REAL TIME HOLDING CONTROL FOR MULTILINE NETWORKS

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ABSTRACT

We introduce a rule based multiline holding criterion for regularity in branch and trunk networks accounting for all passenger groups. On the shared transit corridor, we consider synchronization at the merging or the diverging stop. The decision between holding for regularity or synchronization is taken by comparing the expected passenger cost of each control action. The proposed criterion is tested through simulation in a synthetic double fork network with different shares of transferring passengers, control schemes for regularity and synchronization. The results show that multiline control outperforms the state of the art schemes at the network level, stemming from benefits occurring at the first part of the route and the shared transit corridor and a 3.5% more stable joint headway compared to the other schemes. Additionally, it is advised to perform the synchronization at the diverging stop, as it proves to result in a more stable transferring time equal to the joint frequency of the corridor while reducing the transfer time variability up to -42.7%.

Keywords: Holding strategy, trunk and branch networks, transfers

INTRODUCTION

The inherent stochastic nature of public transport operations is a continuous challenge for service providers. Real time control assists in limiting the negative externalities that are interwoven with highly variable travel times and passenger demand.

Control strategies have been classified spatially (station, interstation and other) by Eberlein et al (*1*), based on the solution approach (analytical solutions and optimization) by Zolfaghari et al (*2*) and based on the objective (headway regulation and waiting time minimization) by Ibarra Rojas et al (*3*). Among such strategies, holding has been shown to be an effective station-based strategy for both bus and rail systems(*4*). The holding criterion varies from schedule adherence and headway adherence to the minimization of passenger cost, and depends on the characteristics of the transit line. To begin with, in the first category holding times refer to scheduled departure times like the early works of Newell and Potts (*5*) and Potts and Tamlin (*6*) and the more recent works from van Oort el al (*7*).

For high frequency lines, the objective is to maintain low headway variability and alleviate bunching. In literature, this has been addressed mostly by rule-based holding strategies that allow departure after a specific threshold (*8–10*) or regulate the headway accounting for both the preceding and the succeeding vehicles (*11–13*). Other approaches worth being mentioned are those of Zhao et al (*14*), that treat buses as agents with a negotiation algorithm, and Bartholdi and Eisenstein (*15*), who adopt quasi-regular headways in order to mitigate bunching phenomena. The last category of holding criteria focuses on minimization of passenger travel times. The two key components to minimize are waiting time and in-vehicle time. Minimization has been addressed using analytical models (*16*), heuristics (*17*) and optimization models (*4*). Gradually, capacity constraints (*2*) and boarding limits (*18*) have been added.

Holding has been combined with other strategies such as stop skipping (*19*), transit signal priority (*20*) and a combination of stop skipping, speed adjustment and boarding limits by Nesheli and Ceder (*21*). It has also been used to synchronize transfers between lines in several works (*22–25*). Holding for synchronization is a first level of interaction and control beyond single line level. Other studies consider the dynamics between lines that share a sequence of common stops. Hernandez et al (*26*) apply multiline holding control for a trunk using game theory. Argote Cabanero et al (*27*) extend the work of Xuan et al (*13*) for shared transit corridors and test it for the city of San Sebastian, Spain. Sanchez Martinez et al (*28*) compare different single line rule-based holding strategies subject to the line and the joint headway for the trunk-and-branch tram network of the city of Boston. Laskaris et al (*29*) introduce a holding criterion for lines merging into a shared transit corridor which includes coordination prior to shared transit corridor and controls jointly the trunk adjusting holding time to passengers experiencing the control action.

So far, the works on controlling multiple lines have been limited and mainly focused on the shared transit corridor. In trunk-and-branch networks there are different passenger groups that interact and are affected differently by decisions taken in favor of single line regularity or the regularity of the joint trunk. In addition, transfer synchronization has not been applied on shared transit corridor stops, thus its effects on the regularity of the trunk has not been investigated.

In this study, we apply a multiline holding criterion for regularity in branch-and-trunk networks consisting of branches prior and after a shared transit corridor. In addition, at the first and the last common stop we combine the regularity criterion with a holding criterion for synchronization. The decision between regularity and synchronization is taken by comparing the passenger cost of each action. The contributions of this paper are twofold: 1) we assess the performance of multiline control compared to single line control and its effect on the cost of every passenger group; and 2) we explore on which common stop synchronization can be feasible and the resulting impact on the regularity of the individual lines. The performance is assessed using simulation for scenarios with different control schemes, demand patterns and cost comparison horizons.

The remainder of the paper is structured as follows: in the next section, the multiline regularity and the synchronization criteria are presented, then the case study employed to assess the performance of the proposed criteria is described, followed by a discussion of the results obtained. In the last section, conclusions are drawn.

METHODOLOGY

Network description

We focus on networks that consist of multiple transit lines and have at least one set of common consecutive stops, which is sufficiently large to be considered in operations as a shared transit corridor (as illustrated in Figure 1). Stops served by a single line are considered as part of a branch. The different stop sets are separated at specific stops (switching stops), where the number of lines operating jointly upstream and downstream is different. Depending on the number of stops prior and after, switching stops are divided into merging and diverging stops. When considering switching stops and how they interact with the different lines, bus stops can be subdivided in three sets: initial branch stops (before a merge), final branch stops (after a diverge) and, in between the two switching stops, shared transit corridor.

Passengers can transfer at any stop of the shared transit corridor. We therefore treat the stops as shared transfer stops as characterized by Hadas and Ceder (*30*), assuming that passengers will not walk to a nearby connecting stop and their transferring time is equal to the walking time between vehicles. Passengers originating at the initial branch need to transfer in order to reach a stop served by a connecting line. Passengers on the shared transit corridor can wait for the line that serves their final destination. Passengers performing trips within the shared transit corridor are assumed indifferent towards the services traversing the shared corridor and will therefore board the first bus arriving at the stop since this choice minimizes their travel time (*31*, *32*).

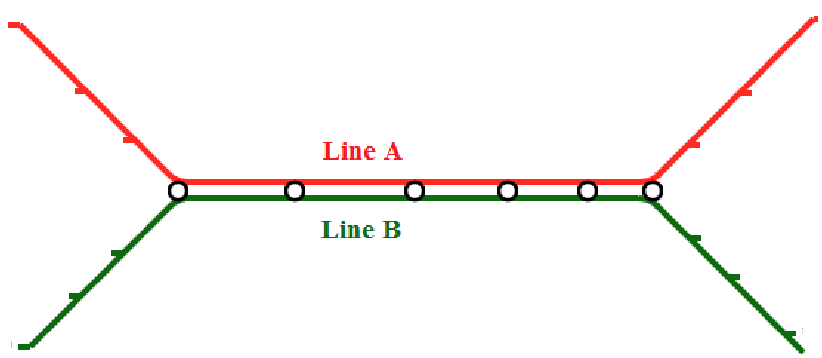

Figure 1 Schematic representation of the network

Assumptions

For the formulation of the criteria, the following assumptions are taken into consideration:

- Vehicles are equipped with AVL technology;

- Historical data for the demand of the lines and transferring passengers are available;
- Capacity constraints are not binding;

Additionally, the current study has the following limitations:

- The transferring criterion is limited to two lines;
- Passengers perform transfers only at a predefined stop at the shared transit corridor.
- One operational direction is considered.

