



The stochastic maintenance location routing allocation problem for rolling stock

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ARTICLE INFO

Keywords:

Facility location
Maintenance routing
Rolling stock
Stochastic programming

ABSTRACT

We study a problem where – given a railway network and different fleets – we have to locate maintenance locations and allocate the fleets to these locations. The allocation of fleets to the maintenance locations complicates the maintenance location routing problem. For each candidate location different facility sizes can be chosen and for each size there is an associated annual facility cost that captures the economies of scale in facility size. Because of the strategic nature of facility location, these facilities should be able to handle changes such as adjustments to the line plan and the introduction of new rolling stock types. We capture these changes by discrete scenarios and we formulate this two-stage stochastic problem as a mixed integer problem. Furthermore, we perform a case study with the Netherlands Railways that provides novel managerial insights by showing that the number of opened maintenance facilities highly depends on the allocation restrictions.

1. Introduction

The maintenance of rolling stock is important to keep railway operations functioning. Without frequent maintenance, many trains would break down leading to the cancellation of trains or even dangerous situations. As a consequence, trains are maintained regularly and when such maintenance is required, the train has to reach a suitable maintenance facility. The accessibility of such a maintenance facility depends on the railway infrastructure and the *line plan*. A *line plan* consists of a set of train lines, where each line is a path in the railway network that is operated with a certain frequency by one rolling stock type. In this paper we study the problem of locating such maintenance facilities while also determining their sizes and the allocation of the rolling stock types to the maintenance facilities.

Facility location decisions are long term obligations, while the line and fleet plan are updated regularly to meet changing passengers demand. As a consequence, any sensible facility location plan must work well under a diverse range of line and fleet plan scenarios. This includes changes in how lines run, up and down-scaling of service frequencies on any given line, the rolling stock types assigned to the lines, and the introduction of new rolling stock types.

To deal with these changes, we formulate the problem as a stochastic maintenance location routing allocation problem for rolling stock (SMLRAP). In the SMLRAP, we seek the optimal locations and sizes of maintenance facilities for rolling stock and the best allocation of the rolling stock types to the maintenance facilities. The objective

consists of minimizing the annual depreciation cost of the facilities and the average annual transportation cost. The annual cost of a facility depends on its location and size. The size of a facility must be chosen from a discrete set that model the economies in scales: a facility which is twice as large costs less than twice as much. As a consequence, it is possible to open a few large facilities to profit from economies of scale or to open multiple smaller facilities to limit the transportation cost.

The maintenance location routing problem for rolling stock was introduced by Tönissen et al. (2019), and extended by Tönissen and Arts (2018). Tönissen and Arts (2018) show that the best strategy is to reduce transportation cost by locating many small facilities instead of opening a few large facilities to profit from economies of scale. However, their paper does not include allocation restrictions of the rolling stock types to the maintenance facilities. These allocation restrictions are based on the fact that each rolling stock type requires specific equipment and resources. Furthermore, a mechanic has to work sufficiently many hours on a specific rolling stock type to retain the qualification for type maintenance. Consequently, there is a restriction on the number of maintenance facilities that each rolling stock can be maintained at. This paper shows that this restriction is very important by a case study with the Netherlands Railways (NS). In addition, these allocation restrictions determine whether the best solution has many small facilities or a few large facilities to benefit from economies of scale.

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The paper starts with a literature review. In Section 3, we formulate the SMLRAP as a mixed integer problem. In Section 4, we explain how we generate our instances and give computational results. Finally, we perform a case study for the NS and provide managerial insights in Section 5.

2. Literature review

For the traditional facility location literature, we refer to the reviews of Daskin (1995) and ReVelle and Eiselt (2005). However, the traditional deterministic facility location literature studies a theoretical problem that often cannot be used to solve real life problems. Most real life facility location problems arise in the combination with other supply chain decisions and contain a great deal of uncertainty.

The combination of facility location with uncertainty is reviewed in the paper of Snyder (2006) and the combination with supply chain decisions in Melo et al. (2009). Since those reviews many papers that include uncertainty and/or supply chain decisions with facility location have been written e.g., Penuel et al. (2010), Álvarez-Miranda et al. (2015) and Kinay et al. (2018). Furthermore, Govindan et al. (2017) provide a review about the closely related problem of supply chain network design under uncertainty.

In this paper we look at facility location in combination with facility sizes that model economies of scale, maintenance routing, and allocation restrictions. Many papers (e.g., Melo et al. (2006), Julka et al. (2007), and Xie et al. (2016)) consider facilities with different facility sizes, but most of them focus on capacity expansion models. Economies of scale in production (Romeijn et al., 2010; Sharkey et al., 2011) or economies in scale for transportation (Lin et al., 2006; Wu et al., 2015) are studied in the literature. However, economies of scale in facility size are only briefly mentioned in Melo et al. (2006) and to our best knowledge only studied in depth by Tönissen and Arts (2018).

