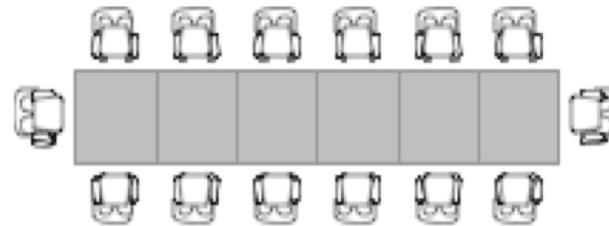


Developing algebraic thinking among students aged 10-14 years: Comparing perspectives of primary and secondary school teachers



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Objective

Present the first results of a questionnaire submitted to:

- primary school teachers
- secondary school mathematics teachers

to collect...

- their respective expectations about the arithmetic-algebra transition;
- their knowledge for teaching « algebraic thinking ».

The analyses will aim to compare the views of teachers from both school levels on algebraic thinking at the primary-secondary school transition.

Early algebra and algebraic thinking

- In the 70s and 80s : Transition between arithmetic and algebra seen as a **rupture**
 - Identify and analyse the students' difficulties in algebra at the beginning of secondary school.
 - How to help students in the beginning of secondary school to overcome the obstacles between arithmetic and algebra.
- Since the late 1990s: Transition between arithmetic and algebra seen as a **continuity** in the learning of numbers and operations
 - Research on « Early Algebra »
 - Develop « algebraic thinking » in students from the primary school level

(Kieran, 1992, 2007)

Early algebra and algebraic thinking

Early algebra

- It is not introducing algebra early,
- but, offer opportunities to enrich, from primary school, the understanding of concepts such as the sense of operations or equality, which are themselves linked to the development of « algebraic thinking » (Carraher & Schliemann, 2007).

Algebraic thinking

Algebraic thinking is not about the use of formal algebraic symbolism but about a particular way of thinking:

1. Relational vision of arithmetic (Carpenter, et al., 2005)

→ sense of operations

→ sense of equality

$$\text{ex: } 47 + 5 = ? + 9$$

2. Presence of an indeterminate (Radford, 2014 and 2018)

Early algebra and algebraic thinking

Presence of an indeterminate and algebraic thinking (Radford, 2014 et 2018) :

Historically, algebra appeared when operations involving indeterminate quantities began to be carried out.

This ability to perform operations on indeterminate quantities is therefore an essential feature of algebra. According to Radford, the characteristics linked to the indeterminate are the following:

- **indeterminacy**: to solve problems involving unknown numbers (unknowns, variables, parameters, etc.) ;
- **denotation**: to name or symbolize these unknown numbers. This denotation can be done in different ways, using the alphanumeric code, but also natural language, gestures or unconventional signs;
- **analyticity**: to treat indeterminate quantities as if they were known, and perform operations with these unknown numbers.

Knowledge for teaching algebra

- Developing algebraic thinking in primary and secondary school requires from teachers of both levels to have a good knowledge for teaching.
- However, there is currently little research on:
 - the specific knowledge for teaching algebra...
 - among primary AND secondary school teachers
- The knowledge for teaching was initially defined by Schulman (1987) (pedagogical content knowledge - PCK) and then adapted to mathematics by Hill, Ball and Schillings (2008) (mathematical content knowledge for teaching), who proposed a model that is used in this study.

Knowledge for teaching mathematics

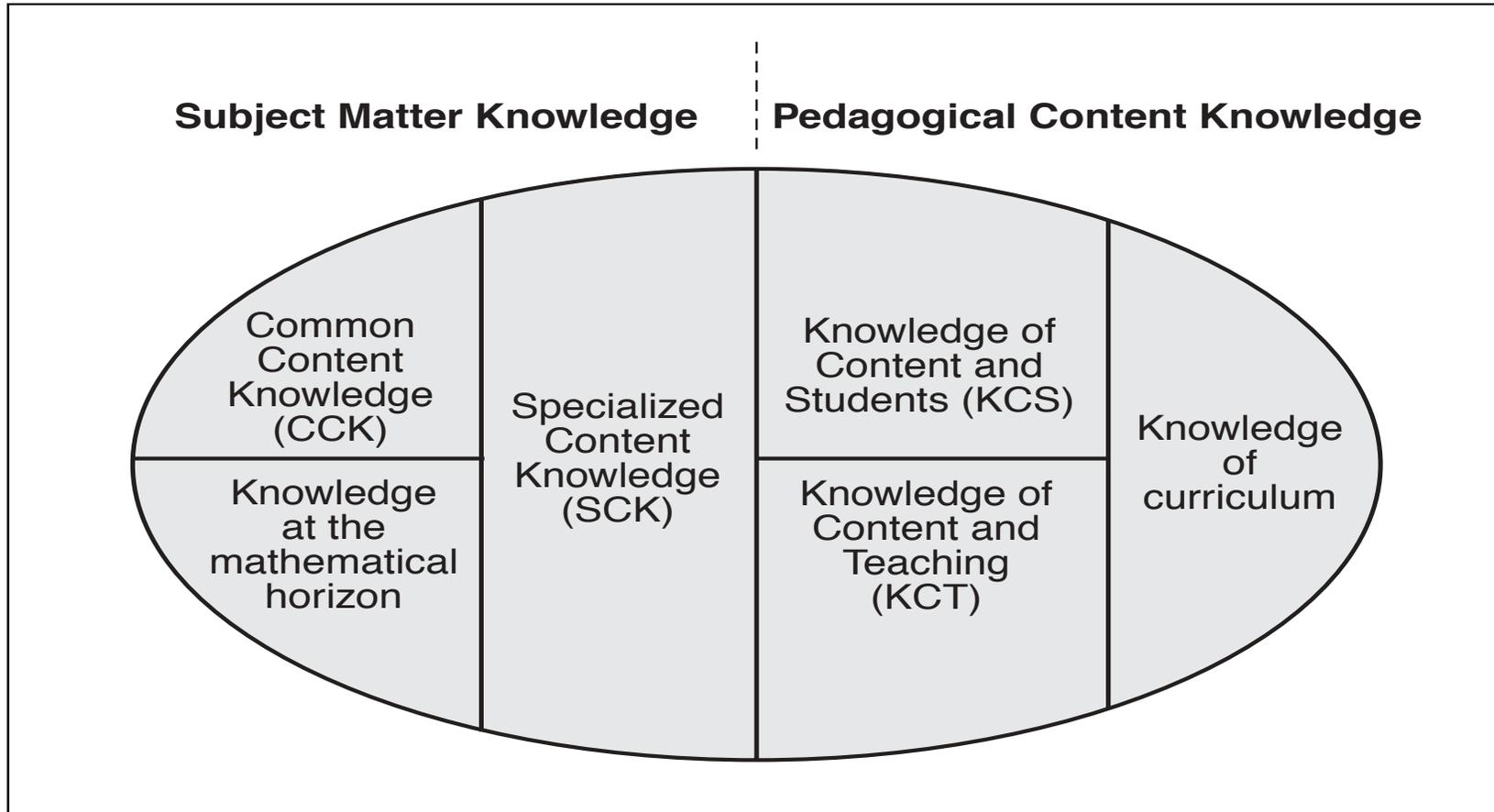


Figure 1. Domain map for mathematical knowledge for teaching.

