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Understanding Hohfeld and Formalizing Legal Rights: The Hohfeldian Conceptions and Their Conditional Consequences

Abstract. Hohfeld's analysis (*Fundamental Legal Conceptions as Applied in Judicial Reasoning*, 1913, 1917) on the different types of rights and duties is highly influential in analytical legal theory, and it is considered as a fundamental theory in AI&Law and normative multi-agent systems. Yet a century later, the formalization of this theory remains, in various ways, unresolved. In this paper I provide a formal analysis of how the working of a system containing Hohfeldian rights and duties can be delineated. This formalization starts from using the same tools as the classical ones by Kanger and Lindahl used, but instead of focusing on the algebraic features of rights and duties, it aims at providing a comprehensive analysis of what these rights and duties actually are and how they behave and at saying something substantial on Power too—maintaining all along the Hohfeldian intentions that these rights and duties are *sui generis* and inherently relational.

Keywords: Rights, Duties, Hohfeld, SDL, ET, State enforcement.

1. Introduction

The Hohfeldian analysis on rights and duties is often referred to and discussed even a century after its birth. This fact shows its importance but also indicates that we are still a bit unsure, on one hand, what was meant by Hohfeld and how this theory could and should be placed on the map of analytical legal theory and our general image of law—and, on the other, how it could and should be formalized. Taking the frame and mission of analytic philosophy seriously, these two issues are obviously connected: formalizing a notion or a theory not only provides a tool of its applicability in formal environments but also clearly helps us understand its meaning. This is what I intend to contribute to in this paper delineating a proposal of the formal representation of a system of rights which is based on Hohfeld's theory. Even if one of my aims is exactly to work as closely to the Hohfeldian intentions

Special Issue: Permissions, Obligations and Beyond
Edited by **Piotr Kulicki** and **Olivier Roy**

Studia Logica
https://doi.org/10.1007/s11225-019-09870-5

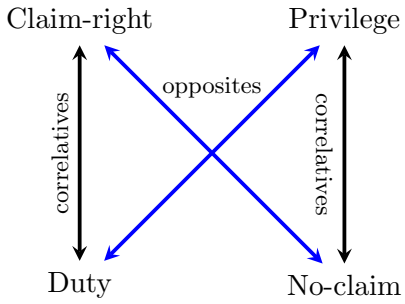
Published online: 16 September 2019

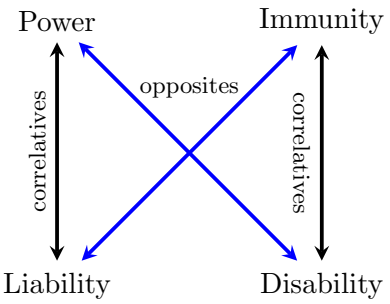
as possible, developing such a formalization sometimes makes it necessary to lean on other theories or theoretical presuppositions. I restrict this reliance to the minimal and only to theories whose spirit corresponds to that of the Hohfeldian analysis. This still means, though, that readers who have different legal theoretical convictions from the ones serving the background of this paper (e.g. who refuse positivism or accept other versions of it) might be reluctant to accept of this formalization as a general model of how legal rights work. This natural limitation comes with the readers' possibly limited acceptance of the Hohfeldian theory itself, too. Nevertheless, even if arguing for the trueness of the Hohfeldian theory is not the primary aim of this paper, with this formal approach I am going to shed light on the impropriety of some often discussed counter-arguments.

2. The Many Faces of 'Right'

The well-known reason why Hohfeld differentiates the types of rights and duties is that he finds that the word 'right' is overused and "even if the difficulty related merely to inadequacy and ambiguity of terminology, its seriousness would nevertheless be worthy of definite recognition and persistent effort toward improvement". This terminological problem involves conceptual obscurity, so bringing it to the fore and showing the possible meanings—the exact legal conceptions—behind it would allow us to see with clarity what we refer to in the various cases: what rights and duties (there) are.

The well-known system of rights (upper line) and duties' (lower line) correlative pairs Hohfeld built can be reconstructed in the following diagrams.





First, let us examine the group on the left. If I have a parcel of land, having privilege to walk through it means that the other person has no claim that I do not go through. This parcel being mine means that the other person has the duty to stay away: I have the claim-right towards him to do so. As the diagram indicates, Claim-right and Duty always go together. The same is

true for the correlative pair of Privilege and No-claim. As the latter's label shows: No-claim is the opposite of having a claim-right. We have a privilege to do something exactly when we don't have a duty to refrain from it; more exactly, as Hohfeld emphasizes: "always, when it is said that a given privilege is the mere negation of a duty, what is meant, of course, is a duty having a content or tenor precisely the opposite to that of the privilege in question." [7]

The group on the right side exhibits a very similar structure; there are various points of difference between the groups, though. Hohfeld already laid down that the rights in the second group are above the rights of the first group in a sense: with Power-rights (but not with the rights in the first group) one does (or does not, in the case of Disability) have the ability to change legal positions; for instance: if I have a parcel, I have a power to sell it, but selling it changes my claim-rights, privileges, and actually my powers connected to it. Therefore the members of the second group are usually considered higher-order rights. Among others, Fitch [3] and Makinson [15] argued for considerations along which the difference seems to be more sophisticated. From Fitch's work we can read[1] that the expressed modalities' nature is different: while we can call the members of the Claim-right group deontic modalities, the Power group involves some kind of capacity, so calling them 'capacitative', rather than 'deontic modalities' might be more exact. Makinson adds that there are structural differences, for example, in their consequences: while if we do something without permission (which is a deontic modality), we have to expect a sanction; by contrast, if we do something without power, we *actually do not do it*. These observations are crucial when formalizing the notions. And even if this point of different consequences is pivotal, this is the area in which it is worth to seek for the source of a formalization of the Hohfeldian rights and duties in a *uniform manner*: I show in this paper how they can be compared and differentiated—and therefore, in a sense, defined—by their conditional consequences.

From the viewpoint of deontic logic, the virtue of the Hohfeldian system is that it handles agency—and this is highly connected to correlativity: according to Hohfeld, someone's right *always* involves someone else's duty, and the other way around. Rights and duties do not exist on their own: being

[1]Fitch is not explicit on this difference but it can be seen from the way he formalizes the modalities.

someone's right and someone else's duty inheres in their essence.[2] And this is a point at which we can start to consider our actual formalization.

3. No Loss of Direction

As Makinson [15] writes: the Hohfeldian theory of rights and duties is "resolutely relational". Hohfeld considers a right relation as a relation between two agents, therefore, we cannot speak either agent's right position without involving the other one. This is a point whose formal representation is (almost[3]) completely missing from the classical Kanger–Lindahl theory of normative positions (see for instance [10–12,14]), according to Makinson [15] and Sergot [19]. Makinson provides some preliminary formal considerations on how directedness should be involved in the formalization of the Hohfeldian theory, and Herrestad and Krogh [6] is the often referred paper introducing the explicit notion of a directed obligation and the indices for the duty-bearer *and* the counterparty that Makinson suggested. They start from an example of a Kangerian atomic type of right, which is a conjunction:

$$O({}_iEA)\,\&\,O({}_jEA)$$

which in his system is to be read as 'a right for j of the type claim, not counter-freedom that A'. The problem is, as it was noted by Makinson [15], each of the two conjuncts may represent an obligation directed at another counterparty than j. Herrestad and Krogh call this as 'the problem of a loss of direction'. Their solution is provided with the same notation Makinson used: writing the bearer's index on the left side of the deontic operator, and the counterparty's one on the right side, and defining as a conjunction:

$${}_iO_j({}_iEA) =_{df} \; {}_iO({}_iEA)\,\&\,O_j({}_iEA)$$

with which there is no loss of direction. As they note, treating ${}_iO_j$ as logical primitive would be sufficient to avoid a loss of direction, but Herrestad and

[2]The question arises whether this is generally true for a legal system as a whole. At this point above I intentionally phrased as "according to Hohfeld": as I clarified in the introduction the primary aim of this paper is providing a formal approach to the theory of Hohfeld. Therefore, if the reader is reluctant to accept the general validity of the Hohfeldian theory, it is better to consider the following formal analysis with that restricted validity. Nevertheless, I do aim to use this formal analysis to show in what follows that this restriction is less reasonable then it now might seem. For the detailed rebuttal of an often raised limitation, see Markovich [16].

