

A Joint Solution for Scheduling and Precoding in Multiuser MISO Downlink Channels

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Abstract—The average performance of the MISO downlink channel, with a large number of users compared to transmit antennas of the base station, depends on the interference management which necessitates the joint design of scheduling and precoding. Unlike the previous works which do not offer a truly joint design, this paper focuses on formulating a problem amenable for the joint update of scheduling and precoding. Novel optimization formulations are investigated to reveal the hidden difference of convex/ concave structure for three classical criteria (weighted sum rate, max-min signal-to-interference plus noise ratio, and power minimization) and associated constraints are considered. Thereafter, we propose a convex-concave procedure framework based iterative algorithm where scheduling and precoding variables are updated jointly in each iteration. Finally, we show the superiority in performance of joint solution over the state-of-the-art designs through Monte-Carlo simulations.

Index Terms—User scheduling, Precoding, multiuser, difference-of-convex, convex-concave procedure

I. INTRODUCTION

With the adoption of full frequency reuse in the next-generation cellular networks, interference among the simultaneously served users becomes a limiting factor thwarting the achievement of near-optimal capacity [2]–[5]. Linear precoding has been largely used to achieve satisfactory interference mitigation at low complexity [6], [7]. Moreover, in a network with a large number of users compared to the number of BS transmit antennas, user scheduling for simultaneous transmission is pivotal for interference management [8], [9]. Optimizing performance in such a network involves the design of precoding variables and user scheduling. Further, different perspectives to network performance motivate the need to investigate multiple figures of merit; these include network throughput, user Quality of Service (QoS) power consumed among others. In this context, we address the joint design of scheduling and precoding problem for multiuser MISO downlink channels in single-cell scenario for the following network optimization design criteria: 1) Maximize the weighted sum rate subject to user’s minimum signal-to-interference plus noise ratio (MSINR), scheduling and power constraints referred to in the sequel simply as WSR. 2) Maximize the MSINR of the scheduled users subject to scheduling and total power constraints henceforth referred to as MMSINR. 3)

Minimize the power utilized subject to scheduling and MSINR constraints henceforth referred to as PMIN.

The aforementioned criteria are designed to improve the complementary aspects of the networks. In all practical wireless systems, a certain minimum received SINR is required for the successful transmission of information. In light of this, to enhance the practical relevance, SINR constraints are introduced in these design criteria. The WSR problem improves the overall throughput of a network while satisfying the scheduling constraint and QoS requirement on the scheduled users. On the contrary, the MMSINR problem improves the performance of the poorest user (in terms of SINR) among those scheduled. Unlike WSR and MMSINR, PMIN optimizes the consumed power while meeting the scheduling and SINR constraints. An elaborate discussion on each design is provided in the subsequent sections.

The joint design of scheduling and precoding, which we simply refer to as joint design, is well studied during the last decade (see [10] and references therein). Most of the existing literature on the joint design can be classified as:

- *Non-iterative decoupled approach*: In this approach, scheduling and precoding are treated as two decoupled problems where usually the users are scheduled according to some criteria followed by precoding [8], [9], [11]–[13].
- *Iterative decoupled approach*: In this approach, scheduling and precoding are still treated as two separate problems. However, scheduling and precoding parameters are refined in each iterate to improve the objective based on the feedback from the previous iterate [14]–[17].
- *Joint formulation with alternate update*: In this approach, the joint design problem is formulated as a function of both scheduling and precoding [18]–[20]. However, these formulations are not amenable for the joint update as the scheduling variables are coupled to precoding variables which inhibits their joint update. Hence, during the solution stage either scheduling constraints are ignored [18] or the scheduling and precoding variables are updated alternatively. [19].

The joint design is a coupled problem where the efficiency of the precoder design depends on the interference among the users which, in turn, is a function of the scheduled users [10]. Hence, the joint update of scheduling and precoding has the potential to achieve better performance over the aforementioned approaches [8], [9], [18], [11]–[17]. The authors in [21] shown the user scheduling and power allocation problem to be

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NP-hard. The joint design problems that are considered in this work encapsulate the problem considered in [21] as a special case. Hence, the considered joint design problems are NP-hard. It is also non-convex due to the constraints on the SINR or rate of scheduled users [14]. Hence, the optimal solution entails an exhaustive search over Boolean space (user scheduling) and further involves the solution of a non-convex precoding problem. The exponential complexity of an exhaustive search for practical system dimensions motivates a shift towards low-complexity achievable solutions. In this context, we quickly review the various relevant works to place ours in perspective.

The joint design problem to maximize the WSR subject to total power constraint, which is referred to as the classical WSR problem, is considered for single cell networks in [8], [9], [11]. The channel orthogonality based scheduling followed by zero-forcing precoding (SUS-ZF) proposed in [9] is proven to be asymptotically optimal for sum rate maximization. However, it is easy to see that SUS-ZF is not optimal for WSR with non-uniform weights and QoS constraints. Similarly, the classical WSR is addressed for multicell networks in [14], [16], [17] and hierarchical networks in [18]. The joint design problem is also considered for MMSINR in [13] and PMIN in [15]. However, scheduling and precoding are not jointly updated in the aforementioned works.

The coupled nature of binary variables with precoding vector appears in many other formulations [22], [23] etc. For example, towards maximizing the weighted sum-rate in a hierarchical network, binary variables associated with users get multiplied to signal power and interference power of SINR [18]. Similarly, in [20], a binary variable is multiplied to the rate of the users in the weighted sum-rate maximization problem. Please note that system models and objectives discussed in [18], [20], [23] are different from each other, and the emphasis is only on the occurrence of the joint design (coupled discrete and continuous) nature that prevails in different designs. The multiplicative nature in previous formulations precludes the joint update of scheduling and precoding. To the best of our knowledge, no prior work exists that update the scheduling and precoding jointly for the aforementioned WSR, MMSINR and PMIN problems. Therefore, we focus on formulating the joint design problem for WSR, MMSINR, and PMIN that facilitates the joint scheduling and precoding solutions.

Revisiting the WSR and MMSINR design problems for fixed scheduled users, it is well-known that the problems are non-convex with difficulty to obtain a global solution. However, efficient suboptimal solutions have been proposed for WSR in [24] and MMSINR in [25], [26] by formulating these as difference-of-convex (DC) programming problems with the help of auxiliary variables and semidefinite programming (SDP) transformations and relaxations. However, the semidefinite relaxations for WSR and MMSINR often lead to non-unity rank solutions from which the approximate rank-1 solutions are extracted [24]–[26]. The rank-1 approximation results in a loss of performance. Moreover, the transformed problems have higher complexity than the original problems due to auxiliary variables and SDP transformations. In this work, we pose the WSR and MMSINR joint design as

DC programming problems without SDP transformation and employing a minimal number of auxiliary variables.

The aforementioned discussion reflects on the novelties of the paper-based both on problem formulation and its solution. The contributions of the paper include:

- The scheduling is handled through the power of the precoding vector of the corresponding user, where non-zero power indicates the user being scheduled (and not scheduled otherwise). Unlike the previous works [18], [20], [23], a binary variable is used for upper bounding the power of the precoding vector. This renders the formulation amenable to the joint design of scheduling and precoding.
- With the help of the aforementioned scheduling, the joint design problem for WSR, MMSINR, and PMIN design criteria are formulated as mixed-integer non-linear programming (MINLP) in a way that would facilitate the joint updates of scheduling and precoding. Here, the nonconvexity of the problem stems from rate and SINRs in the objective and constraints.
- The binary nature of the problem due to scheduling constraints is addressed by relaxing the binary variables into real values. This is followed by penalizing the objective with a novel entropy-based penalty function to promote a binary solution for the scheduling variables. This step transforms the optimization into a continuous non-convex problem.
- Unlike the classical DC formulation using SDP transformation [24]–[26], a novel useful reformulation of the objective and/or SINR constraints are proposed to manipulate the joint design as DC programming without SDP transformation.
- Further, a convex-concave procedure (CCP) based iterative algorithm is proposed for WSR, MMSINR and PMIN DC problems. A procedure is proposed to find a feasible initial point, which is sufficient for these algorithms to converge to a stationary point [27].
- Subsequently, the per iteration complexity of the CCP based algorithms, is discussed. Further, a fast convergence behavior of the proposed algorithms is observed through extensive simulations. Finally, the efficiency of the proposed DC reformulations is compared to the decoupled solutions using the Monte-Carlo simulations.

The rest of the paper is organized as follows. Section II presents the system model and problem formulation of WSR, MMSINR, and PMIN problem. The reformulations and algorithm are proposed for WSR in Section III, MMSINR in Section IV and PMIN in Section V respectively. Section VI presents simulation results, followed by conclusions in Section VII.

Notation: Lower or upper case letters represent scalars, lower case boldface letters represent vectors, and upper case boldface letters represent matrices. $\|\cdot\|$ represents the Euclidean norm, $|\cdot|$ represents the cardinality of a set or the magnitude of a scalar, $(\cdot)^H$ represents Hermitian transpose, $(\cdot)^T$ represents transpose, $\underset{a}{\overset{b}{\cdot}}$ represents a choose b , and $\mathbb{R}\{\cdot\}$ represents real operation, $\mathbb{E}\{\cdot\}$ represents expectation operator

and ∇ represents the gradient.

II. SYSTEM MODEL

Consider the downlink transmission of a single cell MISO system with N users in a cell and a BS with $M(\leq N)$ antennas. Let $\mathbf{h}_i \in \mathbb{C}^{M \times 1}$, $\mathbf{w}_i \in \mathbb{C}^{M \times 1}$ and x_i denote the downlink channel, precoding vector and data of user i respectively. The BS is assumed to transmit independent data to utmost M among N users and $E\{|x_i|^2\} = 1; \forall i$. Further, let n_i be the noise at user i ; the noise realizations at all users are assumed to be independent and characterized as additive white complex Gaussian with zero mean and variance σ^2 . Further, it is assumed that perfect channel state information of all the users is available at BS and that the user channels are constant during the transmission. Let y_i be the noisy received signal of the user i and $\mathbf{y} = [y_1 \dots y_N]^T$, $\mathbf{H} = [\mathbf{h}_1 \dots \mathbf{h}_N]^H$, $\mathbf{W} = [\mathbf{w}_1 \dots \mathbf{w}_N]$, $\mathbf{x} = [x_1 \dots x_N]^T$, $\mathbf{n} = [n_1 \dots n_N]^T$. The received signal vector \mathbf{y} of all users is given by,

$$\mathbf{y} = \mathbf{H}\mathbf{W}\mathbf{x} + \mathbf{n}; \quad (1)$$

Towards defining the WSR problem mathematically, let $\mathcal{T} = \{1, \dots, N\}$ be the set containing indices of all users and \mathcal{K} be a subset of \mathcal{T} with cardinality less than or equal to M . Further, let \mathcal{K} be the collection of all the possible subsets of type \mathcal{K} ; clearly, the cardinality of \mathcal{K} is $C_N^M = \sum_{i=0}^M C_N^i$. With the notations defined, the WSR problem is defined as,

$$\begin{aligned} \mathcal{P}_{\text{WSR}} : \quad & \max_{\forall \mathcal{K} \in \mathcal{K}} \max_{\mathbf{W}_{\mathcal{K}}} \sum_{i \in \mathcal{K}} i R_i \quad (2) \\ & \text{s.t. } R_i \geq \gamma_i; \forall i \in \mathcal{K}; \\ & \sum_{i \in \mathcal{K}} \|\mathbf{w}_i\|_2^2 \leq P_T; \\ & \text{precoding problem for selected users} \\ & \text{Joint scheduling and Precoding problem} \end{aligned}$$

where $\gamma_i = \frac{1}{2 + \sum_{j \neq i \in \mathcal{K}} |\mathbf{h}_i^H \mathbf{w}_j|^2}$, $R_i = \log_2(1 + \gamma_i)$ and $\gamma_i \geq 0$ are the SINR, rate and minimum rate requirement of the user i respectively and \mathcal{K} is the set of scheduled users. Further $\gamma_i \in \mathbb{R}^+$ denotes the weight for i th user offering design flexibility, P_T is the total available power, and $\mathbf{W}_{\mathcal{K}} = \{\mathbf{w}_i\}_{i=1}^{|\mathcal{K}|}$ is the matrix containing the precoding vectors of users of set \mathcal{K} .

