

Managing Strategic Inventories under Investment in Process Improvement

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Abstract

In supplier-retailer interactions, the retailer may carry inventories strategically as a bargaining mechanism to induce the supplier to drop the future wholesale price. As per Anand et al. (2008), the introduction of strategic inventories always benefits the supplier and possibly also the retailer if the holding cost is sufficiently low (due to the contract-space-expansion effect). Is such a move beneficial for the supply chain agents in the presence of process improvement efforts? Such efforts—initiated by suppliers—ultimately reduce production cost and may translate into lower wholesale prices as well as lower consumer prices. We find that strategic inventories may stimulate investment in process improvement when the holding cost is high (as it encourages the supplier to further reduce future cost to eliminate the need for strategic inventories), but may suppress such investment when the holding cost is low (as strategic inventories are cheap to stock and hence cannot be eliminated). Our key result, contrary to the existing literature, is that strategic inventories may be harmful to both supply chain agents in the presence of process improvement. In that case, the supplier effectively over-invests in process improvement efforts, inducing the retailer to reduce the stock of strategic inventories, while reversing the benefits of the contract-space-expansion effect. We also consider variations to the model, whereby the supplier may delay his investment decision, the holding cost may be a function of the wholesale price set by the supplier, consumers may behave strategically, and the planning horizon may consist of multiple periods.

Keywords: Supply Chain Management; Process Improvement; Strategic Inventories.

1 Introduction

Firms are constantly engaged in improving their internal processes in order to reduce the unit cost of production. New technologies and opportunities allow firms to take advantage of emerging solutions that facilitate future reductions in the cost of their operations. For instance, 3D printing bears a significant potential for firms in the manufacturing sector to transform their processes, ultimately allowing them to have a cheaper and a more efficient production system (examples include GE or the PSA Group, a French automotive firm, see Fortune, 2016). Cost reduction efforts are not limited to adoption of new technologies and can also emerge as an outcome of traditional process

management methods. Indeed, according to a cost management survey, streamlining business processes turned out to be one of the main tactical approaches for Fortune 1000 firms to remain competitive (Deloitte, 2013). One such example is the continuous improvement program at John Deere, which seeks to embrace lean processes and further engage suppliers in order to reduce the overall cost of the end products. The benefits of such investments in new technologies and improved processes may not be immediate, as the integration and implementation requires an overhaul of the design (of the product and/or the process), may be time consuming, and possibly may need to wait until the facility can be shut down.¹

Due to their nature, such process improvement and cost reduction efforts usually require long lead times. Namely, investment into such efforts are made well in advance before the outcome of the impact on the cost reduction are realized (Li and Wan, 2016). Recognizing that in such environments firms find it difficult to engage in long-term contracts (Li and Wan, 2016; Tirole, 1986),² firms oftentimes engage in new contractual arrangements once the new transactional costs are determined.

To circumvent, and to some degree divert potential consequences, suppliers who are involved in such efforts will preannounce their investments into process improvement to garner the attention and proper reaction of their immediate customers along the supply chain—the retailers. Being aware of such a renegotiation opportunity, retailers may then carry inventories strategically as a bargaining chip against their suppliers (Anand et al., 2008). Specifically, by holding to some inventories, retailers can force their suppliers to lower the wholesale price when the new contract is signed. On the one hand, carrying inventories is a costly friction (the *inventory-drain* effect in Anand et al., 2008); however, on the other hand, this inventory allows the retailer to source using two different prices: the original wholesale price from the stock of strategically-carried inventory and the newly negotiated wholesale price from the supplier (the *contract-space-expansion* effect in Anand et al., 2008). When the latter effect dominates, which occurs for a large range of holding

¹In the automotive sector, for instance, such changes may occur only once a new model is introduced or once a new facility is built. However, new methods allow producers now to continuously introduce such improvements in their manufacturing process (see <https://eu.detroitnews.com/story/business/autos/foreign/2017/11/07/toyota-cuts-production-costs-record-research-budget/107431598/>).

²Tirole (1986) highlighted two aspects that limit firms from engaging in long-term contracts in such environments: lack of commitment power that may result in contract breaches and renegotiations, and the potential uncertainty associated with the outcomes of the efforts and hence the inability to identify an appropriate contract. While in this manuscript we abstract away from the issues pertaining to uncertainly, many of the challenges persist.

cost values, the level of double marginalization is reduced, and the supply chain is better off. While the supplier cannot eliminate strategic inventories held by the retailer, he can control their level via the first period wholesale price. These results have been derived by Anand et al. (2008) in the absence of process improvement. Accordingly, our interest is in the effect of this preannounced commitment into process improvement on the inventories carried by the retailer and the corresponding performance of both supply chain agents.

In the presence of process improvement, the product's unit cost is likely to decrease over time. The supplier may thus have an incentive to pass on some of the savings to the retailers in a later stage, in order to induce them to increase their purchased amount (rather than decrease the purchased amount as the retailer can also make use of available stock). Namely, the supplier might have an incentive to commit to process improvement efforts—as an additional leverage to affect strategic inventories—thereby indicating a future drop in cost, and hence in wholesale price, which could signal to the retailer that stocking strategic inventories is not necessary. Consequently, we raise the question: do retailers still strategically stock inventories when their suppliers are engaged in process improvement efforts? Alternatively, do strategic inventories stimulate or discourage investment in process improvement efforts?

Our analysis highlights the importance of the *delayed cost reduction effect*. When the retailer contemplates carrying strategic inventories, such an option may suppress investment in process improvement when the cost of holding inventory is sufficiently low (in relation to the cost of the cost reduction), but may stimulate investment in such process improvement when the holding cost of inventory is sufficiently high. The intuition is that with a low holding cost, the threat of holding inventories strategically is high, which suppresses the supplier's incentive to invest. In such a case, some inventories may be carried and the investment in process improvement will apply to a smaller quantity of units that will be purchased by the retailer. Alternatively, when the holding cost is high, the quantities stocked are naturally reduced, thereby increasing the incentive of the supplier to invest in reducing the unit cost in the future. Furthermore, in such a case, the supplier invests more than in the absence of such inventories as the supplier seeks to suppress the retailer's incentive to stock strategic inventories entirely.

This delayed cost reduction effect goes beyond the intricate relationship between inventories and process improvement. As the level of inventories decreases, some of the inventory-drain burden is

relieved while completely *reversing* the benefits stemming from contract-space-expansion effect. In other words, process improvement induces the retailer to decrease the levels of inventories thereby reducing the contract space, effectively making both the supplier and the retailer worse off when the holding cost is sufficiently high.

We further consider and discuss variations to the model. Specifically, we explore *(i)* whether the supplier shall actually commit to the investment in process improvement or shall he delay the decision? *(ii)* whether replacing the fixed holding cost with a holding cost that is a function of the wholesale price (which is set by the supplier) alters the insights of our analysis; *(iii)* the impact induced by the presence of strategic consumers who may wait for lower prices in the second period; and *(iv)* the implications of additional periods in the planning horizon.

2 Literature Review

Our research links process improvement decisions to strategic inventory decisions, in settings that may be characterized by strategic consumers. Process improvement has been intensively studied in the operations management and industrial organization literature, including process improvement decisions in supply chains. Much work has been done on settings with downstream competition, focusing on issues such as the decision of the buyer (or retailer) to outsource production and process improvement (Gilbert et al., 2006), a shared supplier's process improvement decision when one of the buyers can integrate with the supplier (Chen and Sappington, 2009), and the supplier's process improvement decision in a context with competing supply chains when a supply chain can integrate (Gupta and Loulou, 1998; Gupta, 2008). Upstream competition between suppliers and supplier process improvement has been considered in Li (2013), for instance.

Regarding the nature of process improvement itself, the literature identifies two main types: process improvement as the result of (i) learning by doing or (ii) deliberate investment. The learning by doing literature typically addresses cost-reducing process improvement in multi-period models, and assumes that the cost reductions in a later time period are the results of production at some earlier point. Gray et al. (2009) model a setting with a contract manufacturer and an original equipment manufacturer (OEM). The OEM can outsource production to the contract manufacturer, while both actors can reduce the unit cost of production as the result of learning

by doing. Following up on this, Li et al. (2015) model a supply chain with one manufacturer and one retailer, and consider the effect of learning by doing on a manufacturer’s inventory decision and the effectiveness of revenue sharing contracts. Shum et al. (2017) study a firm engaged in a two-period dynamic pricing game with strategic consumers and uncontrolled process improvement between the periods. Importantly, next to learning by doing they include cost reduction as a result of some random technology advancement.

Process improvement investment papers have considered multi-period approaches. Using a Markov Decision Process, Fine and Porteus (1989) determine a firm’s optimal process improvement investment policy. In each decision epoch, a small process improvement (such as a setup cost reduction) can be realized. Li and Rajagopalan (2008) identify optimal process improvement investment policies based on a multi-period real-options model. In their model, process improvement investments increase the knowledge of the process. If successful, process improvement may lead to a higher probability of success of future investments, as well as higher product quality and cash flows. Most papers in this stream, however, focus on the immediate effect of the investment in single period settings, using game-theoretic frameworks. In the early work of d’Aspremont and Jacquemin (1988) a duopoly was considered in which a process improvement competition stage preceded a quantity competition stage, while in Veldman et al. (2014) process improvement and duopolistic quantity competition take place simultaneously (after the observation of managerial incentive contracts for process improvement). Process improvement investment papers taking a supply chain point of view either assume that process improvement investment decisions may precede the supplier’s wholesale pricing decision (Bernstein and Kök, 2009; Ge et al., 2014) or let the investment and wholesale pricing decisions take place simultaneously, as in Ha et al. (2017). In contrast to the learning by doing stream—where the cost reduction effects of learning are typically postponed to a next time period—it seems that in the process improvement investment stream the investment effectuates as soon as possible.

As stated earlier, it is well recognized that process improvement projects are lengthy, often with uncertain outcomes (Li and Wan, 2016; Li and Arreola-Risa, 2017). In the body of the paper, we abstract away from the realization of uncertainty and similar to many contributions, we assume deterministic outcomes (see, e.g., Chu and Sappington, 2007, Laffont and Tirole, 1986, Rogerson, 2003, Yenipazarli, 2017, and a review by Laffont and Tirole, 1993). Nevertheless, papers differ with

respect to the timing of the contract elements. For instance, while Rogerson (1992) assumes the contract is signed before efforts take place, Dasgupta (1990) and Piccione and Tan (1996) assume contracting takes place after the effort choices are made.

The key contribution of our research to the literature on process improvement is the consideration of strategic inventories. While generally this literature abstracts away from the concept of inventories, we explicitly account for their presence, which can play an instrumental role in the interaction between the two supply chains agents, as they—when carried strategically by the retailer—may circumvent actions taken by the supplier and, quite importantly, may affect his investment in process improvement.

The literature on strategic inventories is limited. Strategic inventories were identified by Anand et al. (2008) who recognized their role in multi-period environments. In their model they show that the retailer has an incentive to stock such inventories in order to force the supplier to set a lower second period wholesale price. More recently, Arya and Mittendorf (2013) consider the mediating role that rebates offered by manufacturers directly to consumers have on strategic inventories. Arya et al. (2014) extend the strategic inventories framework to incorporate decentralized decision making in procurement and inventory control, while Mantin and Jiang (2017) let strategic inventories deteriorate over time. Liu et al. (2012) include a commitment by the retailer through an ex ante announced price markup (on top of the wholesale price) and a price protection policy by the manufacturer. Also they show that unique solutions exist for extended (finite) time horizons, and consider other forms of demand functions. An interesting treatment is offered by Hartwig et al. (2015) who test the effect of strategic inventories on supply chain performance by conducting an empirical study in a lab environment. They show that strategic inventories have a positive effect on performance above and beyond that projected by theory—this is driven by the fact that the presence of strategic inventories induce the buyer and the seller to reduce the payoff inequalities. More recently, Roy et al. (2018) have discussed the implications of inventory visibility (i.e., whether the manufacturer can observe the amount of strategic inventories carried by the retailer) which, they show, may increase or decrease the amount of inventory strategically carried by the retailer.

We complement the literature on strategic inventories by accounting for the well-established notion of process improvement. Such investments, broadly intended to reduce the unit cost, may or may not be translated into lower wholesale price, and hence could alter the retailer’s incentive

to carry strategic inventories.

3 Modeling Framework

In this section we introduce the modeling framework where the supplier may invest in cost-reducing process improvement and the retailer may carry strategic inventories. In §5 we extend the framework by including the possibility of facing strategic consumers.

Using the framework of Anand et al. (2008) with dynamic price contracts as a workhorse, we consider a two-period setting and a simple supply chain consisting of a supplier and a retailer. In each of the periods the supplier sets a wholesale price and the retailer decides the order quantity. Further, a new cohort of consumers arrives in each of the periods. The demand stemming from these consumers follows a linear relationship such that $p_i = a - q_i$, $i \in \{1, 2\}$, where p_i and q_i are the price and demand, respectively, in period i . Throughout the paper we let $a = 1$. We assume that after each period, the retailer sells all products offered to the market. The timeline of events, which is depicted in Figure 1, is as follows.

Period 1: Supplier The supplier decides whether or not to invest in process improvement, which reduces the cost of producing a product c in the second period by x . Without loss of generality, we normalize c to zero.³ The investment cost of this process improvement, $\frac{1}{2}\gamma x^2$, is incurred in the first period. The process improvement cost parameter γ measures the supplier’s improvement capability. It is common in the literature to model process improvement investments as quadratic functions to allow for decreasing returns to scale and limited organizational investment budget (d’Aspremont and Jacquemin, 1988; Gupta and Loulou, 1998; Veldman et al., 2014). Throughout the paper we assume that $\gamma > \frac{1}{2}$.⁴ The supplier also sets the first period wholesale price, w_1 . We study a linear wholesale pricing scheme as linear prices are widely adopted in practice (Sluis and De Giovanni, 2016), and these schemes allow

³In case of a model with a market size a and constant cost of production c we can always choose the parameter values of a , c such that $c > x$ and all other (sufficient second-order and positivity) conditions are met. See the Online Appendix A for the details of such a more general model. Moreover, in the case with $a = 1$ and $c = 0$ it is easy to verify that in the cases we present in the next section, in equilibrium $x < 1$ always.

⁴This bound is derived from the case with process improvement (see §4.2), and ensures that the second-stage wholesale price is positive. Sufficient second-order conditions and positivity conditions are given in Online Appendix B. As we show there, there may be stricter lower bounds on γ in the case with strategic consumers, which is presented in §5.