Maintenance routing for rolling stock is studied by Anderegg et al. (2003), Maróti and Kroon (2005, 2007) and for aviation by Gopalan and Talluri (1998), Sarac et al. (2006), Liang et al. (2015) and many others. The combination of maintenance routing and facility location for aviation is studied by Feo and Bard (1989) and Gopalan (2014), for locomotives by Xie et al. (2016), and for rolling stock by Tönissen et al. (2019) and Tönissen and Arts (2018). The paper of Tönissen et al. (2019), includes the maintenance routing for rolling stock in an aggregate way into a facility location model. They model the maintenance location routing problem as two-stage robust optimization and stochastic programming problems and provide a Benders decomposition and a scenario addition algorithm to solve the models to optimality. This problem was extended by Tönissen and Arts (2018) to include unplanned maintenance, economies of scale in facility size and recoveries of the facility location decisions. Our paper extends these papers further by including allocation restrictions of the rolling stock types to the maintenance facilities. These allocation restrictions are required to efficiently apply the model to practice and consequently the results of this paper have important practical implications.

3. The maintenance location routing model

The two-stage stochastic maintenance location allocation routing problem consist of two stages. The first stage decisions are the locations of the facilities, their sizes and the allocation of rolling stock types to facilities. The assignment of rolling stock to facilities is constrained because each rolling stock type can be allocated to at most K different maintenance facilities. These assignment constraints are imposed because each facility needs specific resources to service a rolling stock type and, more importantly, mechanics needs to retain their certification to be allowed to work a specific rolling stock type. The certification can only be retained when mechanics work a sufficiently large number of hours on a rolling stock type.

The second-stage decisions consist of finding optimal routings of the train units to the maintenance facilities. Planned maintenance typically occurs once every half year up to every month. The transport from the train lines to the maintenance facility is done by *interchanging* the destinations of two train units of the same rolling stock type that are at the same end station. The train units continue on each other's train line after such an interchange. Train units that require maintenance are interchanged until they reach a train line connected to a maintenance facility. Whether such an interchange is possible depends on the operational rolling stock schedule and the shunting infrastructure of the end stations. In our two-stage stochastic model, these restrictions are modeled by putting a restriction on the number of interchanges that can occur annually at any given station. A detailed description and operational maintenance routing model for the NS that includes these restrictions can be found in Maróti and Kroon (2005, 2007).

Deadheading, which is driving a train without passengers, is used for the remaining trip when a maintenance facility cannot be reached via these interchanges. Deadheading is expensive and the deadheading cost consists of driving (train driver, fuel etc.) and disservice costs because the train is not available for public transport. Unplanned maintenance occurs when a train unit fails in the field. The failed train unit has to deadhead to the maintenance facility to be repaired. The deadheading of unplanned maintenance is even more expensive, because it cannot be planned in advance and because the train unit sometimes has to be towed.

Like Tönissen and Arts (2018), we formulate the SMLRAP described above as a flow model. This flow model is based on a directed graph in which the lines and candidate facilities are represented by nodes, and the interchanges and deadheading possibilities by arcs. In the next section, we explain how such a graph can be built and how it can be extended to deal with multiple scenarios. In Section 3.2 this graph is used for our mixed integer formulation that provides us with the first stage decisions and the second stage decisions for each scenario.

3.1. Constructing the maintenance routing graph

Given is a physical rail network $G_P = (N_P, E_P)$, consisting of rails E_P , all stations N_P . Next we are given a discrete set of scenarios D , in which each scenario defines a line plan. A line plan consists of a set of lines $L^d, \forall d \in D$, with for each line, two end stations, the type of rolling stock that operates the line, and the *unplanned and planned maintenance frequency* of maintenance visits that originate from this line. Furthermore, a line plan determines the unplanned and planned deadheading cost for each line to each facility, and the set of possible interchanges with for each interchange a coordination cost. Finally, we are given a set of candidate maintenance locations $C \subseteq N_P$.

Fig. 1 shows on the left-hand side an example of a physical rail network graph containing the end stations and in the middle and on the right-hand side two line plans for two different scenarios. There are two train types in the example shown in Fig. 1. The first, denoted by a , is a regional train, stopping at every station, while train type b is an intercity train that skips the small stations. An example of an interchange in the right-hand side of Fig. 1 is line (U, Z, a) to line (Z, V, a) , while an interchange from (U, Z, a) to line (Z, Y, b) is not possible because the rolling stock types do not match. Also note that the number of end stations is different between the line plans: station Z is an end station in the right hand-side of Fig. 1, while it is an in-between station of the line (U, X, b) in the line plan in the middle.

The maintenance routing graph is a directed flow graph, $G_M = (N_M, A_M)$ that is constructed by the following steps:

- For each line $l \in L^d \forall d \in D$, two nodes are made one for the planned maintenance and one for the unplanned maintenance. The set of planned maintenance nodes for scenario d is denoted by N_{PL}^d and for the unplanned maintenance nodes we have N_{UL}^d . Furthermore, we define $N_L^d = N_{PL}^d \cup N_{UL}^d$.

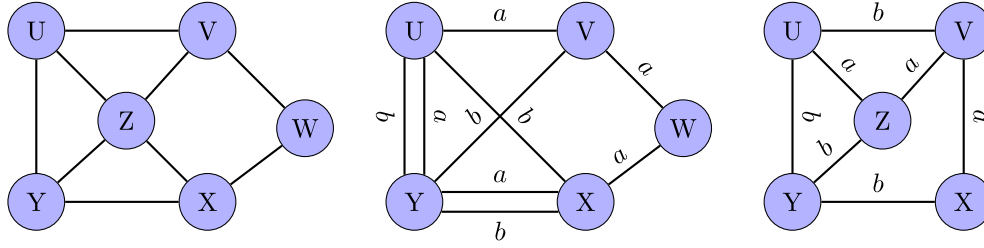


Fig. 1. The physical rail network on the left and two possible line plans.

