2017) to raise the taxes on energy products, we increase the initial level of taxation by 100% every 10 years. The first increase is implemented in 2025, the last in 2055.

2.5.1.4 Energy performance requirements for new buildings

In Luxembourg, the energy efficiency of new residential buildings is prescribed by law since November 2007. This initial building code dictated that as of 1 January 2008, all new constructions needed to have at least the energy efficiency energy class D. The standard was then increased to energy class B $[A]$ in 1 July 2012 [1 January 2015]. This is why in the proportion $\mathrm{PRN}_{j,e_j}(\tau)$ of new constructions, $j \in \{B, A, \mathrm{ZEB}, \mathrm{PEB}\}$ in 2014 $\big[j \in \{A, \mathrm{ZEB}, \mathrm{PEB}\}$ as of 2015$\big]$.

Once again, an extra scenario in which building codes are further tightened is included: as of 1 January 2030 [2045] the standard ZEB [PEB] becomes mandatory for new constructions.

2.5.2 Possible future instruments

This subsection discusses policies that we believe the most interesting for Luxembourg.

2.5.2.1 Remediation duty for existing buildings

As most Member States, Luxembourg does not specify minimum energy efficiency standards for existing residential buildings (BBSR, 2016). The only obligation with regard to existing dwellings is to respect minimum material standards when retrofitting. However, in Luxembourg's neighbouring countries (Germany and France), stricter requirements on the existing building stock do exist. In Germany, partial renovation is, for example, mandatory after the acquisition of an existing building: if a dwelling was bought or inherited after 1 February 2002 the new owners must either insulate the roof or the top floor ceiling. As regards France, the National Assembly adopted on 26 May 2015 a bill which stipulates that every dwelling must be retrofitted until 2025 if its overall primary energy consumption is above $330\,\mathrm{kWh}\,/\mathrm{m}^2\,/\mathrm{year}$; in the LuxHEI model this corresponds to a house of energy efficiency class G. The bill also dictates that as of 2030, all dwellings must be retrofitted before they can be listed for rent or sale.

In light of the energy saving and CO_2 mitigation potential of the existing building stock (Petersdorff et al., 2006; Tommerup and Svendsen, 2006) and the rules deployed in two of Luxembourg's three bordering countries, we included a remediation duty in the LuxHEI model: as of 2022 (considered as the closest possible year for implementation), all residential buildings that are listed for rent or sale must be retrofitted to an overall primary energy class above H. To ensure effectiveness of this tool, the regulation is gradually tightened by one energy class every five years: buildings whose inhabitants switch must at least reach class F as of 2027, E as of 2032, and D $[C]$ as of 2037 [2042].

To model this policy, we begin by considering that without remediation duty, a fraction ζ of the proportion $\mathrm{X}_{i,D,e_i}(\tau)$ of retrofits of class i dwellings is induced by an inhabitant switch. On the contrary, whenever the instrument is applied, all inhabitant switches are followed by a retrofit. To avoid double counting when the remediation duty is in force, the fraction $\zeta\,\mathrm{X}_{i,D,e_i}(\tau)$ must be subtracted from $\mathrm{X}_{i,D,e_i}(\tau)$ and Equation (2.7) of the model must thus be changed to:

$$\mathrm{TRANS}_{i,f\,;\,D,e_i}(\tau) = (1-\gamma_{i,D,e_i}(t))S_{i,D,e_i}(t)\,\big(Z_{i,D,e_i}(\tau) + (\mathrm{X}_{i,D,e_i}(\tau)\,(1-\zeta))\big)\,\mathrm{PR}_{i,f\,;\,D,e_i}(\tau),$$

for all $i < i_{\min}$ and all $f \geq i_{\min}$. The percentage $Z_{i,D,e_i}(\tau)$[23] is the proportion of owner-occupied or rented dwellings that change occupancy in the year τ and the class $i_{\min}$ is the required lowest efficiency class. More specifically, if $i < i_{\min}$ and $f < i_{\min}$, we set $\mathrm{PR}_{i,f\,;\,D,e_i}(\tau) = 0$ and if $i \geq i_{\min}$, then $f > i_{\min}$ and so we leave the original formula unchanged.

2.5.2.2 Carbon tax

In 2005, the world's first and largest international emissions trading system, the European Union Emissions Trading System (EU ETS), was implemented to reduce greenhouse gas (GHG) emissions in the EU. Yet, this system covers only about 45% of the EU's GHG emissions, since it does not cap the volume of gases emitted by the agriculture, residential and transportation sectors. Instead, binding national targets are fixed for these three sectors through the Effort Sharing Decision (ESD).

To meet these targets in a cost-effective way, a carbon tax is often recommended by environmental economists (Pearce, 1991; Gerlagh and Van der Zwaan, 2006; Ghalwash, 2007). As several attempts to introduce an EU-wide carbon tax failed, we consider in our model a national carbon tax that applies a uniform price to emissions from all sources and sectors (Bruvoll and Larsen, 2004; Lin and Li, 2011). For this purpose, we set the initial level of the tax equal to the price of the EU ETS allowances (Weisbach, 2012), and increase the level over time to enhance the reduction of $\mathrm{CO_2}$ emissions progressively (Peck and Teisberg, 1992). More precisely, the tax is based on the predicted annual price increase of EU ETS certificates (Capros et al., 2016). The carbon tax (in the residential sector) will thus increase from 15 €/t $\mathrm{CO_2}$ in the starting year 2020 of the policy tool to 33 €/t $\mathrm{CO_2}$ in 2030 and 89 €/t $\mathrm{CO_2}$ in 2050. In the second half of the century, carbon emissions are projected to decrease (Chakravorty et al., 1997; OECD et al., 2015) and the level of the tax is estimated to decline (Vollebergh, 2014). We therefore assume that the carbon tax comes down to 80 €/t $\mathrm{CO_2}$ [72 €/t $\mathrm{CO_2}$] in 2055 [2060].

The modeling of the carbon tax is similar to the encoding of the energy tax: the price of the tax (in € per kWh) is added to the energy price $P_e(t)$ of the corresponding carrier e.

[23] The data of Eurostat (2017) shows that $Z_{i,D,e_i}(\tau)$ is 1.3% [6%] for owner-occupied [rented] dwellings.

2.6 Results and discussion

We now evaluate the policy tools of Section 2.5 in a similar manner as in Amstalden et al. (2007); Köppel and Ürge-Vorsatz (2007); McCormick and Neij (2009); Giraudet et al. (2011) and Knobloch et al. (2019). To this end, different model scenarios are generated: firstly, the model is run without any instrument (this baseline scenario serves as a benchmark for the following evaluation), secondly, each instrument is put in force individually (the original and extended forms of the existing policy tools are examined) and, thirdly, various bundles of instruments are studied (the bundle of all existing initial tools, the bundle of all existing extended tools and the bundle of all existing and possible future tools). After each run, the scenario is assessed with regard to its environmental and economic effectiveness and its potential to help achieving the Luxembourgish energy and emission targets is determined. Actually, in order to contribute to the EU's 20-20-20 strategy, Luxembourg must decrease by 20% its final energy consumption (in comparison with the 2007 level) as well as its CO_2 emissions from sectors outside the EU ETS (in comparison with the 2005 levels). In the period 2021-2030, emission cuts of even 40% must be achieved (relative to the 2005 levels). Although the national targets are not limited to a single sector, the second part of our assessment is interesting since the building sector offers one of the greatest potentials to decrease energy consumption and CO_2 emissions (Schulz and Mavroyiannis, 2012; TIR Consulting Group LLC and Grand Duchy of Luxembourg Working Group, 2016), and this at comparatively low costs (Levine et al., 2007; Schimschar et al., 2011).

Table 2.4 – Parameter overview

Parameter	Signification	Setting	Literary basis
α	percentage of the initial retrofitting costs that cannot be decreased by experience	20%	Giraudet et al. (2012)
l	learning-by-doing rate	10%	Weiss et al. (2010); Giraudet et al. (2012)
N	average lifetime of a retrofit	35 years	Ministry of the Economy of Luxembourg and Lichtmeß (2014)
$\mathcal{G}$	green value	33%	Högberg (2013)
β	constant percentage of the initial intangible costs	20%	Giraudet et al. (2012)
μ	information acceleration rate	25%	Giraudet et al. (2012)
M	average lifetime of a carrier switch	20 years	Ministry of the Economy of Luxembourg and Lichtmeß (2014)

A detailed overview of the parameters that were not yet specified but used in the model-runs, can be found in the table above.

2.6.1 Baseline projection

The model projects for 2060 a total final energy consumption of 4,122 GWh (-9% compared to 2014) and total $\mathrm{CO_2}$ emissions of 734,600 $\mathrm{t\,CO_2}$ (-20% compared to 2014) (see Appendix 2.7, Table 2.5 and Figure 2.3). The major growth of the residential building stock (Figure 2.1) is due to the projected increase of the Luxembourgish population: based on the 3% GDP growth scenario of Haas and Peltier (2017) and the baseline population projections of Eurostat, we can assert that Luxembourg will face the largest population growth rate (approximately 98% between 2014 and 2060) among all EU Member States. At the end of the model's projection period, the (aggregated) new building stock (NBS) therefore corresponds to 66% of the total building stock. On this basis, savings in energy consumption and $\mathrm{CO_2}$ emissions can only be achieved through changes in the sector's energy efficiency or energy sufficiency. As concerns the gains in energy efficiency, they are mainly realized via a transformation of the existing building stock (EBS), that is, by means of demolitions and retrofits, and the construction of a highly efficient NBS. More precisely, in 2060, the final energy consumption per square meter has fallen by 61% (in comparison with 2014) (Table 2.5), 69% of the total dwellings have at least the energy class B (compared to 7% in 2014) and 32% of the households make use of solar thermal energy to support their heating system (Table 2.7, Figures 2.5 and 2.6). Such large energy efficiency increases are, however, followed by a significant rebound effect: by the end of the projection period the adjustment factor raised by 38% (Table 2.6 and Figure 2.2). This is due to the fact that all along the modeling period, the continuously increasing share of dwellings with a low $\rho_{k,e}$ compensates the natural raise of the energy price $P_e(t)$, thus generating a shrinking of the adjustment factor's independent variable $\rho_{k,e}P_e(t)$ and hence a higher overall factor $F(t)$. Concerning the country's energy objectives, none of them can be achieved in the residential building sector (Table 2.9): the 2020 levels of final energy consumption ($\mathrm{CO_2}$ emissions) are 29% (7%) higher than the 2007 levels (2005 levels) and the 2030 emissions are projected to decrease only by 4% instead of 40% (compared to 2005).

Notice that ignoring the effects of climate change, the green value and the dynamic evolution of the new constructions' building type, changes the outcomes of the baseline scenario for the worse. Actually, compared to the baseline projection that contains all new features (see above), we end up with a total final energy consumption that is 8% higher (4,478 GWh) and total CO_2 emissions that are increased by 10% (805,600 t CO_2). This is inter alia due to the fact that at the end of the projection period, the total number of retrofits has decreased by 50% (compared to the projection with all new features), only 0.2% [0.0%] of the total building stock correspond to ZEB [PEB] (compared to 0.8% [0.5%]), and just 27% make use of solar thermal energy to support their heating system (compared to 32%).

2.6.2 Evaluation of the individual energy policy tools

In the following evaluation, the effects of the 10 single-instrument scenarios (see above) are compared to the baseline scenario. The ranking 1–10 means "most effective–least effective" and the description of the results starts at "rank 10" and ends with "rank 1".

2.6.2.1 Places 10 to 7: Subsidy schemes

No significant variations (about 1% at most) from the baseline are observed in the initial and extended forms of the subsidized loan and capital grant scenarios.

From an ecological and economic viewpoint, the initial subsidized loan scenario is the least effective. Compared to the baseline projection virtually no reduction effects can be observed: additional energy and emission savings of 0.01% (Appendix 2.7, Table 2.5, Figures 2.3 and 2.4) are realized, for a benefit to cost ratio[24] of 101 € per kWh saved (Table 2.8).

In the enhanced version of the subsidized loan scenario, while energy and emission savings increase slightly (−0.02%; Table 2.5, Figures 2.3 and 2.4), cost-effectiveness deteriorates (+26%; Table 2.9).

A bit better results are achieved in the scenario with the initial form of the capital grants: energy savings (−0.17%) and emission reductions (−0.47%) can be observed in 2060 (Table 2.5), for a benefit to cost ratio of 74 € per kWh saved (Table 2.8). Although the instrument induces gains in energy efficiency (−0.21% of conventional energy consumption in 2060), a decrease of the adjustment factor is observed (−0.05% in 2060) (Table 2.6 and Figure 2.2). This small prebound effect is mainly due to the somewhat greater use of

[24]The benefit to cost ratio corresponds to the difference of the 2060 final energy consumption of the baseline scenario and the 2060 final energy consumption of the corresponding policy tool scenario, divided by the financial incentives accumulated during the policy's application period.

electricity, an energy carrier with a comparatively high energy price. Compared to the baseline scenario, a small decrease in accumulated retrofits is also observed at the end of the projection period (-0.05%; Table 2.6 and Figure 2.7). This is due to the fact that capital grants boost the number of retrofits during their application period so that less lucrative retrofitting options remain after that period.

Although the environmental effectiveness of capital grants more than doubles in the enhanced scenario (Table 2.5), its cost-effectiveness changes for the worse (+40%; Table 2.8).

2.6.2.2 Places 6 and 5: Energy taxes

Next ranks the initial [extended] form of the energy tax scenario: state revenues of 14€ [8€] per kWh saved (Table 2.8) come along with a decreased total final energy consumption (-0.34% [-1.61%]) and decreased total $\mathrm{CO_2}$ emissions (-0.37% [-1.77%]) (Table 2.5, Figures 2.3 and 2.4) in 2060. On the contrary to the initial and extended forms of the capital grant scenario, the savings rather come from a better energy sufficiency (adjustment factor in 2060: -0.17% [-0.81%]) than from gains in energy efficiency (conventional energy consumption in 2060: -0.02% [-0.08%]) (Table 2.6). The prebound effect induced by the increased price $P_e(t)$ of most carriers e [largely] offsets the rebound effect caused by the slightly more efficient building stock. At the end of the projection, no substantial deviation from the baseline can be observed in the total number of retrofitted dwellings and in the performance of the total final building stock (Tables 2.6 and 2.7).

2.6.2.3 Places 4 and 3: Remediation duty and carbon tax

Significant savings are reached under the remediation duty and the carbon tax. Compared to the baseline, the remediation duty [carbon tax] reduces the final energy consumption by 4.58% [5.38%] and the carbon dioxide emissions by 5.28% [6.04%] (Appendix 2.7, Table 2.5, Figures 2.3 and 2.4). Although both possible future instruments result in comparable savings, the way in which they are achieved is different.

Among all 10 single-instrument scenarios, the remediation duty generates naturally the largest increase in retrofitted buildings (+39% in 2060; Table 2.6 and Figure 2.7). From Table 2.6 we get that this not only induces the second-highest increase in energy efficiency (conventional energy consumption in 2060: -8.01%) but also the second-highest rebound effect (adjustment factor in 2060: $+1.17\%$). We furthermore observe in Figure 2.7 that each tightening of the remediation duty causes a prompt raise in annual retrofits. Yet, after the last tightening of the regulation, the prompt raise in retrofits is followed by a steady drop. Similarly to the phenomenon observed in the capital grant scenarios, annual demolitions and retrofits shrink the number of buildings that are affected by the remediation duty. Relative to the baseline, a visibly higher share of solar energy is achieved by the policy tool, and while it decreases the share of energy efficiency classes below E, an increase in the share of the classes E, D and C is observed (Table 2.7, Figures 2.5 and 2.6). From a governmental perspective the scenario generates no direct expenses or revenues (Table 2.8).

In contrast to this but comparable to the effects already observed in the energy tax scenarios, the carbon tax generates state revenues of 10 € per kWh saved (Table 2.8) and realizes its savings (conventional energy consumption in 2060: -0.35%) less through performance improvements but rather through a more conscious heating behavior; implying the strongest decrease of the adjustment factor (-3.00% in 2060) (Table 2.6). Moreover, no significant variations from the baseline are observed in the quantity of retrofits as well as in the share of energy classes and carriers in the total final building stock (Table 2.6, Figures 2.5, 2.6 and 2.7).

2.6.2.4 Places 2 and 1: Performance requirements

In the initial form of the scenario with energy performance requirements for new buildings, energy conservation (total final energy consumption in 2060: -2.06%) is below the savings of the remediation duty and the carbon tax scenarios (Table 2.5 and Figure 2.3). The scenario's emission mitigation (total $\mathrm{CO_2}$ emissions in 2060: -10.04%) are, however, well above those of the two previous scenarios, so that the mean value of energy and emission savings becomes the second-highest among all 10 single-instrument scenarios (Table 2.5 and Figure 2.4). In this case, energy conservation is achieved through the joint decrease of the 2060 conventional energy consumption (-0.86%) and the adjustment factor (-2.03%) (Table 2.6). The regulation implies that as of 2015 only 3 performance classes are allowed for new buildings (A, ZEB and PEB); a large majority goes for energy efficiency class A: 62.7% of the buildings in the total stock of 2060 have class A, compared to 3% with class ZEB or PEB (Table 2.7). In comparison with the baseline there is also a trend for more solar thermal energy (34.2% of the total final building stock) (Table 2.7).

By far the best environmental-effectiveness is reached in the extended form of this policy tool: not only does the instrument realize major energy savings (final energy consumption in 2060: -32.87%) but also does it achieve massive carbon dioxide reductions (total CO_2 emissions in 2060: -48.62%) (Table 2.5, Figures 2.3 and 2.4). These major savings particularly stem from a much more efficient NBS, which induces the largest decrease of the total conventional energy consumption in 2060 (-27.53%; Table 2.6). The share of dwellings in the total final stock with an energy efficiency class above B is equal in the initial and extended forms of the building code but a clearly higher share of ZEB (12.6%) and PEB (26.7%) exists in the latter form (Table 2.7, Figures 2.5 and 2.6). Note that although the NBS increases continuously, the greater construction of PEB (as of 2030) results in a strong decrease of the NBS' final energy consumption (Figure 2.8), induced by the new buildings' energy production. Remarkable is also that in 2060 more than half of the total building stock uses solar thermal energy to support their heating system (Table 2.7 and Figure 2.6). These large performance improvements are followed by a comparably large rebound effect: with a raise of 3.60% in 2060, the highest increase of the adjustment factor is observed in this scenario (Table 2.6 and Figure 2.2). Besides that, no direct state revenues or expenses are generated by the policy tool (Table 2.8).

While none of the single-instrument scenarios achieves the country's energy and climate objectives in time, the extended form of the energy performance requirements for new buildings is the only tool that accomplishes the 2020 energy targets as well as the 2030 emission goals (not in time but) in the course of the projection period (Table 2.9). The latter goal (-40% of total CO_2 emissions compared to 2005) is fulfilled in 2050, and the energy target (-20% of total final energy consumption compared to 2007) is realized in 2055.

2.6.3 Evaluation of combined energy policy tools

In the previous section we observed that the 10 single-policy instruments have different ecological and financial impacts. To accumulate the advantages of these instruments, policy makers typically combine the tools in packages and apply them simultaneously. In addition to the standard evaluation of the scenarios' effectiveness (see above), we now evaluate whether or not instruments generate synergistic effects when applied concurrently. Therefore 3 multiple-instrument scenarios are run: the first [second] consists of the existing instruments in their initial [extended] form and the third corresponds to the second, except that we consider the initial form of the energy tax and include the two possible future instruments.

2.6.3.1 Bundle 1

From an environmental viewpoint, running the model with the policy mix that is currently applied by the Luxembourgish state results in a decrease of the total final energy consumption (-2.45% in 2060) and CO_2 emissions (-10.53% in 2060) that is almost identical to the aggregated decrease of the corresponding single-instrument scenarios (Appendix 2.7, Table 2.5, Figures 2.3 and 2.4). In this first instrument package, the effects of capital grants subsidized loans, and the energy tax are insignificant, so that the observed savings mainly stem from the performance requirements. Many outcomes of the multiple-instrument scenario are hence similar to those of the building code scenario: energy conservation comes (in the bundle, just as in the building code) from an interplay of better energy efficiency (total conventional energy consumption in 2060: -1.05%; Table 2.6) and energy sufficiency (adjustment factor in 2060: -2.14%; Table 2.6 and Figure 2.2). Also the share of energy classes and carriers in the total final building stock is nearly identical (in the bundle and in the building code) (Table 2.7, Figures 2.5 and 2.6).

From a financial viewpoint, we are interested in the (negative) balance of state revenues minus state expenses. The absolute value of this balance is greater than the aggregated balances of the individual policy tools. The state revenues of the policy package are below those of the energy tax scenario (due to a lower energy consumption) and the state expenses are greater than the accumulated expenses of both the capital grant and subsidized loan scenario (due to a higher share of eligible dwellings). With a benefit to cost ratio of 5 € per kWh (Table 2.8) the scenario's higher energy savings lead to a significantly better economic effectiveness.

Apart from this, the 2020 emission target is realized during the projection period (in 2047) (Table 2.9).

2.6.3.2 Bundle 2

Further improvements in environmental and economic effectiveness are achieved in the second policy package scenario: compared to the 2060 baseline levels, energy savings of 34.60% and carbon dioxide reductions of 50.71% (Table 2.5, Figures 2.3 and 2.4) are realized.

Environmentally, similar to Bundle 1, the savings of Bundle 2 are almost identical to the aggregated decreases of the corresponding single-instrument scenarios (Table 2.5, Figures 2.3 and 2.4). The presence of capital grants, subsidized loans and the energy tax in the package leads to a greater energy efficiency than in the extended building code scenario (total conventional energy consumption in 2060: -28.13%; Table 2.6) and to a lower rebound effect (adjustment factor in 2060: $+2.90\%$; Table 2.6 and Figure 2.2).

Compared to Bundle 1, the balance of state revenues and expenses more than quadruples. Due to the much larger energy savings in Bundle 2, the scenario yet generates a benefit to cost ratio of 2 € per kWh saved (Table 2.8), which means that the economic effectiveness is further increased.

In Bundle 2, even though the energy and emission targets are not achieved in time, they are all achieved within the projection period (Table 2.9): the 2020 energy objective is realized in 2054, and the 2020 [2030] emission goal in 2037 [2044].

2.6.3.3 Bundle 3

The highest and most cost-effective energy and emission savings are realized in the multiple-instrument scenario of Bundle 3: for state expenses of less than 1 € per kWh saved (Table 2.8), the 2060 final energy consumption decreases by 42.37% and the $\mathrm{CO_2}$ emissions by 59.53% (Table 2.5, Figures 2.3 and 2.4).

Despite the very large performance improvement of the total building stock (total conventional energy consumption in 2060: -36.32%; Table 2.6), the scenario projects a relatively low rebound effect (adjustment factor in 2060: $+2.40\%$; Table 2.6 and Figure 2.2). This is mostly due to the carbon tax, an instrument that realizes most of its savings through a better energy sufficiency. Moreover, the higher quantity of accumulated annual retrofits in 2060 (about $+39\%$; Table 2.6 and Figure 2.7) is due to the incorporation of the remediation duty. This instrument, together with the performance requirements for new constructions, is primarily responsible for the share of efficient energy classes and carriers in the total building stock of 2060, which is the highest among all 10 scenarios (Table 2.7, Figures 2.5 and 2.6).

Similarly to Bundle 2, all national energy and emission targets are reached belatedly (Table 2.9): while a decrease of the final energy consumption by 20% is reached in 2048, emissions mitigation of 20% [30%] are realized in 2034 [2040].

2.7 Conclusion

With a focus on the Grand Duchy of Luxembourg, the present chapter evaluates the influence of energy policy tools on final energy consumption and direct CO_2 emissions in the residential building sector. For this purpose, we develop an advanced version LuxHEI of the French hybrid energy-economy model Res-IRF (Giraudet et al., 2012), which we also customized to the truly specific characteristics of Luxembourg.[25] The LuxHEI model is an energy policy model that is based on economic principles and that takes into account, for instance, global warming, the green value, sustainable energy efficiency classes and energy carriers, as well as a limited availability of carriers. Based on our model's results, four principal conclusions can be drawn. Firstly, we observe that building codes generate the largest energy conservation and mitigation of carbon dioxide emissions, without requiring direct government spending. Secondly, environmental effectiveness is achieved differently depending on the instrument type: while subsidy schemes and regulations mainly affect the building stock's energy efficiency, taxes usually induce a more conscious heating behavior. Thirdly, when used simultaneously, policy tools neither counteract nor generate direct synergistic effects but their individual impacts are more or less added-up. Therefore the policy package with the greatest number of instruments (Bundle 3) also generates the largest effects. Fourthly, in none of the evaluated policy scenarios the national energy and emission targets are achieved on time.

Although we encoded quite a few new features (for example, more sophisticated behavioral factors) to increase our dynamic simulation model's level of realism, it remains a stylized illustration of the real world. This means that modeling assumptions (for example, about households' decision-making behavior and the evolution of the new dwellings' surface or building type) and parameterization hypotheses (for example, about climate change or population growth) are still subject to uncertainty. Changing these suppositions likely affects the scenarios' outcomes to a certain extent. As concerns the barriers to energy efficiency, not all of them are fully integrable in a model (especially those of behavioral nature) and the LuxHEI model's projections may therefore be somewhat optimistic. Apart from this, we did not directly encode the impact of communicative policy tools, which tend to nudge households to behave in a more environmentally conscious way. Even if the energy saving potential of such instruments is relatively small (Gillingham et al., 2018),

[25] We again wish to point out that whenever the data is sufficient to realize the above calibrations and parameterizations, then the LuxHEI model also allows to perform the present study for any other country.

their absence in the model may induce a bit too pessimistic results; thus counteracting the preceding limitation. We are hence confident that the model's predictions are fairly accurate.

The policy recommendations that accompany our analysis are in compliance with other studies (Schaefer et al., 2000; Köppel and Ürge-Vorsatz, 2007; Weiss et al., 2012). More specifically, in the case of Luxembourg, the policy advice can be phrased as follows. Because all instruments have their pros and cons, and induce higher overall effectiveness when applied concurrently, a suitable combination of energy policy tools is advisable for the Luxembourgish residential building sector. In this policy mix, regulatory instruments should play a central role as they have the potential to strongly decrease the sector's energy consumption and CO_2 emissions at low governmental costs. Even though such standards are easier to enforce for new buildings (Köppel and Ürge-Vorsatz, 2007), efforts should persist to ensure the implementation of the remediation duty for existing buildings. In addition, our simulations confirm that regulations don't encourage households to go beyond the standard's requirements; the threshold of these two regulatory instruments should thus be raised regularly. With the two latter instruments being included into the national policy mix, the further presence of capital grants and subsidized loans is essential. These financial instruments allow low-income homes to meet the standard's demands and incite households to go beyond the threshold. However, our results indicate that the design of these subsidy schemes is decisive for the tool's cost-effectiveness: the instrument's application period should be limited relative to the product's market dynamics and eligible households should be specified. To curb the rebound effect that is induced by these four instruments, taxes should not be omitted in the country's policy mix. Considering the overall effectiveness of evaluated tax instruments, we advise the government to focus on the implementation of a national carbon tax (and hereby set an important example for other EU Member States), and to maintain the energy tax rates at the required minimum level of the ETD. To reduce the adverse effects of such a taxation policy, that is, falling economic growth or competitiveness of heavy energy-using industries, the revenues of the tax should be used to promote energy conservation (Callan et al., 2009) (for example, by using the revenues to fund a part of the subsidy schemes).

Future research should focus on the set-up of an alternative modeling method where one considers the dwellings' thermal insulation class instead of their energy efficiency class and where one adds information about the buildings' heating system into the model.

That way, the determination of the building stock's final space heating energy demand could be improved and dwellings that realize energy savings by solely replacing their heating system could be taken into account. Additionally, there is room for a better representation of households' behavioral patterns, for example, the decision-making behavior and the adjustment factor could be developed by including additional socio-economic variables into the model. Such modifications should be realized once the needed data on Luxembourgish households are available.

