Relay Selection and Resource Allocation for SWIPT in Multi-User OFDMA Systems

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***Abstract*—We investigate the resource allocation and relay selection in a two-hop relay-assisted multi-user Orthogonal Frequency Division Multiple Access (OFDMA) network, where the end-nodes support Simultaneous Wireless Information and Power Transfer (SWIPT) employing a Power Splitting (PS) technique. Our goal is to optimize the end-nodes' power splitting ratios as well as the relay, carrier and power assignment so that the sum-rate of the system is maximized subject to harvested energy and transmitted power constraints. Such joint optimization with mixed integer non-linear programming structure is combinatorial in nature. Due to the complexity of this problem, we propose to solve its dual problem which guarantees asymptotic optimality and less execution time compared to a highly-complex exhaustive search approach. Furthermore, we also present a heuristic method to solve this problem with lower computational complexity. Simulation results reveal that the proposed algorithms provide significant performance gains compared to a semi-random resource allocation and relay selection approach and close to the optimal solution when the number of OFDMA sub-carriers is sufficiently large.**

***Index Terms*—Relay selection, Resource Allocation, Orthogonal Frequency Division Multiple Access (OFDMA), Simultaneous Wireless Information and Power Transfer (SWIPT), Cooperative Communications.**

I. Introduction

COOPERATIVE relaying is a promising technology where the information delivery from a source node to a user node is assisted via one or several intermediate nodes, referred to as relay nodes. In doing so, the network coverage is extended, fading effects are alleviated, and the whole network can be rolled out cost-effectively and rapidly [1], [2]. Besides, cooperative relaying has emerged as a promising alternative to alleviate the burden of installing multiple antennas on size-limited terminals [3].

Orthogonal Frequency Division Multiple Access (OFDMA) [4] is one of the most popular air interface for wireless access (e.g., LTE-A and WiMAX) given its spectral efficiency characteristics and its ability to handle frequency-selective fading. High data demands and energy requirements can be met by combining OFDMA and cooperative relaying [5]. However, optimal relay selection in relay-assisted OFDMA is not a straight forward problem since it depends highly on the OFDMA power and sub-carrier allocation across different hops. The selection of the most appropriate relay is one challenging problem by itself in general cooperative relaying scenarios. There are various protocols proposed in the literature to choose the best relay among a collection of available relays so that the performance of a single transmit-relay-receive link is optimized [6]–[8] where the respective strategies are to choose the best relay depending on its geographic position [6], maximization of end-to-end SNR [7], or based on the strongest bottleneck link (i.e., the weaker channel between the source-relay and relay-user channels) [8].

Recent studies have dealt with the joint optimization of relay selection and resource allocation in a single-user OFDMA relay network [9], [10], while the multi-user extension of such a scenario was considered in [5] wherein the performance benefits of channel-dependent OFDMA sub-carrier pairing are illustrated for multi-hop systems. Considering a single source, multi-hop relaying and single user scenario, an OFDM-based selective relaying scheme was proposed in [11], where superior performance benefits of Selective OFDMA relaying with only symbol detection at each relay was shown over Selective OFDM relaying where decoding of whole OFDM block is required at each hop. However, all of these works does not take into account the energy efficiency aspects.

With the rapid development in wireless communication over recent years, the power demand to operate the wireless devices has increased manifold [12]. Simultaneous Wireless Information and Power Transmission (SWIPT) is an upcoming and promising technique to meet these requirements [13]. In this direction, several types of SWIPT receiver architectures were presented in [14], namely: separated, time-switching (TS), power-splitting (PS), and integrated architectures, where superior performance of PS-based SWIPT receiver is established. Implementation of SWIPT-OFDMA systems has attracted due attention recently [15], [16]. The incorporation of OFDMA with SWIPT does not only preserves its existing advantages, but simultaneous charging of multiple devices along with information exchange can be achieved. However, due to the rate-energy (R-E) trade-off [17], the information rate cannot be optimized without diminishing the harvested energy and vice-versa. In addition, the information and particularly the energy cannot be sent from one place to another over large distances. In this context, it has been proposed that the use of relays may come in handy for such cases [18]–[22].

There is limited work available in the literature addressing relay selection to enable SWIPT in multi-user OFDMA (MU-OFDMA) scenarios [23]–[26]. A simple relay selection

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algorithm in MU-OFDMA cooperative Cognitive Radio (CR) networks is analyzed in [23] to enhance the normalized sum-rate of secondary users (SUs) while a novel relay selection method for cooperative communication networks is proposed in [24] using fuzzy logic. Further, a relay selection problem based on co-channel interference is studied in [25], where the relays are assumed to employ the decode-and-forward (DF) protocol. It is noteworthy that [24] and [25] present a multi-user scenario but both the works neither consider OFDMA nor the energy harvesting aspects. On the other hand, performance benefits of relay selection for SWIPT is shown in [26], where a trade-off between the information transfer to the target receiver and harvested energy at the remaining receivers is established. In [27], a relay selection method based on either maximum sum-throughput or minimum outage probability is proposed to facilitate the communication process between two transceivers in a full-duplex mode. However, the multi-user case is not investigated in [26], [27]. In [28], Guo *et al.* consider energy constrained relay nodes within cooperative clustered WSNs and provide optimal strategies to prolong their lifetime using ambient RF signals which, however, do not incorporate the multicarrier setting. Similar to [28], the authors in [29] present a framework where two sources communicate with each other via two way DF and SWIPT-enabled relaying devices. Considering a SWIPT relay system, the authors in [30] proposed an optimal energy efficiency based joint resource allocation and relay selection scheme. Optimal transmission schemes (static and dynamic) for joint time allocation and power distribution are designed in [31], where single DF relay (with SWIPT capabilities) assists a source to transfer information to a destination. With a multiple-antenna source terminals and single-antenna energy harvesting (EH) relay, the performance analysis for an analog network coding based two-way relaying system is investigated in [32]. Noticeably, [28]–[32] considers energy harvesting at the relay nodes, and not at the end-users.

In contrast to the above studies, this paper investigates the joint optimization of single relay selection from a pool of candidate relays, carrier assignment for the two-hop links, power allocation and power-splitting (PS) ratio optimization in a two-hop relay-assisted MU-OFDMA network with SWIPT. The relays employ the amplify-and-forward (AF) protocol which takes into account the channelization and sub-carrier switching to demultiplex, frequency convert and multiplex again, unlike its standard operation. We focus on the single relay selection over the multiple relay selection as the latter involves significant complexity in terms of control and synchronization among the relay nodes. We formulate the resource allocation and relay selection problem to maximize the total system throughput by satisfying the individual users' energy harvesting constraints while respecting the individual source and relays' transmit power limits. This is an extremely challenging problem due to the complexity caused by the joint optimization of several network resources, which requires an exhaustive analysis within the full search space. In order to circumvent this tedious and unaffordable optimization, we propose a) an extremely time-efficient and asymptotically optimal algorithm which yields nearly optimal results for high number of sub-carriers, and b) a heuristic method with good performance and even lesser computational complexity. Numerical results are presented, which show that the proposed low complexity schemes offer better performance than the one achieved with a semi-random resource assignment approach, where the relay and sub-carriers are randomly assigned followed by an optimal allocation of power and PS ratios.

This work builds on the authors' previous publications [18], [33]. In [18], the optimal transceiver design and relay selection for SWIPT in a single-transmitter single-receiver two-hop cooperative network is considered; while in [33] a single transmitter and multiple users scenario is considered with frequency-selective fading and multi-carrier transmission (without incorporation of cooperative systems). Moreover, a linear energy harvesting model was examined in [18], [33] under a more generalized channel model while we do analysis by assuming a non-linear energy harvesting model in this work, under a more practically feasible scenario. The main contributions of this paper are four-fold, listed as follows

1) Firstly, we consider a dual-hop scenario where multiple AF-relays facilitate SWIPT from single source to multiple users with the help of OFDMA carriers in both the hops. Each end-user employs the PS-based SWIPT architecture[1] having a non-linear energy harvesting model. Our aim is to optimize and save network resources while easing the synchronization process amongst the relay nodes.
2) Secondly, we formulate an optimization problem for relay selection, carrier assignment in the two-hops for carrier pairing, power allocation, and the PS ratio for each user in order to maximize the overall sum-rate of the system subjected to transmit power and harvested energy constraints. One possible solution is proposed based on an exhaustive search mechanism within the full feasibility region, which imposes intractability at large dimensions due to extremely high (exponential) time-complexity.
3) Thirdly, we propose an asymptotically optimal solution by considering the dual of the aforementioned problem which reduces the computational complexity to polynomial time. Besides, we also present a heuristic solution with good performance and even lesser time-complexity. Additionally, we derive closed-form expressions for the power metrics by using the Karush-Kuhn-Tucker (KKT) conditions, and consequently determine respective PS ratios at the users.
4) Finally, we demonstrate the effectiveness of the proposed methods where significant gains are observed in comparison with a semi-random resource allocation and relay selection approach. In this vein, the impacts of varying the key system parameters is observed via numerical results. Additionally, the benefits of the proposed techniques are provided and some possible future directions of this work are discussed.

The remainder of this paper is organized as follows. Section

[1]Please note that most of the aforementioned works consider EH at the relays. It is important to mention that for relays that are a part of the infrastructure with their own power supply (as in this paper), EH at the end-users is crucial and this aspect has not been investigated widely. A practical example of the proposed framework can be its implementation in an indoor scenario (e.g., office workspace), where single source is located in the middle of a corridor and different AF relays are placed near the office doors so as to extend the SWIPT coverage inside the specific offices. In general, the role of the relays is of extreme importance in case of hindrance, which is highly experienced in indoor scenarios.

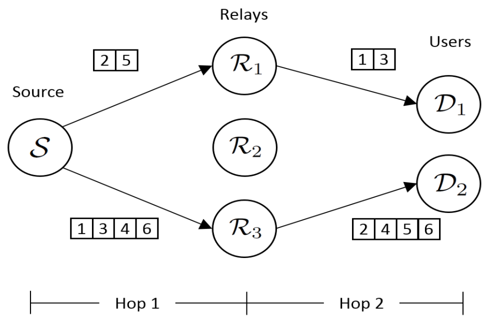

Fig. 1: System model for multi-user relay-assisted OFDMA-based communications with SWIPT.

II introduces the system model. The problem formulation for sum-rate maximization of all users is presented in Section III. The proposed asymptotically optimal and heuristic solutions are illustrated in Sections IV and V, respectively. Numerical results are shown in Section VI, followed by concluding remarks in Section VII.