- A source S is made that is connected with a directed arc to each node in $\bigcup_{d \in D} N_L^d$.
- Arcs between the line nodes from $N_{PL}^d \forall d \in D$ are created whenever an interchange between these lines is possible (the lines have a common end station and there is positive interchange capacity at that end station). The cost of these arcs are the interchange coordination costs. The set of these interchange arcs is denoted by $A_I^d \forall d \in D$.
- A node is made for every candidate facility and we denote the set of these nodes as N_C . Furthermore, each node in N_C is connected to the sink \mathcal{T} with an arc.
- An arc to each facility is created for each node $n \in N_L^d \forall d \in D$. The cost of this arc is the deadheading cost of the line to the facility. The cost of the arc can be 0 when the line is connected to the facility and deadheading is not necessary to reach that facility. The set of all incoming facility arcs for scenario d is denoted as A_C^d .

The number of nodes and arcs in the flow graph G_M is polynomial in the number of lines, end stations and scenarios. This can be easily shown as the number of nodes in the flow graph is equal to $\sum_{d \in D} |N_L^d| + |N_C| + 2$ and the number of arcs is equal to $\sum_{d \in D} (|N_L^d| + |N_L^d| |N_C| + |A_I^d|) + |N_C|$, where $|A_I^d|$ is bounded by $|N_{PL}^d| (|N_{PL}^d| - 1)$. Note that the flow from different scenarios can never mingle in the graph G_M , and as a result we do not have to distinguish flow of the difference scenarios. Consequently, there is a non-overlapping S - \mathcal{T} path for each scenario $d \in D$ that is equivalent to the total route of interchanges and deadheading for an annual maintenance frequency from a train line to the maintenance facility.

In Fig. 2, we demonstrate how to create a maintenance routing flow graph for a small example with only one scenario. The line plan of that scenario is depicted on the left-hand side. In this example, we again show the rolling stock type by using letters and we numbered the lines such that the associated line node can easily be found in the maintenance routing flow graph. We assume that an interchange is possible between line 0 and 1, and $N_C = \{A, B, E\}$, where A and B are facilities located at end stations A and B, while E is a candidate facility that is located elsewhere. The figure on the right-hand side depicts the flow network with unplanned (U) and planned (P) line nodes.

Note that only interchanges followed by deadheading directly to the maintenance facility are allowed in the graph G_M . Deadheading followed by interchanges can easily be allowed in the graph by creating an arc from every planned maintenance line node to every other planned maintenance line. That arc represents the deadheading from one line to another, that can be followed by any combination of interchanges and deadheading until the maintenance facility is reached. The reason that we exclude these kind of routes is that they are very expensive because they cause imbalances in the number of train units per line, which are quite difficult to resolve in practice.

3.2. The mixed integer programming formulation

The size of a facility is expressed by the number of maintenance visits that it can process annually. The workload generated by a planned

maintenance visit is thus set at 1 and that of an unplanned maintenance visit as $u \in \mathbb{R}^+$. The total annual workload of the entire line plan for the current situation is denoted by M . The sizes of a facility at location $n \in N_C$ are denoted by the set Q^n . A tuple $i \in Q^n$, consists of a size q_{ni} that represents the annual workload that a facility can handle and the annual facility cost c_{ni} for facility location n . Furthermore, each rolling stock type $r \in R$, where R is the set containing all rolling stock types, can be maintained by at most K_r different facilities. The first-stage decisions are represented by Y and X . Y contains the binary decision variables $y_{ni} \in \{0, 1\} \forall n \in N_C, i \in Q^n$ that is 1 when a facility of size i is opened at location n and 0 otherwise. X contains the binary decision variables $x_{nr} \in \{0, 1\} \forall n \in N_C, r \in R$ that is 1 when rolling stock type r is allocated to facility n and 0 otherwise.

The maintenance frequency for line l and scenario d is defined by the parameter m_l^d , and n_l^d is the node associated with line l for scenario d . The set of end stations for scenario d is given by S^d , and $g_s^d \in \mathbb{R}_0^+$ is a restriction on the annual number of interchanges at end station $s \in S^d$ for scenario $d \in D$. The number of annual interchanges for scenario d is restricted by the parameter $G^d \in \mathbb{R}_0^+$. The flow through arc a associated with the annual maintenance frequency from line $l \in N_L^d$, in scenario $d \in D$, is represented by the second-stage decision variable $z(a) \in \mathbb{R}_0^+$. For example, $z(1, 7)$ represents the frequency of interchanges from line 1 to line 7, while we also know to which scenario and rolling stock type line 1 and 7 belongs.