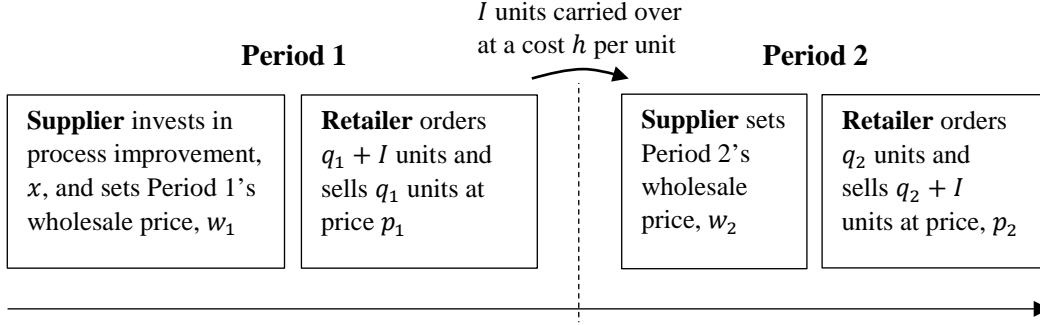


Figure 1: Timeline of events.

us to isolate the strategic effects of process improvement and inventories.

Period 1: Retailer The retailer decides how many units to purchase in the first period. This amount corresponds to two sub-decisions: how many to sell in the first period, q_1 , at a price $p_1 (=1 - q_1)$, and how many to carry over from the first period over to the next, I , while incurring a holding cost of h per unit. Similar to Anand et al. (2008), we assume throughout the paper that $0 < h < \frac{1}{4}$.⁵

Period 2: Supplier The supplier sets the second period's wholesale price, w_2 .

Period 2: Retailer The retailer purchases q_2 units and sells a total of $q_2 + I$ units at a price p_2 .

The retailer's profit in the second period is given by

$$\Pi_{R,2} = (q_2 + I)p_2 - q_2w_2,$$

where $p_2 = 1 - (q_2 + I)$. The retailer's total profit is expressed as

$$\Pi_R = q_1p_1 - (q_1 + I)w_1 - Ih + \Pi_{R,2}.$$

Similarly, the supplier's profit in the second period is given by

$$\Pi_{S,2} = q_2(w_2 + x),$$

⁵The lower bound ensures that holding inventories is costly whereas the upper bound is required to ensure feasibility of stocking inventories.

and the supplier’s total profit is

$$\Pi_S = (q_1 + I)w_1 - \frac{1}{2}\gamma x^2 + \Pi_{S,2}.$$

Throughout the paper we assume that the process improvement investment yields the intended unit cost reduction with absolute certainty. Naturally process improvement projects might fail due to circumstances beyond the supplier’s control. Assuming uncertainty in the success rate of the project, however, does not critically affect our results.⁶

Finally, in our model, the market parameters as well as the holding and process improvement cost parameters are common knowledge to both supply chain agents.⁷ We solve the model by backward induction to yield the optimal decisions of the retailer and the supplier.

4 Model analysis

We carry out the analysis in several steps. We first highlight, separately, the role of strategic inventories in the absence of process improvement—which is essentially the seminal result of Anand et al. (2008)—and the role of process improvement in the absence of strategic inventories. We then proceed by analyzing the complete model incorporating the combined effects of these two decisions. Accordingly, we revisit the decisions made by the retailer and supplier, respectively, to assess whether the logic is sustained.

⁶Similar to Veldman et al. (2014) we can let θ denote the probability of success of the process improvement project. If the stochastic variable y denotes the uncertain unit cost reduction of the supplier, we have that $y(\theta, x)$ is either x with probability θ or 0 with probability $1 - \theta$. The supplier’s expected profits in the second period can be written as $\bar{\Pi}_{S,2} = q_2(w_2 + \theta x)$, while his expected total profit becomes $\bar{\Pi}_S = (q_1 + I)w_1 - \frac{1}{2}\gamma x^2 + \bar{\Pi}_{S,2}$. By setting $\theta x = z$, we can write $\bar{\Pi}_S = (q_1 + I)w_1 - \frac{1}{2}\left(\frac{\gamma}{\theta^2}\right)z^2 + q_2(w_2 + z)$. Letting $\frac{\gamma}{\theta^2} = \gamma_n$, all outcomes become functions of γ_n (among others). Clearly γ_n decreases in θ so the effect of θ on the outcomes can be easily obtained when knowing the effect of γ_n . Moreover, note that both in the deterministic and stochastic case the retailer responds to the *outcomes* of the process improvement project. Therefore no additional assumptions are needed in terms of the retailer’s knowledge of θ (or any other probability distribution parameters), γ , or any potential uncertainty related to γ .

⁷While we assume a deterministic market size, one can also consider stochasticity with respect to the market size. Specifically, following Gümüş et al. (2013), assume the market size (i.e., the demand intercept) in the second period, is either high ($1 + \theta$) or low ($1 - \theta$) with probability $\lambda_{1+\theta}$ and $\lambda_{1-\theta}$, respectively, such that $\lambda_{1+\theta} + \lambda_{1-\theta} = 1$. As in Gümüş et al. (2013), for expositional simplicity, let $\lambda_N = \frac{1}{2}$ for $N \in \{1 + \theta, 1 - \theta\}$, and the corresponding expressions only change by a constant that is a function of θ . Thus, all results follow through.

4.1 When are strategic inventories profitable in the absence of process improvement?

To understand the role of inventories in the interaction between the retailer and his supplier, we isolate this decision by assuming that no investment is made, or possible, in process improvement. Thus, we compare two scenarios: in the first, the retailer does not consider carrying strategic inventories, and in the second, this option is evaluated by the retailer. This is essentially the analysis that was carried out by Anand et al. (2008). In the absence of inventories, the problem trivially becomes a repeated single period setting, where the supplier sets the wholesale price to $\frac{1}{2}$ and the retailer responds by ordering a quantity of $\frac{1}{4}$ in each of the periods. Accordingly, using a superscript N to denote profits in the case without process improvement and strategic inventories, the retailer and supplier profits over the two periods are $\Pi_R^N = \frac{1}{8}$ and $\Pi_S^N = \frac{1}{4}$, respectively.

Once the retailer carries inventories, then he induces the supplier to reduce the wholesale price set in the second period. Specifically, as the model is solved backwards, it is easy to see that $w_2 = \frac{1}{2} - I$. Hence, the retailer has an incentive to stock inventories in order to force the supplier to reduce the future wholesale price. The retailer carries inventories only if the benefits of wholesale price reduction exceed their holding cost. Solving backwards, we have that the retailer's optimal inventory choice is $I = \frac{1}{2} - \frac{2}{3}(w_1 + h)$. That is, the inventory the retailer carries decreases in the holding cost as well as in the first period wholesale price. That is, the retailer recognizes the power of strategic inventories in affecting the future wholesale price and responds to the wholesale price set by the supplier in the first period. The supplier then realizes the importance of w_1 in affecting the retailer's decision and sets $w_1 = \frac{9-2h}{17}$, which induces the retailer to carry strictly positive inventory levels, as the optimal inventory level is given by $I = \frac{5}{34} - \frac{10h}{17}$. This is important, as the supplier raises the wholesale price above the single period optimal price of $\frac{1}{2}$, while the second period wholesale price, $w_2 = \frac{6+10h}{17}$, is always below $\frac{1}{2}$ (and hence less than w_1). The resulting profits of the supplier and retailer are, respectively,

$$\Pi_S^S = \frac{8h^2 - 4h + 9}{34} \quad (1)$$

and

$$\Pi_R^S = \frac{304h^2 - 118h + 155}{1156}, \quad (2)$$

where the superscript S refers to the scenario where strategic inventories may be carried in the absence of process improvement. It can be verified that in the presence of strategic inventories, the retailer is better off only when $h < \frac{21}{152} \approx 0.138$, while the supplier is always better off (see Proposition 1 in Anand et al. 2008). Hence, as long as the holding cost is not too high, the retailer has a strong incentive to stock inventories as a strategic instrument in the dynamic interaction with the supplier. When the holding cost is sufficiently high (but below $\frac{1}{4}$), the retailer might seek to commit to not stocking strategic inventories at all. However, given the dynamic nature of the interaction, the retailer's commitment might not be credible as once the supplier has set the first period wholesale price, it is always in the best interest of the retailer to stock some inventories strategically and, hence, he cannot avoid the profit loss when the holding cost is sufficiently high.

4.2 When is process improvement profitable in the absence of strategic inventories?

To isolate the effect of process improvement, we abstract away from strategic inventories and consider two scenarios. In the first, no investment is considered (and hence this coincides with the benchmark case considered in the previous subsection), and in the second we let the supplier evaluate this option. When the supplier considers the option of improving the process, it can reduce the unit cost with x per unit between the first and second period, by investing an amount of $\frac{1}{2}\gamma x^2$. In that case, in the absence of strategic inventories, the supplier sets $w_1 = \frac{1}{2}$ and chooses $x = \frac{1}{4\gamma-1} > 0$, which results in $w_2 = \frac{2\gamma-1}{4\gamma-1} > 0$. Note that $w_1 > w_2$. Hence, the first period wholesale price is independent of the investment in the process improvement, and only the second period wholesale price is affected—it increases in the process improvement cost parameter, γ . Letting the superscript P denote the case of process improvement in the absence of strategic inventories, the resulting profits of the retailer and supplier are $\Pi_R^P = \frac{32\gamma^2-8\gamma+1}{16(4\gamma-1)^2}$ and $\Pi_S^P = \frac{8\gamma-1}{8(4\gamma-1)}$, respectively, indicating that process improvement makes both the retailer and the supplier always better off.

4.3 Process improvement and strategic inventories

Could strategic inventories hinder process improvement? As we have seen, the retailer always stocks strategic inventories as a bargaining chip against the supplier. If such inventories are kept, the incentive of the supplier to invest in process improvement, and hence further lowering w_2 , could be diminished. At the same time, we have observed that both are better off due to process improvement, and hence it is to their mutual benefit to make sure such investments are made. Alternatively, could the threat of strategic inventories stimulate investment in process improvement? The retailer could use inventories as an instrument to further encourage the supplier to stimulate investment in process improvement. By investing in process improvement in the first period, the supplier implicitly commits to lower wholesale prices in the second period, which may induce the retailer to lower strategic inventories. Accordingly, the interaction between the decisions made by the supplier and retailer, respectively, are revisited in this section. We start by solving the complete model and then we proceed to highlight the impact of the two decisions.

4.3.1 Analysis of process improvement and strategic inventory levels

The analysis is similar to that carried in §4.1 with the addition of the supplier's choice of process improvement investments in the first period. The characterization of the equilibrium outcomes is summarized in the following statement, where the superscript PS indicates the current scenario where both process improvement and strategic inventories are part of the consideration set of the supplier and retailer, respectively. All proofs can be found in Online Appendix C.

Proposition 1. Define $\hat{h} \equiv \frac{5(4\gamma-3)}{8(10\gamma-3)}$. When the supplier may invest in process improvement and the retailer may stock strategic inventories, in equilibrium:

$$I^{PS} = \begin{cases} \frac{-80\gamma h + 20\gamma + 24h - 15}{2(68\gamma - 33)} & h < \hat{h} \\ 0 & otherwise \end{cases} \quad (3)$$

$$x^{PS} = \begin{cases} \frac{2(5+14h)}{68\gamma-33} & h < \hat{h} \\ \frac{1}{4\gamma-1} & otherwise \end{cases} \quad (4)$$

$$\{w_1^{PS}, w_2^{PS}\} = \begin{cases} \left\{ \frac{4(-2\gamma h + 9\gamma + 2h - 4)}{68\gamma - 33}, \frac{2(-20\gamma h + 12\gamma + 13h - 7)}{68\gamma - 33} \right\} & h < \hat{h} \\ \left\{ \frac{1}{2}, \frac{2\gamma - 1}{4\gamma - 1} \right\} & \text{otherwise} \end{cases} \quad (5)$$

$$\{p_1^{PS}, p_2^{PS}\} = \begin{cases} \left\{ \frac{-8\gamma h + 104\gamma + 8h - 49}{2(68\gamma - 33)}, \frac{40\gamma h + 92\gamma - 26h - 47}{2(68\gamma - 33)} \right\} & h < \hat{h} \\ \left\{ \frac{3}{4}, \frac{3\gamma - 1}{4\gamma - 1} \right\} & \text{otherwise.} \end{cases} \quad (6)$$

The resulting profits of the retailer and the supplier are, respectively,

$$\Pi_S^{PS} = \begin{cases} \frac{2(8\gamma h^2 - 4\gamma h - h^2 + 9\gamma + 4h - 4)}{-33 + 68\gamma} & h < \hat{h} \\ \frac{8\gamma - 1}{8(4\gamma - 1)} & \text{otherwise} \end{cases} \quad (7)$$

and

$$\Pi_R^{PS} = \begin{cases} \frac{2432\gamma^2 h^2 - 944\gamma^2 h - 1504\gamma h^2 + 1240\gamma^2 + 1344\gamma h + 346h^2 - 1160\gamma - 295h + 295}{2(-33 + 68\gamma)^2} & h < \hat{h} \\ \frac{32\gamma^2 - 8\gamma + 1}{16(4\gamma - 1)^2} & \text{otherwise.} \end{cases} \quad (8)$$

It is evident that the retailer's choice of strategic inventories (I) depends on the holding cost, h , and the process improvement cost parameter, γ . Specifically, there exists a threshold holding cost (which is a function of γ) above which inventories are not carried strategically any longer. The elimination of strategic inventories by process improvement investment is a new result that complements that of Anand et al. (2008).

Furthermore, in the parameter area where strategic inventories are carried, we notice that the possibility of using process improvement as a commitment device to lower wholesale prices in the second period, has an overall dampening effect on strategic inventories. The following proposition summarizes.