Unlike the WSR design, scheduling of exactly $K(\leq M)$ users is considered in MMSINR formulation. This is because constraining the scheduling to utmost K users always leads to the trivial solution of scheduling only one user and an elaborate discussion is provided at the beginning of Section IV. Let \mathcal{S} be a subset of \mathcal{T} with cardinality equal to K . Let \mathcal{S} be the collection of all the possible subsets of type \mathcal{S} ; clearly, the cardinality of \mathcal{S} is C_N^K . Letting γ_i to the MSINR requirement of user i , $\forall i$, the design problem for MMSINR can then be defined as,

$$\begin{aligned} \mathcal{P}_{\text{MMSINR}} : \quad & \max_{\mathcal{S} \subseteq \mathcal{T}} \max_{\mathbf{W}_{\mathcal{S}}} \min_{i \in \mathcal{S}} \{ \gamma_i \} \quad (3) \\ & \text{s.t. } \sum_{i \in \mathcal{S}} \|\mathbf{w}_i\|_2^2 \leq P_T; \end{aligned}$$

$$\gamma_i \geq \gamma_i; i \in \mathcal{S};$$

$$\begin{aligned} & \text{precoding problem for selected users} \\ & \text{Joint scheduling and Precoding problem} \end{aligned}$$

where $\gamma_i \in \mathbb{R}^+$ is weight and $\mathbf{W}_{\mathcal{S}} = \{\mathbf{w}_i\}_{i=1}^{|\mathcal{S}|}$ is the matrix containing the precoding vectors of users in the set \mathcal{S} . Notice that to accommodate the fairness in the designs, weights or priority factors are introduced through γ_i and γ_i in WSR and MMSINR problems respectively. Various fairness metrics are proposed in the literature, e.g. fairness in terms of rates and allocated power are considered at the physical layer. We refer to [28] and references therein for details on fairness.

Finally, towards defining the PMIN problem, scheduling exactly $K(\leq M)$ users is considered for the same reason mentioned in MMSINR. With notations defined for MMSINR criteria, the PMIN problem is defined as:

$$\begin{aligned} \mathcal{P}_{\text{PMIN}} : \quad & \min_{\mathcal{S} \subseteq \mathcal{T}} \min_{\mathbf{W}_{\mathcal{S}}} \sum_{i \in \mathcal{S}} \|\mathbf{w}_i\|_2^2 \quad \text{s.t. } \gamma_i \geq \gamma_i; i \in \mathcal{S}; \quad (4) \\ & \text{PMIN problem for selected users} \\ & \text{Joint user scheduling and PMIN problem} \end{aligned}$$

The inner optimization in (2), (3), and (4) solves the precoding problem for the scheduled users. The outer optimization, on the other hand, ensures scheduling users with a maximum objective value among all scheduling possibilities. Notice that the inner and outer optimization are coupled - the design of precoder depends on the selected set of users, while the user scheduling depends on the objectives in (2), (3) and (4) which, in turn, are functions of the precoder [29].

Towards proposing low-complexity algorithms, we begin by addressing the user scheduling through the norm of precoding vectors as given below,

$$\|\mathbf{w}_i\|_2 = \begin{cases} 0; & \text{user not selected,} \\ \neq 0; & \text{user selected.} \end{cases} \quad (5)$$

The zero norm of \mathbf{w}_i ensures that all elements of \mathbf{w}_i are zero. Hence, the user i is not scheduled. Similarly, the non-zero norm of the precoder vector \mathbf{w}_i indicates user i being scheduled with an assigned power of $\|\mathbf{w}_i\|_2^2$. In the sequel, we focus on the design of low-complexity solutions to the joint design using (5) to achieve better performance than the decoupled designs.

III. WEIGHTED SUM RATE MAXIMIZATION

In (2), the weighted sum rate objective is considered to improve the overall weighted throughput of the network. Thus, WSR problem schedules only the users who contribute to maximizing the objective. Given sufficient resources, the WSR design schedules close to M users as the objective increases linearly with the number of scheduled users; on the other hand, scheduling of few users with high SINRs only contributes logarithmically to the objective. Hence, the constraint of scheduling utmost of M users is considered as opposed to the harder constraint of scheduling to exactly M users. Besides, the design is flexible to favor users by increasing the corresponding weights i.e., γ_i to relatively larger values over

the users. The minimum rate constraints preclude scheduling of the users whose rates are not in the range of interest. Since the scheduling of zero users is also included in the feasible set, the problem (2) is always feasible. In the sequel, the WSR problem (i.e., (2)) is transformed as a DC programming problem through a sequence of novel reformulations and low-complexity sub-optimal algorithms within the framework of CCP.

A. Joint Design Problem Formulation: WSR

Letting \mathcal{K} to be the set of scheduled users, a tractable formulation of (2) using (5) is,

$$\begin{aligned} \mathcal{P}_1^{\text{WSR}} : & \max_{\mathbf{W}; \forall \mathcal{K} \in \mathcal{K}} \sum_{i=1}^{\mathcal{N}} R_i & (6) \\ \text{s.t. } C_1 : & \|\mathbf{w}_1\|_2 + \dots + \|\mathbf{w}_N\|_2 \leq M; \\ C_2 : & \sum_{i=1}^{\mathcal{N}} \|\mathbf{w}_i\|_2^2 \leq P_T; \\ C_3 : & R_i \geq \gamma_i; i \in \mathcal{K}; \end{aligned}$$

Remarks:

- It is clear from (5) and the definition of ℓ_0 norm, that the constraint C_1 imposes restrictions on the total number of selected users to utmost M . We refer to this constraint as the user scheduling constraint throughout this section.
- The constraint C_2 precludes the design from using a transmission power greater than P_T .
- The constraint C_3 imposes a minimum rate required for the scheduled users.

A Novel MINLP formulation: The problem $\mathcal{P}_1^{\text{WSR}}$ is combinatorial due to the constraint C_1 and C_3 , and non-convex due to the objective and constraints C_1 and C_3 . Towards addressing the combinatorial nature, we let δ_i and τ_i to be the binary scheduling variable and slack variable associated with user i respectively, and $\delta = [\delta_1; \dots; \delta_N]^T$, $\tau = [\tau_1; \dots; \tau_N]^T$ and $\tau_i \leq 2^{\delta_i} - 1; \forall i$. With the defined notations, a tractable formulation of C_1 and C_3 of $\mathcal{P}_1^{\text{WSR}}$ then takes the form,

$$\begin{aligned} \mathcal{P}_2^{\text{WSR}} : & \max_{\mathbf{W}; \delta} f(\delta; \tau), \sum_{i=1}^{\mathcal{N}} \delta_i \log(\delta_i) & (7) \\ \text{s.t. } C_1 : & \delta_i \in \{0; 1\}; \forall i; \\ C_2 : & \|\mathbf{w}_i\|_2^2 \leq P_T \delta_i; \forall i; \\ C_3 : & \sum_{i=1}^{\mathcal{N}} \delta_i \leq M; \\ C_4 : & \sum_{i=1}^{\mathcal{N}} \|\mathbf{w}_i\|_2^2 \leq P_T; \\ C_5 : & 1 + \tau_i \geq \delta_i; \forall i; \\ C_6 : & \tau_i \geq 1 + \delta_i - \tau_i; \forall i; \end{aligned}$$

Remarks:

- The binary nature of δ_i (i.e., C_1) together with C_2 determines the scheduling of users. In other words, $\delta_i = 0$ leads to a precoding vector containing all zero entries.

Similarly $\tau_i = 1$ leads to $\|\mathbf{w}_i\|_2^2 \leq P_T$ which is a trivial upper bound compared to C_4 . Hence the constraint C_2 along with C_1 contributes only to the scheduling aspects of the problem.

- From the objective and constraint C_5 , the variable τ_i provides a lower bound for $1 + \delta_i$.
- The constraint C_6 ensures MSINR or rate constraint of the scheduled users. If user i is scheduled i.e., $\delta_i = 1$, from C_6 , $\tau_i \geq 1 + \delta_i$. Similarly, for an unscheduled user i , C_6 becomes $\tau_i \geq 1$. In fact for $\delta_i = 0$, constraint is met with equality i.e., $\tau_i = 1$ due to C_2 .
- It is easy to see that, at the optimal solution, the constraints C_5 and C_6 are met with equality.

Novelty of $\mathcal{P}_2^{\text{WSR}}$: Novelty of $\mathcal{P}_2^{\text{WSR}}$ lies in the formulation of scheduling constraint, C_2 . This reformulation is vital to the facilitation of the joint update of δ and \mathbf{W} as discussed in the sequel. Notice that this formulation differs from those in the literature ([18], [20], [23], [30], [31], etc) where the scheduling constraint is handled by a binary slack variable which multiplies either the precoding vector or the rate of the user, to control the user scheduling. This multiplication not only makes the constraints non-convex but also makes it difficult to obtain the joint update of Boolean and continuous variables due to the coupling of variables. Moreover, the constraints C_5 and C_6 help to reformulate the objective as a concave function and connects the minimum rate constraints to the objective. This reformulation is crucial as it facilitates the reformulation of $\mathcal{P}_2^{\text{WSR}}$ as DC programming problem without resorting to SDP transformations [24], [32]–[34].

B. A Novel DC reformulation: WSR

A novel rearrangement of SINR constraint C_5 in $\mathcal{P}_2^{\text{WSR}}$ that transforms $\mathcal{P}_2^{\text{WSR}}$ as a DC programming problem without SDP transformation is,

$$\mathcal{P}_3^{\text{WSR}} : \max_{\mathbf{W}; \delta} f(\delta; \tau), \sum_{i=1}^{\mathcal{N}} \delta_i \log(\delta_i) \quad (8)$$

$$\text{s.t. } C_1; C_2; C_3; C_4 \text{ and } C_6 \text{ in (7)}$$

$$C_5 : \mathcal{I}_i(\mathbf{W}) - \mathcal{G}_i(\mathbf{W}; \delta_i) \leq 0; \forall i;$$

where $\mathcal{I}_i(\mathbf{W}) = \frac{P_i}{\sum_{j=1}^N \|\mathbf{h}_i^H \mathbf{w}_j\|^2}$ and $\mathcal{G}_i(\mathbf{W}; \delta_i) = \frac{P_i}{2 + \sum_{j=1}^N \|\mathbf{h}_i^H \mathbf{w}_j\|^2}$. Notice that $\mathcal{I}_i(\mathbf{W})$ is convex in \mathbf{W} , and for $\delta_i > 0$, $\mathcal{G}_i(\mathbf{W}; \delta_i)$ is also jointly convex in \mathbf{W} and δ_i . Hence, (8) is a DC programming problem with combinatorial constraint C_1 . This is the first attempt at reformulating the WSR towards a tractable form without resorting to SDP methods or use of additional slack variables thereby rendering a low-complexity solution to the problem.

Beyond SDP based DC formulation: Notice that for fixed δ , the problem $\mathcal{P}_3^{\text{WSR}}$ becomes a classical WSR maximization problem subject to SINR and total power constraints [24], [32]–[34]. The problem $\mathcal{P}_3^{\text{WSR}}$ is non-convex due to the constraint C_5 . Although, for fixed δ (i.e. fixed δ), the constraint C_5 in $\mathcal{P}_3^{\text{WSR}}$ is formulated as a second-order cone programming (SOCP) constraint [6], a similar SOCP transformation of C_5 is not known when δ is variable. On the other

- Step 2: Solve the following optimization:

$$\begin{aligned} \mathcal{P}_{\text{FESWSR}} : \{\hat{\mathbf{W}}\} : \text{find } \mathbf{W} & \quad (13) \\ \text{s.t. } \mathcal{C}_1 : \|\mathbf{w}_i\|_2^2 \leq \hat{\gamma}_i P_T; \forall i; & \\ \mathcal{C}_2 : \frac{\mathbf{h}_i^H \mathbf{w}_i}{\sum_{j \neq i} \mathbf{h}_i^H \mathbf{w}_j} \geq \frac{\hat{\gamma}_i}{\sqrt{\hat{\gamma}_i - 1}}; \forall i; & \\ \mathcal{C}_3 : \mathcal{R}\{\mathbf{h}_i^H \mathbf{w}_i\} \geq 0; \forall i; & \\ \mathcal{C}_4 : \mathcal{I}\{\mathbf{h}_i^H \mathbf{w}_i\} = 0; \forall i; & \\ \mathcal{C}_5 : \|\mathbf{W}\|_2^2 \leq P_T; & \end{aligned}$$

- Step 3: If $\mathcal{P}_{\text{FESWSR}}$ is feasible go to step 4 else update $\hat{\gamma}_i = \hat{\gamma}_i$ and go to step 2.
- Step 4: Let $\hat{\mathbf{W}}$ be the solution of $\mathcal{P}_{\text{FESWSR}}$. Choose $\hat{\gamma}_i$ such that $1 + \hat{\gamma}_i \leq \hat{\gamma}_i \leq 1 + \hat{\gamma}_i$ where $\hat{\gamma}_i$ is the SINR of the user i calculated using $\hat{\mathbf{W}}$.