[3]Some considerations have been made, though, on whether the intended reading of Kangerian formulae can count as the representation of relationality, see [15].

Krogh want to preserve the undirected version of obligation for cases where it is uncertain whether there is a counterparty at all (or if yes, then that who she is).

However, discussing relationality and directionality of Hohfeldian rights and duties usually stops after considering the correlative pair of Claim-right and Duty: other Hohfeldian rights and duties are not even discussed as being or not being directed. So in this way, we, indeed, lose the directions that Hohfeld strictly assigned to his fundamental legal conceptions. The Claim-right-Duty pair may be considered even more fundamental than the others,[4] Hohfeld is very consistent about this relationality, though: all four pairs of legal positions are described strictly as relational ones. This is crucial in understanding them.

To show how directionality influences the formal representation of rights and duties, let's use the propositional letter F to refer to a 'given state of affairs', and let's use a simple 'sees to it that' sentential operator with the notation E_x, where x is a variable indicating the agent who "acts", with the proposition(al letter) in its argument ('E_xF' is standing for 'x sees to it that F'). Also, let's use sentential modal operators for the Hohfeldian right positions denoting them with (mostly) the rights' initial letters: **CR** is for Claim-right, **O** is for Duty, **PR** is for Privilege, **NC** is for No-claim, **P** is for Power, **L** is for Liability, **I** is for Immunity, and **D** is for Disability. In order to express their being assigned to agents, we use agents (variables) as indices, also the notion of directedness that Herrestad and Krogh [6] introduced, but with the notation Sergot [19] uses. As we will see, it will be needed not merely in the case of Duty, but also for the other active right positions, that is, in cases where the agent whose right position we consider and the one whose action we consider are the same.[5]

Let's start with putting the equivalence to the first correlative pair, Claim-right and Duty:

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[4]Since, for instance, one could say that the second pair, Privilege and No-claim, is described only as the lack of the given duty and the given claim; and Power and the related rights are "different".

[5]This differentiation between active and passive rights and duties is already expressible with these notations and syntactic constraints on them: we can say that in the case of active rights, i.e. where the right owner (or duty bearer) is the same as the actor, the deontic and the action operator have to have the same indices on to be a well-formed formula; meanwhile in the case of passive rights they have to be different. This means that, for instance, $\mathbf{CR}_xE_xF$ and $\mathbf{PR}_xEyF$ are syntactically wrong—as it seems reasonable to think that having claim-right on my own act (against myself) does not make sense *as a legal construction*.

$$\mathbf{CR}_x E_y F \Leftrightarrow \mathbf{O}_{y \to x} E_y F \tag{1}$$

It is nicely visible why the obligation on the right has to be directed in order to be Duty, that is, a correlative pair of Claim-right: without the notation of directedness, x would not occur in that part of the formula.
Let's move to Privilege and No-claim:

$$\mathbf{PR}_x E_x F \Leftrightarrow \mathbf{NC}_y \neg E_x F \tag{2}$$

There is a problem with this formula: its left side does not mention the other party, the party who lacks a claim-right against x. The problem might be more obvious if we use the directed obligation (Duty) to define what Privilege is:

$$\mathbf{PR}_x E_x F \Leftrightarrow \neg \mathbf{O}_{x \to y} \neg E_x F \tag{3}$$

This directionality in the obligation is not visible on the left-hand side, suggesting this way that *there is no duty* at all. If we would like to use a separate notation, an abbreviation for having a privilege, we have to indicate that it is indeed—only—*a* privilege: missing *a* duty (not necessarily all possible duties), the duty which is a directed one, toward the person who has, therefore, no claim-right against x, as it shortly will be shown formally. It may seem strange how a *lack* of something could bear the property of being directed, since if I do not have a duty to refrain from something, that privilege of mine sounds rather general and not directed. But Hohfeld's example is revealing: "Suppose that X, being already the legal owner of the salad, contracts with Y that he (X) will never eat this particular food. With A, B, C, D and others no such contract has been made. One of the relations now existing between X and Y is, as a consequence, fundamentally different from the relation between X and A. As regards Y, X has no privilege of eating the salad; but as regards either A or any of the others, X has such a privilege. It is to be observed incidentally that X's right that Y should not eat the food persists even though X's own privilege of doing so has been extinguished." This 'as regards a given agent' is crucial in understanding the notion of correlativity, and thus crucial in formalizing these correlative notions. Hohfeld is clear on this issue: "a right is an affirmative claim against another, and a privilege is one's freedom from the right or claim of another". The way we wrote in formulas above, the equivalences would only hold in the case where we have only two agents. If it is not the case—as it is usually not—then further explicit notation is needed: let's use the symbol $\succ$ to

express who the agent is whose claim-right we are free from[6] when we have *a* privilege:

$$\mathbf{PR}_{x\succ y}E_xF \Leftrightarrow \neg\mathbf{O}_{x\rightarrow y}\neg E_xF \tag{4}$$

This way the equivalence holds, as it does for No-claim:

$$\mathbf{PR}_{x\succ y}E_xF \Leftrightarrow \mathbf{NC}_y\neg E_xF \tag{5}$$

This is the case with the pair of Power and Liability too. Power is an active right since the right-owner and the agent of the action is the same. Therefore, an undirected power won't be equivalent to a liability which bears its directedness in its formal representation; that is, the following formula does not hold:

$$\mathbf{P}_xE_xF \Leftrightarrow \mathbf{L}_yE_xF \tag{6}$$

but the one below does:

$$\mathbf{P}_{x\rightarrow y}E_xF \Leftrightarrow \mathbf{L}_yE_xF \tag{7}$$

It may seem strange at first glance that Power is a relational thing. This feeling might come from that, in everyday life, we usually refer to Power as something which is not relational: we usually say that *someone* has *the* power to do something, that is, the only thing we mention is the right-owner. But—as it has been emphasized—the Hohfeldian system is consequently relational: with the power we can change *someone's* rights or duties. In the US, a state has the power to summon to the jury each citizen registered in the given state—which sounds like a general Power, but this is a superficial observation. On the one hand, conditions excusing from being a juror can be indicated in a form previously sent to prospective jurors, which is actually a reference to their *immunity*, that is, that the given citizen has no Hohfeldian liability. And on the other hand, no US state has the power to put a duty on me—since I am a Hungarian citizen. Most of the given state's citizens have liability regarding this action of their state, though: each of them has a situation in which their duty of serving as a juror can be created.

The other reason that might make the reader reluctant to accept the relationality of Power is that there is a very close, still distinct notion called 'power' in the close literature: Searle's *institutional power*. The point in this latter is exactly the generality it involves: the general acceptance that

[6]The choice of the symbol $\succ$ is intended to reflect the "similarity" between its form and the notion being free *from* something; this use has no relation to its usual use.

makes the counts-as rules working. The formal representations of the Searlean power happens together with counts-as rules (see for instance Jones and Sergot [9] and others listed in Grossi and Jones [5]), they focus on the counts-as relation behind Power, talking about the special context by which some "normal" action or utterance (like saying something) realizes another action—a "special", legally relevant one (like declaring, ordering, etc.). The legal relevance means that the new action or its result state of affair is something that all of us recognize in its role. If a registrar declares a couple married, we all recognize that arising marriage as a legal construction (institution). But this Power is the Searlean one, not the Hohfeldian. What was really important in Power to Hohfeld is that it changes (more precisely, can change) legal positions. The fact that the given legal positions change provides the result that a generally valid legal construction arises, so the two notions of Power are very close, still: from the Hohfeldian viewpoint (which is inside the law) it is about the relation between two persons, that is, directed positions. And the question where directedness or relationality comes into the picture is: *who* can change *whose* legal position? It needs to be emphasized that we are already within the law: the statements we discuss are all legal(ly relevant) sentences, so—while the Hohfeldian Power also has a strong relation to the counts-as norms—there would be no point in focusing on the counts-as-created context anyway; we are already in it, therefore, this paper does not consider how brute facts become institutional ones, only (institutional[7]) facts of a given legal system, like someone's right, other's duty and their logical relation.