Appendix: Figures and Tables

Table 2.5 – Final energy consumption and direct CO_2 emissions (total building stock)

Scenario	Final energy consumption (kWh)			Final energy consumption per square meter (kWh/m^2)	Direct CO_2 emissions ($t\ CO_2$)			Direct CO_2 emissions per square meter ($t\ CO_2/m^2$)
2014		4546019090		156		915428		0.031
	2030	2045	2060	2060	2030	2045	2060	2060
Baseline	4602715670	4387523790	4122038610	60	889680	813213	734600	0.011
	Deviation from the baseline:				Deviation from the baseline:			
Individual energy policy tools								
1. Existing instruments—initial form								
Capital grants	-0.14%	-0.15%	-0.17%	60	-0.38%	-0.42%	-0.47%	0.011
Subsidised loans	-0.01%	-0.01%	-0.01%	60	-0.01%	-0.01%	-0.01%	0.011
Energy tax	-0.54%	-0.42%	-0.34%	60	-0.54%	-0.44%	-0.37%	0.011
Energy performance requirements	-0.49%	-1.08%	-2.06%	59	-3.66%	-6.39%	-10.04%	0.010
2. Existing instruments—extended form								
Capital grants	-0.19%	-0.34%	-0.37%	60	-0.64%	-0.96%	-1.05%	0.011
Subsidised loans	-0.01%	-0.02%	-0.02%	60	-0.01%	-0.03%	-0.02%	0.011
Energy tax	-1.06%	-1.23%	-1.61%	59	-1.06%	-1.30%	-1.77%	0.010
Energy performance requirements	-1.32%	-15.43%	-32.87%	40	-4.18%	-20.27%	-48.62%	0.005
3. Possible future instruments								
Remediation duty	-1.13%	-3.34%	-4.58%	57	-1.30%	-3.79%	-5.28%	0.010
Carbon tax	-3.71%	-5.67%	-5.38%	57	-3.79%	-6.06%	-6.04%	0.010
Combined energy policy tools								
Bundle 1	-1.09%	-1.56%	-2.45%	58	-4.33%	-6.97%	-10.53%	0.010
Bundle 2	-2.43%	-16.87%	-34.60%	39	-5.46%	-21.99%	-50.71%	0.005
Bundle 3	-6.64%	-24.58%	-42.37%	35	-9.92%	-30.47%	-59.53%	0.004

Table 2.6 – Number of retrofits, Conventional energy consumption, and Adjustment factor (total building stock)

Scenario	Number of retrofits			Conventional energy consumption (kWh)			Adjustment factor		
				2014					
				6445741560			0.72		
	up to 2030	up to 2045	up to 2060	2030	2045	2060	2030	2045	2060
Baseline	46474	113250	195755	5961251570	5392410660	4764806730	0.91	0.97	0.9986
	Deviation from the baseline:								
Individual energy policy tools									
1. Existing instruments—initial form									
Capital grants	0.03%	0.00%	-0.05%	-0.19%	-0.20%	-0.21%	-0.07%	-0.06%	-0.05%
Subsidised loans	0.00%	0.00%	0.00%	-0.02%	-0.02%	-0.02%	0.00%	0.00%	0.00%
Energy tax	0.00%	0.00%	0.00%	-0.01%	-0.02%	-0.02%	-0.32%	-0.22%	-0.17%
Energy performance requirements	0.00%	0.00%	0.00%	-0.08%	-0.31%	-0.86%	-1.40%	-1.70%	-2.03%
2. Existing instruments—extended form									
Capital grants	0.06%	0.04%	-0.04%	-0.29%	-0.54%	-0.53%	-0.17%	-0.13%	-0.12%
Subsidised loans	0.00%	0.00%	0.00%	-0.03%	-0.05%	-0.04%	0.01%	0.01%	0.01%
Energy tax	0.00%	0.00%	0.00%	-0.01%	-0.04%	-0.08%	-0.63%	-0.63%	-0.81%
Energy performance requirements	0.00%	0.00%	0.00%	-0.68%	-11.72%	-27.53%	-1.26%	0.88%	3.60%
3. Possible future instruments									
Remediation duty	28.69%	42.96%	39.28%	-2.26%	-6.13%	-8.01%	0.56%	1.07%	1.17%
Carbon tax	0.00%	0.00%	0.00%	-0.03%	-0.17%	-0.35%	-2.30%	-3.18%	-3.00%
Combined energy policy tools									
Bundle 1	0.06%	0.01%	-0.04%	-0.26%	-0.51%	-1.05%	-1.67%	-1.87%	-2.14%
Bundle 2	0.10%	0.07%	-0.02%	-0.95%	-12.32%	-28.13%	-1.86%	0.31%	2.90%
Bundle 3	28.56%	42.71%	38.93%	-3.29%	-18.60%	-36.32%	-3.15%	-1.00%	2.40%

Table 2.7 – Share of dwellings per energy class and carrier (total building stock of 2060)

Scenario	Share of energy efficiency classes in the total building stock											Share of energy carriers in the total building stock							
	I	H	G	F	E	D	C	B	A	ZEB	PEB	Oil	Gas	Electricity	Pellets	Oil + Solar	Gas + Solar	Electricity + Solar	Pellets + Solar
	2014																		
	8.9%	28.3%	22.1%	15.1%	15.2%	2.1%	1.3%	5.3%	1.8%	0.0%	0.0%	35.6%	58.0%	4.0%	1.0%	0.4%	0.6%	0.5%	0.0%
	2060																		
Baseline	0.5%	2.5%	6.4%	7.5%	6.6%	5.1%	2.8%	21.6%	45.7%	0.8%	0.5%	17.3%	34.2%	10.7%	5.8%	8.7%	10.9%	8.5%	3.9%
Individual energy policy tools																			
1. Existing instruments—initial form																			
Capital grants	0.5%	2.5%	6.4%	7.5%	6.6%	5.2%	2.8%	20.8%	46.4%	0.9%	0.5%	17.3%	33.7%	10.7%	5.8%	8.8%	11.0%	8.7%	4.0%
Subsidised loans	0.5%	2.5%	6.4%	7.5%	6.6%	5.1%	2.8%	21.6%	45.7%	0.8%	0.5%	17.3%	34.2%	10.7%	5.8%	8.7%	10.9%	8.5%	3.9%
Energy tax	0.5%	2.5%	6.4%	7.5%	6.6%	5.1%	2.8%	21.6%	45.7%	0.8%	0.5%	17.3%	34.2%	10.7%	5.8%	8.7%	10.9%	8.5%	3.9%
Energy performance requirements	0.5%	2.5%	6.4%	7.5%	6.6%	5.1%	2.8%	2.9%	62.7%	1.9%	1.1%	18.5%	29.8%	9.6%	7.8%	8.9%	10.3%	9.8%	5.4%
2. Existing instruments—extended form																			
Capital grants	0.5%	2.4%	6.3%	7.5%	6.6%	5.2%	2.8%	20.2%	47.0%	0.9%	0.5%	16.9%	32.9%	10.8%	5.9%	9.1%	11.4%	8.9%	4.1%
Subsidised loans	0.5%	2.5%	6.4%	7.5%	6.6%	5.2%	2.8%	21.6%	45.7%	0.8%	0.5%	17.3%	34.2%	10.7%	5.8%	8.7%	10.9%	8.5%	3.9%
Energy tax	0.5%	2.5%	6.4%	7.5%	6.6%	5.2%	2.8%	21.6%	45.7%	0.8%	0.5%	17.3%	34.0%	10.7%	5.8%	8.8%	11.0%	8.5%	3.9%
Energy performance requirements	0.5%	2.5%	6.4%	7.5%	6.6%	5.1%	2.8%	2.9%	26.4%	12.6%	26.7%	13.6%	24.0%	4.7%	3.3%	4.6%	5.7%	42.1%	2.0%
3. Possible future instruments																			
Remediation duty	0.4%	1.6%	4.9%	6.6%	6.7%	5.9%	4.6%	22.1%	45.9%	0.8%	0.5%	16.4%	33.3%	10.7%	5.8%	9.2%	12.1%	8.5%	3.9%
Carbon tax	0.5%	2.5%	6.4%	7.5%	6.6%	5.2%	2.8%	21.6%	45.7%	0.8%	0.5%	16.9%	33.7%	10.7%	5.8%	9.0%	11.5%	8.5%	3.9%
Combined energy policy tools																			
Bundle 1	0.5%	2.5%	6.4%	7.5%	6.6%	5.2%	2.8%	2.8%	62.7%	1.9%	1.1%	18.3%	29.5%	9.6%	7.8%	9.0%	10.4%	9.9%	5.4%
Bundle 2	0.5%	2.4%	6.3%	7.5%	6.6%	5.2%	2.8%	2.9%	26.5%	12.6%	26.8%	13.0%	23.0%	4.8%	3.4%	5.1%	6.4%	42.4%	2.1%
Bundle 3	0.4%	1.5%	4.8%	6.6%	6.6%	5.9%	4.7%	3.4%	26.7%	12.5%	26.8%	11.5%	21.3%	4.8%	3.4%	5.9%	8.7%	42.4%	2.1%

Table 2.8 – Benefit to cost ratio (in € ; from a governmental perspective)

Scenario	Direct revenues (during instrument's time period)	Direct expenses (during instrument's time period)	Balance	Benefit to cost ratio (€/kWh saved)
Baseline	0.00	0.00	0.00	0.00
Individual energy policy tools				
1. Existing instruments—initial form				
Capital grants	0.00	506140246.29	-506140246.29	-74.10
Subsidised loans	0.00	38398742.03	-38398742.03	-100.51
Energy tax	199761810.82	0.00	199761810.82	14.33
Energy performance requirements	0.00	0.00	0.00	0.00
2. Existing instruments—extended form				
Capital grants	0.00	1871412395.10	-1871412395.10	-123.79
Subsidised loans	0.00	123503822.62	-123503822.62	-135.61
Energy tax	531075380.17	0.00	531075380.17	8.02
Energy performance requirements	0.00	0.00	0.00	0.00
3. Possible future instruments				
Remediation duty	0.00	0.00	0.00	0.00
Carbon tax	2251340323.00	0.00	2251340323.00	10.15
Combined energy policy tools				
Bundle 1	189813712.28	734329463.99	-544515751.71	-5.40
Bundle 2	451734598.71	2745141617.20	-2293407018.49	-1.61
Bundle 3	1776900472.89	2926571465.93	-1149670993.04	-0.66

Table 2.9 – Energy consumption (comp. to 2007), CO_2 emissions (comp. to 2005)

Scenario	Final energy consumption compared to 2007			Direct CO_2 emissions compared to 2005		
	2020 Target : -20%	2030	2060	2020 Target: -20%	2030 Target: -40%	2060
Baseline	28.79%	18.97%	6.55%	6.76%	-4.44%	-21.10%
Individual energy policy tools						
1. Existing instruments—initial form						
Capital grants	28.68%	18.81%	6.37%	6.46%	-4.80%	-21.47%
Subsidised loans	28.79%	18.96%	6.54%	6.76%	-4.45%	-21.11%
Energy tax	27.80%	18.33%	6.19%	5.96%	-4.96%	-21.39%
Energy performance requirements	28.37%	18.39%	4.35%	4.63%	-7.94%	-29.02%
2. Existing instruments—extended form						
Capital grants	28.66%	18.75%	6.16%	6.44%	-5.05%	-21.93%
Subsidised loans	28.79%	18.96%	6.53%	6.76%	-4.45%	-21.12%
Energy tax	27.80%	17.71%	4.84%	5.96%	-5.46%	-22.49%
Energy performance requirements	28.37%	17.41%	-28.47%	4.63%	-8.44%	-59.46%
3. Possible future instruments						
Remediation duty	28.79%	17.63%	1.67%	6.76%	-5.68%	-25.26%
Carbon tax	28.79%	14.56%	0.82%	6.76%	-8.06%	-25.86%
Combined energy policy tools						
Bundle 1	27.37%	17.68%	3.94%	3.75%	-8.58%	-29.41%
Bundle 2	27.35%	16.09%	-30.32%	3.73%	-9.66%	-61.11%
Bundle 3	27.35%	11.07%	-38.60%	3.73%	-13.92%	-68.07%

Figure 2.1 – Evolution of the existing and new building stock

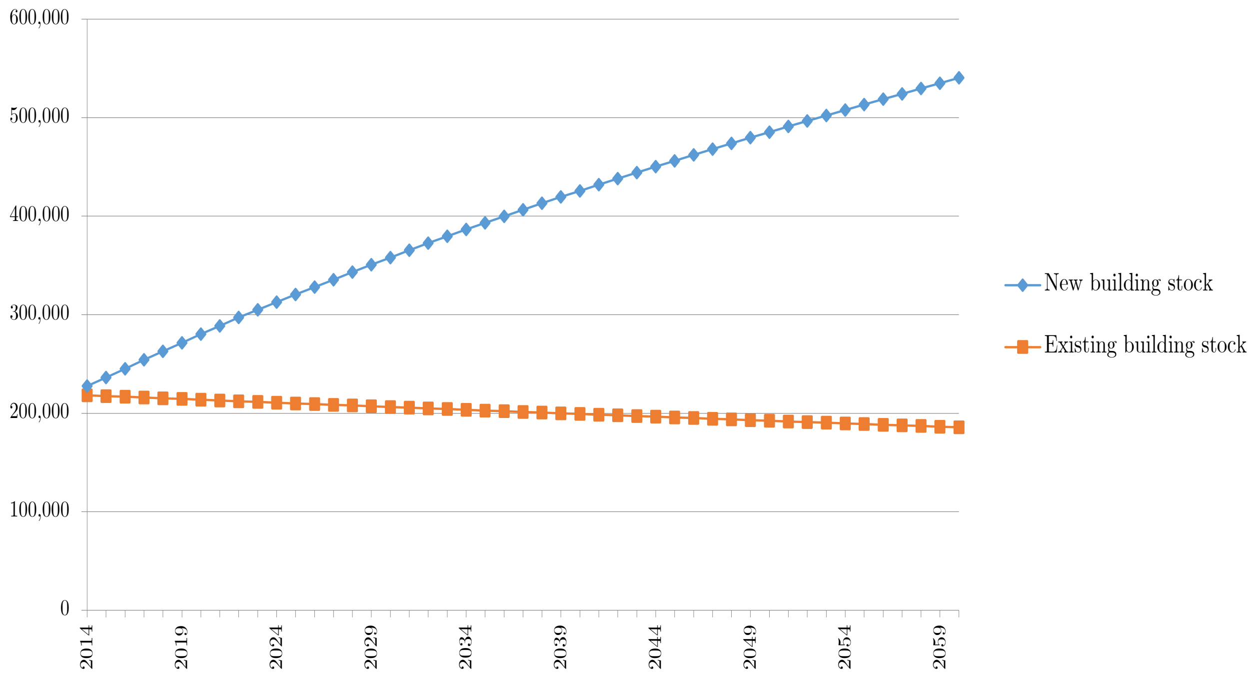


Figure 2.2 – Adjustment factor (total building stock)

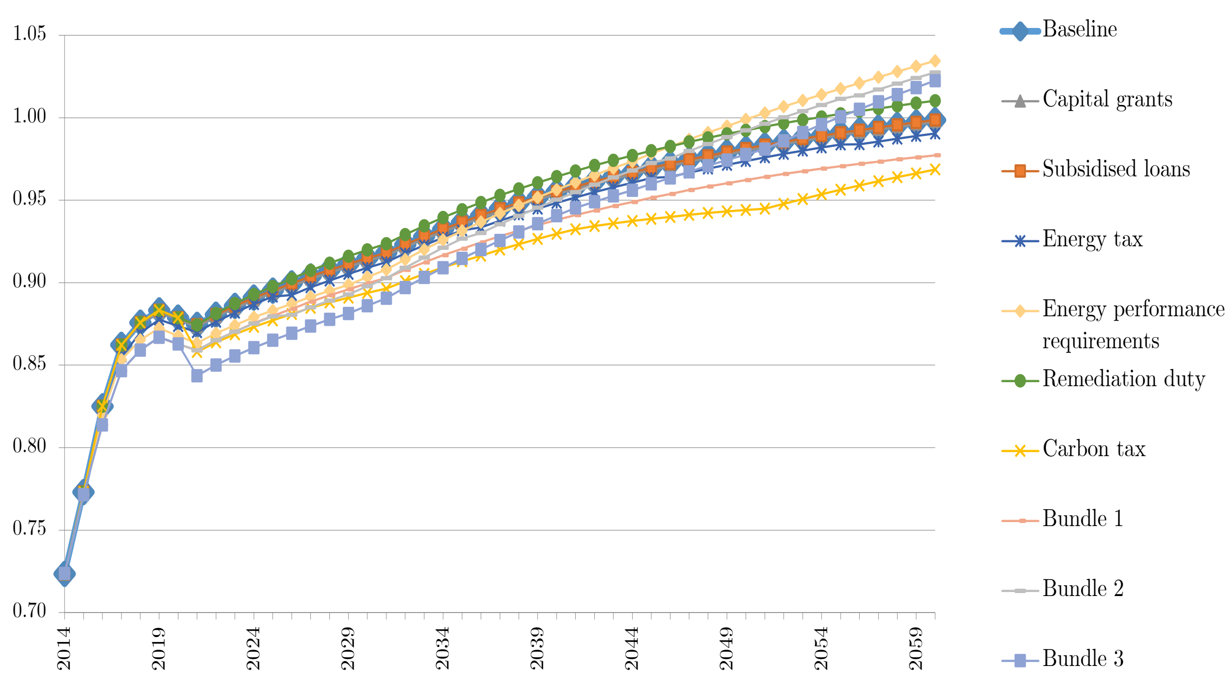


Figure 2.3 – Final energy consumption (kWh) (total building stock)

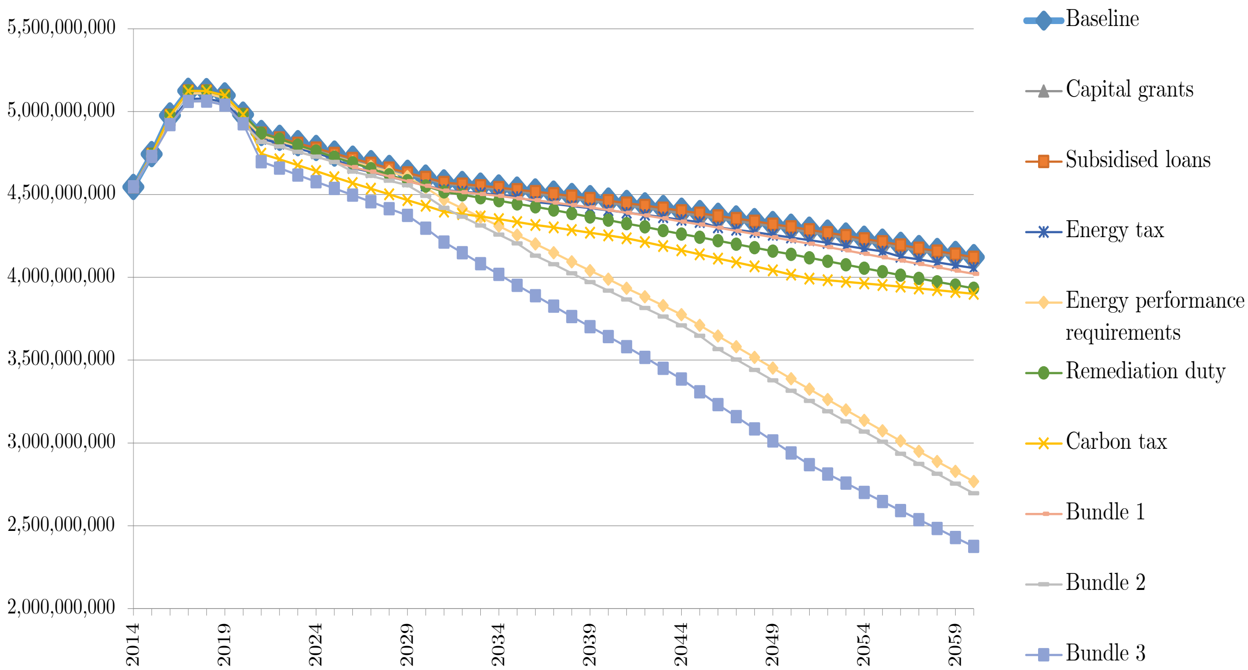


Figure 2.4 – Direct CO_2 emissions (t CO_2) (total building stock)

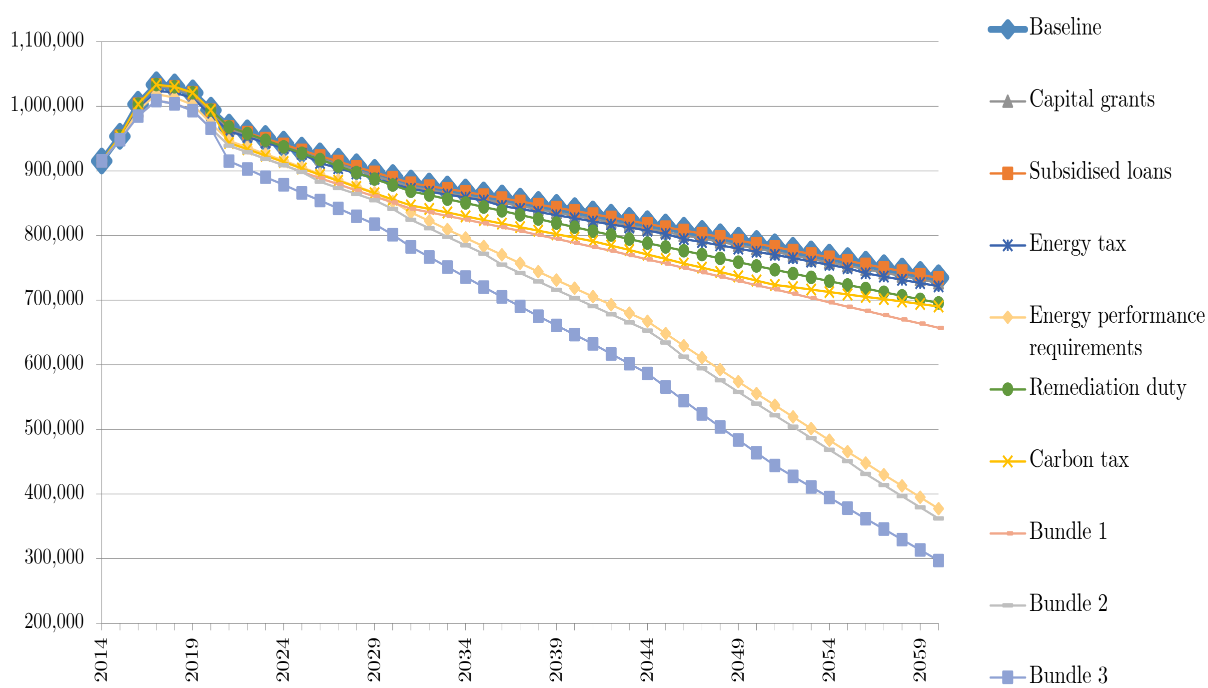


Figure 2.5 – Number of dwellings per energy class (total building stock of 2060)

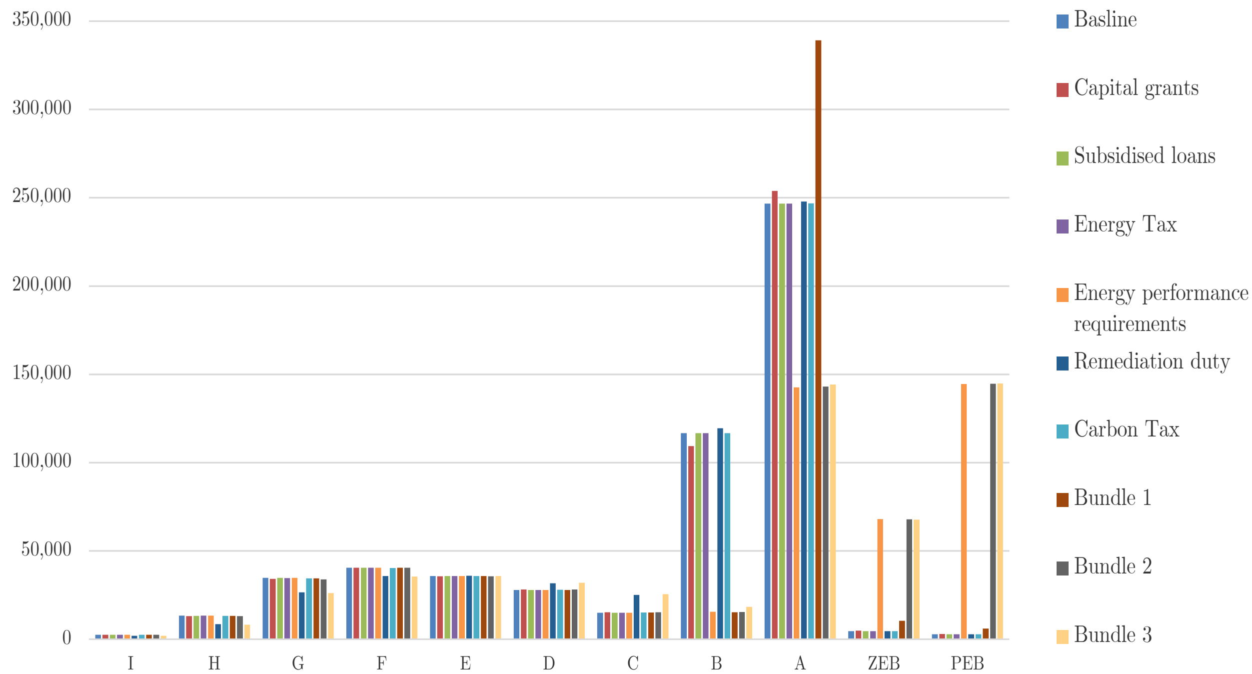


Figure 2.6 – Number of dwellings per energy carrier (total building stock of 2060)

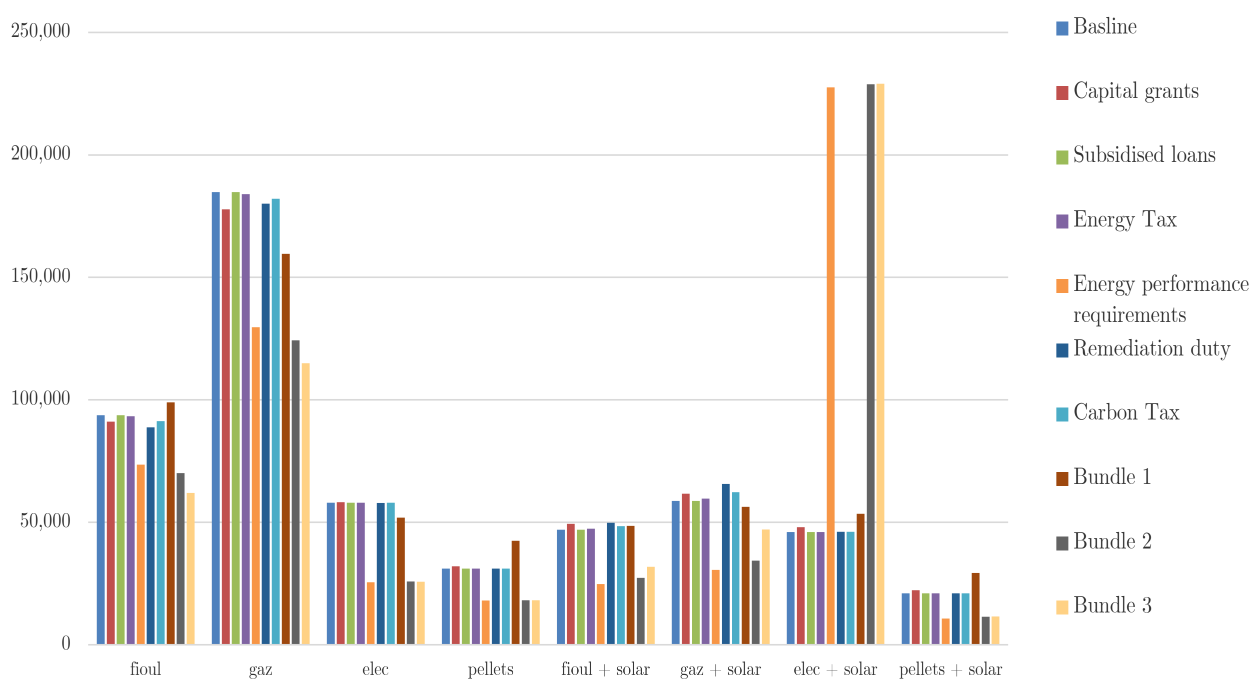


Figure 2.7 – Number of retrofitted dwellings

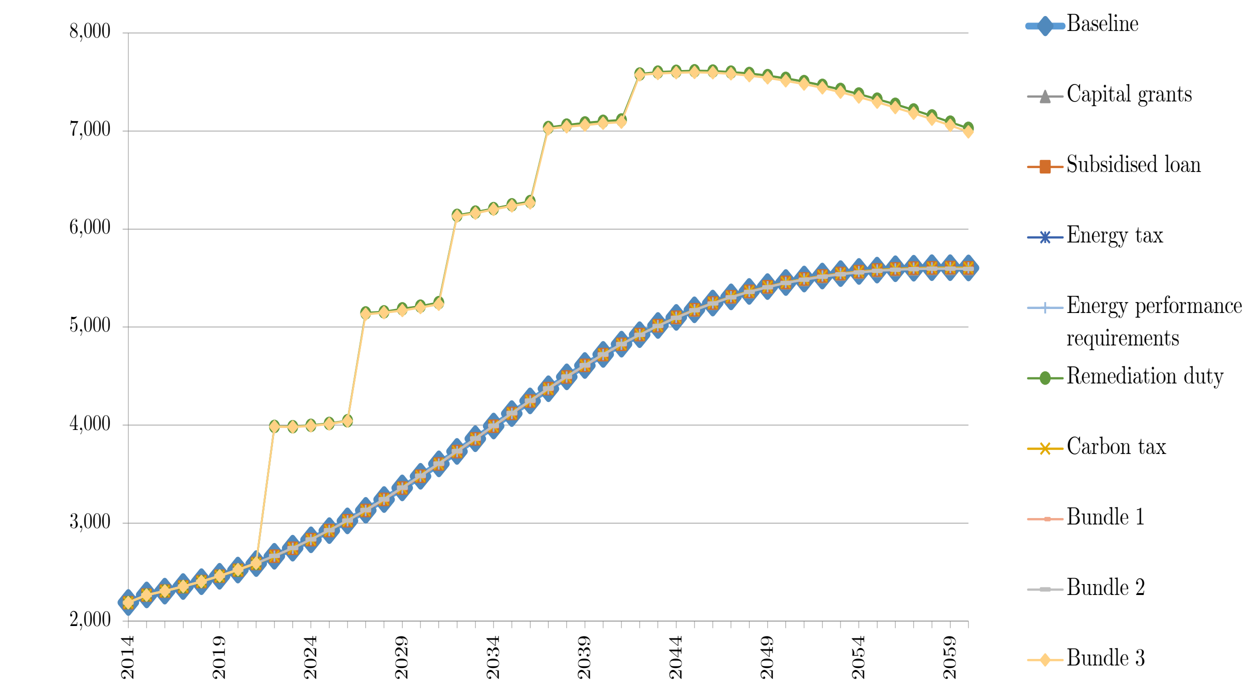
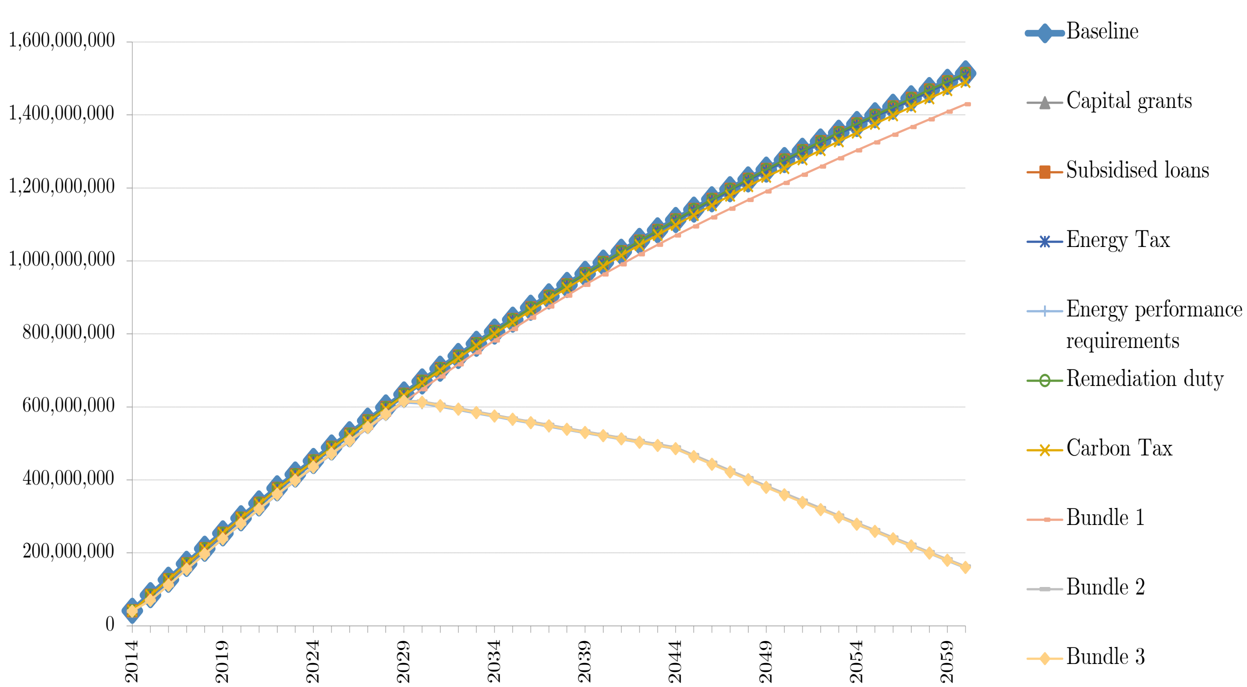


Figure 2.8 – Final energy consumption (kWh) (new building stock)



Chapter 3

The U.S.–China Market for Rare Earth Elements: A Dynamic Game View

3.1 Introduction

In December 2017, the U.S. Department of the Interior published a list of "critical" mineral commodities, that is, minerals that are "essential to the economic and national security of the United States" and whose "supply chain is vulnerable to disruption" (Federal Register, 2017). Among the 35 critical minerals that are listed in this executive order, nearly half correspond to so-called rare earth elements (REEs).