Model of Hill, Ball, et Schillings (p. 377, 2008)

Knowledge for teaching algebra

The few studies on knowledge for teaching algebra of primary and secondary school teachers show that:

- **Primary school teachers** may have a limited knowledge of the algebraic implications of the activities they offer in the classroom and, while they appear to understand students' thinking and mistakes, may have difficulty explaining their causes and proposing effective scaffolding

(Chick & Harris, 2007; Tanisli & Kose, 2013).

- **Secondary school teachers**, while mastering the content, may lack pedagogical knowledge. They would be reluctant to accept the variety of approaches and would offer little potentially effective help in dealing with students' mistakes (Demonty, Vlassis & Fagnant, 2018).

METHODS

An exploratory study

- On-line questionnaire aimed at primary and secondary school teachers
 - Closed Items: QCM
 - Open questions
- Collecting
 - Teachers' knowledge for teaching algebra
 - Teachers' expectations

- Sample:

	Primary T.	Secondary T.
In the very beginning of the questionnaire	41	97
First questions	27	80
Last questions	8	28

- Teachers were either personally contacted or contacted by some responsables of the Ministry of Education
- The issue of early algebra is not in the primary and secondary school curricula

Measuring knowledge for teaching

Three main types of questions asked around **three kinds of activities** :

1. Judge the interest of the activities for primary ? for secondary students?
 - *Mathematical knowledge of content and teaching*
2. Comment or grade correct or incorrect students' productions.
 - *Mathematical knowledge of content and students*
 - *Content knowledge*
3. Predict the potential difficulties of the questions asked in the activities
 - *Mathematical knowledge of content and students*

Measuring knowledge for teaching

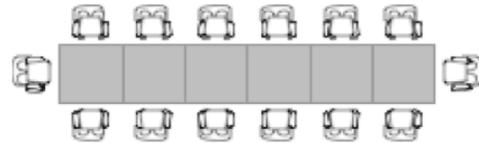
Three types of activities at the heart of algebraic thinking:

1. Generalization activities based on linear patterns

(Demonty, Vlassis , 2018; Radford, 2008)

Chairs and tables

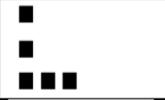
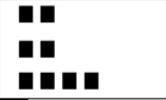
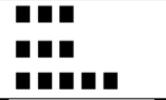
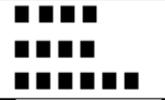
Jules' parents are organising a birthday party for him. They contact 'Monsieur Boulèfrite', the caterer. He has some small square tables. He suggests setting them out side by side to make one long table at which all the guest will sit, as shown below:



Write a rule that can be used to find the number of chairs for any number of tables that you have.

Sequence of squares

This is a sequence of squares:

				
Pattern 1	Pattern 2	Pattern 3	Pattern 4	Pattern 5

Continue the sequence ... How many squares will there be in drawing number 5?

Develop a rule to determine the number of squares with respect to the drawing's number

Measuring knowledge for teaching

2. Mental arithmetic activity: The faulty calculator (Demonty & Vlassis, 2018)

Faulty calculator: We must make a set of calculations, but the calculator doesn't work properly. Each time there is a faulty button.

Find another calculation to input to obtain the same result!

137 + 66 =

()	C	÷
7	8	9	×
4	5	6	-
1	2		+
	0	,	=

843 - 79 =

()	C	÷
7	8		×
4	5	6	-
1	2	3	+
	0	,	=

3. Problem of unequal sharing (Demonty & Vlassis, 2018)

There are three boxes with sweets. The second box has 5 more sweets than the first box, and the third box has 7 more sweets than the first. Overall, there are 24 sweets. How many sweets are there in each of the boxes?

Measuring expectations

1. What exercises do you think that primary school students need to be able to do to facilitate the algebra learning at the beginning of secondary school?

Complete according to your degree of agreement with the proposed exercise

Strongly disagree	Disagree	Slightly disagree	Slightly agree	Agree	Strongly agree
0	1	2	3	4	5

A total of 10 exercises was proposed:

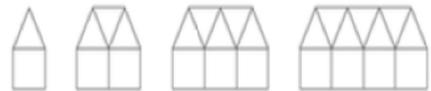
- 6 informal ones

Examples:

Fill out an equality as follows: $8 + 4 = ? + 5$

Answer the questions in the following activity:

Look at this sequences of figures composed of squares and triangles:



Determine the number of triangles of the figure composed of 7 squares.

Propose a message that allows you to calculate the number of triangles regardless of the number of squares.

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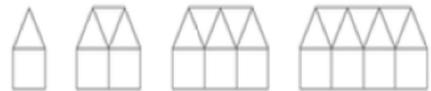
- 6 informal ones
- 3 formal ones (with formal symbolism)

Examples:

Fill out an equality as follows: $8 + 4 = ? + 5$

Answer the questions in the following activity:

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Determine the number of triangles of the figure composed of 7 squares.

Propose a message that allows you to calculate the number of triangles regardless of the number of squares.

Propose a formula that allows you to calculate the number of triangles regardless of the number of squares.

Solving a simple equation

Measuring expectations

1. What exercises do you think that primary school students need to be able to do to facilitate the algebra learning at the beginning of secondary school?

Complete according to your degree of agreement with the proposed exercise

Strongly disagree	Disagree	Slightly disagree	Slightly agree	Agree	Strongly agree
0	1	2	3	4	5

A total of 10 exercises was proposed:

- 6 informal ones
- 3 formal ones (with formal symbolism)
- 1 assertion

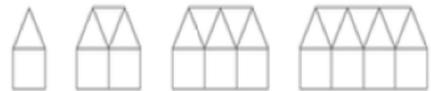
Examples:

Fill out an equality as follows: $8 + 4 = ? + 5$

No exercise in particular, algebra learning only begins in secondary school

Answer the questions in the following activity:

Look at this sequences of figures composed of squares and triangles:



Determine the number of triangles of the figure composed of 7 squares.

Propose a message that allows you to calculate the number of triangles regardless of the number of squares.