II. System Model

We consider a two-hop cooperative network of K fixed relays $(\mathcal{R}_1, \ldots, \mathcal{R}_k, \ldots, \mathcal{R}_K)$ assisting a source $\mathcal{S}$ that can transmit both information and energy to L users $(\mathcal{D}_1, \ldots, \mathcal{D}_\ell, \ldots, \mathcal{D}_L)$, where $L \leq K$ in general. The source, relays and end-user nodes are equipped with single antenna and each relay operates in a time-division half-duplex mode using the AF protocol. Each user is connected to the source via single relay, which is not shared with other users to avoid control channel and synchronization overhead. It is worth mentioning that the scenario considering only single relay with multiple antennas may be beneficial in some situations not only in terms of low synchronization overhead but also distance dependent effects for energy harvesting. This however limits the system performance and in this regard, multiple relays distributed within the network may help cater to the need of users with even lower synchronization overhead and higher efficiency. The transmission from the source to relays and from relays to users is based on OFDMA scheme. In particular, the available bandwidth on both the hops is divided into N sub-carriers ($1 \leq n \leq N$ for the first hop and $1 \leq n' \leq N$ for the second hop) in which the channel is assumed to be frequency-flat. The channel coefficient of the first hop between $\mathcal{S}$ and the $\mathcal{R}_k$ on the n-th sub-carrier is denoted as $h_{1,n,k}$, whereas the channel coefficient of the second hop between the $\mathcal{R}_k$ and the $\mathcal{D}_\ell$ on the n'-th sub-carrier is denoted as $h_{2,n',k,\ell}$. We assume that the users are out of reach of the source and therefore do not receive the direct signal during the first hop. We consider a sub-carrier pairing approach, where the sub-carriers of the first and second hops are paired such that each relay $\mathcal{R}_k$ employs a set of sub-carrier pairs. A sub-carrier pair is defined as the (n, n') sub-carriers such that the relay receives from sub-carrier n, amplifies it, and forwards it on the sub-carrier n'. Note that n and n' may be equal or not. We assume that each pair can only be assigned to one relay but one relay can employ multiple pairs. Fig. 1 illustrates the considered scenario and a particular sub-carrier pairing example for a network with $K = 3$ relays, $D = 2$ users and $N = 6$ sub-carriers.

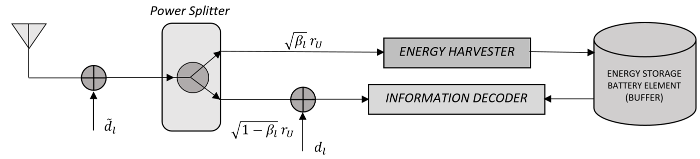

Fig. 2: Receiver architecture based on PS scheme.

The transmit power on the n-th sub-carrier at the source $\mathcal{S}$ in the first hop is denoted as $p_{1,n}$, while the relay $\mathcal{R}_k$ retransmits the received signal by applying the following amplification coefficient

$$\hat{g}_{(n,n'),k} = \sqrt{\frac{p_{2,n',k}}{p_{1,n} \left|h_{1,n,k}\right|^2 + \sigma_k^2}} \tag{1}$$

which ensures that the relay transmit power on the n'-th sub-carrier is $p_{2,n',k}$. In (1), σ_k^2 denotes the noise power at the $\mathcal{R}_k$ relay. The total available power at the source $\mathcal{S}$ and at the relay $\mathcal{R}_k$ is fixed as P_S and $P_{\mathcal{R},k}$, respectively.

On the other hand, the receivers at the users are capable of decoding information as well as harvesting energy according to a PS SWIPT architecture. In particular, the received signal at the user $\mathcal{D}_\ell$ is split into two streams, with the power splitting ratio β_ℓ, as shown in Fig. 2. The fraction $\sqrt{\beta_\ell}$ of the received signal is used for energy harvesting, while the rest is sent to the information decoder. For simplicity, we assume a normalized transmission time for each hop so that the terms *energy* and *power* can be used interchangeably. In practice, the antenna noise $\tilde{d}_\ell \in \mathcal{CN}(0, \sigma^2_{\tilde{d}_\ell})$ has a negligible impact on both the information receiving and energy harvesting, since $\sigma^2_{\tilde{d}_\ell}$ is generally much smaller than the noise power introduced by the baseband processing circuit, and thus even lower than the average power of the received signal [34]. As a consequence, the antenna noise $\tilde{d}_\ell$ is neglected in this paper. In addition, we assume that the relays are a part of the infrastructure having their own power supply and, therefore, they do not need to harvest energy from the received signals.

Let us denote $a_{n,\ell} \in \{0,\ 1\}$ as the binary variable for selection of $\mathcal{D}_\ell$ with $a_{n,\ell} = 1$ indicating $\mathcal{D}_\ell$ is assigned the sub-carrier n in the first hop and $a_{n,\ell} = 0$ otherwise. To comply with the fact that a sub-carrier can only be assigned to a unique user, we impose the following constraint,

$$\sum_{\ell=1}^{L} a_{n,\ell} = 1, \quad \forall n. \tag{2}$$

Assume $\phi_{(n,n')} \in \{0,\ 1\}$ denotes the indicator for sub-carrier pairing, where $\phi_{(n,n')} = 1$ means that sub-carrier n in the first hop is paired with sub-carrier n' in the second hop and $\phi_{(n,n')} = 0$ otherwise. Since each sub-carrier can be paired with one and only one sub-carrier, the binary variable $\phi_{(n,n')}$ must satisfy the following constraints

$$\sum_{n=1}^{N} \phi_{(n,n')} = 1, \ \sum_{n'=1}^{N} \phi_{(n,n')} = 1, \quad \forall (n, n'). \tag{3}$$

TABLE I: Annotations

Symbol	Definition
$\mathbf{a}$	Binary matrix ($N \times L$) for sub-carrier–user assignment
$\boldsymbol{\beta}$	Set of PS ratios at each user : $\{\beta_\ell\}$ where $\ell = 1, \ldots, L$
$\tilde{d}_\ell$	Antenna noise at the ℓ-th user
d_ℓ	Baseband processing circuit noise at the ℓ-th user
$\mathcal{D}_\ell$	Denotes the ℓ-th user where $\ell = 1, \ldots, L$
E_ℓ	Energy harvested at the ℓ-th user node (using a linear or a non-linear energy harvester)
E_ℓ^{Linear}	Energy harvested using a linear energy harvester at the ℓ-th user node
$E_\ell^{\text{Non Linear}}$	Energy harvested using a non-linear energy harvester at the ℓ-th user node
$\epsilon^{(t)}$	Step size corresponding to the subgradient method
η_ℓ	Demanded harvested energy at the ℓ-th user node
$\hat{\gamma}_{(n,n'),k,\ell}$	The effective approximated SNR seen at the decoding branch of the ℓ-th user for (n, n') sub-carrier pair over the $\mathcal{S} \to \mathcal{R}_k \to \mathcal{D}_\ell$ link
$\hat{g}_{(n,n'),k}$	Amplification coefficient of the k-th relay
$h_{1,n,k}$	The channel coefficient of the first hop between $\mathcal{S}$ and the k-th relay on the n-th sub-carrier
$h_{2,n',k,\ell}$	The channel coefficient of the second hop between the k-th relay and the ℓ-th on the n'-th sub-carrier
K	Total number of available relays
L	Total number of end-user nodes
$\boldsymbol{\Lambda}$	Denotes the vectors of the dual variables associated with the individual source and relays' power constraints : $(\lambda_S, \lambda_{R,1}, \ldots, \lambda_{R,K})$, during the power allocation step
$\boldsymbol{\mu}$	Denotes the vectors of the dual variables associated with the individual source and relays' power constraints : $(\mu_S, \mu_{R,1}, \ldots, \mu_{R,K})$, during the power refinement step
n	Sub-carrier index for the first hop where $n = 1, \ldots, N$
n'	Sub-carrier index for the second hop where $n' = 1, \ldots, N$
N	Total number of OFDMA sub-carriers in a hop
$p_{1,n}$	The transmit power on the n-th sub-carrier at the source $\mathcal{S}$ in the first hop
$p_{2,n',k}$	The transmit power on the n'-th sub-carrier at the k-th relay in the second hop
$\boldsymbol{\phi}$	Binary matrix ($N \times N$) for sub-carrier pairing
$P_{\mathcal{R},k}$	Total available power at the relay $\mathcal{R}_k$
P_S	Total available power at the the source $\mathcal{S}$
$\mathcal{R}_k$	The k-th relay where $k = 1, \ldots, K$
R_ℓ	Overall spectral efficiency at the ℓ-th user
$\rho(n)$	Denotes the sub-carrier index in the second hop optimally paired with the sub-carrier n in the first hop, for $n = 1, \ldots, N$
$\sigma^2_{\tilde{d}_\ell}$	Antenna noise power at the ℓ-th user
$\sigma^2_{d_\ell}$	Baseband processing circuit noise power at the ℓ-th user
σ^2_k	Noise power at the k-th relay
$\mathbf{s}$	Binary matrix ($K \times L$) for relay–user coupling
$\mathcal{S}$	The transmit source
S_k	Set of active sub-carrier pairs $(n, \rho(n))$ assigned on relay k for user ℓ
θ, φ	Constants corresponding to the non-linear energy harvester (Typically $\theta = 1500$ and $\varphi = 0.0022$)
ζ	The energy conversion efficiency of the receiver

For the relay–user coupling, we define $s_{k,\ell} = \{0,\ 1\}$, where $s_{k,\ell} = 1$ means that $\mathcal{R}_k$ is selected for $\mathcal{D}_\ell$, $s_{k,\ell} = 0$ otherwise. One relay is assumed to assist single user. Therefore, the following relay–user constraints must be satisfied

$$\sum_{k=1}^{K} s_{k,\ell} = 1, \quad \forall \ell, \tag{4}$$

$$\sum_{\ell=1}^{L} s_{k,\ell} \leq 1, \quad \forall k. \tag{5}$$

The effective SNR seen at the decoding branch of the ℓ-th user for (n, n') sub-carrier pair over the $\mathcal{S} \to \mathcal{R}_k \to \mathcal{D}_\ell$ link is given by

$$\begin{aligned}\gamma_{(n,n'),k,\ell} &= \frac{(1-\beta_\ell) p_{1,n} |h_{1,n,k} \hat{g}_{(n,n'),k} h_{2,n',k,\ell}|^2}{(1-\beta_\ell)\sigma_k^2 |h_{2,n',k,\ell} \hat{g}_{(n,n'),k}|^2 + \sigma^2_{d_\ell}} \\ &= \frac{(1-\beta_\ell)\left(\frac{p_{1,n}|h_{1,n,k}|^2}{\sigma_k^2}\right)\left(\frac{p_{2,n',k}|h_{2,n',k,\ell}|^2}{\sigma^2_{d_\ell}}\right)}{1 + \left(\frac{p_{1,n}|h_{1,n,k}|^2}{\sigma_k^2}\right) + \left(\frac{p_{2,n',k}|h_{2,n',k,\ell}|^2}{\sigma^2_{d_\ell}}\right)}. \end{aligned} \tag{6}$$

The above expression can be further simplified and re-written as follows

$$\gamma_{(n,n'),k,\ell} = \frac{(1-\beta_\ell)\gamma_{1,n,k}\gamma_{2,n',k,\ell}}{1+\gamma_{1,n,k}+(1-\beta_\ell)\gamma_{2,n',k,\ell}}, \quad (7)$$

where $\gamma_{1,n,k} = \frac{|h_{1,n,k}|^2 p_{1,n}}{\sigma_k^2}$, and $\gamma_{2,n',k,\ell} = \frac{|h_{2,n',k,\ell}|^2 p_{2,n',k}}{\sigma_{d_\ell}^2}$, with $\sigma_{d_\ell}^2$ being the baseband processing circuit noise power at $\mathcal{D}_\ell$. Further, we adopt the following approximation to make the analysis more tractable in the high SNR regime

$$\hat{\gamma}_{(n,n'),k,\ell} \approx \frac{(1-\beta_\ell)\gamma_{1,n,k}\gamma_{2,n',k,\ell}}{\gamma_{1,n,k}+(1-\beta_\ell)\gamma_{2,n',k,\ell}}, \quad (8)$$

since $\frac{|h_{1,n,k}|^2}{\sigma_k^2} >> 1$ and $\frac{|h_{2,n',k,\ell}|^2}{\sigma_{d_\ell}^2} >> 1$, therefore the unity term in the denominator of (7) may be ignored thereby causing a negligible impact on the system performance considering SWIPT.