Rare earth elements are a group of 17 chemical elements in the periodic table (Connelly et al., 2005)[1] for which no backstop technology exists.[2] Owing to their very special electrochemical, luminescent and magnetic properties, these elements are key raw materials for advanced technologies, used worldwide in crucial fields such as the energy, military, automotive, or communication sectors (Goodenough et al., 2018). Despite their many common properties, each REE has its particular characteristics, making them unsubstitutable between each other, that is, different usages require specific REEs. For this reason, the ongoing launch and disappearance of products in the technology market also results in a continuously changing demand for the individual elements. However, matching the supply of individual REEs with their varying demand is a big challenge that can lead to

[1]The 15 lanthanides (at. no. 57–71) plus scandium (at. no. 21) and yttrium (at. no. 39).

[2]Following the definition of Nordhaus et al. (1973), "the backstop technology is a set of processes that (1) is capable of meeting the demand requirements, and (2) has a virtually infinite resource base".

significant market imbalances and thus to large price fluctuations (Schlinkert and van den Boogaart, 2015). This challenge is often referred to as the "balancing problem" (Neary and Highley, 1984; Binnemans et al., 2013) and comes from the fact that, on the one hand, rare earths do not occur in nature as native metals but are contained in different concentrations and proportions, together with other REEs, in certain minerals,[3] and, on the other hand, the individual REEs cannot be mined separately. Meeting the demand for all REEs therefore induces supply surpluses of some elements.

Although the global demand for rare earths has so far always been met, ensuring their future availability recently became a central challenge: while the annual difference between the overall rare earths supply and demand used to be largely positive, it started decreasing in 2005 and even became negative in 2010 (Massari and Ruberti, 2013).

A first reason for this most recent challenge is the steeply growing demand for REEs. More precisely, after their discovery in the 18th century, the demand for rare earth elements only became significant as of the mid-20th century and has been increasing ever since (Zhou et al., 2017). To date, as governments and industries worldwide strive to mitigate the devastating effects of climate change, the demand for green technologies (for example, electric and hybrid vehicles, wind turbines, solar panels and rechargeable batteries) is expected to face tremendous growth over the coming decades (Dudley, 2017). With the manufacturing of these sustainable products being heavily dependent on REEs,[4] the emergence of this sector is assumed to greatly contribute to the ongoing increase in the demand for rare earths (Alonso et al., 2012).

A second reason behind the availability challenge is the monopolistic structure of the rare earths supply market: from World War II until the 1960s, the demand for REEs was mainly covered by India, Brazil and South Africa; between the 1960s and the late 1980s, the United States (U.S.) took over the lead in world production; by the 1990s, after eliminating its competitors by practicing dumping prices, China then became the world's largest producer of REEs (Gordon B. Haxel and Orris, 2002) and covered 98% of global production until 2010 (Chakhmouradian and Wall, 2012). This monopolistic market situation is, however, not due to the common assumption that REEs are particularly rare by any geological measure: unlike their name suggests, these elements have an abundance in the earth's crust that is

[3]For example, bastnaesite, monazite, xenotime, or loparite. Note that the concentration and proportion of the individual elements varies with the type of mineral and the deposit's geographical location.

[4]Especially on neodymium and dysprosium (Chu, 2010).

comparable to that of copper, lead, nickel and zinc (Krishnamurthy and Gupta, 2015). Thus, although China holds most (about one third) of the world's proven rare earths reserves, it is not the only country where these elements occur (Zhou et al., 2017). What has actually contributed to China's dominant market position is that the lower operational costs in its rare earths industry (unregulated and unlicensed mines), as well as the environmental risks associated with the mining and processing of these elements, have prevented other countries from harvesting their national deposits (Mancheri, 2015).

Even though the growing demand for REEs and the dependency on China's rare earths exports are no new phenomena, they were globally ignored until 2010, when China withheld exports to Japan; resulting in extensive media coverage and market panic. Note that this was not the only time that the country imposed restrictions on its exports (Mancheri, 2015). While China argues that their export cuts are meant to preserve the environment and its natural resources (Trujillo, 2015), others believe that they serve the purpose of encouraging producers that rely on REEs to relocate their facilities to China (Mancheri, 2015). In 2012, the U.S., the European Union and Japan therefore officially filed a complaint with the World Trade Organization (WTO), claiming that China's export restraints violate WTO rules. Two years later, the WTO Appellate Body ruled in favor of the prosecutors and concluded that China had broken free trade agreement. As a consequence, China had to remove its export tariffs and quotas (Trujillo, 2015). Nevertheless, till today, their exports rely on licenses and remain quite unstable.

This precarious market situation recently triggered certain countries to declare REEs as critical to national interests and to contemplate the possibility of resuming their own end-to-end manufacturing lines (Chu, 2010; Chapman et al.; British Geological Survey, 2015; Federal Register, 2017). The most recent and concrete plan comes from the U.S., who despite its great need for REEs, relies almost entirely on rare earths from China: the Mountain Pass Mine in California is currently the only active mine in the country. While this Californian mine supplied most of the world's REEs in the 1980s, it nowadays operates at just a fraction of its potential capacity and ships its rare earths compounds to China for processing. However, in the context of the ongoing U.S.-China trade battle, China saddled America's ore shipments with higher tariffs. In addition, a recent visit of the Chinese President to one of the country's major rare earths facilities, as well as different Chinese reports of state-controlled entities, raised concerns about a new potential embargo on REEs exports to America (Partington, 2019; Hornby and Sanderson, 2019). Due to these growing threats, the U.S. Department of Commerce published an action plan in June 2019, which aims at securing the domestic supply of these economically and militarily vital materials (U.S. Department of Commerce, 2019); primarily by turning the U.S. once again into a major producer of rare earth elements.

In view of the above, we can thus conclude that REEs differ from most other non-renewable resources in four different ways: although (1) REEs are essential for sustainable technologies; (2) there exists no backstop technology to replace rare earths; (3) the current dominant supplier, China, can make use of its powerful economic and political forces to resist international pressure and defend its leading position; and (4) rather than to simply act as a competitive fringe member, the U.S. aims at becoming a true rival to China. On this account, REEs call for specific attention in a customized study.

In order to do so, the present work examines the optimal committed alignment, that is, the optimal open-loop strategies of China and the U.S., in both competitive and cooperative rare earths markets. Special emphasis is placed on providing information about (1) the optimal time for the U.S. to trigger its resource extraction in a non-cooperative environment, (2) the impact of China's entry-announcement reaction on the competitors' strategic orientations and revenues, and (3) whether or not a cooperation agreement makes both countries better off.

For these purposes, we firstly set up a competitive continuous-time model with two periods and search for open-loop strategies. Using the backward induction method, we work backwards in time and start by defining the duopolistic Nash equilibrium of the second period, where each country irrevocably commits itself to a supply path that maximizes its discounted profit in consideration of the other country's strategy. Subsequently, these outcomes serve to define China's optimal monopolistic extraction path in the first period. Secondly, we set up a cooperative continuous-time model, where both countries supply the market with rare earth elements right from the outset and search for open-loop strategies of a joint optimal control problem.

Our results show that without arbitrage opportunities, (1) China should optimally shrink its rare earths stock to the same level as the one of the U.S., before the latter enters the market, (2) unlike the U.S., China does not benefit from an early entry and is hence most likely to postpone the entry timing by coupling a restrained extraction approach during its monopolistic position with a continuous supply rate at the U.S.'s non-cooperative production launch, and (3) when the two countries agree to cooperate no Pareto improvement occurs.

The remainder of the chapter is organized as follows. Section 3.2 presents the related literature. Section 3.3 and 3.4 solve the non-cooperative and the cooperative game, respectively: both sections start with a description of the model set-up, followed by the deviation of the countries' rare earths extraction rates, the market price and the countries' payoffs. Section 3.4 completes the study with a comparison of each country's non-cooperative and cooperative behaviors. The last section concludes and presents strategy recommendations.

3.2 Related literature

The present section highlights more precisely, why the existing model settings on exhaustible resources are unsuited in the field of rare earths supply games.

Firstly, after Hotelling (1931) informally discussed the extraction behavior in monopolistic and perfectly competitive resource markets, four decades passed until Salant (1976) introduced the theory of dynamic games into the field of exhaustible resources. Under a cartel versus price-taking fringe setting, where players are assumed to act concurrently and where zero extraction costs are considered, the author scrutinizes the producers' extraction behavior and offers an open-loop strategic Nash equilibrium. The former work is subsequently extended by Ulph and Folie (1980), who include production costs for both the cartel and the members of the fringe. At about the same time, Gilbert (1978) also investigates a cartel-fringe extraction game. As opposed to the two previous settings, Gilbert's model integrates the backstop technology concept of Nordhaus et al. (1973) and presumes the cartel to play before the price-taking fringe. The reason why the above model settings are, however, improper for rare earths games is because we do not consider the situation where some small fringe members try to compete with a dominant supplier but rather do we regard the newcomer to be a serious competitor to the incumbent producer.

Secondly, regarding the latter backstop technology in Gilbert (1978), many other resource models we resort to, similarly include a perfect substitute in their setting: Gilbert et al. (1978) analyze the optimal price strategy of a monopolist with limited reserves, who is threatened by the potential entry of competitors that own a backstop technology; Stiglitz (1976), Stiglitz and Dasgupta (1982) and Dasgupta et al. (1982) study what effects market structures have on the resource exploitation rate and on the backstop's timing of innovation; Stiglitz and Dasgupta (1981a,b) assess how uncertainty about the discovery date of a substitute affects the rate of resource extraction under diverse market structures; and, in a more recent contribution, Harris et al. (2010) examines resource depletion in a competitive framework, where a backstop technology is considered to solve the nonzero-sum Cournot game. Notwithstanding, the absence of a backstop technology in our modeling remains perfectly justified because a substitute is yet to be found for rare earth elements.

Thirdly, the only studies to analyze open-loop exhaustion orders in an oligopolistic resource game and thus assume a market structure similar to that of the present chapter, are Lewis and Schmalensee (1980), Loury (1986) and Benchekroun et al. (2009). While all three frameworks assume their players to have heterogeneous stock sizes, Loury (1986) exceptionally considers uniform production costs. As this modeling mismatch allowed both other authors to define circumstances where the rule of Herfindahl (1967) is violated, that is, low cost deposits may not be exhausted first, extraction costs should carefully be included when studying resource-depletion orders. Nevertheless, with our focus being on different issues, these costs are, similarly to the pioneer work of Salant (1976), not properly modeled at this early state.

3.3 Non-cooperative game

In this section, we present a detailed description of the continuous-time dynamic model where players are contenders that act selfishly. Beyond that, we illustrate the solutions to the optimal control problems and draw some initial conclusions.

3.3.1 The model setting

Consider country C the monopolistic rare earths supplier of a first period $I = [0, T^*)$. At the start of this period, where time $t = 0$, two important actions take place: (1) country C commits to a supply strategy that cannot be easily reconsidered or modified; and (2) country A announces its plan to enter the supply market at the optimal time $t = T^*$. By implication, while A's entry marks the beginning of the second period $II = [T^*, +\infty)$, it also changes the market from a monopolistic structure to a duopolistic one. Here, at the start, where time $t = T^*$, each country pledges itself to a supply path that is the optimal response to its competitor's path.

So far, the vast majority of contributions in the exhaustible resource literature considers open-loop strategies, that is, the players commit to a supply path at the outset, which, over the course of the game, cannot be updated to the varying state of the reserve. On the one side, this typical modeling choice is attributable to the supply side of non-renewable resource markets. More specifically, the providers' supply volumes are generally constrained by the existing infrastructure as supply changes necessitate additional investments in either production or storage capacities. On the other side, the frequent design of open-loop frameworks is supported by the markets' demand side. Actually, committing to a supply path right at the outset of the game leads to a stable investment environment, which keeps the importing countries and industries from looking for alternative resources or backstop technologies, and thus prevents the market price from dropping. Beyond that, most of the buyers count on long-term contracts with the suppliers to prevent resource shortages (Liski and Montero, 2014). For example, REEs are not exchanged in spot or futures markets but mainly through long-term contracts between state-owned utilities and manufacturers (Chu, 2010). In view of these facts, open-loop commitment can be considered a proper strategy choice for most exhaustible resource markets.

For the current REEs study, as a first choice of strategy space, we thus follow the tradition and consider open-loop commitment instead of Markovian feedback strategies. This decision is the more natural because even though Markovian strategic Nash equilibria are subgame perfect, they are independent of the initial reserves. Yet, in the entry game of REEs, the real initial state of the game may be important.

In the following, we consider the price function of Stiglitz (1976):

$$P(t) = a\,Q^{\alpha-1}(t), \quad \alpha \in \ (0,1), \tag{3.1}$$

where $\frac{1}{1-\alpha}$ is the price elasticity of demand, a is a positive constant and where at time t, the total market supply $Q(t)$ is the sum of A's supply $q_A(t)$ and C's supply $q_C(t)$. Since country A withholds its REEs extraction until the market entry, its first-period supply $q_A^I(t)$ is zero.

Additionally, we presume that at time t, the supply of country $i \in \{A, C\}$ equals the amount of resources that is extracted from its remaining, privately-owned, reserves $R_i(t)$ and that $R_C(0) >> R_A(0)$.

As concerns country C's revenue, aggregated over the two modeling periods, it is defined by the ensuing problem

$$\begin{aligned}\Pi_C &= \max_{q_C(t)} \int_0^{+\infty} \mathrm{e}^{-rt} P(Q(t))\, q_C(t)\, \mathrm{d}t \\ &= \max_{q_C^I(t)} \int_0^{T^*} \mathrm{e}^{-rt} P^I(q_C^I(t))\, q_C^I(t)\, \mathrm{d}t + \max_{q_C^{II}(t)} \int_{T^*}^{+\infty} \mathrm{e}^{-rt} P^{II}(Q^{II}(t))\, q_C^{II}(t)\, \mathrm{d}t,\end{aligned}$$

subject to the initial reserves and extraction rates (precised later on), where e^{-rt} is a time-preference factor with rate r. Country A's problem is very similar, except that there exits no revenue in the first period.

Within this framework, we argue that country A's rare earths extraction will not:

- start immediately at $T^* = 0$. The explanation behind this assumption is related to the findings of Loury (1986) and Harris et al. (2010), which illustrate that smaller reserves are depleted before larger ones. In fact, with the countries' reserves not being commonly owned, an entry at the outset puts country A in a disadvantaged position as C can push down the price P by making use of its much bigger reserves to flood the duopolistic market with REEs. While this remains strategically reasonable for country C, mainly because it can make up for the initial losses once A has left the market, it is not for country A. More precisely, in view of C's excess supply, A has basically two options: (1) it decides to counterbalance the price collapse by decreasing its own supply q_A. In this case, A's reduced market supply, however, deprives the country from meeting its original objective, that is, to act as an independent rival with real market power and not just a weak member of a small fringe; and (2) it chooses to meet its original objective of becoming a powerful competitor and thus increases its supply to the point where it can defy country C. Yet, this supply behavior not only generates a quick depletion of A's REEs reserves but also further worsens the price deterioration;

- be postponed until $T^* = +\infty$, as this prevents A from extracting its REEs and thus from making any profit;

- be held back until C has depleted its stock at some finite time $T^* < +\infty$. Actually, since C is a rational player who acts in its own best interest, the argument raised in the first point implies that the present scenario is not going to occur. Indeed, since country C has a great interest in avoiding that country A becomes the superior supplier, it will most likely halt its extraction q_C at the latest when $R_C(t) = R_A(0)$.

This reasoning leads us to suppose that country A's rare earths extraction starts at some time $T^* \in (0, +\infty)$, where country C's reserves $R_C(T^*) = R_C^* > 0$.

In the following subsections, this monopoly-to-duopoly game is solved by using backward induction. That way, we demonstrate the existence of T^* and define its exact value.

3.3.2 Rare earths extraction and market prices under duopoly

In order to determine the optimal entry condition for country A, we start with the second period. Here the non-cooperative suppliers' revenue is

$$\pi_i^{II}(R_C^*, T^*) = \max_{q_i^{II}(t)} \int_{T^*}^{+\infty} \mathrm{e}^{-rt} P^{II}(Q^{II}(t))\, q_i^{II}(t)\, \mathrm{d}t,$$

subject to the reserve constraint

$$\int_{T^*}^{+\infty} q_i^{II}(t)\, \mathrm{d}t \leq R_i(T^*) = \begin{cases} R_C^*, \text{ for } i = C \\ R_A(0) \text{ given, for } i = A \end{cases}, \quad q_i^{II}(t) \geq 0,$$

the extraction rate

$$\dot{R}_i(t) = -q_i^{II}(t),$$

and the aggregate supply

$$Q^{II}(t) = q_A^{II}(t) + q_C^{II}(t), \quad t \in [T^*, +\infty).$$

The Lagrangian function of country i is set up as follows:

$$\mathcal{L}_i^{II}\left(q_i^{II}(t), \lambda_i^{II}(t), \alpha_i^{II}\right) = P^{II}(Q^{II}(t)) q_i^{II}(t) - \lambda_i^{II}(t) q_i^{II}(t) - \alpha_i^{II}\left(R_i(T^*) - \int_{T^*}^{t} q_i^{II}(\tau)\, \mathrm{d}\tau\right),$$

where $\lambda_i^{II}(t)$ is country i's shadow price at time $t \in [T^*, +\infty)$, that is, the change in its discounted revenue resulting from an extra unit of its remaining reserves $R_i(t)$, and where α_i^{II} is the static Lagrange multiplier. From the standard first-order conditions (FOCs), for which the second-order sufficient conditions hold as well, we get that the shadow price $\lambda_i^{II}(t)$ grows at interest rate r:

$$\lambda_i^{II}(t) = \lambda_i^{II}(T^*)\mathrm{e}^{r(t-T^*)}, \tag{3.2}$$

with the initial condition $\lambda_i^{II}(T^*)$ being determined by means of the transversality condition.

Moreover, from the FOCs and the partial derivative of country i's instantaneous revenue function $RV_i^{II}(t) = P^{II}(t)q_i^{II}(t)$ with respect to its optimal supply path $q_i^{II}(t)$, we obtain

$$\frac{\partial RV_i^{II}(t)}{\partial q_i^{II}(t)} = \lambda_i^{II}(t) = a\left(Q^{II}(t)\right)^{\alpha-1}\left(1 - \frac{(1-\alpha)q_i^{II}(t)}{q_A^{II}(t) + q_C^{II}(t)}\right). \tag{3.3}$$

Equation (3.3) enables us to express the relationship between the two countries' optimal extraction rates in terms of the shadow values:

$$q_A^{II}(t) = \frac{\lambda_C^{II}(T^*) - \alpha\lambda_A^{II}(T^*)}{\lambda_A^{II}(T^*) - \alpha\lambda_C^{II}(T^*)}\, q_C^{II}(t). \tag{3.4}$$

Equation (3.4) thus suggests that if the competitors value their stock identically at the moment of A's entry, then their duopolistic extraction rates are equal and vice versa. If this is not the case, the country that puts the lowest hypothetical price on its remaining resources at T^* is also the one with the highest second-period supply. Next, as the competitors' revenue is positively related to the total amount of supplied REEs, we argue that both countries have strong financial incentives to fully exhaust their initial reserves $R_i(T^*)$ over the second-period planning horizon. Under this assumption, integrating Equation (3.4) over $[T^*, +\infty)$ gives

$$\lambda_A^{II}(T^*) = \frac{R_C^* + \alpha R_A(0)}{R_A(0) + \alpha R_C^*}\, \lambda_C^{II}(T^*). \tag{3.5}$$

Obviously, at the starting point of the second-period, the ratio of the shadow values $\frac{\lambda_A^{II}(T^*)}{\lambda_c^{II}(T^*)}$ solely depends on the initial duopolistic reserves and the price-elasticity parameter α.

The combination of (3.4) and (3.5) then yields that the optimal extraction ratio checks

$$\frac{q_A^{II}(t)}{q_C^{II}(t)} = \frac{R_A(0)}{R_C^*},\ \forall t \geq T^*. \tag{3.6}$$

The ratio of the two countries' supply is thus constant over time and determined only by their initial stocks $R_i(T^*)$ at T^*.

Furthermore, combining (3.3) and (3.6) allows to deduce the marginal revenue of C

$$\frac{\mathrm{d}RV_C^{II}(t)}{\mathrm{d}q_C^{II}(t)} = \alpha P^{II}(t) = \lambda_C^{II}(t) + q_C^{II}(t)\, a(\alpha-1)\left(q_A^{II}(t)+q_C^{II}(t)\right)^{\alpha-2}\frac{R_A(0)}{R_C^*}, \qquad (3.7)$$

where $P^{II}(t) = a\left(\frac{R_A(0)}{R_C^*}+1\right)^{\alpha-1} q_C^{II}(t)$. Equation (3.7) thus yields C's optimal extraction rate

$$q_C^{II}(t) = \frac{R_C^*}{R_A(0)+R_C^*}\left(\frac{R_A(0)+R_C^*}{R_A(0)+\alpha R_C^*}\right)\left(\frac{\lambda_C^{II}(T^*)}{a}\right)^{\frac{1}{\alpha-1}} \mathrm{e}^{\frac{r(t-T^*)}{\alpha-1}}. \qquad (3.8)$$

Finally, by integrating (3.8) over $[T^*, +\infty)$, we get both countries' optimal choice and the market price. The ensuing proposition presents our results.

Proposition 1. *Suppose that country A enters the market at time $T^* \in (0,+\infty)$, where C's reserves $R_C(T^*) = R_C^* > 0$. Then for any $t \geq T^*$, the second-period open-loop strategic Nash equilibrium supply of C and A are given, respectively, by*

$$q_C^{II}(t) = \frac{r}{1-\alpha}\, R_C^*\, \mathrm{e}^{\frac{r(t-T^*)}{\alpha-1}}, \qquad (3.9)$$

and

$$q_A^{II}(t) = \frac{r}{1-\alpha}\, R_A(0)\, \mathrm{e}^{\frac{r(t-T^*)}{\alpha-1}}. \qquad (3.10)$$

The market price is

$$P^{II}(t) = a\left(\frac{r}{1-\alpha}\left(R_A(0)+R_C^*\right)\right)^{\alpha-1} \mathrm{e}^{r(t-T^*)}. \qquad (3.11)$$

The detailed proof of this proposition can be found in the appendix.

Equations (3.9) and (3.10) show that at time T^*, the supply of both countries $q_i^{II}(T^*)$ depends positively on their initial reserves $R_i(T^*)$, that is, the smaller the country's reserve, the less rare earths it is willing to extract. As the supply is a decreasing function of t, this behavior remains unchanged over time. Moreover, with the market price being a decreasing function of the supply, the opposite effects can be observed in Equation (3.11). This equation also gives country C's trigger reserve R_C^* as a decreasing function of the trigger price P^*:

$$R_C^*(P^*) = \frac{1-\alpha}{r}\left(\frac{P^*}{a}\right)^{\frac{1}{\alpha-1}} - R_A(0), \qquad (3.12)$$

so that the later A enters the market, the higher is P^* and the lower will be R_C^*.

Furthermore, due to the absence of arbitrage opportunity at T^*, we must have

$$\lambda_A(T^*) = \lambda_C(T^*). \tag{3.13}$$

By combining Equations (3.2) and (3.5), Equation (3.13) is satisfied if and only if $R_C^* = R_A(0)$. The next corollary states this finding.

Corollary 1. *When country A enters the rare earths supply competition at time T^*, C's reserves must check*

$$R_C(T^*) = R_C^* = R_A(0).$$

The above analysis mathematically proves the intuition that country A enters the competition at some time $T^* \in (0, +\infty)$. Besides that, it shows that country A should hold off its market entry until C's supply has shrunk its stock to the same level as that of A. This, however, also implies that A has no influence on the time of its entry but totally depends on the monopolist's extraction behavior. In other words, the quicker country C extracts its REEs in the first period, the faster the reserves of C and A coincide and the earlier country A enters the market. For this reason, let us now turn to the monopolistic supplier C and study the impacts of its extraction behavior on the optimal entry timing T^*.

3.3.3 Rare earths extraction and market prices under monopoly

This subsection defines the monopolist's unalterable extraction commitment on the basis of how the production launch of country A affects C's supply at time T^*.

Overall, two scenarios are considered: (1) country C is incapable of changing (or unwilling to change) its supply volume at the moment of A's entry, so that C's supply is continuous at T^*: $q_C^I(T^*) = q_C^{II}(T^*)$. We suppose rigidity in the production process behind this entry reaction, for example, country C is unable to rapidly adjust the workforce or machinery utilization within its manufacturing plants or inelastic supply and distribution networks complicate the modification of product procurement and delivery. Either way, whenever A enters the game in this scenario, the total offer of REEs will instantly increase, causing the market price to decrease at T^*: $P^I(T^*) > P^{II}(T^*)$; (2) country C is capable of planning and controlling its rare earths supply chain, so that at A's entry, C can actually decrease its market supply. In addition, country C wants to prevent the market price from dropping at the newcomer's entry, such that in this case the price is continuous at T^*: $P^I(T^*) = P^{II}(T^*)$.

For this to be satisfied, C must, at T^*, decrease its offer by the quantity of A's supply: $q_C^I(T^*) - q_A^{II}(T^*) = q_C^{II}(T^*)$. In the following, the two scenarios will be examined more closely.

In both cases, the monopolist faces the same price function of Equation (3.1) and commits to a supply path that maximizes its revenue, which is given by

$$\pi_C^I(R_C(0) - R_C^*, T^*) = \max_{q_C^I(t)} \int_0^{T^*} \mathrm{e}^{-rt} P^I(q_C^I(t))\, q_C^I(t)\, \mathrm{d}t,$$

subject to

$$\int_0^{T^*} q_C^I(t)\, \mathrm{d}t = R_C(0) - R_C^*, \quad R_C(0) \text{ given}, \quad q_C^I(t) \geq 0,$$

and

$$\dot{R}_C(t) = -q_C^I(t), \quad t \in [0, T^*).$$

By applying the standard optimal control calculations of Section 3.3.2, we find that at time $t \in [0, T^*)$, country C's shadow price is

$$\lambda_C^I(t) = \lambda_C^I(T^*)\mathrm{e}^{r(t-T^*)}, \quad t \in [0, T^*), \tag{3.14}$$

where the entry level of the shadow value $\lambda_C^I(T^*)$ comes form the second period of the game, and that its marginal revenue is

$$\frac{\mathrm{d}RV_C^I(t)}{\mathrm{d}q_C^I(t)} = a\alpha \left(q_C^I(t)\right)^{\alpha-1} = \alpha P^I(t) = \lambda_C^I(t). \tag{3.15}$$

Substituting (3.14) into (3.15) and rearranging terms gives C's first-period supply function:

$$q_C^I(t) = \left(\frac{\lambda_C^I(T^*)}{a\alpha}\mathrm{e}^{r(t-T^*)}\right)^{\frac{1}{\alpha-1}}. \tag{3.16}$$

From here we can determine the first-period supply and market price in both cases. Henceforth, the continuous supply [price] scenario is denoted by O [P].

Remark 1. *Since country C's supply reaction at the entry is likely to affect the first-period outcomes, we wish to emphasize that each finding prior to Equation* (3.17) *actually depends on the subscript $\theta \in \{O, P\}$. Accordingly, the supply and price functions must, for instance, be viewed as $q_{i,\theta}$ and P_θ, respectively, the shadow prices as $\lambda_{i,\theta}$ and the entry time as T_{θ^*}.*

Scenario O: Continuous supply

In this case, country C's commitment to keep its supply volume continuous at the time of country A's market entry yields that:

$$\lambda^I_{C,O}(T^*_O) = a\alpha \left(\frac{r}{1-\alpha} \, R^*_C \right)^{\alpha-1} . \tag{3.17}$$

Substituting (3.17) into (3.16) gives the results of the next proposition.