Propose a formula that allows you to calculate the number of triangles regardless of the number of squares.

Solving a simple equation

Research questions

RQ 1 – What is the **knowledge for teaching** of primary and secondary school teachers about algebraic thinking?

→ KC and Teaching (judging the interest of an activity)

→ KC and Students (commenting and grading students productions)

Is there a **difference** between the teachers of both levels?

RQ 2 – What are the **expectations** of primary and secondary school teachers about the arithmetic-algebra transition?

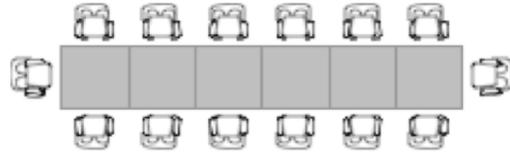
Is there a **difference** between the teachers of both levels?

RESULTS

TEACHERS' KNOWLEDGE FOR TEACHING

Judging the interest of an activity – KC and Teaching

Jules' parents are organising a birthday party for him. They contact 'Monsieur Boulèfrite', the caterer. He has some small square tables. He suggests setting them out side by side to make one long table at which all the guest will sit, as shown below:



Write a rule that can be used to find the number of chairs for any number of tables that you have.

Objective: Do the primary and secondary teachers identify the algebraic implications of an activity?

Number of answers

	Interest for primary school	Interest for secondary school
Primary T.	11	-
Secondary T.	42	47

Judging the interest of an activity – KC and Teaching

Interest of the activity « Chairs and tables » **for primary school**

Opinions of secondary and primary school teachers

	Primary T.		Secondary T.	
	N	%	N	%
Algebraic thinking/generalization	2	18	7	18
Modeling/abstraction	2	18	5	12
Problem solving /logical thinking	6	55	18	44
Practical context	-	-	3	7
No interest (too early/too difficult)	1	9	3	7
Others	-	-	5	12
Total	11	100	41	100

Judging the interest of an activity – KC and Teaching

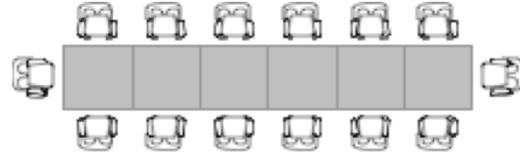
Interest of the activity « Chairs and tables » for secondary school

Opinions of secondary school teachers

	primaire		secondaire	
	N	%	N	%
Algebraic thinking/generalization	-	-	29	62
Modeling/abstraction	-	-	8	17
Problem solving /logical thinking	-	-	3	6
Practical context	-	-	1	2
No interest (too early/too difficult)	-	-	1	2
Others	-	-	5	11
Total	-	-	47	100

Judging students' productions

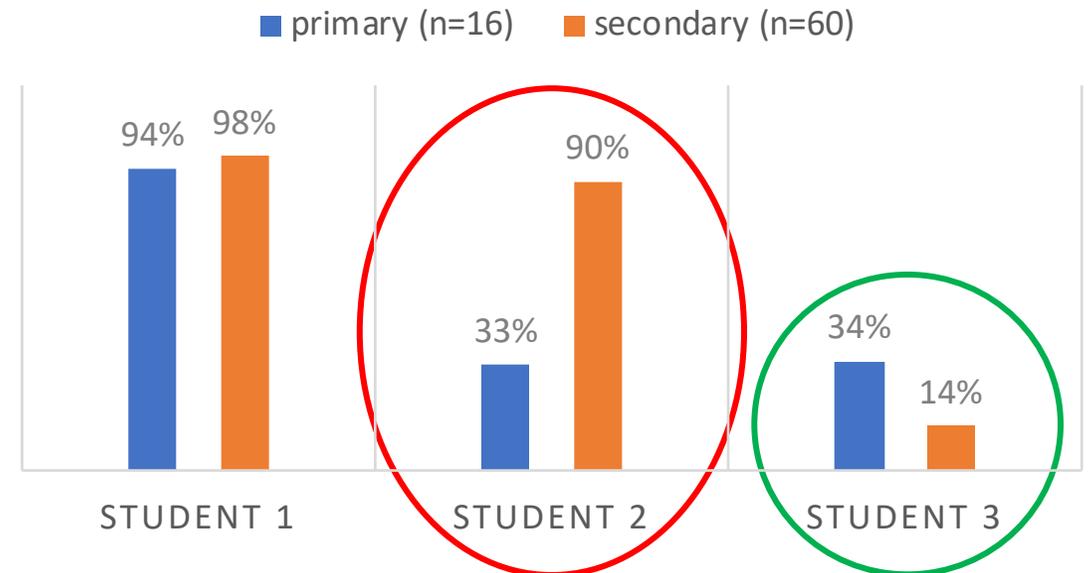
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Write a rule that can be used to find the number of chairs for any number of tables that you have.

According to you, which student(s) give a correct solution?

	Visual presentation	Rule suggested
Student 1		Number of chairs = (number of tables x 2) + 2 <i>Correct solution</i>
Student 2		Number of chairs = (number of tables - 2) x 2 + 6 <i>Correct solution</i>
Student 3		Each time you add a table, you have 2 more chairs Number of chairs = Number of tables + 2 <i>Incorrect solution</i>

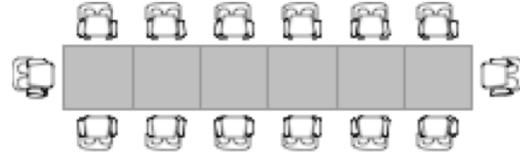


But the solution of Student 2 is considered as correct only by 33% of the primary school teachers

The solution of Student 3 is false, however 34% of primary school teachers find it correct and even 14% of secondary school teachers

