Time-Switching based Simultaneous Wireless Transmission of Information and Energy (Wi-TIE) for Relaying Systems with Caching Architecture

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Abstract. In this chapter, we investigate the performance of a time switching (TS) based energy harvesting model for cache-assisted simultaneous wireless transmission of information and energy (Wi-TIE). In the considered system, a relay which is equipped with both caching and energy harvesting capabilities helps a source to convey information to a destination. Based on the time-splitting mechanism, we analyze the effect of caching on the system performance in terms of stored energy at the relay and the relay-destination link throughput. In particular, two optimization problems are formulated to maximize the energy stored at the relay and the relay-destination throughput. By using KKT method, closed-form solution are obtained for both the problems. Finally, the performance of the proposed design under various operating conditions and parameter values is illustrated using numerical results.

Keywords: Simultaneous wireless transmission of information and energy (Wi-TIE), relay systems, caching, decode-and-forward (DF), energy harvesting (EH).

1 Introduction

The exponential increase in the usage of wireless devices like smart-phones, wearable gadgets, or connected vehicles, has not only posed substantial challenges to meet the performance and capacity demands [1], but also presented some serious environmental concerns with alarming CO_2 emissions [2]. By the end of 2020, this number is expected to grow manifold and is speculated to cross 50 billion [3]. Recent developments in the upcoming paradigm of Internet-of-Things (IoT) emphasize on the interconnection between equipments, commodities, functionalities, and customers, with or without slightest human mediation [4]. Since most of these connecting operations involve wireless sensor nodes or equivalent battery-limited devices that may not be continuously powered, energy becomes

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a sparse and pivotal resource and hence management of energy becomes crucial. In order to address the aforementioned issues, emerging technologies like energy harvesting [5], traffic-aware service provisioning [6], e.g., optimized protocol [7], caching [8], etc., and wireless multi-casting [9] shows great promise.

The increasing number of devices provides an alluring opportunity of relaying the desired signal from the transmitter to the far placed receiver and vice-versa. The nodes facilitating the relay techniques may adopt various strategies to expand the network coverage area and diversity gains [10–12]. In this context, *regenerative* (e.g., decode-and-forward (DF) [13, 14]), and *non-regenerative* (e.g., amplify-and-forward (AF) [15]) relay protocols are two of the widely adopted strategies. A relay employing the DF protocol is capable of decoding the information from the received signal and then transmits the re-encoded message to the end-receiver. On the other hand, the relay operating on the AF protocol directly amplifies the received noisy signal with a suitable weight and this amplified signal is then forwarded to the end-receiver. Considering the aspect of high secrecy rate [16] and comparison of average performance between the two modes in terms of capacity measure [17], the regenerative relaying strategy (i.e., DF protocol) shows great promise with paramount practical interest.

Efficient power management is another key objective of future wireless network that is considered for improving the availability and traffic capacity. In this vein, simultaneous wireless transmission of information and energy (Wi-TIE) is a new paradigm which allows concurrent data reception and energy harvesting from the same Radio Frequency (RF) input signal. Considering rapid drainage of battery sources in wireless devices, it has almost become essential to take up such techniques in order to compensate for this cause. It is noteworthy that Wi-TIE has huddled ample attention in the recent years and is expected to serve the emerging technologies at large [18, 19]. Since the conventional receiver architectures are capable of performing information decoding with focus on increasing the data rate only and are unable to harvest energy, therefore this calls for an alteration in the receiver architecture to support Wi-TIE [20]. In order to address this issue, the researchers in [21, 22] have proposed two receiver architectures namely time switching (TS) and power splitting (PS) schemes. In TS, the received signal is switched between the information decoder and energy harvester such that for a first few fraction of the time period, all of the signal power is utilized for information decoding while for the remaining fraction of time period, all of the signal power is used for harvesting energy. However in PS, the received signal power is distributed between the information decoder and energy harvester so that a fraction of received signal power is used for information decoding while the rest is utilized for harvesting energy simultaneously.

Congestion in the network traffic is often observed during the peak hours while the resources are in idle state during the off-peak hours [23]. The cause of this network blockage is mainly due to the fact that replicas of a common content may be demanded by various mobile users [8]. This phenomenon is generally referred to as content diversity [24] or content reuse [25]. Exploiting the plentiful network resources during off-peak hours is one of the ways to reduce the congestion in the network traffic. Duplication of the content, also termed as content placement or caching, is carried out during the off-peak hours by leveraging the information from the memories distributed across the network. Thus, caching does not only brings content near to the users by retrieving the content beforehand but also reduces the network blockage significantly with improved experience as anticipated by the end-user [25–27].

A generic concept outlining joint energy harvesting (EH) and caching was recently proposed as GreenDelivery Framework in [2] and for the IoT technologies in [4]. An online energy-efficient power control scheme for EH with caching capability was developed by the authors in [28]. However, these works highlight a general overview of EH with caching and does not consider the aspect of Wi-TIE along with caching in RF regime using relay systems. In this chapter, we focus on developing a framework to realize the implementation of Wi-TIE with caching architecture to support future technologies. Intuitively, PS scheme cannot be considered as one of the best receiving modes due to its complex hardware design challenges of the power splitter [22, 29]. Moreover, the incorporation of caching architecture also pose a significant demand in terms of meeting the requirements of the device size. Consequently, we consider that a relay employs the TS scheme along with an additional caching architecture for the sake of tractability in hardware constraints [30]. In this regard, *we envision that caching with Wi-TIE will come in handy to comply with both the capacity and energy needs concurrently.*

The main contributions of this chapter are four-folds, listed as follows

1. We investigate the performance of cache-assisted Wi-TIE relaying systems under time-sharing scheme and DF relaying protocol. To best of the authors' knowledge, a detailed study of Wi-TIE with caching in relaying systems has not been addressed so far in the literature.
2. To proceed, we study the problem for maximization of the link throughput between the relay and the destination subjected to minimum energy stored at the relay and the quality-of-service (QoS) constraints. By using the KKT, closed-form solution of the formulated problem is obtained.
3. Next, we address the problem to maximize the energy stored at the relay subject to minimum data demand by the destination and the stored energy constraint at the relay, in addition to the QoS constraint. Following the similar trend as above, we choose another convention for time distribution within a time period, and achieve the closed-form solutions using the KKT conditions.
4. Finally, we investigate the solutions obtained for the preceding problems by using simulation results and provide an analysis of the corresponding results. We illustrate the influence of parameters variations on the proposed system in addition. Furthermore, we discuss about the practical implementation and future directions for the proposed work.

The remainder of this chapter is organized as follows. In Section 2, we introduce the system model and define the relevant variables to develop the analytical framework. The problem formulations for maximization of link throughput between the relay and destination, and maximization of energy stored at the relay are presented in Section 3 and Section 4, respectively. In Section 5, we demonstrate some simulation results to illustrate the performance of the proposed relaying system with Wi-TIE and caching architectures. Finally, we conclude this chapter in Section 6.

2 System Model

We consider a generic TS based Wi-TIE system in which a DF relay equipped with caching and Wi-TIE capabilities helps to convey information from one source to a destination, as depicted in Fig. 1. Due to limited coverage, there is no direct connection between the source and the destination. The considered model can find application on the downlink where the base station plays the source's role and sends information to a far user via a small- or femto- cell base station. The relay node is equipped with an information decoder, an energy harvester, and a cache in order to store or exchange information as shown in Fig. 2. Assume the overall time interval for establishing a reliable communication to be 'T' seconds. The link between the transmitter and relay with cache is active for some fraction of T seconds, while the link between the relay and destination is active for the remaining part. At the relay, the TS scheme is employed so that the received signal is first provided to the the energy harvester for some fraction of the time allocated for transmitter-relay communication link, and then to information decoder for the remaining fraction. This type of mechanism is also referred in literature as harvest-then-forward protocol [31].