Remarks:

- Notice that the updates of $\hat{\gamma}_i$ are always feasible. Different $\hat{\gamma}_i$ in step 1 which satisfy the constraint \mathcal{C}_1 and \mathcal{C}_3 in $\mathcal{P}_{\text{FESWSR}}$ may lead to different FIPs. Similarly, different choices of $\hat{\gamma}_i \in (0; 1)$ in step 1 may also lead to different FIPs.
- The optimization problem in Step 2 is only a function of \mathbf{W} since $\hat{\gamma}_i$ is fixed a priori and $\hat{\gamma}_i$ can be calculated easily from the solution of $\mathcal{P}_{\text{FESWSR}}$ as given in step 4.
- Following [6], the MSINR constraint, i.e. $\hat{\gamma}_i \geq \hat{\gamma}_i$ is reformulated as a second-order cone (SOC) constraint as given in \mathcal{C}_2 with the help of \mathcal{C}_3 and \mathcal{C}_4 .
- If $\mathcal{P}_{\text{FESWSR}}$ in step 2 is in-feasible for $\hat{\gamma}_i$ in step 1, update $\hat{\gamma}_i$ as given in step 3 and repeat step 2. This is repeated until $\mathcal{P}_{\text{FESWSR}}$ in step 2 becomes feasible.
- If the initial iterates fail to result an non-zero based initial feasible point, the proposed method eventually lead to $\hat{\gamma}_i = 0$ and thus $\mathcal{P}_{\text{FESWSR}}$ in step 2 becomes feasible with $\hat{\mathbf{W}} = 0$. Hence, the proposed methods always results an FIP. By initializing $\hat{\gamma}_i$ close to 0, FIP can be obtained in fewer iterations.
- The FIP obtained by this procedure may not be feasible for the original WSR problem \mathcal{P}_{WSR} in (2) unless $\mathcal{P}_{\text{FESWSR}}$ becomes feasible for $\{\hat{\gamma}_i \in \{0; 1\}\}_{i=1}^N$ satisfying $\sum_{i=1}^N \hat{\gamma}_i \leq M$.
- Although the FIP obtained by this method is not feasible for \mathcal{P}_{WSR} , the final solution obtained by JSP-WSR with this FIP becomes a feasible for \mathcal{P}_{WSR} since the solution satisfies the scheduling and SINR constraints of \mathcal{P}_{WSR} .

Letting $\mathcal{P}_5^{\text{WSR}}(k)$ be the objective value of the problem $\mathcal{P}_5^{\text{WSR}}$ at iteration k , the pseudo code of JSP-WSR for the joint design problem is given in algorithm 1.

E. Complexity: WSR

The computational complexity of JSP-WSR depends on the complexities of iterative procedures proposed in Section III-C and Section III-D. The proposed JSP-WSR in Section III-C is a CCP based iterative algorithm; hence, the complexity of the algorithm depends on complexity of the sub-problems $\mathcal{P}_5^{\text{WSR}}$. The convex problem $\mathcal{P}_5^{\text{WSR}}$ has $(NM + 2N)$ decision variables and $(2N + 1)$ convex constraints and $2N + 1$ linear

Algorithm 1 JSP-WSR

Input: $\mathbf{H}; [1; \dots; N]; P_T; \hat{\gamma}_i; \mathbf{W}^0; \hat{\gamma}_i = 0; k = 1;$
Output: $\mathbf{W};$
while $|\mathcal{P}_5^{\text{WSR}}(k) - \mathcal{P}_5^{\text{WSR}}(k-1)| \geq \epsilon$ **do**
 Convexification: Convexify the problem (10)
 Optimization: Update $(\mathbf{W}; \hat{\gamma}_i)$ by solving $\mathcal{P}_5^{\text{WSR}}$
 Update : $\mathcal{P}_5^{\text{WSR}}(k); \hat{\gamma}_i; k$
end while

constraints. Hence, the computational complexity of $\mathcal{P}_5^{\text{WSR}}$ is $\mathcal{O}((NM + 2N)^3(4N + 2))$ [36]. Similarly, the computational complexity of the proposed procedure in Section III-D to obtain a FIP depends on the per iteration complexity of $\mathcal{P}_{\text{FESWSR}}$. $\mathcal{P}_{\text{FESWSR}}$ is a convex problem with MN decision variables, $2N + 1$ convex constraints and $2N$ linear constraints. Hence, the computational complexity of $\mathcal{P}_{\text{FESWSR}}$ is $\mathcal{O}((MN)^3(4N + 1))$ [36].

IV. MAX MIN SINR

In this section, we focus on the development of a low-complexity algorithm for the MMSINR problem defined in (3). Dropping a user with low SINR clearly improves MSINR. It also reduces the interference to the other users and the power of the dropped user can be used to further improve the MSINR of other users. Hence, the constraint of scheduling utmost K users leads to the global solution which has highest MSINR which is achieved by scheduling only one user. To avoid this, scheduling exactly K users is considered for MMSINR design. Besides the scheduling constraint, the MSINR requirements of the scheduled users are also considered. Without the MSINR requirement, the design becomes superficial as the solution might include zero SINR or SINR values which are not usable in practice.

Infeasibility of MMSINR: The infeasibility of the problem due to the MSINR requirement is explained in [6] for fixed set of users. Similarly, it may not be possible to find exactly K users while satisfying an arbitrarily chosen MSINR, power and system dimension constraints [6]; this renders the problem (3) infeasible. In this work, it is assumed that problem $\mathcal{P}_{\text{MMSINR}}$ has at least one feasible solution for the given scheduling and MSINR constraints. Considering this, a low-complexity sub-optimal algorithm using the frame work of CCP is developed for the MMSINR problem in the sequel.

A. Joint Design Problem Formulation: MMSINR

A tractable mathematical formulation of (3) is,

$$\begin{aligned} \mathcal{P}_1^{\text{MM}} : \max_{\mathbf{W}} \min_{i \in \{1; \dots; N\}} \{ \hat{\gamma}_i \} & \quad (14) \\ \text{s.t. } \mathcal{C}_1 : \|\mathbf{w}_1\|_2; \dots; \|\mathbf{w}_N\|_2 \leq K; & \\ \mathcal{C}_2 : \sum_{i=1}^N \|\mathbf{w}_i\|_2^2 \leq P_T; & \\ \mathcal{C}_3 : \hat{\gamma}_i \geq 1 - \epsilon; \forall i & \end{aligned}$$

where $\mathbf{1}$ is an indicator function with $\mathbf{1} \|\mathbf{w}_i\|_2 = 0$ if $\|\mathbf{w}_i\|_2 = 0$ otherwise $\mathbf{1} \|\mathbf{w}_i\|_2 = 1$.

The SINR γ_i is non-convex and piece-wise minimum of $\{\gamma_i\}_{i=1}^N$ is also non-convex. So, $\mathcal{P}_1^{\text{MM}}$ maximizes a non-convex objective subject to a combinatorial constraint C_1 ; this is generally a NP-hard problem. Moreover obtaining a global solution to $\mathcal{P}_1^{\text{MM}}$ requires an exhaustive search over all the possible sets and solving the classical MMSINR problem for each set.

A Novel Reformulation: In the classical MMSINR problem, for the predefined selected users, SINRs of all users is addressed with a slack variable, say s , that lower bounds γ_i $\forall i$ i.e., $\{\gamma_i\}_{i=1}^N \geq s$ [37], [38]. However, this approach cannot be applied to the present joint design problem since there always exist $N - K$ unscheduled users whose SINR is identically zero. Therefore, lower bounding all $\{\gamma_i\}_{i=1}^N$ with s , makes the problem trivial and the solution, say s^* , is always zero. Letting β_i to be a binary variable associated to user i and \mathcal{S} to be the set of scheduled users and adopting the epigraph formulation, an equivalent formulation of $\mathcal{P}_1^{\text{MM}}$ is,

$$\begin{aligned} \mathcal{P}_2^{\text{MM}} : \max_{\mathbf{W}; s} \quad & s & (15) \\ \text{s.t. } C_1 : \quad & \beta_i \in \{0, 1\}; \forall i; \\ C_2 : \quad & \|\mathbf{w}_i\|_2^2 \leq \beta_i P_T; \\ & \mathcal{X}^N \\ C_3 : \quad & \sum_{i=1}^N \beta_i = K; \\ & \mathcal{X}^N \\ C_4 : \quad & \|\mathbf{w}_i\|_2^2 \leq P_T; \\ & i=1 \\ C_5 : \quad & \gamma_i \geq s; \forall i; \\ C_6 : \quad & \gamma_i \geq s; \forall i; \end{aligned}$$

Remarks:

- Constraint C_5 is the MSINR constraint equivalently written with the help of s .
- The variable s in C_6 is active only when $\beta_i = 1$. For example, when user i not scheduled i.e., $\beta_i = 0$, its SINR is lower bounded by 0 which is always satisfied by the SINR definition. Similarly, when user i scheduled i.e., $\beta_i = 1$, its SINR is lower bounded by s . Thus the maximization of s optimizes the MSINR of only the scheduled users.

B. A Novel DC reformulation: MMSINR

The problem $\mathcal{P}_2^{\text{MM}}$ is a MINLP where the non-convexity is due to constraints C_5 and C_6 , while the combinatorial nature is due to constraint C_1 . Similar to constraint C_5 of $\mathcal{P}_4^{\text{WSR}}$, constraint C_5 of the problem $\mathcal{P}_2^{\text{MM}}$ can be formulated as a DC constraint. However, the same approach cannot be applicable to constraint C_6 in $\mathcal{P}_2^{\text{MM}}$ as β_i and s are both variables. Moreover, to the best of our knowledge DC reformulation of constraints of type C_6 in $\mathcal{P}_2^{\text{MM}}$ is not known. In this section, a novel procedure is proposed to transform constraints of type C_6 in $\mathcal{P}_2^{\text{MM}}$ as DC constraints which involves the change of variable s by $\frac{1}{t}$ followed by rearrangement as described below,

$$1 + \beta_i \gamma_i \geq 1 + \frac{1}{t} \Rightarrow \mathcal{L}_i(\mathbf{W}; t) - \mathcal{H}_i(\mathbf{W}; i; t) \leq 0; \quad (16)$$

where $\mathcal{I}_i(\mathbf{W}) = \sum_{j \neq i} \|\mathbf{h}_i^H \mathbf{w}_j\|^2$, $\mathcal{H}_i(\mathbf{W}; i; t) = \frac{\mathcal{I}_i(\mathbf{W}) + \beta_i \|\mathbf{h}_i^H \mathbf{w}_i\|^2}{t + \beta_i}$ and $\mathcal{L}_i(\mathbf{W}; t) = \frac{\mathcal{I}_i(\mathbf{W})}{t}$. Notice that, given $t > 0$, $\mathcal{L}_i(\mathbf{W}; t)$ is jointly convex in \mathbf{W} and t and $\mathcal{H}_i(\mathbf{W}; i; t)$ is also jointly convex in $\mathbf{W}; i$ and t . Hence, (16) is a DC constraint.

Towards addressing combinatorial constraint C_1 , following the approach in Section III, the binary constraint β_i is relaxed to a box constraint between 0 and 1 and β_i is penalized with $P(\beta_i)$. Letting $\mathcal{J}_i(\mathbf{W}; i; t) = \frac{\mathcal{I}_i(\mathbf{W}) + \beta_i \|\mathbf{h}_i^H \mathbf{w}_i\|^2}{1 + \beta_i t}$, for the sake of completion, with the help of variable t , (16) and penalization approach proposed in Section III, the problem $\mathcal{P}_2^{\text{MM}}$ is reformulated as,

$$\begin{aligned} \mathcal{P}_3^{\text{MM}} : \min_{\mathbf{W}; t} \quad & t - \rho_2 P(\beta_i) & (17) \\ \text{s.t. } C_1 : \quad & 0 \leq \beta_i \leq 1; \forall i; \\ C_2 : C_3 : C_4 \text{ in (15);} \\ C_5 : \quad & \mathcal{I}_i(\mathbf{W}) - \mathcal{J}_i(\mathbf{W}; i; t) \leq 0; \forall i; \\ C_6 : \quad & \mathcal{L}_i(\mathbf{W}; t) - \mathcal{H}_i(\mathbf{W}; i; t) \leq 0; \forall i; \\ C_7 : \quad & t > 0; \end{aligned}$$

where $\rho_2 \in \mathcal{R}^+$ is a penalty parameter of the design.