The spectral efficiency achieved by the decoding branch of the (n,n') sub-carrier pair over the $\mathcal{S} \to \mathcal{R}_k \to \mathcal{D}_\ell$ link can thus be expressed as

$$R_{(n,n'),k,\ell} = \frac{1}{2}\ln\left(1+\hat{\gamma}_{(n,n'),k,\ell}\right), \quad (9)$$

where the factor $1/2$ is introduced to compensate for the two time slots of the considered relay assisted communication. The overall spectral efficiency at $\mathcal{D}_\ell$ is

$$R_\ell = \sum_{i=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} s_{k,\ell}a_{n,\ell}\phi_{(n,n')}R_{(n,n'),k,i}. \quad (10)$$

On the other hand, the energy harvested at the harvesting branch for the (n,n') sub-carrier pair over the $\mathcal{S} \to \mathcal{R}_k \to \mathcal{D}_\ell$ link is given by

$$E_{(n,n'),k,\ell} = \zeta\beta_\ell[|\hat{g}_{(n,n'),k}h_{2,n',k,\ell}|^2(p_{1,n}|h_{1,n,k}|^2+\sigma_k^2)], \quad (11)$$

where ζ is the energy conversion efficiency of the receiver. Furthermore, for a linear energy harvester, the overall harvested energy at $\mathcal{D}_\ell$ considering all intended sub-carriers is given by

$$E_\ell^{\text{Linear}} = \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} E_{(n,n'),k,\ell}. \quad (12)$$

It is noteworthy that all the transmitted OFDM subcarriers are utilized for energy harvesting purpose. Using only the intended sub-carriers for energy harvesting purpose is an inefficient process as the energy available in sub-carriers allocated to other users may potentially be harvested [16].

From a practical viewpoint, it has been well established in the literature that the energy harvesting operation by the energy harvesters is non-linear in nature [35]–[37]. It is apparent from such studies that for small values of energy detection at the energy harvester, the gap between the performances of non-linear and linear energy harvesting models cannot be ignored. Besides, detection of high energy signals where the harvested energy is large or even larger than the demand, the linear energy harvesting model is incapable of capturing the characteristics of the energy harvesters. Thus, this calls for the adoption of a non-linear EH model due to its practical feasibility. In this regard, we define the energy harvested at the receiver [35], [36] as follows

$$E_\ell^{\text{Non Linear}} = \beta_\ell \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} \frac{\mathcal{M}}{1-\psi}\cdot \left(\frac{1}{1+\exp(-\theta p_{2,n',k}|h_{2,n',k,\ell}|^2+\theta\varphi)}-\psi\right), \quad (13)$$

where $\psi \triangleq \frac{1}{1+\exp(\theta\varphi)}$, the constant $\mathcal{M}$ is obtained by determining the maximum harvested energy on the saturation of the energy harvesting circuit, and θ and φ are specific for the capacitor and diode turn-on voltage metrics at the EH circuit. Practically, a standard curve-fitting tool based on analytical data may be used to decide the appropriate values of $\mathcal{M}$, θ, and φ [35].

Concerning the utilization of the harvested energy at each user, there are many interesting ways to do so. The harvested energy may be utilized by the user to recharge its battery or it may be stored for later use. Simultaneous powering to each information decoding sub-block is also a possibility. In order to convey its future energy harvesting demands to the central controller (which is responsible for performing all the computational and intimation task) using adequate signaling/feedback, the user may utilize the harvested energy for such purposes. This kind of mechanism may require a separate framework and analysis, as provided in [38]. However, any of the aforementioned methods may be adopted (as per the requirement) to utilize the harvested energy at the end-user.

Our goal is to optimize the network resources so as to maximize the total end-nodes' sum-rate under a set of harvested energy and transmitted power constraints. In the following section, we formulate the corresponding optimization problem and discuss about the possible solutions.

III. Problem Formulation

We consider the problem of relay selection, carrier assignment in the two-hops for carrier pairing, power allocation, and the computation of the optimal PS ratio at each user that maximizes the effective sum-rate of the end-user nodes, while ensuring that the harvested energy at each user is above a given threshold and that the total transmit power does not exceed a given limit. As such, the variables to be optimized are: relay–user coupling $\mathbf{s} = \{s_{k,l}\}$, sub-carrier–user assignment $\mathbf{a} = \{a_{n,\ell}\}$, sub-carrier pairing $\boldsymbol{\phi} = \{\phi_{(n,n')}\}$, power allocation in the two-hops $\mathbf{p} = \{p_{1,n}, p_{2,n',k}\}$, and PS ratio at each user $\boldsymbol{\beta} = \{\beta_\ell\}$, where $n = 1,2,\ldots,N$, $n' = 1,2,\ldots,N$, $k = 1,2,\ldots,K$, and $\ell = 1,2,\ldots,L$. We assume that a central controller has access to channel state information (CSI) and rate/energy demands to solve such an optimization problem and informs the concerned nodes about the resulting allocation through a separate channel and appropriate signaling. Mathematically, the overall optimization problem can be represented as

$$(P1): \max_{\{\mathbf{p},\mathbf{a},\boldsymbol{\phi},\mathbf{s},\boldsymbol{\beta}\}} \quad \sum_{\ell=1}^{L} R_\ell \tag{14}$$

$$\text{subject to}: \quad (C1): E_\ell \geq \eta_\ell, \ \ell = 1,\ldots,L, \tag{15}$$

$$(C2): \sum_{\ell=1}^{L} a_{n,\ell} = 1, \ n = 1,\ldots,N, \tag{16}$$

$$(C3): \sum_{n=1}^{N} \phi_{(n,n')} = 1, \ n' = 1,\ldots,N, \tag{17}$$

$$(C4): \sum_{n'=1}^{N} \phi_{(n,n')} = 1, \ n = 1,\ldots,N, \tag{18}$$

$$(C5): \sum_{k=1}^{K} s_{k,\ell} = 1, \ \ell = 1,\ldots,L, \tag{19}$$

$$(C6): \sum_{\ell=1}^{L} s_{k,\ell} \leq 1, \ k = 1,\ldots,K, \tag{20}$$

$$(C7): \sum_{n=1}^{N} p_{1,n} \leq P_S, \tag{21}$$

$$(C8): \sum_{n'=1}^{N}\sum_{\ell=1}^{L}\sum_{n=1}^{N} s_{k,\ell} a_{n,\ell} \phi_{(n,n')} p_{2,n',k} \leq P_{\mathcal{R},k}, \ k = 1,\ldots,K, \tag{22}$$

$$(C9): 0 \leq \beta_\ell \leq 1, \ \ell = 1,\ldots,L, \tag{23}$$

where E_ℓ is the harvested energy via linear or non-linear energy harvester (for further analysis, we assume $E_\ell = E_\ell^{\text{Non Linear}}$, unless specified otherwise), (21) and (22) represents the source power constraint and the relay power constraint, respectively. It is clear that $(P1)$ is a non-linear mixed integer programming problem involving computation of the optimal solutions $\{\mathbf{p}^*, \mathbf{a}^*, \boldsymbol{\phi}^*, \mathbf{s}^*, \boldsymbol{\beta}^*\}$ with the total of $(K \cdot L)^{N!}$ possibilities of relay–user coupling, sub-carrier–user assignment, and sub-carrier pairing. These concurrent assignments become extremely complicated as the values of K, L, and N become large. In the following section, we present an asymptotic solution with appreciably lesser time complexity and good performance in comparison to the highly complex exhaustive search technique.

IV. Proposed Asymptotically Optimal Solution

As mentioned previously, solving for $(K \cdot L)^{N!}$ possibilities of relay–user coupling, sub-carrier–user assignment, and sub-carrier pairing is a complicated process to obtain an optimal solution considering large values of K, L and N. In addition, discrete nature of the binary variables' assignment $(\mathbf{s}, \mathbf{a}, \boldsymbol{\phi})$ when coupled with the power and harvested energy constraints, makes the problem very difficult to solve. In order to simplify this tedious computation task, we seek for other methods which does not only provide asymptotically optimal results, but also have far lesser time complexity in comparison to the aforementioned exhaustive search method. As shown in [39], under a certain aspect called the time-sharing condition, the duality gap of a non-convex resource allocation optimization problem is negligible. Furthermore, [39] proves that the time-sharing condition is satisfied for practical multi-user spectrum optimization problems in multi-carrier systems for sufficiently large number of sub-carriers. From our optimization problem formulation, it is clear that the time-sharing condition is definitely satisfied (for sufficiently large N) and hence the dual method can be applied to obtain an asymptotically optimal solution in this regard. In the following sub-sections, we illustrate using the dual method that $(P1)$ can be solved in three stages with a polynomial time-complexity which is significantly less than its counterpart optimal exhaustive search method. In this vein, we provide an asymptotic technique based on the block-coordinate descent approach for joint optimization of intended variables.

A. Dual Problem Formulation

To proceed, we define $\mathcal{D}$ as the set of all possible relay–user coupling $\mathbf{s} = \{s_{k,\ell}\}$, sub-carrier–user assignment $\mathbf{a} = \{a_{n,\ell}\}$, sub-carrier pairing $\boldsymbol{\phi} = \{\phi_{(n,n')}\}$, the PS ratio $\boldsymbol{\beta} = \{\beta_\ell\}$, all satisfying (16), [(17), (18)], [(19), (20)], and [(15), (23)], respectively. We also define $\mathcal{P}(\mathbf{a}, \boldsymbol{\phi}, \mathbf{s}, \boldsymbol{\beta})$ as the set of all power allocations $\mathbf{p} = \{p_{1,n}, p_{2,n',k}\}$ for given relay–user coupling, sub-carrier–user assignment, sub-carrier pairing, and PS ratio $(\mathbf{s}, \mathbf{a}, \boldsymbol{\phi}, \boldsymbol{\beta})$ that satisfy $p_{1,n} \geq 0$, $p_{2,n',k} \geq 0$ for $s_{k,\ell} a_{n,\ell} \phi_{(n,n')} = 1$ and $\beta_\ell \in \mathcal{D}$, and $p_{1,n} = p_{2,n',k} = 0$ for $s_{k,\ell} a_{n,\ell} \phi_{(n,n')} = 0$ and $\beta_\ell \in \mathcal{D}$. Then, the Lagrange dual function of problem $(P1)$ can be written as

$$g(\boldsymbol{\Lambda}) \triangleq \max_{\substack{\mathbf{p} \in \mathcal{P}(\mathbf{a},\boldsymbol{\phi},\mathbf{s},\boldsymbol{\beta}) \\ \{\mathbf{a},\boldsymbol{\phi},\mathbf{s},\boldsymbol{\beta}\} \in \mathcal{D}}} \mathcal{L}(\mathbf{p}, \mathbf{a}, \boldsymbol{\phi}, \mathbf{s}, \boldsymbol{\beta}; \boldsymbol{\Lambda}) \tag{24}$$

where the Lagrangian is expressed as

$$\mathcal{L}(\mathbf{p}, \mathbf{a}, \boldsymbol{\phi}, \mathbf{s}, \boldsymbol{\beta}; \boldsymbol{\Lambda}) = \sum_{\ell=1}^{L} R_\ell + \lambda_S \Big(P_S - \sum_{n=1}^{N} p_{1,n} \Big) + \sum_{k=1}^{K} \lambda_{R,k} \Big(P_{R,k} - \sum_{n'=1}^{N} p_{2,n',k} \Big), \tag{25}$$

with $\boldsymbol{\Lambda} = (\lambda_S, \lambda_{R,1}, \ldots, \lambda_{R,K}) \geq 0$ denoting the vectors of the dual variables associated with the individual power constraints. Hence, the dual optimization problem is

$$(P2): \min_{\boldsymbol{\Lambda}} \quad g(\boldsymbol{\Lambda}) \tag{26}$$

$$\text{subject to}: \quad \boldsymbol{\Lambda} \geq 0. \tag{27}$$

Since it is explicit that a dual function is always convex by definition [40], therefore the gradient or subgradient-based methods can be used to minimize $g(\boldsymbol{\Lambda})$ with guaranteed convergence. Let $\mathbf{p}^*(\boldsymbol{\Lambda})$ denote the optimal power allocation in (24) at dual point $\boldsymbol{\Lambda} = (\lambda_S, \lambda_{R,1}, \ldots, \lambda_{R,K})$, then the subgradient of $g(\boldsymbol{\Lambda})$ can be derived as follows

$$\Delta\lambda_S = P_S - \sum_{n=1}^{N} p^*_{1,n}(\boldsymbol{\Lambda}), \tag{28}$$

$$\Delta\lambda_{R,k} = P_{R,k} - \sum_{n'=1}^{N} p^*_{2,n',k}(\boldsymbol{\Lambda}), k = 1,\ldots,K. \tag{29}$$