After all these, probably it is not surprising to the reader that I claim that Disability, which is an active duty (just like the specific Hohfeldian Duty itself as in having the same agent as the duty-bearer and the actor), also needs a refinement in its formal representation to show its directedness—and so the equivalence with Immunity which already shows its relationality in our language set-up:

$$\mathbf{I}_x E_y F \Leftrightarrow \mathbf{D}_{y \to x} E_x F \tag{8}$$

At Section 5 we will come back to the question whether this resolute relationality of the Hohfeldian theory means that the rights we feel undirected—for example because we think of them as absolute rights, like in the case of property—cannot be explained by the Hohfeldian conceptions.

[7] A legal system has only institutional facts: even "brute" or "natural" facts arise in their institutional—declared, confirmed, proved, established, etc.—form.

4. An Image of a System of Rights

So far so good, but these equivalences do not say anything substantial about how these rights and duties "behave" in the legal system, or what they mean, only that they come in pairs. If we want to provide a description of what Hohfeldian rights and duties look like and what consequences they have we can start with a simple language whose notations we used above. The classical formalizations by Kanger and Lindahl (more precisely, the rephrasing of these theories by Makinson [15] and Sergot [19]) used a very simple non-normal modal logic "behind" the operator E_x which logic Chellas [2] called ET. It contains only the rule of interchangeability of equivalent propositions in the operator's scope and the axiom T in order to have successful actions. Behind the deontic modalities, they used SDL—so do I.[8]

4.1. Language and Semantics

Let's start with the following language set-up:

Definition 1. Our modal language is given by

$$p \in \Phi \mid \phi \,\&\, \psi \mid \neg\phi \mid \bot \mid E_a\phi \mid \mathbf{O}_{a\to b}\phi$$

for $a, b \in A$ finite set of agents, where Φ is the set of propositional letters.

Definition 2. For a set W of possible worlds and set A of agents write

$$\mathfrak{F} = \langle W, f_a, R^O_{a,b}\rangle_{a,b\in A}$$

where $f_a : \wp(W) \to \wp(W)$ is a function and $R^O_{a,b} \subseteq W^2$ is a binary relation.

Models are structures

$$\mathfrak{M} = \langle W, f_a, R^O_{a,b}, V\rangle_{a,b\in A}$$

where V is a valuation function for atomic propositions: $V : \Phi \to \wp(W)$

Definition 3. For $\mathfrak{M} = \langle W, f_a, R^O_{a,b}, V\rangle_{a,b\in A}$ and $w \in W$ we let

- $V(\bot) = \emptyset$
- $w \models p \Leftrightarrow w \in V(p)$ for propositional letters $p \in \Phi$.

[8] Using SDL carries with it the usual problems, but in this paper I do not address them—partly because I join Sergot [19] in arguing that inadequacies of SDL's often criticized axioms are "relatively benign" in this topic; but mainly in order to keep the paper on the track of its original focus. Ultimately, SDL scarcely will be the axiomatic background of the perfect formal theory of normative positions, but one step at a time.

- $w \models \varphi \,\&\, \psi \Leftrightarrow w \models \varphi$ AND $w \models \psi$.
- $w \models \neg\varphi \Leftrightarrow w \not\models \varphi$.

It is convenient to extend the valuation V to arbitrary formulas:

$$V(\varphi) := \{w : \mathfrak{M}, w \models \varphi\}$$

and we add the following:

- $w \models E_a\varphi \Leftrightarrow w \in f_a(V(\varphi))$
- $w \models \mathbf{O}_{a\to b}\varphi \Leftrightarrow \forall w'(wR^O_{a,b}w' \Rightarrow w' \models \varphi)$

Constraints

- constraint on $f : f_a(X) \subseteq X$ for all $X \subseteq W$, in particular $f_a(V(\varphi)) \subseteq V(\varphi)$
- constraint on $R^O_{a,b}$: $\forall w \exists w'\ wR^O_{a,b}w'$

4.2. Formalizing Claim-Right and Duty

While in their formal representations Kanger and Lindahl concentrated on the algebraic features of the structures that can be built from the composition elements [a given state of affairs and its negation, doing and its negation (refraining), and obligation and its negation (permission)] with providing atoms of a Boolean algebra, this language—with some refinements—can be used to describe some inherent features of Hohfeldian rights and duties, too. What we need to keep in mind, though, is that Hohfeld refuses to provide proper definition ('proper' in a sense that reducing the conceptions—definiendum—to something else—definiens)[9] because he considers these conceptions *sui generis*. Therefore, we do not lean on the will theory or the interest theory.[10]

What we lean on, though, is an insight Makinson [15] uses tacitly when providing an informal suggestion on who the counterparty of a directed obligation is:
x bears an obligation to y that F under the system N of norms
iff

[9]Hohfeld uses the expression 'formal definition', but we have no reason to think that the formal approach itself is what he refuses.

[10]These two theories are the common referential points in legal theory defining what rights are. The interest theorists—the first of which usually Bentham is identified—grasp the point of a right in the interest of the right-holder, while the will theorists—Kant, Hart, Kelsen, etc.—see the right-holder's control as the crucial point. For explanation and further references see for instance [22].

in the case that F is not true then y has the power under the code N to initiate legal action against x for non-fulfillment of F

This tradition of defining the counterparty is called *claimant* theory[11] as it identifies the counterparty with the claimant. Makinson lays down the informal definition-like description of the counterparty, but by this suggestion, Makinson (and other advocates of claimant theory, like Wellman [21]) provide a kind of definition of what a directed obligation is (and considering the correlation and the directedness of the obligation, they practically define the claim-right of y also). Makinson himself calls attention to the problem of the material conditional on the right-hand side of the biconditional: in this form it would cause that if x sees to it that F (which would make the conditional sentence vacuously true), it would be an obligation of him to see to it that (because of the biconditional). This is something we obviously do not want. And there is another difficulty with this—intuitive-sounding—definition (as Sergot [19] points out), the right-left direction of the biconditional: y can initiate a legal action against anyone without having a claim-right originally. It's just that he won't win the case. Sergot suggests that involving some kind of expectation of success in the definition could be a solution. I think the solution is somewhere else (which, however, can be considered as an explanation of how the expectation of success comes into the picture).

We can provide a comprehensive description of what a directed obligation (and, therefore, a claim-right) is with using the notion of state enforcement—that the Makinsonian definition uses, too.