The problem $\mathcal{P}_3^{\text{MM}}$ maximizes a convex objective subject to convex and DC constraints. Hence $\mathcal{P}_3^{\text{MM}}$ is a DC problem and a CCP based algorithm could be solved with an FIP obtained from Section IV-D. However, the strict equality constraint C_3 in $\mathcal{P}_3^{\text{MM}}$; limits the update of the β_i . In order to allow the flexibility in choosing β_i , the following problem is considered instead,

$$\mathcal{P}_4^{\text{MM}} : \min_{\mathbf{W}; t} \quad t - \rho_2 P(\beta_i) + \sum_{i=1}^N \beta_i - K \rho_A \quad (18)$$

$$\text{s.t. } C_1; C_2; C_4; C_5; C_6; C_7 \text{ in (17);}$$

where $\rho_A \in \mathcal{R}^+$ is a penalty parameter. It is easy to see that choosing the appropriate ρ_A (usually higher value) ensures the equality constraint. The problem $\mathcal{P}_4^{\text{MM}}$ is also a DC problem and a CCP based algorithm, JSP-MMSINR, is proposed in the sequel to solve it efficiently.

C. JSP-MMSINR: A Joint Design Algorithm

In this section, we propose a CCP framework based iterative algorithm to the problem $\mathcal{P}_4^{\text{MM}}$, which is referred to as JSP-MMSINR, wherein the JSP-MMSINR executes the following Convexification and Optimization steps in each iteration:

- Convexification: Let $(\mathbf{W}; t)^{k-1}$ be the estimates of $\mathbf{W}; i; t$ in iteration $k-1$. In iteration k , the concave part of C_5 and C_6 for user i in $\mathcal{P}_4^{\text{MM}}$ i.e., $-\mathcal{H}_i(\mathbf{W}; i; t)$ and $-\mathcal{J}_i(\mathbf{W}; i; t)$ are replaced by its affine approximation around $(\mathbf{W}; t)^{k-1}$ which is given by,

$$\begin{aligned} & \mathcal{H}_i(\mathbf{W}; i; t)^{k-1}, -\mathcal{H}_i(\mathbf{W}; i; t)^{k-1} \\ & \geq -R \nabla^H \mathcal{H}_i(\mathbf{W}; i; t)^{k-1} \left(\frac{\mathbf{w}_i - \mathbf{w}_i^{k-1}}{t - t^{k-1}} \right) \sum_{l=1}^N \mathbf{w}_l^{k-1} \geq \end{aligned}$$

$$\begin{aligned} & \mathcal{J}_i(\mathbf{W}; t)^{k-1}, -\mathcal{J}_i(\mathbf{W}; t)^{k-1} \\ & \geq -R \nabla^H \mathcal{J}_i(\mathbf{W}; t)^{k-1} \left\{ \mathbf{w}_i - \mathbf{w}_i^{k-1} \right\}_{i=1}^N \geq 0; \\ & \geq \end{aligned} \quad (19)$$

where $\nabla \mathcal{H}_i(\mathbf{W}; t)^{k-1}$ and $\nabla \mathcal{J}_i(\mathbf{W}; t)^{k-1}$ are the evaluated gradients of $\mathcal{H}_i(\mathbf{W}; t)$ and $\mathcal{J}_i(\mathbf{W}; t)$ at $(\mathbf{W}; t)^{k-1}$ respectively. The expressions for $\nabla \mathcal{H}_i(\mathbf{W}; t)^{k-1}$ and $\nabla \mathcal{J}_i(\mathbf{W}; t)^{k-1}$ can be obtained by following (11). Similarly, the first order Taylor series approximation of the objective in $\mathcal{P}_4^{\text{MM}}$ after ignoring the constant terms,

$$\mathcal{F}(t; \mathbf{W}) = t - \sum_{i=1}^N \nabla \mathcal{P}_i^{k-1} + \sum_{i=1}^N \mathcal{A}_i - KA \quad (20)$$

- Optimization: The update $(\mathbf{W}; t)^k$ is obtained by solving the following convex problem:

$$\begin{aligned} & \mathcal{P}_5^{\text{MM}} : \max_{\mathbf{W}; t} \mathcal{F}(t; \mathbf{W}) \\ & \text{s.t. } C_1; C_2; C_3; C_4 \text{ in (18)} \\ & C_5 : \mathcal{I}_i(\mathbf{W}; t) + \mathcal{J}_i(\mathbf{W}; t)^{k-1} \leq 0; \forall i; \\ & C_6 : \mathcal{L}_i(\mathbf{W}; t) + \mathcal{H}_i(\mathbf{W}; t)^{k-1} \leq 0; \forall i; \end{aligned}$$

D. Feasible Initial Point: MM-SINR

Notice that JSP-MMSINR is a CCP framework based algorithm and hence a FIP is sufficient for the algorithm to converge to a stationary point [27]. Unlike WSR problem, obtaining a trivial FIP to the problem $\mathcal{P}_4^{\text{MM}}$ is difficult as initializing \mathbf{W} to all zeros results in zero SINR for all the users and thus $t = 0$ where later is the violation of the constraint C_5 . However, one may find a FIP by the following iterative procedure.

- Step 1: Initialize $\hat{\mathbf{W}} = \hat{\mathbf{A}}$ that satisfies constraints C_1 and C_3 in $\mathcal{P}_4^{\text{MM}}$.
- Step 2: For a fixed t , ignoring the constraints dependent on t , $\mathcal{P}_4^{\text{MM}}$ can be reformulated as a convex problem by [6] or [7]. This would result similar formulation as shown in $\mathcal{P}_{\text{FESWSR}}$; hence omitted due to space limitation. Let $\hat{\mathbf{W}}$ be the solution from this step.
- Step 3: Exit the loop if $\hat{\mathbf{W}}$ from step 2 is feasible and $t^0 = \frac{1}{\min_i \{i-j\}}$ else set $t = \hat{t}$ and continue to step 2.

Remarks:

- By construction, the initial $\hat{\mathbf{W}}$ from step 1 is always feasible to $\mathcal{P}_4^{\text{MM}}$.
- Efficient algorithms to solve the convex precoding problem in step 2 is proposed in [6] and [7], and is solvable globally using tools like CVX.
- The number of iterations that are needed to obtain a FIP from above procedure depends on $\hat{\mathbf{A}}$, \hat{t} and K . Suppose, if the initial $\hat{\mathbf{A}} \approx 0$, a FIP is obtained in one iteration with high probability. Similarly, if the initial $\hat{\mathbf{A}} \approx 1$ the solution from above can be infeasible in the initial iterations. For

the latter case, smaller \hat{t} leads to a FIP in few iterations and larger \hat{t} takes longer iterations to find a FIP.

Notice that a FIP obtained from this process is only feasible to problem $\mathcal{P}_4^{\text{MM}}$ but not to the problem $\mathcal{P}_{\text{MMSINR}}$ since it violates scheduling constraint and binary constraint of (5). However, appropriate adaptation of the penalty parameters β_2 and β_3 (e.g. monotonic increment) ensures that the obtained final solution from algorithm 2 is always feasible to $\mathcal{P}_{\text{MMSINR}}$.

Letting $\mathcal{P}_5^{\text{MM}}(k)$ be the objective value of the problem $\mathcal{P}_5^{\text{MM}}$ at iteration k , the pseudocode of JSP-MMSINR for the joint design problem is given in algorithm 2.