We define the information transfer rates as: R_1 from the source to the relay, R_2 from the cache to the relay, and R_3 from the relay to the destination. Let $h_{S,R}$ denote the channel gain between the source and the relay and $h_{R,D}$ denote

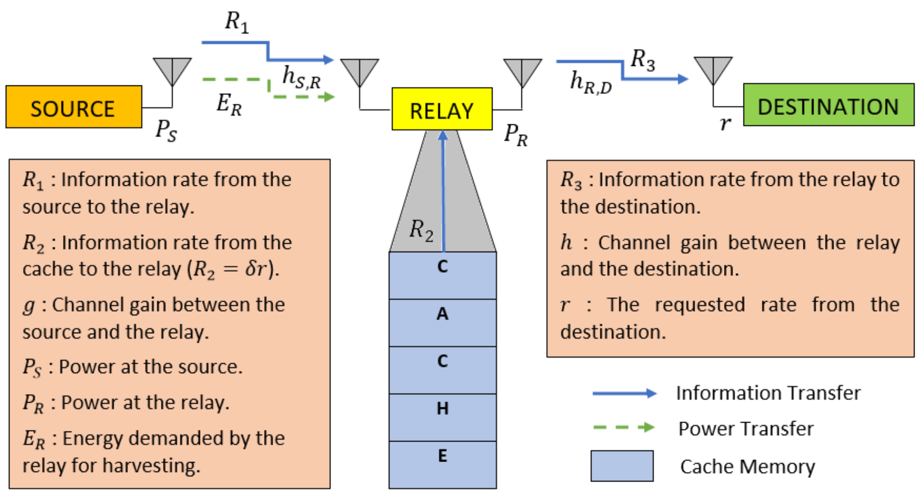

Fig. 1. System model for Wi-TIE with caching.

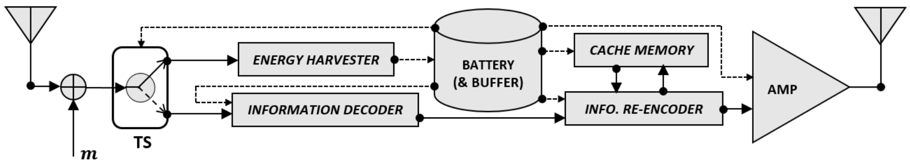

Fig. 2. Proposed DF relay transceiver design for hybrid Wi-TIE and caching with time switching (TS) architecture.

the channel gain between the relay and the destination. Following the similar trend, we define $d_{S,R}$ and $d_{R,D}$ as the distance between the source and relay, and the distance between the relay and destination, respectively. Furthermore, P_S and P_R denote the transmit power at the source and the relay, respectively, and E_R denotes the harvested energy at the relay. E_R denotes the energy harvested by the relay.

Assuming the case where the transmitter provides both information and energy to the relay. Then, the signal received at the relay when the transmitter transmits the symbols $x \in \mathbb{C}$, such that $\mathbb{E}\{|x|^2\} = 1$ where $\mathbb{E}\{\cdot\}$ and $|\cdot|$ denotes the statistical expectation and the norm respectively, is given by

$$y_R = \sqrt{P_S}\ d_{S,R}^{-\vartheta/2}\ h_{S,R}\ x\ +\ n_R, \tag{1}$$

where ϑ is the path loss exponent, n_R is the additive white Gaussian noise (AWGN) at the relay which is an independent and identically distributed (i.i.d.) complex Gaussian random variable with zero mean and variance $\sigma_{n_R}^2$.

Similarly, the signal received at the destination node transmitted by the relay is given by

$$y_D = \sqrt{P_R}\ d_{R,D}^{-\vartheta/2}\ h_{R,D}\ \tilde{x}\ +\ n_D, \tag{2}$$

where $\tilde{x}$ is the transmit symbol from the relay which can be either the decoded source symbol or a symbol from the cache such that $\mathbb{E}\{|\tilde{x}|^2\} = 1$, and n_D is the AWGN at the destination node which is an i.i.d. complex Gaussian random variable with zero mean and variance $\sigma_{n_D}^2$.

The effective signal-to-noise ratio (SNR) at the relay is given by

$$\gamma_{S,R} = \frac{P_S\ d_{S,R}^{-\vartheta}\ |h_{S,R}|^2}{\sigma_{n_R}^2}. \tag{3}$$

By using the Gaussian code book, the achievable information rate on the source-relay link is given by

$$R_1 = B\ \log_2(1 + \gamma_{S,R}), \tag{4}$$

where B is the channel bandwidth.

In addition to the information sent from the source, the relay has access to the information stored in its cache to serve the destination. For robustness, we assume that the relay does not have information about content popularity. Therefore, it will store $0 \leq \delta \leq 1$ parts of every file in its cache [23]. For convenience, we call δ as the caching coefficient throughout the paper. This caching scheme will serve as the lower bound benchmark since the content popularity in practice might not be uniform but, e.g., Zipf distribution.

When the destination requests a file from the library, δ parts of that file is already available at the relay's cache. In other words, the relay's cache can provide, in addition to the source-relay link, an information rate

$$R_2 = \delta r, \tag{5}$$

where r is the information rate demanded by the destination.

The effective SNR at the destination is given by

$$\gamma_{R,D} = \frac{P_R \, d_{R,D}^{-\vartheta} \, |h_{R,D}|^2}{\sigma_{n_D}^2}. \tag{6}$$

The achievable information rate at the destination is given by

$$R_3 = B \, \log_2(1 + \gamma_{R,D}). \tag{7}$$

To proceed, we assume a convention for allocation of time fractions for: *i)* energy harvesting at the relay, *ii)* information decoding at the relay, and *iii)* information decoding at the destination. Assuming the overall time interval for establishing a reliable communication to be 'T' seconds, we adopt the following notations. The link between the transmitter and relay with cache is considered to be active for a fraction of βT seconds, while the link between the relay and destination is active for the remaining $(1-\beta)T$, where $0 \leq \beta \leq 1$. As mentioned earlier, since the relay adopts a TS type of scheme for Wi-TIE, we assume that the energy harvesting at the relay takes place for a fraction of $\alpha\beta T$ seconds and the information decoding at the relay takes place for a fraction of $(1-\alpha)\beta T$ seconds, where $0 \leq \alpha \leq 1$. The convention for time interval mentioned above can be expressed as in Fig. 3. For simplicity, we assume normalized time to use energy and power interchangeably without loss in generality.

The harvested energy at the relay is given by

$$E_R = \zeta\alpha\beta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2), \tag{8}$$

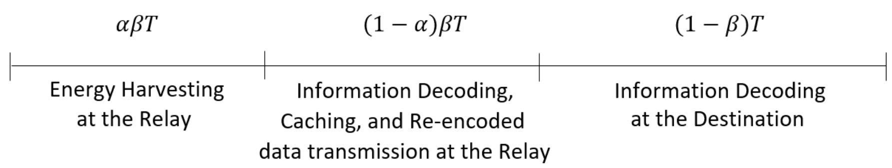

Fig. 3. Convention assumed for distribution of time to investigate the Rate Maximization Problem.

where ζ is the energy conversion efficiency of the receiver.

In the following couple of sections, we address the problems for maximizing the serving information rate between the relay and the destination, and maximizing the energy stored at the relay, respectively.

3 Maximization of the serving information rate

In this section, we consider the problem of maximization of the serving information rate between the relay and the destination, while ensuring that the harvested energy at the destination is above a given threshold and that the total transmit powers at the transmitter and relay does not exceed a given limit. The corresponding optimization problem (P1) can be expressed as

$$(P1): \max_{\alpha,\beta,P_R} \quad (1-\beta)R_3 \tag{9}$$

$$\text{subject to}: \quad (C1): (1-\alpha)\beta(R_1+R_2) \geq (1-\beta)R_3, \tag{10}$$

$$(C2): (1-\beta)P_R \leq E_R + E_{ext}, \tag{11}$$

$$(C3): 0 < P_S \leq P^{\star}, \tag{12}$$

$$(C4): 0 \leq \alpha \leq 1, \tag{13}$$

$$(C5): 0 \leq \beta \leq 1. \tag{14}$$

where E_{ext} is the external energy required at the relay for further transmission of the signal, and $P^{\star}$ is the maximum power limit at the transmitter. This is a non-linear programming problem involving joint computations of α, β, and P_R, which introduces intractability. Therefore, we propose to solve this problem using the Karush-Kuhn-Tucker (KKT) conditions.