Regularity Criterion

The holding criterion was introduced by Laskaris et al (*33*) and is derived from a generalized passenger travel time function, consisting of waiting time and in-vehicle time as presented in previous studies (*29*, *33*). The general form of the holding criterion is given below:

$$t_{ijk}^{hold,reg}=\max\left\{\theta_1\frac{\left[\left(t_{jk+1}^{exit}-t_{ijk}^{exit}\right)-\left(t_{ijk}^{exit}-t_{jk-1}^{exit}\right)\right]}{2}+\theta_2\frac{\left[\left(t_{ijk+1}^{exit}-t_{ijk}^{exit}\right)-\left(t_{ijk}^{exit}-t_{ijk-1}^{exit}\right)\right]}{2}\right.$$
$$\left.+\theta_3\frac{\left[\left(\tilde{t}_{i,j^{switch},k}^{exit}-\tilde{t}_{i,j^{switch},k-1}^{exit}\right)-\left(\tilde{t}_{i,j^{switch},k+1}^{exit}-\tilde{t}_{i,j^{switch},k}^{exit}\right)\right]}{2}-\frac{\beta^{inveh}q_{ijk}}{2\beta^{wait}\Lambda_j},0\right\} \tag{1}$$

with

t_{ijk}^{hold}	the holding time for trip k of line i at stop j in [time units];
t_{ijk}^{exit}	the departure (exit) time in [time units];
$\hat{t}_{ijk}^{exit}$	the expected departure time from the next switching stop in [time units];
q_{ijk}	the occupancy of trip k of line i at stop j in [passengers];
Λ_j	the sum of the arrival rates from current stop j until the end of the route in [passengers/time unit];

Formula (1) sets the holding time as a function of the stop set currently visited, the passenger demand, and the transition between the stop sets of the network. The first two terms are introduced to regularize the headway of the line and the shared transit corridor, considering the passenger demand that is affected by the corresponding headway. The third term has the objective of smoothening the transition between different stop sets by estimating the expected departure time from the next switching stop downstream and ensuring that the vehicles will initiate their independent operation with lower headway variability. The fourth and final term is the ratio between the passengers on board and the sum of the arrival rates from the current and the remaining downstream stops until the end of the line. This passenger ratio is subtracted from the holding time, calculated by the previous terms, in order to limit the effect on other passenger groups.

Each term in Equation (1) is weighted by the ratio between the corresponding passenger segment and the total demand. Furthermore, the weights include a decay function based on the distance to the next switching stop to avoid controlling when relying on estimations with lower accuracy. The terms that regulate the headways (joint and line) at the current stop of the corridor share the same distance weight, compared to the projection term in the equation. A parameter α, set to 0.5, is applied to both to demonstrate their equal contribution to the estimation of holding time to regulate both headways.

$$
\begin{aligned}
\theta_1 &= \frac{\sum_{o=j}^{N_{J_c}} \sum_{d=o+1}^{N_{J_c}} \lambda_{o,d}}{\sum_{o=j}^{N_{J_c}} \sum_{d=o+1}^{N_{J_c}} \lambda_{o,d} + \sum_{o=j}^{N_{J_c}} \sum_{d=o+1}^{N_{J}} \lambda_{o,d} + \sum_{o=crit}^{N_{J}} \sum_{d=o+1}^{N_{J}} \lambda_{o,d}} + (\alpha)\left(1 - \frac{1}{j^{switch} - j}\right) \\
\theta_2 &= \frac{\sum_{o=j}^{N_{J_c}} \sum_{d=o+1}^{N_{J}} \lambda_{o,d}}{\sum_{o=j}^{N_{J_c}} \sum_{d=o+1}^{N_{J_c}} \lambda_{o,d} + \sum_{o=j}^{N_{J_c}} \sum_{d=o+1}^{N_{J}} \lambda_{o,d} + \sum_{o=crit}^{N_{J}} \sum_{d=o+1}^{N_{J}} \lambda_{o,d}} + (1-\alpha)\left(1 - \frac{1}{j^{switch} - j}\right) \\
\theta_3 &= \frac{\sum_{o=crit}^{N_{J}} \sum_{d=o+1}^{N_{J}} \lambda_{o,d}}{\sum_{o=j}^{N_{J_c}} \sum_{d=o+1}^{N_{J_c}} \lambda_{o,d} + \sum_{o=j}^{N_{J_c}} \sum_{d=o+1}^{N_{J}} \lambda_{o,d} + \sum_{o=crit}^{N_{J}} \sum_{d=o+1}^{N_{J}} \lambda_{o,d}} + \left(\frac{1}{j^{switch} - j}\right)
\end{aligned}
\tag{2}
$$

Transferring Criterion

We apply a transfer criterion as presented by Gavriilidou and Cats (*25*). The authors apply the following criterion to a single stop, given different levels of information on the passenger demand. In line with the formulation of the regularity criterion, we assume that passenger information is based on historical data on boarding, alighting and transferring passengers.

The holding time needed for synchronization is set equal to the difference between current time and the expected arrival of the next vehicle of the connecting line and is given by the following formula:

$$t_i^{hold,sync} = \left(\tilde{t}_{i+1,j}^{arrival} - t^{current}\right) + \tau^{transfer} \tag{3}$$

with

$\tilde{t}_{i+1,j}^{arrival}$	the expected arrival time of the following vehicle of the connecting line i+1 at stop j in [time units]
$t^{current}$	current time in [time units]; and
$\tau^{transfer}$	minimum transferring time between vehicles in [time units].

Passenger Cost Comparison

At each of the shared transit corridor stops, holding aims to provide instructions to the driver in terms of dwell time in order to minimize the cost. Therefore, the decision to hold for regularity (Equation (1)) or for synchronization (Equation (3)) is based on the minimum passenger cost:

$$Pax\_Cost = \beta_{wait} c^{wait} + \beta_{transfer} c^{transfer} + \beta_{held} c^{held} \tag{4}$$

Passenger cost consists of all different components of passenger travel time. Waiting time cost c^{wait} is the product of half of the predicted headway between consecutive arrivals and the arrival rate of the passengers at the current and the downstream stops of the rolling horizon:

$$c^{wait}=\delta\sum_{m=j}^{\mu}\frac{1}{2}\left(t_{imk}^{arr}+t_{ijk}^{hold,reg}-t_{imk}^{exit}\right)\Lambda_{\mu}+(\delta-1)\sum_{m=j}^{\mu}\frac{1}{2}\left(t_{imk}^{arr}+t_{ijk}^{hold,sync}-t_{imk}^{exit}\right) \quad (5)$$

Where μ is the number of subsequent stops considered for the comparison of passenger cost and δ a dummy variable which is equal to 1 for waiting time cost with holding for regularity and zero when holding for synchronization. Transfer cost $c^{transfer}$ (if the current vehicle will not be held for synchronization) is the time transferring passengers have to wait until the next arrival of the desired downstream line:

$$c^{transfer}=\delta\left[\tilde{p}^{transfer}\left(\tilde{t}_{i+1,j}^{arrival}-t_{i,j}^{arrival}\right)\right] \quad (6)$$