Algorithm 2 JSP-MMSINR

Input: $\mathbf{H}; [-1; \dots; -N]; P_T; \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9, \beta_{10}, \beta_{11}, \beta_{12}, \beta_{13}, \beta_{14}, \beta_{15}, \beta_{16}, \beta_{17}, \beta_{18}, \beta_{19}, \beta_{20}, \beta_{21}, \beta_{22}, \beta_{23}, \beta_{24}, \beta_{25}, \beta_{26}, \beta_{27}, \beta_{28}, \beta_{29}, \beta_{30}, \beta_{31}, \beta_{32}, \beta_{33}, \beta_{34}, \beta_{35}, \beta_{36}, \beta_{37}, \beta_{38}, \beta_{39}, \beta_{40}, \beta_{41}, \beta_{42}, \beta_{43}, \beta_{44}, \beta_{45}, \beta_{46}, \beta_{47}, \beta_{48}, \beta_{49}, \beta_{50}, \beta_{51}, \beta_{52}, \beta_{53}, \beta_{54}, \beta_{55}, \beta_{56}, \beta_{57}, \beta_{58}, \beta_{59}, \beta_{60}, \beta_{61}, \beta_{62}, \beta_{63}, \beta_{64}, \beta_{65}, \beta_{66}, \beta_{67}, \beta_{68}, \beta_{69}, \beta_{70}, \beta_{71}, \beta_{72}, \beta_{73}, \beta_{74}, \beta_{75}, \beta_{76}, \beta_{77}, \beta_{78}, \beta_{79}, \beta_{80}, \beta_{81}, \beta_{82}, \beta_{83}, \beta_{84}, \beta_{85}, \beta_{86}, \beta_{87}, \beta_{88}, \beta_{89}, \beta_{90}, \beta_{91}, \beta_{92}, \beta_{93}, \beta_{94}, \beta_{95}, \beta_{96}, \beta_{97}, \beta_{98}, \beta_{99}, \beta_{100}, \beta_{101}, \beta_{102}, \beta_{103}, \beta_{104}, \beta_{105}, \beta_{106}, \beta_{107}, \beta_{108}, \beta_{109}, \beta_{110}, \beta_{111}, \beta_{112}, \beta_{113}, \beta_{114}, \beta_{115}, \beta_{116}, \beta_{117}, \beta_{118}, \beta_{119}, \beta_{120}, \beta_{121}, \beta_{122}, \beta_{123}, \beta_{124}, \beta_{125}, \beta_{126}, \beta_{127}, \beta_{128}, \beta_{129}, \beta_{130}, \beta_{131}, \beta_{132}, \beta_{133}, \beta_{134}, \beta_{135}, \beta_{136}, \beta_{137}, \beta_{138}, \beta_{139}, \beta_{140}, \beta_{141}, \beta_{142}, \beta_{143}, \beta_{144}, \beta_{145}, \beta_{146}, \beta_{147}, \beta_{148}, \beta_{149}, \beta_{150}, \beta_{151}, \beta_{152}, \beta_{153}, \beta_{154}, \beta_{155}, \beta_{156}, \beta_{157}, \beta_{158}, \beta_{159}, \beta_{160}, \beta_{161}, \beta_{162}, \beta_{163}, \beta_{164}, \beta_{165}, \beta_{166}, \beta_{167}, \beta_{168}, \beta_{169}, \beta_{170}, \beta_{171}, \beta_{172}, \beta_{173}, \beta_{174}, \beta_{175}, \beta_{176}, \beta_{177}, \beta_{178}, \beta_{179}, \beta_{180}, \beta_{181}, \beta_{182}, \beta_{183}, \beta_{184}, \beta_{185}, \beta_{186}, \beta_{187}, \beta_{188}, \beta_{189}, \beta_{190}, \beta_{191}, \beta_{192}, \beta_{193}, \beta_{194}, \beta_{195}, \beta_{196}, \beta_{197}, \beta_{198}, \beta_{199}, \beta_{200}, \beta_{201}, \beta_{202}, \beta_{203}, \beta_{204}, \beta_{205}, \beta_{206}, \beta_{207}, \beta_{208}, \beta_{209}, \beta_{210}, \beta_{211}, \beta_{212}, \beta_{213}, \beta_{214}, \beta_{215}, \beta_{216}, \beta_{217}, \beta_{218}, \beta_{219}, \beta_{220}, \beta_{221}, \beta_{222}, \beta_{223}, \beta_{224}, \beta_{225}, \beta_{226}, \beta_{227}, \beta_{228}, \beta_{229}, \beta_{230}, \beta_{231}, \beta_{232}, \beta_{233}, \beta_{234}, \beta_{235}, \beta_{236}, \beta_{237}, \beta_{238}, \beta_{239}, \beta_{240}, \beta_{241}, \beta_{242}, \beta_{243}, \beta_{244}, \beta_{245}, \beta_{246}, \beta_{247}, \beta_{248}, \beta_{249}, \beta_{250}, \beta_{251}, \beta_{252}, \beta_{253}, \beta_{254}, \beta_{255}, \beta_{256}, \beta_{257}, \beta_{258}, \beta_{259}, \beta_{260}, \beta_{261}, \beta_{262}, \beta_{263}, \beta_{264}, \beta_{265}, \beta_{266}, \beta_{267}, \beta_{268}, \beta_{269}, \beta_{270}, \beta_{271}, \beta_{272}, \beta_{273}, \beta_{274}, \beta_{275}, \beta_{276}, \beta_{277}, \beta_{278}, \beta_{279}, \beta_{280}, \beta_{281}, \beta_{282}, \beta_{283}, \beta_{284}, \beta_{285}, \beta_{286}, \beta_{287}, \beta_{288}, \beta_{289}, \beta_{290}, \beta_{291}, \beta_{292}, \beta_{293}, \beta_{294}, \beta_{295}, \beta_{296}, \beta_{297}, \beta_{298}, \beta_{299}, \beta_{300}, \beta_{301}, \beta_{302}, \beta_{303}, \beta_{304}, \beta_{305}, \beta_{306}, \beta_{307}, \beta_{308}, \beta_{309}, \beta_{310}, \beta_{311}, \beta_{312}, \beta_{313}, \beta_{314}, \beta_{315}, \beta_{316}, \beta_{317}, \beta_{318}, \beta_{319}, \beta_{320}, \beta_{321}, \beta_{322}, \beta_{323}, \beta_{324}, \beta_{325}, \beta_{326}, \beta_{327}, \beta_{328}, \beta_{329}, \beta_{330}, \beta_{331}, \beta_{332}, \beta_{333}, \beta_{334}, \beta_{335}, \beta_{336}, \beta_{337}, \beta_{338}, \beta_{339}, \beta_{340}, \beta_{341}, \beta_{342}, \beta_{343}, \beta_{344}, \beta_{345}, \beta_{346}, \beta_{347}, \beta_{348}, \beta_{349}, \beta_{350}, \beta_{351}, \beta_{352}, \beta_{353}, \beta_{354}, \beta_{355}, \beta_{356}, \beta_{357}, \beta_{358}, \beta_{359}, \beta_{360}, \beta_{361}, \beta_{362}, \beta_{363}, \beta_{364}, \beta_{365}, \beta_{366}, \beta_{367}, \beta_{368}, \beta_{369}, \beta_{370}, \beta_{371}, \beta_{372}, \beta_{373}, \beta_{374}, \beta_{375}, \beta_{376}, \beta_{377}, \beta_{378}, \beta_{379}, \beta_{380}, \beta_{381}, \beta_{382}, \beta_{383}, \beta_{384}, \beta_{385}, \beta_{386}, \beta_{387}, \beta_{388}, \beta_{389}, \beta_{390}, \beta_{391}, 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\beta_{623}, \beta_{624}, \beta_{625}, \beta_{626}, \beta_{627}, \beta_{628}, \beta_{629}, \beta_{630}, \beta_{631}, \beta_{632}, \beta_{633}, \beta_{634}, \beta_{635}, \beta_{636}, \beta_{637}, \beta_{638}, \beta_{639}, \beta_{640}, \beta_{641}, \beta_{642}, \beta_{643}, \beta_{644}, \beta_{645}, \beta_{646}, \beta_{647}, \beta_{648}, \beta_{649}, \beta_{650}, \beta_{651}, \beta_{652}, \beta_{653}, \beta_{654}, \beta_{655}, \beta_{656}, \beta_{657}, \beta_{658}, \beta_{659}, \beta_{660}, \beta_{661}, \beta_{662}, \beta_{663}, \beta_{664}, \beta_{665}, \beta_{666}, \beta_{667}, \beta_{668}, \beta_{669}, \beta_{670}, \beta_{671}, \beta_{672}, \beta_{673}, \beta_{674}, \beta_{675}, \beta_{676}, \beta_{677}, \beta_{678}, \beta_{679}, \beta_{680}, \beta_{681}, \beta_{682}, \beta_{683}, \beta_{684}, \beta_{685}, \beta_{686}, \beta_{687}, \beta_{688}, \beta_{689}, \beta_{690}, \beta_{691}, \beta_{692}, \beta_{693}, \beta_{694}, \beta_{695}, \beta_{696}, \beta_{697}, \beta_{698}, \beta_{699}, \beta_{700}, \beta_{701}, \beta_{702}, \beta_{703}, \beta_{704}, \beta_{705}, \beta_{706}, \beta_{707}, \beta_{708}, \beta_{709}, \beta_{710}, \beta_{711}, \beta_{712}, \beta_{713}, \beta_{714}, \beta_{715}, \beta_{716}, \beta_{717}, \beta_{718}, \beta_{719}, \beta_{720}, \beta_{721}, \beta_{722}, \beta_{723}, \beta_{724}, \beta_{725}, \beta_{726}, \beta_{727}, \beta_{728}, \beta_{729}, \beta_{730}, \beta_{731}, \beta_{732}, \beta_{733}, \beta_{734}, \beta_{735}, \beta_{736}, \beta_{737}, \beta_{738}, \beta_{739}, \beta_{740}, \beta_{741}, \beta_{742}, \beta_{743}, \beta_{744}, \beta_{745}, \beta_{746}, \beta_{747}, \beta_{748}, \beta_{749}, \beta_{750}, \beta_{751}, \beta_{752}, \beta_{753}, \beta_{754}, \beta_{755}, \beta_{756}, \beta_{757}, \beta_{758}, \beta_{759}, \beta_{760}, \beta_{761}, \beta_{762}, \beta_{763}, \beta_{764}, \beta_{765}, \beta_{766}, \beta_{767}, \beta_{768}, \beta_{769}, \beta_{770}, \beta_{771}, \beta_{772}, \beta_{773}, \beta_{774}, \beta_{775}, \beta_{776}, \beta_{777}, \beta_{778}, \beta_{779}, \beta_{780}, \beta_{781}, \beta_{782}, \beta_{783}, \beta_{784}, \beta_{785}, \beta_{786}, \beta_{787}, \beta_{788}, \beta_{789}, \beta_{790}, \beta_{791}, \beta_{792}, \beta_{793}, \beta_{794}, \beta_{795}, \beta_{796}, \beta_{797}, \beta_{798}, \beta_{799}, \beta_{800}, \beta_{801}, \beta_{802}, \beta_{803}, \beta_{804}, \beta_{805}, \beta_{806}, \beta_{807}, \beta_{808}, \beta_{809}, \beta_{810}, \beta_{811}, \beta_{812}, \beta_{813}, \beta_{814}, \beta_{815}, \beta_{816}, \beta_{817}, \beta_{818}, \beta_{819}, \beta_{820}, \beta_{821}, \beta_{822}, \beta_{823}, \beta_{824}, \beta_{825}, \beta_{826}, \beta_{827}, \beta_{828}, \beta_{829}, \beta_{830}, \beta_{831}, \beta_{832}, \beta_{833}, \beta_{834}, \beta_{835}, \beta_{836}, \beta_{837}, \beta_{838}, \beta_{839}, \beta_{840}, \beta_{841}, \beta_{842}, \beta_{843}, \beta_{844}, \beta_{845}, \beta_{846}, \beta_{847}, \beta_{848}, \beta_{849}, \beta_{850}, \beta_{851}, \beta_{852}, \beta_{853}, \beta_{854}, \beta_{855}, \beta_{856}, \beta_{857}, \beta_{858}, \beta_{859}, \beta_{860}, \beta_{861}, \beta_{862}, \beta_{863}, \beta_{864}, \beta_{865}, \beta_{866}, \beta_{867}, \beta_{868}, \beta_{869}, \beta_{870}, \beta_{871}, \beta_{872}, \beta_{873}, \beta_{874}, \beta_{875}, \beta_{876}, \beta_{877}, \beta_{878}, \beta_{879}, \beta_{880}, \beta_{881}, \beta_{882}, \beta_{883}, \beta_{884}, \beta_{885}, \beta_{886}, \beta_{887}, \beta_{888}, \beta_{889}, \beta_{890}, \beta_{891}, \beta_{892}, \beta_{893}, \beta_{894}, \beta_{895}, \beta_{896}, \beta_{897}, \beta_{898}, \beta_{899}, \beta_{900}, \beta_{901}, \beta_{902}, \beta_{903}, \beta_{904}, \beta_{905}, \beta_{906}, \beta_{907}, \beta_{908}, \beta_{909}, \beta_{910}, \beta_{911}, \beta_{912}, \beta_{913}, \beta_{914}, \beta_{915}, \beta_{916}, \beta_{917}, \beta_{918}, \beta_{919}, \beta_{920}, \beta_{921}, \beta_{922}, \beta_{923}, \beta_{924}, \beta_{925}, \beta_{926}, \beta_{927}, \beta_{928}, \beta_{929}, \beta_{930}, \beta_{931}, \beta_{932}, \beta_{933}, \beta_{934}, \beta_{935}, \beta_{936}, \beta_{937}, \beta_{938}, \beta_{939}, \beta_{940}, \beta_{941}, \beta_{942}, \beta_{943}, \beta_{944}, \beta_{945}, \beta_{946}, \beta_{947}, \beta_{948}, \beta_{949}, \beta_{950}, \beta_{951}, \beta_{952}, \beta_{953}, \beta_{954}, \beta_{955}, \beta_{956}, \beta_{957}, \beta_{958}, \beta_{959}, \beta_{960}, \beta_{961}, \beta_{962}, \beta_{963}, \beta_{964}, \beta_{965}, \beta_{966}, \beta_{967}, \beta_{968}, \beta_{969}, \beta_{970}, \beta_{971}, \beta_{972}, \beta_{973}, \beta_{974}, \beta_{975}, \beta_{976}, \beta_{977}, \beta_{978}, \beta_{979}, \beta_{980}, \beta_{981}, \beta_{982}, \beta_{983}, \beta_{984}, \beta_{985}, \beta_{986}, \beta_{987}, \beta_{988}, \beta_{989}, \beta_{990}, \beta_{991}, \beta_{992}, \beta_{993}, \beta_{994}, \beta_{995}, \beta_{996}, \beta_{997}, \beta_{998}, \beta_{999}, \beta_{1000}$

Output: $\mathbf{W}; t$

while $|\mathcal{P}_5^{\text{MM}}(k) - \mathcal{P}_5^{\text{MM}}(k-1)| \geq \epsilon$ **do**

Convexification: Convexify the problem (19)

Optimization: Update $(\mathbf{W}; t)^k$ by solving $\mathcal{P}_5^{\text{MM}}$

Update : $\mathcal{P}_5^{\text{MM}}(k); t; k;$

end while

E. Complexity: MM-SINR

Similar to JSP-WSR, the computational complexity of JSP-MMSINR depends on the complexities of iterative procedures proposed in Section IV-C and Section IV-D. The proposed JSP-MMSINR in Section III-C is a CCP based iterative algorithm; hence, the complexity of the algorithm depends on complexity of the sub-problems $\mathcal{P}_5^{\text{MM}}$. The problem $\mathcal{P}_5^{\text{MM}}$ has $(NM + N + 1)$ decision variables, $2N + 1$ convex and $2N + 1$ linear constraints, hence the computational complexity of $\mathcal{P}_5^{\text{MM}}$ is $\mathcal{O}((NM + N + 1)^3(4N + 2))$. Similarly, the computational complexity of the procedure in Section IV-D depends on the per iteration complexity of problem in step 2 which is a convex problem with MN decision variables, $2N + 1$ convex constraints and $2N$ linear constraints. Hence, the computational complexity of problem in step2 in Section IV-D is $\mathcal{O}(MN)^3(4N + 1)$ [36].

V. POWER MINIMIZATION

In this section, we consider the joint design problem with the objective of minimizing the sum power consumed at the BS subject to scheduling of K users whose MSINR requirement is met. As mentioned previously, scheduling utmost K users leads to the trivial solution of no users being scheduled which results in zero consumed power.

A. Joint Design Problem Formulation: PMIN

Similar to Section IV, the user scheduling is handled through the norm of the precoder as shown in (5). With the help of (5) and notations defined, and letting \mathcal{S} to be the set of scheduled users, a tractable formulation of $\mathcal{P}_{\text{PMIN}}$ solely as a function of precoding vectors as follows:

$$\mathcal{P}_1^{\text{PMIN}} : \min_{\mathbf{W}; \mathcal{S}} \sum_{i \in \mathcal{S}} \|\mathbf{W}_i\|_2^2 \quad (21)$$

$$\begin{aligned} \text{s.t. } C_1: & \quad \|\mathbf{w}_1\|_2 \dots \|\mathbf{w}_N\|_2 = K; \\ C_2: & \quad i \geq \tilde{\gamma}_i; \quad i \in \mathcal{S}; \end{aligned}$$

The problem $\mathcal{P}_1^{\text{PMIN}}$ is combinatorial due to the constraints C_1 and C_2 and also non-convex due to $\{i\}_{i=1}^N$ in constraint C_2 . Letting $\beta \in \mathcal{R}^+$ to be a constant, a mathematically tractable formulation that allows us to design a low-complexity algorithm is

$$\begin{aligned} \mathcal{P}_2^{\text{PMIN}}: & \quad \min_{\mathbf{W};} \|\mathbf{W}\|_2^2 \quad (22) \\ \text{s.t. } C_1: & \quad i \in \{0,1\}; \quad \forall i; \\ C_2: & \quad \|\mathbf{w}_i\|_2^2 \leq \beta; \quad \forall i; \\ C_3: & \quad \sum_{i=1}^N i = K; \\ C_4: & \quad i \geq \tilde{\gamma}_i; \quad \forall i; \end{aligned}$$

Remarks:

- For $i = 1$, β in C_2 provides upper bound on the power of user i . Moreover, a lower bound on β would be the total system power.