We define $\delta_{in}^d(n)$ and $\delta_{out}^d(n)$ as the set of ingoing and outgoing arcs of node n for scenario d in graph G_M . In addition, we let the index P and U denote the planned and unplanned maintenance subset, respectively, and when we use the index r we only include the subset of arcs that belong to rolling stock type r . As defined in Section 3.1, A_I^d is the set of interchange arcs and $A_C^d = \bigcup_{n \in N_C} \delta_{in}^d(n)$, the set of incoming candidate facilities arcs. Furthermore, we define A_s^d as the set of arcs representing the interchanges at end station s for scenario d and when we drop the index d for a set, this is shorthand notation for taking the union of the sets for all scenarios, e.g., $\delta_{out}(n) = \bigcup_{d \in D} \delta_{out}^d(n)$ for any $n \in N_M$.

The cost of arc a is $c(a)$, which is only defined for arcs in the set $\bigcup_{d \in D} A_I^d \cup A_C^d$. Finally, the weights $w_d \forall d \in D$ denote the expected fraction of time that a scenario is used during the life time of the facilities. We can now formulate the SMLRAP:

$$(SMLRAP) \quad \min \sum_{n \in N_C} \sum_{i \in Q^n} c_{ni} y_{ni} + \sum_{d \in D} w_d \sum_{a \in A_I^d \cup A_C^d} c(a) z(a)$$

$$\text{s.t.} \quad \sum_{i \in Q^n} y_{ni} \leq 1 \quad \forall n \in N_C, \quad (1)$$

$$\sum_{n \in N_C} x_{nr} \leq K_r, \quad \forall r \in R, \quad (2)$$

$$x_{nr} \leq \sum_{i \in Q^n} y_{ni} \quad \forall n \in N_C, \quad \forall r \in R, \quad (3)$$

$$\sum_{a \in \delta_{in}^d(n)} z(a) + u \sum_{a \in \delta_{out}^d(n)} z(a) \leq x_{nr} |D| \max_{i \in Q^n} q_{ni} \quad \forall n \in N_C, \quad \forall r \in R, \quad (4)$$

$$\sum_{a \in \delta_{in}^d(n)} z(a) + u \sum_{a \in \delta_{out}^d(n)} z(a) \leq \sum_{i \in Q^n} y_{ni} q_{ni} \quad \forall d \in D, \quad \forall n \in N_C, \quad (5)$$

$$\sum_{a \in \delta_{in}(n)} z(a) = \sum_{a \in \delta_{out}(n)} z(a) \quad \forall n \in N_M \setminus \{S, \mathcal{T}\}, \quad (6)$$

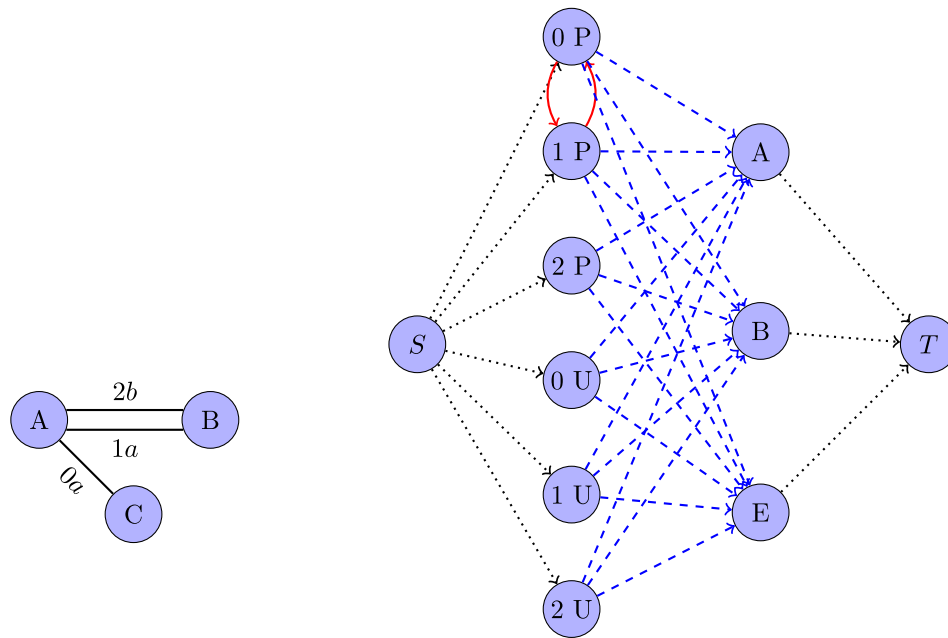


Fig. 2. Left a line planning possibility and right the resulting flow graph ($G_M = (N_M, A_M)$). The arcs from and to the source and sink are dotted black, the interchange arcs (A_i^d) solid red and the arcs to the facilities (A_i^c) are dashed blue.