Proposition 2. *Process improvement suppresses the incentive to hold strategic inventories ($I^S \geq I^{PS}$). Further, the introduction of process improvement completely eliminates strategic inventories when $h > \hat{h}$.*

This is an important result as it differs from the case where investment in process improvement is absent. Recall that in the benchmark case, which follows the model of Anand et al. (2008), the retailer carries strategic inventories in the entire feasible range, i.e., whenever $h < \frac{1}{4}$. However,

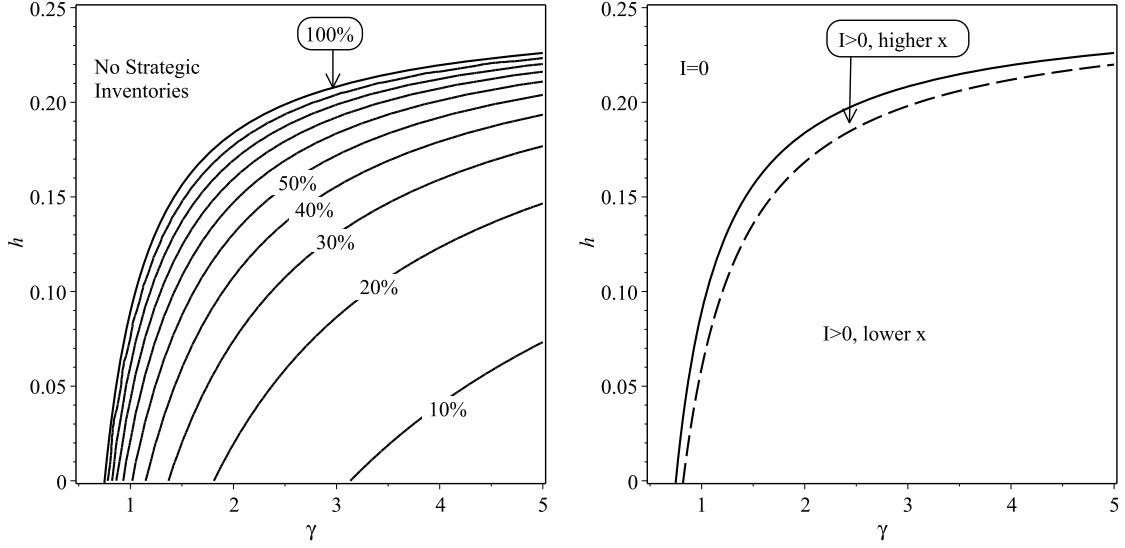
now due to the process improvement, the retailer effectively eliminates such inventories if the holding cost is sufficiently high, or alternatively, when the process improvement cost parameter, γ , is sufficiently low.

This result is displayed graphically in Figure 2a. Quite naturally, the effect diminishes in γ , since higher values of γ imply a higher cost of investment in process improvement, which reduces the supplier's investment and therefore reduces the cost reduction in the second period, thereby limiting overall the retailer's incentive to reduce the amount of strategic inventories stocked.

It is also evident that the supplier will always invest in process improvement (that is, x is always positive). However, the effect of the presence of strategic inventories on investment in process improvement is not uniformly positive or negative. The following proposition highlights the surprising effect of strategic inventories on process improvement levels (recall the definition of \hat{h} in Proposition 1).

Proposition 3. *For $h < \frac{28\gamma-23}{28(4\gamma-1)}$, strategic inventories suppress investment in process improvement ($x^{PS} < x^P$), whereas for $\frac{28\gamma-23}{28(4\gamma-1)} < h < \hat{h}$ strategic inventories stimulate investment in process improvement ($x^{PS} > x^P$). When $h > \hat{h}$, since no strategic inventories are carried any longer, they do not alter the investment in process improvement (i.e., $x^{PS} = x^P$).*

This result is illustrated in Figure 2b. The figure illustrates the dampening effect of strategic inventories on process improvement investments in the largest part of the parameter area where strategic inventories are carried. In this parameter area the supplier will not use process improvement to completely offset the use of strategic inventories by the retailer. From the retailer's perspective, the negative implications of inventory carryover are more than compensated by the wholesale price reduction in the second period, which is the result of the combined effects of process improvement and the strategic effect of inventories. From a comparative statics viewpoint, we see that strategic inventories monotonically decrease in h while process improvement increases in h . For large enough h , given γ , low strategic inventory levels incentivize the supplier to increase, rather than decrease process improvements levels, beyond the level set at zero inventories.



(a) Impact of process improvement on strategic inventories (% reduction) (b) Impact of strategic inventories on process improvement

Figure 2: Process improvement and strategic inventories.

Recall from an earlier footnote that we could assume a probability of success of the investment θ . Knowing that $\frac{\partial I}{\partial \gamma} = \frac{36(5+14h)}{(68\gamma-33)^2} > 0$ the effects of uncertain investment outcomes can be easily assessed. That is, strategic inventories increase as the success probability decreases, which is a straightforward result.

4.3.2 Is investment in process improvement beneficial in the presence of strategic inventories?

We have observed that the supplier will always invest in process improvement, whether the retailer will stock strategic inventories or not. From a profit point of view, process improvement is clearly beneficial to both supply chain agents when no strategic inventories are carried. In the presence of strategic inventories, we have seen in Proposition 2 that process improvement suppresses strategic inventories. As the supplier has a Stackelberg position when it comes to process improvement investments, and the retailer will always benefit from process improvement due to lower wholesale prices in the second stage, we would expect that process improvement will benefit both the supplier and the retailer in the presence of strategic inventories. Proposition 4 confirms this intuition.

Proposition 4. *Assume the retailer considers carrying strategic inventories. Then, process im-*

provement investments will make both the supplier and the retailer better off (i.e., $\Pi_S^{PS} > \Pi_S^S$ and $\Pi_R^{PS} > \Pi_R^S$).

This complements the discussion from §4.2, supporting the notion that the supplier's investment in process improvement is beneficial to both the supplier and the retailer regardless of whether the retailer considers, or not, to carry strategic inventories. Hence, one can conclude that the supplier always has the incentive to invest in process improvement. We next explore the impact of strategic inventories assuming the supplier makes such an investment in improving its processes.

4.3.3 Are strategic inventories beneficial in the presence of process improvement?

We have noted that in the case without process improvement (§4.1), the retailer is better off with strategic inventories only when the holding cost is sufficiently low ($h < 0.138$) while the supplier is always better off. Does the same qualitative insight hold in the presence of process improvement? That is, does it hold true that the consideration of strategic inventories by the retailer will always make the supplier better off? Recall that the supplier will always make a strictly positive process improvement investment, whether or not strategic inventories are carried, and that the retailer's incentive to stock strategic inventories is diminished (Proposition 2). Comparing the retailer's and supplier's profits in the two cases where the supplier invests in process improvement—when strategic inventories are absent (i.e., Π_R^P and Π_S^P) vs. when they are considered by the retailer (i.e., Π_R^{PS} and Π_S^{PS})—we establish the range of parameters where the retailer and supplier benefit from stocking strategic inventories when the supplier invests in process improvement.

Proposition 5. *Assume the supplier invests in process improvement. Then, stocking strategic inventories (which occurs when $h < \hat{h}$) will make the supplier better off (i.e., $\Pi_S^{PS} > \Pi_S^P$) when $h < f(\gamma) \equiv \frac{32\gamma^2 - 40\gamma + 8 - \sqrt{272\gamma^2 - 200\gamma + 33}}{4(32\gamma^2 - 12\gamma + 1)}$, and worse off otherwise (i.e., $\Pi_S^{PS} < \Pi_S^P$), and will make the retailer better off (i.e., $\Pi_R^{PS} > \Pi_R^P$) when $h < g(\gamma) \equiv \frac{3776\gamma^3 - 6320\gamma^2 + 2524\gamma - 295 - \sqrt{X}}{4(1216\gamma^2 - 752\gamma + 173)(4\gamma - 1)}$, and worse off otherwise (i.e., $\Pi_R^{PS} < \Pi_R^P$), where X is defined in the proof.*

While it is not too surprising that the retailer can be worse off due to strategic inventories when the supplier invests in process improvement, the fact that the supplier can be worse off is a new result. Specifically, in §4.1 we have seen that in the absence of process improvement, strategic inventories always benefit the supplier. Further, as process improvement is always profit improving

(see §4.2), one might expect that strategic inventories in the presence of investment in process improvement will still benefit the supplier. Figure 3 illustrates how the supplier's, the retailer's, as well as the supply chain's profits are affected due to the carrying of strategic inventories by the retailer, along with the threshold $\hat{h} \equiv \frac{5(4\gamma-3)}{8(10\gamma-3)}$, above which no strategic inventories are carried (in which case there is no difference in the profit of the different parties).

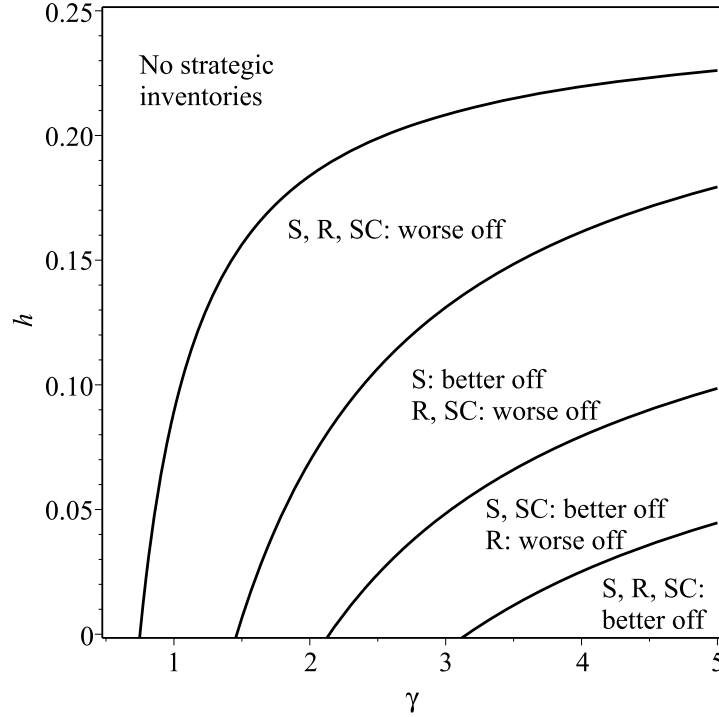


Figure 3: Change in profit due to strategic inventories in the presence of process improvement (S: Supplier, R: Retailer, SC: Supply Chain).

What drives this surprising result? Recall that in the absence of process improvement, the introduction of strategic inventories has benefited the supplier as they have improved the channel coordination between the retailer and the supplier. Despite the holding cost incurred by the retailer (the inventory-drain effect), holding strategic inventories allows the retailer to source at two prices (w_1 and w_2), thereby increasing the space of alternatives faced by the retailer (the contract-space-expansion effect). This latter effect reduces the level of double marginalization and benefits the supplier, who, in effect, controls the inventory carried by the retailer. This effect also benefits the retailer as long as it dominates the inventory-drain effect, the cost of which is incurred by the retailer. Once the supplier invests in process improvement, another element enters the equation: the

delayed cost reduction effect, i.e., the implicit commitment to unit cost reduction over time. In the absence of strategic inventories, the supplier—being the Stackelberg leader—is able to fully benefit from investment in process improvement; however, the introduction of strategic inventories may weaken the potential to do so. The first effect that occurs is over-investment in process improvement. As can be observed from Figure 2b, the supplier invests more in process improvement due to the threat of strategic inventories when holding costs are high enough. But the fact that this reduces his profits suggests that the supplier would actually fare better under lower process improvement levels.⁸ Such over-investment induces even lower second period wholesale prices, and while the retailer still stocks strategic inventories, the combined effect hurts the supplier. More important is the direct effect of process improvement on the level of strategic inventories. Due to the investment in process improvement, the supplier signals to the retailer that he is committed to reductions in the product’s unit cost and hence the need to stock inventories strategically diminishes. By reducing the level of inventories, the retailer relieves some of the inventory-drain burden, but at the same time he completely reverses the benefits of the contract-space-expansion effect. That is, process improvement induces the retailer to stock lower levels of inventories which reduce the contract space, effectively making both the supplier and the retailer worse off when h is sufficiently high.

Finally, it may be noted that given any value of γ , there always exists an h above which the supplier is worse off. This can be illustrated by analyzing the various thresholds from Proposition 5 when γ approaches infinity (letting process improvement levels approach but not converge to zero). Specifically, the supplier is worse off when $h > f(\gamma)|_{\gamma \rightarrow \infty} = 0.25$, which is the upper limit of h in Anand et al. (2008). Finally, the retailer is worse off when $h > g(\gamma)|_{\gamma \rightarrow \infty} = \frac{21}{152} \approx 0.138$, which again is the threshold from Anand et al. (2008) in the case of no process improvement.

4.4 Summary and discussion

Table 1 provides an overview of the various cases under consideration, contingent on whether or not process improvement and strategic inventories are carried.

⁸Note that the area where process improvement increases due to strategic inventories (see Figure 2b) is fully part of the area where the supplier is worse off (see Figure 3).

Table 1: Case overview.

		Process improvement investment?	
		NO	YES
Strategic inventories possible?	NO	case N $\{\pi_R^N, \pi_S^N\}$	case P $\{\pi_R^P, \pi_S^P\}$
	YES	case S $\{\pi_R^S, \pi_S^S\}$	case PS $\{\pi_R^{PS}, \pi_S^{PS}\}$

Let us briefly reflect on how the profits of the two supply chain actors change, when the supplier decides to invest in process improvement, or the retailer decides to carry strategic inventories. As we noted in §4.2 and Proposition 4, an investment in process improvement by the supplier is beneficial for both parties, whether strategic inventories are not carried at all (i.e., $\mathbf{N} \rightarrow \mathbf{P}$) or are indeed carried ($\mathbf{S} \rightarrow \mathbf{PS}$). The effects of strategic inventories are less straightforward. Carrying strategic inventories in the absence of process improvement ($\mathbf{N} \rightarrow \mathbf{S}$) mirrors the results given by Anand et al. (2008). That is, the supplier always benefits from strategic inventories, whereas the retailer benefits only when holding costs are low enough (see §4.1). When the supplier invests in process improvement, the profit implications of inventory carryover ($\mathbf{P} \rightarrow \mathbf{PS}$) are not immediately clear. Specifically, the retailer’s profit increases only when holding costs are low and process improvement cost (characterized by γ) is high. Interestingly, we see that the supplier may be worse off, whereas he generally benefits from strategic inventories when no process improvement investments are made.

Finally, we might wonder whether the supplier or the retailer could be better off by making no investments in process improvement and have no strategic inventory carryover at all, when we compare this to the case where both are strictly positive ($\mathbf{N} \rightarrow \mathbf{PS}$). As the supplier makes the first move by committing to process improvement (and sets w_1 to manipulate purchasing behavior), it might not come as a surprise that he always fares better under \mathbf{PS} .