Denoting $\Delta\mathbf{\Lambda} = (\Delta\lambda_S, \Delta\lambda_{R,1}, \ldots, \Delta\lambda_{R,K})$, the dual variables are updated as: $\mathbf{\Lambda}^{(t+1)} = \mathbf{\Lambda}^{(t)} + \epsilon^{(t)}\Delta\mathbf{\Lambda}$. Using the step size $\epsilon^{(t)}$ following the diminishing step size policy, the subgradient method above is guaranteed to converge to the optimal dual variables $\mathbf{\Lambda}^*$. Such an update method is of polynomial computational complexity in the number of dual variables $K + 1$.

B. Optimizing Primal Variables at a Given Dual Point

Computation of the dual function $g(\mathbf{\Lambda})$ involves determining the optimal $\{\mathbf{p}^*, \mathbf{a}^*, \boldsymbol{\phi}^*, \mathbf{s}^*, \boldsymbol{\beta}^*\}$ at the given dual point $\mathbf{\Lambda}$. In the following, we present the detailed derivation of the optimal primal variables in three phases. Before that, let us rewrite $g(\mathbf{\Lambda})$ in (24) as follows

$$g(\mathbf{\Lambda}) = \max_{\substack{\mathbf{p}\in\mathcal{P}(\mathbf{a},\boldsymbol{\phi},\mathbf{s},\boldsymbol{\beta})\\ \{\mathbf{a},\boldsymbol{\phi},\mathbf{s},\boldsymbol{\beta}\}\in\mathcal{D}}} \sum_{\ell=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} \mathcal{L}_{(n,n'),k,\ell} + \lambda_S P_S + \sum_{k=1}^{K}\lambda_{R,k}P_{R,k}, \quad (30)$$

where

$$\mathcal{L}_{(n,n'),k,\ell} \triangleq R_\ell - \lambda_S p_{1,n} - \lambda_{R,k} p_{2,n',k}. \quad (31)$$

1) Optimal Power Allocation for Given Relay–User Coupling, Sub-carrier–User Assignment, Sub-carrier Pairing, and PS Ratio: Here we analyze the optimal power allocation $\mathbf{p}^*$ for given relay–user coupling ($\mathbf{s}$), sub-carrier–user assignment ($\mathbf{a}$), sub-carrier pairing ($\boldsymbol{\phi}$), and the PS ratio ($\boldsymbol{\beta}$). Suppose that a sub-carrier pair (n, n') is valid and assigned to the user ℓ, k-th relay assigned to the user ℓ in a frame of transmission time, i.e., $s_{k,\ell}a_{n,\ell}\phi_{(n,n')} = 1$. The optimal power allocation over this sub-carrier pair – relay – user unit $((n, n'), k, \ell)$ can be determined by solving the following problem

$$(P3): \max_{\{p_{1,n}, p_{2,n',k}\}} \quad \mathcal{L}_{(n,n'),k,\ell} \quad (32)$$
$$\text{subject to}: \quad p_{1,n} \geq 0, \quad (33)$$
$$p_{2,n',k} \geq 0. \quad (34)$$

It can be easily shown that $\mathcal{L}_{(n,n'),k,\ell}$ is a concave function of $(p_{1,n}, p_{2,n',k})$. Applying the KKT conditions [40], we obtain the optimal power allocation

$$p^*_{1,n} = \begin{cases} c_{(n,n'),k,\ell}\, p^*_{2,n',k}, & \text{if } p^*_{2,n',k} > 0, \\ \left(\frac{1}{\lambda_S} - \frac{1}{x_1}\right)^+, & \text{if } p^*_{2,n',k} = 0, \end{cases} \quad (35)$$

and $p_{2,n',k}$ is given in (36) at the top of next page, where $c_{(n,n'),k,\ell}$ is given in (37) with $x_1 = \frac{|h_{1,n,k}|^2}{\sigma_k^2}$, $x_2 = \frac{|h_{2,n',k,\ell}|^2}{\sigma_{d_\ell}^2}$ and $(\chi)^+ = \max(0, \chi)$.

It is indicated from (35) and (36) that for this particular path $((n, n'), k, \ell)$, if $(1 - \beta_\ell)^2 x_1 x_2^2 \leq 2\lambda_S(c_{(n,n'),k,\ell}x_1 + (1 - \beta_\ell)x_2)^2$, no power is assigned in the second hop. The inequality above can be further simplified and expressed as $f_1(\lambda_S) \leq f_2(\lambda_{R,k})$, where $f_1(\cdot)$ and $f_2(\cdot)$ are functions of λ_S and $\lambda_{R,k}$, respectively, with corresponding simplified expressions. From an economic outlook, the dual variables λ_S and $\lambda_{R,k}$ can be elucidated as the power prices at the source and k-th relay, respectively. After re-arranging the dual variables, the aforementioned inequality can be further re-written as $f_2^{-1}(\lambda_{R,k}) \leq f_1^{-1}(\lambda_S)$. Correspondingly, $f_1^{-1}(\lambda_S)$ can be viewed as the SNR gain at the user conceived by the source via the indirect link per cost metric, and $f_2^{-1}(\lambda_{R,k})$ the gain of SNR at the user created by the k-th relay per cost metric. In order to maximize the SNR gain at the user for a given tight comprehensive fee on the transmit power, it can be interpreted that all of the payment should be given to the source if $f_2^{-1}(\lambda_{R,k}) \leq f_1^{-1}(\lambda_S)$ or $f_1(\lambda_S) \leq f_2(\lambda_{R,k})$. Alternatively, if $f_1(\lambda_S) \geq f_2(\lambda_{R,k})$, then non-zero power allocation should be made at the source and k-th relay using the connecting parameter $c_{(n,n'),k,\ell}$ in (35).

2) Optimization of Relay–User Coupling, Sub-carrier–User Assignment, and Sub-carrier Pairing: Substituting the optimal power allocation expression (35) and (36) in (31) to eliminate the power variables and then into (30), we can obtain an alternative expression of the dual function as defined in (38) at the top of next page.

Here, the function $\mathcal{J}_{(n,n'),k,\ell}(\mathbf{\Lambda})$ is defined in (39) at the top of next page. Based on (38), we next determine the relay–user pairing $\mathbf{s}^*$, optimal sub-carrier–user assignment $\mathbf{a}^*$, and the sub-carrier pairing $\boldsymbol{\phi}^*$. In this regard, we propose the following methods to obtain suitable solutions.

Suppose (n, n') is a valid sub-carrier pair in the given sub-carrier pairing scheme $\boldsymbol{\phi}$, i.e., $\phi_{(n,n')} = 1$. Then, it is obvious that the optimal relay selected for the ℓ-th user corresponding to this sub-carrier pair should be the one having maximum value of $\mathcal{J}_{(n,n'),k,\ell}(\mathbf{\Lambda})$ in (39). That is

$$s^*_{k,\ell} = \begin{cases} 1, & k = k(n,n') = \arg\max_k \mathcal{J}_{(n,n'),k,\ell}, \\ 0, & \text{otherwise.} \end{cases} \quad (40)$$

Similarly, it is obvious that the optimal sub-carrier allocated to the ℓ-th user corresponding to the $s^*_{k,\ell}$ obtained above, and for this sub-carrier pair, should be the one having maximum value of $\mathcal{J}_{(n,n'),k,\ell}(\mathbf{\Lambda})$ in (39). That is

$$a^*_{n,\ell} = \begin{cases} 1, & \ell = \ell(n,n') = \arg\max_\ell \mathcal{J}_{(n,n'),k,\ell}, \\ 0, & \text{otherwise.} \end{cases} \quad (41)$$

Therefore, the function $\mathcal{J}_{(n,n'),k,\ell}(\mathbf{\Lambda})$ defined in (39) serves as the optimal criterion for relay–user coupling as well as sub-carrier–user assignment.

Substituting $\mathbf{s}^*$ and $\mathbf{a}^*$ computed above, into (38), we obtain the corresponding dual function

$$g(\mathbf{\Lambda}) = \max_{\{\boldsymbol{\phi},\boldsymbol{\beta}\}\in\mathcal{D}} \sum_{\ell=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N} \phi_{(n,n')}\mathcal{J}_{(n,n')} + \lambda_S P_S + \sum_{k=1}^{K}\lambda_{R,k}P_{R,k}, \quad (42)$$

where $\mathcal{J}_{(n,n')} \triangleq \mathcal{J}_{(n,n'),k(n,n'),\ell(n,n')}(\mathbf{\Lambda})$. Define $N \times N$ profit matrix $\mathbf{J} = [\mathcal{J}_{(n,n')}]$. In order to maximize the objective in

$$p_{2,n',k}^{*}=\begin{cases}\Bigg[\Big(\frac{1}{c_{(n,n'),k,\ell}(1-\beta_\ell)x_1x_2}\Big)\Big(\frac{(1-\beta_\ell)^2x_1x_2^2-2\lambda_S(c_{(n,n'),k,\ell}x_1+(1-\beta_\ell)x_2)^2}{2\lambda_S(c_{(n,n'),k,\ell}x_1+(1-\beta_\ell)x_2)}\Big)\Bigg]^+, \\ \quad \text{if} \quad (1-\beta_\ell)^2x_1x_2^2>2\lambda_S(c_{(n,n'),k,\ell}x_1+(1-\beta_\ell)x_2)^2, \\ 0, \quad \text{if} \quad (1-\beta_\ell)^2x_1x_2^2\le 2\lambda_S(c_{(n,n'),k,\ell}x_1+(1-\beta_\ell)x_2)^2.\end{cases} \tag{36}$$

$$c_{(n,n'),k,\ell}=\sqrt{\frac{\lambda_{R,k}(1-\beta_\ell)x_2}{\lambda_S x_1}}. \tag{37}$$

$$g(\mathbf{\Lambda})=\max_{\{\mathbf{a},\boldsymbol{\phi},\mathbf{s},\boldsymbol{\beta}\}\in\mathcal{D}}\sum_{i=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N}a_{n,i}\phi_{(n,n')}s_{k,i}\mathcal{J}_{(n,n'),k,i}(\mathbf{\Lambda})+\lambda_S P_S+\sum_{k=1}^{K}\lambda_{R,k}P_{R,k}. \tag{38}$$

$$\mathcal{J}_{(n,n'),k,i}(\mathbf{\Lambda})\triangleq\frac{1}{2}\sum_{\ell=1}^{L}\ln\left(1+\frac{(1-\beta_\ell)^2x_1x_2^2-2\lambda_S(c_{(n,n'),k,\ell}x_1+(1-\beta_\ell)x_2)^2}{2\lambda_S(c_{(n,n'),k,\ell}x_1+(1-\beta_\ell)x_2)^2}\right)+(-c_{(n,n'),k,\ell}\lambda_S \\ -\lambda_{R,k})\cdot\left(\frac{1}{c_{(n,n'),k,\ell}(1-\beta_\ell)x_1x_2}\right)\left(\frac{(1-\beta_\ell)^2x_1x_2^2-2\lambda_S(c_{(n,n'),k,\ell}x_1+(1-\beta_\ell)x_2)^2}{2\lambda_S(c_{(n,n'),k,\ell}x_1+(1-\beta_\ell)x_2)}\right). \tag{39}$$

(42), we should pick exactly one element in each row and each column of matrix $\mathbf{J}$ such that the sum of profits is as large as possible. Clearly, this is a standard linear assignment problem and can be efficiently solved by the Hungarian method [41], whose complexity is $\mathcal{O}(N^3)$.