The expected number of transferring passengers is estimated as the product between the fraction of alighting passengers *a* at the transferring stop and the sum of the arrival rates transferring from one line to another:

$$\tilde{p}^{transfer}=a\sum_{m=1}^{j}\sum_{n=j}^{N_{i+1}}\lambda_{m,n} \quad (7)$$

Finally, the cost of held passengers c^{held} is the product of the passengers on board and the additional time of the control action they experience:

$$c^{held}=\delta\left(t^{hold,reg}q\right)+(\delta-1)\left(t^{hold,sync}q\right) \quad (8)$$

All components are weighted according to results of previous studies (*34*, *35*) for a given comparison horizon. The comparison horizon for the cost of the waiting passengers is set to the number of remaining downstream common stops. Regularity and synchronization criteria can be paired at any stop of the shared transit corridor.

$$t^{hold}=\begin{cases} t^{hold,reg} & Pax\_Cost^{reg} < Pax\_Cost^{sync} \\ t^{hold,sync} & Pax\_Cost^{reg} \geq Pax\_Cost^{sync} \end{cases} \quad (9)$$

CASE STUDY

The holding criterion presented above is tested for a generic network consisting of two lines operating in one direction as depicted in Figure 1. Both lines consist of 30 stops, and have the same stop sets. The first ten stops of each line serve the passengers within the initial branch, followed by ten successive stops within the shared transit corridor, which is the set of common stops, and finally the last ten stops of each line compose the final branch of the lines. Both lines have the same frequency of 10 min.

The vehicles on each line are dispatched so that an ideal joint frequency of 5 min is planned for the common part and vehicles arrive alternately at the merging stop. Vehicles complete trips in one direction only, without being assigned for another trip to the opposite direction. The demand profiles of the lines are given in Figure 2. The majority of the demand is concentrated in the shared transit corridor while the two branches and the traversing passenger groups have similar demand shares.

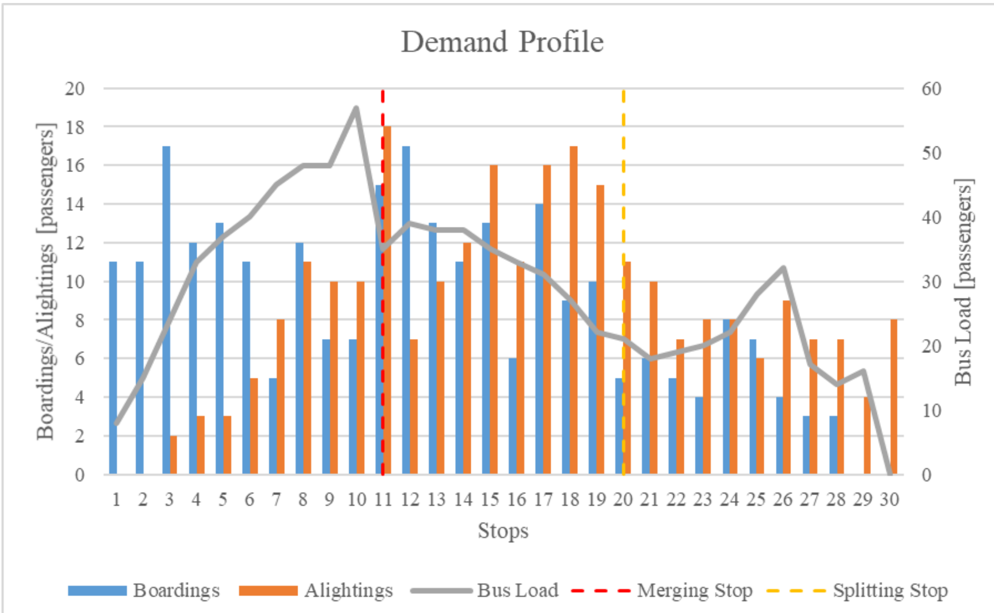

Figure 2 Demand Profiles of Lines 1 and 2

For this experimental setup, one common stop is chosen as control point for both regularity and synchronization.

Scenarios

The first division of the scenarios is based on the two general passenger groups that benefit from the regularity criterion and the transferring criterion, respectively. In order to assess the effect of synchronization on the network, three scenarios with different shares of transferring passengers are tested corresponding to 5%, 10% and 15% of the total demand. Passengers are transferring only from line 1 to line 2, in order to capture potential differences in performance between the first and the connecting line. The size of passenger groups affected by the regularity criterion remains unchanged through the different scenarios. The dynamics of the subgroups of passengers travelling to different parts of the network are assessed for the different parts of the network.

An important factor is the stop where the choice between the regularity and transferring criterion is made. Two different cases are tested: in the first case, holding for synchronization is enabled at the merging stop and in the second case at the diverging stop of the corridor. Three different control schemes are compared: the first is a do-nothing scenario (NC) where no control action is taken. The second scheme is a single line rule based holding control strategy (EH) by Cats et al (*12*) that regularizes the headway subject to both the preceding and the succeeding vehicle while limiting the maximum allowed headway to a specific share of the planned headway.

Finally, the proposed cooperative passenger cost (CPC) criterion of Equation 1 is used. With CPC, at the merging and the diverging stop, we allow also holding for synchronization. The decision between holding for regularity or synchronization is taken by comparing the passenger cost using Equation 4. Three different scenarios are considered based on the number of subsequent stops taken into account when comparing the passenger cost. Passenger cost is compared at the current stop only, for five downstream stops and for ten downstream stops. The five stop horizon corresponds to half of the length of the downstream stop set, while the ten stop horizon represents the full length.

In summary, the scenarios are divided in two categories based on the stop wherein either synchronization or regularity criterion are applied, in three further categories based on the control scheme chosen (NC-EH-CPC). A final subdivision is performed for CPC scenarios alone, based on the horizon chosen for comparison. For the sake of understanding, the scenario names bear the same form SxCyz, where x refers to the stop that passengers can transfer at (1:merging stop,2:diverging stop) and y to the control scheme used (1:No Control, 2: Even Headway and 3: CPC). For scenarios with CPC, z refers to the horizon (number of stops) used to compute the passenger cost (1: One stop 2: Five Stops and 3: Ten Stops). The scenarios are tested for three different levels of transferring passenger demand.

All scenarios are simulated using the mesoscopic transit simulator BusMezzo (*36*). BusMezzo has been previously used to evaluate holding strategies (*37*). The simulator includes a set of implemented holding strategies which are called after the completion of the dwell time to calculate holding time before giving the departure time of the vehicle at each stop. In order to apply coordinated control, all interacting lines should be taken into consideration. After the completion of dwell times, the first step of the controller is to retrieve all lines sharing the same control strategy. The routes of the lines are then compared stop by stop in order to find the set of consecutive stops. If the number of consecutive stops is equal or greater than the minimum number for them to be considered a shared transit corridor (as pre-specified by the user) then the merging stop and the diverging stop are defined, otherwise both lines are treated as individual, according to Laskaris et al (2016). The remaining stops sets are then characterized following their relation to the identified shared transit corridor (initial branch - final branch). Regularity holding is adopted according to the characterization of the current stop.

If synchronization between lines is allowed, the expected arrival of the next vehicle of the connecting line is estimated by summing the scheduled riding time between the last visited stop and the transfer stop.