Judging students' productions

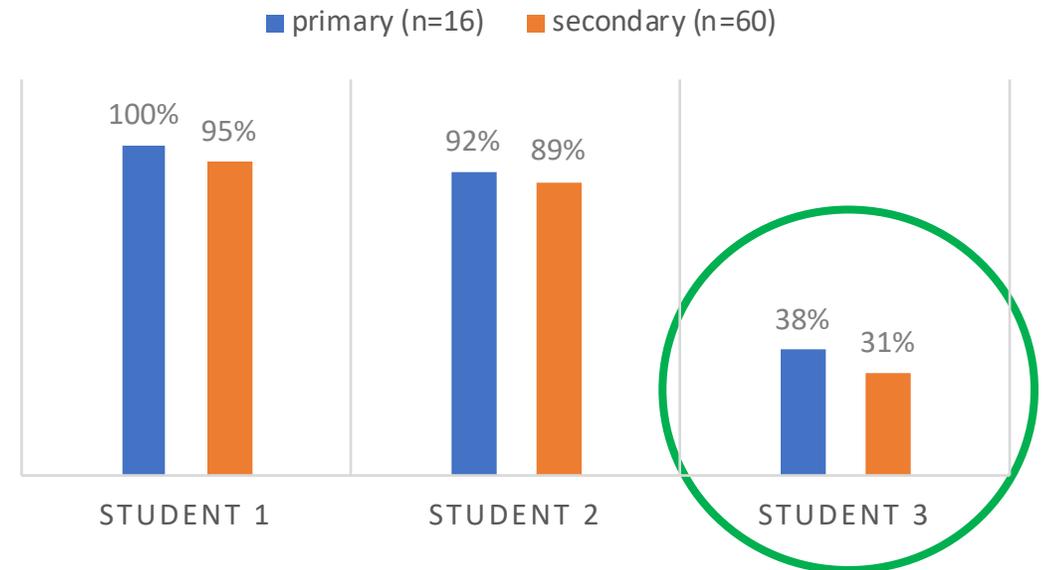
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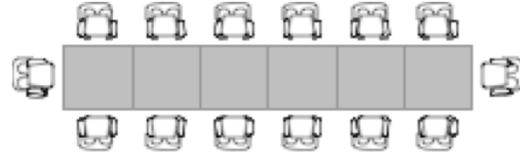


The wrong solution of Student 3 is considered as algebraic by approximately 1/3 of the teachers (primary and secondary school).

Why is the opinion of the teachers so different with respect to the 3 solutions, since in each the students relate the number of chairs to the number of tables?

Judging students' productions

Jules' parents are organising a birthday party for him. They contact 'Monsieur Boulèfrite', the caterer. He has some small square tables. He suggests setting them out side by side to make one long table at which all the guest will sit, as shown below:



Write a rule that can be used to find the number of chairs for any number of tables that you have.

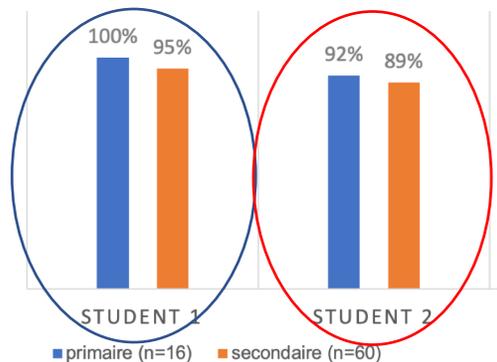
According to you, which student(s) give an algebraic solution?

Why are solutions 1 and 2 "recognized as algebraic"?

Student 1		Number of chairs = (number of tables x 2) + 2
Student 2		Number of chairs = (number of tables - 2) x 2 + 6

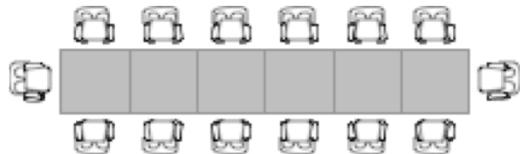
This expression seems to be considered totally algebraic because it is **a reduced expression** involving the indeterminate (number of tables)

This expression seems to be considered globally algebraic because it is **an unreduced expression** involving the indeterminate (number of tables)



Judging students' productions

Jules' parents are organising a birthday party for him. They contact 'Monsieur Boulèfrite', the caterer. He has some small square tables. He suggests setting them out side by side to make one long table at which all the guest will sit, as shown below:



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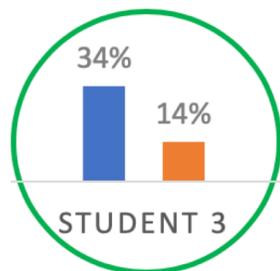
According to you, which student(s) give an algebraic solution?

Why was solution 3 less recognized as algebraic?

Student 3		Each time you add a table, you have 2 more chairs Number of chairs = Number of tables + 2
-----------	--	--

This sentence denotes an « arithmetical generalization » (Radford, 2008): « We add 2 each time » (arithmetical sequence, $r = 2$)

It seems that the student wanted to express the number of chairs according to the number of tables. It is an incorrect translation of his reasoning (sentence above)



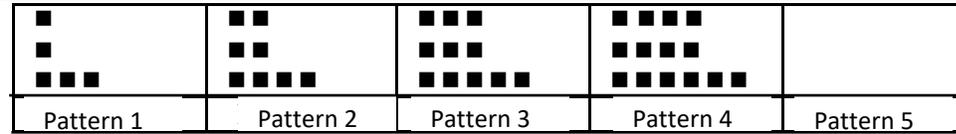
■ primaire (n=16) ■ secondaire (n=60)

The incorrect solution of Student 3 is considered algebraic by about 1/3 of the primary and secondary teachers.

Grading students' productions

Sequence of squares

This is a sequence of squares:



Continue the sequence ... How many squares will there be in drawing number 5?

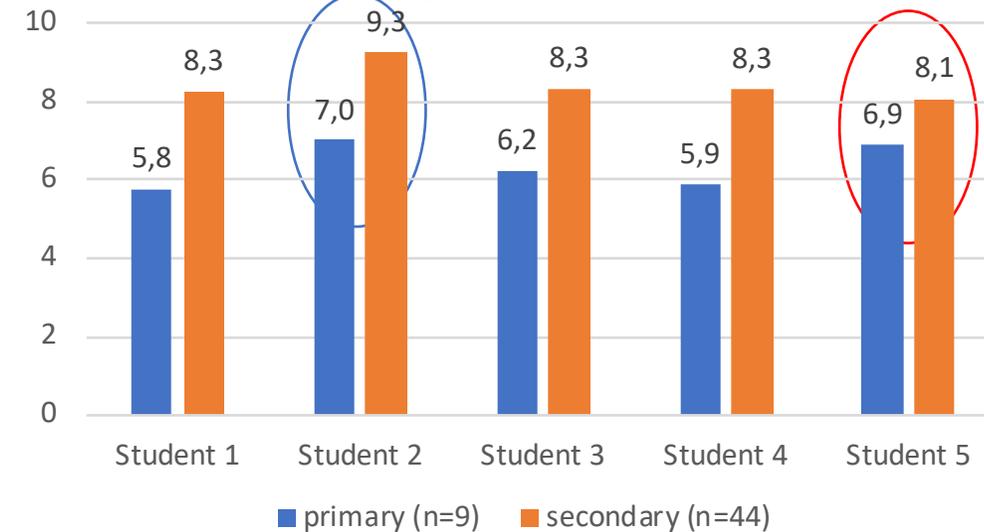
Develop a rule to determine the number of squares with respect to the drawing's number

Rate student productions out of 10

	Production
Student 1	$1 \cdot n + 2 + 2 \cdot n$
Student 2	$3n + 2$
Student 3	$n + 2 + 2n$
Student 4	$(n + 2) \cdot 3 - 4$
Student 5	3 times the pattern number + 2

(it wasn't detailed to the teachers that the 5 productions are correct.)