The story is different for the retailer. To be precise, there exists an area Ω within the feasible parameter region where the retailer does not benefit from the combined use of process improvement and strategic inventories.⁹ So even though the retailer benefits from process improvement $\mathbf{N} \rightarrow \mathbf{P}$,

⁹The Ω area is illustrated in Online Appendix D.

retailer profits drop below the profits obtained in N , when in addition strategic inventories are carried ($P \rightarrow PS$). As Ω is characterized (among others) by high γ the benefits obtained from a small process improvement investment are clearly outweighed by the profit decrease due to strategic inventories. Table 2 summarizes.

Table 2: Profit analysis.

Profit analysis			
Case	The Supplier ..	The Retailer ..	Reference
$N \rightarrow P$	<i>always benefits</i>	<i>always benefits</i>	§4.2
$S \rightarrow PS$	<i>always benefits</i>	<i>always benefits</i>	§4.3.2, Proposition 4
$N \rightarrow S$	<i>always benefits</i>	<i>benefits only if $h < 0.138$</i>	§4.1
$P \rightarrow PS$	<i>benefits only if $h < f(\gamma)^\dagger$</i>	<i>benefits only if $h < g(\gamma)^\ddagger$</i>	§4.3.3, Proposition 5
$N \rightarrow PS$	<i>always benefits</i>	<i>benefits only if $h, \gamma \notin \Omega$</i>	This section

[†] $f(\gamma)$ is monotonically increasing in γ , with $f(\gamma)|_{\gamma \rightarrow \infty} = 0.25$.

[‡] $g(\gamma)$ is monotonically increasing in γ , with $g(\gamma)|_{\gamma \rightarrow \infty} = 0.138$.

Note that we generally observe that the supplier always invests in process improvement. We also saw earlier that strategic inventory generally suppresses process improvement investments. We can consider whether this also implies that the gains from investing in process improvement are higher when no strategic inventories are carried, compared to the situation where inventories are carried. Indeed, comparing firm profits on the path $N \rightarrow P$ with $P \rightarrow PS$, it is clear that both the supplier and retailer benefit the most from process improvement when inventories are absent.

5 Robustness and Extensions

We consider several important extensions and robustness analyses. In §5.1 we let the supplier delay his investment announcement. In §5.2 we study the implications of having the holding cost be a function of the wholesale price. §5.3 accounts for the potential presence of strategic consumers. §5.4 visits the possibility of having longer horizons.

5.1 Delayed Investment Announcement

In our main model we have assumed that the supplier commits to the process improvement investment ahead of any interaction between the supplier and the retailer. One can challenge this choice and argue that the supplier could be better off by replacing this commitment with an option that

could be exercised by the supplier at the end of the first period (while still affecting the unit cost of the second period).¹⁰ Does this dramatically alter the decisions and outcomes predicted by our core model? Or, more specifically, is it in the best interest of the supplier to commit early to an investment in process improvement? In this section we explore the alterations due to this change in the sequence of events.

In terms of the model we assume that the supplier sets the process improvement level in the second period simultaneously with the decision w_2 , while bearing the investment cost in the second period as well. Equilibrium outcomes are given in Online Appendix E. The next proposition essentially captures the comparison between this delayed investment case, and the **PS** case presented in § 4.3.

Proposition 6. *When the supplier may invest in process improvement and the retailer may stock strategic inventories, then delaying the announcement of process improvement investments (**D**) has the following effects: Process improvement investments (i) increase if strategic inventories are carried in both **D** and **PS** cases ($x^D|_{I^D, I^{PS}>0} > x^{PS}|_{I^D, I^{PS}>0}$), (ii) are identical if no inventories are carried at all in both cases ($x^D|_{I^D, I^{PS}=0} = x^{PS}|_{I^D, I^{PS}=0}$), (iii) either increase or decrease (based on a threshold inventory holding cost level) if inventories are only carried in the **PS** case ($x^D|_{I^D=0, I^{PS}>0} \neq x^{PS}|_{I^D=0, I^{PS}>0}$). Furthermore, the delayed investment decision reduces strategic inventories ($I^D < I^{PS}$).*

Our first observation is that under the delayed investment decision, the area where strategic inventories are carried decreases. That is, inventories are carried if $h < \hat{h}^D$, while we can verify that $\hat{h}^D < \hat{h}$. This yields three sub-areas. For every $h < \hat{h}^D$, process improvement increases when the investment decision is delayed. Now, the supplier has both the wholesale price and process improvement level in the second stage at his disposal, which results in higher process improvement levels. If $h > \hat{h}$ then inventories are absent in both cases. This eliminates the strategic effect of delaying the announcement of process improvement, resulting in identical process improvement levels. Finally, if $\hat{h}^D < h < \hat{h}$, then inventories are carried only in the **PS** case. The fact that in this

¹⁰Alternatively, the supplier could make an investment in the first period and reduce costs in both periods. Such an option, however, could increase cost reduction levels while simultaneously incentivizing the retailer to carry strategic inventories. Although this case is interesting in its own right, the focus of this paper is on delayed implementation of process improvement investments. Hence, we will not elaborate on the case where the investment already effectuates in the first period.

area process improvement may either increase or decrease is a result that mimics Proposition 3 and Figure 2b: for $\frac{28\gamma-23}{28(4\gamma-1)} < h < \hat{h}$ process improvement decreases because of the delayed investment decision while for $\hat{h}^D < h < \frac{28\gamma-23}{28(4\gamma-1)}$ process improvement increases.

Next, observe that less inventory is carried. While on the one hand the supplier cannot use the process improvement announcement in the first period to signal cost reductions in the second period (eliminating the strategic effect of process improvement over time), it can better align wholesale prices to eliminate strategic inventories.

Given this new equilibrium, is the retailer better off compared to the **PS** case? On the one hand, the retailer gives up some of his bargaining capacity and, evidently, he ends up paying more for the units in the second period despite the larger investment in process improvement, while, on the other hand, he benefits from reduced stocking levels and lower first period wholesale price. We find that the latter effect dominates, as the retailer is always better off under the delayed commitment setting. In this setting it seems that the delayed announcement acts as an intermediary to reduce the effect of double marginalization, at least from the retailer's perspective, such that the retailer is better off overall. This further supports some of the results derived from the main model presented in Propositions 2 and 4.

While the retailer is always better off, does this also hold true for the supplier? After all, the supplier can benefit from elimination of inventories which facilitates greater investment in process improvement and hence greater savings. However, we have also noticed that strategic inventories are not necessarily the most favorable alternative of the supplier. As suggested already, the delayed announcement which results in lower inventories indeed provides the retailer with some relief from his inventory-drain burden but at the same time this completely reverses the benefits of the contract-space-expansion effect. Yet, while this still benefits the retailer, with this reduction in strategic inventories the supplier ends up being worse off when the process improvement cost parameter γ is sufficiently high. This is illustrated in Figure 4.¹¹

¹¹In our setting in this subsection, we assume that γ does not change if the decision is delayed. However, in practice, such a delay comes at a cost in the form of a higher γ value (as the implementation need to be expedited). This will certainly bear a negative implication on the decision to delay the decision, which, as indicated Figure 4, is already tilting in favor of pre-announcing this commitment, rather than delaying to a later point in time.

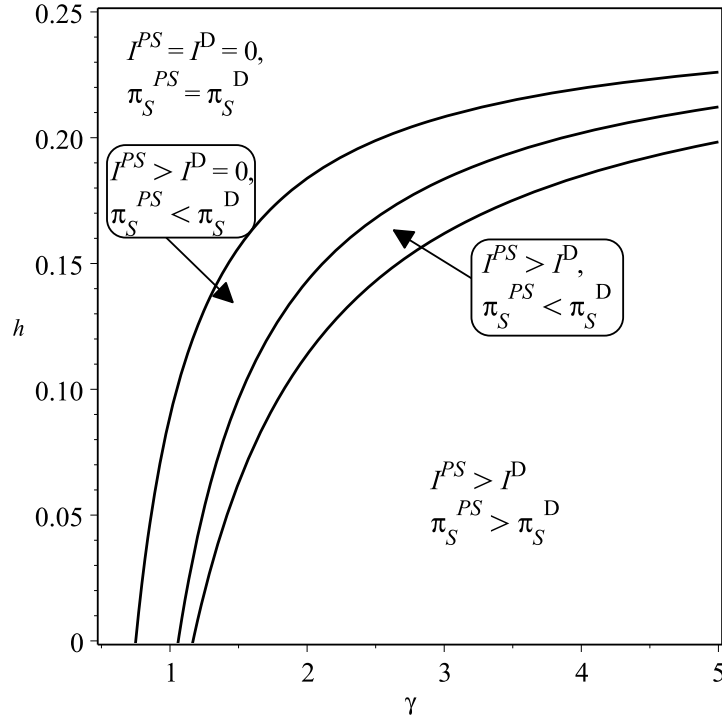


Figure 4: Effects of delayed investment on supplier profits (the superscript D indicates the delayed announcement case).

5.2 Wholesale price-dependent holding cost

Thus far, and consistent with the strategic inventories literature, we have assumed that the holding cost is exogenous. However, it is not unusual to find examples where the holding cost is a function of the cost of the units carried as inventories, and more specifically from the retailer's perspective, as a function of the wholesale price of the good. While one can easily replace the holding cost per unit with a holding cost that is the product of holding cost rate and the wholesale price of the good, a question emerges: will this change the outcome of the model and analysis? Intuitively, if the supplier can affect the retailer's holding cost, then he has another lever to induce the retailer to make decisions that are better aligned with the supplier's objective. At the same time, this may limit the supplier's actions as a higher wholesale price may discourage the retailer from stocking inventory, which may actually harm the supplier (recall, e.g., from §4.1 that the supplier is better off when the retailer carries strategic inventories). We explore the impact of such wholesale price-dependent holding cost in this subsection.¹²

¹²We thank the anonymous referees for suggesting this extension.

We first consider the seminal setting studied by Anand et al. (2008). That is, we seek to explore whether inventory decisions are altered in the absence of process improvement considerations. Recall, in the absence of process improvement, the optimal inventory amounted to $I^S = \frac{5}{34} - \frac{10h}{17}$. Now, when we replace the holding cost with iw_1 and resolve the model, we find that the optimal inventory is given by $I^{Sh} = \frac{(2i+5)(2i-1)}{2(4i^2-4i-17)}$, with superscript h denoting the current case with the revised holding cost. Returning to the base model, we replace h with iw_1 where w_1 is the wholesale price which solves $w_1 = \frac{9-2iw_1}{17}$. This yields $I^S = \frac{5-10i}{34+4i}$. Since the model with percentage holding cost requires $i < 0.5$, we can prove that $I^{Sh} \geq I^S$ with equality holding only for $i \in \{0, 0.5\}$. This is an interesting result suggesting that the supplier induces the retailer to stock a higher level of inventory through the manipulation of the wholesale price as compared with the base model.

In the more general case, when process improvement is present, we have that $I^{PS} = \left[\frac{-80\gamma h + 20\gamma + 24h - 15}{2(68\gamma - 33)} \right]^+$. Similarly, we replace h with iw_1 and solve $w_1 = \frac{4(-2\gamma iw_1 + 9\gamma + 2iw_1 - 4)}{68\gamma - 33}$ for w_1 , ultimately giving rise to $I^{PS} = \left[-\frac{40\gamma i - 20\gamma - 8i + 15}{2(8\gamma i + 68\gamma - 8i - 33)} \right]^+$. When the holding cost is a function of the wholesale price we have $I^{PSh} = \left[\frac{64\gamma^4 i^2 + 128\gamma^4 i - 48\gamma^3 i^2 - 80\gamma^4 - 96\gamma^3 i + 12\gamma^2 i^2 + 144\gamma^3 + 24\gamma^2 i - \gamma i^2 - 76\gamma^2 - 2\gamma i + 15\gamma - 1}{2(64\gamma^4 i^2 - 64\gamma^4 i - 48\gamma^3 i^2 - 272\gamma^4 + 128\gamma^3 i + 12\gamma^2 i^2 + 368\gamma^3 - 68\gamma^2 i - \gamma i^2 - 168\gamma^2 + 14\gamma i + 31\gamma - i - 2)} \right]^+$. We find that the former is lower than I^{PS} for any γ value less than about 7. This means that for reasonable values of the process improvement cost parameter we obtain the opposite result. Namely, the supplier induces the retailer to stock lower quantities of strategic inventories, meaning that the availability of investment in process improvement dramatically alters the interaction between the two supply chain agents. For larger γ values, we find that this reduction of inventory occurs only for a sufficiently low holding cost rate or a sufficiently large holding cost rate, whereas for intermediate values, strategic inventories will marginally increase.

These opposing results regarding the impact of the holding cost rate on strategic inventories raise an important question: do the insights derived in our core analysis still hold? Omitting the analysis, especially as expressions are less tractable, we can show that all qualitative results hold through.

5.3 Strategic Consumers

Thus far we have assumed that consumers behave myopically, in the sense that they only respond to the price they observe upon their arrival. In practice, however, consumers may develop some expectations about future prices, and they may delay their purchase if they expect prices to drop

in the future.¹³

Let us consider the prices realized in the previous section. Careful inspection of the prices leads us to the following conclusion:

Corollary 1. *When the supplier may invest in process improvement and the retailer may stock strategic inventories (case **PS**), $p_1 > p_2$.*

The possibility of lower market prices in the second period raises an opportunity for consumers to behave strategically. Strategic consumers are willing to wait for a later period if they expect the future price to be lower, which is exactly what we see in the **PS** case. For simplicity of exposition, we assume that strategic consumer behavior is characterized by full patience such that these consumers perceive the good purchased in the second period as equally good as that purchased in the first period (see, e.g., Mersereau and Zhang 2012).¹⁴ Hence, if the strategic consumers expect the second period price to be lower than the first period price, they will wait for the second period. However, if they expect the price to increase over time, then they will all purchase immediately and will not wait. In line with Mersereau and Zhang (2012), we further assume that a fraction α of the consumers are strategic.¹⁵

In the presence of strategic consumers, several interesting challenges emerge. Will the decreasing price path from the **PS** case be maintained, or will the supplier and retailer circumvent strategic consumers and will try to encourage them to purchase early? How will the prices change if the

¹³There is an expansive literature that considers the presence of strategic consumers—those consumers who take into account future realizations of prices and act upon their price and product availability expectations. Generally, the literature is in agreement on the detrimental effects induced by the presence of strategic consumers (e.g., for a review of monopoly models in the presence of strategic consumers, see Kremer et al. 2017). Accordingly, numerous contributions have explored methods of counteracting the presence of such consumers, for example, via price commitments or presentation strategies (see the review by Aviv et al. 2009). One such approach that can actually benefit the retailer is proposed by Aviv and Wei (2014), who suggest firms to offer reward mechanisms that incentivize customers to purchase early. According to Li et al. (2014), strategic consumers can be beneficial in the context of airline pricing, as their patience allow the airline to occasionally drop the price thereby segmenting between different consumers types.