A DC reformulation: The problem $\mathcal{P}_2^{\text{PMIN}}$ is an MINLP due to combinatorial constraint C_1 and non-convex constraint C_4 . Similar to WSR and MMSINR problems, using the DC formulation of constraint C_4 and penalization method for C_1 , the DC formulation of the problem $\mathcal{P}_2^{\text{PMIN}}$ is,

$$\begin{aligned} \mathcal{P}_3^{\text{PMIN}}: & \quad \min_{\mathbf{W};} \|\mathbf{W}\|_2^2 - \sum_{i=1}^N \beta P(i) \quad (23) \\ \text{s.t. } C_1: & \quad 0 \leq i \leq 1; \quad \forall i; \\ C_2; & \quad C_3 \text{ in (22)}; \\ C_4: & \quad \mathcal{I}_i(\mathbf{W}) - f_i(\mathbf{W}; i); \quad \forall i; \end{aligned}$$

where $\beta \in \mathcal{R}^+$ is the penalty parameter and $f_i(\mathbf{W}; i) = \frac{\mathcal{I}_i(\mathbf{W}) + |\mathbf{h}_i^H \mathbf{w}_i|^2}{1 + \tilde{\gamma}_i}$.

The problem $\mathcal{P}_3^{\text{PMIN}}$ is a DC problem which can be solved using CCP. However, finding a FIP becomes difficult as for chosen β , $\mathcal{P}_3^{\text{PMIN}}$ may become infeasible [6]. For the ease of finding an FIP, the constraint C_2 in $\mathcal{P}_4^{\text{PMIN}}$ is relaxed and penalized as follows:

$$\begin{aligned} \mathcal{P}_4^{\text{PMIN}}: & \quad \min_{\mathbf{W};} \|\mathbf{W}\|_2^2 - \sum_{i=1}^N \beta P(i) + \sum_{i=1}^N \beta (i - K) \quad (24) \\ \text{s.t. } & \quad C_1; C_2; C_4 \text{ in (23)} \end{aligned}$$

where $\beta > 0$ is penalty parameter. Notice that for the appropriate β , equality constraint is ensured. Moreover, The problem $\mathcal{P}_4^{\text{PMIN}}$ is a DC problem which solvable using CCP.

B. Joint Design Algorithm: PMIN

In this section, following the CCP framework proposed in Section IV-C, the CCP based algorithm for PMIN is proposed. The proposed joint scheduling and precoding (JSP)

for PMIN (JSP-PMIN) algorithm executes the following two steps iteratively until the convergence:

- **Convexification:** Let $(\mathbf{W}; i)^{k-1}$ be the estimates of $(\mathbf{W}; i)$ in iteration $k-1$. In iteration k , the concave part of C_3 in $\mathcal{P}_4^{\text{PMIN}}$ for user i i.e., $-f_i(\mathbf{W}; i)$ is replaced by its affine approximation around the estimate of $(\mathbf{W}; i)^{k-1}$ which is given by,

$$\begin{aligned} f(\mathbf{W}; i) & \approx -f(\mathbf{W}; i)^{k-1} - \nabla^H f(\mathbf{W}; i)^{k-1} (\mathbf{w}_i - \mathbf{w}_i^{k-1}) \\ R: & \quad \{ \mathbf{w}_i - \mathbf{w}_i^{k-1} \}_{i=1}^N \end{aligned} \quad (25)$$

- **Optimization:** Update $(\mathbf{W}; i)^k$ is obtained by solving the following convex problem:

$$\begin{aligned} \mathcal{P}_5^{\text{PMIN}}: & \quad \min_{\mathbf{W};} \|\mathbf{W}\|_2^2 + \sum_{i=1}^N \beta (i - K) \quad (26) \\ \text{s.t. } C_1: & \quad 0 \leq i \leq 1; \quad \forall i; \\ C_2: & \quad \|\mathbf{w}_i\|_2^2 \leq \beta; \quad \forall i; \\ C_3: & \quad \mathcal{I}_i(\mathbf{W}) + f(\mathbf{W}; i)^{k-1} \leq 0; \quad \forall i; \end{aligned}$$

The convex problem $\mathcal{P}_5^{\text{PMIN}}$ has $(NM + N)$ decision variables, $2N$ convex and $2N$ linear constraints, hence the computational complexity of $\mathcal{P}_5^{\text{PMIN}}$ is $\mathcal{O}((NM + N)^3 (4N))$.

C. Feasible Initial Point: PMIN

An initial feasible point, which suffices the convergence of JSP-PMIN to a stationary point [27], for the problem $\mathcal{P}_5^{\text{PMIN}}$ is obtained by the following iterative procedure.

- Step 1: Initialize $\hat{\mathbf{W}} = \hat{\mathbf{W}}^0$ that satisfies C_1 and C_3 in $\mathcal{P}_4^{\text{PMIN}}$.
- Step 2: The precoding problem of $\mathcal{P}_4^{\text{PMIN}}$ for fixed $\hat{\mathbf{W}}$ can be reformulated as a convex problem by [6] or [7]. Let $\hat{\mathbf{W}}$ be the solution from this step.
- Step 3: Exit the loop if $\hat{\mathbf{W}}$ is feasible (see [6]) else set $\hat{\mathbf{W}} = \hat{\mathbf{W}}^1$ and continue to step 2.

Notice that a FIP obtained above may not be feasible to $\mathcal{P}_2^{\text{PMIN}}$ since it may violate binary and scheduling constraints. However, the adopted penalty methods ensure the scheduling and binary constraints. Hence, the final solution obtained from 3 is always a feasible solution to $\mathcal{P}_2^{\text{PMIN}}$.

Letting $\mathcal{P}_5^{\text{PMIN}}(k)$ be the objective value of the problem $\mathcal{P}_5^{\text{PMIN}}$ at iteration k , The pseudo code of the algorithm is illustrated in the algorithm 3.

VI. SIMULATION RESULTS

A. Simulation Setup

In this section, we evaluate the performance of the proposed algorithms for the MMSINR, WSR and PMIN problems. The system parameters and benchmark scheduling method discussed in this paragraph are common for all the figures. Entries of the channel matrix, i.e., $\{h_{ij}\}$ s are drawn from the complex normal distribution with zero mean and unit variance

Algorithm 3 JSP-PMIN

Input: $\mathbf{H}; \gamma_1, \dots, \gamma_N; \gamma_2, \gamma_3 = 0, \mathbf{W}^0, \gamma_3 = 0, k = 1;$
Output: $\mathbf{W};$
while $|\mathcal{P}_5^{\text{PMIN}}(k) - \mathcal{P}_5^{\text{PMIN}}(k-1)| \geq \epsilon$ **do**
 Convexification: Convexify the problem (19)
 Optimization: Update $\mathbf{W}^k; k$ by solving $\mathcal{P}_5^{\text{PMIN}}$
 Update : $\mathcal{P}_5^{\text{PMIN}}(k); k$
end while

and noise variances are considered to be unity i.e., $\sigma^2 = 1$. Simulation results in all the figures are averaged over 500 different channel realizations (CRs). The penalty parameter γ_1 is initialized to 0.5 and incremented as $\gamma_1 = 1.1 \gamma_1$ until $\gamma_1 \leq 10$. For all the simulations of MMSINR and PMIN, K is chosen as M . By the nature of MMSINR (PMIN) design, dropping the user with the lowest SINR (higher power) leads to a better objective. This phenomenon continues until it drops $N - M$ users and can not drop any further due to the scheduling constraint. Since, this naturally enforces the binary nature of $\gamma_2, \gamma_3 = 0$ ($\gamma_3 = 0$) in MMSINR (PMIN) still yields the binary γ_1 which is shown Section VI-D and VI-E. Hence, γ_2 and γ_3 are fixed zero in all iterations. The penalty parameters γ_1 and γ_2 are initialized to 0.01 and incremented as $\gamma_1 = 1.2 \gamma_1$ and $\gamma_2 = 1.2 \gamma_2$ in each iteration until $\gamma_1 \leq 20$ and $\gamma_2 \leq 20$.

B. Benchmark algorithms

To evaluate the performance of the proposed JSP algorithms - due to the lack of a comparable joint solution - the following benchmarks (iterative decoupled solutions that execute the following steps in sequence) are devised:

- In step 1, users are scheduled according to proposed weighted semi-orthogonal user scheduling (WSUS) or exhaustive search-based user scheduling (ES) or random user scheduling (RUS). The considered WSUS is an extension of the SUS algorithm proposed in [9]. In SUS, the users are selected sequentially based on the orthogonality of their channels with those of already scheduled ones. In WSUS, orthogonality indices calculated according to SUS are multiplied with their associated weights and the user with the highest weighted orthogonality index is scheduled. This process is repeated until M users are scheduled.
- In step 2, the precoding problem for the scheduled users is solved by the following methods:
 - It is easy to see that, retaining only the terms corresponding to scheduled users by substituting corresponding γ_i s to 1 (rest are made zero) and ignoring the constraint solely dependent on γ_i s in (9), (18) yields the DC formulation of the precoding problem for the scheduled users for WSR and MMSINR respectively. These problems can be solved using CCP with a FIP obtained from $\mathcal{P}_{\text{FESWSR}}$ and $\mathcal{P}_{\text{FESPMIN}}$ by substituting corresponding γ_i s with 1. RUS, ES, SUS and WSUS combined with this proposed WSR is simply referred to as RUS-WSR, ES-WSR, SUS-WSR, and WSUS-WSR respectively and for MMSINR as RUS-

TABLE I: Summary of the acronyms of different benchmark algorithms based on design criteria.

Design criteria	Scheduling schemes	Precoding schemes
WSR: weighted sum rate	SUS: Semi orthogonal user scheduling	WSR: proposed precoding; WSR-Z: proposed precoding with a trivial FIP; RWSR: reference precoding [24]
MMSINR: Max-Min SINR	WSUS: Weighted SUS	MMSINR: proposed precoding scheme
PMIN: Power Minimization	RUS: Random user scheduling	PMIN: reference precoding in [7]

MMSINR, ES-MMSINR, SUS-MMSINR, and WSUS-MMSINR respectively. Similarly, RUS, ES, SUS and WSUS based scheduling followed by the SDP based power minimization proposed in [7] is used for PMIN precoding problem and is referred to simply as RUS-PMIN, ES-PMIN, SUS-PMIN, and WSUS-PMIN respectively. For the ease of reference, acronyms used for benchmark (BM) algorithms are tabulated in Table I.

- An SDR version of DC formulation proposed in [24] also used for solving the precoding for the scheduled users in WSR case as a reference hence is referred to as RWSR. WSUS combined with RWSR is referred to as WSUS-RWSR.
- In step 3: If the precoding problem in step 2 is infeasible exit the loop else drop the user with least orthogonality and repeat step 2 for an updated set of scheduled users. However, the precoding problems for MMSINR and PMIN are assumed to be feasible.