The final solution can be postulated according to the following theorem

THEOREM 1: If $\lambda_1 \neq 0 \implies G(x, P_R) = 0$; $\lambda_2 \neq 0 \implies H(x, P_R) = 0$, then we obtain the following optimal values

$$P_R = \Big(\exp\big(\mathcal{W}(\mathcal{A}\exp(-\log^2(2)) + \log(2)) + \log^2(2)\big) - 1\Big)\left(\frac{\sigma^2_{n_D}}{d^{-\vartheta}_{R,D}\ |h_{R,D}|^2}\right), \tag{15}$$

where $\mathcal{A} = \frac{(\ln(2)d^{-\vartheta}_{R,D}|h_{R,D}|^2)(\zeta(P_S d^{-\vartheta}_{S,R}|h_{S,R}|^2+\sigma^2_{n_R}))}{\sigma^2_{n_D}}$ and $\mathcal{W}(\cdot)$ is the Lambert W function [32].

$$\beta = \frac{\varphi - E_{ext}(B\log_2(1+\gamma_{S,R}) + \delta r)}{\varphi - \zeta(P_S d^{-\vartheta}_{S,R}|h_{S,R}|^2 + \sigma^2_{n_R})(B\log_2(1+\gamma_{S,R}) + \delta r)}, \tag{16}$$

where $\varphi = P_R(B\log_2(1+\gamma_{S,R}) + \delta r) - \zeta(P_S d^{-\vartheta}_{S,R}|h_{S,R}|^2 + \sigma^2_{n_R})B\log_2(1+\gamma_{R,D})$.

$$\alpha = \frac{(1-\beta)P_R - E_{ext}}{\zeta\beta(P_S d^{-\vartheta}_{S,R}|h_{S,R}|^2 + \sigma^2_{n_R})}. \tag{17}$$

The detailed analysis to obtain the optimal solutions mentioned above, is provided in the Appendix A of this chapter. ■

Thus, we find that the objective in (9) is maximized using the solution proposed above. However, the solution cannot be guaranteed as the global optimal due to non-linearity of the problem. In the next section, we formulate another problem to maximize the energy stored at the relay.

4 Maximization of the Energy Stored at the Relay

Herein, we formulate an optimization problem to maximize the energy stored at the relay, while ensuring that the requested rate between relay-destination is above a given threshold and that the total transmit powers at the transmitter and relay do not exceed a given limit. To proceed, we assume that the overall time interval for establishing a reliable communication to be 'T' seconds as in the previous case. Following notations are assumed in order to make the problem formulation more tractable. The link between the transmitter and relay with cache is considered to be active for a fraction of $(\theta + \phi)T$ seconds, while the link between the relay and destination is active for the remaining $(1 - (\theta + \phi))T$, where $0 \leq \theta + \phi \leq 1$. As mentioned earlier, since the relay adopts a TS type of scheme for Wi-TIE, we assume that the energy harvesting at the relay takes place for a fraction of θT seconds and the information decoding at the relay takes place for a fraction of ϕT seconds. The convention for selection of time interval in order to analyze the Stored Energy Maximization Problem is as expressed in Fig. 4. For simplicity, we assume normalized time to use energy and power interchangeably without loss in generality, same as in the previous case.

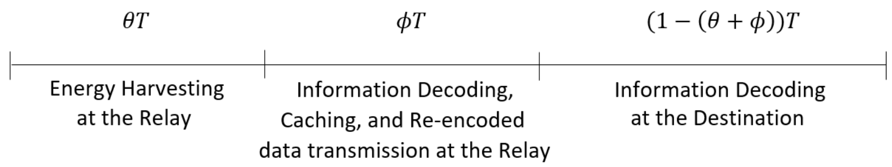

Fig. 4. Convention assumed for distribution of time to investigate the Stored Energy Maximization Problem.

The harvested energy at the relay is given by

$$E_R = \zeta\theta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2), \tag{18}$$

where ζ is the energy conversion efficiency of the receiver, as defined previously.

We now consider the problem of maximization of the energy stored at the relay, while ensuring that the requested rate between relay-destination is above a given threshold and that the total transmit powers at the transmitter and relay does not exceed a given limit. The corresponding optimization problem (P2) can be expressed as

$$
\begin{aligned}
(P2):\ & \max_{\theta,\phi,P_R} \quad [\zeta\theta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2+\sigma_{n_R}^2)-(1-(\theta+\phi))P_R]^+ && (19)\\
\text{subject to}:\ & (C1): \phi(R_1+R_2)\geq(1-(\theta+\phi))R_3, && (20)\\
& (C2): (1-(\theta+\phi))P_R\leq E_R+E_{ext}, && (21)\\
& (C3): (1-(\theta+\phi))R_3\geq r, && (22)\\
& (C4): 0<P_S\leq P^\star, && (23)\\
& (C5): 0\leq\theta+\phi\leq 1. && (24)
\end{aligned}
$$

where $(x)^+=\max(0,x)$, E_{ext} is the external energy required at the relay for further transmission of the signal, and $P^\star$ is the maximum power limit at the transmitter. This is a non-linear programming problem involving joint computations of θ, ϕ, and P_R, which introduces intractability. Therefore, we propose to solve this problem using the Karush-Kuhn-Tucker (KKT) conditions. The two possible solution are as mentioned in the following theorems, respectively

THEOREM 2: If $\lambda_1\neq 0 \implies G(\theta,\phi,P_R)=0$; $\lambda_2=0 \implies H(\theta,\phi,P_R)\neq 0$; $\lambda_3\neq 0 \implies I(\theta,\phi,P_R)=0$, then we obtain the following optimal values

$$P_R^\dagger=(\nu-1)\left(\frac{\sigma_{n_D}^2}{d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right), \tag{25}$$

$$\phi^\dagger=\frac{r}{B\log_2(1+\gamma_{S,R})+\delta r}, \tag{26}$$

$$\theta^\dagger=1-r\left(\frac{1}{B\log_2(1+\gamma_{S,R})+\delta r}+\frac{1}{B\log_2\left(1+\frac{P_R^\dagger d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2}\right)}\right), \tag{27}$$

where

$$\nu=\exp\left(\mathcal{W}(-\mathcal{A}\exp(-\log^2(2))+\log(2))+\log^2(2)\right) \tag{28}$$

with $\mathcal{A}=\ln(2)-\left(\frac{\zeta}{\sigma_{n_D}^2}\right)(\ln(2)d_{R,D}^{-\vartheta}|h_{R,D}|^2)(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2+\sigma_{n_R}^2)$. ■

THEOREM 3: If $\lambda_1\neq 0 \implies G(\theta,\phi,P_R)=0$; $\lambda_2\neq 0 \implies H(\theta,\phi,P_R)=0$; $\lambda_3\neq 0 \implies I(\theta,\phi,P_R)=0$, then the following values are optimal

$$P_R^*=(\eta_L-1)\left(\frac{\sigma_{n_D}^2}{d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right), \tag{29}$$

$$\phi^*=\frac{r}{B\log_2(1+\gamma_{S,R})+\delta r}, \tag{30}$$

$$\theta^*=\frac{rP_R^*-E_{ext}B\log_2\left(1+\frac{P_R^* d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2}\right)}{\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2+\sigma_{n_R}^2)}, \tag{31}$$

where η_L = Largest Root of $[\mathcal{A} + B\log_2(\eta)\big(\mathcal{B} + \mathcal{C}\eta + \mathcal{D}B\log_2(\eta)\big) = 0]$, with $\mathcal{A} = a \cdot b \cdot r$, $\mathcal{B} = -a \cdot b - b \cdot r \cdot \left(\frac{\sigma^2_{n_D}}{d^{-\vartheta}_{R,D}|h_{R,D}|^2}\right) + a \cdot r$, $\mathcal{C} = b \cdot r \cdot \left(\frac{\sigma^2_{n_D}}{d^{-\vartheta}_{R,D}|h_{R,D}|^2}\right)$, and $\mathcal{D} = -b \cdot E_{ext}$, where $a = \zeta(P_S d^{-\vartheta}_{S,R}|h_{S,R}|^2 + \sigma^2_{n_R})$, and $b = B\log_2(1+\gamma_{S,R}) + \delta r$. ∎

The detailed analysis to obtain the optimal solutions mentioned in the two theorems above, is provided in the Appendix B of this chapter. ∎

To summarize the solutions obtained above, we propose the following algorithm to maximize the stored energy in the relay supporting Wi-TIE - Caching system (MSE-WC Algorithm)

Algorithm. *MSE-WC Algorithm*

Input: The parameters $h_{S,R}$, $h_{R,D}$, δ, r, and E_{ext}.
Output: The maximized value of energy stored at the relay: $\{E_S\}$.