¹⁴One can further model strategic consumers as having a lower utility due to waiting. For instance, letting v denote a consumer’s valuation, the immediate utility is given by $v - p_1$ whereas the utility from buying in the second period is discounted by a factor δ due to waiting, for example, since one needs to invest time following the price or the reduced time during which the product can be used. Thus, the delayed utility is given by $\delta(v - p_2)$ and the strategic consumer compares the two utilities upon deciding in the first period whether to buy or to wait.

¹⁵Some argue that consumers can choose whether to behave strategically or myopically, as is the case in the modeling framework of Aflaki et al. (2016). Further, note that since our setting is deterministic, we abstract away from rationing and stock out considerations, see, e.g., Liu and Van Ryzin (2008). More generally, our model in this section is closely related to Shum et al. (2017). We extend their framework in several dimensions. Importantly, we consider a supply chain consisting of a retailer and a supplier, and account for the presence of strategic inventories. In addition, we let the supplier invest in process improvement efforts. As such, our paper is the first to (1) consider the use of process improvement as a tool for the supplier to eliminate strategic inventories, and (2) study the interaction between strategic consumers and strategic inventories.

decreasing path is maintained? Namely, knowing that strategic consumers will wait, will the two prices be farther apart (to increase segmentation) or will they be closer to each other? Further, if prices decrease over time, will the retailer alter the amount of inventories carried over and will the supplier change his investment in process improvement?

To address these challenging questions, let D_{sw} denote the size of the first period cohort that strategically wait until the second period. Note that $D_{sw} \in \{0, \alpha\}$. That is, if consumers expect $p_1 \leq p_2$, then none of them wait and hence $D_{sw} = 0$, whereas if they expect $p_1 > p_2$, then $D_{sw} = \alpha$ as all strategic consumers wait.¹⁶ Thus, the inverse demand function in the second period is given by $p_2 = 1 - \frac{1}{1+D_{sw}}(q_2+I)$, as the base demand in the second period increases by the additional D_{sw} units and the slope of the inverse demand function is adjusted accordingly. The corresponding retailer's profit in the second period is (letting the superscript SS refer to Strategic inventories and Strategic consumers) $\Pi_{R,2}^{SS} = (q_2+I)p_2 - q_2w_2$, which yields $q_2 = \frac{1}{2}(1-w_2)(1+D_{sw}) - I$. The supplier's profit in the second period is $\Pi_{S,2}^{SS} = (w_2+x)q_2 = \frac{w_2+x}{2}((1-w_2)(1+D_{sw}) - I)$, which is maximized for $w_2 = \frac{1-x}{2} - \frac{I}{1+D_{sw}}$. The retailer's total profit is given by $\Pi_R^{SS} = q_1(p_1 - w_1) - I(w_1 + h) + \Pi_{R,2}^{SS}$. Note that due to the strategic waiting of customers, we have $q_1 = (1-p_1)(1-D_{sw})$. The supplier's total profit is $\Pi_S^{SS} = w_1(q_1 + I) + \Pi_{S,2}^{SS} - \frac{1}{2}\gamma x^2$.

Equilibrium outcomes are given in Online Appendix F. The first insight is important as it reveals that the declining pricing path is preserved (i.e., $p_1^{SS} > p_2^{SS}$). That is, the supply chain's members do not discourage strategic waiting and all of the strategic consumers wait for the second period to take advantage of the lower price (see appendix for additional details). The second insight pertains to the behavior of prices. Interestingly, we find that the behavior of the two prices with respect to the proportion of strategic consumers is not monotonic. Specifically, we find that if the holding cost is sufficiently low, then both prices increase in α ; within some intermediary range of h values p_1 increases in α whereas p_2 decreases in α ; and for sufficiently high holding cost p_1 is independent of α while p_2 decrease in α . We elaborate more and demonstrate this behavior in the online appendix.

The presence of waiting consumers has further implications for the decisions and interactions

¹⁶Technically speaking, strategic consumers' decision of whether to buy in the first period or to wait involves their utility. Specifically, a consumer with valuation v compares his utility from the first period, $v - p_1$, with the utility gained from waiting $\delta(v - p_2)$, where δ reflects the discount factor or the consumer patience, which is assumed to equal to 1 (as we limit our attention to perfectly patient strategic consumers). Thus, if $p_1 > p_2$ then $v - p_1 < \delta(v - p_2)$ and all strategic consumers simply wait, as we state in the text.

between the supplier and the retailer. Since these consumers wait for the second period, any investment into process improvement will take effect on a larger volume of consumers, thereby incentivizing the supplier to increase the investment, which, in turn could induce the retailer to stock less inventory, or none at all, as the second period wholesale price will ultimately decrease. Indeed, we find that the parameter space for carrying strategic inventories diminishes as \hat{h}^{SS} —the threshold below which strategic inventories are carried—decreases in α . However, quite importantly, inventories are still carried, and their levels may even be increased. Quite naturally, the supplier increases the investment in process improvement.¹⁷

Proposition 7. *(i) The parameter range where strategic inventories are carried is decreasing in α . That is, $\frac{\partial \hat{h}^{SS}}{\partial \alpha} < 0$. Further, (ii) inventory may decrease in α , increase in α , or both. Also, inventory increases in γ . Finally, (iii) x increases in α when strategic inventories are carried ($h < \hat{h}^{SS}$) as well as when no strategic inventories are carried ($h \geq \hat{h}^{SS}$).*

The behavior of inventory is illustrated in Figure 5. We see that the inventory generally decreases in α , which is as expected as the importance of postponing the retailer’s purchasing decision increases as more consumers will buy in the second period. Yet, it is quite puzzling that inventory may actually increase. This occurs when γ is particularly high, for sufficiently low levels of strategic consumers. Our intuition is that when the process improvement cost parameter is high, then the investment is rather limited and hence the reduction in the wholesale price is quite limited. To ensure reduction in the wholesale price, the retailer ends up increasing the inventory by a small amount to induce the supplier to invest more and reduce the wholesale price. Once there are sufficiently many strategic consumers, the benefit of investing in process improvement is evident and consequently inventory levels drop.

¹⁷We shall only note that the innovation choice increases in the fraction of strategic consumers, with a small drop once the transition occurs from carrying strategic inventories to not carrying them.

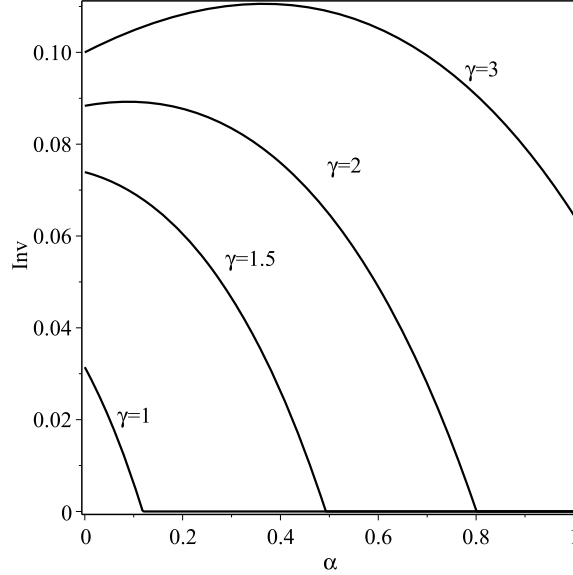


Figure 5: Optimal Inventory as a function of the fraction of strategic consumers; $h = 0.05$

Revisiting Propositions 2 and 3, we now assess the interaction between process improvement and strategic inventories in the presence of strategic consumers. We have the following statement, which reveals that, qualitatively, the nature of the interaction between the two factors does not alter when $\alpha > 0$.

Proposition 8. (i) process improvement suppresses strategic inventories if $\alpha > 0$ (i.e., $I^{SS}|_{x=0} > I^{SS}|_{x>0}$). (ii) For $\hat{h}^{SS} > h > \bar{h} \equiv -\frac{(\alpha^2+4\alpha(\gamma+6)-28\gamma+23)}{4(\alpha-7)(\alpha-4\gamma+1)}$ strategic inventories stimulate process improvement (i.e., $x^{SS}|_{ISS>0} > x^{SS}|_{ISS=0}$), whereas if $h < \bar{h}$, strategic inventories suppress process improvement (i.e., $x^{SS}|_{ISS>0} < x^{SS}|_{ISS=0}$).

Lastly, we note that the profits of both supply chain partners strictly increase in the presence of strategic consumers. This is driven by the fact that the patience exhibited by consumers allow the supplier to commit to a larger investment in process improvement to take advantage of the larger volume of consumers that will visit the retailer in the second period. By further reducing the unit cost of the good, larger gains can be realized. The dynamics of the interaction between the retailer and the supplier persist, but to a lesser degree as, in general, the retailer stocks lower levels of inventory in expectation of lower future wholesale price.

5.4 Longer Horizons

One of the modeling assumptions that can be challenged relates to the fact that we encompass both the inventory stocking decision as well as investment in R&D within a single framework. More specifically, that we allocate the same “weight” to the inventory holding decision, which may be perceived as a short-term decision, and R&D investment, which may be perceived as a long-term decision. While it may be true that these two decisions are generally made on different time scales, our perspective, as in Anand et al. (2008) relates to the strategic aspect of inventory stocking decisions. Namely, the amount of inventory that a retailer needs to be carried over from one planning period to the next when decisions of strategic magnitude are carried out by the supplier. It is natural that a supplier, as in Anand et al. (2008), does not change his wholesale price before every order is made by the retailer, and to the same degree, the inventory decision in our model is not at the operational level, rather, it reflects the amount to be carried over when major changes occur. This is very much in line with papers such as Arya et al. (2014), who focuses on the interplay between strategic inventories and the decision of a multi-divisional buying firm to centralize or decentralize buying activities. Such organization structure decisions are likely to take place on similar time scales as the one central to our paper.

Yet, to consider the interplay between short and long term decisions, assume each period in our setting is composed of two (or more) sub-periods. Thus, at the beginning of the first period investment in R&D takes place followed by the sub-periods in which the original and constant production cost holds. At the final sub-period of the first period, the retailer takes a strategic inventory decision—how many units to hold before a new production cost takes effect. Then the players enter the second period, the unit cost is realized and the operational interaction between the two agents persists. Regardless of the sub-period to sub-period interplay between the supplier and the retailer, our paper captures the key trade-off between the two strategic decisions in this setting: the supplier’s investment in R&D at the beginning of the first period, and the retailer’s inventory decision before the second period.

What happens between the sub-periods? It depends on the assumption relating to the wholesale price. If the wholesale price is the same in each sub-period of the same period, then pricing during this period is essentially according to a commitment contract and hence no inventory is carried

by the retailer between sub-periods (see Anand et al., 2008). However, if the wholesale price is re-announced in each of the sub-periods, then we essentially resort to the basic model of Anand et al. (2008) with the results directly applying to this setting.

To conclude, our framework focuses on the strategic component of inventories while truly applying to the strategic nature of R&D investment, even above and beyond that captured by traditional R&D models. For instance, in the seminal paper by d'Aspremont and Jacquemin (1988), R&D decisions are made in the first stage immediately followed by capacity decisions, whereas our framework is more consistent with reality where there is a delay between the R&D decision (or investment) and the timing during which the effects implied by this investment take place.

6 Discussion

Recognizing the strategic role of firms' sourcing processes, managers must be aware of how to affect their supply chain partner's decisions. In supplier-retailer supply chains, retailers may stock inventories strategically to induce the supplier to reduce their products' wholesale price at a later point in time. Specifically, as the strategic inventories allow the retailer to source either from his own stock or from the supplier, the supplier competes against his own products, forcing him to reduce the wholesale price. Although previous research has shown that such strategic inventories can improve supply chain performance (Anand et al., 2008), the supplier may wonder how to dampen such unwanted competition and consider other options to maintain high selling prices over stretched periods of time. In this paper, we focus on the effect of the supplier's process improvement investments, which reduce the unit production cost and allow the supplier to profitably reduce wholesale prices. Accordingly, we explore whether investments in process improvement can eliminate strategic inventories in the supply chain.

Our analysis reveals several important insights. First and foremost, we find that process improvement suppresses and can even completely eliminate strategic inventories. This is a new result that sheds light on the interaction between the two supply chain partners and reflects the strategic interplay between these two factors. Importantly, strategic inventories may stimulate process improvement when the holding cost is sufficiently high, and suppress it otherwise. This may relate to the retailer's incentive to stock strategic inventories. Are these strategic inventories profitable

in the context of process improvement? We find that they may hurt both the retailer (which is a natural result in light of Anand et al., 2008) as well as the supplier. The latter result is more surprising and can be attributed to the reversal of the contract-space-expansion effect due to the reduced stocking of strategic inventories.

In addition, we have explored several extensions to the model. Explicitly, we have studied the supplier's incentive to delay his investment announcement, revealing that he may be worse off (in particular when the improvement cost parameter, γ , is sufficiently high, implying a high cost for process improvement); and we have considered the impact induced by the presence of (perfectly) strategic consumers, suggesting that both the retailer and the manufacturer are better off as they can gravitate demand to a later period while taking advantage of the reduced cost. We have also discussed the robustness of the model. In particular, we find that, qualitatively, all results still hold if the holding cost is endogenous (i.e., wholesale price-dependent) rather than exogenous, and we have outlined the implications of longer horizons.

Managerially, our work stresses the importance of cost-reducing process improvement, especially in the presence of strategic inventories carried by the retailer. Both partners need to account for the interplay between the factors influencing their decision making and properly foresee the strategic response of their counterpart. Our work highlights a novel and intricate strategic interaction while abstracting away from several aspects that may prevail in practice. For instance, the interaction between supplier and retailer may be governed by a Stackelberg (leader-follower) type of setting, or the two partners may bargain over the magnitude of the process improvement investment and prices. Further, the supplier may be limited in his manufacturing capacity whereas the retailer may have a limited storage capacity. Finally, the supplier may have private information about his cost structure (including his planned process improvement efforts or the result of process improvement projects) and could consider whether or not to share this information truthfully with the retailer while, at the same time, the retailer may possess private information about sales and inventory levels (as is the case in Roy et al., 2018). Such decisions, which depend on many factors (e.g., the value of the holding cost as in Roy et al., 2018, whether the supplier offers a menu of contracts, and the prevailing contract mechanism), are left for future exploration.