Proposition 2. *If country C is unable to adjust its supply when country A enters the market at time T^*_O, then for any $t \in [0, T^*_O)$, the optimal open-loop supply path of country C is given by*

$$q^I_{C,O}(t) = \frac{r}{1-\alpha} \, R^*_C \; \mathrm{e}^{\frac{r(t-T^*_O)}{\alpha-1}} , \tag{3.18}$$

and the market price is

$$P^I_O(t) = a \left(\frac{r}{1-\alpha} \, R^*_C \right)^{\alpha-1} \mathrm{e}^{r(t-T^*_O)} . \tag{3.19}$$

Obviously, the monopolistic supply follows the same functional form as C's supply in the second period (see Equation (3.9)).

To define the optimal time T^*_O for country A to enter the competition, we take integrals over $[0, T^*_O)$ on both sides of the dynamic equation $\dot{R}_C(t) = -q^I_{C,O}(t)$ and get the proposition below.

Proposition 3. *If country C is unable to adapt its supply at A's market entry, then the entry happens at time*

$$T^*_O = \frac{1-\alpha}{r} \ln \left(\frac{R_C(0)}{R_A(0)} \right) . \tag{3.20}$$

Unsurprisingly, the entry time depends, besides the price elasticity of demand and the time preference, primarily on the initial reserve ratio $\frac{R_C(0)}{R_A(0)}$. The observation that it is yet independent of country C's entry reserves R^*_C is because the absence of supply change at the entry, makes the latter variable become irrelevant.

Scenario P: Continuous price

In the previous case, the continuous supply commitment of country C implies that at the time of country A's supply launch, the aggregated supply of the two countries is found to be larger than the single supply of C. By implication, country A's entry is characterized by a drop in both the market price and hence C's instantaneous revenue. In order to avoid, however, a loss in instantaneous profits (per unit of market supply), the present commitment of country C is assumed to include a supply decrease at the entry. Since this drop is designed to balance out A's supply, it prevents the aggregated supply and the market price from changing. Putting this into effect, yields the ensuing results.

Proposition 4. *Suppose country C commits to decrease its supply when country A enters the market at time T_P^*. Then for any $t \in [0, T_P^*)$, the optimal open-loop supply path of country C is given by*

$$q_{C,P}^I(t) = \frac{r}{1-\alpha} \left(R_A(0) + R_C^*\right) \, \mathrm{e}^{\frac{r(t-T_P^*)}{\alpha-1}}, \tag{3.21}$$

and the market price is

$$P_P^I(t) = a \left(\frac{r}{1-\alpha} (R_A(0) + R_C^*) \right)^{\alpha - 1} \mathrm{e}^{r(t-T_P^*)}. \tag{3.22}$$

Naturally, the monopolistic market price follows the same functional form as the market price in the second period (see Equation (3.11)).

Again, integrating the dynamic equation $\dot{R}_C(t) = -q_{C,P}^I(t)$ over $[0, T_P^*)$ on both sides gives us the optimal time T_P^* for country A to enter the market. The solution is illustrated in the subsequent proposition.

Proposition 5. *If country C's commitment includes altering its supply at country A's market entry, then the entry happens at time*

$$T_P^* = \frac{1-\alpha}{r} \ln \left(\frac{R_C(0) + R_A(0)}{R_C^* + R_A(0)} \right). \tag{3.23}$$

In contrast to the continuous supply scenario O, the above entry time does not depend on the initial reserves ratio but on the ratio of aggregated initial reserves and aggregated entry reserves $\frac{R_C(0)+R_A(0)}{R_C^*+R_A(0)}$. This is hardly surprising because, while the continuous supply assumption implies that C's supply remains unaltered by A's entry, such that only the initial reserves matter, the present continuous price assumption requires that C specifically aims at avoiding an instantaneous profit loss due to a price collapse. The fact that this is achieved by keeping the initial aggregate entry supply of the two countries at one level with C's terminal first-period supply, explains the dependency on the aggregated entry reserves.

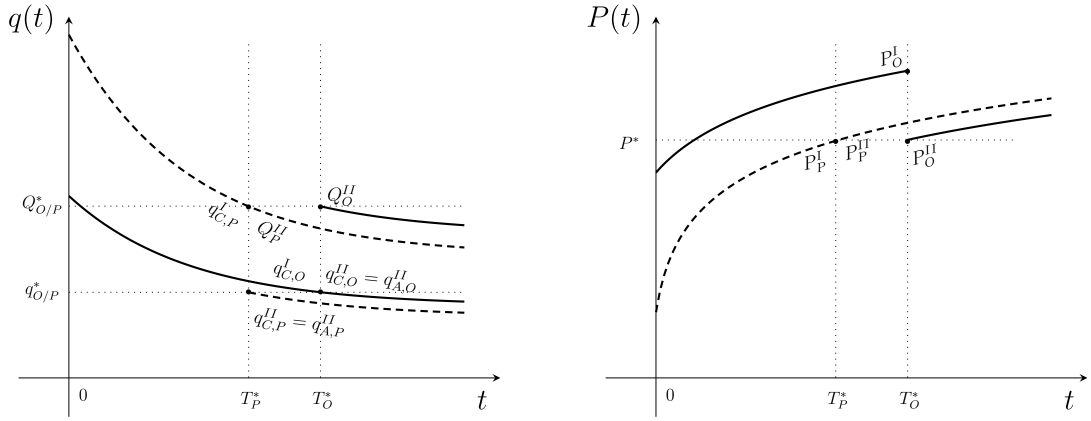
The following conclusion is straightforward by comparing the results of Proposition 3 and 5.

Corollary 2. $T_O^* > T_P^*$.

The finding in Corollary 2 is attributable to the fact that the monopolist extracts its REEs faster if the price, instead of the supply, is continuous, that is, $q_{C,O}^I(t) < q_{C,P}^I(t), \forall t \leq T_P^*$. At first sight, it may indeed seem peculiar that country C's cautious intention at the outset of scenario P, that is, to avoid instantaneous profit losses by committing to a supply decrease at the moment of A's entry, leads to a less cautious first-period extraction. The reasons for this seemingly surprising result are as follows: whenever C's supply commitment is continuous, the country keeps its offer at a conservative level to ensure that the supply growth at the entry does not cause a collapse of the market price; otherwise, if C's commitment includes discontinuity in supply, the country can more easily balance out market fluctuations and hence adopts a more adventurous monopolistic behavior. In addition, as a consequence of the different supply paths in the first period, the monopolistic market price of the continuous price scenario P is below that of the continuous supply scenario O, that is, $P_O^I(t) > P_P^I(t),\ \forall t \leq T_P^*$. This is illustrated in Figure 3.1 below.

Nevertheless, when comparing the second-period outcomes, we observe a switch in the extraction behavior: as from the start of the competition, the originally more conservative supply of O dominates the supply of P, that is, $q_{i,O}^{II}(t) > q_{i,P}^{II}(t),\ \forall t \geq T_O^*$ (see Figure 3.1). This finding can partly be explained by the scenario's contrasting entry reactions: while the continuous supply assumption of O compels the extraction to remain steady at the entry, the continuous price assumption of P forces the exploitation to drop, which emerges to let the supply of P fall below the one of O. The explanation is completed by the fact that the countries' duopolistic extraction attitudes are, apart from the entry timing, comparable (see Proposition 1). More precisely, as this yields similar supply-decrease behaviors, their extraction curves are prevented from crossing, that is, the supply of P remains below the one of O throughout the differential game.

Figure 3.1 – Supply and price differences: O, P



3.3.4 Financial effects of the incumbent's reaction to a newcomer's market entry

In this subsection, we extend the above analysis by focusing on the revenue of both market participants. In fact, we study how country C's response to A's market entry affects the payoffs and determine which reaction is most profitable. For this purpose, the present section starts by determining the countries' first- and second-period revenues in both scenarios.

In the first period, the profit of country C is given by

$$\pi^I_{C,\theta} = \int_0^{T_{\theta *}} \mathrm{e}^{-rt} P^I_\theta(t)\; q^I_{C,\theta}(t)\,\mathrm{d}t,$$

where $\theta = \{O, P\}$. From the findings of Propositions 2 and 3, as well as Corollary 1, it follows that whenever C decides to keep its supply continuous at the entry, its revenue is

$$\begin{aligned}\pi^I_{C,O} &= a\left(\frac{r}{1-\alpha}\right)^{\alpha-1}(R^*_C)\alpha\left(\mathrm{e}^{\frac{\alpha r T^*_O}{1-\alpha}} - \mathrm{e}^{-rT^*_O}\right)\\ &= a\left(\frac{r}{1-\alpha}R_C(0)\right)^{\alpha-1}(R_C(0)-R_A(0))\,.\end{aligned} \tag{3.24}$$

In contrast, when the market entry causes a drop in C's supply, its revenue is

$$\begin{aligned}\pi^I_{C,P} &= a\left(\frac{r}{1-\alpha}\right)^{\alpha-1}(R_A(0)+R^*_C)\alpha\left(\mathrm{e}^{\frac{\alpha r T^*_P}{1-\alpha}}-\mathrm{e}^{-rT^*_P}\right)\\ &= a\left(\frac{r}{1-\alpha}(R_C(0)+R_A(0))\right)^{\alpha-1}(R_C(0)-R_A(0)).\end{aligned} \tag{3.25}$$

Since C is the only supplier in the first period, country A's revenue is $\pi^I_{A,\theta}=0$.

The outcomes of Equations (3.24) and (3.25) yield that the monopolistic revenue of scenario O beats the one of scenario P in terms of profitability: $\pi^I_{C,O} > \pi^I_{C,P}$. Hence, while the positive relation between supply and profit results in higher instantaneous revenues in scenario P when the two periods coexist, that is, $\pi^I_{C,P}(t) > \pi^I_{C,O}(t),\ \forall t \leq\ T^*_P$, they do not make off for the additional profits that scenario O generates over (T^*_P, T^*_O). From a first-period perspective, a more cautious monopolistic extraction behavior should thus be envisaged by country C.

Regarding the second period, the profit of country i is given by

$$\pi^{II}_{i,\theta} = \int_{T_{\theta^*}}^{+\infty} \mathrm{e}^{-rt} P^{II}_\theta(t)\ q^{II}_{i,\theta}(t)\,\mathrm{d}t,$$

where $i \in \{A, C\}$. Based on the findings of Proposition 1 and Corollary 1, it follows that under scenario O, both countries' revenue is

$$\pi^{II}_{A,O} = \pi^{II}_{C,O} = a\,R_A(0)\left(\frac{r}{1-\alpha}2R_C(0)\right)^{\alpha-1}, \tag{3.26}$$

whereas under scenario P, it is

$$\pi^{II}_{A,P} = \pi^{II}_{C,P} = a\,R_A(0)\left(\frac{r}{1-\alpha}\left(R_C(0)+R_A(0)\right)\right)^{\alpha-1}. \tag{3.27}$$

This time, the second-period revenue of Equation (3.26), discounted to the present value, is below that of Equation (3.27): $\pi^{II}_{i,O} < \pi^{II}_{i,P}$. Similarly to our observation in the first period, the higher instantaneous profits of scenario O over $[T^*_O, +\infty)$ do not balance out the extra revenue that scenario P generates over $[T^*_P, T^*_O)$. In other words, country A will be better off if the monopolistic supplier exploits its reserves rapidly, so that its entry is shifted further forward and the competition is characterized by a restrained extraction behavior.

For country C, however, given that the entry-reactions have opposite effects on its first- and second-period revenues, its optimal strategy decision can only be determined through the aggregated revenue

$$\Pi_{C,\theta} = \pi^{I}_{C,\theta} + \pi^{II}_{C,\theta}. \tag{3.28}$$

Here, substituting the revenues of both periods into Equation (3.28) yields that C's aggregated revenue is greater in scenario O: $\Pi_{C,O} > \Pi_{C,P}$. Accordingly, even if C's supply commitment would allow for some flexibility at the entry of country A, the country has no financial incentive to adjust the supply at the entry T_{θ^*}. Rather, it should postpone A's entry by minimizing the monopolistic market supply, followed by a continuous extraction at the entry and a modest duopolistic supply path. The subsequent proposition summarized the latter results.

Proposition 6. *Suppose that A's optimal entry time is T_{θ^*}, where $\theta \in \{O, P\}$. Then*

(1) $\pi^{I}_{C,O} > \pi^{I}_{C,P}$;

(2) $\pi^{II}_{A,\theta} = \pi^{II}_{C,\theta}$;

(3) $\pi^{II}_{i,O} < \pi^{II}_{i,P},\ i = \{A, C\}$;

(4) $\Pi_{C,O} > \Pi_{C,P},\ \Pi_{A,O} < \Pi_{A,P},\ \textit{with}\ \Pi_{A,\theta} = \pi^{II}_{A,\theta}$.

As the non-cooperative game turns out to be sub-optimal for country A, the coming section analyzes whether or not cooperation can lead to a Pareto improvement.

3.4 Cooperative game

This section introduces a continuous-time dynamic model where the players can cooperate and coordinate their strategies. In addition, we present our findings and compare them with the competitive ones in Section 3.3.

3.4.1 The model and its results

Consider a model[5] where countries A and C start to cooperate right at the beginning of the game and suppose that their cooperation lasts forever. In this case, the time in

[5]We keep the notations of Section 3.3 and the subscript J denotes the cooperative scenario.

the joint period $t \in [0, +\infty)$, where at time 0 both countries collectively and definitively commit themselves to a joint supply strategy $q_J(t)$ that maximizes their joint revenue RV_J. Furthermore, given that no differences in the exploitation or manufacturing processes of the two countries are considered in the current model, their joint revenue RV_J is split based on the share of their initial reserves $R_i(0)$.[6] Apart from that, the price function $P_J(t)$ of this game is identical to that in the non-cooperative game (see Equation (3.1)). The cooperative suppliers' joint revenue is thus given by

$$RV_J = \max_{q_J(t)} \int_0^{+\infty} \mathrm{e}^{-rt} P_J(q_J(t))\, q_J(t)\, \mathrm{d}t,$$

subject to

$$\int_0^{+\infty} q_J(t)\, \mathrm{d}t = R_C(0) + R_A(0) = R_J(0) \text{ given}, \quad q_J(t) \geq 0,$$

and

$$\dot{R}^J(t) = -q_J(t), \quad t \in [0, +\infty).$$

The standard process of solving optimal control problems reveals that the shadow price is

$$\lambda_J(t) = \lambda_J(0)\mathrm{e}^{rt}, \tag{3.29}$$

with $\lambda_J(0)$ being determined by the transversality condition, and that the marginal revenue is

$$\frac{\mathrm{d}RV_J(t)}{\mathrm{d}q_J(t)} = a\alpha \left(q^J(t)\right)^{\alpha-1} = \alpha P_J(t) = \lambda_J(t). \tag{3.30}$$

Moreover, substituting (3.29) into (3.30) and rearranging terms leads to the joint supply function:

$$q_J(t) = \left(\frac{\lambda_J(0)\mathrm{e}^{rt}}{a\alpha}\right)^{\frac{1}{\alpha-1}}. \tag{3.31}$$

Again, if we consider that the joint reserve is completely exploited over $[0, +\infty)$, integrating the dynamic equation $\dot{R}^J(t) = -q_J(t)$ over $[0, +\infty)$ gives us the shadow price at the beginning of the game:

$$\lambda_J(0) = a\alpha \left(\frac{r}{1-\alpha}(R_C(0) + R_A(0))\right)^{\alpha-1}. \tag{3.32}$$

[6]Note that this is equivalent to assuming that their joint supply q_j is split based on their initial reserves $R_i(0)$.

From here it is enough to substitute (3.32) into (3.31) to find the optimal rare earths extraction and market price in the cooperative game. The results are presented in the ensuing proposition.

Proposition 7. *Suppose that countries A and C start their cooperation at the beginning of the game, and assume that it lasts forever. Then the optimal joint supply is given by*

$$q_J(t) = \frac{r}{1-\alpha}(R_C(0) + R_A(0))\, \mathrm{e}^{\frac{rt}{\alpha-1}},$$

and the market price is

$$P_J(t) = a\left(\frac{r}{1-\alpha}(R_C(0) + R_A(0))\right)^{\alpha-1} \mathrm{e}^{rt}.$$

The results of the last proposition allow us to determine the joint revenue:

$$RV_J = a\left(\frac{r}{1-\alpha}\right)^{\alpha-1} (R_C(0) + R_A(0))\alpha. \tag{3.33}$$

Based on the competitors' initial stocks, we get from Equation (3.33) that the revenue of country i is

$$RV_{i,J} = \frac{R_i(0)}{R_A(0) + R_C(0)} RV_J = a\ R_i(0)\left(\frac{r}{1-\alpha}(R_C(0) + R_A(0))\right)^{\alpha-1}, \tag{3.34}$$

where $i = \{A, C\}$. In order to better define the particularities of the cooperative behavior, the following subsection compares the above outcomes with the ones of Subsection 3.3.4.

3.4.2 Competitive and cooperative comparison

The crucial difference between the games in Sections 3.3 and 3.4 is that, unlike under competition, where A's extraction is withheld until $t = T^* > 0$, the cooperation agreement allows for A's supply to be triggered immediately at $t = 0$. Although this supply is proportional to the countries' initial reserves and hence higher for C than for A, the deal implies that over the first period $I = (0, T_{\theta^*})$, where $\theta \in \{O, P\}$, A's cooperative resource extraction is greater than the one under competition:

$$q_{A,J}(t) = \frac{R_A(0)}{R_C(0) + R_A(0)}\, q_J(t) > q^I_{A,\theta}(t) = 0,\ \forall t < T_{\theta^*}. \tag{3.35}$$

Yet, when turning to the second period $II = [T_{\theta^*}, +\infty)$, the situation changes. In fact, due to A's entry eagerness that emerges from the initial supply reluctance under competition, when finally cranking up its rare earths extraction processes at $T^*_{\theta^*}$, the market supply emerges to be greater than the one under cooperation:

$$q_{A,J}(t) = \frac{R_A(0)}{R_C(0) + R_A(0)}\, q_J(t) < q^{II}_{A,\theta}(t),\ \forall t \geq T_{\theta^*}. \tag{3.36}$$

On the contrary, country C supplies the market with REEs as from $t = 0$ in both games and the relation between its non-cooperative and cooperative supply emerges to depend on the scenario under evaluation. Therefore the remainder of this subsection consists of a consecutive comparison between the results of scenario J and the ones of O and P.

Comparison 1: Scenarios J and O

Despite the framework discrepancies, country C's supply commitment is identical in scenarios O and J:

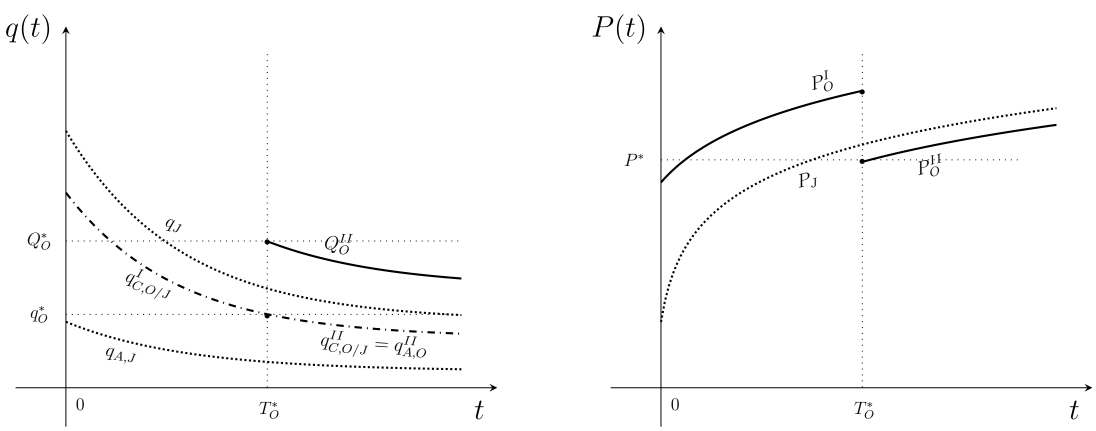
$$q_{C,J}(t) = \frac{R_C(0)}{R_C(0) + R_A(0)}\, q_J(t) = q_{C,O}(t),\ \forall t \in (0, +\infty). \tag{3.37}$$

This similarity can be attributed to the fact that in O, country C does actually not behave particularly competitively. Indeed, C's monopolistic and duopolistic supply follows the same functional form in scenario O (see Equations (3.9) and (3.18)) and remains unchanged at A's sudden entry. In other words, country C does not behave any differently when loosing its monopolistic position and having to truly compete with A.

Regarding the first period, Equations (3.35) and (3.37) indicate that A's immediate supply launch in J provokes a higher total supply, that is, $q_J(t) > Q^I_O(t),\ \forall t < T^*_O$, such that, the market price becomes more consumer-friendly: $P_J(t) < P^I_O(t),\ \forall t < T^*_O$. The latter observation emerges because, unlike in J, scenario O is still characterized by a monopolistic market structure, which typically turns out to be sub-optimal for consumers. On the one hand, country C's equal supply behaviors in (3.37) and the higher competitive market price in O, thus start off provoking greater instantaneous revenues: $RV_{C,J}(t) < \pi^{I}_{C,O}(t), \ \forall t < T^*_O$. On the other hand, country A's first-period supply absence in O generates zero revenues, which implies that the cooperation agreement proves itself advantageous: $RV_{A,J}(t) > \pi^{I}_{A,O}(t), \ \forall t < T^*_O$.

Concerning the second period, Equations (3.36) and (3.37) show that A's more offensive exploitation behavior in O results in a higher total supply, that is, $q_J(t) < Q^{II}_O(t), \ \forall t \geq T^*_O$, and hence in a lower market price: $P_J(t) > P^{II}_O(t), \ \forall t \geq T^*_O$. This confirms the intuition that when both countries supply the market with REEs, the competition should produce more consumer-friendly outcomes. In view of country C's supply in (3.37) and the previous market price it becomes straightforward that, from a second-period viewpoint, C should aim for a cooperative market structure: $RV_{C,J}(t) > \pi^{II}_{C,O}(t), \ \forall t \geq T^*_O$. Furthermore, since the higher supply in (3.36) asserts itself against the lower market price, the exact opposite can be observed for country A: $RV_{A,J}(t) < \pi^{II}_{A,O}(t), \ \forall t \geq T^*_O$. Figure 3.2 illustrates these observations.[7]

Figure 3.2 – Supply and price differences: J, O



Owing to the opposite relation between country i's first- and second-period revenues, the analysis is completed by comparing their aggregated revenues of Equations (3.28) and (3.34). The subsequent proposition concludes.

Proposition 8. *Suppose countries C and A cooperate and supply the market as of $t = 0$. Then, compared to the aggregated revenues under competition, where at $t = T^*_O > 0$, the supply of A begins and the one of C remains continuous, the total cooperative revenues of A and C are,*

[7] The presented supply discrepancies correspond to a situation where $\frac{R_A(0)}{R_A(0)+R_C(0)} = \frac{1}{4}$.

respectively,

$$RV_{A,J} > \Pi_{A,O},$$

and

$$RV_{C,J} < \Pi_{C,O}.$$

In light of these findings, we can conclude that, in comparison with the competitive game O, cooperation does not lead to a Pareto improvement: while country A is better off in the cooperative game, the opposite holds true for country C. Consequently, C is advised against sharing the market with A under a cooperative supply agreement that starts at the outset.

To evaluate if the previous findings vary when changing the non-cooperative game, the outcomes of scenario J are now compared with those of P.

Comparison 2: Scenarios J and P

As opposed to scenario O, country C's monopolistic extraction in P is found to be greater than the cooperative one: $q_{C,J}(t) < q^{I}_{C,P}(t),\ \forall t < T^{*}_{P}$. Nevertheless, due to A's immediate entry in scenario J, the total supply emerges to be identical under non-cooperative and cooperative interactions, that is, $q_J(t) = Q_P(t),\ \forall t \in (0, +\infty)$, so that the same applies for the market price:

$$P_J(t) = P_P(t),\ \forall t \in\ (0, +\infty). \tag{3.38}$$

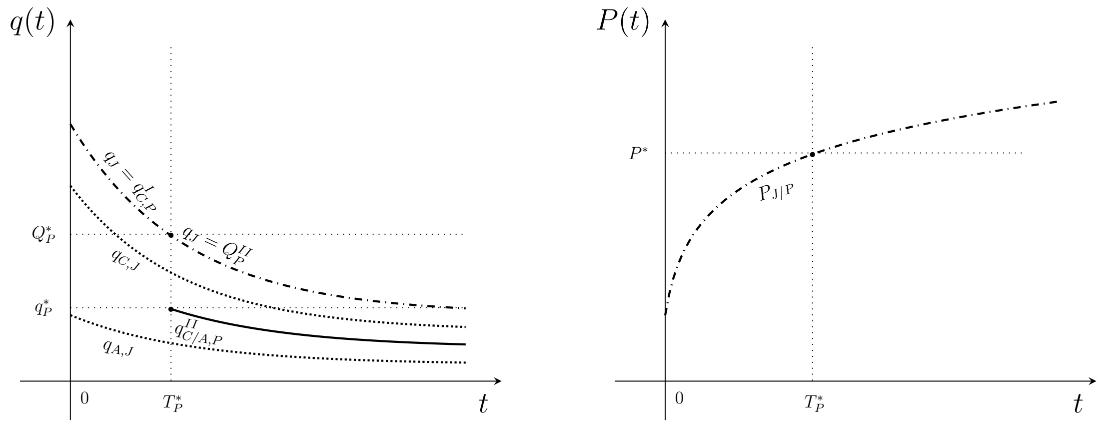
The latter observation originates from the fact that C's supply decrease at the entry intends to prevent the market price from dropping when the actual competition begins, that is, from becoming more consumer-friendly at the entry.

By implication, while the first period is, from country C's standpoint, characterized by higher instantaneous payoffs in P than in J, that is, $RV_{C,J}(t) < \pi^{I}_{C,P}(t),\ \forall t < T^{*}_{P}$, the inverse holds true for country A; notably because of its lacking monopolistic supply in P: $RV_{A,J}(t) > \pi^{I}_{A,P}(t),\ \forall t < T^{*}_{P}$.

In the second period, the latter outcomes are again reversed. Actually, C's supply drop at A's entry in P results in a lower extraction than in J: $q_{C,J}(t) > q^{II}_{C,P}(t),\ \forall t \geq T^{*}_{P}$. However, similarly to the discovery in O, country A's initial production abstinence in P implies that its supply dominates the one in J as from the market entrance: $q_{A,J}(t) <\ q^{II}_{A,P}(t),\ \forall t \geq\ T^{*}_{P}$.

From the uniform market prices in Equation (3.38) it thus follows that the supply discrepancies yield greater instantaneous revenues under cooperation for country C, that is, $RV_{C,J}(t) > \pi^{II}_{C,P}(t),\ \forall t \geq T^*_P$, and lower ones for country A: $RV_{A,J}(t) < \pi^{II}_{A,P}(t),\ \forall t \geq T^*_P$. These findings are presented in Figure 3.3.[8]

Figure 3.3 – Supply and price differences: J, P



Given that we, once more, find polar relations between country i's first- and second-period revenues, only the aggregated revenues of Equations (3.28) and (3.34) allow to conclude the present study. The next proposition presents our findings.

Proposition 9. *Suppose countries C and A cooperate and supply the market as of $t = 0$. Then, compared to the aggregated revenues under competition, where at $t = T^*_O > 0$, the supply of A begins and the one of C drops to keep the market price continuous, the total cooperative revenues of A and C are, respectively,*

$$RV_{A,J} = \Pi_{A,P},$$

and

$$RV_{C,J} = \Pi_{C,P}.$$

Proposition 9 hence states that country C's [A's] cooperative losses [gains] before the entry are exactly outbalanced by the cooperative gains [losses] after the entry. In other words, although the countries are not worse off when cooperating, they are also not better off, that is, no Pareto improvement occurs.

In closing, it is worth mentioning that the set-up of the above model can be changed such that the cooperation starts at A's entry time T_{θ^*}. Similarly to the previous setting, the coun-

[8]The different extraction rates correspond to a situation where $\frac{R_A(0)}{R_A(0)+R_C(0)} = \frac{1}{4}$.

tries' joint supply q_{J^*} is split based on the share of their initial reserves $R_i(T_{\theta^*})$. However, with the countries' rare earths reserves being equal at T_{θ^*}, that is, $R_A(0) = R_C(T_{\theta^*})$ (see Corollary 1), they meet at eye level and have no reason to behave any differently when cooperating or competing. Consequently, both of their aggregated revenues are found to be equal: $RV_{i,J^*} = \Pi_{i,\theta}$.

3.5 Conclusion

The present chapter studies both the non-cooperative and cooperative games, played by two countries, epitomized by China and the U.S., that wish to optimally position themselves in the rare earths supply market. The first goal of the study consists in determining the ideal time for a potential new supplier, the U.S., to trigger its national rare earths extraction in a competitive environment. For this purpose, we set up a non-cooperative continuous-time model with two periods and search for open-loop strategies. Using backward induction, we start by searching for Nash equilibrium strategies in the second period, where, at the outset, the U.S. enters the rivalry and each country commits itself irreversibly to an extraction path that leads to the best possible payoff—taking into account the other country's strategy. Thereafter, we pursue our second objective, which is to define China's optimal supply commitment in the first period, where it still holds a monopolistic position. Here China's competitive extraction path is specified under two assumptions: (1) its supply volume is adjustable at the moment of the U.S.'s market entry; and (2) its supply inflexibility does not allow for instantaneous volume changes. Subsequently, the countries' first-best supply behaviors are identified by means of their aggregated revenues. In third place, we then analyze whether or not a Pareto improvement occurs when both countries decide to cooperate as from the moment of the U.S.'s entry announcement. To this end, a cooperative continuous-time model is set up, where, at the outset, the countries put their reserves together and jointly choose an optimal extraction path that maximizes their joint revenue. Later on, the countries' non-cooperative and cooperative revenues are compared to draw conclusions.

Our findings firstly show that the U.S.'s optimal entry strategy is to hold back with the competitive production launch until its reserves coincide with those of China. By implication, China determines the entry timing through the speed of its monopolistic resource extraction: the faster [slower] its exploitation, the earlier [later] its stock is reduced to the level of the U.S., where the entry is triggered. Secondly, China's first-period extraction increases with the adaptability of its production processes: discontinuous supply-commitment cause the competitor to act more adventurously. Our results also show that while the U.S. would benefit from an early entry, the opposite holds true for China as its aggregated revenues positively depend on the length of the monopolistic period. This suggests that under competition, China should stick to a conservative monopolistic extraction and keep its supply both unchanged at the entry and modest over the second period. Thirdly, when evaluating the optimal strategies of the cooperative game, we find that a joint extraction does not lead to a Pareto improvement. In fact, although the cooperation deal with China proves itself financially interesting for the U.S., the former will be worst off and is hence still advised to prevent the U.S. from entering at an early stage.