Let $\rho(n)$ denote the sub-carrier index in the second hop optimally paired with the sub-carrier n in the first hop, for $n=1,\ldots,N$. Then, the optimal sub-carrier pairing variables can be expressed as

$$\phi_{n,n'}^{*}=\begin{cases}1, & n'=\rho(n),\\ 0, & \text{otherwise.}\end{cases} \tag{43}$$

3) Power Splitting Ratio: In order to optimize the values of $\boldsymbol{\beta}$, we substitute (43) in (42) and obtain the following dual function

$$g(\mathbf{\Lambda})=\max_{\{\boldsymbol{\beta}\}\in\mathcal{D}}\sum_{\ell=1}^{L}\sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{n'=1}^{N}\mathcal{J}_{(n,n')}+\lambda_S P_S \\ +\sum_{k=1}^{K}\lambda_{R,k}P_{R,k}. \tag{44}$$

It is clear that $\mathcal{J}_{(n,n')}$ is a decreasing function of $\boldsymbol{\beta}$. Therefore, the equality in (15) must hold. This yields the following solution

$$\beta_\ell^{*}=\frac{\eta_\ell}{\sum_{\forall k}\sum_{\forall n'}\sum_{\forall n}\frac{\mathcal{M}}{1-\psi}\cdot\left(\frac{1}{1+\exp(-\theta p_{2,n',k}^{*}|h_{2,n',k,\ell}|^2+\theta\varphi)}-\psi\right)}. \tag{45}$$

Combining the above three phases together, we have obtained the optimal primal variables $\{\mathbf{p}^*,\mathbf{a}^*,\boldsymbol{\phi}^*,\mathbf{s}^*,\boldsymbol{\beta}^*\}$ for given dual variables $\mathbf{\Lambda}$. Suppose that the complexity of dual variable updates mentioned in Section IV-A and overall computation for such operation is in the order of $(K\cdot L\cdot N^5)^\alpha$. Then, the computational complexity of solving the dual problem (24) using the asymptotic approach is of $\mathcal{O}(N^3\cdot(K\cdot L\cdot N^5)^\alpha)$. This extremely reduced time-complexity implies a significant advantage over the exhaustive search method with immensely high computational complexity involving a span over the whole feasible regime with $(K\cdot L)^{N!}$ possibilities of relay–user coupling, sub-carrier–user assignment, and sub-carrier pairing.

C. Refinement of Power Allocation and PS Ratio

In this section, we now focus on obtaining the optimal solution for the corresponding primal problem in (14) after having acquired the optimal dual point $\mathbf{\Lambda}^*$ using the method as illustrated above. Because of a certain duality gap for fixed N, the optimal $\mathbf{p}^*(\mathbf{\Lambda}^*)$, $\mathbf{s}^*(\mathbf{\Lambda}^*)$, $\mathbf{a}^*(\mathbf{\Lambda}^*)$, $\boldsymbol{\phi}^*(\mathbf{\Lambda}^*)$, and $\boldsymbol{\beta}^*(\mathbf{\Lambda}^*)$ may not completely lie within the feasibility set. Therefore to address this concern, we initially attain the solution for the relay–user coupling, sub-carrier-relay assignment, sub-carrier pairing and $\{\mathbf{s}^*(\mathbf{\Lambda}^*),\ \mathbf{a}^*(\mathbf{\Lambda}^*),\ \boldsymbol{\phi}^*(\mathbf{\Lambda}^*)\}$, by applying the methods projected in the prior sub-sections. Next, we refine the power allocation $\mathbf{p}$ to meet the power constraints (21) and (22), and the PS ratio $\boldsymbol{\beta}$ to meet the PS ratio constraint (23) of the primal problem. It is also worth mentioning that this technique is asymptotically optimal due to the vanishing duality gap for adequately high values of N.

Let S_k denote the set of active sub-carrier pairs $(n,\rho(n))$ assigned on relay k for user ℓ. By substituting the value of $\{\mathbf{s}^*(\Lambda^*),\mathbf{a}^*(\Lambda^*),\boldsymbol{\phi}^*(\Lambda^*)\}$ into (14), the power refinement problem can be equivalent to the following power allocation problem, which is convex

$$(P4): \max \sum_{\ell=1}^{L} \hat{R}_\ell \tag{46}$$

$$\text{subject to}: \ (C1): E_\ell \geq \eta_\ell, \ \ell = 1, \ldots, L, \tag{47}$$

$$(C2): \sum_{(n,\rho(n))\in S_k} p_{1,n} \leq P_S, \ k = 1, \ldots, K, \tag{48}$$

$$(C3): \sum_{(n,\rho(n))\in S_k} p_{2,n',k} \leq P_{\mathcal{R},k}, \ \forall k \text{ with } S_k \neq \emptyset, \tag{49}$$

$$(C4): 0 \leq \beta_\ell \leq 1, \ \ell = 1, \ldots, L, \tag{50}$$

where

$$\hat{R}_\ell \triangleq \frac{1}{2} \sum_{i=1}^{L} \sum_{k=1}^{K} \sum_{(n,\rho(n))\in S_k} \ln\Big(1 + \tilde{\gamma}_{n,\rho(n),k,i}\Big), \tag{51}$$

with $\tilde{\gamma}_{n,\rho(n),k,i} = \frac{(1-\beta_i)x_{1,n}p_{1,n}x_{2,\rho(n),k}p_{2,\rho(n),k}}{x_{1,n}p_{1,n}+(1-\beta_i)x_{2,\rho(n),k}p_{2,\rho(n),k}}$, $x_{1,n} = \frac{|h_{1,n,k}|^2}{\sigma_k^2}$ and $x_{2,\rho(n),k} = \frac{|h_{2,\rho(n),k,\ell}|^2}{\sigma_{d_\ell}^2}$.

Define $\mathcal{D}$ as the set of all non-negative $\{p_{1,n}\}$, $\{p_{2,\rho(n),k}\}$ and β_ℓ. The Lagrangian of $(P4)$ over $\mathcal{D}$ is

$$\begin{aligned}\mathcal{H}(\mathbf{p},\boldsymbol{\beta};\boldsymbol{\mu}) = &\sum_{\ell=1}^{L} \hat{R}_\ell + \mu_S\Big(P_S - \sum_{(n,\rho(n))\in S_k} p_{1,n}\Big) \\ &+ \sum_{k=1}^{K} \mu_{R,k}\Big(P_{R,k} - \sum_{(n,\rho(n))\in S_k} p_{2,\rho(n),k}\Big),\end{aligned} \tag{52}$$

with $\boldsymbol{\mu} = (\mu_S, \mu_{R,1}, \ldots, \mu_{R,K}) \geq 0$ being the vectors of the dual variables associated with the individual power constraints. Then, the dual function is

$$g(\boldsymbol{\mu}) \triangleq \max_{\{\mathbf{p},\boldsymbol{\beta}\}\in\mathcal{D}} \mathcal{H}(\mathbf{p},\boldsymbol{\beta};\boldsymbol{\mu}). \tag{53}$$

Next, we intend to perform the power allocation and PS ratio refinements using the previously computed values of relay–user coupling, sub-carrier–user assignment, and optimal sub-carrier pairing. In the following sub-sections, we present the methods for the refinement of the power allocation and the corresponding PS ratio.

1) Optimal Power Allocation for a given PS Ratio: Assuming a given PS ratio $\boldsymbol{\beta} \in \mathcal{D}$ is valid. Then, employing the KKT conditions to compute the dual function at given dual point, it can be readily seen that the optimal power allocation follows the same expression as in (35) and (36). We rewrite them as follows for ease of presentation

$$p_{1,n}^* = \begin{cases} c_{(n,\rho(n)),k,\ell}\, p_{2,\rho(n),k}^*, & \text{if } \ p_{2,\rho(n),k}^* > 0, \\ \left(\frac{1}{\mu_S} - \frac{1}{x_{1,n}}\right)^+, & \text{if } \ p_{2,\rho(n),k}^* = 0, \end{cases} \tag{54}$$

where $p_{2,\rho(n),k}^*$ and $c_{(n,\rho(n)),k,\ell}$ are shown at the top of next page. The Lagrange multipliers μ_S and $\mu_{R,k}$ can be updated using the sub-gradient method and are chosen so as to satisfy the individual power constraints in (48) and (49) respectively.

2) Optimal PS Ratio: Substituting (55) in (47) and assuming that equality holds at the optimal point, we obtain the following

$$\beta_\ell^* = \frac{\eta_\ell}{\sum \frac{\mathcal{M}}{1-\psi} \cdot \left(\frac{1}{1+\exp(-\theta p_{2,\rho(n),k}^* |h_{2,\rho(n),k,\ell}|^2 + \theta\varphi)} - \psi\right)}, \tag{57}$$

where $(n, \rho(n)) \in S_k$, $k = 1, 2, \ldots, K$, and $\ell = 1, 2, \ldots, L$.

The Lagrange multipliers in $\boldsymbol{\mu}$ are chosen to meet the power and PS ratio constraints in (47), (48), (49) and (50), and can be updated by the subgradient method. It can be easily observed that the computational complexity of the power refinement process is far more smaller than that of solving the dual problem. Thus, the overall complexity of our asymptotic solution is $\mathcal{O}(N^3(K \cdot L \cdot N^5)^\alpha)$.

V. Proposed Heuristic Solution

In this section, we propose a sub-optimal low-complexity technique to solve the problem in $(P1)$. Intuitively, the major factor impacting the system performance is the decision on the appropriate selection of the intended variables **s**, **a**, and $\boldsymbol{\phi}$. In the proposed asymptotic solution, computation of **s** and **a** follows a disjoint type of mechanism, which imposes some extra time-complexity. In this context, we provide a heuristic solution based on joint sequential assignment of the relay and sub-carrier pair that maximizes the equivalent $\mathcal{J}_{(n,n'),k,\ell}(\boldsymbol{\Lambda})$ metric in (39) to further reduce the time-complexity. The process for realizing this method is same as in the previous section, except for the step IV-B2).