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Managing Strategic Inventories under Investment in Process Improvement—Supplementary Materials (Online Appendix)

A Model with market size a and constant cost of production c

In case **P**, the sufficient second-order conditions in the first stage of the first period yield $\gamma > \frac{1}{4}$. In equilibrium we have $q_1^* = \frac{a-c}{4}$ and $q_2^* = \frac{(a-c)\gamma}{4\gamma-1}$. Clearly we need the condition $a > c$ for quantities to be strictly positive, which is an assumption frequently encountered in the literature. Furthermore we have $x^* = \frac{a-c}{4\gamma-1}$, which is strictly positive given the assumptions described. An additional restriction is that unit cost cannot be negative. That is, $c - x^* \geq 0$. Writing out gives $4\gamma c - a \geq 0$. In other words, γ and c should be large enough, while a should be small enough. Moreover, given that $\gamma > \frac{1}{4}$ there always exist parameter values for a and c that ensure the condition $4\gamma c - a \geq 0$ holds. This condition also ensures $w_2^* = \frac{2\gamma(a+c)-a}{4\gamma-1}$ is strictly positive, while $w_1^* = \frac{a+c}{2}$ is strictly positive always.

In case **S**, which is the case derived from Anand et al. (2008), we can easily obtain expressions as functions of a and c . Following a previous line of reasoning again yields the condition $a > c$. The limit on the holding cost parameter for strictly positive inventory levels changes to $h < \frac{a-c}{4}$, showing how the parameter range for carrying inventories changes with a and c . It is not hard to see that all equilibrium outcomes are strictly positive, i.e., $q_1^* = \frac{1}{17}(4a - 4c + h) > 0$, $q_2^* = \frac{1}{17}(3a - 3c + 5h) > 0$, $w_1^* = \frac{1}{17}(9a + 8c - 2h) > 0$, $w_2^* = \frac{1}{17}(6a + 11c + 10h) > 0$.

In case **PS**, we again have $a > c$ and can show with some manipulations that all equilibrium outcomes are positive, regardless of the values of a , c , i.e., $q_1^* = \frac{8\gamma(4a-4c+h)-17a+17c-8h}{136\gamma-66} > 0$, $q_2^* = \frac{4\gamma(3a-3c+5h)-2a+2c+h}{68\gamma-33} > 0$, $x^* = \frac{2(5a-5c+14h)}{68\gamma-33} > 0$, $w_1^* = \frac{4a(9\gamma-4)+c(32\gamma-17)-8(\gamma-1)h}{68\gamma-33} > 0$, $w_2^* = \frac{-24a\gamma+14a-44c\gamma+19c-40\gamma h+26h}{33-68\gamma} > 0$. From the condition $c - x^* > 0$ we have $\gamma > \frac{10a+23c+28h}{68c}$, which is stricter than the well-known condition in this case, given by $\gamma > \frac{33}{68}$. Strategic inventories are positive if $\gamma > \frac{15(a-c)-24h}{20(a-c)-80h}$, which is stricter than the condition $\gamma > \frac{10a+23c+28h}{68c}$ for some parameter values. We can rewrite $\gamma > \frac{15(a-c)-24h}{20(a-c)-80h}$ to $h < \frac{5(a-c)(4\gamma-3)}{80\gamma-24}$, showing that a and c affect the upper limit on the holding cost in a straightforward manner.

B Relevant conditions

In the case with strategic inventories (case **S**), we have $\frac{\partial^2 \Pi_{R,2}}{\partial q_2^2} < 0$, $\frac{\partial^2 \Pi_{S,2}}{\partial w_2^2} < 0$, $\frac{\partial^2 \Pi_R}{\partial I^2} < 0$, $\frac{\partial^2 \Pi_R}{\partial q_1^2} < 0$ and $\frac{\partial^2 \Pi_S}{\partial w_1^2} < 0$. In the second stage (where the retailer determines q_1 and I), the determinant of the Hessian matrix is strictly positive to guarantee negative definite solutions. In terms of positivity of the equilibrium solutions, it is straightforward to verify that all solutions are strictly positive if $I > 0$, which is the case if $h > \frac{1}{4}$.

In the case with process improvement (case **P**), we have $\frac{\partial^2 \Pi_{R,2}}{\partial q_2^2} < 0$, $\frac{\partial^2 \Pi_{S,2}}{\partial w_2^2} < 0$, $\frac{\partial^2 \Pi_R}{\partial q_1^2} < 0$, $\frac{\partial^2 \Pi_S}{\partial x^2} = \frac{1}{4} - \gamma$ and $\frac{\partial^2 \Pi_S}{\partial w_1^2} < 0$. In the first stage (where the supplier determines x and w_1), the determinant of the Hessian matrix is strictly positive if $\gamma > \frac{1}{4}$, which ensures $\frac{\partial^2 \Pi_S}{\partial x^2} < 0$ as well. Inspecting positivity of the outcomes we have $w_2 > 0$ if $\gamma > \frac{1}{2}$. This condition ensures that all other solutions are strictly positive.

In the case with process improvement and strategic inventories (case **PS**), we have $\frac{\partial^2 \Pi_{R,2}}{\partial q_2^2} < 0$, $\frac{\partial^2 \Pi_{S,2}}{\partial w_2^2} < 0$, $\frac{\partial^2 \Pi_R}{\partial I^2} < 0$, $\frac{\partial^2 \Pi_R}{\partial q_1^2} < 0$, $\frac{\partial^2 \Pi_S}{\partial x^2} = \frac{4}{9} - \gamma$ and $\frac{\partial^2 \Pi_S}{\partial w_1^2} < 0$. In the second stage (where the retailer determines q_1 and I), the determinant of the Hessian matrix is always strictly positive. In the first stage (where the supplier determines x and w_1), the determinant of the Hessian matrix is strictly positive if $\gamma > \frac{33}{68}$, which also ensures $\frac{\partial^2 \Pi_S}{\partial x^2} < 0$. Strategic inventories are positive if $\gamma > \frac{3(5-8h)}{20(1-4h)}$. This condition ensures that $\gamma > \frac{33}{68}$ for $0 < h < \frac{1}{4}$, and ensures that all other equilibrium solutions are strictly positive.

We continue with the case with process improvement, strategic inventories and strategic consumers (case **SS**). We have $\frac{\partial^2 \Pi_{R,2}}{\partial q_2^2} < 0$, $\frac{\partial^2 \Pi_{S,2}}{\partial w_2^2} < 0$, $\frac{\partial^2 \Pi_R}{\partial I^2} < 0$, $\frac{\partial^2 \Pi_R}{\partial q_1^2} < 0$, $\frac{\partial^2 \Pi_S}{\partial x^2} = \frac{4(1+\alpha)}{9} - \gamma$ and $\frac{\partial^2 \Pi_S}{\partial w_1^2} < 0$. In the second stage (where the retailer determines q_1 and I), the determinant of the Hessian matrix is always strictly positive. In the first stage (where the supplier determines x and w_1), the determinant of the Hessian matrix is strictly positive if $\gamma > \bar{\gamma} \equiv \frac{\alpha^2 + 34\alpha + 33}{4(17-\alpha)}$, which also ensures $\frac{\partial^2 \Pi_S}{\partial x^2} < 0$. Note that $\bar{\gamma} = \frac{33}{68}$ if $\alpha = 0$ and that $\bar{\gamma}$ is increasing in α . Strategic inventories are positive if $h < \hat{h}^{SS} \equiv \frac{1}{4} - \frac{3(\alpha+1)(\alpha+3)}{8((10-2\alpha)\gamma + (\alpha-3)(\alpha+1))}$. Rewriting this to $\gamma > \hat{\gamma} \equiv \frac{(1+\alpha)(15-8h(3-\alpha)+\alpha)}{4(1-4h)(5-\alpha)}$, we observe that always $\hat{\gamma} > \bar{\gamma}$ and that all other equilibrium solutions are strictly positive if $\gamma > \hat{\gamma}$.

When strategic inventories are zero, all relevant sufficient second-order conditions are satisfied if $\gamma > \frac{1+\alpha}{4}$ and all equilibrium solutions are positive if $\gamma > \frac{1+\alpha}{2}$.

C Proofs

Proof of Proposition 1. We solve the model using backwards induction. The retailer's second period profit is given by $\Pi_{R,2} = (q_2 + I)(1 - q_2 - I) - q_2 w_2$, and the profit maximizing order quantity is $q_2^* = \frac{1}{2}(1 - w_2) - I$. The supplier's second period profit is given by $\Pi_{S,2} = q_2(w_2 + x)$ which is maximized at $w_2^* = \frac{1}{2}(1 - x) - I$. Proceeding with the first period, we maximize the retailer's total profit, $\Pi_R = -\frac{3}{4}I^2 + (-h - \frac{1}{4}x - w_1 + \frac{3}{4})I - q_1^2 - w_1 q_1 + q_1 + \frac{(x+1)^2}{16}$, subject to $q > 0$ and $I \geq 0$. Thus, we have the following Lagrangian: $L_R(q, I) = \Pi_R + \lambda_{R1}q + \lambda_{R2}I$, which yields two solutions.

When the inventory constraint is not binding, we have $I = \frac{1}{2} - \frac{x}{6} - \frac{2(w_1+h)}{3}$ and $q_1 = \frac{1-w_1}{2}$, which is positive as long as $w_1 < 1$. It worth noting that $dI/dx < 0$, that is, the incentive to stock inventories decreases in the investment in process improvement. Given these retailer's decisions, we maximize the supplier's profit, $\Pi_S = -\frac{17}{18}w_1^2 + \frac{(4h+5x+18)w_1}{18} + \frac{(-9\gamma+4)x^2}{18} - \frac{4xh}{9} + \frac{2h^2}{9}$ subject to $w_1 > 0$ and $x \geq 0$. Similarly, we solve the supplier's Lagrangian: $L_S(q, I) = \Pi_S + \lambda_{S1}x + \lambda_{S2}w_1$, which yields two solutions. When x is not binding, we have $x = \frac{2(5+14h)}{68\gamma-33}$ and $w_1 = \frac{4(-2\gamma h+9\gamma+2h-4)}{68\gamma-33}$. We observe that x is never binding, as $\gamma > \frac{1}{2}$ and $0 < h < \frac{1}{4}$ implying $x > 0$.

When the inventory constraint is binding, which occurs when $h > \frac{5(4\gamma-3)}{8(10\gamma-3)}$, we have $I = 0$ and $q = \frac{1-w_1}{2}$. Solving the supplier's problem, we have $w_1 = \frac{1}{2}$ and $x = \frac{1}{4\gamma-1}$ when the x constraint is not binding. Again, we observe that x is never binding, as $\gamma > \frac{1}{2}$ implying $x > 0$.

Plugging these optimal values in I , w_2 , p_1 , p_2 , and profit functions, gives rise to the expressions in the proposition. □

Proof of Proposition 2. Equating strategic inventory levels in the case with and without process improvement and solving yields only solutions in the space with negative h . We will suppress any additional details. □

Proof of Proposition 3. Equating process improvement levels in the case with and without strategic inventories yields the solution $h = \frac{28\gamma-23}{28(4\gamma-1)}$. Comparing the solution with the upper boundary for positive strategic inventories $h = \frac{5(4\gamma-3)}{8(10\gamma-3)}$, we have $\frac{28\gamma-23}{28(4\gamma-1)} < \frac{5(4\gamma-3)}{8(10\gamma-3)}$ for any $\gamma > \frac{1}{2}$. Some numerical checks in the areas where $h < \frac{28\gamma-23}{28(4\gamma-1)}$ and $\frac{28\gamma-23}{28(4\gamma-1)} < h < \frac{5(4\gamma-3)}{8(10\gamma-3)}$ suffice to complete the proof. □

Proof of Proposition 4. Equating supplier/retailer profits in the case with process improvement

(with non-zero strategic inventories)—as given in (7) and (8)— and without process improvement (with non-zero strategic inventories)—as given in (1) and (2)—and solving yields only solutions in the space with negative h . We will suppress any additional details.

Note that in the area $\frac{5(4\gamma-3)}{8(10\gamma-3)} < h < \frac{1}{4}$ inventories are carried in case **S** but not in case **PS**. This implies that firm profits in case **PS** are equal to the profits in case **P**. Equating firm profits in case **S** with the profits in case **P** we see that the main finding, which is that process improvement benefits the supply chain agents, still holds if inventories are not carried in case **PS**. \square

Proof of Proposition 5. Consider the supplier's profit. Comparing the profit when $h < \frac{5(4\gamma-3)}{8(10\gamma-3)}$ with the profit when no strategic inventories are carried gives rise to the threshold $h_{1,2} = \frac{32\gamma^2 - 40\gamma + 8 \pm \sqrt{272\gamma^2 - 200\gamma + 33}}{4(32\gamma^2 - 12\gamma + 1)}$. We can show that h_1 always exceeds the threshold $h = \frac{5(4\gamma-3)}{8(10\gamma-3)}$ in the area with non-negative h . Thus, we have that when $\frac{5(4\gamma-3)}{8(10\gamma-3)} > h > h_2$ the supplier is worse off when the retailer carries strategic inventories. If $h < h_2$, the supplier is better off with strategic inventories. Note that this condition is only relevant if $\gamma > \frac{17+\sqrt{41}}{16} \approx 1.46$.

Consider the retailer's profit. Comparing the profit when $h < \frac{5(4\gamma-3)}{8(10\gamma-3)}$ with the profit when no strategic inventories are carried gives rise to the threshold $h_{1,2} = \frac{3776\gamma^3 - 6320\gamma^2 + 2524\gamma - 295 \pm \sqrt{X}}{4(1216\gamma^2 - 752\gamma + 173)(4\gamma-1)}$, with $X = 1183744\gamma^6 + 17790976\gamma^5 - 34972416\gamma^4 + 27232384\gamma^3 - 10748192\gamma^2 + 2054712\gamma - 132858$. As before, we have $h_1 > \frac{5(4\gamma-3)}{8(10\gamma-3)} > h_2$. \square

Proof of Proposition 6. In this case, all sufficient second-order conditions and positivity conditions are satisfied if $\gamma > 0.631$.