This point is worth spending some time on, though, since choosing state enforcement as a (the) basic feature of legal rights might need some justification, even if it has deep historical and philosophical roots. It is well to clarify some issues in its defense. In this paper, only *legal* rights and duties are considered. The Hohfeldian theory has a relevance in ethics, since obligations and rights can be *moral* ones; it also find applications, as Sergot [19] writes, "in other areas, such as the specification of aspects of computer systems, as a contribution to the formal theory of organisations in the analysis of notions such as responsibility, entitlement, authorisation and delegation, and in the field of multi-agent systems." But the original, and, therefore,

[11] Claimant theory is not the only approach of the counterparty: Herrestad and Krogh [6] consider themselves as proponents of the benefit theory with focusing on who the beneficiary is of a directed obligation when defining counterparty. I do not commit myself to any of these two: on the one hand, I think the differentiation—even if it has legal philosophical background—is artificial; and on the other hand, these commitments of the mentioned authors did not make any difference in the formal representation of rights.

primary context of the Hohfeldian conceptions is law. A second argument in defense of building of state enforcement when describing the essence of rights is that the possibility of seeking remedy in court—and binding the very *existence* of the rights to it—actually is a well-grounded, fundamental expectation in western legal culture. The *Declaration of the Rights of Man and of the Citizen*, being approved by the National Assembly of France, August 26, 1789, is not just the crucial document of the French Revolution, but also the establishing pioneer of the European chartal constitutionalization. Its articles[12] list the rights we still consider as the most important ones regarding ourselves as humans and citizens. Some of these are general liberty, the right to property (Article 2), equality before the law (Article 6) or free speech (Article 11). But there are two articles in which the reader finds some "meta-requirements": Article 12 says that "To guarantee the Rights of Man and of the Citizen a public force is necessary" while in Article 16 we find that "Any society in which no provision is made for guaranteeing rights (...), has no Constitution." Article 12 declares the necessity of state enforcement of rights listed above and below it, while Article 16 clarifies this necessity's level: not ensuring the existence of state enforcement of rights has no constitution, a society in which the observance of the law is not assured has no constitution at all—and since the constitution is the means of ensuring rights by providing them a legal frame, in such a state there are no legal rights. Meanwhile the Declaration considers human rights and political rights, which are (to be) regarded as complex rights of the *sui generis* ones described by Hohfeld, we have no reason to debate that the *notion of a right* as such involves this possibility of enforcement as a requirement of being a *legal* right.[13]

But let's consider this issue a bit more generally: among norms, the possibility of seeking remedy in court is the *differentia specifica* of legal ones. (At first glance, it might seem intuitive to regard *sanction* as the hallmark of a legal norm, but that would not be true: for Catholic people, going to Hell constitutes a sanction in the case of religious norms; also, being ostracized can be highly unattractive, which is a reason behind following social norms.

[12] The English translation presented here is from the Digital Public Library of America: https://www.dp.la/primary-source-sets/sources/889.

[13] Jospeh Raz mentions cases in [17] that are referred to as 'rights' but they are unenforceable. I consider this kind of referring to these rules as metaphors: they might put some moral burden on the "obliged" agent to act according to the "requirement", but a rule without defining the (legal) consequences of its violation is—as Roman law called it—a *lex imperfecta*. I think these are not rights (and duties) in the legal sense, rather some directives with no legal effect and effective, real legal relevance.

But none of these norms has the state behind them.) And while rights and, especially, duties as notions can be associated with morality and ethics also, Hohfeld himself was a jurist and called these notions fundamental *legal* conceptions *as applied in judicial reasoning*, that is, it is himself who puts these notions in a judicial context. Therefore it seems natural, reasonable, and justified to capture their essence by building on their specificity making them *legal rights and duties*.[14]

So I use the insight from the Makinsonian definition of involving state enforcement, but I do not use Power in it. Instead, I leave that Claim-right play the main role in the state enforcement since this is the case in law. We need to introduce into the language an agent constant j standing for judiciary,[15] and a sentential operator C with the intended meaning that the leeway of a given state of affairs is compensated. The proposal formally looks like this:

$$\mathbf{O}_{x\to y}E_xF \leftrightarrow \neg E_xF \to \mathbf{CR}_yE_jE_x\mathrm{C}F \tag{9}$$

that is, x has a duty toward y to see to it that F if and only if whenever x does not fulfill his duty, y will have a claim-right against the judiciary to see to it that x see to it that F is compensated. Involving judiciary is the point in describing what state enforcement means: in the case of non-fulfillment the original two-sided right relation becomes a new one;[16] a new claim-right (and, therefore, a new duty) arises: a new claim-right of the original counterparty but this time against the judiciary to enforce *the original duty*—as we probably would intuitively phrase it. But what is enforced cannot really be the original duty: the whole process only starts *if* the original duty is not fulfilled. Even if it was to pay a given amount, and what is enforced is paying the same amount, the states of affairs with which we refer to these are not the same since the original payment had a deadline—by which it has not

[14]Of course, I have no intention to claim that this is the only valid approach to rights (or to law in general). Even within legal positivism, whose original traditions (from John Austin) would justify such an approach, there are more recent leading schools (see, for example, H. L. A. Hart) having adversative theses.

[15]I discuss later whether it can be considered a 'judge', a judicial instant (that is, rather the role, not a particular person) instead.

[16]This new right relation might be considered as not two- but three-sided in a sense: the new claim-right-duty relation is a two-sided one, of course, between the original right-owner and the judiciary as the new duty bearer, but the original duty-bearer is still present, too: the state enforcement happens with a judgement which means the the judiciary has a power to change the original duty-bearer's normative positions, that is, he has a liability considering the judiciary's action of judgement.

been completed. (We, of course, need to interpret the compensation *broadly* containing those states of affairs which are similar enough[17] to the original, and not only the "classical" 'compensation' by which we usually mean monetary compensation.[18]) The point is that state enforcement is embodied by the claim-right that arises: the claim-right whose owner is the same as that of the original claim-right was, but the agent against whom it is there is a new one, the judiciary.

Why without Power? We have at least three good reasons. First of all, this way the problem with the Makinsonian definition does not arise: it is always true that if someone has a claim-right against the judiciary to enforce an—unfulfilled—original duty then this agent has (had) the original claim-right against the original duty-bearer. Second, there are cases where it is not the counterparty who has the power to initiate a legal action in the case of non-fulfillment. These cases—where the legal process is started by particular authorities *ex officio*—are mostly in public law, which is not the typical area of analyzing the Hohfeldian theory, but as it will be shown soon, a perfectly adequate one to do so. Using Power in the definition would rule these cases out of speaking about rights at all. The third reason is to leave Power out from defining what a claim-right is that if we define (counterparties of) Claim-right and Duty with Power, we lose a crucial difference between the ability of having rights and the ability to change them. This difference is very well represented in civil law countries' legal terminology as having distinct, well defined terms for them: for instance 'jogképesség' and 'cselekvőképesség' in Hungarian, 'Rechtsfähigkeit' and 'Handlungsfähigkeit' in German, they are 'capacidad de obrar' and 'capacidad de goce' in Spanish, or 'zdolność prawna' and 'zdolność do czynności prawnych' in Polish (all respectively); meanwhile the English legal terminology is not precise and transparent on this issue, maybe 'legal capacity' is the closest expression to describe the ability to have rights, and 'legal competence' or 'capacity to act' to describe the ability to change them. Each person has the first one, but not all of us have the second: children and people lacking mental soundness partly or completely lack the capacity to act.

[17]It is not straightforward how to give 'compensation' formally since it highly depends on what the original state of affairs was, how other states of affairs can be related to it, what the judiciary sees as similar enough, etc. So at this point let's assume that we know what compensation is, and that there is always one.

[18]I postpone, though, developing the formal details to further work, let's just take it for granted at this moment that we know what compensation means.

Does this mean then that Power has no role in *how a claim-right*—more precisely the ensuring of claim-rights—*works*? It absolutely does not mean that. It is not only a necessary factor of initiating a legal action—by which the state enforcement can be triggered in cases where the process does not start ex officio—, but also the necessary factor of how state enforcement happens: the judge has a power to impose a duty on the person involved in the legal action and this is the means of punishment in criminal cases, too (which will be discussed soon). But the fact that Power is the crucial element of state enforcement (both in triggering and in carrying it out) does not mean that we should involve it in the description of *what a claim-right is*. Initiating a legal action—for which power is needed, indeed—is about asking the judiciary to decide the given case (justly). What the formula (9) above is intended to show is the deep structure about consequences of having a duty (or a claim-right on the other side), this is *how it is* in the law,[19] it is *legal metaphysics*.[20]

And if it is legal metaphysics, it is natural to involve a legal necessity operator—that solves the problem of the material conditional indicated in the case of the Makinsonian definition:

$$\mathbf{O}_{x \to y} E_x F \leftrightarrow \Box(\neg E_x F \to \mathbf{CR}_y E_j E_x \mathrm{C} F) \tag{10}$$

This legal necessity operator comes with the logic S5 (and its classical semantics) since what we want to describe with it is really the way the things (rights and duties) are in law.