1. : Initialize: $\zeta \in (0,1]$, $P_T \in (0, \varepsilon P_{Max}]$, $0.5 < \varepsilon < 1$, $\sigma^2_{n_R} = 1$, and $\sigma^2_{n_D} = 1$.
2. : Compute $P^{\dagger}_R$, $\phi^{\dagger}$, and $\theta^{\dagger}$ using (25), (26), and (27) respectively.
3. : Define: $E^{\dagger}_S = \zeta\theta^{\dagger}(P_S d^{-\vartheta}_{S,R}|h_{S,R}|^2 + \sigma^2_{n_R}) - (1 - (\theta^{\dagger} + \phi^{\dagger}))P^{\dagger}_R$.
4. : Compute P^*_R, ϕ^*, and θ^* using (29), (30), and (31) respectively.
5. : Define: $E^*_S = \zeta\theta^*(P_S d^{-\vartheta}_{S,R}|h_{S,R}|^2 + \sigma^2_{n_R}) - (1 - (\theta^* + \phi^*))P^*_R$.
6. : $E_S = \max(E^{\dagger}_S, E^*_S)$.
7. : **return** E_S.

The algorithm proposed above returns the maximized value of the objective function as its output. First, we initialize all the necessary values as indicated in 1). Then, we compute the optimal values of $P^{\dagger}_R$, $\phi^{\dagger}$, and $\theta^{\dagger}$ in 2), and define the energy stored at the relay in 3). 2) and 3) corresponds to the solutions obtained for Case VI during the analysis. Similarly, we find the optimal values of P^*_R, ϕ^*, and θ^* in 4), and define the energy stored at the relay in 5) accordingly. 4) and 5) corresponds to the solutions obtained for Case VIII during the analysis. Next, we find the maximum of the two computed local optimal solutions for the energy stored at the relay, which in turn maximizes the objective function. It should also be noted that the solutions proposed in (P2) for maximizing the energy stored at the relay are not necessarily global optimum, as the problem is non-linear in nature. However, the KKT conditions guarantees the local optimal solutions.

5 Simulation Results

In this section, we evaluate the performance of the proposed system for the solutions presented in this chapter. We consider a total bandwidth of $B = 1$ MHz, $\zeta = 0.80$, $\sigma^2_{n_R} = 0$ dBW, and $\sigma^2_{n_D} = 0$ dBW. Throughout the simulations, we assume that the channel coefficients are i.i.d. and follows Rayleigh distribution.

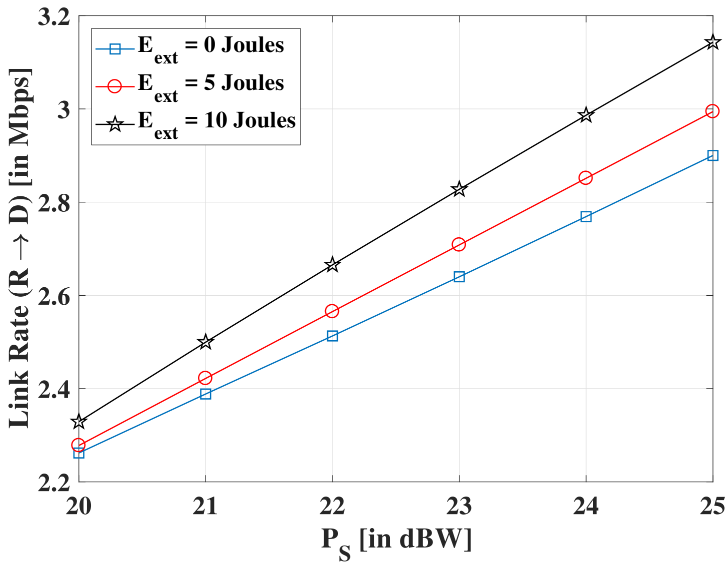

Fig. 5. Rate versus total transmit power at the source (P_S) for different values of E_{ext} assuming $r = 1$ Mbps $\delta = 0.2$.

The path loss is considered to be 3 dB. All the results are evaluated over 500 Monte-Carlo random channel conditions.

Fig. 5 and Fig. 6 illustrates the results corresponding to the solutions proposed for the rate maximization problem. It is observed from Fig. 5 that the rate at the destination is maximized using the solutions proposed in this chapter. It is seen that the rate keeps increasing with increasing transmit power values at the source (P_S), keeping $r = 1$ Mbps and $\delta = 0.2$. On the other hand, it is clear that with increasing values of E_{ext}, the rate increases non-linearly. Fig. 6 depicts the plot for the rate at the destination versus the caching gain of the relay with $E_{ext} = 31.6$ Joules and $r = 2$ Mbps. It is shown that for increasing values of P_S, the rate keeps increasing considerably. It is observed that the relay-destination rate increases with increasing caching gain coefficient values.

Fig. 7 presents the stored energy at the relay as a function of the source's transmit power and different external energy values. It is shown from the results that the source transmit power has large impacts on the stored energy at the relay. In particular, increasing the source's transmit power by 2 dBW will double the stored energy at the relay. It is also observed that increasing the external energy can significantly improve the stored energy at high P_S values. However, when P_S is small, increasing E_{ext} does not bring considerable improvement. This is because at low P_S values, most of the time is used for information transfer from the source to the relay.

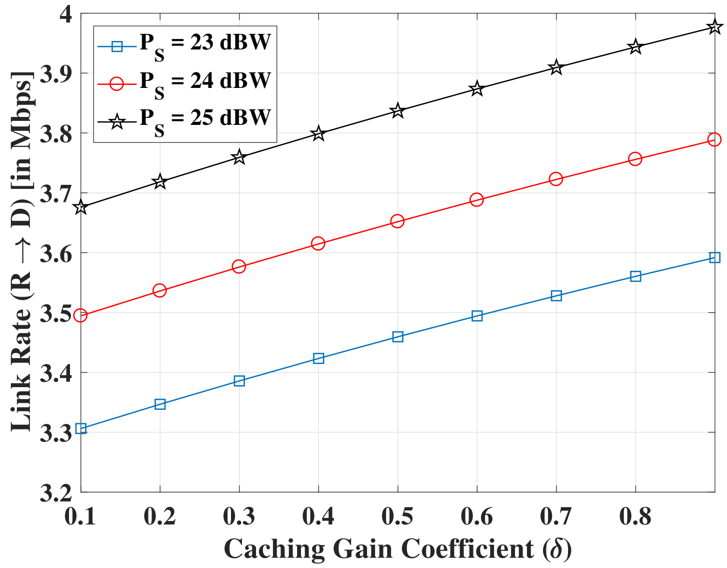

Fig. 6. Rate versus the caching gain (δ) for different values of P_S assuming $r = 2$ Mbps and $E_{ext} = 31.6$ Joules.