Recall that in the **PS** case we need $h < \frac{5(4\gamma-3)}{8(10\gamma-3)}$ for strictly positive strategic inventories. Define $\hat{h}^D = \frac{2(80\gamma^4 - 144\gamma^3 + 76\gamma^2 - 15\gamma + 1)}{(4\gamma-1)^2(40\gamma^2 - 24\gamma + 3)}$. For $h < \hat{h}^D$ we have strictly positive strategic inventories in the delayed investment case. Clearly $\hat{h}^D < \frac{5(4\gamma-3)}{8(10\gamma-3)}$ for every $\gamma > \frac{1}{2}$.

Solving $I^{PS} = I^D$ yields $h = -\frac{2(992\gamma^4 - 1040\gamma^3 + 388\gamma^2 - 58\gamma + 3)}{2944\gamma^4 - 2272\gamma^3 + 520\gamma^2 - 20\gamma - 3}$. The right-hand side of this function is negative for any $\gamma > \frac{1}{2}$, showing that the solution to $I^{PS} = I^D$ does not switch sign in the area where $0 < h < \hat{h}^D$. A numerical check suffices to show that in that area, $I^{PS} > I^D$.

For parameter values $\hat{h}^D < h < \frac{5(4\gamma-3)}{8(10\gamma-3)}$ no strategic inventories are carried in the delayed investment case, while in the **PS** case strictly positive inventories are carried. Solving $x^{PS} = x^D$ in this area yields $h = \frac{28\gamma-23}{28(4\gamma-1)}$, which is strictly in between \hat{h}^D and $\frac{5(4\gamma-3)}{8(10\gamma-3)}$. For $\hat{h}^D < h < \frac{28\gamma-23}{28(4\gamma-1)}$ we have $x^D > x^{PS}$, while for $\frac{28\gamma-23}{28(4\gamma-1)} < h < \frac{5(4\gamma-3)}{8(10\gamma-3)}$, $x^{PS} > x^D$.

Solving $x^{PS} = x^D$ yields $h = \frac{2(544\gamma^4 - 896\gamma^3 + 588\gamma^2 - 154\gamma + 13)}{4352\gamma^4 - 6080\gamma^3 + 2472\gamma^2 - 344\gamma + 13}$. The right-hand side of this function is larger than \hat{h}^D for any $\gamma > 0.631$, showing that the solution to $x^{PS} = x^D$ does not switch sign in the area where $0 < h < \hat{h}^D$. A numerical check suffices to show that in that area, $x^D > x^{PS}$.

Solving $p_1^{PS} = p_1^D$ and $w_1^{PS} = w_1^D$ yields $h = \frac{2(208\gamma^4 - 384\gamma^3 + 188\gamma^2 - 28\gamma + 1)}{576\gamma^4 - 272\gamma^3 + 12\gamma^2 - 2\gamma + 1}$. The right-hand side of this function is below \hat{h}^D for $1.06035 < \gamma < 1.30902$. In this area $p_1^D > p_1^{PS}$ and $w_1^D > w_1^{PS}$. Otherwise, $p_1^{PS} > p_1^D$ and $w_1^{PS} > w_1^D$.

Solving $p_2^{PS} = p_2^D$ and $w_2^{PS} = w_2^D$ yields $h = -\frac{2(224\gamma^4 - 72\gamma^3 - 100\gamma^2 + 48\gamma - 5)}{3648\gamma^4 - 4176\gamma^3 + 1496\gamma^2 - 182\gamma + 5}$. The right-hand side of this function does not cross the area where $0 < h < \hat{h}^D$. A numerical check suffices to show that in that area, $p_2^D > p_2^{PS}$ and $w_2^D > w_2^{PS}$. \square

Proof of Corollary 1. When $h \geq \frac{5(4\gamma-3)}{8(10\gamma-3)}$, the first period price exceeds that of the second period since always $\frac{3\gamma-1}{4\gamma-1} < \frac{3}{4}$. When $h < \frac{5(4\gamma-3)}{8(10\gamma-3)}$, the first period price exceeds that of the second period when $h < \frac{6\gamma-1}{24\gamma-17}$. Notice that the condition $h < \frac{5(4\gamma-3)}{8(10\gamma-3)}$ is relevant only when $\gamma > \frac{3}{4}$, and that $\frac{6\gamma-1}{24\gamma-17} > \frac{1}{4}$ for any $\gamma > \frac{3}{4}$. This implies that whenever strategic inventories are carried, $p_1 > p_2$. \square

Proof of Proposition 7. (i) Differentiating the threshold, $\frac{\alpha^2 + 4\alpha\gamma + 16\alpha - 20\gamma + 15}{8(3 - \alpha^2 + 2\alpha\gamma + 2\alpha - 10\gamma)}$, with respect to α gives $\frac{3(\alpha^2\gamma + 3\alpha^2 - 10\alpha\gamma + 6\alpha - 23\gamma + 3)}{4(\alpha^2 - 2\alpha\gamma - 2\alpha + 10\gamma - 3)^2}$, which is negative when $\frac{5\gamma - 3 - 2\sqrt{12\gamma^2 + 9\gamma}}{\gamma + 3} < \alpha < \frac{5\gamma - 3 + 2\sqrt{12\gamma^2 + 9\gamma}}{\gamma + 3}$. Note that $\frac{5\gamma - 3 - 2\sqrt{12\gamma^2 + 9\gamma}}{\gamma + 3}$ is negative for every $\gamma > 0$ and that $\frac{5\gamma - 3 + 2\sqrt{12\gamma^2 + 9\gamma}}{\gamma + 3} > 1$ for every $\gamma > \frac{33}{68}$. Hence, the threshold is decreasing in α .

(ii) We only need to consider the case of $h < \hat{h}^{SS}$. Solving $\frac{\partial I}{\partial \alpha} = 0$ yields two solutions for γ , expressed in terms of α and h . Expressions are large and therefore omitted. It is straightforward to show that the first solution has no relevant intersection with the lower boundary of the feasible parameter region (given by γ as functions of α and \hat{h}^{SS}), so that a numerical check suffices to show that this solution is not within the feasible parameter region. The second solution also has no relevant intersection with the lower boundary of the feasible parameter region, but lies strictly above it. This suggests that, given a γ value, $\frac{\partial I}{\partial \alpha}$ has either no or one sign change. Some numerical samples show that for low γ values (e.g., $\gamma = 1$) we have $\frac{\partial I}{\partial \alpha} < 0$, whereas for larger γ values (e.g., $\gamma = 3$) $\frac{\partial I}{\partial \alpha}$ shifts from increasing to decreasing in the α direction. For even larger γ values and low h values (e.g., $\gamma = 7$, $h = 0.05$), we observe that $\frac{\partial I}{\partial \alpha} > 0$.

Solving $\frac{\partial I}{\partial \gamma} = 0$ we only need to consider the case of $h < \hat{h}^{SS}$. It is straightforward to show that inventory increases in γ since $\frac{\partial I}{\partial \gamma} = \frac{12(\alpha+1)^2(\alpha+3)(2(7-\alpha)h+5)}{(\alpha^2+4\alpha\gamma+34\alpha-68\gamma+33)^2} > 0$.

(iii) When no inventories are carried, $\frac{\partial x}{\partial \alpha} = \frac{4\gamma}{(\alpha-4\gamma+1)^2} > 0$. When inventories are carried, $\frac{\partial x}{\partial \alpha} = \frac{2(8\alpha^2\gamma h+80\alpha^2h-272\alpha\gamma h+5\alpha^2+160\alpha h+872\gamma h+10\alpha+360\gamma+80h+5)}{(\alpha^2+4\alpha\gamma+34\alpha-68\gamma+33)^2}$. This expression is positive when $h = 0$, since $\frac{\partial x}{\partial \alpha}|_{h=0} = \frac{2(5\alpha^2+10\alpha+360\gamma+5)}{(\alpha^2+4\alpha\gamma+34\alpha-68\gamma+33)^2} > 0$. We will show that $\frac{\partial x}{\partial \alpha}$ is increasing in h . $\frac{\partial x^2}{\partial \alpha h} = \frac{16(\alpha^2\gamma+10\alpha^2-34\alpha\gamma+20\alpha+109\gamma+10)}{(\alpha^2+4\alpha\gamma+34\alpha-68\gamma+33)^2} > 0$ since the denominator is linearly increasing in γ and strictly positive when $\gamma = \frac{33}{68}$. \square

Proof of Proposition 8. process improvement suppresses strategic inventories if $\alpha > 0$. Does strategic inventories suppress process improvement? Equating the two process improvement levels and solving yields the solution $\bar{h} = -\frac{(\alpha^2+4\alpha(\gamma+6)-28\gamma+23)}{4(\alpha-7)(\alpha-4\gamma+1)}$, which is strictly below \hat{h}^{SS} . For $\hat{h}^{SS} > h > \bar{h}$ strategic inventories stimulate process improvement. For $h < \bar{h}$, strategic inventories suppress process improvement. Thus the results given in Propositions 3 and 2 do not qualitatively change if $\alpha > 0$. \square

D Illustrating the range of Omega

Figure 6 demonstrates the region Ω , i.e., the range of h and γ values under which a transition from case N to case PS is harmful to the retailer. This region is defined as the range of h values such that $\underline{h} < h < \bar{h}$, where \underline{h} and \bar{h} are functions of γ , with \underline{h} (resp., \bar{h}) monotonically decreasing (resp., increasing) in γ and converging to 0.138 (resp., 0.25)—the corresponding values from Anand et al. (2008). We shall note that \underline{h} and \bar{h} are feasible only when $\gamma > 21.351$.

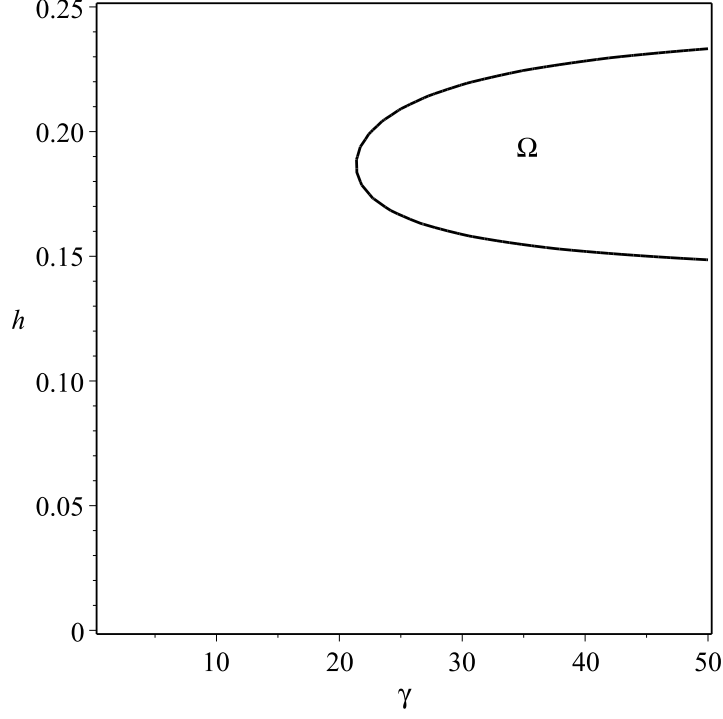


Figure 6: Range of Ω

E Delayed Investment Announcement

When the supplier delays the process improvement investment announcement to period 2, then in equilibrium

$$I^D = \begin{cases} \frac{2-640\gamma^4 h+160\gamma^4+704\gamma^3 h-288\gamma^3-280\gamma^2 h+152\gamma^2+48\gamma h-30\gamma-3h}{4(272\gamma^4-368\gamma^3+168\gamma^2-31\gamma+2)} & h < \hat{h}^D, \\ 0 & \text{otherwise,} \end{cases} \quad (9)$$

$$x^D = \begin{cases} \frac{(4\gamma-1)(40\gamma^2 h+24\gamma^2-24\gamma h-16\gamma+3h+2)}{2(272\gamma^4-368\gamma^3+168\gamma^2-31\gamma+2)} & h < \hat{h}^D, \\ \frac{1}{4\gamma-1} & \text{otherwise,} \end{cases} \quad (10)$$

$$[w_1^D, w_2^D] = \begin{cases} \left[\frac{2-32\gamma^4(2h-9)+128\gamma^3(h-3)-4\gamma^2(17h-44)+2\gamma(7h-16)+h}{2(272\gamma^4-368\gamma^3+168\gamma^2-31\gamma+2)}, \frac{(4\gamma-1)(2\gamma-1)(8\gamma^2(5h+3)-8\gamma(3h+2)+3h+2)}{2(272\gamma^4-368\gamma^3+168\gamma^2-31\gamma+2)} \right] & h < \hat{h}^D, \\ \left[\frac{1}{2}, \frac{2\gamma-1}{4\gamma-1} \right] & \text{otherwise,} \end{cases} \quad (11)$$

$$[p_1^D, p_2^D] = \begin{cases} \left[\frac{6-64\gamma^4(h-13)+32\gamma^3(4h-35)-4\gamma^2(17h-128)+14\gamma h-94\gamma-h}{4(272\gamma^4-368\gamma^3+168\gamma^2-31\gamma+2)}, \right. \\ \left. \frac{\gamma^4(320h+736)+\gamma^3(1008-432h)+\gamma^2(208h+472)-\gamma(42h+90)+3(h+2)}{4(272\gamma^4-368\gamma^3+168\gamma^2-31\gamma+2)} \right] & h < \hat{h}^D, \\ \left[\frac{3}{4}, \frac{3\gamma-1}{4\gamma-1} \right] & \text{otherwise,} \end{cases} \quad (12)$$

where $\hat{h}^D \equiv \frac{2(80\gamma^4-144\gamma^3+76\gamma^2-15\gamma+1)}{(40\gamma^2-24\gamma+3)(4\gamma-1)^2}$. The resulting profits of the retailer and the supplier are, respectively,

$$\Pi_S^D = \begin{cases} \frac{(8\gamma-1)(4\gamma-1)^3 h^2 - 4h(4\gamma-1)^2(4\gamma^2-6\gamma+1) + 4(12\gamma^2-8\gamma+1)^2}{8(272\gamma^4-368\gamma^3+168\gamma^2-31\gamma+2)^2} & h < \hat{h}^D, \\ \frac{8\gamma-1}{8(4\gamma-1)} & \text{otherwise,} \end{cases} \quad (13)$$

and

$$\Pi_R^D = \begin{cases} \frac{\Delta_1 h^2 - \Delta_2 h + \Delta_3}{8(272\gamma^4-368\gamma^3+168\gamma^2-31\gamma+2)^2} & h < \hat{h}^D, \\ \frac{32\gamma^2-8\gamma+1}{16(4\gamma-1)^2} & \text{otherwise,} \end{cases} \quad (14)$$

where

$$\Delta_1 = (9728\gamma^6 - 18368\gamma^5 + 13928\gamma^4 - 5384\gamma^3 + 1116\gamma^2 - 118\gamma + 5)(1 - 4\gamma)^2,$$

$$\Delta_2 = 4(15104\gamma^8 - 45056\gamma^7 + 54560\gamma^6 - 35440\gamma^5 + 13552\gamma^4 - 3132\gamma^3 + 429\gamma^2 - 32\gamma + 1), \text{ and}$$

$$\Delta_3 = 4(19840\gamma^8 - 51968\gamma^7 + 56928\gamma^6 - 34176\gamma^5 + 12392\gamma^4 - 2800\gamma^3 + 387\gamma^2 - 30\gamma + 1).$$

F Strategic Consumers

We first derive the equilibrium outcomes. Relying on Corollary 1, we assume that in the presence of strategic consumers, we also have $p_1 > p_2$. Below we show that this assumption is satisfied and hence constitutes an equilibrium.