Future research should focus on reformulating the countries' strategies to Markovian ones. Instead of assuming that the countries commit forever to an extraction path at the beginning of the game, this would allow them to modify their strategies with respect to time and the current value of rare earths reserves. Furthermore, a Stackelberg type leader-follower differential game, would be an interesting extension of the current study. Here the countries would no more act concurrently by guessing each other's supply path but the follower would determine its supply strategy after knowing the one of the leader, who in turn anticipated the follower's response for its own strategy. Last but not least, one could change the model's demand function and empirically determine its parameters. Comparing these results with those of the present chapter then allows to asses, theoretically and numerically, in how far different assumptions about the countries' supply behavior, the decision-making process and the market demand will affect the above conclusions.

Appendix: Proof of Corollary 1

The second-period optimal control problem for both countries is

$$\pi_i^{II}(R_C^*, T^*) = \max_{q_i^{II}(t)} \int_{T^*}^{+\infty} \mathrm{e}^{-rt} P^{II}(Q^{II}(t))\, q_i^{II}(t)\, \mathrm{d}t,$$

subject to

$$\int_{T^*}^{+\infty} q_i^{II}(t)\, \mathrm{d}t \leq R_i(T^*) = \begin{cases} R_C^*, \text{ for } i = C \\ R_A(0) \text{ given, for } i = A \end{cases}, \quad q_i^{II}(t) \geq 0,$$

and

$$\dot{R}_i(t) = -q_i^{II}(t), \quad t \in \ [T^*, +\infty).$$

Since the first constraint of the optimization problem is equivalent to

$$\int_{T^*}^{t} q_i^{II}(\tau)\, \mathrm{d}\tau \leq R_i(T^*)\,, \ t \in [T^*, +\infty),$$

the Lagrangian is set up as follows:

$$\mathcal{L}_i^{II}\left(q_i^{II}(t), \lambda_i^{II}(t), \alpha_i^{II}\right) = P^{II}(Q^{II}(t)) q_i^{II}(t) - \lambda_i^{II}(t) q_i^{II}(t) - \alpha_i^{II} \left(R_i(T^*) - \int_{T^*}^{t} q_i^{II}(\tau)\, \mathrm{d}\tau\right),$$

where $\lambda_i^{II}(t)$ is the shadow price and α_i^{II} is the static Lagrange multiplier. The standard first-order conditions (FOCs) are

$$\begin{cases} \dot{\lambda}_i^{II}(t) = r\, \lambda_i^{II}(t), \\ \frac{\partial}{\partial q_i^{II}(t)} \mathcal{L}_i^{II}\left(q_i^{II}(t), \lambda_i^{II}(t), \alpha_i^{II}\right) = \frac{\partial}{\partial q_i^{II}(t)} RV_i^{II}(t) - \lambda_i^{II}(t) \ + \ \alpha_i^{II} \frac{\partial}{\partial q_i^{II}(t)} \int_{T^*}^{t} q_i^{II}(\tau)\, \mathrm{d}\tau = 0, \\ \alpha_i^{II} \geq 0, \ \ R_i(T^*) - \int_{T^*}^{t} q_i^{II}(\tau)\, \mathrm{d}\tau \geq 0, \ \ \alpha_i^{II} \left(R_i(T^*) - \int_{T^*}^{t} q_i^{II}(\tau)\, \mathrm{d}\tau\right) = 0, \end{cases}$$

where the revenue of country $i \in \{A, C\}$ is $RV_i^{II}(t) = P^{II}(t)\, q_i^{II}(t)$. Based on the remarks in Subsection 3.3.1, it is not optimal for i to exhaust its resources in finite time, thus $R_i(T^*) > \int_{T^*}^{t} q_i^{II}(\tau)\, \mathrm{d}\tau$, and hence $\alpha_i^{II} = 0$.

The first FOC yields that i's shadow price $\lambda_i^{II}(t)$ of its remaining reserve $R_i(t)$ grows at interest rate r:

$$\lambda_i^{II}(t) = \lambda_i^{II}(T^*)\mathrm{e}^{r(t-T^*)}, \tag{3.39}$$

where the initial condition $\lambda_i^{II}(T^*)$ comes from the transversality condition.[9] The second FOC and the fact that $\alpha_i^{II} = 0$ show that the shadow price does actually correspond to:

$$\frac{\partial RV_i^{II}(t)}{\partial q_i^{II}(t)} = \lambda_i^{II}(t). \tag{3.40}$$

Furthermore, since the revenue of country i is

$$RV_i^{II}(t) = P^{II}(t)q_i^{II}(t) = a\left(Q^{II}(t)\right)^{\alpha-1} q_i^{II}(t) = a\left(q_A^{II}(t) + q_C^{II}(t)\right)^{\alpha-1} q_i^{II}(t),$$

the partial derivative of its revenue function with respect to its supply path is

$$\frac{\partial RV_i^{II}(t)}{\partial q_i^{II}(t)} = a\left(Q^{II}(t)\right)^{\alpha-1}\left(1 - \frac{(1-\alpha)q_i^{II}(t)}{q_A^{II}(t) + q_C^{II}(t)}\right) = \lambda_i^{II}(t), \tag{3.41}$$

which yields

$$\frac{1 - \frac{(1-\alpha)q_i^{II}(t)}{q_A^{II}(t)+q_C^{II}(t)}}{1 - \frac{(1-\alpha)q_j^{II}(t)}{q_A^{II}(t)+q_C^{II}(t)}} = \frac{\lambda_i^{II}(T^*)}{\lambda_j^{II}(T^*)} = \frac{\lambda_i^{II}(t)}{\lambda_j^{II}(t)}, \tag{3.42}$$

where $i, j \in \{A, C\}$ and $i \neq j$. After rearranging Equation (3.42), we find

$$q_A^{II}(t) = \frac{\lambda_C^{II}(T^*) - \alpha\lambda_A^{II}(T^*)}{\lambda_A^{II}(T^*) - \alpha\lambda_C^{II}(T^*)}\, q_C^{II}(t). \tag{3.43}$$

When integrating Equation (3.43) over $[T^*, +\infty)$ and by assuming that over $[T^*, +\infty)$ the total REEs reserve is exhausted, as from an economical viewpoint it is not optimal to leave some elements in the deposit (no market value), we get

$$\frac{R_A(0)}{R_C(T^*)} = \frac{\lambda_C^{II}(T^*) - \alpha\lambda_A^{II}(T^*)}{\lambda_A^{II}(T^*) - \alpha\lambda_C^{II}(T^*)}, \tag{3.44}$$

that is,

$$\lambda_A^{II}(T^*) = \frac{R_C^* + \alpha R_A(0)}{R_A(0) + \alpha R_C^*}\, \lambda_C^{II}(T^*).$$

[9]Note that all second-order sufficient conditions are satisfied as well.

Combining (3.43) and (3.44) yields

$$\frac{q_A^{II}(t)}{q_C^{II}(t)} = \frac{R_A(0)}{R_C^*}, \ \forall t \geq T^*. \tag{3.45}$$

In view of Equation (3.45), the price function in (3.1) is

$$P^{II}(t) = a\left(q_A^{II}(t) + q_C^{II}(t)\right)^{\alpha-1} = a\left(\frac{R_A(0)}{R_C^*} + 1\right)^{\alpha-1}\left(q_C^{II}(t)\right)^{\alpha-1} \tag{3.46}$$

$$\left[P^{II}(t) = a\left(1 + \frac{R_C^*}{R_A(0)}\right)^{\alpha-1}\left(q_A^{II}(t)\right)^{\alpha-1}\right],$$

and the revenue of country C $[A]$ is

$$RV_C^{II}(t) = P^{II}(t)q_C^{II}(t) = a\left(\frac{R_A(0)}{R_C^*} + 1\right)^{\alpha-1}(q_C^{II}(t))^{\alpha} \tag{3.47}$$

$$\left[RV_A^{II}(t) = P^{II}(t)q_A^{II}(t) = a\left(1 + \frac{R_C^*}{R_A(0)}\right)^{\alpha-1}(q_A^{II}(t))^{\alpha}\right].$$

Consequently, country i's marginal revenue thus, on the one hand, corresponds to:

$$\frac{\mathrm{d}RV_i^{II}(t)}{\mathrm{d}q_i^{II}(t)} = \alpha P^{II}(t). \tag{3.48}$$

On the other hand, the findings of (3.41) and (3.45) imply that the derivative of country C's revenue function with respect to its supply path leads to:

$$\frac{\mathrm{d}RV_C^{II}(t)}{\mathrm{d}q_C^{II}(t)} = \lambda_C^{II}(t) + q_C^{II}(t)\, a(\alpha-1)\left(q_A^{II} + q_C^{II}(t)\right)^{\alpha-2}\frac{R_A(0)}{R_C^*}. \tag{3.49}$$

$$\left[\text{resp.} \quad \frac{\mathrm{d}RV_A^{II}(t)}{\mathrm{d}q_A^{II}(t)} = \lambda_A^{II}(t) + q_A^{II}(t)\, a(\alpha-1)\left(q_A^{II} + q_C^{II}(t)\right)^{\alpha-2}\frac{R_C^*}{R_A(0)}\right].$$

Combining (3.48) and (3.49) yields

$$\lambda_C^{II}(t) = \frac{q_A^{II}(t) + \alpha q_C^{II}(t)}{q_A^{II}(t) + q_C^{II}(t)}\, P^{II}(t),$$

and based on Equation (3.45), this is

$$\lambda_C^{II}(t) = \frac{R_A(0) + \alpha R_C^*}{R_A(0) + R_C^*} \, P^{II}(t) \tag{3.50}$$

$$\left[\lambda_A^{II}(t) = \frac{R_C^* + \alpha R_A(0)}{R_A(0) + R_C^*} \, P^{II}(t)\right].$$

When substituting (3.46) into (3.50), we obtain

$$q_C^{II}(t) = \frac{R_C^*}{R_A(0) + R_C^*} \left(\frac{R_A(0) + R_C^*}{R_A(0) + \alpha R_C^*}\right) \left(\frac{\lambda_C^{II}(T^*)}{a}\right)^{\frac{1}{\alpha-1}} \mathrm{e}^{\frac{r(t-T^*)}{\alpha-1}}. \tag{3.51}$$

Integrating Equation (3.51) over $[T^*, +\infty)$ yields

$$\lambda_C^{II}(T^*) = a \left(\frac{R_A(0) + \alpha R_C^*}{R_A(0) + R_C^*}\right) \left(\frac{r}{1-\alpha} \left(R_A(0) + R_C^*\right)\right)^{\alpha-1}. \tag{3.52}$$

After substituting (3.52) into (3.51), we find the extraction rate of country C:

$$q_C^{II}(t) = \frac{r}{1-\alpha} \, R_C^* \, \mathrm{e}^{\frac{r(t-T^*)}{\alpha-1}}, \tag{3.53}$$

which, combined with Equation (3.46), yields the duopoly market price of REEs:

$$P^{II}(t) = a \left(\frac{r}{1-\alpha} \left(R_A(0) + R_C^*\right)\right)^{\alpha-1} \mathrm{e}^{r(t-T^*)}. \tag{3.54}$$

Then, by substituting (3.53) into (3.45), we get the extraction rate of country A:

$$q_A^{II}(t) = \frac{R_A(0)}{R_C^*} \, q_C^{II}(t) = \frac{r}{1-\alpha} \, R_A(0) \, \mathrm{e}^{\frac{r(t-T^*)}{\alpha-1}}. \tag{3.55}$$

This finishes the proof. The results are presented in Proposition 1 of Subsection 3.3.2.

Chapter 4

Dynamic Rare Earth Elements Game under Markovian Feedback Strategies

4.1 Introduction

In their seminal contribution, both Reinganum and Stokey (1985) and Dockner et al. (2000) call attention to the choice of strategy space when modeling dynamic games as they potentially stipulate equilibrium outcomes. In this field, strategic behaviors are generally represented by: (1) open-loop strategies, where each player irreversibly commits itself to a prospective path of actions right at the outset of the game; and (2) Markovian strategies, where each player, instead of committing, continuously customizes its current rule of actions to the observed value of the state variables.

Hitherto, the economic literature on non-renewable resources games, however, almost exclusively focuses on analyzing open-loop strategies. The reasons for this are twofold: one is linked to the supply side and the other one to the demand side of the market. In fact, since the utilities' supply levels are from a technical viewpoint limited by the disposable extraction and distribution capacities, quick supply enlargements are knotty, if not impossible, to realize once manufacturing plants and associated infrastructure have been installed. Additionally, the probable absence of arbitrage opportunities in competitive resource markets (Shahidehpour et al., 2003), that is, the lack of situations where utilities can capitalize on price discrepancies between markets or time periods, makes deliberate storage of extracted resources unprofitable. Under stable market conditions, utilities are therefore counseled to fully tap their practical supply potential, which, in light of the abovedescribed sluggish growth potential, comes close to a delivery commitment. The other crucial argument in favor of open-loop commitments is that most exhaustible resource suppliers have long-term contracts with the demand side in order to avoid bottlenecks in the buyers' production processes (Liski and Montero, 2014).

Although the latter supply-demand relationship makes open-loop commitments an appropriate strategy choice for limited resource markets, Clemhout and Wan (1991) and Dockner and Sorger (1996) consider the Markovian supply dependency on current stocks a more realistic representation of extractive firms' behavior. Beyond that, due to the subgame perfectness that characterizes Markovian equilibria by construction, Markovian decision rules are regarded as more robust than open-loop paths (Dockner et al., 2000; Van Long et al., 1999). For instance, open-loop Nash equilibria state that each country's selected supply path is the optimal feasible response to the other country's supply path—as long as the latter does not deviate from its initial strategy, that is, strategies are optimal only when looking from the initial date and/or state of the game. In contrast to this, Markovian Nash equilibria guarantee optimal supply rules also if the competitor deviates from its initial strategy, that is, strategies are optimal when viewed from any date/state of the game.

Nonetheless, Markovian behaviors find extremely little application in non-renewable resource models; probably by dint of their typical intractability (Dockner et al., 2000). Indeed, besides Eswaran and Lewis (1985), who provide explicit value functions of both an open-loop and a Markovian problem with zero extraction costs, Salo and Tahvonen (2001) are, to our best knowledge, the sole to analytically offer Nash equilibrium Markovian strategies for an oligopolistic market. Thereafter, Groot et al. (2003) derive a feedback Stackelberg equilibrium of a cartel-fringe problem, where the fringe acts as price taker, and fix the time-inconsistency issue of the open-loop Stackelberg equilibrium of Gilbert (1978); discussed in Newbery (1981). More recently, under a similar cartel versus price-taking fringe setting as Salant (1976), Benchekroun and Withagen (2012) explicitly state Nash equilibrium Markovian strategies, which turn out to coincide with Salant's open-loop Nash equilibria.

Against this backdrop, the present work first and foremost aspires to extend the above literature by investigating the Markovian-strategic orientations in the rare earth elements (REEs) market. Note that this intend makes the study complementary to our earlier work in Chapter 3, where we already introduced the prevailing problems in the REEs supply market. For the readers' convenience, let us, however, recap that although

China successfully defended its quasi-monopolistic position in this market since the 1990s, the U.S. lately threatened to fall back on its much smaller rare earths reserves to again launch national production and become a significant market player. This situation is reproduced in a continuous-time model, comprised of a first monopolistic period that is switched for a second duopolistic period when the potential rival translates its entry threat into action. Yet, in consequence of the REEs market's particularities and the resulting specific framework assumptions, the techniques applied in the existing literature prove themselves of no great benefit to solve our problems for Markovian Nash equilibrium strategies: (1) even though Salo and Tahvonen (2001) solve a duopolistic problem that is similar to ours, their setting considers economic instead of physical resource depletion, that is, exploitation does not continue for an infinite time but stops once marginal extraction costs coincide with the choke price. The issue with this economic-depletion formulation is that it evokes a dependency on the terminal condition, comparable to the Markovian-conflicting initial-condition dependency, which renders the subgame perfectness of the Nash equilibrium questionable. Moreover, the indispensability of REEs for the economic well-being and domestic security of countries worldwide, together with the non-existence of backstop technologies à la Nordhaus, makes the imposition of a choke price or a finite ending point of supply inappropriate for our analysis. As an alternative, we make the application of Markovian feedback strategies possible by considering physical REEs exhaustion over an infinite modeling horizon; (2) the solving approaches of Groot et al. (2003) are inapplicable in our study because, firstly, they search for a Stackelberg equilibrium, that is, their players do not act concurrently as under our Nash equilibrium concept but one player leads by acting first and the other one subsequently follows and, secondly, one of their players is considered to be a price taker, which is incompatible with the fact that both of our players pursue serious market power; (3) analogically to finding the heterogeneous Nash equilibrium in Zou (2016), the model of Benchekroun and Withagen (2012) allows for the Nash equilibrium to be found by utilizing the conjecturing technique, that is, to solve the problem by guessing the fringe's optimal behavior. Nevertheless, despite the guessing approach's frequently deployed in differential games (Van Long, 2011), it is less suited for symmetric games like ours. Instead, what eventually enables us to find closed-form Markovian solutions is guessing the form of the problems' Bellman value function. With the second main priority of this chapter consisting in analyzing possible differences between open-loop and Markovian strategies, we finally compare the Markovian outcomes of the present study with the results of our earlier work in Chapter 3, where we inter alia study the non-cooperative U.S.-China game under open-loop strategies.

From the above comparison it follows that whenever China acts alone in the market and is confronted with both the same initial and terminal conditions, then its behavior is not affected by the way its decision-making process is set up. In opposition to this, if China and the U.S. simultaneously supply the market with REEs, then different strategy spaces generate different outcomes, that is, open-loop and Markovian Nash equilibria show significant disparities. Actually, the more flexible Markovian decision rules initially lead to less conservative extraction attitudes, which in turn yield both consumer-friendly market prices and supplier-friendly instantaneous revenues. During the course of the game, the latter situation is, however, inverted and, based on the competitors' current REEs reserves (U.S. Geological Survey, 2020), open-loop path strategies eventually turn out to be the more lucrative strategy choice; at least from the U.S.'s viewpoint. From China's perspective, the latter only applies if its supply remains unchanged when the U.S. finally joins the REEs market. On the contrary, if China attempts to prevent the market price from falling at the U.S.'s market entry by dropping its supply, then Markovian strategies win and are most probable to define the competitors' actions throughout the two-period game.

The rest of the chapter looks as follows. After a description of the model's specific features, Section 4.2 proceeds by depicting the utilized problem-solving technique and the analytical solutions of the Markovian (Nash equilibrium) strategies. By means of our results in Chapter 3, Section 4.3 compares and discusses the behavioral and financial divergences provoked by the application of alternative strategy spaces. Section 4.4 closes with concluding remarks.

4.2 The model and rare earths extractions

Suppose country C's monopolistic position of the first period $I = [0, T^*)$ to be jeopardized at time $t = 0$ by country A's announcement to enter the rare earths supply market at $t = T^*$. Since C's privately owned initial REEs reserves $R_C(0)$ are much larger than those of its potential competitor, that is, $R_C(0) >> R_A(0)$, the entry timing T^* is crucial for A's success. More precisely, given that the entry triggers the duopolistic supply competition of the second period $II = [T^*, +\infty)$, A's market entrance is presumed to be postponed until some time $T^* \in (0, +\infty)$, where C's reserves $R_C(T^*) = R_C^* > 0$. The existence of T^* shall be proven in the coming sections.

Beyond that, we let the market price be determined by the isoelastic inverse demand function of Stiglitz (1976):

$$P(Q) = a\, Q^{\alpha-1}, \tag{4.1}$$

where $a > 0$, $\alpha \in (0, 1)$ so that price elasticity of demand $\frac{1}{1-\alpha} > 1$ and where the total market supply Q is comprised of A's supply q_A and C's supply q_C, with A's first period supply q_A^I being zero.

Ultimately, extracted REEs are assumed to be entirely supplied to the market and both countries' profit functions are considered free of extraction costs.

Reasoning backwards in time, our analysis begins by defining the second-period Nash equilibrium Markovian decision rules of countries A and C. Following the setting of Benchekroun and Withagen (2012) and taking into account that the implementation of a Markovian behavior can be a tricky task in competitive markets, particularly because resource owners generally strive to hide stock-size information from their rivals (Gerlagh and Liski, 2014), we here confine the typical Markovian stock dependency on the decision-makers own reserves.

Subsequently, after having solved the duopoly problem, we study country C's first-period Markovian and open-loop supply behaviors. The reason for including an open-loop commitment into this period is that C may: (1) not be able to immediately adopt a Markovian strategy as from A's entry-announcement at $t = 0$. Instead, C could first have to negotiate contracts of different time periods with the demand side or have to develop its supply chain management before attaining the required flexibility for the second-period Markovian game; and (2) not be willing to permanently observe the state of its reserves and revise its supply strategy accordingly, as it believes these efforts to not be worthwhile in the absence of competition.

By implication, as mentioned in the introduction, these two strategy spaces result in solutions of different nature: (1) C's open-loop choice variable $q_{C,S}^I(t)$ depends on time t only, where S denotes the first-period open-loop outcomes when the second period is characterized by a Markovian behavior; and (2) C's Markovian choice variable $q_{C,M}^I(R_C, t)$ depends on time t and the observed state of its reserves at that time $R_C = R_C(t)$, where M denotes the first-period outcomes when both periods are characterized by a Markovian behavior.

With this in mind, the next subsection solves the second-period optimization problem in Markovian strategy spaces.

4.2.1 Markovian Nash equilibrium strategies

In the duopolistic period II, where $t \in [T^*, +\infty)$, both competitors' revenue is given by

$$\pi_i^{II}(R_C^*, T^*) = \max_{q_i^{II}(R_i,t)\geq 0} \int_{T^*}^{+\infty} e^{-r(t-T^*)} P^{II}\big(Q^{II}(R_i,t)\big)\, q_i^{II}(R_i,t)\ \mathrm{d}t,$$

subject to the reserve constraint

$$\int_{T^*}^{+\infty} q_i^{II}(R_i,t)\ \mathrm{d}t \leq R_i(T^*) = \begin{cases} R_A(0)\ \text{given, if}\ \ i = A, \\ R_C^*,\ \text{if}\ \ i = C, \end{cases}$$

the extraction rate

$$\dot{R}_i(t) = -q_i^{II}(R_i,t)\,,$$

and the aggregated supply

$$Q^{II}(R_i,t) = q_A^{II}(R_i,t) + q_C^{II}(R_i,t)\,,$$

where e^{-rt} is a time-preference factor with rate r.

Since the above optimization problem explicitly depends on t only through the time-preference factor, we can henceforth consider stationary Markovian Nash equilibria (Dockner et al., 2000; Kamien and Schwartz, 1981). As a result, the stationary Hamilton-Jacobi-Bellman (HJB) equation of country i in the second time period becomes

$$r\,W_i(R_i) = \max_{q_i^{II}} \left\{P^{II}(Q)\, q_i^{II} - W_i'(R_i)\, q_i^{II}\right\}, \tag{4.2}$$

where $q_i^{II} = q_i^{II}(R_i)$, where $W_i(R_i)$ is country i's Bellman value function and where $W_i'(R_i) = \frac{\mathrm{d}W_i(R_i)}{\mathrm{d}R_i}$. From the first-order condition (FOC) of the right hand side of Equation (4.2),[1] it follows that

$$W_i'(R_i) = a\ \left(q_i^{II} + q_j^{II}\right)^{\alpha-2}\ \left(\alpha q_i^{II} + q_j^{II}\right). \tag{4.3}$$

[1] All second-order sufficient conditions are verified.

Moreover, as no arbitrage opportunity should exist at the entry time T^*, we have

$$W_i'(R_i(T^*)) = W_j'(R_j(T^*)).$$

As depicted in Appendix A, this allows to proof the results of the subsequent lemma.

Lemma 1. *Suppose that each country plays Markovian strategies and that they both entirely exhausts their initial reserves $R_i(T^*)$ over $[T^*, +\infty)$. Then the ratio of both countries' extraction rates remains unchanged:*

$$\frac{q_A^{II}(R_A(t))}{q_C^{II}(R_C(t))} = \frac{R_A(0)}{R_C^*}, \ \forall t \geq T^*.$$

Furthermore, if the market is free of arbitrage opportunities when country A launches its production at time $T^* \in (0, +\infty)$, then country C's reserves at T^* must check

$$R_C(T^*) = R_C^* = R_A(0). \tag{4.4}$$

Although the finding of Equation (4.4) empowers country C to determine the start of the competition through the speed of its monopolistic extraction, it eventually enables both countries to compete at eye level: none of them benefits from an advantageous position. Lemma 1 therefore implies that all conditions are fulfilled to search for a symmetric Nash equilibrium (Dockner et al., 2000). Accordingly, the FOC of Equation (4.3) becomes

$$q_i^{II}(R_i) = \left(\frac{W_i'(R_i)}{2^{\alpha-2}a(1+\alpha)}\right)^{\frac{1}{\alpha-1}}. \tag{4.5}$$

Substituting (4.5) into the HJB equation of (4.2) then allows to conjecture that the value function will be of the form

$$W_i(R_i) = \beta R_i \alpha,$$

where β is to be determined. From this guess, it follows that the HJB equation becomes

$$r\beta R_i^{\alpha} = a2^{\alpha-1}\left(\frac{\beta\alpha R_i^{\alpha-1}}{2^{\alpha-2}a(1+\alpha)}\right)^{\frac{\alpha}{\alpha-1}} - \beta\alpha R_i^{\alpha-1}\left(\frac{\beta\alpha R_i^{\alpha-1}}{2^{\alpha-2}a(1+\alpha)}\right)^{\frac{1}{\alpha-1}}. \tag{4.6}$$

When solving Equation (4.6) with respect to β, we find

$$\beta = 2^{\alpha-2}a\left(\frac{1-\alpha}{r}\right)^{1-\alpha}\left(\frac{1+\alpha}{\alpha}\right)\alpha. \tag{4.7}$$

By substituting the parameter of Equation (4.7) into the FOC of (4.5), we obtain the results of the proposition below. As C's monopolistic extraction determines the starting time of the duopolistic competition (see Equation (4.4)), the second-period Markovian outcomes are hereafter denoted by the subscript of the first-period: $\phi \in \{M, S\}$.

Proposition 1. *Suppose that both competitors play Markovian strategies and that the market entry of country A takes place at time $T\phi^* \in (0, +\infty)$. Then for any $R_i(t)$ and for any $t \geq T\phi^*$, the second-period Markovian strategic Nash equilibrium supply of A and C are given, respectively, by*

$$q^{II}_{A,\phi}(R_A(t)) = \frac{r}{1-\alpha}\frac{1+\alpha}{\alpha}R_A(t),$$

and

$$q^{II}_{C,\phi}(R_C(t)) = \frac{r}{1-\alpha}\frac{1+\alpha}{\alpha}R_C(t). \tag{4.8}$$

The market price is

$$P\phi^{II}(t) = a\left(\frac{r}{1-\alpha}\frac{1+\alpha}{\alpha}\ (R_A(t) + R_C(t))\right)^{\alpha-1}. \tag{4.9}$$

Furthermore, using the country i's supply function of Proposition 1 in the dynamic equation $\dot{R}_i(t) = -q^{II}_{i,\phi}(R_i(t))$ and solving the resulting differential equation gives the reserve function

$$R_i(t) = R_i(T\phi^*)\, e^{\frac{r}{\alpha-1}\frac{1+\alpha}{\alpha}(t-T\phi^*)}. \tag{4.10}$$

Substituting Equations (4.10) into (4.9) then yields

$$P\phi^{II}(t) = a\left(\frac{r}{1-\alpha}\frac{1+\alpha}{\alpha}\,(R_A(0) + R^*_C)\right)^{\alpha-1} \mathrm{e}^{r\frac{1+\alpha}{\alpha}(t-T\phi^*)},\ \forall t \geq T\phi^*. \tag{4.11}$$

The above observation, that the price of REEs grows faster than at the rate r, comes from the fact that Hotelling's rule does not hold in an environment where competitors play Markovian strategies (Benchekroun and Gaudet, 2003; Gaudet, 2007). Subsection 4.3.2 further discusses this issue.

To complete the Markovian monopoly-to-duopoly game, the next subsection defines country C's Markovian supply behavior in the monopolistic period. Henceforth the subscript M is used to indicate the Markovian results of this optimal control problem.

4.2.2 Markovian optimal control strategies

In this first period I, where $t \in [0, T_M^*)$, country C chooses a supply strategy that maximizes the following revenue

$$\pi_{C,M}^I(R_C(0) - R_C^*, T_M^*) = \max_{q_{C,M}^I(R_C,t)\geq 0} \int_0^{T_M^*} \mathrm{e}^{-rt} P_M^I\big(q_{C,M}^I(R_C,t)\big)\, q_{C,M}^I(R_C,t)\ \mathrm{d}t,$$

subject to

$$\int_0^{T^*} q_{C,M}^I(R_C,t)\ \mathrm{d}t = R_C(0) - R_C^*, \quad R_C(0) \text{ given},$$

and

$$\dot{R}_C(t) = -q_{C,M}^I(R_C,t)\,.$$

The stationary HJB equation to this problem is

$$rV(R_C) = \max_{q_{C,M}^I} \left\{ P_M^I(q_{C,M}^I)\, q_{C,M}^I - V'(R_C)\, q_{C,M}^I \right\},$$

and yields the FOC below

$$q_{C,M}^I(R_C) = \left(\frac{V'(R_C)}{a\alpha}\right)^{\frac{1}{\alpha-1}}.^2 \tag{4.12}$$

We again conjecture that the value function will be of the form

$$V(R_C) = BR_C\alpha, \tag{4.13}$$

where B is to be determined. Based on the latter guess, the FOC of Equation (4.12) becomes

$$q_{C,M}^I(R_C) = \left(\frac{B}{a}\right)^{\frac{1}{\alpha-1}} R_C. \tag{4.14}$$

Substituting the FOC of (4.14) and the guess of (4.13) into the above HJB equation leads to:

$$rBR_C\alpha = a\left(\left(\frac{B}{a}\right)^{\frac{1}{\alpha-1}} R_C\right)^{\alpha-1} \left(\frac{B}{a}\right)^{\frac{1}{\alpha-1}} R_C - B\alpha R_C^{\alpha-1}\left(\frac{B}{a}\right)^{\frac{1}{\alpha-1}} R_C. \tag{4.15}$$

[2] The second-order sufficient conditions hold as well.