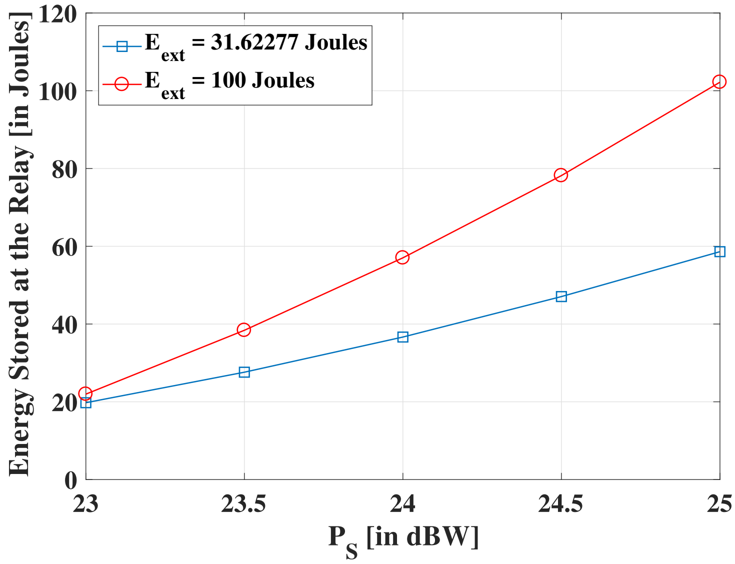

Fig. 7. Energy stored at the relay versus total transmit power at the source (P_S) for various values of E_{ext} with $\delta = 0.9$ and $r = 2.5$ Mbps.

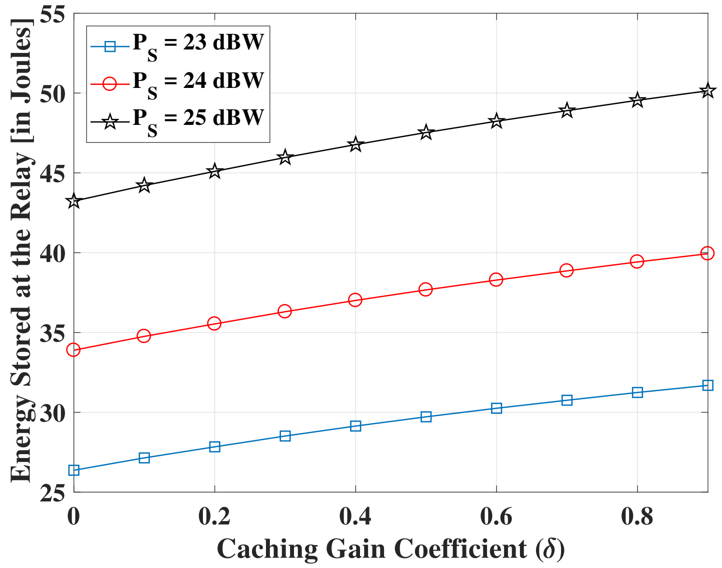

Fig. 8. Energy stored at the relay versus the caching gain (δ) for different values of P_S assuming $E_{ext} = 31.6$ Joules and $r = 2.5$ Mbps.

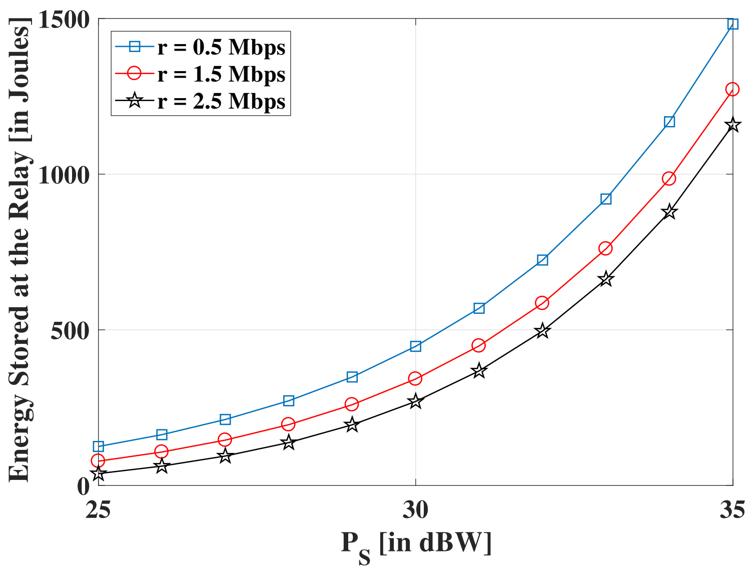

Fig. 9. Energy stored at the relay versus the total transmit power at the source (P_S) for different values of r with $\delta = 0.2$ and $E_{ext} = 31.6$ Joules.

Fig. 8 depicts the plot of the energy stored at the relay as a function of the cache capacity δ, with $E_{ext} = 31.6$ Joules and $r = 2.5$ Mbps. The case with $\delta = 0$ implies that there is no caching at the relay. It is shown that caching helps to increase the saved energy at the relay for all P_S values. And the increased stored energy are almost similar for different P_S. This is because of the linear model of the caching system in (5).

Fig. 9 presents an evaluation of the energy stored at the relay against the increasing values of P_S. It is seen that the energy stored at the relay keeps increasing with increasing transmit power values at the source (P_S), for $\delta = 0.2$ and $E_{ext} = 31.6$ Joules. On the other hand, it is clear that with increasing values of r, the energy stored at the relay decreases non-linearly. This is due to the fact that in order to meet the demand of requested rate at the destination, more energy would be required for resource allocation at the relay which utilizes the harvested energy.

6 Conclusion

In this chapter, we proposed and investigated a novel time switching (TS) based Wi-TIE with caching architecture for half duplex relaying systems where the relay employs the DF protocol. We addressed the problem to maximize the data throughput between the relay and destination under constraint on minimum energy stored at the relay; and for maximizing the energy stored at the relay under constraints on minimum rate and harvested energy, guaranteeing a good performance in both the cases with regards to the QoS constraints. Besides, both the problems were formulated according to two separate yet distinct conventions over the time period. We presented the closed-form solutions for the proposed relay system to enable Wi-TIE with caching. With the help of simulations, we illustrated the results corresponding to the solutions obtained for the aforementioned problems with parameter variations. This work can be further extended to many fascinating directions like selection of the best relay out of given multiple relays, multiuser and multicarrier scenario, and relaying with full duplexing mode.

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Appendix A: Analysis of different possibilities from KKT Conditions for Data Maximization Problem

The Lagrangian corresponding to ($P2$) can be denoted as follows

$$\mathcal{L}(\alpha,\beta,P_R;\lambda_1,\lambda_2,\lambda_3,\lambda_4) = F(\alpha,\beta,P_R) - \lambda_1\ G(\alpha,\beta,P_R) - \lambda_2\ H(\alpha,\beta,P_R) - \lambda_3\ I(\alpha,\beta,P_R) - \lambda_4\ J(\alpha,\beta,P_R), \quad (32)$$

where

$$F(\alpha,\beta,P_R) = (1-\beta)B\log_2(1+\gamma_{R,D}), \quad (33)$$
$$G(\alpha,\beta,P_R) = (1-\beta)B\log_2(1+\gamma_{R,D}) - (1-\alpha)\beta[B\log_2(1+\gamma_{S,R}) + \delta r] \leq 0, \quad (34)$$
$$H(\alpha,\beta,P_R) = (1-\beta)P_R - \zeta\alpha\beta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2) - E_{ext} \leq 0, \quad (35)$$
$$I(\alpha,\beta,P_R) = \alpha - 1 \leq 0, \quad (36)$$
$$J(\alpha,\beta,P_R) = \beta - 1 \leq 0. \quad (37)$$

For optimality, $\nabla\mathcal{L}(\alpha,\beta,P_R;\lambda_1,\lambda_2,\lambda_3,\lambda_4) = 0$. Thus, we can represent the equations for satisfying the optimality conditions as

$$\frac{\partial\mathcal{L}(\alpha,\beta,P_R;\lambda_1,\lambda_2,\lambda_3,\lambda_4)}{\partial\alpha} \implies -\lambda_1[\beta(B\log_2(1+\gamma_{S,R}) + \delta r)] - \lambda_2[-\zeta\beta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)] - \lambda_3 = 0, \quad (38)$$

$$\frac{\partial\mathcal{L}(\alpha,\beta,P_R;\lambda_1,\lambda_2,\lambda_3,\lambda_4)}{\partial\beta} \implies -B\log_2(1+\gamma_{R,D}) - \lambda_1[-B\log_2(1+\gamma_{R,D})] - (1-\alpha)[B\log_2(1+\gamma_{S,R}) + \delta r] - \lambda_2[-P_R - \zeta\alpha(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)] - \lambda_4 = 0, \quad (39)$$

$$\frac{\partial\mathcal{L}(\alpha,\beta,P_R;\lambda_1,\lambda_2,\lambda_3,\lambda_4)}{\partial P_R} \implies \frac{\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2 + P_R d_{R,D}^{-\vartheta}|h_{R,D}|^2} - \lambda_1\left(\frac{\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2 + P_R d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right) - \lambda_2 = 0. \quad (40)$$