It might be worth saying a few words about the nature of the sentence $\neg E_x F$. As I emphasized earlier, in this theory we are within the law: even if this sentence seems to lack of any legal notion or modality, we need to consider it as it describes a legal, or institutional fact. What I mean by that is the following: the proper reading of the antecedent is that it has been established in court that x did not see to it that F. Not the brute fact is what we are interested in: it needs to be proven, or, we might also say that it needs to be decided by the judge[21] that this is what (did not) happen(ed). This finding of the judge which shifts such a fact legally relevant, or institutional.

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[19] Or *how it should be* in law—as Hohfeld had some difficulty with identifying his theory as descriptive or normative—at least his paper's reader surely has, we accordingly can phrase the proposal in two ways.

[20] I would like to thank John F. Horty for suggesting this description of what I show with the formulas.

[21] One might find it convincing if I refer to that this is considered this way in everyday life, too. Just think about the short dialogue between Agent Cooper and Audrey Horne about the presumed murderer of Laura Palmer in the end of Twin Peaks series one:

But, since we are already within the frame of a given legal system, we do not need to add anything to make it visibly institutional.[22]

4.2.1. Rights in Criminal Law Private law, however, is not the only area of law in which rights are handled. Public law is a branch in which it is a bit more difficult to detect the Hohfeldian correlativity and directedness: it is not surprising that rights like the right to life is a frequent example of counterarguments against the Hohfeldian system's generality. But it also can be shown with our formal tools that rights protected by criminal law are types of Claim-right, too.

$$\bigwedge_{x\in A} \mathbf{CR}_x \neg \bigvee_{y\in A} E_y F \leftrightarrow \Box \bigwedge_{y\in A} \left(E_y F \rightarrow \bigwedge_{x\in A} \mathbf{CR}_x E_j \mathrm{S}_y F \right) \tag{11}$$

that is, in the case of criminal law, all of us have a claim-right against everyone else ($x \neq y$) that no one commit felonies (no one sees to it that a given felony is committed), which means, that it is necessary that if anyone commits a felony then all of us have a claim-right against the judiciary to punish (sanction, S) the perpetrator[23] for seeing to it that F, that is for committing that felony.[24] This structure identifies crimes: those actions are pronounced as felonies whose lack all of us have a claim-right to—against all of us. The fact that in criminal law the public prosecutor, that is, the state's (the society's) representative stands against the accused agent shows that everybody's claim-right has been infringed (by not fulfilling refraining from committing the felony)—let's just recall how the court clerk pronounces a trial in the US when it starts: "The People of the State of (say) New

Footnote 21 continued

Audrey: Did you arrest him?

Cooper: Yeah.

Audrey: Did he do it?

Cooper: That's for a court to *decide*.

[22] If one thinks of this sentence this way, implying that "we are already in the court room", it must be easier to accept that this right-structure description does not need the power to initiate a legal action.

[23] y, that is, the index in S_y shows the subject of the sanction, not the agent who executes it; the details of the formal description of what a sanction is are postponed to further work, let's take it for granted at this point that we know what a sanction is.

[24] The formula can be written in this way, too:

$$\bigwedge_{x\in A} \bigwedge_{y\in A} \mathbf{CR}_x \neg E_y F \leftrightarrow \Box \bigwedge_{y\in A} \left(E_y F \rightarrow \bigwedge_{x\in A} \mathbf{CR}_x E_j \mathrm{S}_y F \right).$$

York versus y". Not the injured party, not the victim of the crime, not his relatives, and not the district attorney, but 'People of the State of New York'. For detailed argument for and about applying the Hohfeldian theory to criminal law, see Markovich [16].[25]

4.3. Formalizing Privilege and No-Claim

Hohfeld defined Privilege and No-claim as the *lack* of the given duty and claim-right, respectively:

$$\mathbf{PR}_{x\succ y}E_xF \Leftrightarrow \neg\mathbf{O}_{x\rightarrow y}\neg E_xF \tag{12}$$

$$\mathbf{NC}_xE_yF \Leftrightarrow \neg\mathbf{CR}_xE_yF \tag{13}$$

We cannot assign a legal (logical) consequence to a lack of a right. But we can put them into the formulas of Claim-right and Duty to check whether they still sound plausible. This is what happens when we put Privilege into the formula of Duty:

$$\neg\mathbf{O}_{x\rightarrow y}E_xF \leftrightarrow \neg\Box(\neg E_xF \rightarrow \mathbf{CR}_yE_jE_x\mathrm{C}F) \tag{14}$$

that is

$$\neg\mathbf{O}_{x\rightarrow y}E_xF \leftrightarrow \Diamond(\neg E_xF \wedge \neg\mathbf{CR}_yE_jE_x\mathrm{C}F) \tag{15}$$

that is, in the case of x having a privilege (from y) not to see to it that F is the case, it can happen that x does not see to it that F and y has no claim-right toward the judiciary that it see to it that x see to it that F is compensated. Why not sure? Does this diamond bother us? It does not: a conjunction without the diamond would mean, for instance, that—in the case of there is no duty of x toward y—x does not see to it that F. This is obviously not something we want to say since it would mean we only do things that we have to.[26]

[25] There is a range of actions conducting which are not felonies but fall under the notion of misdemeanor where committing is punished by the police (or sometimes other authorities with specific competence and jurisdiction). For these, it might be reasonable to give a specific—parallel—formula where the role of the judiciary is played by the police/these special authorities in order to get a more precise picture but the structure is the same.

[26] There might be readers who find (15) or the diamond itself too weak as it is compatible with y having the claim-right in the overwhelming majority of the cases, as long as there is one case where y doesn't have it, which criticism I should accept, but it seem to be the price we pay for using some legal necessity.

4.3.1. Protected Freedoms/Vested (Civil) Liberties It is often raised whether Hohfeld was right in taking Privilege as a fundamental legal conception, which is not even "something" but a lack of something, and not considering protected freedoms instead: where the other party is forbidden to interfere with what we do. These are what Bentham [1] called 'vested liberties', but the fact that this is what von Wright [23] called simply as a 'right' shows that its fundamentality is often raised. Nowadays we usually refer to these rights as 'civil liberties' (**CL**)—an obvious example of these is free(dom of) speech. We can show with this language that a civil liberty is a special type of a claim-right, a claim-right against everyone else not to stop me in speaking freely:

$$\mathbf{CL}_x E_x F \Leftrightarrow \mathbf{CR}_x \neg \bigvee_{y \in A} E_y \neg E_x F \tag{16}$$

That is, Hohfeld was right to take Claim-right (and its negation) as fundamental: other types of rights, like a civil liberty, can be expressed with it.[27]

4.3.2. Constraints on Iteration and the System of Appeal In the formulae (9)–(15) we used the agent constant j with the intended meaning 'judiciary' to talk about judiciary as a whole as an agent. To prevent an unwanted infinite regress we should include a restriction on the variable assignment not letting j to be put in the place of x (or y, depending on the formula). This is not the only solution, though.