Solving this model backwards, yields the threshold $\hat{h} = \frac{\alpha^2+4\alpha\gamma+16\alpha-20\gamma+15}{8(3-\alpha^2+2\alpha\gamma+2\alpha-10\gamma)}$ below which strategic inventories are carried, and none otherwise. Accordingly, we have process improvement and prices as prescribed by (16) and (18), respectively. The expressions of the order quantities are omitted due to their length.

We next verify that the assumption that $p_1 > p_2$ is satisfied. When $h \geq \frac{\alpha^2+4\alpha\gamma+16\alpha-20\gamma+15}{8(3-\alpha^2+2\alpha\gamma+2\alpha-10\gamma)}$ we have that $[p_1, p_2] = \left[\frac{3}{4}, \frac{\alpha-3\gamma+1}{\alpha-4\gamma+1} \right]$. Since p_2 is decreasing in α (because $\frac{\partial p_2}{\partial \alpha} = \frac{\gamma}{(\alpha-4\gamma+1)^2}$), we estimate p_2 when it obtains the highest values, that is, when $\alpha = 0$. Since $p_2|_{\alpha=0} = \frac{3\gamma-1}{4\gamma-1} < \frac{3}{4}$ for

every $\gamma < \infty$, we conclude that $p_1 < p_2$. When $h < \frac{\alpha^2+4\alpha\gamma+16\alpha-20\gamma+15}{8(3-\alpha^2+2\alpha\gamma+2\alpha-10\gamma)}$ we have that $p_1 - p_2 = \frac{(-\alpha^2-18\alpha+24\gamma-17)h+\alpha-6\gamma+1}{\alpha^2+4\alpha\gamma+34\alpha-68\gamma+33}$, which is decreasing in γ since $\frac{\partial(p_1-p_2)}{\partial\gamma} = \frac{2(\alpha+13)(1+\alpha)(2\alpha h-14h-5)}{(\alpha^2+4\alpha\gamma+34\alpha-68\gamma+33)^2} < 0$. Hence, we evaluate $p_1 - p_2$ at $\gamma \rightarrow \infty$. We have $(p_1 - p_2)|_{\gamma \rightarrow \infty} = \frac{3(4h-1)}{2(\alpha-17)}$, since this expression is decreasing in h we further estimate it at the largest value h obtained in this range, that is, when $h = \hat{h}$, giving $\lim_{\gamma \rightarrow \infty} (p_1 - p_2)|_{h=\hat{h}} = 0$. Hence, the assumption that $p_1 > p_2$ is satisfied.

In the presence of strategic consumers the equilibrium outcomes are given by:

$$I^{SS} = \begin{cases} \frac{(1+\alpha)(8\alpha^2h-16\alpha\gamma h+\alpha^2+4\alpha\gamma-16\alpha h+80\gamma h+16\alpha-20\gamma-24h+15)}{2(\alpha^2+4\alpha\gamma+34\alpha-68\gamma+33)} & h < \hat{h}^{SS}, \\ 0 & otherwise, \end{cases} \quad (15)$$

$$x^{SS} = \begin{cases} \frac{2(1+\alpha)(2\alpha h-14h-5)}{\alpha^2+4\alpha\gamma+34\alpha-68\gamma+33} & h < \hat{h}^{SS}, \\ \frac{1+\alpha}{4\gamma-\alpha-1} & otherwise, \end{cases} \quad (16)$$

$$[w_1^{SS}, w_2^{SS}] = \begin{cases} \left[\frac{-4(2\alpha^2h-2\alpha\gamma h+4\alpha h-2\gamma h-4\alpha+9\gamma+2h-4)}{\alpha^2+4\alpha\gamma+34\alpha-68\gamma+33}, \frac{-2(3\alpha^2h-4\alpha\gamma h-10\alpha h+20\gamma h-7\alpha+12\gamma-13h-7)}{\alpha^2+4\alpha\gamma+34\alpha-68\gamma+33} \right] & h < \hat{h}^{SS}, \\ \left[\frac{1}{2}, \frac{\alpha-2\gamma+1}{\alpha-4\gamma+1} \right] & otherwise, \end{cases} \quad (17)$$

$$[p_1^{SS}, p_2^{SS}] = \begin{cases} \left[\frac{(1-8h)\alpha^2+(8\gamma h+4\gamma-16h+50)\alpha+8\gamma h-104\gamma-8h+49}{2(\alpha^2+(4\gamma+34)\alpha-68\gamma+33)}, \frac{(1-6h)\alpha^2+(8\gamma h+4\gamma+20h+48)\alpha-40\gamma h-92\gamma+26h+47}{2(\alpha^2+(4\gamma+34)\alpha-68\gamma+33)} \right] & h < \hat{h}^{SS}, \\ \left[\frac{3}{4}, \frac{\alpha-3\gamma+1}{\alpha-4\gamma+1} \right] & otherwise, \end{cases} \quad (18)$$

where $h < \hat{h}^{SS} \equiv \frac{1}{4} - \frac{3(\alpha+1)(\alpha+3)}{8((10-2\alpha)\gamma+(\alpha-3)(\alpha+1))}$ defines the area where inventory is strictly positive.

The profit expressions are omitted due to their length.

We have the following insight:

Corollary 2. *Similar to the base model, in the presence of strategic consumers, $p_1 > p_2$.*

Proof of Corollary 2. Follows from (18). □

The second insight pertains to the behavior of prices. Interestingly, we find that the behavior of the two prices with respect to the proportion of strategic consumers is not monotonic.

Corollary 3.

$$\frac{\partial[p_1^{SS}, p_2^{SS}]}{\partial\alpha} = \begin{cases} [> 0, > 0] & h < \min\{\hat{h}^{SS}, h^{p2}\}, \\ [> 0, < 0] & h^{p2} < h < \hat{h}^{SS}, \\ [= 0, < 0] & \text{otherwise,} \end{cases} \quad (19)$$

where $h^{p2} \equiv \frac{-7\alpha^2+24\alpha\gamma+48\gamma^2-14\alpha-96\gamma-7}{16(\alpha^2\gamma+7\alpha^2-28\alpha\gamma+12\gamma^2+14\alpha-5\gamma+7)}$. Further, $\frac{\partial[p_1^{SS}-p_2^{SS}]}{\partial\alpha} > 0$ in each of the regions defined in (19).

Proof of Corollary 3. Solving $\frac{\partial p_1}{\partial\alpha} = 0$ we have $h^{p1} \equiv \frac{(6\gamma-\alpha-1)(2\alpha+3\gamma+2)}{(5\alpha-139)(\alpha+1)\gamma+32(\alpha+1)^2+72\gamma^2}$. We verify whether h^{p1} is in the feasible range given by $0 \leq h \leq \hat{h}^{SS}$ for any $\alpha \in (0, 1)$ and $\gamma > \frac{33}{68}$. Solving $h^{p1} = \hat{h}^{SS}$ yields $\gamma_1 = \frac{16(1+\alpha)}{61-5\alpha}$ and $\gamma_2 = \frac{\alpha^2+34\alpha+33}{4(17-\alpha)}$. Clearly $\gamma_1, \gamma_2 > 0$ for $\alpha \in (0, 1)$. Considering the first solution, we have that $h^{p1}|_{\gamma=\gamma_1} = -\frac{5(\alpha+7)(\alpha-17)}{8(5\alpha^2-44\alpha+23)}$, which yields an asymptote at $\alpha = \frac{1}{5}(22 - 3\sqrt{41}) \approx 0.558125$. It is easy to verify that $h^{p1}|_{\gamma=\gamma_1} > 0.25$ for $\alpha \in \{0, 0.558125.. \}$, while $h^{p1}|_{\gamma=\gamma_1} < 0$ for $\alpha \in (0.558125.., 1)$. Considering the second solution, we have that $h^{p1}|_{\gamma=\gamma_2} = \frac{5}{2(\alpha-7)} < 0$. Thus, we can conclude that the plane given by h^{p1} does not cross the feasible range given by $0 \leq h \leq \hat{h}^{SS}$, $\alpha \in (0, 1)$, $\gamma > \frac{33}{68}$. In that range, the derivative $\frac{\partial p_1}{\partial\alpha}$ does not switch sign, and it is now straightforward to check that $\frac{\partial p_1}{\partial\alpha} > 0$.

We now proceed with the FOC of p_2 . Solving the FOC, $\frac{\partial p_2}{\partial\alpha} = \frac{(-16\gamma h-112h-7)\alpha^2+((448h+24)\gamma-224h-14)\alpha+(-192h+48)\gamma^2+(80h-96)\gamma-112h-7}{(\alpha^2+(4\gamma+34)\alpha-68\gamma+33)^2} = 0$, for h , we have that this derivative is positive when $h < h^{p2} \equiv \frac{-7\alpha^2+24\alpha\gamma+48\gamma^2-14\alpha-96\gamma-7}{16(\alpha^2\gamma+7\alpha^2-28\alpha\gamma+12\gamma^2+14\alpha-5\gamma+7)}$, noting that the denominator is always positive (this can be observed by equating the denominator to 0 and solving for α and the resulting two roots are both greater than 1 for $\gamma > 0$).

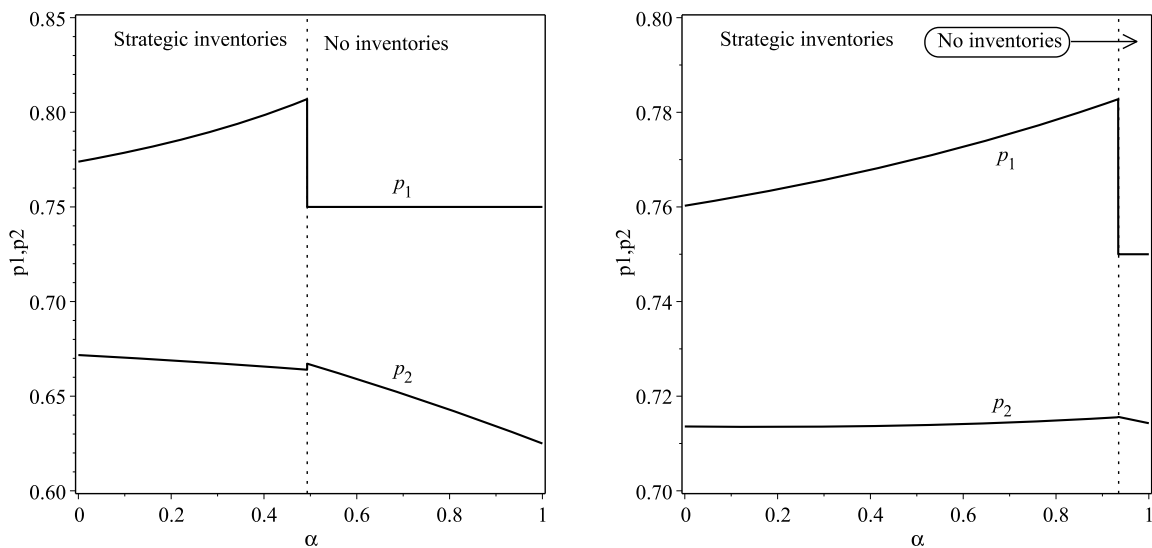
Consider the case of $h \geq \hat{h}^{SS}$. It is evident that p_2 is independent of α , hence $\frac{\partial p_2}{\partial\alpha} = 0$. $\frac{\partial p_2}{\partial\alpha} = -\frac{\gamma}{(\alpha-4\gamma+1)^2} < 0$.

Lastly, showing that $\frac{\partial[p_1^{SS}-p_2^{SS}]}{\partial\alpha} > 0$ follows the same arguments. \square

Generally, the first period price increases in the proportion of strategic consumers whenever inventories are carried. This indicates that the diversion of the strategic consumers to the later period allows the retailer to improve the segmentation by targeting consumers with higher valuations in the first period. Further, while the second period price might increase or decrease in the proportion of strategic consumers (depending on the value of h), the separation between the

first and second period prices always increases in α in each of the regions. This occurs due to the aforementioned segmentation coupled with the reduction in unit cost (due to greater investment in process improvement as discussed below) that is mostly passed on to the consumers.

The different behaviors of prices are illustrated in the two panels of Figure 7. Consider first Figure 7a. The dashed vertical line indicates the value of α such that $h = \hat{h}^{SS}$. To the left of this dashed vertical line, the value of h is such that $h^{p_2} < h < \hat{h}^{SS}$. Accordingly, we observe how p_1 increases in the proportion of strategic consumers whereas p_2 decreases in this proportion. To the right of the dashed line, no inventories are carried, and then the first period price is fixed in α whereas the second period price decreases in α . Consider Figure 7b. To the right of the dashed line, the value of h is such that $h < \hat{h}^{SS} < h^{p_2}$. Accordingly, two prices increase in α . As before, to the right of \hat{h}^{SS} , p_1 is fixed in α whereas the second period price decreases in α .



(a) $\gamma = 1.5$, $h = 0.05$, with $\hat{h}^{SS} = 0.05$ at $\alpha \approx 0.493$

(b) $\gamma = 4$, $h = 0.15$, with $\hat{h}^{SS} = 0.15$ at $\alpha \approx 0.934$

Figure 7: Prices as a function of the fraction of strategic consumers