$$z(a) = m_i^d \quad \forall d \in D, \quad \forall l \in N_l^d, \quad a \in \delta_{in}^d(m_i^d) \setminus A_l, \quad (7)$$

$$\sum_{a \in A_i^c} z(a) \leq g_s^d \quad \forall d \in D, \quad \forall s \in S^d, \quad (8)$$

$$\sum_{a \in A_i^d} z(a) \leq G^d \quad \forall d \in D, \quad (9)$$

$$x_{nr} \in \{0, 1\} \quad \forall n \in N_C, \quad \forall r \in R, \quad (10)$$

$$y_{ni} \in \{0, 1\} \quad \forall n \in N_C, \quad \forall i \in Q^n, \quad (11)$$

$$z(a) \geq 0 \quad \forall a \in A_M. \quad (12)$$

We minimize the cost of opening the facilities and the expected maintenance routing cost over all scenarios. Constraints (1) guarantee that each facility can be opened with at most 1 size. Constraints (2) ensure that rolling stock type r can be maintained at most at K_r facilities. Constraints (3) guarantee that we can only allocate rolling stock types to opened facilities and constraints (4) guarantee that rolling stock type r can only be maintained at a facility n when matching resources are installed. Constraints (5) restrict the number of annual planned and unplanned maintenance visits that can be assigned to opened facility (n, i) with size q_{ni} . Constraints (6) are the flow conservation constraints, while Constraints (7) guarantee that every maintenance visit is assigned to a facility. Constraints (8) and (9) are the end station and budget interchange capacity constraints.

4. Computational results

4.1. Instance generation

We generate the instances based on data gained from the NS. We assume that the candidate locations are always located at the end stations. We have 59 end stations which all can be used as candidate locations. When we generate instances with a certain number of candidate facilities, these candidate facilities are randomly chosen from these 59 end stations. The facility costs are an estimation of the average annual cost of land, the necessary infrastructure and the maintenance facility itself including all side buildings. Furthermore, we

Table 1

Cost increases for locations compared to a facility with size $1/3M$.

Size	1/12 M	1/8 M	1/6 M	1/4 M	1/3 M	1/2 M	2/3 M	M	4/3 M
Factor	0.29	0.42	0.54	0.77	1.00	1.45	1.90	2.78	3.65

either decrease or increase the facility cost based on the average land price of the province that a location is in. To create the cost for the different sizes, we multiply the cost estimation for each location with the factors depicted in Table 1. The factors for the sizes are estimated with the square root safety staffing rule (Halfin and Whitt, 1981): When the size of a facility is increased by a factor x , the needed safety size C to deal with uncertainty is only increased by a factor \sqrt{x} . Consequently, the cost increases by a factor $\frac{x+C\sqrt{x}}{1+C} < x$. The required safety stock for a standard location of size $1/3 M$ is estimated to be approximately 0.21.

The network interchange budget G^d is $\mathcal{U}(0.25M, M) \forall d \in D$, the unplanned maintenance factor u is set to 0.25, and K is uniformly randomly generated between 1 and 4. All scenarios are based on four basic line plans. These basic line plans are: the current situation (2015),¹ an estimation of 2018, and two possibilities for approximately 2025. The future line plans are based on the plan “Beter en Meer” (Prorail and NS, 2014), a commercial plan made by the NS and Prorail. The purpose of the plan is to cater to the growing numbers of passengers. These basic line plans contain all the lines (97, 97, 99, and 100 lines), the rolling stock type serving the line, and an estimate of the number planned yearly maintenance visits per line. Scenarios are made by picking such a basic line plan, and slightly altering the planned maintenance frequency and rolling stock types. The altered planned maintenance frequency for each line of the line plan is generated from a triangular distribution. The planned maintenance frequency of the basic plan is the mode of this distribution. Furthermore, we assume that the number of maintenance visits can decrease by 32.5% and increase by 75%, due to uncertainty in the number of maintenance visits each train unit requires each year and the number of passengers using a certain line. A

¹ The data was gathered in 2015.

Table 2
Scenario results for the NS instances.

Scenarios	Opened	Cost (M/year)	Time (min.)	Fails
1	2–5 (3.8)	22.2	0.06	0
2	2–5 (3.4)	21.9	0.23	0
4	2–4 (3.0)	22.6	1.3	0
8	2–4 (2.9)	23.2	5.0	0
16	2–4 (2.7)	23.3	27.9	0
32	2–4 (2.5)	23.4	73.0	2
64	2–4 (2.6)	23.9	152.2	5
128	2–4 (2.9)	24.2	180.0	10

maximum of 20% of the rolling stock types of the lines can be swapped with each other. Moreover, the unplanned maintenance frequency for a line is the same as the planned maintenance frequency, as they occur approximately equally often for the NS.

4.2. Benchmark experiments

Experiments show that the number of sizes of the facilities or the number of basic scenarios that are used to create the scenarios have no significant influence on the solution time of the instances. The solution time is mainly impacted by the number of candidate facilities and to a lesser degree by the number of scenarios.

For 5, 10, 20, 40, and 59 candidate facilities the average solution time over 10 instances is shown in Fig. 3. Note that throughout this paper we use \lg to represent the binary logarithm (\log_2). The number of scenarios is increased by a factor of 2, each time that 8 or more out of 10 instances could be solved within an hour. Otherwise, the experiments were stopped. With those conditions we can solve up to 128 (2^7) scenarios when we have 5 candidate facilities, but only instances with 1 scenario when we include all 59 candidate facilities.

When the additional assignment constraint of a maximum of K facilities is removed (and consequently also the associated binary decision variables) the instances are more easier to solve (more than one order of magnitude decrease in solution time). These results are shown in Fig. 4 and in this case we can solve up to 512 (2^9) scenarios when there are 5 candidate locations and 8 (2^3) scenarios when there are 59 candidate locations. Consequently, these allocation decisions significantly increase the solution time of the instances.