For the calculation of passenger cost, the length of the comparison horizon in terms of number of stops is needed as input. The regularity criterion's passenger cost is derived from the expected headways along the horizon and the historical arrival rates for the passengers at stops. For the transferring passenger cost, the number of transferring passengers is also given in arrival rates via an input file. The arrival rates for the transferring passengers result from the number of transferring passengers as recorded in the no control scenario.

RESULTS

Corridor Results

The shared transit corridor can be considered as the most important stop set, since the majority of the demand is generated or travels through this part of the network. Passengers on this part of the network benefit from the joint frequency of vehicles from different lines. Table 1 shows the coefficient of variation of the joint headway on the shared transit corridor for all different scenarios. Undeniably, CPC yields the lowest variability by accounting for all lines that mutually interact. The only exception appears in the scenario which allows synchronization at the last stop, combined with cost comparison accounting for the diverging stop alone. The significantly high variability of the joint headway is a result of the number of synchronization events that take place at the last stop.

TABLE 1 Coefficient of Variation of Joint Headway

Joint Headway		Merging Stop (S_1)	Diverging Stop (S_2)
5% Transferring Passengers	C_1	0.504	0.491
	C_2	0.412	0.401
	C_{31}	0.398	0.403
	C_{32}	0.388	0.392
	C_{33}	0.389	0.393
10% Transferring Passengers	C_1	0.521	0.490
	C_2	0.406	0.381
	C_{31}	0.393	0.419
	C_{32}	0.409	0.388
	C_{33}	0.403	0.387
15% Transferring Passengers	C_1	0.518	0.515
	C_2	0.435	0.427
	C_{31}	0.401	0.460
	C_{32}	0.399	0.423
	C_{31}	0.399	0.401

Two representative examples of scenario with 15% share of transferring passengers with synchronization allowed at the merging and the diverging stop, respectively, are illustrated in Figure 3. Recall that this scenario has the highest share of transferring passengers. The progression of the coefficient of variation of joint headways along the shared transit corridor is plotted against the corridor stops. With CPC, coordination between lines initiates at the branches and vehicles enter the shared transit corridor with a lower coefficient of variation. CPC manages to maintain low variability for the majority of the corridor stops until the point where a transition to single line operation begins. The most notable difference between the two scenarios is the behavior at the stop where synchronization is allowed. While synchronization rarely occurs in the merging stop scenario, this is the most frequent control decision in the diverging stop scenario. Vehicles held for synchronization must therefore wait for a time equal to the joint headway, which increases the level of variability accordingly. However, since this happens at the last common stop, it does not affect the joint operation.

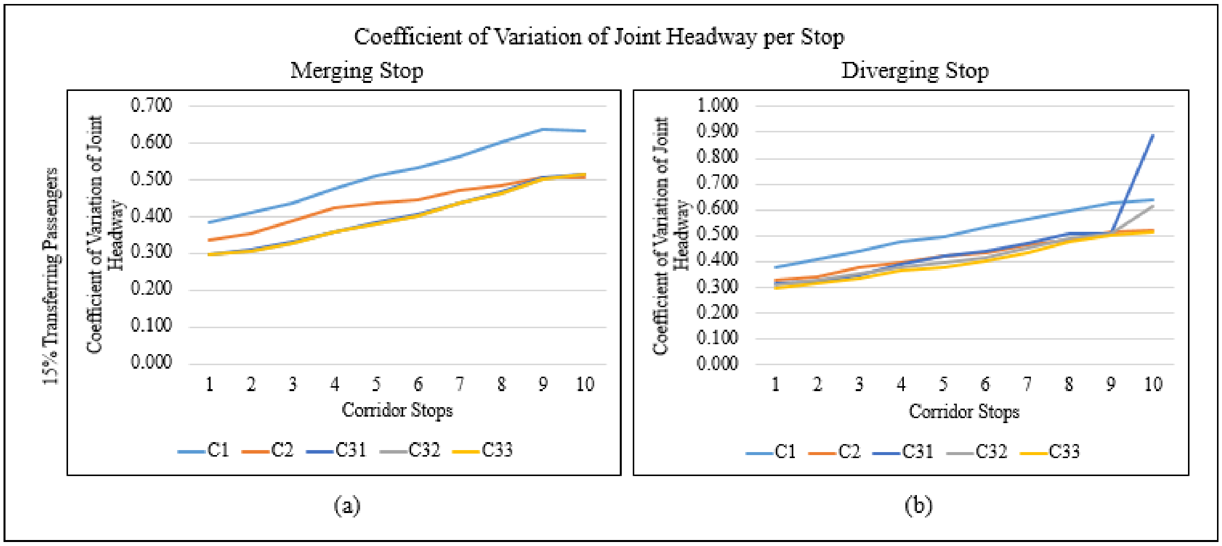

Figure 3 Coefficient of Variation of Joint Headway per Stop

The passenger costs for all passengers using the shared transit corridor are summarized in Table 2. CPC outperforms NC and EH in terms of waiting time in all scenarios. NC results in the lowest in-vehicle time because the reduction of waiting times in the holding control scenarios comes at the cost of an increased in-vehicle time. Among the scenarios where control is applied, CPC scenarios results to lower in vehicle time compared to EH scenarios. Again, when synchronization is chosen more frequently, it requires longer holding times and this is noticeable in scenario S_2C_{31}. The in-vehicle delay is significantly higher compared to the scenarios with different horizons and control schemes.

TABLE 2 Passenger cost at the shared transit corridor

	5% Transferring Passengers			10% Transferring Passengers			15% Transferring Passengers		
	Waiting Time [sec]	In vehicle time [sec]	Travel Time [sec]	Waiting Time [sec]	In vehicle time [sec]	Travel Time [sec]	Waiting Time [sec]	In vehicle time [sec]	Travel Time [sec]
S_1C_1	169.7	151.4	490.9	167.6	152.2	487.4	176.1	152.2	504.5
S_1C_2	163.3	154.0	480.6	161.5	155.0	477.9	166.8	155.1	488.8
S_1C_{31}	161.6	152.5	475.6	161.2	153.9	476.4	164.3	153.0	481.5
S_1C_{32}	161.2	152.1	474.5	161.2	154.2	476.7	164.1	152.9	481.1
S_1C_{33}	162.0	152.7	476.7	161.5	153.9	476.8	164.1	152.9	481.1
S_2C_1	173.3	150.8	497.4	172.9	151.3	497.0	173.6	152.3	499.4
S_2C_2	165.9	153.0	484.8	164.1	154.1	482.4	164.8	154.8	484.4
S_2C_{31}	164.1	154.8	482.9	154.0	161.2	469.2	149.9	163.5	463.4
S_2C_{32}	163.9	152.8	480.7	163.6	153.5	480.6	159.8	156.8	476.3
S_2C_{33}	163.9	152.9	480.6	164.4	153.2	482.1	160.9	154.2	476.0

Line Level Results

The coefficient of variation of headway per stop is shown in Figure 4 for line 1 and in Figure 5 for line 2. The performance improves significantly compared to NC and as expected EH is the most effective strategy regulating single line operation. With CPC, the evolution of the variability index follows similar behavior with the lines in merging and diverging fork networks (*29*, *33*). For both lines, the variability of the headway on the branch prior to the shared transit corridor starts increasing when the coordination is prioritized, and the regularity of the joint operation becomes more important. At the end of the corridor, the criterion shifts again from joint operation to single line operation and the loss of performance cannot be recovered until the end of the line. An interesting trend is apparent for the line that is held for synchronization at the diverging stop. In the scenarios with high shares of transferring passengers, scenario S2C33 shows a significant reduction in the coefficient of variation at the diverging stop compared to the scenarios bearing other cost comparison horizons. This scenario has the lowest share of holding for synchronization. No comparable effects for the connecting line (Line 2) can be seen. Line 2 performance is comparable to that of a diverging line as in (*33*), with a loss of performance due to the transition and a late recovery in the final branch. Line 2 is held only for regularity and the transferring passengers at the diverging stop are treated similarly to passengers travelling from corridor to branch.