Average mark up to 10



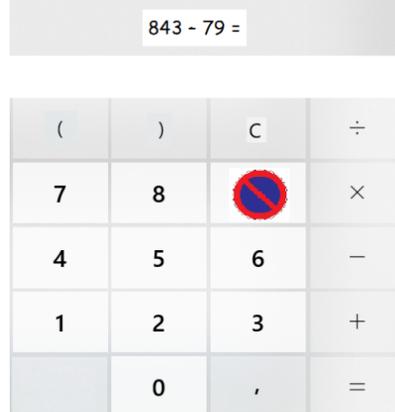
These 5 productions are correct, but:

- 1- Few teachers give the maximum mark of 10
- 2- Primary school teachers give marks clearly lower than those of secondary
- 3- Production 2 (the indeterminate variable labelled by the letter) is the one that gets better marks even though Production 5 is similar (considering its reasoning). Only **primary teachers** give similar marks to **answers 2 and 5**.

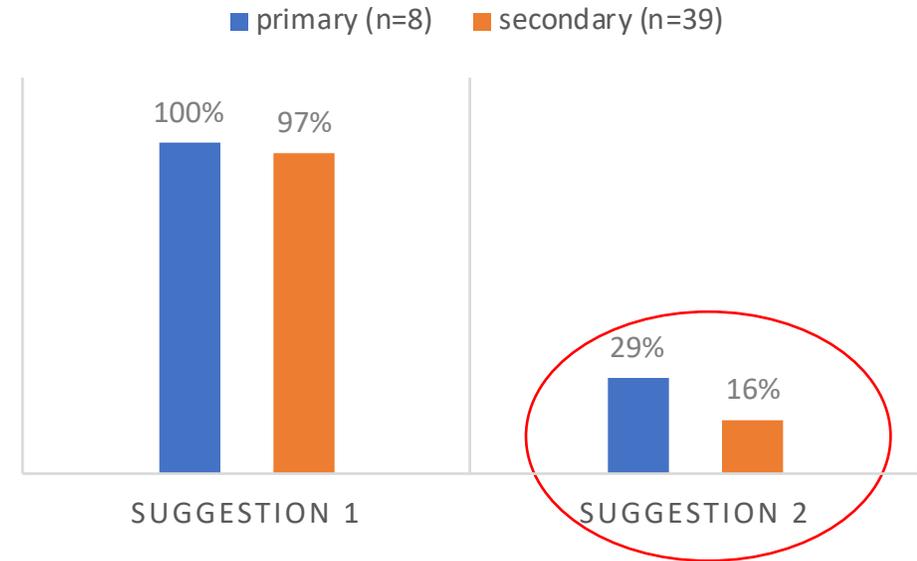
Secondary teachers seem to consider that the reduced expression (the indeterminate variable labelled by the letter) is more algebraic

Judging students' productions

Faulty calculator: We must make a set of calculations, but the calculator doesn't work properly. Each time there is a faulty button. Find another calculation to input to obtain the same result!



These are the suggestions from the students. Do they seem correct to you?



Suggestion 2 is wrong, but 29% of primary school and 16% of secondary school teachers find it correct.

Suggestion 1

❖ 137 + 66 =

Notre calcul :

147 + 56

Explication

On a ajouté 10 à 137 alors on les retire à 66.

we add 10 to 137 so we remove them to 66.

Correcte Compensation

applied to the addition

(Correct relational calculation)

$137 + 10 - 10 + 66 = 147 + 56$

Suggestion 2

❖ 843 - 79 =

Notre calcul :

842 - 80

Explication :

On a ajouté 1 à 79 alors on retire 1 à 843.

we add 1 to 79 so we remove 1 to 843.

Incorrecte Compensation

Applied to the subtraction

(Incorrect relational calculation)

$843 - 1 - 79 + 1 \dots\dots(842 - 80)$

Keeping the gap

(Correct relational calculation)

$(843 - 1) - (79 - 1) = 843 - 1 - 79 + 1 \dots\dots (842 - 78)$

Judging students' productions

There are three boxes with sweets. The second box has 5 more sweets than the first box, and the third box has 7 more sweets than the first. Overall, there are 24 sweets. How many sweets are there in each of the boxes?

Solution 1

Arithmetic strategy (trial and error)

Si la 1^{ère} boîte contient 10 bonbons,
 La 2^e boîte contient : $10 + 5 = 15$ bonbons
 La 3^e boîte contient : $10 + 7 = 17$ bonbons
 Et $10 + 15 + 17 = 42$ bonbons : C'est trop

Si la 1^{ère} boîte contient 7 bonbons,
 La 2^e boîte contient : $7 + 5 = 12$ bonbons
 La 3^e boîte contient : $7 + 7 = 14$ bonbons
 Et $7 + 12 + 14 = 33$ bonbons : C'est trop

Si la 1^{ère} boîte contient 4 bonbons,
 La 2^e boîte contient : $4 + 5 = 9$ bonbons
 La 3^e boîte contient : $4 + 7 = 11$ bonbons
 Et $4 + 9 + 11 = 24$ bonbons : C'est bon

On a donc : la 1^{ère} boîte qui contient 4 bonbons, la 2^e boîte, 9 bonbons et la 3^e boîte, 11 bonbons.

Solution 2

Formal algebraic strategy

1^{ère} boîte : x
 2^e boîte : $x + 5$
 3^e boîte : $x + 7$
 $x + x + 5 + x + 7 = 24$
 $3x + 12 = 24$
 $3x = 12$
 $x = 4$

Et donc,
 la 1^{ère} boîte qui contient 4 bonbons,
 la 2^e boîte contient 9 bonbons
 la 3^e boîte contient 11 bonbons.

Solution 3

Arithmetic-algebraic strategy

1^{ère} boîte : x
 2^e boîte : $x + 5$
 3^e boîte : $x + 7$

$24 - 12 = 12$
 $12 : 3 = 4$
 $x = 4$

Et donc,
 la 1^{ère} boîte contient 4 bonbons,
 la 2^e boîte contient 9 bonbons
 la 3^e boîte contient 11 bonbons.