5. Case study: NS

We see from the benchmark instances that instances with many candidate facilities are hard to solve. However, many of the 59 end stations are not serious candidates for a maintenance facility. When we limit the list of candidate facilities to the most likely candidates, a list of 12 end stations remains. This “shortlist” contains end stations that are on strategic locations throughout the country. These strategic locations are around an urban agglomeration in the Netherlands where most train traffic is concentrated and at the extremities of the rail network. These 12 end stations are used as candidate locations for the remainder of the case study.

5.1. Scenarios

We start with 10 instances with one scenario, and double the number of scenarios in each next set. In Table 2 we report the minimum and maximum number of opened facilities, followed by the average number of opened facilities between parentheses. Furthermore, we denote the average total cost in millions per year, the average solution time, and the number of instances that could not be solved within 180 min. When an instance is not solved within the 180 min time limit, we report the best found integer solution, and report 180 min as solution time.

The average number of facilities seems to slightly decrease with the number of scenarios. We expect that the reason for this is that a solution

Table 3
PVSS for the NS instances.

Name	Tcost (M/year)	Fcost (M/year)	Cost (M/year)	PVSS
SMLRAP	17.8	5.8	23.3	0
EVCS ($\alpha = 1.0$)	19.3	6.4	25.7	9.2
EVCS ($\alpha = 1.5$)	19.3	7.3	26.6	12.1
EVCS ($\alpha = 2.0$)	19.3	8.1	27.4	14.8
EVCS ($\alpha = 3.0$)	19.3	9.8	29.1	19.8

with only a few large facilities has a more stable performance for the different scenarios than a solution with many small facilities. The trade-off between solution stability and solution time seems to be best at 16 scenarios. Consequently, we use 16 scenarios for our remaining experiments.

5.2. The value of the stochastic solution

The value of the stochastic solution (VSS) is a common measure within the stochastic programming literature (Birge and Louveaux, 1997). The VSS is defined as the difference between the expectation of the expected value solution and the optimal objective value of the two-stage stochastic programming problem. In our problem, where each scenario contains a graph, the best equivalent to the expected value solution is the first-stage solution of solving the SMLRAP with one single scenario, which is the best estimate of the current situation. We then use this first-stage solution as input for the SMLRAP with multiple scenarios, i.e., we fix the allocation and facility decisions and based on these decisions an optimal maintenance routing is found for each scenario. We define the optimal objective value of this program as the expected value of the current situation (EVCS). Consequently, we now have: $VSS = EVCS - SMLRAP$. Because, the VSS depends on the scale of the objective value, we increase the interpretability by defining the percentage value of the stochastic solution (PVSS): $\frac{VSS}{EVCS} \cdot 100\%$. The PVSS can be seen as the expected percentage of cost savings of solving the SMLRAP with a sufficient number of scenarios instead of a deterministic model based only on the current situation.

For our experiments we use the instances with 16 scenarios from Section 5.1. However, for these instances the EVCS cannot be calculated, as it is infeasible for each of the instances. This is not a surprise as most of our scenarios are growth scenarios and many of the scenarios introduce new rolling stock types. Consequently, infeasibility is caused by capacity problems and by the new rolling stock types that are not assigned to any facility. To deal with this, we allow limited recovery to the first-stage solution that is used as input for calculating the EVCS. We allow recovery by upgrading opened facilities to a higher capacity and we allow the allocation of the new rolling stock types to these facilities. We define the cost of upgrading a facility $n \in N_C$ from capacity i to capacity j as $\alpha(c_{nj} - c_{ni})$, where $\alpha \geq 1$. The α represents the potential additional cost of upgrading an existing facilities compared to building a facility of that size immediately. This value largely depends on the time that upgrading the facilities and installing the resources for the new rolling stock types takes. Consequently, when recovery is required and it is executed poorly, e.g., because it is not incorporated into the planning at all, the potential cost and damage (e.g., deferred maintenance and loss of public image) could be enormous.

The current situations first-stage solution opens two facilities with a capacity of $1/8M$ and M . For the 10 instances with the 16 scenarios, these capacities are upgraded to at most $1/3M$ and $4/3M$, respectively. We report the average transportation cost (Tcost), average facility cost (Fcost), the average total cost in millions per year, and the average PVSS, for the SMLRAP solution followed by the EVCS results for $\alpha \in \{1.0, 1.5, 2.0, 3.0\}$ in Table 3

From Table 3 it can be seen that even when we can recover without additional cost (which is very unlikely) we can already save 9.2%. When we increase α to 3.0 the cost savings increase to 19.8%. Finally,