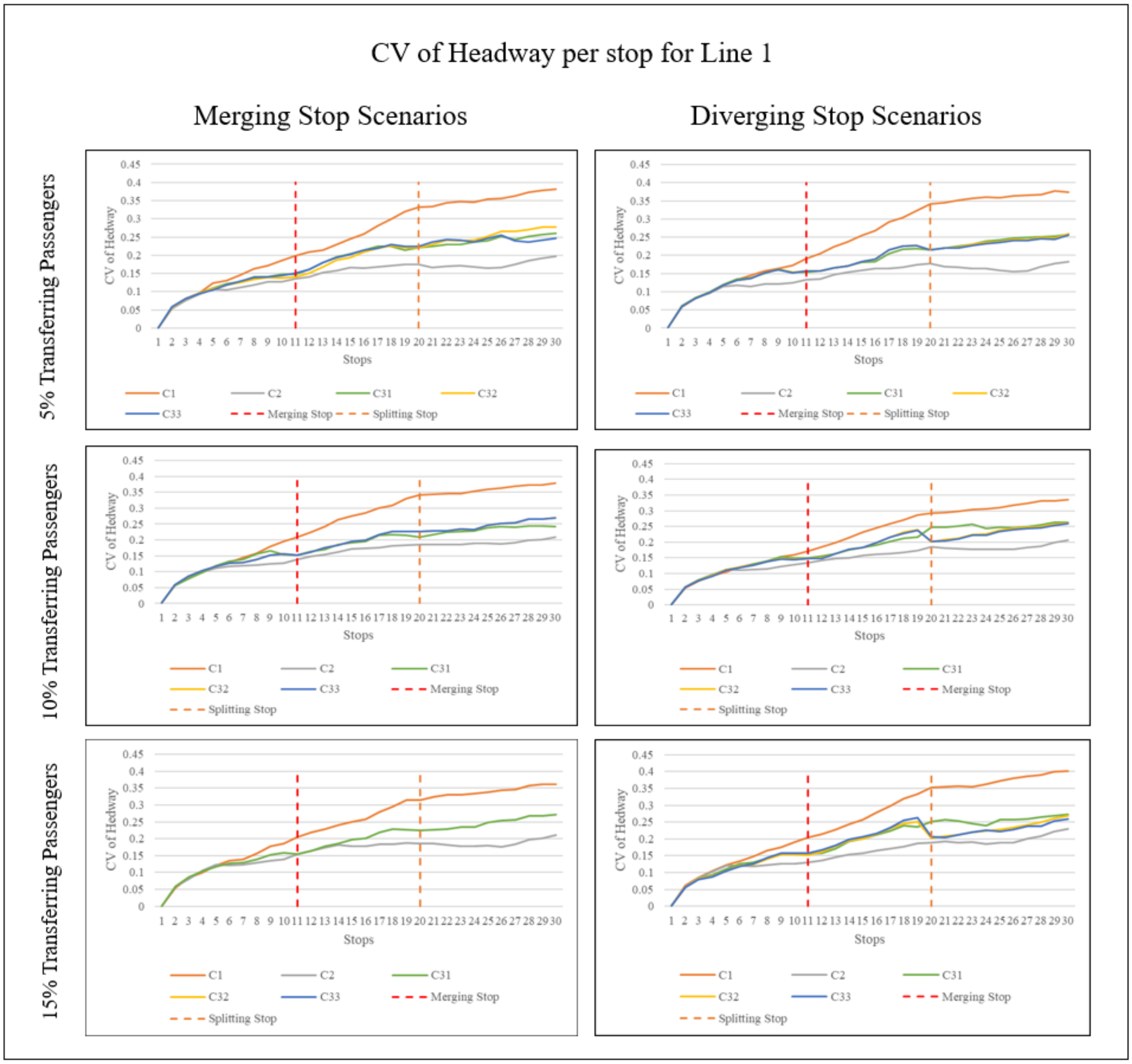

Figure 4 Coefficient of Variation of Headway per Stop of Line 1

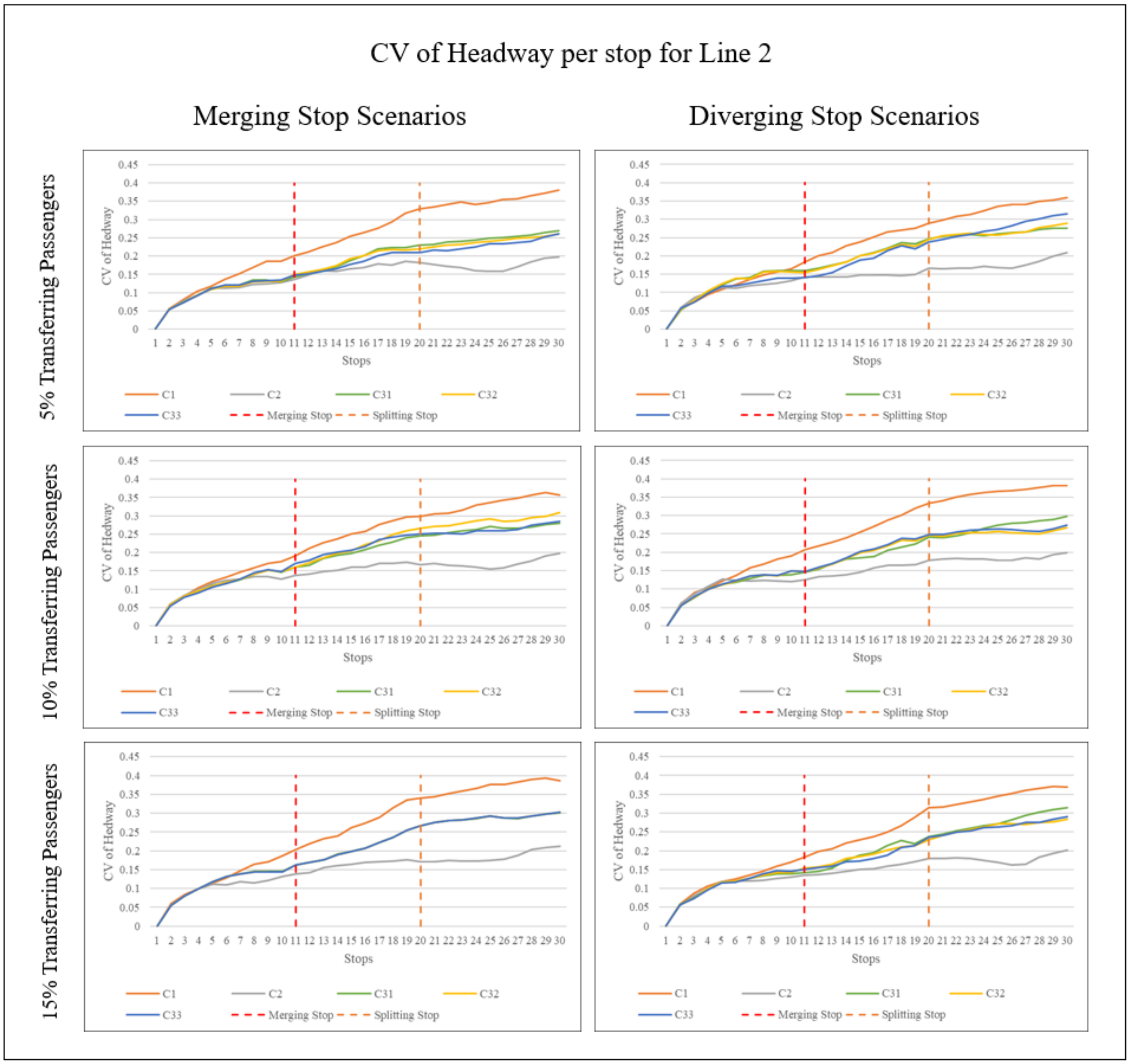

Figure 5 Coefficient of Variation of Headway per Stop of Line 2

Table 3 shows the passenger costs for line 1 and line 2. As expected, EH outperforms CPC in terms of line regularity for both lines, meeting its objective. Compared to NC, CPC achieves better results in terms of waiting time with a lower cost for the on-board passengers. In vehicle time with CPC slightly increases or remains at the same level as NC. The only exception is the S_2 scenario set for 15% of transferring passengers. Due to the high number of synchronization events, passengers on line 1, who are favored by the regularity of the line, are in turn penalized by an additional time waiting for line 2.

TABLE 3 Passenger costs for Line 1 and Line 2

	5% Transferring Passengers			**10% Transferring Passengers**			**15% Transferring Passengers**		
	Waiting Time [sec]	In vehicle time [sec]	Travel Time [sec]	Waiting Time [sec]	In vehicle time [sec]	Travel Time [sec]	Waiting Time [sec]	In vehicle time [sec]	Travel Time [sec]
	Line 1								
S_1C_1	314	144.2	772.2	315.2	144.4	774.9	314	144.4	772.3
S_1C_2	304.2	146	754.4	308.3	147.4	764	304.4	147.1	755.9
S_1C_{31}	308.6	145.4	762.5	310.2	145.9	766.3	310.2	145.3	765.7
S_1C_{32}	307.3	145	759.5	310.5	146	767.1	310.2	145.2	765.6
S_1C_{33}	309	145.6	763.5	310	146.2	766.3	310.2	145.2	765.6
S_2C_1	314.1	143.9	772.2	313.4	144.3	771.1	314.4	144.7	773.4
S_2C_2	304.3	145.7	754.2	308.3	146.3	762.9	305.2	147.1	757.6
S_2C_{31}	308.6	146.7	764	309.1	151.2	769.3	310.4	153.2	774
S_2C_{32}	308.9	145.7	763.5	306.7	145	758.5	309.4	147.5	766.4
S_2C_{33}	308.9	145.7	763.5	306.7	144.8	758.3	308.6	146	763.2
	Line 2								
S_1C_1	310.2	143.7	764.1	310.7	144.1	765.5	315.4	144.7	775.4
S_1C_2	305.4	146.7	757.4	304.7	146.6	756	304.9	146.7	756.5
S_1C_{31}	308.4	145	761.7	308.1	145.9	762.1	310.1	144.9	765.1
S_1C_{32}	308.4	144.9	761.6	308.1	145.6	761.7	309.9	145	764.8
S_1C_{33}	309.3	145.2	763.8	308.6	145.5	762.8	309.9	145	764.8
S_2C_1	311.8	143.8	767.3	316.6	144.3	777.4	312.2	144.6	769
S_2C_2	305.8	145.9	757.5	307	146.1	760.1	303.5	146.5	753.4
S_2C_{31}	310.4	145.5	766.4	309.5	145.1	764.1	308.1	145.3	761.4
S_2C_{32}	308.6	146.7	764	309.1	151.2	769.3	310.4	153.2	774
S_2C_{33}	308.9	145.7	763.5	306.7	145	758.5	309.4	147.5	766.4