Solving Equation (4.15) with respect to B then yields

$$B = \left(\frac{1-\alpha}{r}\right)^{1-\alpha} a. \tag{4.16}$$

When substituting Equation (4.16) back into the initial guess of Equation (4.13), we find

$$V(R_C) = \left(\frac{1-\alpha}{r}\right)^{1-\alpha} a\, R_C^{\alpha}. \tag{4.17}$$

Next, combining Equation (4.17) with the FOC of (4.15) allows to determine the proposition below.

Proposition 2. *Suppose that country C applies a Markovian behavior in both time periods and that country A enters the market at time $T_M^* \in (0, +\infty)$. Then for any $R_C(t)$ and for any $t \leq T_M^*$, C's optimal monopolistic Markovian supply is given by*

$$q_{C,M}^{I}(R_C(t)) = \frac{r}{1-\alpha} R_C(t), \tag{4.18}$$

and the market price corresponds to:

$$P_M^I(R_C(t)) = a \left(\frac{r}{1-\alpha} R_C(t)\right)^{\alpha-1}. \tag{4.19}$$

By substituting Equation (4.18) into the dynamic equation of the above problem we get

$$R_C(t) = R_C(0)\, \mathrm{e}^{\frac{r}{\alpha-1}t},\ \forall t \leq T_M^*. \tag{4.20}$$

Hence, combining Equations (4.20) and (4.19) with the price function of (4.1) returns

$$P_M^I(t) = a \left(\frac{r}{1-\alpha} R_C(0)\right)^{\alpha-1} \mathrm{e}^{rt}.$$

The fact that here the market price increases at rate r over time, suggest that Hotelling's rule holds under Markovian strategies when there is no competition.

Furthermore, considering Equation (4.20) at time $t = T_M^*$ yields the entry timing of the ensuing proposition.

Proposition 3. *If country C plays Markovian strategies in both time periods, then the entry takes place at time*

$$T_M^* = \frac{1-\alpha}{r} \ln\left(\frac{R_C(0)}{R_A(0)}\right). \tag{4.21}$$

The entry timing is here primarily affected by the initial condition, that is, the countries' initial reserve ratio $\frac{R_C(0)}{R_A(0)}$. In fact, Equation (4.21) yields that if $R_A(0) = R_C(0)$, the entry time $T_M^* = 0$ and so the duopolistic competition starts immediately. On the contrary, if, for example, $R_C(0) > R_A(0)$, then $T_M^* > 0$, which entails that the country with the initially larger reserves is already in the supply market when the duopolistic competition starts. This observation confirms our initial intuition that the country with the smaller reserves should optimally postpone its market entry until the country with the larger reserves has reduced its stock to a level that makes both of their reserves coincide.

The coming subsection studies the monopolistic open-loop supply of country C, when Markovian strategies characterize the second period; via the backstop induction method. Hereinafter, these optimal control results are indicated by the subscript S.

4.2.3 Open-loop optimal control strategies

In this case, the revenue of the first time period, where $t \in [0, T_S^*)$, is determined by

$$\pi_{C,S}^I\left(R_C(0) - R_C^*, T_S^*\right) = \max_{q_{C,S}^I(t)\geq 0} \int_0^{T_S^*} \mathrm{e}^{-rt} P_S^I\big(q_{C,S}^I(t)\big)\, q_{C,S}^I(t)\, \mathrm{d}t,$$

subject to

$$\int_0^{T^*} q_{C,S}^I(t)\, \mathrm{d}t = R_C(0) - R_C^*, \quad R_C(0) \text{ given},$$

and

$$\dot{R}_C(t) = -q_{C,S}^I(t).$$

Given that the first constraint of the above problem can also be written as

$$\int_0^{\tau} q_{C,S}^I(t)\, \mathrm{d}t \leq R_C(0) - R_C^*, \ \tau \in [0, T_S^*],$$

the Lagrangian is

$$\mathcal{L}_C^I\Big(q_{C,S}^I(t), \lambda_C^I(t), \alpha_C^I\Big) = P_S^I\big(q_{C,S}^I(t)\big)\, q_{C,S}^I(t) - \lambda_C^I(t)\, q_{C,S}^I(t) - \alpha_C^I\Big(R_C(0) - R_C^* - \int_0^\tau q_{C,S}^I(t)\,\mathrm{d}t\Big),$$

where $\lambda_C^I(t)$ is the shadow price and α_C^I is the static Lagrange multiplier. This yields the following standard first-order conditions (FOCs)

$$\begin{cases} \dot{\lambda}_C^I(t) = r\,\lambda_C^I(t), \\ \frac{\partial}{\partial q_{C,S}^I(t)}\mathcal{L}_C^I\big(q_{C,S}^I(t), \lambda_C^I(t), \alpha_C^I\big) = \frac{\mathrm{d}}{\mathrm{d}q_{C,S}^I(t)} RV_{C,S}^I(t) - \lambda_C^I(t) \,+\, \alpha_C^I \frac{\mathrm{d}}{\mathrm{d}q_{C,S}^I(t)} \int_0 \tau q_{C,S}^I(t)\,\mathrm{d}t = 0, \\ \alpha_C^I \geq 0, \;\; R_C(0) - R_C^* - \int_0^\tau q_{C,S}^I(t)\,\mathrm{d}t \geq 0, \;\; \alpha_C^I\left(R_C(0) - R_C^* - \int_0^\tau q_{C,S}^I(t)\,\mathrm{d}t\right) = 0, \end{cases}$$

where country C's monopolistic revenue $RV_{C,S}^I(t) = P_S^I\big(q_{C,S}^I(t)\big)\, q_{C,S}^I(t)$.[3] In the case where $\tau < T_S^*$, we have $R_C(0) - R_C^* > \int_0^\tau q_{C,S}^I(t)\,\mathrm{d}t$, and therefore the constant $\alpha_C^I = 0$.

Furthermore, we get from the first FOC that the shadow price corresponds to:

$$\lambda_C^I(t) = \lambda_C^I(T_S^*)\mathrm{e}^{r(t-T_S^*)}, \tag{4.22}$$

where the transversality condition gives the initial condition $\lambda_C^I(T_S^*)$.

When substituting the fact that $\alpha_C^I = 0$ into the second of the above FOCs, we find that the marginal revenue is given by

$$\frac{\mathrm{d}RV_{C,S}^I(t)}{\mathrm{d}q_{C,S}^I(t)} = \lambda_C^I(t). \tag{4.23}$$

In view of the revenue function $RV_{C,S}^I(t) = P_S^I\big(q_{C,S}^I(t)\big)\, q_{C,S}^I(t)$ and the price function (4.1), we obtain that the marginal revenue can also be written as

$$\frac{\mathrm{d}RV_{C,S}^I(t)}{\mathrm{d}q_{C,S}^I(t)} = a\alpha\left(q_{C,S}^I(t)\right)^{\alpha-1} \tag{4.24}$$

Combining Equations (4.22), (4.23) and (4.24) then yields

$$q_{C,S}^I(t) = \left(\frac{\lambda_C^I(T_S^*)}{a\alpha}\mathrm{e}^{r(t-T_S^*)}\right)^{\frac{1}{\alpha-1}}, \; \forall t \leq T_S^*, \tag{4.25}$$

[3]All second-order sufficient conditions are satisfied.

so that at $t = T_S^*$,

$$\lambda_C^I(T_S^*) = a\alpha(q_{C,S}^I(T_S^*))^{\alpha-1}. \tag{4.26}$$

As we now consider country C's supply to remain unaffected by country A's entry at T_S^*, that is, $q_{C,S}^I(T_S^*) = q_{C,S}^{II}(T_S^*)$, the Markovian supply function of Proposition 1 enters the above problem as transversality condition. This hypothesis is in line with country C's sluggish attitude in the present scenario: instead of a rapid and offensive switch to a competitive Markovian behavior at A's entry-notice, C decides upon a least-effort approach and initially delays the switch from open-loop to Markovian strategies until being left with no other choice. In this context, quick and aggressive reactions are therefore not to be expected from country C.

Based on the above hypothesis, we combine Equations (4.8), (4.10) and (4.26) and find that

$$\lambda_C^I(T_S^*) = a\alpha\left(\frac{r}{1-\alpha}\frac{1+\alpha}{\alpha}R_C^*\right)^{\alpha-1}. \tag{4.27}$$

Substituting Equation (4.27) into (4.25) then leads to results of the next proposition.

Proposition 4. *Suppose country C adopts an open-loop strategy in the first period, followed by a Markovian one in the second period, and keeps its supply volumes unchanged when country A enters the market at T_S^*. Then for any $t \le T_S^*$, C's optimal monopolistic open-loop supply path is given by*

$$q_{C,S}^I(t) = \frac{r}{1-\alpha}\frac{1+\alpha}{\alpha}R_C^*\,\mathrm{e}^{\frac{r(t-T_S^*)}{\alpha-1}}, \tag{4.28}$$

and the market price corresponds to:

$$P_S^I(t) = a\left(\frac{r}{1-\alpha}\frac{1+\alpha}{\alpha}R_C^*\right)^{\alpha-1}\mathrm{e}^{r(t-T_S^*)}.$$

Moreover, substituting (4.28) into the dynamic equation $\dot{R}_C(t) = -q_{C,S}^I(t)$ of the above problem and subsequently taking integrals over $[0, T_S^*)$ on both sides of the resulting equation, yields the optimal timing for country A's entry. The coming proposition presents this solution.

Proposition 5. *If the same conditions as in Proposition 4 apply, then the entry takes place at time*

$$T_S^* = \frac{1-\alpha}{r}\ln\left(\frac{R_A(0)+\alpha R_C(0)}{(1+\alpha)R_A(0)}\right).$$

With the case under study substantially differing to the one in Subsection 4.2.2, the emergence of a different entry time was to be expected. However, as the present continuous supply assumption leaves country C's extraction unaffected by A's entry, the entry timing is again independent of any entry values. Instead it depends, similarly to scenario M, only on the price elasticity of demand, the time preference and the countries' initial REEs reserves.

4.3 Extended comparison

The present section analyzes how the sophisticated Markovian decision rules of the above scenarios perform in comparison to the more plain open-loop strategies. For this purpose, we refer to our findings in Chapter 3, where we already solved the following problem

$$\Pi_i = \max_{q_i^I(t)\geq 0} \int_0^{T^*} \mathrm{e}^{-rt} P^I(q_i^I(t))\, q_i^I(t)\,\mathrm{d}t + \max_{q_i^{II}(t)\geq 0} \int_{T^*}^{+\infty} \mathrm{e}^{-rt} P^{II}(Q(t))\, q_i^{II}(t)\,\mathrm{d}t,$$

subject to

$$\int_0^{+\infty} q_i(t)\,\mathrm{d}t \leq R_i(0),\ R_i(0)\ \text{given},$$

and

$$\dot{R}_i(t) = -q_i(t),\ t \in (0, +\infty),$$

for open-loop strategies.

One initial observation can be derived by comparing the optimal entry conditions under both open-loop and Markovian strategy spaces. Here we observe that the requirement for the successful execution of country A's market entry is not affected by the strategies applied within the periods. Instead, what will likely be shaped by the type of strategy is the countries' exploitation speed and thus the point in time at which the optimal entry condition is satisfied, that is, when country C's extraction has reduced its reserve levels to that of country A.

Before now tackling the main part of this section's comparison, which consists of evaluating the behavioral and financial variations between alternative strategy concepts, we define the countries' first- and second-period Markovian revenues by means of the findings in Section 4.2.

On that basis, substituting the first-period outcomes of Subsections 4.2.2 and 4.2.3, respectively, into

$$\pi^{I}_{C,\phi} = \int_{0}^{T\phi^*} \mathrm{e}^{-rt} P\phi^{I}\big(q^{I}_{C,\phi}\big)\, q^{I}_{C,\phi}\, \mathrm{d}t, \tag{4.29}$$

where $\phi \in \{M, S\}$, yields the monopolistic revenue of country C: (1) when Markovian strategies are played in both periods

$$\pi^{I}_{C,M} = a\,(R_C(0) - R^*_C)\left(\frac{r}{1-\alpha} R_C(0)\right)^{\alpha-1};$$

and (2) when open-loop strategies are played in the first period and Markovian ones in the second period

$$\pi^{I}_{C,S} = a\,(R_C(0) - R^*_C)\left(\frac{r}{\alpha(1-\alpha)}\,(\alpha R_C(0) + R^*_C)\right)^{\alpha-1}.$$

As regards the second period, the revenue of country $i \in \{A, C\}$, discounted to $t = 0$, is determined by

$$\pi^{II}_{i,\phi} = \int_{T\phi^*}^{+\infty} \mathrm{e}^{-rt} P\phi^{II}(Q(R_i,t))\, q^{II}_{i,\phi}(R_i,t)\ \mathrm{d}t. \tag{4.30}$$

Substituting Propositions 1 and 3 into Equation (4.30) shows that when the duopolistic Markovian period is already preceded by a Markovian behavior, both players' second-period profit is

$$\pi^{II}_{i,M} = a\,2^{\alpha-2}\,R^*_C\left(\frac{1+\alpha}{\alpha}\right)\alpha\left(\frac{r}{1-\alpha}\,R_C(0)\right)^{\alpha-1}.$$

On the contrary, when the duopolistic Markovian period is preceded by an open-loop commitment, we obtain from the findings of Propositions 1 and 5 that Equation (4.30) reads as follows:

$$\pi^{II}_{i,O} = a\,2^{\alpha-2}\ \left((1+\alpha)\,R^*_C\right)\left(\frac{1}{\alpha}\right)\alpha\left(\frac{r}{1-\alpha}\ (R^*_C + \alpha R_C(0))\right)^{\alpha-1}.$$

Evaluating the revenues of the four scenarios M, O, P and S then yields the findings of Table 4.1, where 1 denotes the highest revenue as well as the earliest entry time and where the aggregated revenue is given by $\Pi_{i,\psi} = \pi^{I}_{i,\psi} + \pi^{II}_{i,\psi}$, with $\psi \in \{M, O, P, S\}$ and $\pi^{I}_{A,\psi}$ being zero.

Table 4.1 – Entry time and revenue ranking

Scenario	**Strategy**		**Entry time**	**Revenue**		**Aggregated revenue**
	period I	period II		period I	period II	period I & II
	C	A & C	T^*	C	A & C	C
O	open-loop	open-loop	3	1	2/3	1
	continuous supply					
P	open-loop	open-loop	2	2	1	3/2
	continuous price					
S	open-loop	Markovian	1	3	3/2	4
	continuous supply					
M	Markovian	Markovian	3	1	4	2/3

The remainder of this section further discusses the origin of these results and draws first conclusions.

4.3.1 First-period analysis

Starting with the first period, we establish from scenarios O and S that optimal control problems of seemingly identical strategy spaces and transversality conditions can lead to different outcomes. In fact, by comparing the monopolistic supply functions of the latter scenarios, we get that whenever country C keeps its supply continuous at A's entry and switches to a Markovian strategy in the second period, instead of maintaining an open-loop behavior in both periods, then its open-loop supply rate of the first-period increases, that is, $q^I_{C,O}(t) < q^I_{C,S}(t), \ \forall t \leq \ T^*_S$, and ends up ranking highest in terms of instantaneous supply levels (see Figure 4.1). This observation results from the application of the backward induction approach: the duopolistic extraction behavior, which varies with the strategy type, defines the value of the first-period transversality condition and thus the way in which the monopolist acts (Dockner et al., 2000). Furthermore, based on the monopolistic supply functions of scenarios O and P, we get that whenever country C applies open-loop strategies in both periods and changes its supply at A's entry, instead of keeping its continuous, then its first-period open-loop supply increases as well, that is, $q^I_{C,O}(t) < q^I_{C,P}(t),\ \forall t \leq T^*_S$, and finishes placing second highest in the supply ranking. Similarly to the previous case, these unequal optimal control outcomes originate from dissimilar transversality conditions. Finally, we discover from scenarios M and O that when country C plays open-loop or Markovian strategies in both periods, imposing a supply continuity in scenario O, while keeping scenario M free of impositions, yields identical transversality conditions and hence equal first-period supply rates, which turn out placing lowest in the ranking. In light of the above, we can therefore conclude that whenever country C modifies its supply behavior at one point in the game, be it through the application of a different strategy or a sudden supply drop, then this more adventurous attitude is also reflected by a less conservative first-period extraction.

Based on the latter findings, the entry times in Table 4.1 are straightforward: since country A's market entry is in each scenario triggered when country C's monopolistic extraction has shrunk its stock to the level of A's initial reserves, that is, when $R^*_C = R_A(0)$, the fastest extraction $q^I_{C,S}(t)$ leads to the earliest entry T^*_S and identical extractions $q^I_{C,M}(t) = q^I_{C,O}(t)$ generate identical entries $T^*_M = T^*_O$ (see Figure 4.1).

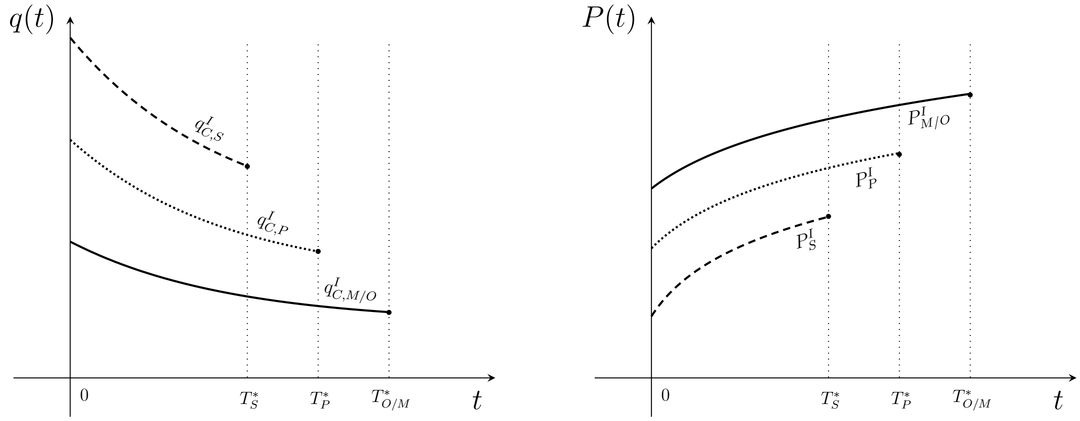
Moreover, given the positive [negative] dependency between the instantaneous supply and revenue [price] in the first period, the more vigorous rare earths extraction of P and S turns out beneficial for suppliers [consumers] for as long as the scenarios' monopolistic periods coexist: $\pi^I_{i,S}(t) > \pi^I_{i,\sigma}(t),\ \left[P^I_S(t) < P^I_\sigma(t)\right],\ \forall t \leq T^*_S$, where $\sigma \in \{M, O, P\}$, and $\pi^I_{C,P}(t) > \pi^I_{C,\zeta}(t),\ \left[P^I_P(t) < P^I_\zeta(t)\right],\ \forall t \leq T^*_P$, where $\zeta \in \{M, O\}$. Nevertheless, when comparing the summed first-period revenues, where $\pi^I_{C,S} < \pi^I_{C,\sigma}$ and $\pi^I_{C,P} < \pi^I_{C,\zeta}$, we get that the additional length of the longer and more conservative periods is sufficient to make up for the instantaneous losses before T^*_S and T^*_P, respectively. This is presented in the next proposition.

Proposition 6. *Suppose that A's optimal entry time is T^*_ψ, where $\psi \in \{M, O, P, S\}$. Then*

$$\pi^I_{C,M} = \pi^I_{C,O} > \pi^I_{C,P} > \pi^I_{C,S}.$$

Consequently, from a first-period perspective, country C should keep its monopolistic supply as low as possible and thus strive for scenario M or O. However, as the latter deduction may change when the second-period revenues are included into the strategy development, the ensuing subsection assesses the duopolistic outcomes.

Figure 4.1 – Monopolistic supply and price differences



4.3.2 Second-period analysis

With regard to the duopolistic revenues in Table 4.1, which correspond to the aggregated revenues of country A, we notice that a Markovian decision rule should not be coupled with a supply increase at the start of the game. This is proven by scenario M, where the Markovian behavior and the entry reaction, that is, $q^I_{C,M}(T^*_M) < q^{II}_{i,M}(T^*_M)$, yield the lowest second-period profit. Instead, when playing Markovian strategies in the competitive period, the supply of country C should rather remain continuous when country A enters the market. The latter situation can be observed in scenario S, where the Markovian strategy and the unchanged supply, that is, $q^I_{C,S}(T^*_S) = q^{II}_{i,S}(T^*_S)$, produce a duopolistic revenue that competes for the second-best position with the revenue of scenario O, where a steady entry supply, that is, $q^I_{C,O}(T^*_O) = q^{II}_{i,O}(T^*_O)$, is practiced with open-loop strategies. The first-best duopolistic outcomes are, however, achieved when the competitors decrease their supply at the beginning of the game and adopt an open-loop behavior. This is seen in scenario P, where a supply commitment, combined with an extraction decline, that is, $q^I_{C,P}(T^*_P) > q^{II}_{i,P}(T^*_P)$, provokes the highest second-period payoffs. These findings are accumulated in the following proposition.

Proposition 7. *Suppose that A's optimal entry time is T^*_ψ, where $\psi \in \{M, O, P, S\}$. Then*

$$\pi^{II}_{i,P} > \pi^{II}_{i,O} \lesseqgtr \pi^{II}_{i,S} > \pi^{II}_{i,M}.$$

To better retrace the reasons behind the observations in Proposition 7, the different scenarios will now be consecutively compared.

Comparison 1: Scenarios (O, P) and (M, S)

For scenarios of identical duopolistic strategy spaces, that is, the sets (O, P) and (M, S), the initially more offensive extraction manner, detected in Subsection 4.3.1, suddenly becomes more cautious as the second period starts: $q^{II}_{i,O}(t) > q^{II}_{i,P}(t),\ \forall t \geq T^*_O$ and $q^{II}_{i,M}(t) > q^{II}_{i,S}(t),\ \forall t \geq T^*_M$. These findings are depicted in Figure 4.2.

Indeed, regarding the set (O, P), while both scenarios are characterized by an open-loop commitment, their entry reactions stand in opposition. Actually, unlike scenario O's supply-continuity assumption, scenario P supposes a continuous market price at A's entry. Since this requires a prompt drop in country C's supply, it is unsurprising that the extraction rate of scenario P falls below the one of scenario O when the competition begins. The fact that it remains there over the infinite competition is, however, related to the identical supply strategies. More specifically, despite the different entry reactions, committing to a supply path when the competition starts, reflects related extraction attitudes that yield comparable supply-decrease behaviors, which in turn keep the supply functions from crossing.

Concerning the set (M, S), the more restrained duopolistic extraction of scenario S results from the fact that, unlike in scenario M, the continuous-supply condition already imposes a certain extraction reluctance at the beginning of the game. Additionally, in scenario S, country C could not utilize the first period to familiarize with the new, more demanding supply strategy, which explains its lower risk appetite and thus its more cautious supply behavior throughout the game.

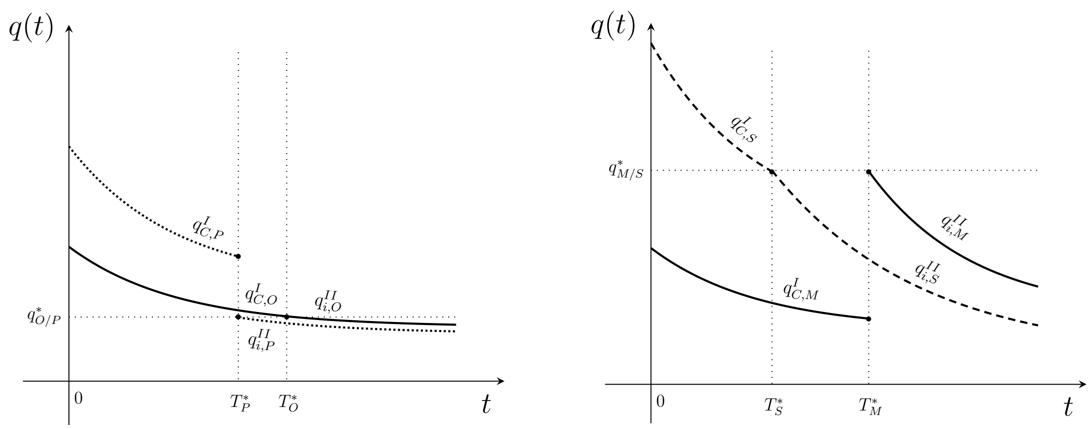
As the second-period revenues also positively [negatively] depend on the amount of extracted REEs, the dominant supply functions yield higher [lower] instantaneous revenues [prices] when both periods coexist: $\pi^{II}_{i,O}(t) > \pi^{II}_{i,P}(t),\ [P^{II}_O(t) < P^{II}_P(t)],\ \forall t \geq T^*_O$ and $\pi^{II}_{i,M}(t) > \pi^{II}_{i,S}(t),\ [P^{II}_M(t) < P^{II}_S(t)],\ \forall t \geq T^*_M$. Yet, evaluating their total duopolistic revenues yields the proposition below.

Proposition 8. *If the duopolistic game is played successively under identical strategy spaces, then a more conservative extraction behavior turns out to eventually pay off:*

$$\pi^{II}_{i,O} < \pi^{II}_{i,P} \text{ and } \pi^{II}_{i,M} < \pi^{II}_{i,S}.$$

Hence, when intending to increase the second-period profits in the sets (O, P) and (M, S), the countries should aim for a restrained supply attitude in the competitive phase.

Figure 4.2 – Supply differences: (O, P) and (M, S), respectively



Since the findings of Proposition 8 do not hold for scenarios of different duopolistic strategy spaces, these scenarios are now more profoundly analyzed in the coming subsections.

Comparison 2: Scenarios M and O

Appraising the opposite second-period strategies of scenarios M and O right at the beginning of the game, that is, at $T^* = T^*_M = T^*_O$, confirms the intuition that the more competitive Markovian behavior leads to a consumer-friendly market price

$$P^{II}_M(T^*) = \left(\frac{\alpha}{1+\alpha}\right)^{1-\alpha} P^{II}_O(T^*) < P^{II}_O(T^*).$$

In combination with a more aggressive rare earths extraction

$$q^{II}_{i,M}(T^*) = \frac{1+\alpha}{\alpha}\, q^{II}_{i,O}(T^*) > q^{II}_{i,O}(T^*), \tag{4.31}$$

this hence provokes a higher instantaneous revenue at the starting point

$$\pi^{II}_{i,M}(T^*) = P^{II}_M(T^*)\, q^{II}_{i,M}(T^*) = \left(\frac{1+\alpha}{\alpha}\right)^{\alpha} \pi^{II}_{i,O}(T^*) > \pi^{II}_{i,O}(T^*), \tag{4.32}$$

such that the Markovian strategies are, initially, also beneficial for suppliers.

Nevertheless, while the higher Markovian supply reactivity is, on the one side, expressed through a more daring extraction behavior at the outset, it is, on the other side, also reflected by a more dynamic supply reduction when advantageous. This means that when comparing the slope of both extraction curves we get that in a first phase, the Markovian extraction decreases quicker than the open-loop one. The proposition below presents this result.

Proposition 9. *Suppose the two-period game is played successively under both Markovian and open-loop strategies and the open-loop supply is continuous at A's market entry T_O^*. Then, in the second period, there exists one and only one time*

$$T_{MO}^{+} = T^{*} + 2\,\frac{\alpha(1-\alpha)}{r}\ln\left(\frac{1+\alpha}{r}\right)$$

where the ratio of the Markovian and open-loop supply derivatives is

$$\frac{\dot{q}_{i,M}^{II}(t)}{\dot{q}_{i,O}^{II}(t)} = \mathrm{e}^{-\frac{r(t-T^{*})}{\alpha(1-\alpha)}}\left(\frac{1+\alpha}{\alpha}\right)^{2}\begin{cases} <1, \ \forall t \in [T^{*}, T_{MO}^{+}), \\ >1, \ \forall t \in (T_{MO}^{+}, +\infty). \end{cases}$$

Consequently, there exists a point in the duopolistic period, where the open-loop and Markovian supply functions cross. This is stated in the following proposition and depicted in Figure 4.3.

Proposition 10. *Suppose the two-period game is played successively under both Markovian and open-loop strategies and the open-loop supply is continuous at A's market entry T_O^*. Then, in the second period, there exists one and only one time*

$$\widehat{T}_{MO} = T^{*} + \frac{\alpha(1-\alpha)}{r}\ln\left(\frac{1+\alpha}{\alpha}\right),$$

where the ratio of Markovian and open-loop supplies is

$$\frac{q_{i,M}^{II}(t)}{q_{i,O}^{II}(t)} = \mathrm{e}^{\frac{r(t-T^{*})}{\alpha(\alpha-1)}}\,\frac{1+\alpha}{\alpha}\begin{cases} >1, \text{ if } \ t \in [T^{*}, \widehat{T}_{MO}), \\ <1, \text{ if } \ t \in (\widehat{T}_{MO}, +\infty), \end{cases} \tag{4.33}$$

and the ratio of the corresponding market prices becomes

$$\frac{P_M^{II}(t)}{P_O^{II}(t)} = \left(\frac{q_{i,M}^{II}(t)}{q_{i,O}^{II}(t)}\right)^{\alpha-1} \begin{cases} < 1, \text{ if } \ t \in [T^*, \widehat{T}_{MO}), \\ > 1, \text{ if } \ t \in (\widehat{T}_{MO}, +\infty). \end{cases}$$

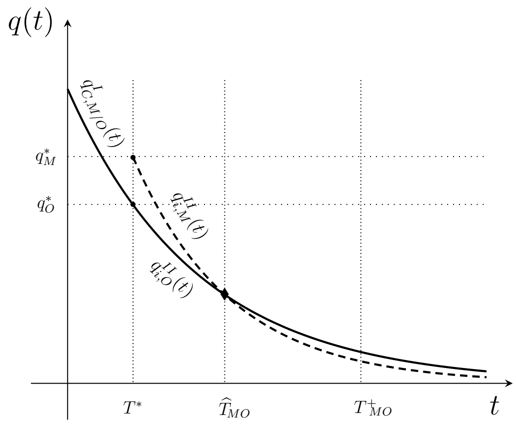
The ratio of the Markovian and open-loop instantaneous revenues is

$$\frac{\pi_{i,M}^{II}(t)}{\pi_{i,O}^{II}(t)} = \left(\frac{q_{i,M}^{II}(t)}{q_{i,O}^{II}(t)}\right)^{\alpha} \begin{cases} > 1, \text{ if } \ t \in [T^*, \widehat{T}_{MO}), \\ < 1, \text{ if } \ t \in (\widehat{T}_{MO}, +\infty). \end{cases} \tag{4.34}$$

In view of the price and revenue functions in Proposition 10, it can be concluded that Markovian strategies perform better from a consumer and supplier perspective, respectively, only as long as the Markovian supply is greater than the open-loop one, that is, until the tipping point $\widehat{T}_{MO}$. Hence, the more laborious Markovian decision rule not strictly prevails over the less sophisticated open-loop strategy but there exists a time after which the situation is reversed.