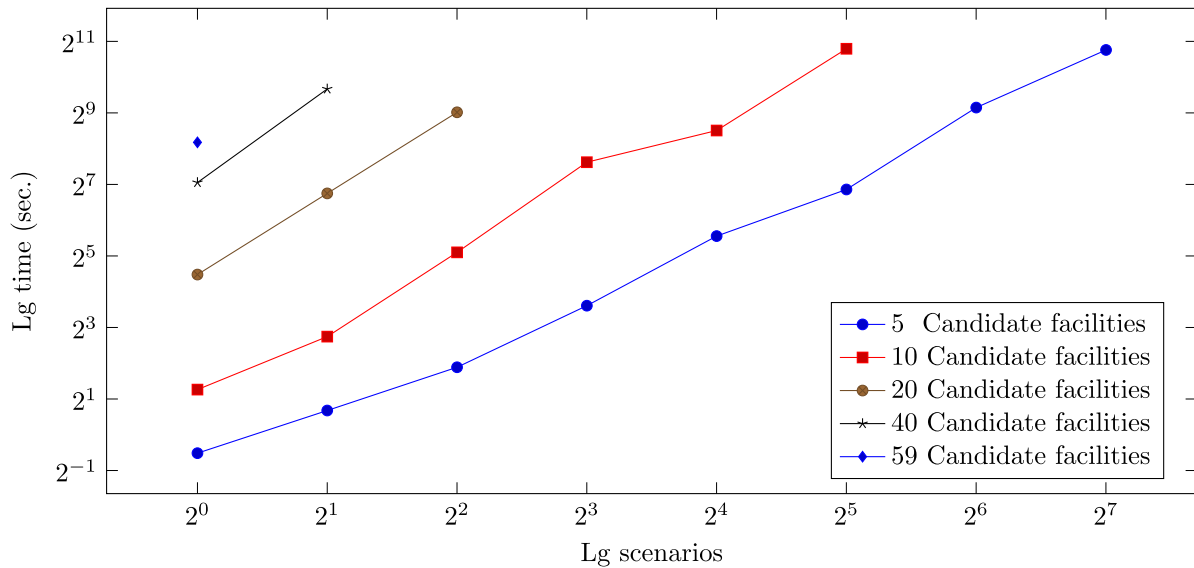


Fig. 3. Computational time for the MIP for different number of scenarios and candidate facilities.

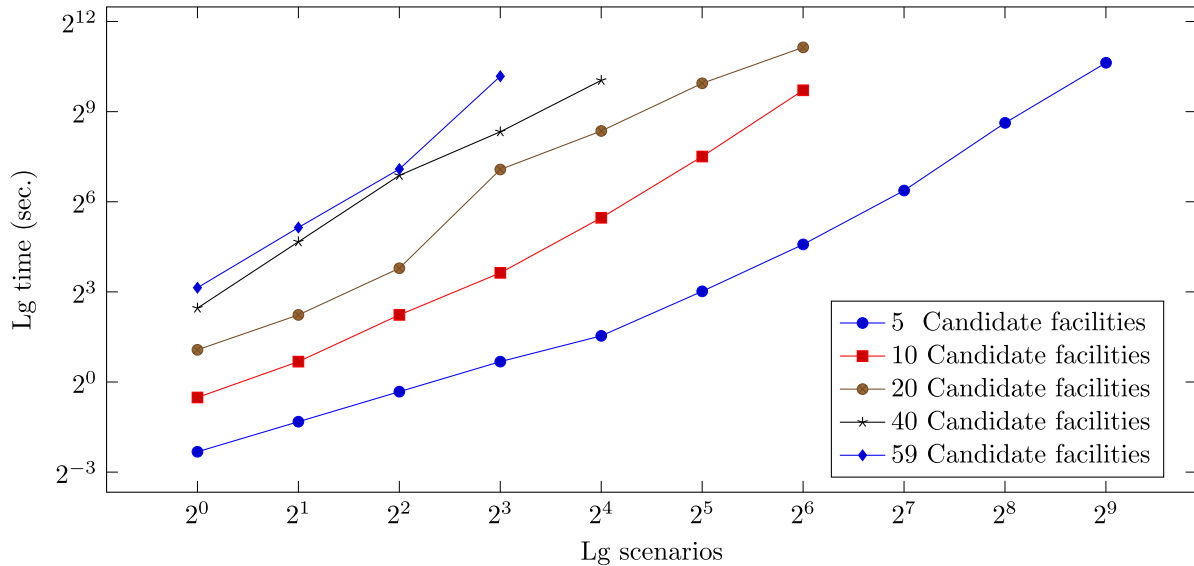


Fig. 4. Behavior of the computational time for the MIP without the K constraint for different number of scenarios at different scales.

Table 4 Sensitivity analysis K for the NS instances.

Name	Opened	Tcost (M/year)	Fcost (M/year)	Cost (M/year)
K = 1	2-3 (2.4)	18.8	6.1	25.0
Current	2-4 (2.7)	17.5	5.8	23.3
G2	3-6 (4.6)	16.0	5.3	21.1
K = 2	5-6 (5.1)	14.5	5.4	19.7
K = 3	5-8 (6.6)	12.5	5.1	17.5
∞	8-12 (10.2)	10.5	5.2	15.6

it can be seen that the transportation cost for these instances are independent of α . Consequently, the cost savings are 9.2% plus the additional cost of recovery.

5.3. Varying the number of allocated facilities

We compare the current situation (1 maintenance facilities for every rolling stock type except the VIRM which can be allocated to two), with

a situation where the rolling stock types with the largest number of train units (VIRM, ICM, SLT, and SNG) can also be allocated to two maintenance facilities (G2). Furthermore, we compare it to the situations where all rolling stock types can be maintained at respectively 2, 3, and an infinite number of maintenance facilities. Note that in our analysis, we only increase K_r in constraints (2) and that consequently any additional cost for increasing the capabilities of the maintenance facilities is outside scope and not taken into consideration.