The conditions for feasibility are as expressed in (34), (35), (36), and (37). Complementary slackness expressions can be represented as follows

$$\lambda_1 \cdot G(\alpha,\beta,P_R) = 0, \quad (41)$$
$$\lambda_2 \cdot H(\alpha,\beta,P_R) = 0, \quad (42)$$
$$\lambda_3 \cdot I(\alpha,\beta,P_R) = 0, \quad (43)$$
$$\lambda_4 \cdot J(\alpha,\beta,P_R) = 0. \quad (44)$$

The conditions for non-negativity are: $\alpha, \beta, P_R, \lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0$. It is clear that if $\lambda_3 \neq 0$, then $I(\alpha, \beta, P_R) = 0$ implying that $\alpha = 1$. Since this is not a feasible solution, therefore $\lambda_3 = 0$. Similarly, it can be shown that $\lambda_4 = 0$. Next, we analyze the remaining cases in order to obtain a feasible solution.

Case I: $\lambda_1 = 0 \implies G(\alpha, \beta, P_R) \neq 0$; $\lambda_2 \neq 0 \implies H(\alpha, \beta, P_R) = 0$

From (38), we find that $\lambda_2 = 0$, and substituting the value in (39) yields

$$-B\log_2(1+\gamma_{R,D}) = 0. \tag{45}$$

It is clear that $\gamma_{R,D} \neq 0$. Hence, this case is not possible.

Case II: $\lambda_1 = 0 \implies G(\alpha, \beta, P_R) \neq 0$; $\lambda_2 = 0 \implies H(\alpha, \beta, P_R) \neq 0$
This case is not acceptable as $B\log_2(1+\gamma_{R,D}) \neq 0$.

Case III: $\lambda_1 \neq 0 \implies G(\alpha, \beta, P_R) = 0$; $\lambda_2 = 0 \implies H(\alpha, \beta, P_R) \neq 0$
From (40), we find that $\lambda_1 = 1$ which implies that $P_S \to 0$. Therefore, this is not a feasible solution.

Case IV: $\lambda_1 \neq 0 \implies G(\alpha, \beta, P_R) = 0$; $\lambda_2 \neq 0 \implies H(\alpha, \beta, P_R) = 0$
For $\lambda_1 \neq 0 \implies G(x, P_R) = 0$; $\lambda_2 \neq 0 \implies H(x, P_R) = 0$, we deduce to the following equations

$$-\lambda_1[\beta(B\log_2(1+\gamma_{S,R}) + \delta r)] + \lambda_2[\zeta\beta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)] = 0, \tag{46}$$

$$\begin{aligned}- B\log_2(1+\gamma_{R,D}) + \lambda_1[B\log_2(1+\gamma_{R,D})] + (1-\alpha)[B\log_2(1+\gamma_{S,R}) + \delta r] \\ + \lambda_2[P_R + \zeta\alpha(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)] = 0,\end{aligned} \tag{47}$$

$$\frac{\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2 + P_R d_{R,D}^{-\vartheta}|h_{R,D}|^2} - \lambda_1\left(\frac{\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2 + P_R d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right) = 0, \tag{48}$$

$$(1-\beta)B\log_2(1+\gamma_{R,D}) - (1-\alpha)\beta[B\log_2(1+\gamma_{S,R}) + \delta r] = 0. \tag{49}$$

$$(1-\beta)P_R - \zeta\alpha\beta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2) - E_{ext} = 0. \tag{50}$$

From (46) and (48) we have

$$\lambda_1 = \frac{\chi_1(\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2))}{\chi_2}, \tag{51}$$

$$\lambda_2 = \frac{\chi_1(B\log_2(1+\gamma_{S,R}) + \delta r)}{\chi_2}, \tag{52}$$

where $\chi_1 = (\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2)$, and $\chi_2 = (\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2)(\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)) + (\sigma_{n_D}^2 + P_R d_{R,D}^{-\vartheta}|h_{R,D}|^2)(B\log_2(1+\gamma_{S,R}) + \delta r)$.

Assuming $\kappa = 1 + \gamma_{R,D}$ and substituting (51) and (52) in (47), we obtain the following equation

$$\mathcal{A} + \kappa[\ln(2) - B\log_2(\kappa)] = 0, \tag{53}$$

where $\mathcal{A} = \frac{(\ln(2)d_{R,D}^{-\vartheta}|h_{R,D}|^2)(\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2+\sigma_{n_R}^2))}{\sigma_{n_D}^2}$. The solution of this equation is obtained in a closed form as follows

$$\kappa = \exp\big(\mathcal{W}(\mathcal{A}\exp(-\log^2(2)) + \log(2)) + \log^2(2)\big), \tag{54}$$

where $\mathcal{W}(\cdot)$ is the Lambert W function [32].

Using (54), (49) and (50), we obtain the following

$$\beta = \frac{\varphi - E_{ext}(B\log_2(1+\gamma_{S,R}) + \delta r)}{\varphi - \zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)(B\log_2(1+\gamma_{S,R}) + \delta r)}, \tag{55}$$

where $\varphi = P_R(B\log_2(1+\gamma_{S,R}) + \delta r) - \zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)B\log_2(1+\gamma_{R,D})$.

$$\alpha = \frac{(1-\beta)P_R - E_{ext}}{\zeta\beta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)}. \tag{56}$$

Appendix B: Analysis of different possibilities from KKT Conditions for Maximization Problem of Energy Stored at the Relay

The Lagrangian for $(P3)$ can be denoted as follows

$$\begin{aligned}\mathcal{L}(\theta,\phi,P_R;\lambda_1,\lambda_2,\lambda_3,\lambda_4) = F(\theta,\phi,P_R) - \lambda_1\ G(\theta,\phi,P_R) - \lambda_2\ H(\theta,\phi,P_R)\\ - \lambda_3\ I(\theta,\phi,P_R) - \lambda_4\ J(\theta,\phi,P_R),\end{aligned} \tag{57}$$

where

$$F(\theta,\phi,P_R) = [\zeta\theta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2) - (1-(\theta+\phi))P_R]^+, \tag{58}$$

$$G(\theta,\phi,P_R) = (1-(\theta+\phi))B\log_2(1+\gamma_{R,D}) - \phi[B\log_2(1+\gamma_{S,R}) + \delta r] \le 0, \tag{59}$$

$$H(\theta,\phi,P_R) = (1-(\theta+\phi))P_R - \zeta\theta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2) - E_{ext} \le 0, \tag{60}$$

$$I(\theta,\phi,P_R) = r - (1-(\theta+\phi))B\log_2(1+\gamma_{R,D}) \le 0, \tag{61}$$

$$J(\theta,\phi,P_R) = (\theta+\phi) - 1 \le 0. \tag{62}$$

For optimality, $\nabla\mathcal{L}(\theta,\phi,P_R;\lambda_1,\lambda_2,\lambda_3,\lambda_4) = 0$. Thus, we can represent the equations for satisfying the optimality conditions as

$$\begin{aligned}\frac{\partial\mathcal{L}(\theta,\phi,P_R;\lambda_1,\lambda_2,\lambda_3,\lambda_4)}{\partial\theta} \implies [\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2+\sigma_{n_R}^2)+P_R]-\lambda_1[-B\log_2(1+\gamma_{R,D})]\\ - \lambda_2[-P_R - \zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)] - \lambda_3[B\log_2(1+\gamma_{R,D})] - \lambda_4 = 0,\end{aligned} \tag{63}$$

$$\begin{aligned}\frac{\partial\mathcal{L}(\theta,\phi,P_R;\lambda_1,\lambda_2,\lambda_3,\lambda_4)}{\partial\phi} \implies P_R-\lambda_1[-B\log_2(1+\gamma_{R,D})-(B\log_2(1+\gamma_{S,R})+\delta r)]\\ - \lambda_2[-P_R] - \lambda_3[B\log_2(1+\gamma_{R,D})] - \lambda_4 = 0,\end{aligned} \tag{64}$$

$$\frac{\partial \mathcal{L}(\theta,\phi,P_R;\lambda_1,\lambda_2,\lambda_3,\lambda_4)}{\partial P_R} \implies -(1-(\theta+\phi))-\lambda_1\left[(1-(\theta+\phi))\left(\frac{\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2+P_R d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right)\right]$$
$$-\lambda_2(1-(\theta+\phi))-\lambda_3\left[-(1-(\theta+\phi))\left(\frac{\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2+P_R d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right)\right]=0. \quad (65)$$