To perform this task, two dependent algorithms are designed, as depicted at the bottom of this page (Algorithm 1) and top of next page (Algorithm 2). Note that Algorithm 1 returns the relay assignment variable **s** but an incomplete version of the carrier related variables $\boldsymbol{\phi}$ and **a**, since only the sub-carrier pairs associated with the assigned relays have been determined. In order to fully determine the sub-carrier pairs and assign them to the corresponding user, we propose a similar approach where the remaining values of $\{\mathcal{J}_{(n,n'),k,\ell}\}$ are sequentially maximized, taking into account the previous relay selection. The carrier refinement algorithm is summarized in Algorithm 2. To facilitate better understanding, visualizations of $\mathcal{J}_{(n,n'),k,\ell}(\boldsymbol{\Lambda})$ metric, and the in-between procedures corresponding to both the algorithms is depicted

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Algorithm 1 Relay and carrier assignment algorithm

Require:
- Number of users/destinations: L
- Number of relays: K
- Number of sub-carriers: N
- Function: $\mathcal{J}_{(n,n'),k,\ell}(\boldsymbol{\Lambda})$

Initialize: Iteration counter: $t = 1$

while t $\neq$ L **do**

  Find the relay, user and sub-carrier pair that maximizes the equivalent $\mathcal{J}_{(n,n'),k,\ell}(\boldsymbol{\Lambda})$ metric of the remaining un-assign possibilities,

$$\{(n^*, n'^*), k^*, \ell^*\} = \max\{\mathcal{J}_{(n,n'),k,\ell}(\boldsymbol{\Lambda})\} \tag{58}$$

  Assign $\phi_{(n^*,n'^*)} = 1$, $s_{k^*,\ell^*} = 1$ and $a_{n^*,\ell^*} = 1$.

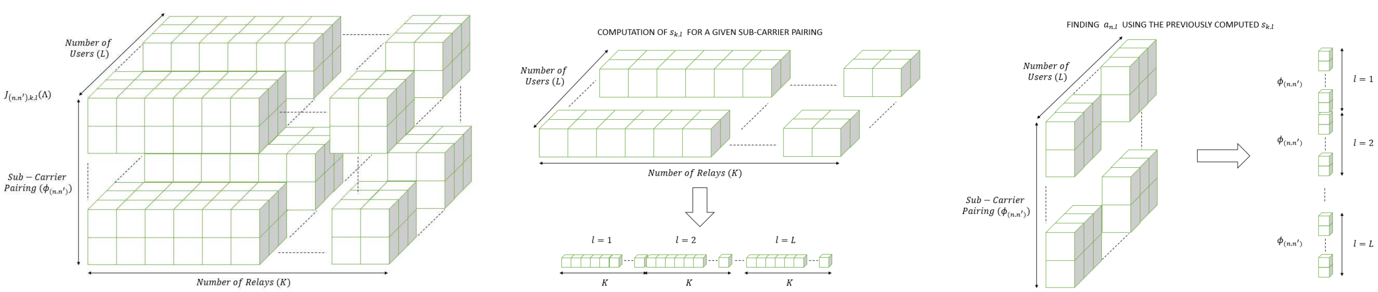
  $t = t + 1$.

end while

Return: Variables: $\boldsymbol{\phi}, \mathbf{s}, \mathbf{a}$.

$$p^*_{2,\rho(n),k} = \begin{cases} \left[\left(\frac{1}{c_{(n,\rho(n)),k,\ell}(1-\beta_\ell)x_{1,n}x_{2,\rho(n),k}}\right)\left(\frac{(1-\beta_\ell)^2 x_{1,n}x^2_{2,\rho(n),k} - 2\mu_S(c_{(n,\rho(n)),k,\ell}x_{1,n}+(1-\beta_\ell)x_{2,\rho(n),k})^2}{2\mu_S(c_{(n,\rho(n)),k,\ell}x_{1,n}+(1-\beta_\ell)x_{2,\rho(n),k})}\right)\right]^+, \\ \quad \text{if} \quad (1-\beta_\ell)^2 x_{1,n}x^2_{2,\rho(n),k} > 2\mu_S(c_{(n,\rho(n)),k,\ell}x_{1,n} + (1-\beta_\ell)x_{2,\rho(n),k})^2, \\ 0, \quad \text{if} \quad (1-\beta_\ell)^2 x_{1,n}x^2_{2,\rho(n),k} \le 2\mu_S(c_{(n,\rho(n)),k,\ell}x_{1,n} + (1-\beta_\ell)x_{2,\rho(n),k})^2. \end{cases} \tag{55}$$

$$c_{(n,\rho(n)),k,\ell} = \sqrt{\frac{\mu_{R,k}(1-\beta_\ell)x_{2,\rho(n),k}}{\mu_S x_{1,n}}}. \tag{56}$$



(a) (b) (c)

Fig. 3: (a) 3-D visualization of the $\mathcal{J}_{(n,n'),k,\ell}(\mathbf{\Lambda})$ matrix, (b) Visualization of Algorithm 1 depicting the steps involved in computation of $s_{k,\ell}$ for a given $\phi_{(n,n')}$, and (c) Visualization of Algorithm 2 depicting the steps involved in computation of $a_{n,\ell}$ using the $s_{k,\ell}$ computed previously in (b).

Algorithm 2 Carrier assignment refinement algorithm

Require:
- Number of users/destinations: L
- Number of relays: K
- Number of sub-carriers: N
- Function: $\mathcal{J}_{(n,n'),k,\ell}(\mathbf{\Lambda})$
- Relay selection: $\mathbf{s}$

Initialize: Iteration counter: $t = L + 1$

while $t \neq N$ **do**

    Find the sub-carrier pair and user that maximizes the equivalent channel gain of the remaining un-assign possibilities,

$$\{(n^*, n'^*), \ell^*\} = \max\{\mathcal{J}_{(n,n'),k,\ell}(\mathbf{\Lambda})\} \tag{59}$$

    Assign $\phi_{(n^*,n'^*)} = 1$ and $a_{n^*,\ell^*} = 1$.

    $t = t + 1$.

end while

Return: Variables: $\boldsymbol{\phi}, \mathbf{a}$.

in Fig. 3, placed at the top of this page. Finally, the power refinement and computation of optimal PS ratios are performed by using the method as in Section IV-C.

Suppose that the complexity of dual variable updates required and overall computation for such operation to perform the above mentioned task is in the order of $(K \cdot L \cdot N^2)^\alpha$. Then, the proposed heuristic method provides a remarkable computational time-complexity advantage of $\mathcal{O}(N^3 \cdot (K \cdot L \cdot N^2)^\alpha)$ over the BF approach with the order of $(K \cdot L)^{N!}$, significantly better than that of the proposed asymptotic approach with $\mathcal{O}(N^3 \cdot (K \cdot L \cdot N^5)^\alpha)$.

VI. Simulation Results

In this section, we present some simulation results to evaluate the performance of the proposed resource allocation and relay selection strategy. The simulations are performed for different parameter values to analyze the efficiency of proposed methods. For simplicity, we assume that the source and all the relays are subjected to the same power constraint i.e., $P_S = P_{R,1} = P_{R,2} = \ldots = P_{R,K}$, and the harvested energy demand at each user is assumed to be same i.e, $\eta_1 = \eta_2 = \ldots = \eta_\ell = \eta$ throughout this paper.

We consider the node distribution as shown in Fig. 4, where a source is placed at a fixed point $(0, 5)$ m, the K relay nodes are placed randomly within a 4 m^2 region inside the coordinates (4,4) and (6,6) between the source and the users' area. The users are placed randomly as well within a 4×10 m^2 area between the coordinates (6,0) and (10,10). For our analysis, we generate several random combinations of relay and user placements from the source, within these specified room dimensions. The ITU Radiocommunication Sector (ITU-R) P.1238 [42] channel model is employed with the central frequency assumption at 1.9 GHz to emulate a wireless broadband network. We consider frequency-selective channel model, with 5-multipath arrivals averaged according to the Poisson process and the root mean square (rms) delay is set to 36.3078 ns as per the aforementioned room dimensions. During the first hop transmission from the source to relays, and the second hop transmission from the selected relays to the intended users, the path-loss model (with shadowing) is adopted for each multipath signal according to the corresponding parameters provided in [42]. The signal fading from the source to the relays and from relays to the users follow the Ricean distribution with K-factor of 3.5. All the OFDM sub-carriers are assumed to experience a flat-fading and the total bandwidth is fixed to 20 MHz. The noise power at the

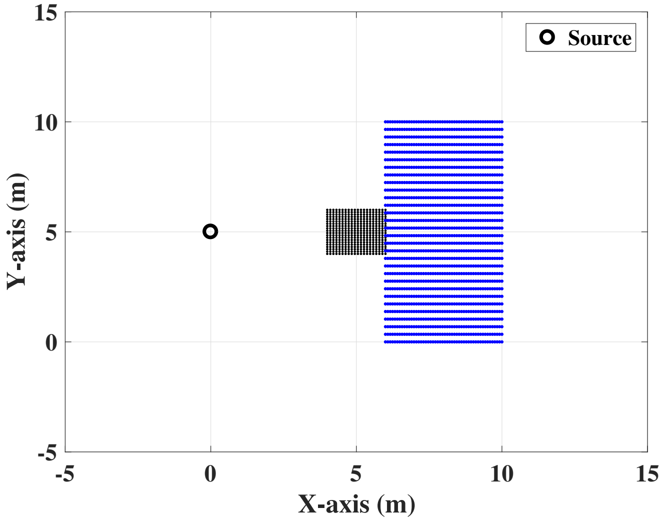


Fig. 4: The simulation scenario comprises of room with dimension of 10×10 m^2 which includes a source placed at (0,5) m, pool of relays within the region (4,4) m and (6,6) m, and users placed inside (6,0) m and (10,10) m.