The control decisions have different effects on each of the six passenger groups in the double fork network. Figure 6 illustrates the relative passenger cost compared to NC at the network level for all scenarios. The biggest gain from CPC is achieved prior to and within the shared transit corridor. The passengers traversing different stops sets are the most crucial passenger groups for CPC, since they experience the control action for regulating the operation of each stop set and the transition between stop sets. The reduction in passenger cost is lower with CPC than with EH. For the scenarios with synchronization at the diverging stop, passenger costs increase significantly in S_2C_{31}. Beside the cost increase for passengers travelling from the initial branch to the final branch, the cost increases also for the passengers travelling from the corridor to the branches in contrast to scenarios S_2C_{32} and S_2C_{33}. EH is superior for the final branch in all scenarios.

Figure 6 Network passenger cost per passenger group

Transfer Waiting Time

Table 4 summarizes the average transfer time and its standard deviation for each scenario. The lowest average transfer time is reported in the NC and EH scenarios. In both instances lines operate independently and there is no corrective action for the joint headway or bunching between lines. On the other hand, on the shared transit corridor CPC aims to maintain or restore regularity based on the joint frequency of the common stop set. Vehicles among different lines are therefore controlled in order to maintain a stable interline headway and reduce interline bunching. As a result, CPC offers a more reliable transfer time due to the corridor control for regularity. S_2C_{32} resulted to 42.7% less variable transfer time compared to S_2C_2. Synchronization has conflicting objectives with respect to regularity in a shared transit corridor, and by applying multiline control and seeking for the optimum for the network a longer but reliable transfer time can be achieved compared to potentially shorter but dramatically unreliable operations, inflicting thus occasionally very long delays.

TABLE 4 Average Transfer Time and Standard Deviation

		C_1		C_2		C_{31}		C_{32}		C_{33}	
		Average Transfer Time [sec]	St Deviation	Average Transfer Time [sec]	St Deviation	Average Transfer Time [sec]	St Deviation	Average Transfer Time [sec]	St Deviation	Average Transfer Time [sec]	St Deviation
Merging Stop S1	5% Transferring Passengers	287.8	39.7	303.2	42.3	288.6	34.5	295.7	29.1	289.1	30.2
	10% Transferring Passengers	288.5	37.3	291.0	34.7	303.6	30.8	293.2	26.5	307.3	28.2
	15% Transferring Passengers	295.5	42.6	285.3	43.2	299.0	25.4	297.0	26.6	299.1	25.4
Diverging Stop S2	5% Transferring Passengers	291.3	34.5	308.0	43.2	304.5	28.8	318.1	24.7	310.8	26.6
	10% Transferring Passengers	287.1	38.2	286.9	34.7	299.0	30.0	297.5	24.8	301.5	30.0
	15% Transferring Passengers	294.7	38.8	282.6	41.3	295.9	27.9	293.6	26.6	288.9	24.5

Controller Decisions

The frequency of each control decision per scenario is summarized in Table 5.