C. WSR Performance Evaluation

In figure 1a, we compare the performance of JSP-WSR as a function of N varying from 15 to 30 in steps of 5 for $M = 10$, $P_T = 10\text{dB}$ and $\gamma_i = 4\text{dB}$, $\forall i$. Weights $\{\gamma_i\}_{i=1}^N$ are randomly drawn from the set $\{\frac{k}{N}\}; k = 1; \dots; N$. In figure 1a, RUS-WSR, SUS-WSR, WSUS-WSR and WSUS-RWSR are the decoupled benchmark algorithms. The JSP-WSR initialized with a trivial solution ($\mathbf{W}^0 = \mathbf{0}, \gamma_0 = 0$) is referred to as JSP-WSR-Z and JSP-WSR initialized with an FIP obtained from Section III-D continues to be referred to as JSP-WSR. From figure 1a, it is clear that the joint solution JSP-WSR outperforms all the other decoupled benchmarks. Although JSP-WSR, RUS-WSR, SUS-WSR, and WSUS-WSR have the same underlying precoding algorithm, JSP-WSR achieves better performance as it jointly updates scheduling and precoding. Considering weights into scheduling in WSUS-WSR improves over SUS-WSR, as shown in figure 1a, but it still outperformed by JSP-WSR. However, the gains diminish as N increases as the probability of finding nearly orthogonal user channels (for the considered Gaussian model) increases; this implies that the user scheduling has minimum impact on performance. Hence, WSUS-WSR performs close to JSP-WSR for N relatively larger than M . However, the gains

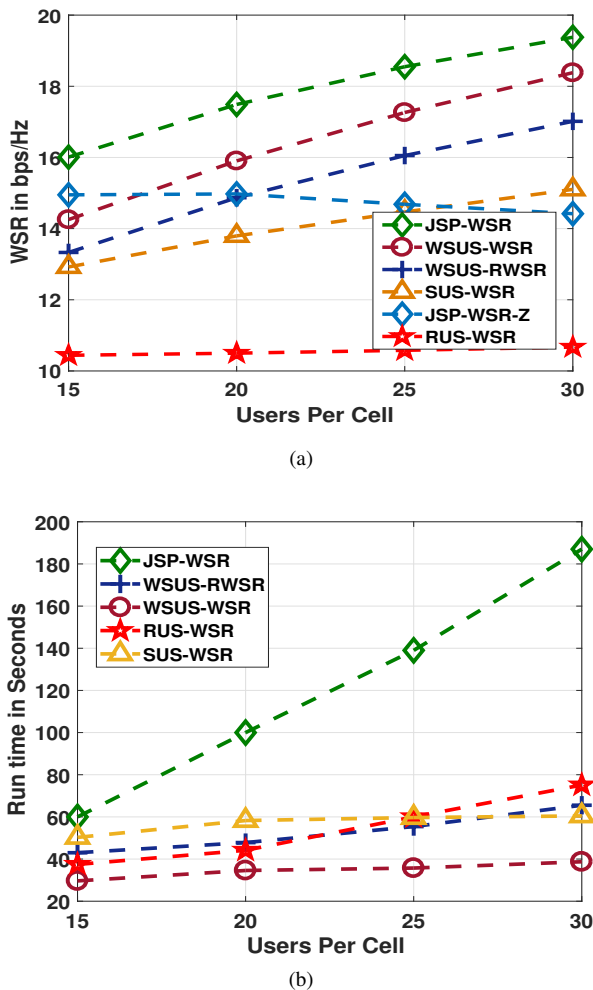


Fig. 1: Comparison of different WSR optimization approaches for $M = 10$, $\{-i = 4\text{dB}\}_{i=1}^N$; $P_T = 10$ dB, and N is varied from 15 to 30 (a) Achieved WSR and (b) algorithm run time

obtained by JSP-WSR even in comparison with WSUS-WSR still amounts up to 28% ($N = 15$). Notice that despite the difference in the rate of growth, all methods benefit from multiuser diversity to improve SR as N increases.

Notice that JSP-WSR and JSP-WSR-Z are identical except the FIPs. JSP-WSR and JSP-WSR-Z are CCP based algorithms hence the performance differentiation depends on FIP. Figure 1a shows that while a poor FIP like $\mathbf{W}^0 = \mathbf{0}$, $\mathbf{0} = \mathbf{0}$ results in worse performance than decoupled solutions, the FIPs from Section III-D achieves better performance. This shows the efficiency of the FIP mechanism detailed in Section III-D. In particular, $\mathbf{W}^0 = \mathbf{0}$, $\mathbf{0} = \mathbf{0}$ is a bad choice since it is the solution that achieves lowest WSR i.e., zero and hence the solutions of JSP-WSR-Z are generally the stationary points around the lowest objective.

Despite having the same WSUS scheduling algorithm and the same FIP for precoding, WSUS-WSR outperforms WSUS-RWSR due to the difference in precoding algorithms as shown in figure 1a. Although classical WSR can be formulated as a DC problem using proposed reformulations and also by the approach in [24], due to the efficiency of proposed reformulations, WSUS-WSR achieves the better objective which is confirmed by figure 1a.

Figure 1b illustrates the complexity of algorithms as a function of running time in seconds. Notice that the running time includes the time to calculate the FIPs and the final solutions. In the decoupled algorithms i.e., WSUS-WSR and WSUS-RWSR the complexity of scheduling algorithms is negligible compared to the latter precoding problem. Since the precoding is always performed on M users, the precoding complexity of RUS-WSR, WSUS-WSR, and WSUS-RWSR is only a function of M . On the contrary, joint design algorithms, JSP-WSR, JSP-WSR-Z operate on N users hence the complexity increases with N . However, due to the efficiency in the design of JSP-WSR, its complexity can be comparable to that of WSUS-WSR for relatively low values of N , e.g. $N = 15$.

In table III, the number of iterations needed for JSP-WSR to converge to a stationary point is illustrated as a function of $M = 10$, $\{-i = 4\text{dB}\}_{i=1}^N$; $P_T = 10$ dB, and N is varied from 15 to 30 (in steps of 5). The per-iteration complexity of JSP-WSR is known to be polynomial and Table III confirms the number of iterations that are need for JSP-WSR to converge is also in $\mathcal{O}(N)$. Figure 1-b shows an approximately linear increase in run-time for JSP-WSR to converge, despite the sub-linear increase in number of iterations. This confirms the predominant contribution of the per-iteration complexity in $\mathcal{P}_5^{\text{WSR}}$ to the total run-time. Moreover, table III confirms that JSP-WSR, in general, has a fast convergence rate (i.e., low-complexity). However, in some scenarios, JSP-WSR may not exhibit fast convergence (i.e., high complexity).

In table II, the performance of JSP-WSR is compared with ES-WSR and WSUS-WSR for $M = 3$, $\{-i = 4\text{dB}\}_{i=1}^N$; $P_T = 10$ dB, and N is varied from 5 to 7 in steps of 1. Although the JSP-WSR is guaranteed to converge only to a stationary point theoretically, the results in table II confirms that these stationary points are indeed high-quality solutions. On the other hand, the shortcomings of the decoupled solution i.e., WSUS-WSR, leads to a large performance gap from both ES-WSR and JSP-WSR.

The performance of the JSP-WSR is illustrated for uniform weighted case i.e. $\{i = 1\}_{i=1}^N$ in figure 2a as a function of N . The performance gain by jointly updating scheduling and precoding in JSP-WSR over the decoupled SUS-WSR and SUS-RWSR is clear from figure 2a. However, as N increases ($N \approx 20$), SUS schedules the users with strong channel gains and least interference; hence SUS-WSR performs close to JSP-WSR. Despite the efficiency of SUS in the region around $N = 20$, SUS-RWSR performs poor due to the inefficiency of the RWSR precoding scheme.

Figure 2b illustrates the convergence behavior of the JSP-WSR and the convergence of β to binary values as a function of iterations. The SR obtained in each iteration is shown by the red curve while the penalized SR is shown by the blue curve. As the FIP of JSP-WSR contains a non-binary β , the solutions obtained in the initial iterations include the non-binary β ; hence, the difference between SR (red curve) and SR plus penalty (blue curve). However, as the penalty factor (λ) increases over the iterations, JSP-WSR favors the solutions with binary β s. As a result, the penalty approaches zero over the iterations i.e., $\mathcal{P}(\beta) \approx 0, \forall i$. This behavior is clear from iteration 8 onwards. Moreover, the convergence behavior of

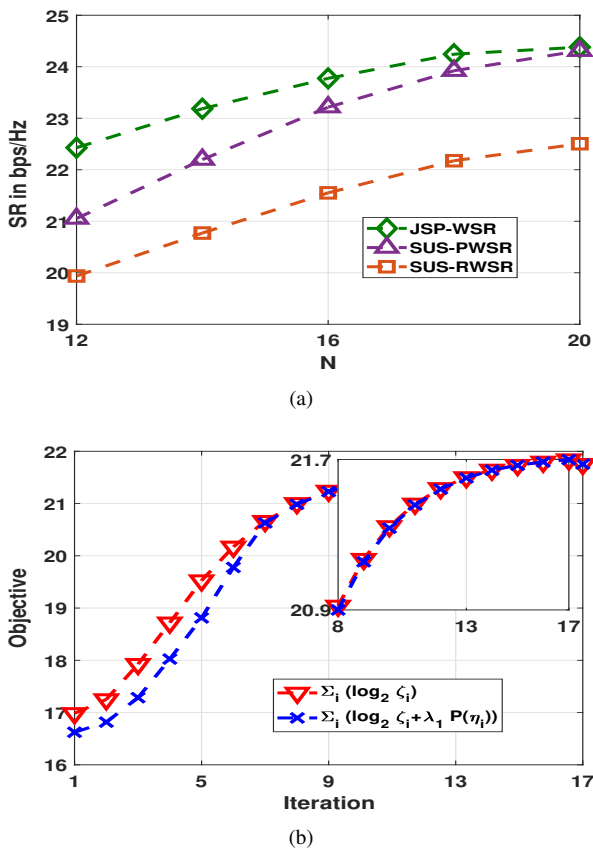


Fig. 2: Comparison of various WSR optimization methods for uniform weighted case with $M = 10$, $\{\gamma_i = 4\text{dB}\}_{i=1}^N$, $P_T = 10\text{dB}$ (a) N varying from 12 to 20. (b) convergence of the JSP-WSR (with penalty) and convergence of ζ_i to binary for $N = 20$.

TABLE II: Comparison of various WSR solutions for $M = 3$, $\{\gamma_i = 4\text{dB}\}_{i=1}^N$; $P_T = 10\text{ dB}$, and N varying from 5 to 7.

Users in cell (N)	weighted sum rate in bps/Hz		
	ES-WSR	JSP-WSR	WSUS-WSR
$N = 5$	5.83	5.67	5.32
$N = 6$	6.51	5.97	5.59
$N = 7$	6.64	6.33	6.02

the JSP-WSR to a stationary point of $\mathcal{P}_5^{\text{WSR}}$ is shown by the convergence of the blue curve which depicts its objective value.

D. MMSINR Performance Evaluation

Figure 3 illustrates the weighted MSINR of the scheduled users (averaged over 500 different CRs and referred to as TABLE III: Convergence rate of JSP-WSR for $M = 10$, $\{\gamma_i = 4\text{dB}\}_{i=1}^N$; $P_T = 10\text{ dB}$ as a function of N .

Number of users in a cell	Average number of iterations to converge
$N = 15$	16
$N = 20$	20.3
$N = 25$	22.5
$N = 30$	24.8

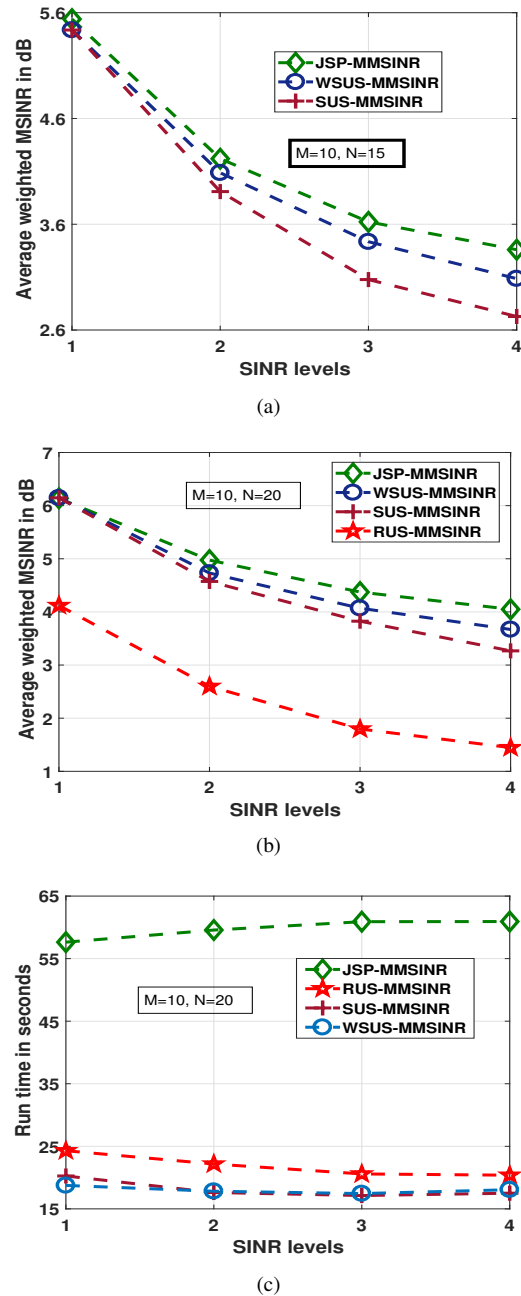


Fig. 3: Comparison of different MMSINR optimization approaches for $P_T = 10\text{dB}$, $\{\gamma_i = 0\text{dB}\}_{i=1}^N$ and SINR levels are varied from 1 to 4 (a) $N = 15$ (b) $N = 20$ (c) algorithm run time.

average weighted MSINR) as a function of SINR levels. For SINR level 1, 2, 3 and 4, the weight γ_i associated with user i is randomly drawn from the sets $\{1\}$, $\{0.5; 1\}$, $\{0.333; 0.6666; 0.9999\}$ and $\{0.25; 0.5; 0.75; 1\}$ respectively. For example, for SINR levels 2, γ_i is randomly selected from $\{0.5; 1\}$. Hence the MMSINR requirement of each user is $\gamma_i=0.5$ or γ_i (also $\gamma_i = 1$). Notice that a higher value of γ_i increases the likeliness of user i being scheduled.