Solution 4

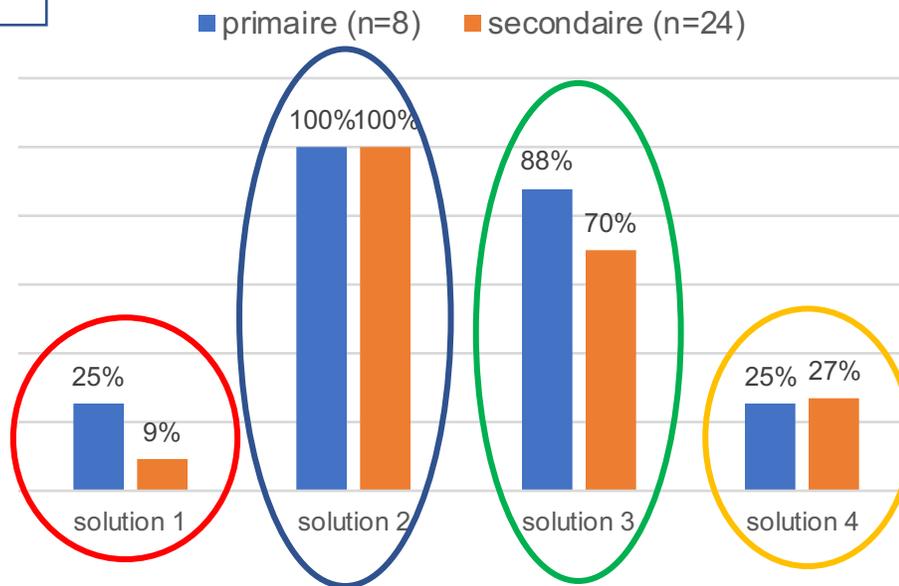
Arithmetic-algebraic strategy

1^{ère} boîte : \square
 2^e boîte : $\square \begin{array}{|c|} \hline 5 \\ \hline \end{array}$
 3^e boîte : $\square \begin{array}{|c|} \hline 7 \\ \hline \end{array}$

$24 - 12 = 12$
 $12 : 3 = 4$

Et donc,
 la 1^{ère} boîte qui contient 4 bonbons,
 la 2^e boîte contient 9 bonbons
 la 3^e boîte contient 11 bonbons.

According to you, which solution(s) can be considered as algebraic(s)?



=> Implementation of the same approach for solutions 3 and 4

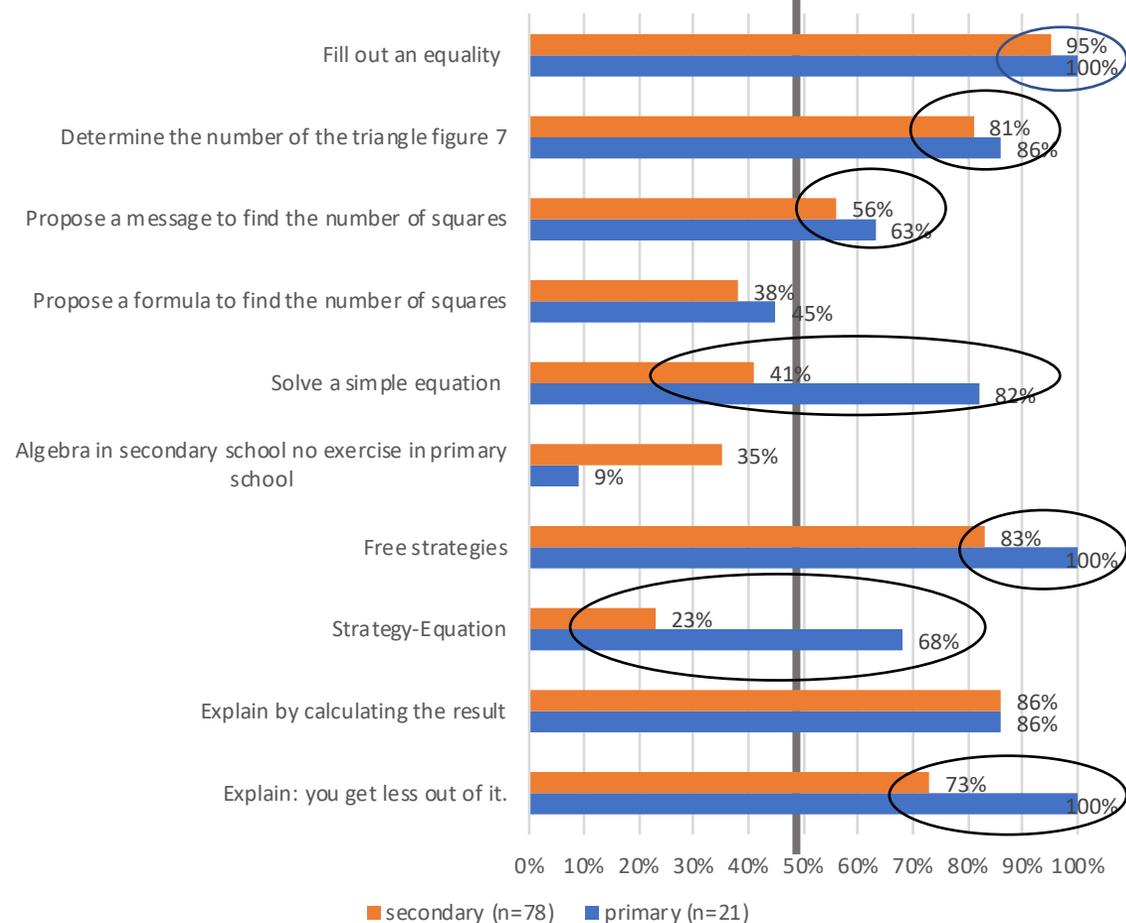
Solution 3 : Arithmetic-algebraic strategy recognition of a common part (undetermined variable represented and labelled by the letter x) then arithmetic calculation

Solution 4 : Arithmetic-algebraic strategy recognition of a common part (undetermined variable represented but not labelled) then arithmetic calculation

TEACHERS' EXPECTATIONS

Teachers' expectations

In your opinion, what exercises should primary students be able to do to facilitate the learning of algebra in early secondary school? (% of agreement : slightly agree + agree + strongly agree)