TABLE 5 Controller Decisions

Comparison Horizon		10 Stops		1 Stop		5 Stops	
		Regularity	Synchronization	Regularity	Synchronization	Regularity	Synchronization
		Merging Stop					
Merging Stop	5% Transferring Passengers	100%	0%	100.0%	0.0%	100%	0%
Merging Stop	10% Transferring Passengers	100%	0%	99.0%	1.0%	100%	0%
Merging Stop	15% Transferring Passengers	100%	0%	99.5%	0.5%	100%	0%
		Diverging Stop					
Diverging Stop	5% Transferring Passengers	95.4%	4.6%	66.4%	33.6%	94.6%	5.4%
Diverging Stop	10% Transferring Passengers	92.4%	7.6%	10.1%	89.9%	85.2%	14.8%
Diverging Stop	15% Transferring Passengers	87.1%	12.9%	1.1%	98.9%	47.2%	52.8%

Controlling for regularity at the beginning of the shared transit corridor is dominant due to the great penalization in terms of passenger cost of the corresponding group if synchronization is selected. Independently from the length of the horizon, the majority of the passengers are concentrated downstream on the shared transit corridor and benefit from the regularization of the system. Synchronization at the merging stop can cause extremely long waiting times for the passengers along the shared transit corridor. Therefore, when the comparison of passenger cost extends beyond the current stop, the system focuses exclusively on maintaining regularity. When comparing the cost at the current stop alone, control for synchronization occurs, but comprises only 1% of the control decisions.

The results are significantly different when synchronization is allowed at the diverging stop as opposed to the merging stop. The comparison horizon extends to the branch stop set. Demand on the branches is lower than on the shared transit corridor, which makes synchronization a feasible option. Similar to synchronization at the merging stop, vehicles are held mostly to restore regularity, but the controller chooses to hold for synchronization even in Scenario 1 with a low share of transferring passengers. The share of synchronization decisions increases with the share of transferring passengers and with the shortening of the comparison horizon. In the most myopic scenario (current stop cost comparison), synchronization is the most frequent choice in scenarios with 10% and 15% share of transferring passengers.

CONCLUSIONS

In this paper, a multiline criterion for regularity is introduced and tested using a synthetic double fork network with two lines. In addition, synchronization is allowed at either the merging or the diverging stop. The criterion for choosing between synchronization and regularity is the resulting passenger cost for each control action.

Overall, the results show that multiline control is beneficial for the network, resulting in a lower overall passenger cost. This result comes from the substantial gains along the shared transit corridor. In line with the results for a merging fork network (*29*), coordination helps to achieve a joint headway with lower variability prior to the common stop set and this is maintained along the corridor. Although the performance of CPC at the individual line level is not as high as single line control, significant cost reduction with lower in vehicle delay for the lines (often a shortcoming of holding control strategies) is achieved compared to no control.

With a high concentration of demand on the shared transit corridor, from a control perspective it is not recommended to favor synchronization over regularity at the merging stop since the expected synchronization cost is very high for the system. At the diverging stop, synchronization becomes feasible and is the dominant choice under a range of demand distribution settings for shorter cost comparison horizons. However, at the diverging stop and after regulating the joint operation, transferring passengers benefit mostly by the low variability of the joint headway and their average transfer time corresponds to the headway of the shared transit corridor.

This study introduces a new holding criterion that can be applied in a common type of transit networks, the branch and trunk one. The criterion is easy to be implemented in real time as it mostly relies on the position of the vehicles in real time and historical data for the passenger demand. Synchronization is also integrated in this control scheme and recommendations are provided on which stop it can be feasible and what will be the consequences to each of the passenger groups. Based on our findings, synchronization should be applied at the last common stop to be feasible and to avoid affecting the joint operation at the common part.

Future research will focus on extending the evaluation of the criterion to a greater number of lines and more transfer options. Moreover, similarly to the work of (*38*), different levels of real time passenger data will be integrated to assess their potential for estimating the actual passenger cost and hence contribute to more informed control decisions.

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AUTHOR CONTRIBUTIONS

The authors confirm contribution to the paper as follows: study conception and design; G. Laskaris, O. Cats, E. Jenelius, M. Rinaldi and F. Viti; analysis and interpretation of results: G. Laskaris; draft manuscript preparation: G. Laskaris, O. Cats, E. Jenelius, M. Rinaldi and F. Viti. All authors reviewed the results and approved the final version of the manuscript.

REFERENCES

1. Eberlein, X. J., N. H. M. Wilson, and D. Bernstein. Modeling Real-Time Control Strategies In Public Transit Operations. In *Computer-Aided Transit Scheduling* (P. N. H. M. Wilson, ed.), Springer Berlin Heidelberg, pp. 325–346.
2. Zolfaghari, S., N. Azizi, and M. Y. Jaber. A Model for Holding Strategy in Public Transit Systems with Real-Time Information. *International Journal of Transport Management*, Vol. 2, No. 2, 2004, pp. 99–110. https://doi.org/10.1016/j.ijtm.2005.02.001.
3. Ibarra-Rojas, O. J., F. Delgado, R. Giesen, and J. C. Muñoz. Planning, Operation, and Control of Bus Transport Systems: A Literature Review. *Transportation Research Part B: Methodological*, Vol. 77, 2015, pp. 38–75. https://doi.org/10.1016/j.trb.2015.03.002.
4. Puong, A., and N. H. Wilson. A Train Holding Model for Urban Rail Transit Systems. *Computer-aided systems in public transport*, 2008, pp. 319–337.
5. Newell, G. F., and R. B. Potts. Maintaining a Bus Schedule. No. 2, 1964.
6. Potts, R. B., and E. A. Tamlin. Pairing of Buses. *Australian Road Research*, Vol. 2, No. 2, 1964.
7. Oort, N. van, J. W. Boterman, and R. van Nes. The Impact of Scheduling on Service Reliability: Trip-Time Determination and Holding Points in Long-Headway Services. *Public Transport*, Vol. 4, No. 1, 2012, pp. 39–56. https://doi.org/10.1007/s12469-012-0054-4.
8. Abkowitz, M. D., and M. Lepofsky. Implementing Headway-Based Reliability Control on Transit Routes. *Journal of Transportation Engineering*, Vol. 116, No. 1, 1990, pp. 49–63. https://doi.org/10.1061/(ASCE)0733-947X(1990)116:1(49).
9. Rossetti, M. D., and T. Turitto. Comparing Static and Dynamic Threshold Based Control Strategies. *Transportation Research Part A: Policy and Practice*, Vol. 32, No. 8, 1998, pp. 607–620.
10. Fu, L., and X. Yang. Design and Implementation of Bus-Holding Control Strategies with Real-Time Information. *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 1791, 2002, pp. 6–12. https://doi.org/10.3141/1791-02.
11. Daganzo, C. F., and J. Pilachowski. Reducing Bunching with Bus-to-Bus Cooperation. *Transportation Research Part B: Methodological*, Vol. 45, No. 1, 2011, pp. 267–277. https://doi.org/10.1016/j.trb.2010.06.005.
12. Cats, O., A. Larijani, H. Koutsopoulos, and W. Burghout. Impacts of Holding Control Strategies on Transit Performance. *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2216, 2011, pp. 51–58. https://doi.org/10.3141/2216-06.
13. Xuan, Y., J. Argote, and C. F. Daganzo. Dynamic Bus Holding Strategies for Schedule Reliability: Optimal Linear Control and Performance Analysis. *Transportation Research Part B: Methodological*, Vol. 45, No. 10, 2011, pp. 1831–1845. https://doi.org/10.1016/j.trb.2011.07.009.
14. Zhao, J., S. Bukkapatnam, and M. M. Dessouky. Distributed Architecture for Real-Time Coordination of Bus Holding in Transit Networks. *IEEE Transactions on Intelligent Transportation Systems*, Vol. 4, No. 1, 2003, pp. 43–51. https://doi.org/10.1109/TITS.2003.809769.
15. Bartholdi III, J. J., and D. D. Eisenstein. A Self-Coördinating Bus Route to Resist Bus Bunching. *Transportation Research Part B: Methodological*, Vol. 46, No. 4, 2012, pp. 481–491. https://doi.org/10.1016/j.trb.2011.11.001.
16. Hickman, M. D. An Analytic Stochastic Model for the Transit Vehicle Holding Problem.