The conditions for feasibility are as expressed in (59), (60), (61), and (62). Complementary slackness expressions can be represented as follows

$$\lambda_1 \cdot G(\theta,\phi,P_R)=0, \quad (66)$$
$$\lambda_2 \cdot H(\theta,\phi,P_R)=0, \quad (67)$$
$$\lambda_3 \cdot I(\theta,\phi,P_R)=0, \quad (68)$$
$$\lambda_4 \cdot J(\theta,\phi,P_R)=0. \quad (69)$$

The conditions for non-negativity are: $\theta,\phi,P_R,\lambda_1,\lambda_2,\lambda_3,\lambda_4 \geq 0$. It is clear that if $\lambda_4 \neq 0$, then $J(\theta,\phi,P_R)=0$ implying that $\theta+\phi=1$. Since this is not a feasible solution, therefore $\lambda_4=0$. Next, we analyze the remaining cases in order to obtain a feasible solution. The analysis is as follows

<u>Case I</u>: $\lambda_1=0 \implies G(\theta,\phi,P_R)\neq 0$; $\lambda_2=0 \implies H(\theta,\phi,P_R)\neq 0$; $\lambda_3=0 \implies I(\theta,\phi,P_R)\neq 0$
From (63) and (64), we find that $P_R=-\zeta(P_S|g|^2+\sigma_{n_R}^2)$ or $P_R=0$ respectively. Since both these solutions cannot be accepted, therefore this case is not possible.

<u>Case II</u>: $\lambda_1=0 \implies G(\theta,\phi,P_R)\neq 0$; $\lambda_2=0 \implies H(\theta,\phi,P_R)\neq 0$; $\lambda_3\neq 0 \implies I(\theta,\phi,P_R)=0$
This case again leads us to the unacceptable solution as in the previous case, therefore this case can be excluded.

<u>Case III</u>: $\lambda_1=0 \implies G(\theta,\phi,P_R)\neq 0$; $\lambda_2\neq 0 \implies H(\theta,\phi,P_R)=0$; $\lambda_3=0 \implies I(\theta,\phi,P_R)\neq 0$
From the optimality conditions, we deduce that $\lambda_2=1$ with no solutions for θ, ϕ, and P_R. Hence, this case is not admissible.

<u>Case IV</u>: $\lambda_1\neq 0 \implies G(\theta,\phi,P_R)=0$; $\lambda_2=0 \implies H(\theta,\phi,P_R)\neq 0$; $\lambda_3=0 \implies I(\theta,\phi,P_R)\neq 0$
Herein, we find that $\lambda_1<0$; which violates the non-negativity condition. Thus, this case is infeasible.

<u>Case V</u>: $\lambda_1=0 \implies G(\theta,\phi,P_R)\neq 0$; $\lambda_2\neq 0 \implies H(\theta,\phi,P_R)=0$; $\lambda_3\neq 0 \implies I(\theta,\phi,P_R)=0$
For this case, we can represent the following equations in their simplified forms

$$[\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2+\sigma_{n_R}^2)+P_R]+\lambda_2[P_R+\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2+\sigma_{n_R}^2)]$$
$$-\lambda_3[B\log_2(1+\gamma_{R,D})]=0, \quad (70)$$

$$P_R+\lambda_2[P_R]-\lambda_3[B\log_2(1+\gamma_{R,D})]=0, \quad (71)$$

$$1+\lambda_2-\lambda_3\left(\frac{\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2+P_R d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right)=0, \quad (72)$$

$$(1-(\theta+\phi))P_R - \zeta\theta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2) - E_{ext} = 0, \tag{73}$$

$$r - (1-(\theta+\phi))B\log_2(1+\gamma_{R,D}) = 0. \tag{74}$$

Solving the above equations, we obtain solutions for P_R as follows

$$P_R = \max(\exp\left(\mathcal{W}(\exp(-\log^2(2)) + \log(2)) + \log^2(2)\right), \\ \exp\left(\mathcal{W}_{-1}(\exp(-\log^2(2)) + \log(2)) + \log^2(2)\right)), \tag{75}$$

where $\mathcal{W}(\cdot)$ is the LambertW function or the product log function and $\mathcal{W}_k(\cdot)$ is the analytic continuation of the product log function [32]. As the solution is independent of P_S, it is reasonable to discard this solution.

Case VI: $\lambda_1 \neq 0 \implies G(\theta,\phi,P_R) = 0$; $\lambda_2 = 0 \implies H(\theta,\phi,P_R) \neq 0$; $\lambda_3 = 0 \implies I(\theta,\phi,P_R) \neq 0$
We can represent the following equations in their simplified forms

$$[\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2) + P_R] + \lambda_1[B\log_2(1+\gamma_{R,D})] \\ - \lambda_3[B\log_2(1+\gamma_{R,D})] = 0, \tag{76}$$

$$P_R + \lambda_1[B\log_2(1+\gamma_{R,D}) + (B\log_2(1+\gamma_{S,R}) + \delta r)] - \lambda_3[B\log_2(1+\gamma_{R,D})] = 0, \tag{77}$$

$$1 + \lambda_2 - \lambda_3\left(\frac{\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2 + P_R d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right) = 0, \tag{78}$$

$$(1-(\theta+\phi))P_R - \zeta\theta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2) - E_{ext} = 0, \tag{79}$$

$$r - (1-(\theta+\phi))B\log_2(1+\gamma_{R,D}) = 0. \tag{80}$$

From (76) and (77), we obtain

$$\lambda_1 = \frac{\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)}{B\log_2(1+\gamma_{S,R}) + \delta r}. \tag{81}$$

Substituting (81) in (78), we find the following

$$\lambda_3 = \left(\frac{\sigma_{n_D}^2 + P_R d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right) + \left(\frac{\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)}{B\log_2(1+\gamma_{S,R}) + \delta r}\right). \tag{82}$$

Substituting (81) and (82) in (77), and assuming $\nu = 1 + \gamma_{R,D}$, we obtain the following equation

$$\nu[B\log_2(\nu) - \ln(2)] + \mathcal{A} = 0, \tag{83}$$

where $\mathcal{A} = \ln(2) - \left(\frac{\zeta}{\sigma_{n_D}^2}\right)(\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2)(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)$.

The solution of the above expression can be expressed as follows

$$\nu = \exp\left(\mathcal{W}(-\mathcal{A}\exp(-\log^2(2)) + \log(2)) + \log^2(2)\right). \tag{84}$$

Consequently, the we obtain the following

$$P_R^{\dagger} = (\nu - 1)\left(\frac{\sigma_{n_D}^2}{d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right). \tag{85}$$

From (79) and (80), and using (84), we obtain

$$\phi^{\dagger} = \frac{r}{B\log_2(1+\gamma_{S,R}) + \delta r}. \tag{86}$$

Finally, substituting (26) in (80), we find the following

$$\theta^{\dagger} = 1 - r\left(\frac{1}{B\log_2(1+\gamma_{S,R}) + \delta r} + \frac{1}{B\log_2\left(1 + \frac{P_R^{\dagger} d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2}\right)}\right), \tag{87}$$

where $P_R^{\dagger}$, $\phi^{\dagger}$, and $\theta^{\dagger}$ are the optimal values obtained for P_R, ϕ, and θ, respectively in this case.