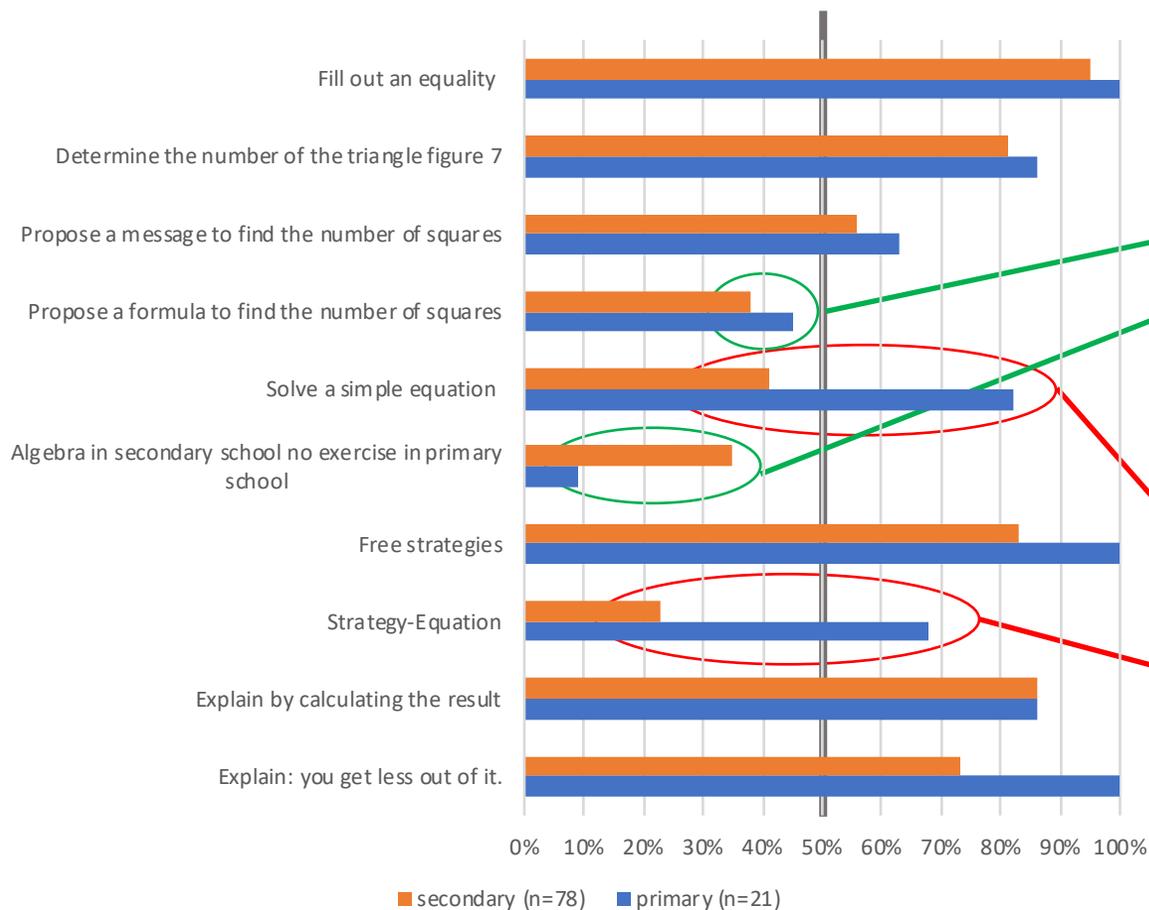


Fill out an equality as follows: $8 + 4 = ? + 5$
Answer the questions in the following activity: Look at this sequences of figures composed of squares and triangles:
Determine the number of triangles of the figure composed of 7 squares.
Propose a message that allows you to calculate the number of triangles regardless of the number of squares.
Propose a formula that allows you to calculate the number of triangles regardless of the number of squares.
Solve a simple equation as shown below:
$\begin{array}{r l} x + 9 = 14 & \\ x = 14 - 9 & - 9 \\ x = 5 & \text{(on retranche 9 des} \\ & \text{deux côtés)} \end{array}$
No exercise in particular, algebraic learning only begins in high school
Have the ability to solve the following type of problem: In a cycle 3 of a primary school, there are 37 students. In Cycle 3.1 class, there are 5 more students than in Cycle 3.2 class. How many students are there per class?
According to a solving strategy of the student's choice?
According to a solving strategy using an equation?
Fill out the following inequality: $750 - 13 \dots 750 - 25$ by the symbol $>$ Based on the results of the 2 operations, explaining that $750 - 13 = 737$, so it's bigger than $750 - 25 = 725$
Explaining that $750 - 13$ is greater than $750 - 25$ because less is taken out at $750 - 13$ than at $750 - 25$.

Primary teachers consider that formal and informal exercises are important while secondary teachers point more informal exercises

Teachers' expectations

In your opinion, what exercises should primary students be able to do to facilitate the learning of algebra in early secondary school? (% of agreement)



Primary and secondary school teachers are rather opposed to:

- asking primary school students to find formulas.
- starting learning algebra only in secondary school (mainly primary teachers)

Secondary school teachers, unlike primary school teachers, are rather opposed to asking primary school students:

- to formally solve equations;
- to solve with a strategy using an equation.

CONCLUSIONS

Regarding teachers knowledge for teaching

1. The content analysis of the primary and secondary school teachers' answers showed that few of them consider the interest of a generalization activity for primary school students' algebraic thinking.
 - At this level, they consider it as a problem-solving activity.
 2. But when teachers have to judge the interest of this type of situation in a closed question (cf. expectations), they generally point out its importance for developing algebraic thinking.
 3. Secondary teachers seem more open to the variety of solutions/formulas. They attribute more often than primary teachers a high grade to different but correct solutions.
 4. For secondary school teachers, and especially primary school teachers, an algebraic solution is a solution involving
 - a letter to identify the unknown (problem solving)
 - a reduced expression (generalization activity)
- => they do not seem to be aware of algebraic thinking as a way of reasoning that does not necessarily involve formal expression.

Regarding teachers' expectations

1. Very few primary school teachers think that algebra only begins at secondary school, while one-third of secondary school teachers think so.
2. A majority of **primary school teachers** think that all the activities, both **informal** and **formal**, are important to facilitate algebra learning.
 - Trace of the current curriculum and mainly of the previous one which advocated teaching equation solving as early as grade 5.
3. A majority of **secondary school teachers** consider only **informal** activities as important to facilitate algebra learning.
 - Formal learning is considered to be a matter for secondary school only.

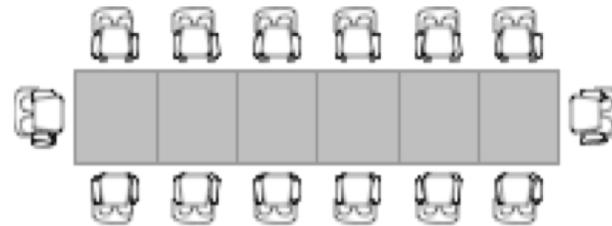
Perspectives

For **professional development programmes**, it would be appropriate ...