Of course, our findings essentially depend on the specific REEs assumptions of an infinite horizon problem without a backstop technology and physical exhaustion of the finite reserves in the very long run. In fact, changing any of these hypotheses will probably lead to different results.

Figure 4.3 – Supply differences: M, O



Before closer analyzing the latter profitability switches, we wish to further explain why the Markovian market price of Equation (4.11) climbs quicker than projected by Hotelling (1931). Actually, the observation of Equation (4.33), that the duopolistic Markovian supply dominates the open-loop one before the turning point $\widehat{T}_{MO}$, entails that the period $[T^*, \widehat{T}_{MO})$ is characterized by a faster resource depletion. The next corollary concludes.

Corollary 3. *At time $\widehat{T}_{MO}$, the remaining reserves of country $i \in \{A, C\}$ under open-loop and Markovian strategies are given, respectively, by*

$$R_{i,O}(\widehat{T}_{MO}) = R_A(0)\, \mathrm{e}^{\frac{r(T^* - \widehat{T}_{MO})}{1-\alpha}},$$

and

$$R_{i,M}(\widehat{T}_{MO}) = \frac{\alpha}{1+\alpha}\, R_{i,O}(\widehat{T}_{MO}) < \frac{1}{2}\, R_{i,O}(\widehat{T}_{MO}).$$

In other words, Corollary 3 connotes that the quicker Markovian supply decrease prior to $T^{\dagger}_{MO}$ lets the supply curves intersect at a moment $\widehat{T}_{MO}$, where the Markovian rare earths reserves are already less than half the ones of the open-loop scenario. By implication, when playing Markovian strategies, significantly less resources are available for extraction over $[\widehat{T}_{MO}, +\infty)$. Given that, despite the lower Markovian supply after the turning point $\widehat{T}_{MO}$, this unbalanced scarcity situation emerges to remain unchanged for as long as the duopolistic period is finite $[T^*, (\eta+1)T^*]$, that is,

$$R_{i,M}((\eta+1)T^*) < R_{i,O}((\eta+1)T^*),$$

where η is some finite number in $\mathbb{R}^*_+$, justifies that the price in Equation (4.11) rises at a rate higher than r. More precisely, although our assumptions deviate from the ones used by Hotelling (1931), the Markovian market price must still comply with the fundamental concept of Hotelling's rule, that is, to reflect the right scarcity value of the resources, in order to send the appropriate rationing signals to the suppliers. On this account, the initially lower Markovian price must at one point catch up with the open-loop one, which is only possible by increasing at a greater rate $r\frac{1+\alpha}{\alpha} > r$.

Coming back to the instantaneous revenues in Proposition 10, we get from Equation (4.34) that the summed Markovian revenue over $[T^*, \widehat{T}_{MO}]$ is higher than the one under open-loop strategies and that the inverse holds true for the time period $[\widehat{T}_{MO}, +\infty]$. This is illustrated in the subsequent corollary.

Corollary 4. *If we consider the time period* $[T^*, \widehat{T}_{MO}]$*, then the total open-loop revenue, discounted to* $t = 0$*, is given by*

$$\begin{aligned}\pi_{i,O}^{[T^*,\widehat{T}_{MO}]} &= \int_{T^*}^{\widehat{T}_{MO}} e^{-rt} P_O^{II}(t)\, q_{i,O}^{II}(t)\, \mathrm{d}t \\ &= e^{-rT^*}\, 2^{\alpha-1} a \frac{1-\alpha}{r} \left(1 - \left(\frac{\alpha}{1+\alpha}\right)^{\alpha}\right) \left(\frac{r}{1-\alpha} R_A(0)\right)^{\alpha},\end{aligned}$$

and the total Markovian revenue corresponds to:

$$\pi_{i,M}^{[T^*,\widehat{T}_{MO}]} = \frac{\left(\frac{1+\alpha}{\alpha}\right)^{\alpha} - \left(\frac{\alpha}{1+\alpha}\right)^{\alpha}}{2\left(1 - \left(\frac{\alpha}{1+\alpha}\right)^{\alpha}\right)}\, \pi_{i,O}^{[T^*,\widehat{T}_{MO}]} > \pi_{i,O}^{[T^*,\widehat{T}_{MO}]}. \tag{4.35}$$

As concerns the period $[\widehat{T}_{MO}, +\infty]$*, the total open-loop revenue, discounted to* $t = 0$*, is*

$$\begin{aligned}\pi_{i,O}^{[\widehat{T}_{MO},+\infty]} &= \int_{\widehat{T}_{MO}}^{+\infty} e^{-rt} P_O^{II}(t)\, q_{i,O}^{II}(t)\, \mathrm{d}t \\ &= e^{-rT^*} \left(\frac{2r}{1-\alpha}\right)^{\alpha-1} \left(\frac{\alpha}{1+\alpha} R_A(0)\right)^{\alpha},\end{aligned}$$

and the total Markovian revenue is

$$\pi_{i,M}^{[\widehat{T}_{MO},+\infty]} = \frac{1}{2}\, \pi_{i,O}^{[\widehat{T}_{MO},+\infty]} < \pi_{i,O}^{[\widehat{T}_{MO},+\infty]}. \tag{4.36}$$

Nevertheless, it is straightforward that when extending the period $[T^*, \widehat{T}_{MO}]$ to some finite time $\overline{T}_{MO} > \widehat{T}_{MO}$, the greater Markovian revenue of Equation (4.35) will progressively be counterbalanced by the lower instantaneous Markovian revenues of Equation (4.34). Ultimately, this process turns out to allow for the initial open-loop losses to be overcompensated, so that in the end, none of the competitors benefits from playing Markovian strategies in the second period. The ensuing proposition outlines this finding.

Proposition 11. *If the duopolistic game is played successively under open-loop strategies with continuous supply at A's entry* T_O^* *and Markovian strategies, then*

$$\pi_{i,M}^{II} < \pi_{i,O}^{II}.$$

Comparison 3: Scenarios M and P

In contrast to the strategy set (M, O), the duopolistic periods of scenarios M and P start at different times, that is, $T_P^* < T_M^*$, and the open-loop extraction of scenario P is below the one of the previously examined scenario O, that is, $q_{i,P}^{II}(t) < q_{i,O}^{II}(t),\ \forall t \geq\ T_O^*$. Despite these discrepancies, the characteristics of the open-loop and Markovian extraction behaviors remain comparable to those in the former comparison and so, similar discoveries are made. As a matter of fact, we observe that the Markovian market prices and instantaneous revenues are profitable for consumers and suppliers, respectively, as from the Markovian entry T_M^* until the more pronounced extraction decrease of scenario M lets the supply functions intersect at some tipping point $\widehat{T}_{MP}$. The proposition below and Figure 4.4 present this outcome.

Proposition 12. *Suppose the two-period game is played successively under both Markovian and open-loop strategies and the open-loop price is continuous at A's market entry T_P^*. Then, in the second period, there exists one and only one time*

$$\widehat{T}_{MP} = \frac{\alpha(1-\alpha)}{r} \ln\left(\frac{1+\alpha}{\alpha}\left(\frac{R_C(0)}{R_A(0)}\right)^{\frac{1+\alpha}{\alpha}} \frac{2R_A(0)}{R_C(0)+R_A(0)}\right) > T_M^*,$$

where the ratio of Markovian and open-loop supplies is

$$\frac{q_{i,M}^{II}(t)}{q_{i,P}^{II}(t)} = \mathrm{e}^{\frac{rt}{\alpha(\alpha-1)}}\,\frac{1+\alpha}{\alpha}\frac{2R_A(0)}{R_C(0)+R_A(0)}\left(\frac{R_C(0)}{R_A(0)}\right)^{\frac{1+\alpha}{\alpha}} \begin{cases} > 1, \text{ if } \ t \in [T_M^*, \widehat{T}_{MP}), \\ < 1, \text{ if } \ t \in (\widehat{T}_{MP}, +\infty), \end{cases}$$

and the ratio of the corresponding market prices becomes

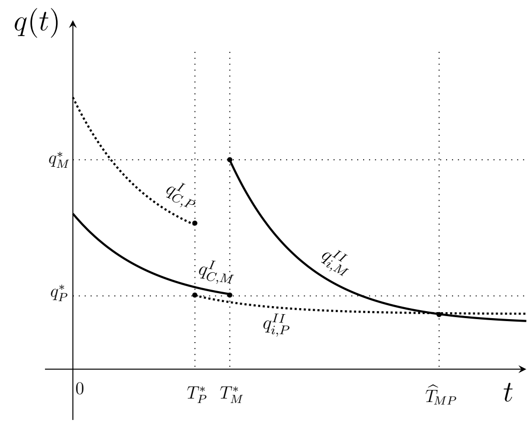
$$\frac{P_M^{II}(t)}{P_P^{II}(t)} = \left(\frac{q_{i,M}^{II}(t)}{q_{i,P}^{II}(t)}\right)^{\alpha-1} \begin{cases} < 1, \text{ if } \ t \in [T_M^*, \widehat{T}_{MP}), \\ > 1, \text{ if } \ t \in (\widehat{T}_{MP}, +\infty). \end{cases}$$

As a result, the ratio of the Markovian and open-loop instantaneous revenues is

$$\frac{\pi_{i,M}^{II}(t)}{\pi_{i,P}^{II}(t)} = \left(\frac{q_{i,M}^{II}(t)}{q_{i,P}^{II}(t)}\right)^{\alpha} \begin{cases} > 1, \text{ if } \ t \in [T_M^*, \widehat{T}_{MP}), \\ < 1, \text{ if } \ t \in (\widehat{T}_{MP}, +\infty). \end{cases} \tag{4.37}$$

On the one hand, Proposition 12 entails that, similarly to the observation in Equation (4.36), the summed Markovian revenue over $[\widehat{T}_{MP}, +\infty)$ corresponds to only half of the open-loop one. On the other hand, however, the earlier start of the duopolistic open-loop scenario P implies that, unlike in Equation (4.35), the sign between the summed second-period revenues before the turning point $\widehat{T}_{MP}$, depends on the elasticity parameter α and the reserve ratio $\frac{R_C(0)}{R_A(0)}$.

Figure 4.4 – Supply differences: M, P



Comparing both revenues numerically yields that the open-loop strategies between $[T_P^*, \widehat{T}_{MP}]$ are more likely to perform better financially than the Markovian strategies between $[T_M^*, \widehat{T}_{MP}]$ as $\frac{R_C(0)}{R_A(0)}$ increases (see Appendix B, Figure 4.7). Whenever this is the case, the open-loop revenue accumulated over $[T_P^*, T_M^*]$ balances out the higher Markovian revenue over $[T_M^*, \widehat{T}_{MP}]$. Otherwise, given that open-loop revenues eventually turn out beneficial (see Proposition 13 below), there must again exist a finite time $\overline{T}_{MP} > \widehat{T}_{MP}$, where the higher instantaneous open-loop profits over $[\widehat{T}_{MP}, +\infty)$ make up for the losses before $\widehat{T}_{MP}$.

Proposition 13. *If the duopolistic game is played successively under open-loop strategies with continuous price at A's entry T_P^* and Markovian strategies, then*

$$\pi_{i,M}^{II} < \pi_{i,P}^{II}.$$

Comparison 4: Scenarios (O, S) and (P, S)

Although the present comparison also studies the duopolistic differences between open-loop and Markovian strategies, the conclusions this time more strongly vary with the elasticity parameter α and the initial reserves $\frac{R_C(0)}{R_A(0)}$.

Actually, concerning the set (O, S), where the competition of the Markovian scenario S starts well before the open-loop one of O, that is, $T_S^* < T_O^*$, it is unclear which of the two scenarios generates more consumer- and supplier-friendly prices and revenues, respectively, when both duopolistic periods exist in parallel. Numerically we find that the open-loop behavior continually scores better than the Markovian one only if the ratio $\frac{R_C(0)}{R_A(0)} \geq 2.5$ (see Appendix B, Figure 4.8). Else, the Markovian behavior could, similarly to the two latter comparisons, initially beat the open-loop one in terms of profitability. Whenever the latter is the case, that is, if the supply functions cross at some tipping point $\widehat{T}_{OS} \in (T_O^*, +\infty)$, we get from Proposition 7 that its timing is decisive for the relation between the total duopolistic payoffs. Indeed, when playing open-loop strategies in the first period and keeping the supply continuous at A's entry, then the probability that higher profits result from sticking to open-loop strategies in the second period, instead of switching to Markovian ones, increases with the ratio $\frac{R_C(0)}{R_A(0)}$ and even becomes certain once $\frac{R_C(0)}{R_A(0)} \geq 2$ (see Appendix B, Figure 4.9):

$$\frac{\pi_{i,O}^{II}}{\pi_{i,S}^{II}} = \begin{cases} \lesseqgtr 1, \text{ if } \frac{R_C(0)}{R_A(0)} < 2, \\ > 1, \text{ if } \frac{R_C(0)}{R_A(0)} \geq 2. \end{cases} \tag{4.38}$$

Consequently, while for $\frac{R_C(0)}{R_A(0)} \in [2, 2.5)$ and $t \in [T_S^*, \widehat{T}_{OS})$, the instantaneous Markovian revenues could theoretically be greater than the open-loop ones, Equation (4.38) illustrates that the accumulated benefits are not sufficient to offset the more profitable open-loop performance after $\widehat{T}_{SO}$.

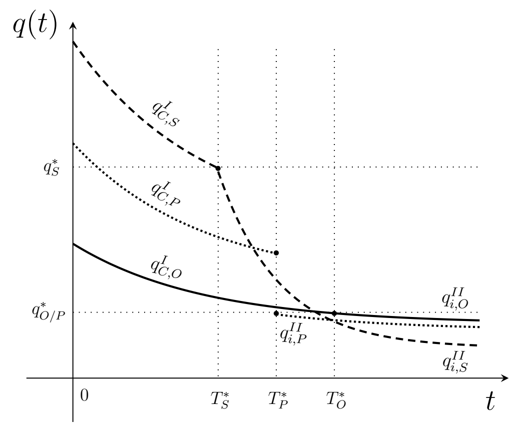
Regarding the set (P, S), where the Markovian competition of scenario S also starts earlier than the open-loop one of P, that is, $T_S^* < T_P^*$, the Markovian strategies invariably yield unfavorable market prices and instantaneous revenue for consumers and suppliers, respectively, as from the start of the open-loop competition at T_P^*, if the ratio $\frac{R_C(0)}{R_A(0)} \geq 3.5$ (see Appendix B, Figure 4.10). Nevertheless, in contrast to the above strategy set (O, S), sticking to an open-loop supply path over the two periods, while dropping the extraction at A's entry, always generates higher duopolistic revenues than changing from open-loop to Markovian strategies in between the periods and keeping the supply continuous at the entry. This finding is illustrated in the proposition below.

Proposition 14. *If the duopolistic game is played successively under open-loop strategies with* *continuous price at A's entry T_P^* and Markovian strategies, then*

$$\pi_{i,P}^{II} > \pi_{i,S}^{II}.$$

The figure below presents the supply differences between scenarios O, P and S, for both $\frac{R_C(0)}{R_A(0)} \in (2.4, 3.5)$ and $\alpha \in (0.39, 0.6)$.

Figure 4.5 – Supply differences: O, P, S



4.3.3 Aggregated analysis

Even though the outcomes of Subsection 4.3.2 show that country A comes off best when scenario P is played, that is, when open-loop strategies are played in both periods and the market price remains continuous at the entry, the latter situation will most probably not occur. The reason therefore is that at the moment of A's entry-announcement $t = 0$, country C opts for an optimal strategy scenario by comparing its own aggregated revenues. Determining the aggregated revenues of country C by means of the previously defined first- and second-period revenues yields the following proposition.

Proposition 15. *If country A's optimal entry time is T_ψ^*, where $\psi \in \{M, O, P, S\}$, then*

$$\Pi_{C,O} > \Pi_{C,M} \lesseqgtr \Pi_{C,P} > \Pi_{C,S}.$$

This signifies that A's first-best scenario P competes from C's perspective for the second-best place with scenario M and should thus not be envisaged. More specifically, the relation between the two aggregated revenues varies with the elasticity parameter α and the initial condition $\frac{R_C(0)}{R_A(0)}$ (see Appendix B, Figure 4.11). Numerical tests show that whenever the monopoly-to-duopoly game is played successively, under either open-loop strategies with continuous supply or Markovian strategies, then

$$\frac{\Pi_{C,P}}{\Pi_{C,M}} = \begin{cases} > 1, \text{ if } \; R_C(0) \leq 2.4\, R_A(0), \\ \lesseqgtr 1, \text{ if } \; 2.4\, R_A(0) < R_C(0) < 3.7\, R_A(0), \\ < 1, \text{ if } \; R_C(0) \geq 3.7\, R_A(0). \end{cases}$$

Furthermore, Proposition 15 connotes that in order to optimize its payoff, country C should select scenario O and sticking to an open-loop supply path in both periods, while keeping its supply continuous at the entry of country A. Finally, we also see that country C should in no case play scenario S and link the application of alternative strategy spaces in both periods with a steady supply at the entry.

4.4 Conclusion

The novelty of this chapter is twofold: firstly, it takes into account the particularities of REEs, for instance, the importance of their availability, the non-existence of substitutes and the presence of a politically and economically strong dominating supplier that holds prevailing reserves; secondly, it offers closed-form expressions of Markovian-strategic interactions. In fact, even tough the commonly used alternative of time-dependent open-loop strategies is acceptable for non-renewable resource games, a more lifelike depiction of market players' decision-making originates from time- and reserve-dependent Markovian strategies. Yet, given that the finding of explicit Markovian solutions requires "a good deal of experience and mathematical creativity" (Dockner et al., 2000), this strategy type has so far received little attention.

With a view to putting the above into practice, we consider a Markovian game, where an incumbent REEs supplier, China, who managed to dominate and control the market for the past three decades, is threatened by a potential newcomer, the U.S., who aspires—despite its substantially scarcer reserves—to become (once again) an independent but mighty competitor. Technically speaking, we set up a two-period continuous-time model with no backstop technology and with resource depletion in the very long-run, and solve both the first-period finite horizon optimal control problem and the second-period infinite horizon differential game, triggered by the U.S.'s production launch, for Markovian (Nash equilibrium) strategies. Since our analysis puts strong focus on gaining further insights about the extend to which model outcomes can be affected by the choice of strategy spaces, we subsequently compare the Markovian findings with the outcomes of our earlier work (Chapter 3), where a similar game is solved for open-loop strategies under opposite transversality conditions. Not only does this practice allow for the discrepancies between the two competitors' extraction manners to be evaluated but also does this permit to determine their overall optimal strategic alignment.

In the latter context, the open-loop and Markovian comparison reveals that: (1) the outcomes of optimal control problems with identical initial and terminal conditions are unaffected by the applied strategy concept; contrarily, (2) differential game Nash equilibrium outcomes are shaped by the choice of strategy, that is, while the more flexible Markovian decision rules initially turn out beneficial for both consumers and suppliers, the situation is gradually reversed as the game proceeds. Obviously, these outcomes rely on the REEs exhaustion assumption in infinite time. Which strategy's summed payoffs eventually prevail in the differential game, occasionally depends, next to the price elasticity of demand, on the countries initial reserves. However, as the U.S. Geological Survey (2020) states that China currently holds thirty times larger reserves than the U.S., our numerical simulations connote that the potential newcomer suffers financial damage from adopting a Markovian behavior. This observation also holds true from China's perspective: although here the overall Markovian performance emerges to be slightly better, the payoffs only rank second-best. The strategy space that is instead found to be optimal for China, and thus most likely to characterize the two-period game, consists of open-loop commitments and a continuous supply at the moment of the U.S.'s market entry.

The extension of the present analysis can be approached in different ways. Since the dependency on China's rare earths exports has led other global players like Australia, Russia and Japan to make plans about enhancing their domestic mining and processing capabilities, our duopolistic model structure could be swapped for a cartel-fringe setting. While following the standard assumption of the competitive fringe being comprised of smaller rare earths suppliers, the cartel would here be imagined to consist of two dominant players, the U.S. and China. With the market then being shared by more players, the inclusion of stock-dependent extraction costs wins in importance. In addition, rather than to consider a Nash equilibrium concept, where players act concurrently, a Stackelberg solution with a leading decision-maker would make good sense; especially when considering a cartel-fringe framework.

Appendix A: Proof of Lemma 1

Recall that the optimal control problem faced by the competitors in the second time period looks as follows:

$$\pi_i^{II}(R_C^*, T^*) = \max_{q_i^{II}(R_i,t)\geq 0} \int_{T^*}^{+\infty} e^{-r(t-T^*)} P^{II}(Q(R_i,t))\ q_i^{II}(R_i,t)\ \mathrm{d}t,$$

subject to

$$\int_{T^*}^{+\infty} q_i^{II}(R_i,t)\ \mathrm{d}t \leq R_i(T^*) = \begin{cases} R_A(0) \text{ given, if } i = A, \\ R_C^*, \text{ if } i = C, \end{cases}$$

and

$$\dot{R}_i(t) = -q_i^{II}(R_i,t)\,,\ t \in [T^*, +\infty).$$

The stationary Hamilton-Jacobi-Bellman (HJB) equation of the above problem is thus given by

$$r\, W_i(R_i) = \max_{q_i^{II}} \left\{ P^{II}(Q)\, q_i^{II} - W_i'(R_i)\, q_i^{II} \right\},$$

which yields the corresponding first-order condition (FOC)

$$W_i'(R_i) = a\ \left(q_i^{II} + q_j^{II}\right)^{\alpha-2}\ \left(\alpha q_i^{II} + q_j^{II}\right).$$

It follows that

$$\frac{W_i'(R_i)}{W_j'(R_j)} = \frac{\alpha q_i^{II} + q_j^{II}}{\alpha q_j^{II} + q_i^{II}},$$

where $i, j \in \{A, C\}$ and $i \neq j$.

Moreover, given that country i's Markovian supply strategy $q_i^{II}(R_i)$ depends only on its own reserves R_i and not on the ones of country j, we can apply the envelope theorem to differentiate both sides of the HJB equation with respect to R_i. This gives

$$r\, W_i'(R_i) = W_i''(R_i)\ \left(-q_i^{II}\right),$$

which, based on the dynamic equation of the above problem, can be rewritten as

$$r = \frac{W_i''(R_i)}{W_i'(R_i)}\, \dot{R}_i. \tag{4.39}$$

From Equation (4.39) it then follows that

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{W_i'(R_i)}\right) = -\frac{W_i''(R_i)}{(W_i'(R_i))^2}\,\dot{R}_i = -\frac{1}{W_i'(R_i)}\,r. \tag{4.40}$$

Finally, solving the differential equation of (4.40) leads to:

$$W_i'(R_i(t)) = W_i'(R_i(T^*))\,\mathrm{e}^{r(t-T^*)}, \tag{4.41}$$

where $W_i'(R_i(T^*)) = \frac{\mathrm{d}W_i(R_i(T^*))}{\mathrm{d}R_i}$, and so

$$\frac{W_i'(R_i(t))}{W_j'(R_j(t))} = \frac{W_i'(R_i(T^*))}{W_j'(R_j(T^*))}. \tag{4.42}$$

At this point, we can combine Equations (4.4) and (4.42) to find that

$$\frac{\alpha q_i^{II} + q_j^{II}}{\alpha q_j^{II} + q_i^{II}} = \frac{W_i'(R_i(T^*))}{W_j'(R_i(T^*))},$$

which, after rearranging, corresponds to:

$$q_i^{II}(R_i(t)) = \frac{\alpha W_i'(R_i(T^*)) - W_j'(R_j(T^*))}{\alpha W_j'(R_j(T^*)) - W_i'(R_i(T^*))}\,q_j^{II}(R_j(t)). \tag{4.43}$$

Under the assumption that both competitors fully exhaust their initial reserves $R_i(T^*)$ over the second-period planning horizon, because from an economical viewpoint it cannot be optimal to leave some REEs in the deposit (no market value), integrating Equation (4.43) over $[T^*, +\infty)$ yields

$$\frac{R_i(T^*)}{R_j(T^*)} = \frac{\alpha W_i'(R_i(T^*)) - W_j'(R_j(T^*))}{\alpha W_j'(R_j(T^*)) - W_i'(R_i(T^*))}, \tag{4.44}$$

which can also be expressed as

$$W_i'(R_i(T^*)) = \frac{\alpha R_i(T^*) + R_j(T^*)}{\alpha R_j(T^*) + R_i(T^*)}\,W_j'(R_j(T^*)). \tag{4.45}$$

Then, after combining Equations (4.43) and (4.44), we have

$$\frac{q_A^{II}(R_A(t))}{q_C^{II}(R_C(t))} = \frac{R_A(0)}{R_C^*},\ \forall t \geq T^*.$$

The conjecture that there is no arbitrage opportunity for any of the countries at the entry time T^*, means that

$$W_i'(R_i(T^*)) = W_j'(R_j(T^*)).$$

From Equations (4.41) and (4.45), we obtain that the above no-arbitrage-condition is satisfied if and only if

$$R_C^* = R_A(0).$$

This finishes the proof of Lemma 1 in Section 4.2.1.

Appendix B: Figures

Figure 4.6 – Revenue tipping point $\overline{T}_{MO}$

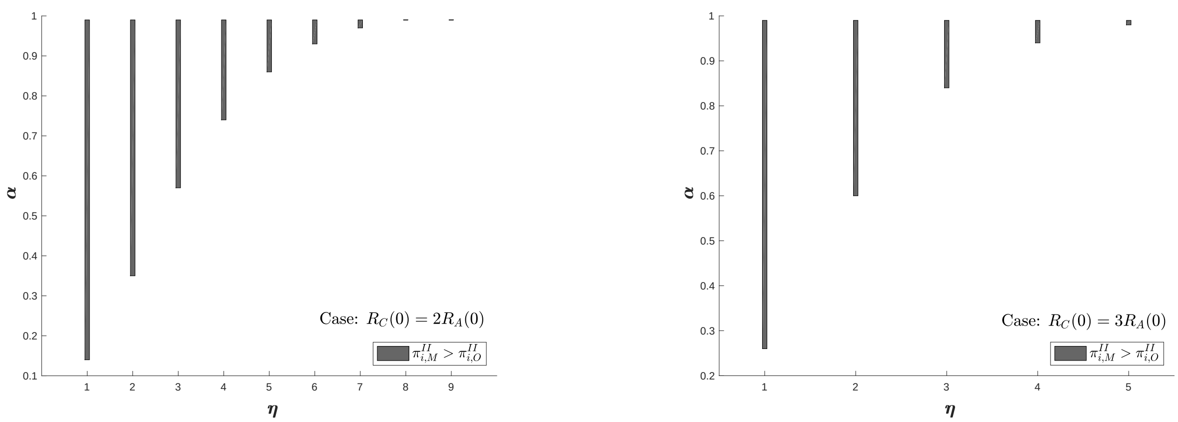


Figure 4.7 – Summed profits before $\widehat{T}_{MP}$

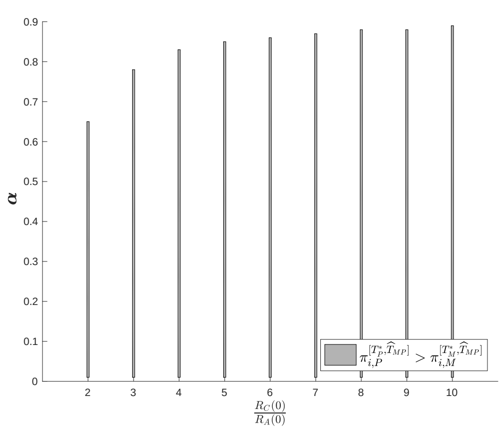


Figure 4.8 – Instantaneous profits ex T_O^*

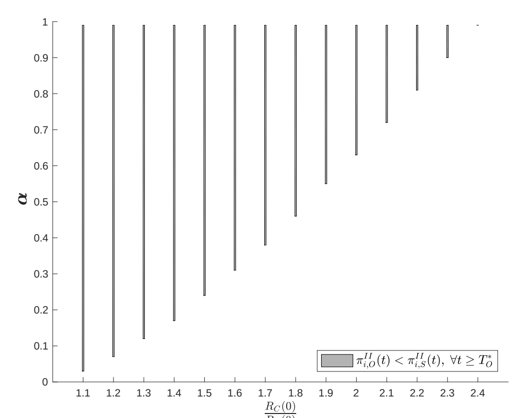


Figure 4.9 – Second-period profits: O, S

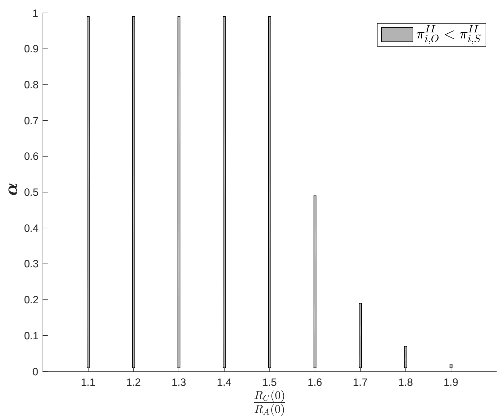


Figure 4.10 – Instantaneous profits ex T_P^*

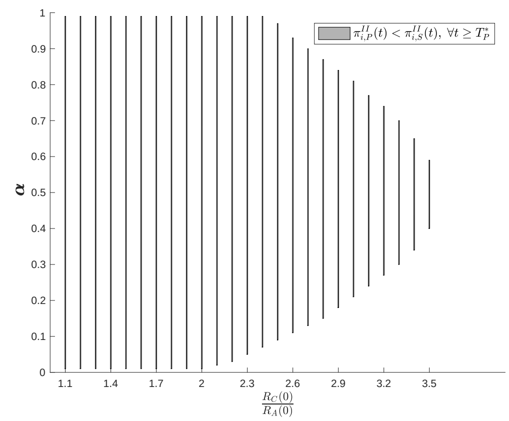
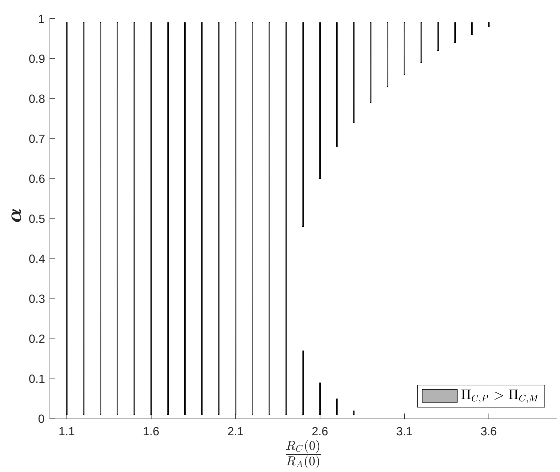


Figure 4.11 – Aggregated profits: M, P



Bibliography

ADEME (2013). *Les chiffres clés du bâtiment*, pages 1–91.