There is another solution, which reflects on the inner hierarchy of the judiciary and the institution of appeal. Appeal plays a crucial role in the rule of law and in state enforcement by aiming at minimizing the fallibility and contingency coming with the human factor in it. The system of appeal can be involved into our formal representation of what a claim-right means in the following way:

$$\begin{aligned} \mathbf{CR}_x E_y F \leftrightarrow \Box(\neg E_y F \rightarrow \mathbf{CR}_x E_j E_y \mathrm{C}F) \\ \wedge \Box(\neg E_j E_y \mathrm{C}F \rightarrow \mathbf{CR}_x (E_J E_j E_y \mathrm{C}F \vee E_J E_y \mathrm{C}F)) \end{aligned} \tag{17}$$

where j and J are a judicial instances with an ordering $j < J$, that is, J stands for the appellate court (or judge). The disjunction in the consequent of the added conditional shows the two possible types of decision of the

[27] The formula can be written in the following way, too:

$$\mathbf{CL}_x E_x F \Leftrightarrow \bigwedge_{y \in A} \mathbf{CR}_x \neg E_y \neg E_x F.$$

appellate court: overwriting the original sentence or remanding the case to the original court for new trial can be the tools for ending up with a decision in accordance with the claim-right. Obviously, infinite regress has to be prevented in this case, too, so we can stop it by saying $E_J E_J F \leftrightarrow E_J F$. This, of course, can be generalized to a longer chain of hierarchy with j_i where $1 \leq i \leq n$ and an ordering $1 < \cdots < n$ and put the constraint on the highest level: $E_{j_n} E_{j_n} \leftrightarrow E_{j_n}$, where the possibility of further appeal ends.

4.4. Formalizing Power and Liability

The formalization of Power and Liability is something that the classical formalization by Kanger and Lindahl practically misses (they use the expression 'power' but not to the Hohfeldian notion). The formal representation of Power has to reflect upon the difference between the two Hohfeldian groups. Hohfeld provides a lot of examples of Power, but the most concrete remark we can use for the formalization is about the correlative concept of Liability: Hohfeld says that "it is a liability to have a duty created". There is Hungarian translation of the Hohfeldian essay by Miklós Szabó [8] in which the point made here can be better comprehended.[28] If we were to "retranslate" strictly literally the Hungarian version to English in order to show why it is more expressive, it would be something like this: Liability is the situation when it can happen that someone imposes a duty on us. What is missing—or at least not emphasized enough—in the original Hohfeldian sentence's literal level is that a liability is a liability even before the duty is created: we have liability not only when it happens, but we have liability already if it is the case that a duty can arise. From the examples we know that it is not only a duty (and a claim-right with it) that can arise from someone using her power, but also a privilege, and a power or an immunity too (with their correlative pairs, of course). This *potential* of Power is the essential point to be grasped. It can be formally described with some kind of if-then structure to keep the case when the power is not used—but is still present. This is good for us since we described (defined) Claim-right with a conditional, too, showing what consequence it has together with an action (actually, its lack). This structure perfectly fits our image of Power: if one uses it, it has legal (logical) consequences—in terms of the other person's rights upon which our agent has the power. We, therefore, can formalize Power by providing a (strict) conditional, just like it happened in the case of Claim-right and Duty. The following formula (where F', F'' ... are states of affairs—which

[28] "Beavatkozásnak kitettség az, ha kötelezettséget róhatnak ránk".

are not independent of F, explanation comes eftsoon; and v', v'' ... are agent variables) describes what we know about Power's working:

$$\mathbf{P}_{x\to y}E_xF^c \leftrightarrow \Box(E_xF^c \to (\mathbf{O}_{y\to v}E_yF' \vee \neg\mathbf{O}_{y\to v'}E_yF'' \vee\mathbf{P}_{y\to v''}E_yF''' \vee \neg\mathbf{P}_{y\to v'''}E_yF'''')) \quad (18)$$

There are some crucial points in this formalization worth noting. We can have a power only on special actions, that's what is indicated with the index c in F^c—shortly I explain its meaning; just like what the connection is between F^c and F'-s (but what we need to see already now that the F', ..., F'''' are different state of affairs, otherwise the consequent part of the conditional would be a tautology). There are some important remarks about the agents in (18): unlike in the case of rights and duties in the first Hohfeldian group, in the case of Power and related rights x and y can be the same since we can have a power to change *our* legal positions (most adults have); and it also can be the case that x and v (or v'...) are the same: creditors might have the power to change how much money someone owes to them (with putting interest on the amount), as they can release the debt, too. But it is not necessarily the case: the policeman can impose a duty on a driver to give priority to another car.

Let's see what F^c and F'-s are. The c in the index is supposed to indicate that we can have a power to act resulting only in states of affairs that are *constituted* by the law: by constitutive rules. By virtue of its importance, the formal representation of constitutive rules, or more precisely, that of counts-as rules is a highly discussed topic in deontic logic. The most important approaches are listed and discussed by Davide Grossi and Andrew J. I. Jones [5]. Two of these approaches refer directly to Power as a benchmark problem: the one referred to mainly with the (we might say) pioneer paper of Jones and Sergot [9] titled *The Formal Characterisation of Institutionalised Power*, and the approach discussed (among others) in the paper of Jonathan Gelati, Guido Governatori, Antonino Rotolo and Giovanni Sartor [4]. But as it has been discussed above: the notion Jones and Sergot discuss is the Searlean power, while the notion discussed by Gelati et al. rather covers Hohfled's one—even if they do not refer to him directly. Both approaches use a conditional logic to grasp the counts-as rules in order to explain how Power works. Jones and Sergot conduct a thorough analysis of count-as rules and their proposed conditional logic discussing (and formally characterizing) cases how brute facts(/actions) and institutional facts(/actions) can be joined by counts-as rules—and the connective $\Rightarrow_s$ with which they represent it—in different combinations. As the s in its index shows they

put a big emphasis on the institution s providing the context, they do not emphasize two things I consider crucial in the case of Power, though: its relationality in terms of agents (obviously, as in the Searlean notion of power it does not even come up) and the fact that the constitutive rule-based actions—on which we can have power at all, and actually, need to have to perform them—are utterances (in a broad sense). As Makinson [15] says discussing Hohfeld's theory: "power, in the sense that we are discussing, is always a power to » perform a legal act « or » create a legal fact « —in other words, to alter, by means of a performative utterance or inscription or some other conventionally recognized gesture or procedure". (I consider the latter as alternatives of an utterance being, in legal cases, also utterances in a broad sense of it—as soon it will be discussed.) Gelati et al. consider both relationality (in using agent indices—that's why they practically cover the Hohfeldian notion) and the relevance of utterances (in using the operator *proc* standing for proclamations), but in [4] they do not provide an analysis on its meaning in terms of semantics. What utterances are these actions? These utterances are about changing someone's rights: imposing them by declaration. These utterances usually (not always, though) use the notion created by the constitutive rules backing the given power: in the case of a purchase, for instance, we do not say things like "from now on, I have the privilege to use and eat this bar of chocolate, you have the claim-right that I pay its price to you, and after paying I will have the power to sell it...", we only say "I buy it"—since we all know what buying something means, it is written in the Civil Code. In these cases, these utterances realize the manageable and effective "technique of presentation" that Alf Ross [18] assigned to constitutive rules. Or, if we did not have the benefit of a proper discipline, we do not even say a word in the shop, just take the chocolate and put the money on the counter. That is, an utterance is understood in a broad sense including implicit conduct, too.[29] Sale, marriage (contract of a marriage), testamentary disposition, eviction, transferring one's interest are all F^c-s, more exactly, $E_x F^c$-s, that is making *legal statements*, that is, utterances with legal consequences. It is crucial to add: seeing to it that a given F^c is the case counts as (really) making a legal statement *only* if we had the power to do so. Otherwise, they are only gibberish without any legal consequences. Even if it is straightforward that there are constitutive rules behind Power, we need to separate them: Power is the thing which *triggers* the working of a constitutive rule (joining an appearance of an institutional

[29] Obviously, this approach invokes the theory of speech acts, but, as it has been shown, I consider these utterances in a broad(er) sense (than they usually are in speech act theories).

fact to its institutional consequences—change in someone's rights[30]). It is one of the necessary conditions that has to be fulfilled in order to have a legal statement, I would say, the most important one.