<u>Case VII</u>: $\lambda_1 \neq 0 \implies G(\theta,\phi,P_R) = 0$; $\lambda_2 \neq 0 \implies H(\theta,\phi,P_R) = 0$; $\lambda_3 = 0 \implies I(\theta,\phi,P_R) \neq 0$
The simplified equations for this case can be represented as follows

$$[\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2) + P_R] + \lambda_1[B\log_2(1+\gamma_{R,D})] + \lambda_2[P_R + \zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)] = 0, \tag{88}$$

$$P_R + \lambda_1[B\log_2(1+\gamma_{R,D}) + (B\log_2(1+\gamma_{S,R}) + \delta r)] + \lambda_2[P_R] = 0, \tag{89}$$

$$1 + \lambda_1\left(\frac{\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2 + P_R d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right) + \lambda_2 = 0, \tag{90}$$

$$(1 - (\theta+\phi))B\log_2(1+\gamma_{R,D}) - \phi[B\log_2(1+\gamma_{S,R}) + \delta r] = 0, \tag{91}$$

$$(1 - (\theta+\phi))P_R - \zeta\theta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2) - E_{ext} = 0. \tag{92}$$

From (89) and (90), and assuming $\mu = 1 + \gamma_{R,D}$, we obtain the following equation

$$\mu(B\log_2(\mu) + B\log_2(1+\gamma_{S,R}) + \delta r - \ln(2)) + \ln(2) = 0. \tag{93}$$

Since the solution of (93) is composed of complex values, therefore this case is not acceptable.

<u>Case VIII</u>: $\lambda_1 \neq 0 \implies G(\theta,\phi,P_R) = 0$; $\lambda_2 \neq 0 \implies H(\theta,\phi,P_R) = 0$; $\lambda_3 \neq 0 \implies I(\theta,\phi,P_R) = 0$
The equations to be used for computation of θ, ϕ, and P_R in this case can be written as

$$[\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2) + P_R] + \lambda_1[B\log_2(1+\gamma_{R,D})] + \lambda_2[P_R + \zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)] - \lambda_3[B\log_2(1+\gamma_{R,D})] = 0, \tag{94}$$

$$P_R + \lambda_1[B\log_2(1+\gamma_{R,D}) + (B\log_2(1+\gamma_{S,R}) + \delta r)] + \lambda_2[P_R] - \lambda_3[B\log_2(1+\gamma_{R,D})] = 0, \quad (95)$$

$$1 + \lambda_1\left(\frac{\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2 + P_R d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right) + \lambda_2 - \lambda_3\left(\frac{\ln(2) d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2 + P_R d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right) = 0, \quad (96)$$

$$(1-(\theta+\phi))B\log_2(1+\gamma_{R,D}) - \phi[B\log_2(1+\gamma_{S,R}) + \delta r] = 0, \quad (97)$$

$$(1-(\theta+\phi))P_R - \zeta\theta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2) - E_{ext} = 0, \quad (98)$$

$$r - (1-(\theta+\phi))B\log_2(1+\gamma_{R,D}) = 0. \quad (99)$$

From (97) and (99), we obtain

$$\phi = \frac{r}{B\log_2(1+\gamma_{S,R}) + \delta r}. \quad (100)$$

Similarly, from (98) and (99), we find

$$\theta = \frac{rP_R - E_{ext}B\log_2(1+\gamma_{R,D})}{\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)}. \quad (101)$$

Substituting (100) and (101) in (99), and assuming $\eta = 1 + \gamma_{R,D}$ we obtain the following equation

$$\mathcal{A} + B\log_2(\eta)[\mathcal{B} + \mathcal{C}\eta + \mathcal{D}B\log_2(\eta)] = 0, \quad (102)$$

where $\mathcal{A} = a\cdot b\cdot r$, $\mathcal{B} = -a\cdot b - b\cdot r\cdot\left(\frac{\sigma_{n_D}^2}{d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right) + a\cdot r$, $\mathcal{C} = b\cdot r\cdot\left(\frac{\sigma_{n_D}^2}{d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right)$, and $\mathcal{D} = -b\cdot E_{ext}$, with $a = \zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)$, and $b = B\log_2(1+\gamma_{S,R}) + \delta r$.

Considering $\mathcal{F}(\eta) = \mathcal{A} + B\log_2(\eta)[\mathcal{B} + \mathcal{C}\eta + \mathcal{D}B\log_2(\eta)]$, we have a nonlinear equation of the type

$$\mathcal{F}(\eta) = 0. \quad (103)$$

Let us assume that η is a simple or one of the multiple roots of (103), and η_0 is an initial point prediction sufficiently near to η. Using the Taylor's series expansion [cite], we can express the following

$$\mathcal{F}(\eta_0) + (\eta-\eta_0)\mathcal{F}'(\eta_0) + \frac{1}{2}(\eta-\eta_0)^2\mathcal{F}''(\eta_0) = 0. \quad (104)$$

In order to solve the nonlinear equation $\mathcal{F}(\eta) = 0$, an alternative equivalence formulation has been used to develop a class of iterative methods. Simplified form of (104) can be re-written as

$$\mathcal{F}''(\eta_0)(\eta-\eta_0)^2 + 2\mathcal{F}'(\eta_0)(\eta-\eta_0) + 2\mathcal{F}(\eta_0) = 0. \quad (105)$$

It is clear that the equation in (105) is of the quadratic form. Hence, the corresponding roots can be expressed as

$$\eta - \eta_0 = \frac{-\mathcal{F}'(\eta_0) \pm \sqrt{[\mathcal{F}'(\eta_0)]^2 - 2\mathcal{F}''(\eta_0)\mathcal{F}(\eta_0)}}{\mathcal{F}''(\eta_0)}. \quad (106)$$

Depending on the sign of proceeding the radical term, the formula in (106) provides the following two possibilities

$$\eta = \eta_0 - \frac{\mathcal{F}'(\eta_0) + \sqrt{[\mathcal{F}'(\eta_0)]^2 - 2\mathcal{F}''(\eta_0)\mathcal{F}(\eta_0)}}{\mathcal{F}''(\eta_0)}. \tag{107}$$

$$\eta = \eta_0 - \frac{\mathcal{F}'(\eta_0) - \sqrt{[\mathcal{F}'(\eta_0)]^2 - 2\mathcal{F}''(\eta_0)\mathcal{F}(\eta_0)}}{\mathcal{F}''(\eta_0)}. \tag{108}$$

Using the fixed point formulations in (107) and (108), and in order to maximize the objective in (19), the following formula for an approximate solution η_{k+1} can be used to find the larger root iteratively [33]

$$\eta_{k+1} = \eta_k - \frac{\mathcal{F}'(\eta_k) - \sqrt{[\mathcal{F}'(\eta_k)]^2 - 2\mathcal{F}''(\eta_k)\mathcal{F}(\eta_k)}}{\mathcal{F}''(\eta_k)}. \tag{109}$$

It should be noted that the denominator of (109) is independent of $\mathcal{F}'(\eta_k)$ which makes it specially fit to find the largest root of the (103).

Since the nonlinear equation in (102) involves the logarithmic terms, the number of iterations required to find the optimal largest root may be higher for the chosen value of η_0. In that case, Halley's method (2) or the modified Chebyshev's method (39) in [34] may also be used to reduce the number of iterations.

Finally, we obtain the following solutions for this case

$$P_R^* = (\eta - 1)\left(\frac{\sigma_{n_D}^2}{d_{R,D}^{-\vartheta}|h_{R,D}|^2}\right), \tag{110}$$

$$\phi^* = \frac{r}{B\log_2(1+\gamma_{S,R}) + \delta r}, \tag{111}$$

$$\theta^* = \frac{rP_R^* - E_{ext}B\log_2\left(1 + \frac{P_R^* d_{R,D}^{-\vartheta}|h_{R,D}|^2}{\sigma_{n_D}^2}\right)}{\zeta(P_S d_{S,R}^{-\vartheta}|h_{S,R}|^2 + \sigma_{n_R}^2)}, \tag{112}$$

where P_R^*, ϕ^*, and θ^* are the optimal values obtained for P_R, ϕ, and θ, respectively in this case.