Alonso, E., Sherman, A. M., Wallington, T. J., Everson, M. P., Field, F. R., Roth, R., and Kirchain, R. E. (2012). Evaluating rare earth element availability: A case with revolutionary demand from clean technologies. *Environmental Science & Technology*, 46(6):3406–3414.

Aminrad, Z., Zakaria, S., and Hadi, A. S. (2011). Influence of age and level of education on environmental awareness and attitude: case study on Iranian students in Malaysian Universities. *The Social Sciences*, 6(1):15–19.

Amstalden, R. W., Kost, M., Nathani, C., and Imboden, D. M. (2007). Economic potential of energy-efficient retrofitting in the Swiss residential building sector: The effects of policy instruments and energy price expectations. *Energy Policy*, 35(3):1819–1829.

Ástmarsson, B., Jensen, P. A., and Maslesa, E. (2013). Sustainable renovation of residential buildings and the landlord/tenant dilemma. *Energy Policy*, 63:355–362.

Balaram, V. (2019). Rare earth elements: A review of applications, occurrence, exploration, analysis, recycling, and environmental impact. *Geoscience Frontiers*, 10(4):1285–1303.

Bazilian, M. D. (2018). The mineral foundation of the energy transition. *The Extractive Industries and Society*, 5(1):93–97.

BBSR (2016). Mietrecht und energetische Sanierung im europäischen Vergleich. *BBSR Online Publikation*, (13):1–146.

Benchekroun, H. and Gaudet, G. (2003). On the profitability of production perturbations in a dynamic natural resource oligopoly. *Journal of Economic Dynamics and Control*, 27(7):1237–1252.

Benchekroun, H., Halsema, A., and Withagen, C. (2009). On nonrenewable resource oligopolies: The asymmetric case. *Journal of Economic Dynamics and Control*, 33(11):1867–1879.

Benchekroun, H. and Withagen, C. (2012). On price taking behavior in a nonrenewable resource cartel-fringe game. *Games and Economic Behavior*, 76(2):355–374.

Binnemans, K., Jones, P. T., Van Acker, K., Blanpain, B., Mishra, B., and Apelian, D. (2013). Rare-earth economics: The balance problem. *JOM*, 65(7):846–848.

Birol, F. et al. (2010). World Energy Outlook 2010. *International Energy Agency*, 1(3):1–738.

Boeters, S. and Koornneef, J. (2011). Supply of renewable energy sources and the cost of EU climate policy. *Energy Economics*, 33(5):1024–1034.

Böhringer, C., Koschel, H., and Moslener, U. (2008). Efficiency losses from overlapping regulation of EU carbon emissions. *Journal of Regulatory Economics*, 33(3):299–317.

Boonekamp, P. G. (2006). Actual interaction effects between policy measures for energy efficiency—a qualitative matrix method and quantitative simulation results for households. *Energy*, 31(14):2848–2873.

Bovenberg, A. L., Goulder, L. H., and Gurney, D. J. (2005). Efficiency costs of meeting industry-distributional constraints under environmental permits and taxes. *RAND Journal of Economics*, 36(4):951–971.

Braun, F. G. (2010). Determinants of households space heating type: A discrete choice analysis for German households. *Energy Policy*, 38(10):5493–5503.

British Geological Survey (2015). Risk list 2015. pages 1–8.

Bruvoll, A. and Larsen, B. M. (2004). Greenhouse gas emissions in Norway: Do carbon taxes work? *Energy Policy*, 32(4):493–505.

Bye, B., Fæhn, T., and Rosnes, O. (2018). Residential energy efficiency policies: Costs, emissions and rebound effects. *Energy*, 143:191–201.

Callan, T., Lyons, S., Scott, S., Tol, R. S., and Verde, S. (2009). The distributional implications of a carbon tax in Ireland. *Energy Policy*, 37(2):407–412.

Capros, P., De Vita, A., Tasios, N., Siskos, P., Kannavou, M., Petropoulos, A., Evangelopoulou, S., Zampara, M., Papadopoulos, D., Nakos, C., et al. (2016). EU Reference Scenario 2016: Energy, transport and GHG emissions—Trends to 2050. pages 1–221.

Carley, K. M. (2009). Computational modeling for reasoning about the social behavior of humans. *Computational and Mathematical Organization Theory*, 15(1):47–59.

Cayla, J.-M., Maizi, N., and Marchand, C. (2011). The role of income in energy consumption behaviour: Evidence from French households data. *Energy Policy*, 39(12):7874–7883.

Cayre, E., Allibe, B., Laurent, M.-H., and Osso, D. (2011). There are people in the house! How the results of purely technical analysis of residential energy consumption are misleading for energy policies. *Proceedings of the ECEEE Summer Study*, Paper 7-277:1675–1683.

Chakhmouradian, A. R. and Wall, F. (2012). Rare earth elements: minerals, mines, magnets (and more). *Elements*, 8(5):333–340.

Chakravorty, U., Roumasset, J., and Tse, K. (1997). Endogenous substitution among energy resources and global warming. *Journal of Political Economy*, 105(6):1201–1234.

Chapman, A., Arendorf, J., Castella, T., Thompson, P., Willis, P., Espinoza, L., Klug, S., and Wichmann, E. Study on critical raw materials at EU level. *Oakdene Hollins*, pages 1–158.

Charlier, D. (2015). Energy efficiency investments in the context of split incentives among French households. *Energy Policy*, 87:465–479.

Chu, S. (2010). Critical materials strategy. *U.S. Department of Energy*, pages 1–165.

Clemhout, S. and Wan, H. (1991). Environmental problem as a commonproperty resource game. In *Dynamic Games in Economic Analysis*, pages 132–154. Springer.

Connelly, N. G., Hartshorn, R. M., Damhus, T., and Hutton, A. T. (2005). *Nomenclature of inorganic chemistry: IUPAC recommendations 2005*. Royal Society of Chemistry.

Dasgupta, P., Gilbert, R. J., and Stiglitz, J. E. (1982). Invention and innovation under alternative market structures: The case of natural resources. *The Review of Economic Studies*, 49(4):567–582.

De Boer, M. and Lammertsma, K. (2013). Scarcity of rare earth elements. *ChemSusChem*, 6(11):2045–2055.

Dockner, E. J., Jorgensen, S., Van Long, N., and Sorger, G. (2000). *Differential games in economics and management science*. Cambridge University Press.

Dockner, E. J. and Sorger, G. (1996). Existence and properties of equilibria for a dynamic game on productive assets. *Journal of Economic Theory*, 71(1):209–227.

Dudley, B. (2017). BP Energy Outlook: 2017 Edition. pages 1–103.

Dutta, T., Kim, K.-H., Uchimiya, M., Kwon, E. E., Jeon, B.-H., Deep, A., and Yun, S.-T. (2016). Global demand for rare earth resources and strategies for green mining. *Environmental Research*, 150:182–190.

Eswaran, M. and Lewis, T. (1985). Exhaustible resources and alternative equilibrium concepts. *Canadian Journal of Economics*, pages 459–473.

European Commission (2010). Energy 2020: A strategy for competitive, sustainable and secure energy. *Communication from the Commission to the European Parliament and others*, pages 1–20.

European Council (2014). Conclusions on 2030 climate and energy policy framework. *EUCO*, 169:14.

Eurostat (2017). European Union Statistics on Income and Living Conditions (EU-SILC).

Falkner, R. (2016). The Paris Agreement and the new logic of international climate politics. *International Affairs*, 92(5):1107–1125.

Fankhauser, S., Hepburn, C., and Park, J. (2010). Combining multiple climate policy instruments: How not to do it. *Climate Change Economics*, 1(03):209–225.

Federal Register (2017). Presidential executive order on a federal strategy to ensure secure and reliable supplies of critical minerals.

Fleiter, T., Worrell, E., and Eichhammer, W. (2011). Barriers to energy efficiency in industrial bottom-up energy demand models—A review. *Renewable and Sustainable Energy Reviews*, 15(6):3099–3111.

Flues, F., Löschel, A., Lutz, B. J., and Schenker, O. (2014). Designing an EU energy and climate policy portfolio for 2030: Implications of overlapping regulation under different levels of electricity demand. *Energy Policy*, 75:91–99.

Gaudet, G. (2007). Natural resource economics under the rule of hotelling. *Canadian Journal of Economics*, 40(4):1033–1059.

Geller, H., Harrington, P., Rosenfeld, A. H., Tanishima, S., and Unander, F. (2006). Policies for increasing energy efficiency: Thirty years of experience in OECD countries. *Energy Policy*, 34(5):556–573.

Gerlagh, R. and Liski, M. (2014). Cake-eating with private information. *CESifo Working Paper Series*.

Gerlagh, R. and Van der Zwaan, B. (2006). Options and instruments for a deep cut in CO_2 emissions: Carbon dioxide capture or renewables, taxes or subsidies? *The Energy Journal*, 27(3):25–48.

Ghalwash, T. (2007). Energy taxes as a signaling device: An empirical analysis of consumer preferences. *Energy Policy*, 35(1):29–38.

Gilbert, R. J. (1978). Dominant firm pricing policy in a market for an exhaustible resource. *The Bell Journal of Economics*, pages 385–395.

Gilbert, R. J., Goldman, S. M., et al. (1978). Potential competition and the monopoly price of an exhaustible resource. *Journal of Economic Theory*, 17(2):319–331.

Gillingham, K., Harding, M., and Rapson, D. (2012). Split incentives in residential energy consumption. *The Energy Journal*, 33(2):37–62.

Gillingham, K., Keyes, A., and Palmer, K. (2018). Advances in evaluating energy efficiency policies and programs. *Annual Review of Resource Economics*, 10:511–532.

Gillingham, K. and Palmer, K. (2014). Bridging the energy efficiency gap: Policy insights from economic theory and empirical evidence. *Review of Environmental Economics and Policy*, 8(1):18–38.

Giraudet, L.-G., Branger, F., Guivarch, C., and Quirion, P. (2015). Global sensitivity analysis of an energy-economy model of the residential building sector. *Environmental Modelling & Software*, 70:45–54.

Giraudet, L.-G., Guivarch, C., and Quirion, P. (2011). Comparing and combining energy saving policies: Will proposed residential sector policies meet French official targets? *The Energy Journal*, 32:213–242.

Giraudet, L.-G., Guivarch, C., and Quirion, P. (2012). Exploring the potential for energy conservation in French households through hybrid modeling. *Energy Economics*, 34(2):426–445.

Goodenough, K. M., Wall, F., and Merriman, D. (2018). The rare earth elements: demand, global resources, and challenges for resourcing future generations. *Natural Resources Research*, 27(2):201–216.

Gordon B. Haxel, J. B. H. and Orris, G. J. (2002). Rare earth elements: Critical resources for high technology. *U.S. Geological Survey*, pages 1–4.

Groot, F., Withagen, C., and De Zeeuw, A. (2003). Strong time-consistency in the cartel-versus-fringe model. *Journal of Economic Dynamics and Control*, 28(2):287–306.

Haas, T. and Peltier, F. (2017). Projections macroéconomiques et démographiques de long terme: 2017-2060. *Bulletin du STATEC*, (3):1–53.

Harris, C., Howison, S., and Sircar, R. (2010). Games with exhaustible resources. *SIAM Journal on Applied Mathematics*, 70(7):2556–2581.

Hausman, J. A. (1979). Individual discount rates and the purchase and utilization of energy-using durables. *The Bell Journal of Economics*, 10(1):33–54.

Herfindahl, O. C. (1967). Depletion and economic theory. *Extractive Resources and Taxation*, pages 63–90.

Högberg, L. (2013). The impact of energy performance on single-family home selling prices in Sweden. *Journal of European Real Estate Research*, 6(3):242–261.

Hornby, L. and Sanderson, H. (2019). Rare earths: Beijing threatens a new front in the trade war. *Financial Times*.

Hörner, M., Cischinsky, H., and Lichtmeß, M. (2016). Analyse der Diskrepanz von Energiebedarf und-verbrauch bei Energiepässen von Wohngebäuden in Luxemburg: Teil 1: Methode der multiplen linearen Regression. *Bauphysik*, 38(3):166–175.

Hotelling, H. (1931). The economics of exhaustible resources. *Journal of Political Economy*, 39(2):137–175.

Hourcade, J.-C., Jaccard, M., Bataille, C., and Ghersi, F. (2006). Hybrid modeling: New answers to old challenges. Introduction to the special issue of 'The Energy Journal'. *The Energy Journal*, 27:1–11.

Huang, P., Zhang, X., and Deng, X. (2006). Survey and analysis of public environmental awareness and performance in Ningbo, China: A case study on household electrical and electronic equipment. *Journal of Cleaner Production*, 14(18):1635–1643.

Huq, S. and Ayers, J. (2007). Critical list: The 100 nations most vulnerable to climate change. *IIED Sustainable Development Opinion*.

Islam, N. and Winkel, J. (2017). Climate change and social inequality. *DESA Working Paper*.

Itard, L. (2008). Towards a sustainable Northern European housing stock: Figures, facts, and future. *IOS Press*, pages 1–213.

Jaccard, M. and Dennis, M. (2006). Estimating home energy decision parameters for a hybrid energy-economy policy model. *Environmental Modeling & Assessment*, 11(2):91–100.

Jaffe, A. B., Newell, R. G., and Stavins, R. N. (2004). Economics of energy efficiency. *Encyclopedia of Energy*, 2:79–90.

Jaffe, A. B. and Stavins, R. N. (1994). The energy-efficiency gap. What does it mean? *Energy Policy*, 22(10):804–810.

Kagermann, H. (2015). Change through digitization—Value creation in the age of Industry 4.0. In *Management of Permanent Change*, pages 23–45. Springer.

Kamien, M. I. and Schwartz, N. L. (1981). *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*. Elsevier North-Holland Publishing Co.

Knobloch, F., Pollitt, H., Chewpreecha, U., and Mercure, J.-F. (2019). Simulating the deep decarbonisation of residential heating for limiting global warming to 1.5 C°. *Energy Efficiency*, 12(2):521–550.

Köppel, S. and Ürge-Vorsatz, D. (2007). Assessment of policy instruments for reducing greenhouse gas emissions from buildings. Report for the UNEP-Sustainable Buildings and Construction Initiative. *United Nations Environment Programme and Central European University*, pages 1–91.

Krishnamurthy, N. and Gupta, C. K. (2015). *Extractive metallurgy of rare earths*. CRC Press.

Lechtenböhmer, S. and Schüring, A. (2011). The potential for large-scale savings from insulating residential buildings in the EU. *Energy Efficiency*, 4(2):257–270.

Lee, W. and Yik, F. (2004). Regulatory and voluntary approaches for enhancing building energy efficiency. *Progress in Energy and Combustion Science*, 30(5):477–499.

Levine, M., Ürge-Vorsatz, D., Blok, K., Geng, L., Harvey, D., Lang, S., Levermore, G., Mongameli Mehlwana, A., Mirasgedis, S., Novikova, A., et al. (2007). Residential and commercial buildings. *Climate Change*, 20:1–17.

Lewis, T. R. and Schmalensee, R. (1980). On oligopolistic markets for nonrenewable natural resources. *The Quarterly Journal of Economics*, 95(3):475–491.

Lichtmeß, M. (2013). Das Problem mit der neutralen Bewertung der Gebäudeenergieeffizienz von Wohngebäuden. *Cahier Scientifique, Revue Technique Luxembourgeoise*.

Lin, B. and Li, X. (2011). The effect of carbon tax on per capita CO_2 emissions. *Energy Policy*, 39(9):5137–5146.

Lindner, M., Maroschek, M., Netherer, S., Kremer, A., Barbati, A., Garcia-Gonzalo, J., Seidl, R., Delzon, S., Corona, P., Kolström, M., et al. (2010). Climate change impacts, adaptive capacity, and vulnerability of European forest ecosystems. *Forest Ecology and Management*, 259(4):698–709.

Liski, M. and Montero, J.-P. (2014). Forward trading in exhaustible-resource oligopoly. *Resource and Energy Economics*, 37:122–146.

Loury, G. C. (1986). A theory of'oil'igopoly: Cournot equilibrium in exhaustible resource markets with fixed supplies. *International Economic Review*, 27(2):285–301.

Mancheri, N. A. (2015). World trade in rare earths, chinese export restrictions, and implications. *Resources Policy*, 46:262–271.

Massari, S. and Ruberti, M. (2013). Rare earth elements as critical raw materials: Focus on international markets and future strategies. *Resources Policy*, 38(1):36–43.

Matteoli, A. (2003). Council Directive 2003/96/EC of 27 October 2003 restructuring the Community framework for the taxation of energy products and electricity. *Official Journal of the European Union*, 33(1):41–70.

McCormick, K. and Neij, L. (2009). Experience of policy instruments for energy efficiency in buildings in the Nordic countries. *Lund University*, pages 1–74.

Ministry of Luxembourg (2007). Mémorial adminstratif n°221 du 30 novembre 2010: Perofrmance énergétique des bâtiments d'habitation. *Officiel Journal of the Grand Duchy of Luxembourg*, pages 3762–3836.

Ministry of the Economy of Luxembourg (2017). Energiepass-Datenbank des Wirtschaftsministeriums Luxemburg.

Ministry of the Economy of Luxembourg and Lichtmeß, M. (2014). Berechnung kostenoptimaler Niveaus von Mindestanforderungen an die Gesamtenergieeffizienz für neue und bestehende Wohn- und Nichtwohngebäude. *Governmental Publication*, pages 1–128.

Ministry of the Economy of Luxembourg, Myenergy, and Ploss, M. (2017). Weiterentwicklung der Gebäuderenovierungsstrategie. Weiterreichende Strategieansätze und Maßnahmen. *Governmental Publication*, pages 1–38.

Myenergy Luxemgourg (2018). Quelles aides financières pour votre projet de construction ou de rénovation. *Myenergy Publication*, pages 1–21.

Neary, C. and Highley, D. (1984). The economic importance of the rare earth elements. *Developments in Geochemistry*, 2:423–466.

Newbery, D. M. (1981). Oil prices, cartels, and the problem of dynamic inconsistency. *The Economic Journal*, 91(363):617–646.

Nordhaus, W. D., Houthakker, H., and Solow, R. (1973). The allocation of energy resources. *Brookings Papers on Economic Activity*, 1973(3):529–576.

OECD, IEA, NEA, and ITF (2015). Aligning policies for a low-carbon economy. *OECD Publishing*, pages 1–242.

Olhoff, A. and Christensen, J. M. (2018). Emissions gap report 2018. *U.N. Environment Program*, pages 1–112.

Pachauri, R. K., Allen, M. R., Barros, V. R., Broome, J., Cramer, W., Christ, R., Church, J. A., Clarke, L., Dahe, Q., Dasgupta, P., et al. (2014). Climate Change 2014: Synthesis Report. Contribution of Working Groups I, II and III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change. *IPCC*, pages 1–151.

Palmer, J. A., Suggate, J., Robottom, I., and Hart, P. (1999). Significant life experiences and formative influences on the development of adults' environmental awareness in the UK, Australia and Canada. *Environmental Education Research*, 5(2):181–200.

Partington, R. (2019). Global markets fall as China prepares to hit back at US in trade war. *The Guardian*.

Pearce, D. (1991). The role of carbon taxes in adjusting to global warming. *The Economic Journal*, 101(407):938–948.

Peck, S. C. and Teisberg, T. J. (1992). CETA: A model for carbon emissions trajectory assessment. *The Energy Journal*, 13:55–77.

Petersdorff, C., Boermans, T., and Harnisch, J. (2006). Mitigation of CO_2 emissions from the EU-15 building stock. Beyond the EU directive on the energy performance of buildings. *Environmental Science and Pollution Research*, 13(5):350–358.

Pizer, W. A. (2002). Combining price and quantity controls to mitigate global climate change. *Journal of Public Economics*, 85(3):409–434.

Pöttering, H.-G. and Necas. P. (2009). 406/2009/EC of the European Parliament and of the Council of 23 April 2009 on the effort of Member States to reduce their greenhouse gas emissions to meet the Community's greenhouse gas emission reduction commitments up to 2020. *Official Journal of the European Union*, 140.

Reinganum, J. F. and Stokey, N. L. (1985). Oligopoly extraction of a common property natural resource: The importance of the period of commitment in dynamic games. *International Economic Review*, pages 161–173.

Reynolds, T. W., Bostrom, A., Read, D., and Morgan, M. G. (2010). Now what do people know about global climate change? Survey studies of educated laypeople. *Risk Analysis: An International Journal*, 30(10):1520–1538.

Rivers, N. and Jaccard, M. (2005). Combining top-down and bottom-up approaches to energy-economy modeling using discrete choice methods. *The Energy Journal*, 26:83–106.

Rohdin, P. and Thollander, P. (2006). Barriers to and driving forces for energy efficiency in the non-energy intensive manufacturing industry in Sweden. *Energy*, 31(12):1836–1844.

Salant, S. W. (1976). Exhaustible resources and industrial structure: A nash-cournot approach to the world oil market. *Journal of Political Economy*, 84(5):1079–1093.

Salo, S. and Tahvonen, O. (2001). Oligopoly equilibria in nonrenewable resource markets. *Journal of Economic Dynamics and Control*, 25(5):671–702.

Sartori, I., Wachenfeldt, B. J., and Hestnes, A. G. (2009). Energy demand in the Norwegian building stock: Scenarios on potential reduction. *Energy Policy*, 37(5):1614–1627.

Schaefer, C., Weber, C., Voss-Uhlenbrock, H., Schuler, A., Oosterhuis, F., Nieuwlaar, E., Angioletti, R., Kjellsson, E., Leth-Petersen, S., Togeby, M., et al. (2000). Effective policy instruments for energy efficiency in residential space heating: An international empirical analysis (EPISODE). *Institut für Energiewirtschaft und Rationelle Energieanwendung*, pages 1–123.

Schimschar, S., Blok, K., Boermans, T., and Hermelink, A. (2011). Germany's path towards nearly zero-energy buildings—Enabling the greenhouse gas mitigation potential in the building stock. *Energy Policy*, 39(6):3346–3360.

Schleich, J. and Gruber, E. (2008). Beyond case studies: Barriers to energy efficiency in commerce and the services sector. *Energy Economics*, 30(2):449–464.

Schleussner, C.-F., Rogelj, J., Schaeffer, M., Lissner, T., Licker, R., Fischer, E. M., Knutti, R., Levermann, A., Frieler, K., and Hare, W. (2016). Science and policy characteristics of the Paris Agreement temperature goal. *Nature Climate Change*, 6(9):827.

Schlinkert, D. and van den Boogaart, K. G. (2015). The development of the market for rare earth elements: Insights from economic theory. *Resources Policy*, 46:272–280.

Schofer, E. and Meyer, J. W. (2005). The worldwide expansion of higher education in the twentieth century. *American Sociological Review*, 70(6):898–920.

Schuler, A., Weber, C., and Fahl, U. (2000). Energy consumption for space heating of West-German households: Empirical evidence, scenario projections and policy implications. *Energy Policy*, 28(12):877–894.

Schulz and Mavroyiannis (2012). Directive 2012/27/EU of the European Parliament and of the Council of 25 October 2012 on energy efficiency, amending Directives 2009/125/EC and 2010/30/EU and repealing Directives 2004/8/EC and 2006/32/EC. *Official Journal of the European Union*, pages 1–55.

Shahidehpour, M., Yamin, H., and Li, Z. (2003). *Market operations in electric power systems: Forecasting, scheduling, and risk management*. John Wiley & Sons.

Simon, H. A. (1955). A behavioral model of rational choice. *The Quarterly Journal of Economics*, 69(1):99–118.

Sorrell, S., Schleich, J., Scott, S., O'Malley, E., Trace, F., Boede, U., Ostertag, K., and Radgen, P. (2000). Reducing barriers to energy efficiency in public and private organizations. *Science and Policy Technology Research (SPRU)*, pages 1–197.

STATEC (2011). Recensement de la Population.

STATEC (2017). Registre des Bâtiments et des Logements (RBL).

Stiglitz, J. E. (1976). Monopoly and the rate of extraction of exhaustible resources. *The American Economic Review*, 66(4):655–661.

Stiglitz, J. E. and Dasgupta, P. (1981a). Market structure and resource extraction under uncertainty. *The Scandinavian Journal of Economics*, 83(2):318–333.

Stiglitz, J. E. and Dasgupta, P. (1981b). Resource depletion under technological uncertainty. *Econometrica*, 49(1):85–104.

Stiglitz, J. E. and Dasgupta, P. (1982). Market structure and resource depletion: A contribution to the theory of intertemporal monopolistic competition. *Journal of Economic Theory*, 28(1):128–164.

Sunikka-Blank, M. and Galvin, R. (2012). Introducing the prebound effect: The gap between performance and actual energy consumption. *Building Research & Information*, 40(3):260–273.

Thollander, P. and Ottosson, M. (2008). An energy efficient Swedish pulp and paper industry—Exploring barriers to and driving forces for cost-effective energy efficiency investments. *Energy Efficiency*, 1(1):21–34.

TIR Consulting Group LLC and Grand Duchy of Luxembourg Working Group (2016). The Third Industrial Revolution Strategy Study for the Grand Duchy of Luxembourg. *Governmental Publication*, pages 121–149.

Tommerup, H. and Svendsen, S. (2006). Energy savings in Danish residential building stock. *Energy and Buildings*, 38(6):618–626.

Train, K. (1985). Discount rates in consumers' energy-related decisions: A review of the literature. *Energy*, 10(12):1243–1253.

Trianni, A. and Cagno, E. (2012). Dealing with barriers to energy efficiency and SMEs: Some empirical evidences. *Energy*, 37(1):494–504.

Trujillo, E. (2015). China—Measures related to the exportation of rare earths, tungsten, and molybdenum. *American Journal of International Law*, 109(3):616–623.

Ulph, A. M. and Folie, G. (1980). Exhaustible resources and cartels: An intertemporal nash-cournot model. *Canadian Journal of Economics*, pages 645–658.

United Nations Framework Convention on Climate Change (2015). Paris Agreement. *United Nations Treaty Collection*.

U.S. Department of Commerce (2019). A Federal Strategy to Ensure Secure and Reliable Supplies of Critical Minerals.

U.S. Geological Survey (2020). Mineral Commodity Summaries—Rare Earths.

Van Long, N. (2011). Dynamic games in the economics of natural resources: a survey. *Dynamic Games and Applications*, 1(1):115–148.

Van Long, N., Shimomura, K., and Takahashi, H. (1999). Comparing open-loop with markov equilibria in a class of differential games. *The Japanese Economic Review*, 50(4):457–469.

Vollebergh, H. (2014). Green tax reform: Energy tax challenges for the Netherlands. *PBL Netherlands Environmental Assessment Agency*, pages 1–50.

Watts, N., Amann, M., Arnell, N., Ayeb-Karlsson, S., Belesova, K., Berry, H., Bouley, T., Boykoff, M., Byass, P., Cai, W., et al. (2018). The 2018 report of the lancet countdown on health and climate change: shaping the health of nations for centuries to come. *The Lancet*, 392(10163):2479–2514.

Weber, L. (1997). Some reflections on barriers to the efficient use of energy. *Energy Policy*, 25(10):833–835.

Wei, S., Jones, R., and De Wilde, P. (2014). Driving factors for occupant-controlled space heating in residential buildings. *Energy and Buildings*, 70:36–44.

Weisbach, D. A. (2012). Carbon taxation in the EU: Expanding the EU carbon price. *Journal of Environmental Law*, 24(2):183–206.

Weiss, J., Dunkelberg, E., and Vogelpohl, T. (2012). Improving policy instruments to better tap into homeowner refurbishment potential: Lessons learned from a case study in Germany. *Energy Policy*, 44:406–415.

Weiss, M., Junginger, M., Patel, M. K., and Blok, K. (2010). A review of experience curve analyses for energy demand technologies. *Technological Forecasting and Social Change*, 77(3):411–428.

Weitzman, M. L. (1974). Prices vs. quantities. *The Review of Economic Studies*, 41(4):477–491.

Zhou, B., Li, Z., and Chen, C. (2017). Global potential of rare earth resources and rare earth demand from clean technologies. *Minerals*, 7(11):203.

Zou, B. (2016). Differential games with (a)symmetric players and heterogeneous strategies. *Journal of Reviews on Global Economics*, 5:171–179.