Transportation Science, Vol. 35, No. 3, 2001, pp. 215–237. https://doi.org/10.1287/trsc.35.3.215.10150.
17. Eberlein, X. J., N. H. M. Wilson, and D. Bernstein. The Holding Problem with Real–Time Information Available. *Transportation Science*, Vol. 35, No. 1, 2001, pp. 1–18. https://doi.org/10.1287/trsc.35.1.1.10143.
18. Delgado, F., J. C. Munoz, and R. Giesen. How Much Can Holding and/or Limiting Boarding Improve Transit Performance? *Transportation Research Part B: Methodological*, Vol. 46, No. 9, 2012, pp. 1202–1217. https://doi.org/10.1016/j.trb.2012.04.005.
19. Sáez, D., C. E. Cortés, F. Milla, A. Núñez, A. Tirachini, and M. Riquelme. Hybrid Predictive Control Strategy for a Public Transport System with Uncertain Demand. *Transportmetrica*, Vol. 8, No. 1, 2012, pp. 61–86. https://doi.org/10.1080/18128601003615535.
20. Koehler, L. A., L. O. Seman, W. Kraus, and E. Camponogara. Real-Time Integrated Holding and Priority Control Strategy for Transit Systems. *IEEE Transactions on Intelligent Transportation Systems*, 2018.
21. Nesheli, M. M., and A. Ceder. Real-Time Public Transport Operations: Library of Control Strategies. *Transportation Research Record: Journal of the Transportation Research Board*, No. 2647, 2017, pp. 26–32.
22. Hall, R., M. Dessouky, and Q. Lu. Optimal Holding Times at Transfer Stations. *Computers & Industrial Engineering*, Vol. 40, No. 4, 2001, pp. 379–397. https://doi.org/10.1016/S0360-8352(01)00039-0.
23. Delgado, F., N. Contreras, and J. C. Munoz. Holding for Transfers. Presented at the Transportation Research Board 92nd Annual Meeting, 2013.
24. Wu, W., R. Liu, and W. Jin. Designing Robust Schedule Coordination Scheme for Transit Networks with Safety Control Margins. *Transportation Research Part B: Methodological*, Vol. 93, 2016, pp. 495–519.
25. Gavriilidou, A., and O. Cats. Reconciling Transfer Synchronization and Service Regularity: Real-Time Control Strategies Using Passenger Data. *Transportmetrica A: Transport Science*, Vol. 0, No. 0, 2018, pp. 1–29. https://doi.org/10.1080/23249935.2018.1458757.
26. Hernández, D., J. C. Muñoz, R. Giesen, and F. Delgado. Analysis of Real-Time Control Strategies in a Corridor with Multiple Bus Services. *Transportation Research Part B: Methodological*, Vol. 78, 2015, pp. 83–105. https://doi.org/10.1016/j.trb.2015.04.011.
27. Argote-Cabanero, J., C. F. Daganzo, and J. W. Lynn. Dynamic Control of Complex Transit Systems. *Transportation Research Part B: Methodological*, Vol. 81, No. Part 1, 2015, pp. 146–160. https://doi.org/10.1016/j.trb.2015.09.003.
28. Fabian, J. J., and G. E. Sánchez-Martínez. Simulation-Based Comparison of Holding Strategies for a Multi-Branch Light Rail Service. Presented at the Transportation Research Board 96th Annual MeetingTransportation Research Board, 2017.
29. Laskaris, G., O. Cats, E. Jenelius, M. Rinaldi, and F. Viti. Multiline Holding Based Control for Lines Merging to a Shared Transit Corridor. *Transportmetrica B: Transport Dynamics*, 2018.
30. Hadas, Y., and A. Ceder. Public Transit Network Connectivity. *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2143, 2010, pp. 1–8. https://doi.org/10.3141/2143-01.
31. Chriqui, C., and P. Robillard. Common Bus Lines. *Transportation Science*, Vol. 9, No. 2, 1975, pp. 115–121. https://doi.org/10.1287/trsc.9.2.115.
32. Marguier, P. H. J., and A. Ceder. Passenger Waiting Strategies for Overlapping Bus Routes.

Transportation Science, Vol. 18, No. 3, 1984, pp. 207–230. https://doi.org/10.1287/trsc.18.3.207.

33. Laskaris, G., O. Cats, E. Jenelius, M. Rinaldi, and F. Viti. A Holding Control Strategy for Diverging Bus Lines. Presented at the Conference on Advanced Systems in Public Transport, Brisbane, Australia, 2018.
34. Wardman, M. Public Transport Values of Time. *Transport Policy*, Vol. 11, No. 4, 2004, pp. 363–377. https://doi.org/10.1016/j.tranpol.2004.05.001.
35. Cats, O., and G. Loutos. Evaluating the Added-Value of Online Bus Arrival Prediction Schemes. *Transportation Research Part A: Policy and Practice*, Vol. 86, 2016, pp. 35–55. https://doi.org/10.1016/j.tra.2016.02.004.
36. Toledo, T., O. Cats, W. Burghout, and H. N. Koutsopoulos. Mesoscopic Simulation for Transit Operations. *Transportation Research Part C: Emerging Technologies*, Vol. 18, No. 6, 2010, pp. 896–908. https://doi.org/10.1016/j.trc.2010.02.008.
37. Cats, O., A. Larijani, Á. Ólafsdóttir, W. Burghout, I. Andréasson, and H. Koutsopoulos. Bus-Holding Control Strategies. *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2274, 2012, pp. 100–108. https://doi.org/10.3141/2274-11.
38. Gavriilidou, A., and O. Cats. Reconciling Transfer Synchronization and Service Regularity: Real-Time Control Strategies Using Passenger Data. *Transportmetrica A: Transport Science*, Vol. 0, No. 0, 2018, pp. 1–29. https://doi.org/10.1080/23249935.2018.1458757.