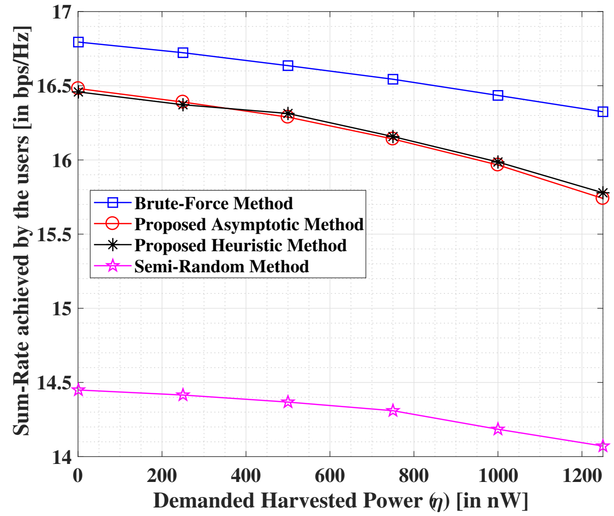


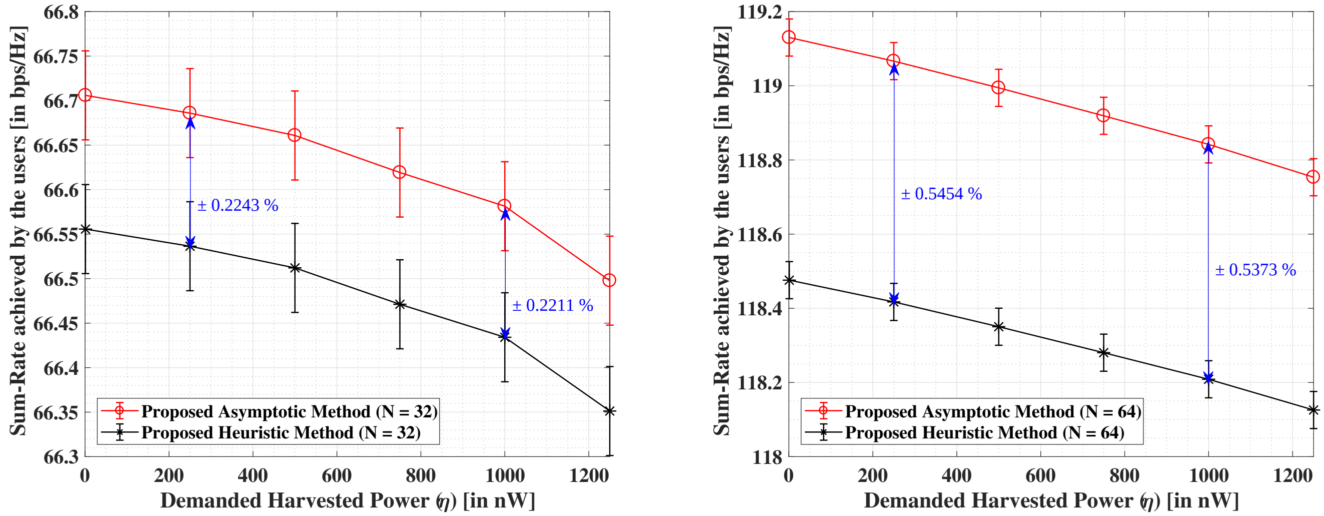
Fig. 5: Sum-rate achieved by the system versus different harvested energy demands for comparison of the algorithms with $P_S = P_{R,1} = P_{R,2} = P_{R,3} = 0.1$ W, $K = 3$, $L = 2$, and $N = 6$ for 50 Monte-Carlo random channels.

relay and user nodes is assumed the same and equal to -110 dBW. Concerning the non-linear EH model, the constants $\mathcal{M}$, θ, and φ are chosen to be $\mathcal{M} = \mathrm{E_{Max}}$ where $\mathrm{E_{Max}}$ is the maximum harvested energy at which the EH circuit is saturated (calculated individually for each user), $\theta = 1500$, and $\varphi = 0.0022$, respectively [35], [36].

As a benchmark, the performance of a semi-random resource allocation and relay selection approach is also presented. This baseline approach operates as follows: (i) Choose any valid combination of $\mathbf{s}$, $\mathbf{a}$, and $\boldsymbol{\phi}$ that satisfy (16), [(17), (18)], and [(19), (20)], respectively. (ii) Perform the OFDMA sub-carrier power allocation using the conventional WF approach [43]. (iii) Compute the PS ratios at all the user nodes and correspondingly the sum-rate of the system. However, the time-complexity of this method is very fast, but using this kind of hit-and-trial approach has the probability of getting the optimal choice as $1/(K \cdot L)^{N!}$. Therefore, this method is considered to provide a sub-optimal solution unless the hit-and-trial method coincides with optimal selection. To this end, it should be noted that due to unavailability of prior work in such a scenario, we use the semi-random scheme and the exhaustive search approach as the benchmarks to analyze the outcomes of the proposed algorithms.

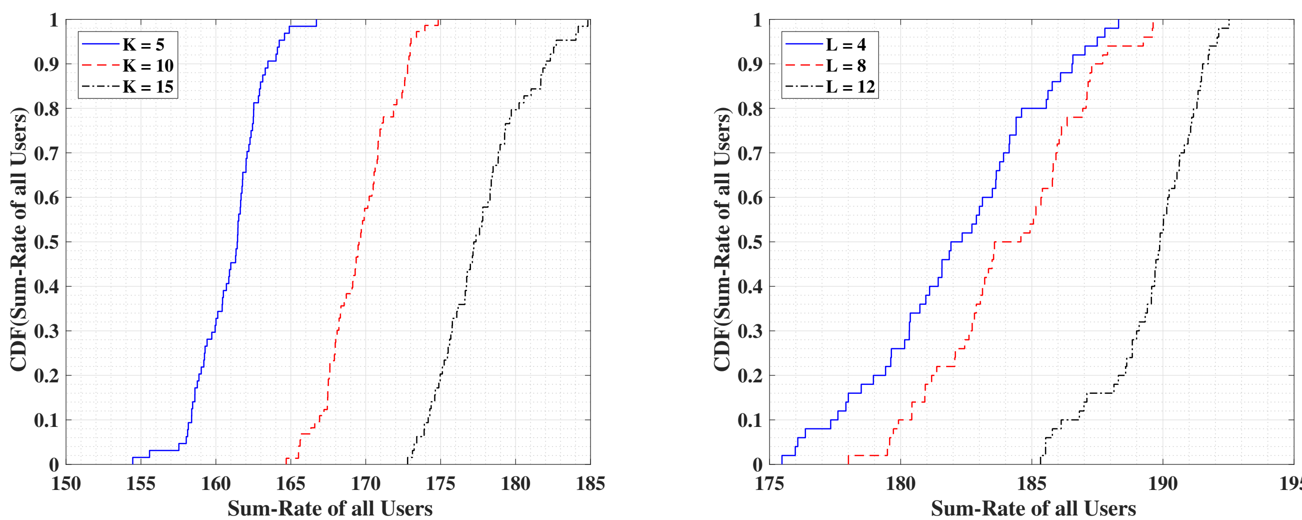
Fig. 5 depicts the effect on sum-rate of all users with increasing harvested energy demands for the proposed solutions. We set $K = 3$, $L = 2$, $N = 6$ and $P_S = P_{R,1} = P_{R,2} = P_{R,3} = 0.1$ W. The results are evaluated and averaged over 100 Monte-Carlo random channel conditions for both the hops. A comparison of the proposed asymptotic and heuristic methods is depicted. For comparison purposes, we illustrate the exhaustive search solution as well. Due to computational limitations, the exhaustive search method is difficult to realize for higher values of N (specifically for $N \geq 6$). It is found that the proposed asymptotically optimal method performs better at lower harvested energy demands with narrower duality gap in comparison to the optimal exhaustive search method. However, this gap widens with growing demands of harvested energy because the proposed algorithm involves joint computations of $\mathbf{s}$, $\mathbf{a}$, and $\boldsymbol{\phi}$ and due to the mixed-integer non-linear structure of this problem, the optimization task becomes cumbersome which reduces the optimality by some margin. Moreover, from the analysis presented in [39], it is clear that the time-sharing condition is not satisfied in case of Asymptotic method at lower N values. This leads to an increasing duality gap between the solutions of primal and dual problems. On the other hand, it is found that the heuristic method performs better than the proposed asymptotic method for higher harvested energy demands at lower N values. However, we cannot claim its asymptotic optimality from the outcomes herein due to sub-optimal results. Higher N values are analyzed in subsequent figures, showing the asymptotic behavior of the proposed method.

We set $K = 10$, $L = 8$, and $P_S = P_{R,1} = \ldots = P_{R,10} = 0.5$ W in Fig. 6(a) with $N = 32$, and in Fig. 6(b) with $N = 64$. We observe that the proposed techniques improve the system performance with the increasing values of N. These results are evaluated and averaged over 50 Monte-Carlo random channel conditions for both the hops, respectively. Due to high time complexity of the exhaustive search case for higher parameter values, we have demonstrated the results using the proposed asymptotic, and heuristic methods only; to prove that the system performance improves considerably for higher values of N. Note that a ± 5 % tolerance limit is set for the error bars indicated in both 6(a) and 6(b). It is found that for $N = 32$, the asymptotic solution performs approximately 0.22% better than the heuristic solution, while for $N = 64$, the asymptotic solution performs approximately 0.54% better than the heuristic solution. Additionally, the improvement in the system performances from $N = 32$ to $N = 64$ for the asymptotic method is of approximately 44% while for the heuristic method, the improvement is approximately 43.8%. From our observation, we find that there is a considerable



(a) (b)

Fig. 6: Sum-rate achieved by the system versus different harvested energy demands for (a) $N = 32$ and (b) $N = 64$, for 50 Monte-Carlo random channel conditions for both the hops, respectively.



(a) (b)

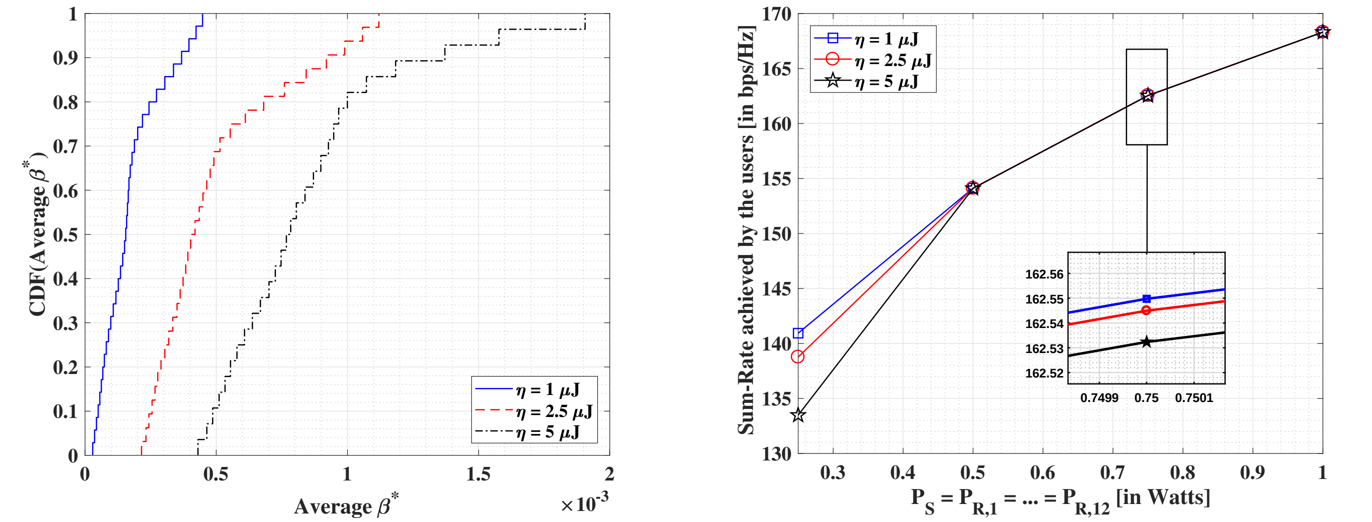
Fig. 7: (a) CDF plot of sum-rate of the system for different values of K with $L = 4$, $N = 64$ and (b) CDF plot of sum-rate of the system for different values of L with $K = 15$, $N = 64$.

improvement in the system performance for increasing values of N. Moreover, it is also noteworthy that the results from the proposed asymptotic algorithm are expected to asymptotically converge with the optimal solution, for sufficiently large value of N (ideally as $N \to \infty$) [39], as the time-sharing condition will be satisfied yielding an asymptotically zero duality gap between the corresponding Lagrange dual and primal problems.