- to clarify respective expectations
- to put forward the idea of algebraic thinking
 - which can be developed from primary school in continuity with secondary school
 - ... and evolve from informal learning in primary school to formal learning in secondary school
- to highlight the importance of the diversity of formulas and solving strategies
 - to anchor the meaning of formal learning (meaning of letter, equality, operations) of secondary school

THANK YOU FOR YOU ATTENTION

Developing algebraic thinking among students aged 10-14 years: Comparing perspectives of primary and secondary school teachers



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Cieaem71 – Braga, Portugal 22 - 26 July 2019



Judging the interest of an activity – KC and Teaching

Interest of the activity « Chairs and tables » **for primary school**

Opinions of secondary and primary school teachers : examples

Pensée algébrique/généralisation/régularité	
Primaire	« Premières expériences avec l'algèbre avec une situation qui pourrait être réelle »
Secondaire	« Situation de la vie courante : initiation à l'algèbre » « De voir que le nombre de chaises dépend de « manière régulière » du nombre de tables. Développer la pensée algébrique des élèves » « Explorer des relations/dépendances entre grandeurs. Mettre en mot des relations fonctionnelles »

Résolution de problèmes /pensée logique	
Primaire	« Promouvoir le développement d'une pensée logique » « Résoudre de façon mathématique une situation du quotidien »
Secondaire	« Exploration, activité de recherche » « Initiation à la recherche dans le cadre d'une activité nouvelle »

Judging the interest of an activity – KC and Teaching

Interest of the activity « Chairs and tables » for secondary school

Opinions of secondary school teachers: Examples

Pensée algébrique/généralisation/régularité	
Secondaire	« Elaboration d'une formule et l'introduction du calcul littéral » « Généraliser la règle en utilisant le calcul littéral » « Savoir généraliser à partir d'observations bien comprises pour des valeurs numériques » « Donner du sens à l'utilisation de variables »

Modélisation/abstraction	
Secondaire	« Apprendre à modéliser une situation » « Donner une modélisation mathématique d'un problème pratique (concret) »

Résolution de problèmes/pensée logique	
Secondaire	« Problème de la vie courante » « Comprendre un problème et le traduire en langage mathématique pour le résoudre efficacement »

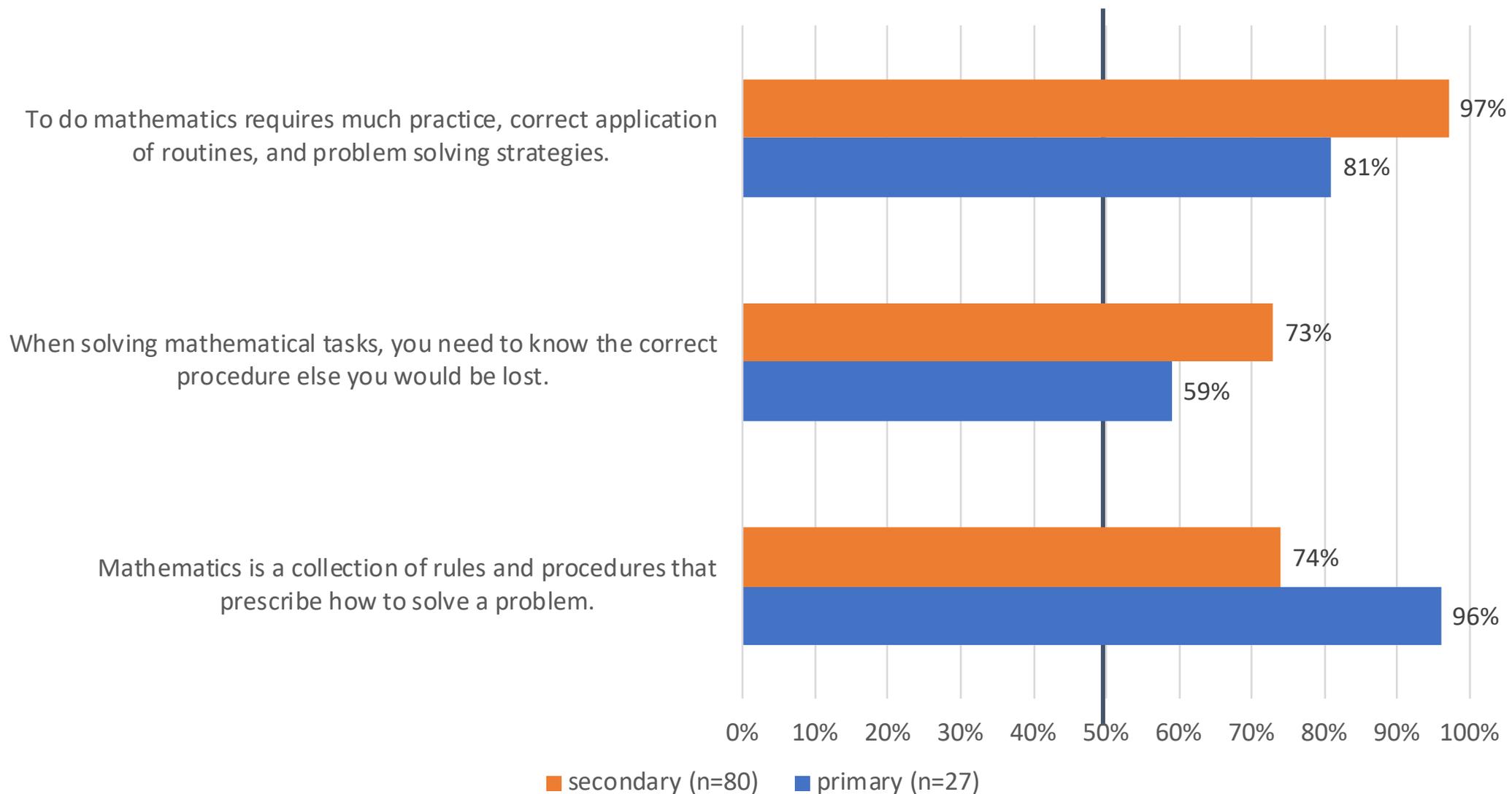
Judging the difficulty of the 3 types of situations KC and Students

Sequence of squares	Difficult for primary school	Difficult for secondary school
Primary T. (N = 13)	30 %	17 %
Secondary T. (N = 60)	56 %	34%
Mental arithmetic activity	Difficult for primary school	Difficult for secondary school
Primary T. (N = 13)	25 %	0 %
Secondary T. (N = 60)	49 %	23 %
Problem of unequal sharing	Difficult for primary school	Difficult for secondary school
Primary T. (N = 13)	50 %	17 %
Secondary T. (N = 60)	57 %	31 %

For each activity, we observe the same phenomena :

Secondary school teachers always consider the activities more difficult, for both primary and secondary students, than primary school teachers

Beliefs about the nature of mathematics



Beliefs about learning mathematics