We generate for each case 10 instances and report the minimum and maximum number of opened facilities, followed by the average number of opened facilities between parentheses in Table 4. Furthermore, we denote the average transportation cost (Tcost), the average facility cost (Fcost), and the average total cost in millions per year.

In Table 4 we see that the number of opened facilities is highly dependent on K . When K is large, many small facilities are opened to limit the transportation cost. However, when K is small, we are forced to open only a few large facilities with economies of scale. It is interesting to note that even though the larger facilities have economies of scale, the total facility cost is cheaper when we build many small

Table 5
Sensitivity analysis unplanned maintenance visits for the NS instances.

Unplanned factor	Opened	Cost (M/year)
0	2–3 (2.2)	7.8
0.25	2–3 (2.5)	11.8
0.5	2–4 (2.7)	15.8
0.75	2–4 (2.8)	19.5
1	2–4 (2.7)	23.3
1.33	2–4 (2.8)	28.4
2	2–4 (3.0)	38.5
4	2–5 (3.6)	68.6

Table 6
Sensitivity analysis number of rolling stock types for the NS instances.

Cost factor	Opened	Cost (M/year)
5 – 6 (current)	2–4 (2.7)	23.3
5 – 6 ($K = 2$)	5–6 (5.1)	19.7
3 – 3 (current)	2–3 (2.2)	23.4
3 – 3 ($K = 2$)	3–5 (4.3)	19.9
1 – 1 (current)	2–3 (2.1)	21.15
1 – 1 ($K = 2$)	2–4 (2.9)	19.0

facilities. The reason for this is that the few facilities with economies of scales are built in the busy, central and expensive areas of the Netherlands, while in the case of many small facilities some of them are built in the less expensive areas of the Netherlands.

Going from the current situation to a situation where all larger rolling stock types are maintained by two facilities, the number of facilities would increase almost twofold and the cost reduction is 9.4%. A comparison with the cases where $K = 3$ and $K = \infty$, gives cost savings of respectively 24.9% and 33.0%.

5.4. Varying unplanned maintenance visits

In this section we increase the number of unplanned maintenance visits with the following factors: 0, 0.25, 0.5, 0.75, 1, 1.33, 2, and 4. The number of planned maintenance visits remains unchanged. Again we generate 10 instances for each situation and we report our results in Table 5.

We see that large cost savings (excluding the cost of the actual maintenance) can be made by decreasing the number of unplanned maintenance visits. Furthermore, the number of unplanned maintenance visits affects the number of facilities. When there is no unplanned maintenance, the number of maintenance facilities is approximately two, while it is almost four when the annual maintenance frequency is multiplied by 4.

5.5. Rolling stock types

The more rolling stock types there are, the less likely it becomes that a train unit can reach a maintenance facility by interchanges. Consequently, by decreasing the number of rolling stock types, the total deadheading cost can be decreased and fewer facilities may be required. Currently, the NS has 5 intercity types and 6 regional rolling stock types that needs to be maintained. We look at the effect of decreasing this number to 3 intercity types and 3 regional types, and 1 intercity type and 1 regional type. We compare these results with the current K and with the case that every rolling stock type is allowed to be maintained by two facilities. Once more, we generate 10 instances for each case and we report our results in Table 6.

When there are more rolling stock types, the difference between the number of facilities is larger between the different K 's. The differences in cost are respectively 15.4%, 15.0%, and 10.2%. The total difference from the current situation to a situation with only 1 rolling stock type

for both regional and intercity transport is 9.2% and 18.5% for the current situation and $K = 2$, respectively.

6. Conclusion

We added the allocation of rolling stock types and allocation restrictions to the two-stage stochastic maintenance location routing problem. This is an important extension because in practice there are restrictions to which rolling stock types can be maintained by which maintenance facilities. These restrictions are caused by the fact that each rolling stock type requires special equipment and matching resources. Sensitivity analysis shows that these allocation restrictions are indeed important as they highly influence the solution. The number of facilities decreases from an average of 10.2 facilities to only 2.4 facilities. Increasing the number of facilities where rolling stock can be maintained yields cost savings up to 33%. Consequently, a trade-off should be made between the cost savings caused by relaxing the allocation restrictions and the required cost to increase the capabilities of the facilities.

The number of rolling stock types only has a small influence on the number of facilities that are opened and it only decreases the cost by approximately 9.2%. Furthermore, decreasing the unplanned maintenance frequency by a factor 2 does not influence the number of opened facilities. However, it does decrease the costs by 32.3%, without taking the additional cost savings in the maintenance cost into consideration. Consequently, decreasing the unplanned maintenance frequency should be a priority.

Finally, our research shows that it is important to include multiple scenarios when locating maintenance facilities. The optimal allocation and facility decisions for the current situation at the NS are infeasible for our 10 test instances. Consequently, a feasible solution can only be achieved by adding (expensive) recovery actions. In our experiments we estimate that the cost savings of including these scenarios is 9.2% plus the additional cost needed for the recovery actions. (These costs are difficult to estimate but may be quite sizeable.)

Acknowledgment

We thank the NS for all the help they have given us while writing this paper.

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