Fig. 7(a) presents the performance of the proposed asymptotic algorithm in terms of the Cumulative Density Function (CDF) of users' sum-rate for different values of K with $L = 4$, $N = 64$ and $P_S = P_{R,1} = P_{R,2} = \ldots = P_{R,K} = 0.5$ W. The results are evaluated and averaged over 75 Monte-Carlo random channel conditions for both the hops. We set $\eta = 2.5$ μW. It is observed that there is a subtle increase in the sum-rate of the system by increasing the number of relays. Intuitively, more relay options provide an additional advantage due to better relay placements which may in-turn improve the system performance significantly.

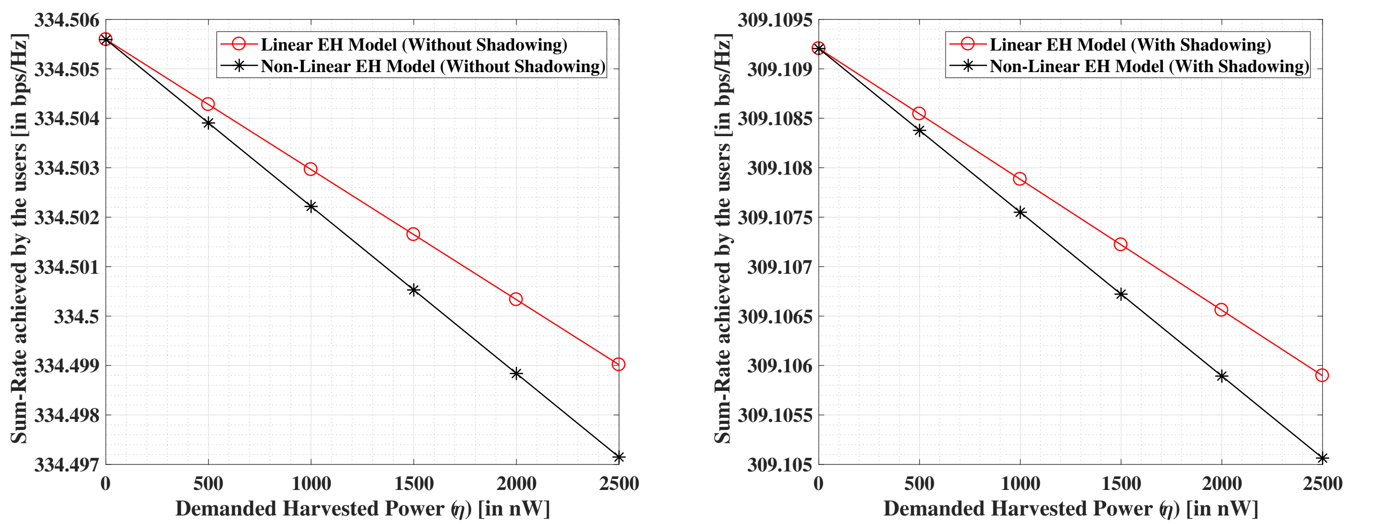
Fig. 7(b) shows the CDF of the sum-rate of the system for different values of L with $K = 15$, $N = 64$ and $P_S = P_{R,1} = \ldots = P_{R,15} = 0.75$ W. The harvested energy demand is assumed to be $\eta = 5$ μW. The results are evaluated using the proposed asymptotic method and are averaged over 50 Monte-Carlo random channel conditions for both the hops. It is observed that the system performance improves by an appreciable margin when there is an increase in the number of users provided that the network resources are sufficient. However, this gap is expected to reduce as the number of relays approaches the number of users.

Fig. 8(a) illustrates the CDF plot of the average PS ratio (β^*) for different values of harvested energy demands using



(a) (b)

Fig. 8: (a) CDF plot of the average PS ratio (β^*) for different harvested energy demands with $K = 12$, $L = 8$, and $N = 64$ and (b) Sum-rate achieved by the system versus different harvested energy demands with $K = 12$, $L = 10$ and $N = 64$.



(a) (b)

Fig. 9: Sum-rate achieved by the system versus different harvested energy demands to demonstrate the difference between the performances of Linear and Non-Linear EH models for : (a) Without Shadowing and (b) With Shadowing, with $K = 10$, $L = 8$ and $N = 128$ for 20 Monte-Carlo random channel conditions.

the proposed asymptotic algorithm. Here, we set $K = 12$, $L = 8$, $N = 64$ and $P_S = P_{R,1} = \ldots = P_{R,12} = 0.5$ W. The results are evaluated and averaged over 50 Monte-Carlo random channel conditions for both the hops. We observe that β^* increases significantly as the harvested energy demand keeps increasing. It is noteworthy that this depiction also accounts for the rate-energy (R-E) trade-off [17] which is well-known in the literature.

Fig. 8(b) depicts the plot of the sum-rate of the system against the different transmit power values at the source and relays. The simulations are performed using the proposed asymptotic method for various harvested energy demands. The results are evaluated and averaged over 20 Monte-Carlo random channel conditions for both the hops assuming $K = 12$, $L = 10$, and $N = 64$. It is noted that with increasing values of harvested energy, the sum-rate of the system decreases. However, with the increasing values of the source and relays' transmit powers, the sum-rate of the system increases considerably. This phenomenon is also an evidence of the R-E trade-off, as mentioned before. It is also noteworthy that once the harvested energy demand is fulfilled at the end-user, the sum-rate of the system nearly converge together when the individual transmit powers are sufficiently large.

In Fig. 9 we show that for $K = 10$, $L = 8$, $N = 128$, $\zeta = 0.8$ (for linear EH model), and $P_S = P_{R,1} = \ldots = P_{R,10} = 0.5$ W, the characteristic feature of a non-linear energy harvester in comparison to the linear one. It is seen that for increasing values of harvested energy demands, both

TABLE II: Sum-Rate (in bps/Hz) comparison of Direct Link only and Asymptotic Methods.

Method \ EH Demand	$\eta_\ell = 1$ nW	$\eta_\ell = 50$ nW	$\eta_\ell = 100$ nW	$\eta_\ell = 150$ nW	$\eta_\ell = 200$ nW
Direct link only	49.1369	11.6324	2.8345	1.3219	0
Proposed (without direct link)	147.1453	147.1448	147.1443	147.1438	147.1433

the linear and non-linear energy harvesting models show a decreasing trend. From Fig. 9(a) and Fig. 9(b), it is clear that there is a significant impact of shadowing on energy harvesting. Specifically, we observe that there is 21.6% (approximately) difference between the results of the EH models for the cases with and without shadowing, when $\eta = 1$ nW.

Considering $K = 10$, $L = 6$, $N = 64$, and $P_S = P_{R,1} = \ldots = P_{R,10} = 0.5$ W, we show in Table II that the contribution from the direct link at the end-users is negligible in comparison to the proposed framework with cooperative relaying. For the case with direct link, separate CSIs are generated according to the ITU-R framework (as in the two aforementioned hops). Then the sub-carriers are allocated based on maximum channel gain followed by WF technique. The PS ratio at each user is computed in a similar manner as for the proposed technique. Each value in Table II is computed after averaging over 20 Monte-Carlo experiments. The results prove that the system with only direct active link is incapable of providing sufficient sum-rate at end-users with increasing EH demands.

VII. Conclusion and Future Directions

In this paper, we have proposed a novel resource allocation and relay selection scheme for cooperative multi-user multi-relay OFDMA networks with SWIPT capabilities at the end-users. The problem formulation for maximization of sum-rate of all the users was found to be non-linear with mixed-integer programming and hence an explicit low-complexity solution could not be obtained. By exploiting the availability of time-sharing criteria and choosing sufficiently large number of sub-carriers, we efficiently solved this combinatorial problem using the dual method with polynomial complexity. In this context, we proposed suitable methods which can noticeably improve the system performance and illustrated the effectiveness of the proposed algorithms via numerical results, where we showed the asymptotic optimality of the solutions for sufficiently large parametric values of the number of OFDM sub-carriers, relays and users. This work can be further extended to many promising directions including the study of current framework with non-orthogonal multiple access (NOMA) instead of OFDM, and incorporation of multi-antenna systems where each device in the system may be assumed to have multiple antennas. The latter case may offer several interesting problems like antenna-selection, and beamform designing for SWIPT in MIMO-OFDM/MIMO-NOMA systems. Additional benefits from contributions by the direct links (corresponding to signals in the first hop) at the end-users may also be leveraged.

Appendix A
Derivation of Optimal Solution $\boldsymbol{p}^*$ in (35) and (36)

Using the short notations: $x_1 = \frac{|h_{1,n,k}|^2}{\sigma_k^2}$, $x_2 = \frac{|h_{2,n',k,\ell}|^2}{\sigma_{d_\ell}^2}$, $p_1 = p_{1,n}$ and $p_2 = p_{2,n',k}$, the derivative of (25) with respect to p_1 is represented as

$$\frac{\partial \mathcal{L}_{(n,n'),k,\ell}}{\partial p_1} = -\lambda_S + \frac{1}{2} \cdot \frac{(1-\beta_\ell)^2 \cdot x_1 \cdot x_2^2 \cdot p_2^2}{(x_1 \cdot p_1 + (1-\beta_\ell) \cdot x_2 \cdot p_2)} \cdot \frac{1}{(x_1 \cdot p_1 + (1-\beta_\ell) \cdot x_2 \cdot p_2 + (1-\beta_\ell) \cdot x_1 \cdot x_2 \cdot p_1 \cdot p_2)}. \tag{60}$$

Similarly, the derivative of (25) with respect to p_2 is represented as

$$\frac{\partial \mathcal{L}_{(n,n'),k,\ell}}{\partial p_2} = -\lambda_{R,k} + \frac{1}{2} \cdot \frac{(1-\beta_\ell) \cdot x_1^2 \cdot x_2 \cdot p_1^2}{(x_1 \cdot p_1 + (1-\beta_\ell) \cdot x_2 \cdot p_2)} \cdot \frac{1}{(x_1 \cdot p_1 + (1-\beta_\ell) \cdot x_2 \cdot p_2 + (1-\beta_\ell) \cdot x_1 \cdot x_2 \cdot p_1 \cdot p_2)}. \tag{61}$$

Firstly, we consider the condition when both p_1^* and p_2^* are positive. Equating (60) and (61), we get

$$\lambda_{R,k} \cdot (1-\beta_\ell) \cdot x_2 \cdot p_2^2 = \lambda_S \cdot x_1 \cdot p_1^2. \tag{62}$$

This yields

$$c = \sqrt{\frac{\lambda_{R,k} \cdot (1-\beta_\ell) \cdot x_2}{\lambda_S \cdot x_1}}. \tag{63}$$

In order to let $p_1^* > 0$, the factor c should be positive, which is true in general. Substituting (63) into (60), we get

$$p_2^* = \left(\frac{1}{c_{(n,n'),k,\ell}(1-\beta_\ell)x_1x_2} \right) \cdot \left(\frac{(1-\beta_\ell)^2 x_1 x_2^2 - (2\lambda_S)(c_{(n,n'),k,\ell}x_1 + (1-\beta_\ell)x_2)^2}{2\lambda_S(c_{(n,n'),k,\ell}x_1 + (1-\beta_\ell)x_2)} \right). \tag{64}$$

If the value of (64) is negative, the p_2^* should be set to zero. For such cases, the optimal power allocation in the first hop should follow the expression of conventional water-filling approach

$$p_1^* = \left(\frac{1}{\lambda_S} - \frac{1}{x_1} \right)^+. \tag{65}$$

Thus, the optimality of solution $\boldsymbol{p}^*$ in (35) and (36) is proved.

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