Therefore, we can show a formal abbreviation (not a well-formed formula of our language!) of what structure a constitutiverule-made state of affairs have:

$$F^c : U(\mathbf{O}_{y\to v}E_yF' \vee \neg\mathbf{O}_{y\to v'}E_yF'' \vee \mathbf{P}_{y\to v''}E_yF''' \vee \neg\mathbf{P}_{y\to v'''}E_yF'''')$$

That is, these are utterances made about someone's[31] right positions. But I can say, even write and sign, in presence of witnesses, that hereby I transfer the ownership of the Trump Tower to the reader, but—despite using all the linguistic phrases described in how a transfer of ownership happens—unfortunately, nothing will happen. Making the utterance is not enough to make a legal statement. Such a person needs to make the utterance with the structure above in order to conduct a (real) legal statement who has the power to do so: I, unfortunately, cannot change Mr. Trump's rights concerning his tower (neither other rights, I'm afraid). We need to preserve this necessary contribution of having a power in describing its relation to making (constitutive rule-based) statements. But what should it look like in the semantics?

4.4.1. A Semantics for Power A step towards a proper semantics for Power might be the following. Power's syntactic form should be preserved such as a modal operator has adding $\mathbf{P}_{a\to b}\phi$ to our language, but its meaning should be different from a modal operator. My proposal to describe Power's semantic nature is involving the structure of a formula used in Public Announcement Logic.[32] Let's say that

$$\mathbf{P}_{a\to b}E_aF^c := [E_aF^c]C_b$$

by which we mean the following:
$\mathfrak{M}, w \models [E_aF^c]C_b$ holds if and only if we have that $\mathfrak{M}, w \not\models E_aF^c$ or that $\mathfrak{M}^P, w \models E_aF^c$ and $\mathfrak{M}^P, w \models C_b$ where the model

$$\mathfrak{M}^P = \langle W^P, f_a^P, R_{a,b}^{PO}, R^{P\Box}, V^P\rangle_{a,b\in A}$$

[30]Please note, again, we are within the law, we only deal with conceptions that are part of the institution of law, more precisely, of a given legal system, that's why we don't need to emphasize the institutional nature.

[31]which can be the same who makes the utterance as we can talk about *our* rights

[32]Let me emphasize that I do not evoke Public Announcement Logic as such with its usual axiomatization since the correspondence between the definition above and formula (18) would only hold with very specific conditions because of the reduction axiom of PAL.

is defined by

- $W^P := \{w' \in W \mid \mathfrak{M}, w' \models E_a F^c\}$—retain only the worlds where $E_a F^c$ is true
  and leave the functions, arrows and valuation the same between/at the remaining worlds:
- $f_a^P(X) = f_a(X)$ for all $X \subseteq W^P$
- $R_{a,b}^{PO} = R_{a,b}^{O} \cap (W^P \times W^P)$
- $R^{P\Box} = R^{\Box} \cap (W^P \times W^P)$
- $w' \in V^P(p)$ if and only if $w' \in V(p)$
  and with C_b we refer to the constitutive rules defining b-s new right position:
- $w \models C_b$ if and only if $w \models \mathbf{O}_{b\to v} E_b F'$
  OR $w \models \neg\mathbf{O}_{b\to v'} E_b F''$
  OR $w \models \mathbf{P}_{b\to v''} E_b F'''$
  OR $w \models \neg\mathbf{P}_{b\to v'''} E_b F''''$

Having Power in the truth-conditions might make the concept seem poorly-defined. We, therefore, might add a clause about "well foundedness" of this definition to solve this (in the sense that eventually the evaluation of terms having operators $\mathbf{P}_{a\to b}$ will lead to a C_b where no more terms with $\mathbf{P}$-s occur), but it is worth noting that this problem does not come from the formal system: it is present in legal philosophy, just think about Kelsen's hierarchy of norms and the problem of a basic norm in [13].

We know which F'-s are involved in the change by making a given legal statement from the legal rules behind: they can be written in the legislative rules, or in contracts; these background constitutive rules serve as functions when assigning a set of F'-s to F^c-s, and these serve the truth conditions of $[E_a F^c]C_b$.

This way we only said about Power that we want to be true in order to be able to have the definition-like description of its working in formula (18). $E_x F^c$, that is, uttering sentences about someone's rights can be true without resulting in his rights' changing. It is Power that triggers such a change.[33]

[33]The semantics provided here for Power solves the problem of how to have the biconditional (18) in the syntax and this has been so far all that we want to tell about Power as this potential is the main feature of it. It raises some further questions, though, about Power's nature, for example it is in question whether such a situation where two people have the power at the same time to the same action concerning the same person can be

4.5. Formalizing Immunity and Disability

With Immunity and Disability, the case is the same as it was with Privilege and No-claim: these legal positions are just the lack of the given liability and power, respectively, that is:

$$\mathbf{I}_x E_y F^c \Leftrightarrow \neg \mathbf{L}_x E_y F^c \tag{19}$$

$$\mathbf{D}_{x \to y} E_x F^c \Leftrightarrow \neg \mathbf{P}_{x \to y} E_x F^c \tag{20}$$

If we put Disability into the formula of Power, we get what we want to get:

$$\neg \mathbf{P}_{x \to y} E_x F^c \leftrightarrow \Diamond (E_x F^c \,\&\, \neg (\mathbf{O}_{y \to v} E_y F' \vee \neg \mathbf{O}_{y \to v'} E_y F'' \vee \mathbf{P}_{y \to v''} E_y F''' \vee \neg \mathbf{P}_{y \to v'''} E_y F'''')) \tag{21}$$

that is, x does not have a power toward y to see to it that F^c if and only if it can happen that x utters the sentence that is supposed to change y's legal position but it does not change. This shows plausibly that making a legal statement, that is, $E_x F^c$ *with its legal consequences* is something for which we need to have a power (toward the agent whose right position would be concerned by the change in which making the legal statement would result).

5. Still No Loss of Direction

I promised to discuss whether the relationality proven in Section 3 means that in the Hohfeldian theory there aren't any undirected rights and duties. It does not have this consequence. We can find some hints even within the Hohfeldian theory itself about how to fit that kind of (undirected, or so called absolute) right conceptions into his framework that, at first sight, seem quite different.

The essay *Fundamental Legal Conceptions* has a second part. It is much less cited or discussed, but it contains an analysis of the difference between *paucital* and *multital* rights.[34] (Hohfeld uses these terms instead of the common terms 'relations in personam' and 'relations in rem', respectively. He does so because he feels the classical expressions misleading—whose clarification takes a remarkable part of the essay, that we won't discuss at all,

Footnote 33 continued
given with this semantics in an intuitive way. I postpone going in details, though, to a next paper.

[34]This is not the only pair of conceptions connected to rights that Hohfeld wanted to analyze, he lists seven pairs at the beginning of the essay section, but his early death stopped him in conducting; unfortunately we can read only the first pair's analysis.

though.) Nigel Simmonds [20] picks two picturing examples of each category: "Suppose that I have a contract with you whereby you are obliged to manufacture a quantity of widgets. I have a claim-right against you and you have a correlative duty to manufacture the widgets. I might have a similar contract with another widget manufacturer, with similar consequences in terms of our claim-rights and duties. However many such contracts I have, however, my claim-rights are essentially limited to a definite number of persons. These are what Hohfeld calls 'paucital' claim-rights. (...) Suppose on the other hand, that I am the owner of Blackacre. I have a claim-right that you should not enter the land without my consent. I have the identical claim-right against your mother, my employer, the Bishop of Ely, and anyone else that you care to mention. Each of these claim-rights is a consequence of my ownership of Blackacre. These are 'multital rights'." Hohfeld gives a short summary description of each type's features: "A paucital right, or claim, is either a unique right residing in a person (or group of persons) and availing against a single person (or single group of persons); or else it is a one of a *few* fundamentally similar, yet separate, rights availing respectively against a few definite persons. A multital right, or claim, is always *one* of a large *class* of *fundamentally similar* yet separate rights, actual and potential, residing in *a single* person (or single group of persons) but availing *respectively* against persons constituting a very large and indefinite class of people." There is another good example of Simmonds [20]: "my claim-right that you should not assault me is a multital right, since it is only one member of a large class of similar rights holding against an indefinite number of people (i.e. I have a right that your mother should not assault me, a right that the Bishop of Ely should not assault me, and so on)", for which Hohfeld would add the example of a patentee's right that any other person shall not manufacture articles covered by the patent.

Why is this analysis of Hohfeld important to us? Because this shows that the way we expressed formally what happens with the Hohfeldian basic notion of Claim-right in criminal law or in the case of civil liberties actually fits perfectly Hohfeld's way of thinking about his conceptions.

At first glance, though, this does not explain the naturality of how Power's directedness fits perfectly the image we have about Power. But only at first glance, since meanwhile the examples and descriptions are only about claim-rights, privileges, powers and immunities can also appear in both multital and paucital versions. As Simmonds [20] adds: "on the Hohfeldian analysis, the owner of Blackacre possesses not a single right over an area of land, but a complex aggregation of claim-rights, powers and immunities. For example

the owner has a series of multital claim-rights that persons should not trespass on the land; a series of multital privileges to enter upon and exploit the land himself; a series of multital powers to transfer title to the land or create lesser interests in it, such as leases or easements; and a series of multital immunities against having his title affected by act of other persons." This description of ownership shows wonderfully that rights we usually think as *absolute* and *not relative* can also be expressed in Hohfeldian terms.

How does this series of power look like formally if F^c is that the title to the land is transferred?

$$\bigwedge_{y \in A} \mathbf{P}_{x \to y} E_x F^c \tag{22}$$

The operation Makinson [15] called 'decountering', that is, the way duty as a directed obligation can be formally changed into an undirected one looks like the following in this language:

$$\bigwedge_{y \in A} \mathbf{O}_{x \to y} \neg E_x F \Leftrightarrow \mathbf{O}_x \neg E_x F \tag{23}$$

The same can be shown in cases of active rights and duties (where the right-owner or the duty-bearer is the same as the person whose action is considered):

$$\bigwedge_{y \in A} \mathbf{PR}_{x \succ y} E_x F \Leftrightarrow \mathbf{PR}_x E_x F \tag{24}$$

$$\bigwedge_{y \in A} \mathbf{P}_{x \to y} E_x F^c \Leftrightarrow \mathbf{P}_x E_x F^c \tag{25}$$

$$\bigwedge_{y \in A} \mathbf{D}_{x \to y} E_x F^c \Leftrightarrow \mathbf{D}_x E_x F^c \tag{26}$$

In the case of passive rights and duties the formal representation of the relation between directed and undirected versions needs a different modification since the other party appears as the index of the action operator. We introduced it in this indexed form in our language, but if we let it be unindexed with the interpretation of 'EF' as 'F is seen to it' than the formulas are straightforward:

$$\mathbf{CR}_x \bigwedge_{y \in A} E_y F \Leftrightarrow \mathbf{CR}_x EF \tag{27}$$

$$\mathbf{NC}_x \bigwedge_{y \in A} E_y F \Leftrightarrow \mathbf{NC}_x EF \tag{28}$$

$$\mathbf{L}_x \bigwedge_{y \in A} E_y F^c \Leftrightarrow \mathbf{L}_x E F^c \tag{29}$$

$$\mathbf{I}_x \bigwedge_{y \in A} E_y F^c \Leftrightarrow \mathbf{I}_x E F^c \tag{30}$$

6. Conclusion and Further Work

As we have seen, even if there are important differences between the two Hohfeldian groups of rights and duties, talking about their conditional consequences in a system of rights can provide a good basis to describe them in a uniform manner. This way of description does not mean proper definitions, but does mean definitions in the sense of providing necessary and sufficient conditions to identify each type of rights and duties according to their "behavior". This image of a system of rights has been delineated using basically the same formal tools as the classical formalizations of the Hohfeldian theory do, but instead of focusing the rights' algebraic features, I aimed at developing a more comprehensive formal account providing some substantial insights on the nature of these conceptions—maintaining all along the Hohfeldian intention to treat the conceptions as *sui generis*.

Talking about the conditional consequences made the uniform manner possible by making the interaction between normative positions and actions (including "refrainings") describable. Using conditional structures for definitional descriptions, though, brings some difficulties. The presented formula describing Power, for example, allows "trivial" powers, that is, does not perfectly express the trigger nature of Power. Overcoming these difficulties is left for further research.

There is a consideration that needs to be made about the role of the state considering a system of rights. We saw that the judiciary is clearly involved in the description of rights (in the case of the first group). But there is another power (now in the Montesquieunian sense) of the state that is often touched by arguments on rights and duties: the legislature. What role does the legislature have, if any? My answer is that the role of legislation is to create a legal system which can be described with a logic in which the biconditionals we provided are valid. At this point, I have to refer to what already has been mentioned about the Hohfeldian theory's and our formalization's status. If these are descriptive theories then they tell how things with rights and duties *are* in the law. If we can imagine that they can be otherwise, then what this theory and its formalization describe is how things with rights and duties *should be* in the law: what legal rights

and duties *should* mean, how their system *should* work. This requirement is the one that has been included in the *Declaration of Rights of Man and of the Citizen* when stating that "Any society in which no provision is made for guaranteeing rights (...), has no Constitution." One can accept that (human) rights exist without this effort of the state, but what they expect is that the state create a legal system which makes these rights *legal rights*. What I have been pursuing with this formal representation is showing how a system of *legal rights* works. And what is needed to have this system is having the biconditionals valid in a logic that describes the given legal system.

Could it be otherwise? Well, the usual comment here[35]—concerning the general relation between logic and law—is that legal validity is actually insensitive to logical validity.[36] A legal system can be legally valid without maintaining consistency among rights and duties. But to say a legal system *really* contains a right means that the legal system obeys a logic having these biconditionals valid. And this is what is expressed in the paragraph cited from the Declaration of the Rights of Man and of the Citizen.

In 2013, Sergot closed his state-of-the-art paper [19] discussing normative positions and the limitations of the Kanger–Lindahl formalization with saying that "there are nevertheless grounds to believe that a more comprehensive formal comprehensive formal account could be developed." The formal representation of the Hohfeldian theory as it has been presented in this paper is, of course, far from perfect and complete yet. The formal handling of compensation, sanction, or states of affairs involved in the definitions in general is to be developed; just like the detailed and axiomatic description of the logic we would like to involve to describe how Power works. These preliminary considerations of the proposal might have been enough, though, to delineate a prospective image of a comprehensive and reasonable formal approach to systems of rights.

Acknowledgements. Open access funding provided by Eötvös Loránd University (ELTE). The research leading to this paper partly has been conducted during visiting fellowships in the Law School of the University of

[35] And this is how far we will go into discussing this issue, since its detailed investigation would take (an)other paper(s).

[36] Here I consider legal validity as the *existence* of a legal norm: it has been created in the prescribed way (procedure), by the agent who has the power to create it. If the prescribed procedure doesn't say anything specific about logical requirements then legal norms can exist (which means that they are valid—as von Wright assumes too from the very beginning) without being consistent.

Edinburgh and in the Centre for Philosophy of Natural and Social Science at London School of Economics and Political Science. I am grateful for the comments on this paper's earlier versions to Marek Sergot, David Makinson, Olivier Roy, Burkhard Schafer, and the anonymous reviewers. The research reported in this paper was supported by the Higher Education Excellence Program of the Ministry of Human Capacities in the frame of Artificial Intelligence research area of Budapest University of Technology and Economics (BME FIKP-MI/FM). Support provided by the research Project K-116191 of the Hungarian Scientific Research Fund is also gratefully acknowledged.

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