The performance of JSP-MMSINR is compared with SUS-MMSINR and WSUS-MMSINR for $M = 10$, $\{\gamma_i = 1 (0\text{ dB})\}_{i=1}^N$, $P_T = 10\text{dB}$ and $N = 15$ in figure 3a and $N = 20$ in figure 3b. It is clear from figure 3a and 3b, that the joint solution JSP-MMSINR improves the performance over the decoupled design RUS-MMSINR, SUS-

TABLE IV: Performance of various MMSINR solutions for $M = 3$, SINR level 4, $P_T = 10$ dB and N varying from 5 to 7

Users in cell (N)	Average MSINR in dB with		
	ES-MMSINR	JSP-MMSINR	WSUS-MMSINR
$N = 5$	5.23	5.15	4.63
$N = 6$	5.95	5.79	5.03
$N = 7$	6.71	6.56	5.99

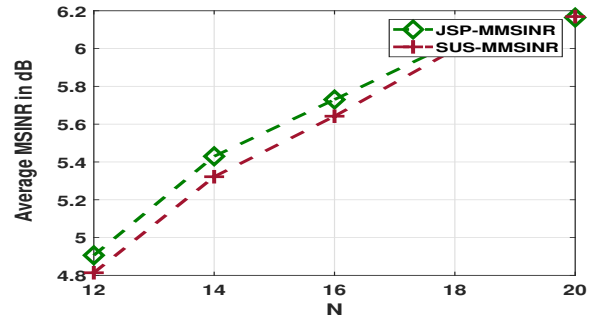
MMSINR, and WSUS-MMSINR. Despite identical underlying precoding scheme in JSP-MMSINR, RUS-MMSINR, SUS-MMSINR, and WSUS-MMSINR, the systematic approach of joint scheduling and precoder update considering the weights helps JSP-MMSINR to achieve better performance. The naive user scheduling based method i.e., RUS-MMSINR clearly performs poorer than other benchmark methods. Although WSUS-MMSINR achieves better performance over SUS-MMSINR by considering the weights into scheduling, it still performs worse than JSP-MMSINR showing the inefficiency of decoupled design. The gains obtained by JSP-MMSINR compared to best performing decoupled method i.e., WSUS-MMSINR amounts up to 10% (figure 3b, SINR level 4).

In figure 3c, the run time of the algorithms is illustrated as a function of SINR levels for $M = 10$, $N = 20$ and $P_T = 10$ dB. Figure 3c shows that the gains of JSP-MMSINR are achieved at the expense of high computational complexity as illustrated in figure 3c. Moreover, the complexity of JSP-MMSINR increases as SINR levels increase. The increase in SINR levels enforces the inclusion of users with higher SINR requirement since the users with lower SINR requirement may not be sufficient to schedule exactly M users. Hence, JSP-MMSINR takes relatively longer time to converge compared lower SINR levels.

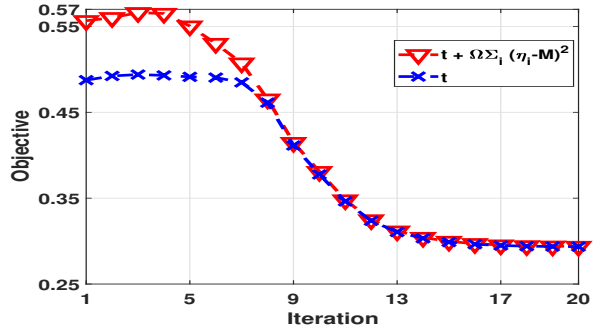
The performance of JSP-MMSINR is illustrated for uniform weighted case i.e., $\{\gamma_i = 1\}_{i=1}^N$ in figure 4 for $M = 10$ and $P_T = 10$ dB. In figure 4a, the average MSINR is illustrated as a function of N varying from 12 to 18 in steps of 2. The superior performance of JSP-MMSINR over SUS-MMSINR is clear from 4a. However, the gains diminish as N increases as the SUS based solution becomes efficient as mentioned previously.

In figure 4b, the convergence behavior of the algorithm and progression towards achieving exact scheduling constraint i.e., $\sum_{i=1}^N \gamma_i = M$ are illustrated as a function of the iteration number. While the blue curve depicts the inverse of MSINR (i.e., t in \mathcal{P}_4^{MM}) achieved over the iteration, the red curve depicts the penalized objective where the penalty aims to satisfy the constraint of scheduling exactly M users. As FIPs violate the exact scheduling constraint, the penalized objective (red curve) is far from the objective (blue curve). However, increasing the penalty parameter ω over the iterations until $\omega \leq 20$ ensures the scheduling constraint. This behavior is observed from iteration 8 in figure 4b as the difference between penalized objective and objective is approximately zero. Moreover, the binary nature of γ_i is also achieved over the iterations due to nature of MMSINR for fixed $\gamma_2 = 0$ in figure 3 and 4.

Table IV compares the performance of JSP-MMSINR,



(a)



(b)

Fig. 4: Comparison of MMSINR for uniform weighted case, $M = 10$, $\gamma_i = 1$; $\gamma_i = 0$ dB, $\forall i$, $P_T = 10$ dB (a) N varies from 12 to 20. (b) Convergence of the JSP-MMSINR (with penalty) and convergence of t to binary for $N = 20$.

WSUS-MMSINR and ES-WSR for $M = 3$, SINR level 4, $P_T = 10$ dB. The relatively similar performance of JSP-MMSINR and ES-MMSINR confirms the efficiency of JSP-MMSINR and high-quality nature of stationary points that the JSP-MMSINR converges to.

E. PMIN Performance Evaluation

The total power consumed by the scheduled users (for each channel realization) is averaged over 500 channel realizations (CRs) which is referred to as average total power per CR. In figure 5, the average total power per CR is depicted as a function of SINR levels for $M = 10$, $N = 15$ in figure 5a and $N = 20$ in figure 5b. The SINR level 1, 2, 3 and 4 (chosen differently than MMSINR design) on the x-axis indicate that γ_i is randomly chosen from the sets $\{1\}$, $\{1;2\}$, $\{1;2;3\}$ and $\{1;2;3;4\}$ for user i respectively. For example, for the SINR level 2, γ_i for user i is randomly chosen from the set $\{1;2\}$.

Figure 5a and 5b clearly show that the joint solution JSP-PMIN outperforms RUS-PMIN, SUS-PMIN and WSUS-PMIN. Although the precoding problem for the scheduled users by RUS, SUS, and WSUS is solved globally using [7], the inefficient scheduling leads to poorer performance compared to JSP-PMIN. On the contrary, the system design in JSP-PMIN helps to gain up to 25% (SINR level 4, figure 5b) in comparison with WSUS-PMIN. In figure 5c, the run time of algorithms is illustrated in seconds as a function of SINR levels. As shown in figure 5c, the performance gains of JSP-MMSINR incur higher computational complexity.

In table V, the performance of JSP-PMIN and WSUS-PMIN is compared with ES-PMIN for $M = 5$, $\gamma_i \in \{0;1;2;3\}$; $\forall i$ for

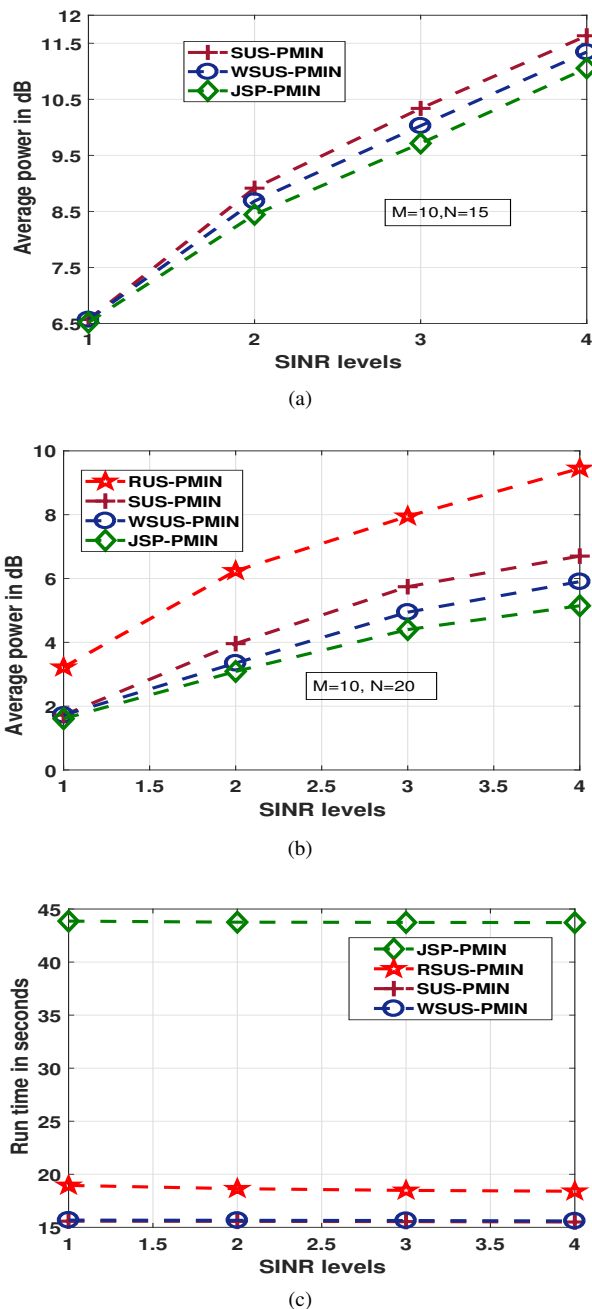


Fig. 5: Comparison of different PMIN optimization approaches for $M = 10$, $P_T = 10$ dB and SINR levels varying from 1 to 4 (a) $N = 15$ (b) $N = 20$ (c) algorithm run time.

different N . Despite the theoretical guarantees of convergence JSP-PMIN only to a stationary point, JSP-PMIN performs close to ES-PMIN as can be observed in table V. This justifies the efficiency of JSP-PMIN approach.

The performance JSP-PMIN for uniform weighted case (i.e., all users with same MSINR requirement) is illustrated in figure 6 for $M = 10$ and $\{\eta_i = 1\}_{i=1}^N$. In figure 6a, the average total power per CR in dB is depicted as a function of N varying from 15 to 30 in steps of 5. The superior performance of JSP-PMIN over SUS-PMIN is clear from figure 6a. However, the gains diminish as N increases as the SUS based scheduling becomes efficient (kindly refer to similar discussion on WSR results).

TABLE V: Performance comparison of various PMIN solutions for $M = 3$, SINR level 4 and N is varied from 5 to 7.

Users in cell (N)	Average power consumed in dB with		
	ES-PMIN	JSP-PMIN	WSUS-PMIN
$N = 5$	5.4477	5.99	6.2448
$N = 6$	5.116	5.6164	5.8771
$N = 7$	3.95	4.5365	5.1270

In figure 6b, the convergence behavior of the JSP-PMIN algorithm (red curve) and the progression towards ensuring the exact scheduling constraint is depicted as a function of iterations for $N = 15$. The FIP may include the solutions that violate exact scheduling constraint due to which the penalized objective and objective differs by a large factor in the initial iterations. However, the increment in the penalty parameter ensures the exact scheduling constraint over the iterations. This is confirmed by figure 6b, as the difference between penalized objective and objective, becomes approximately zero. For the reasons at the beginning of this section, $\epsilon_3 = 0$ still achieves the binary nature of η_i over iterations.

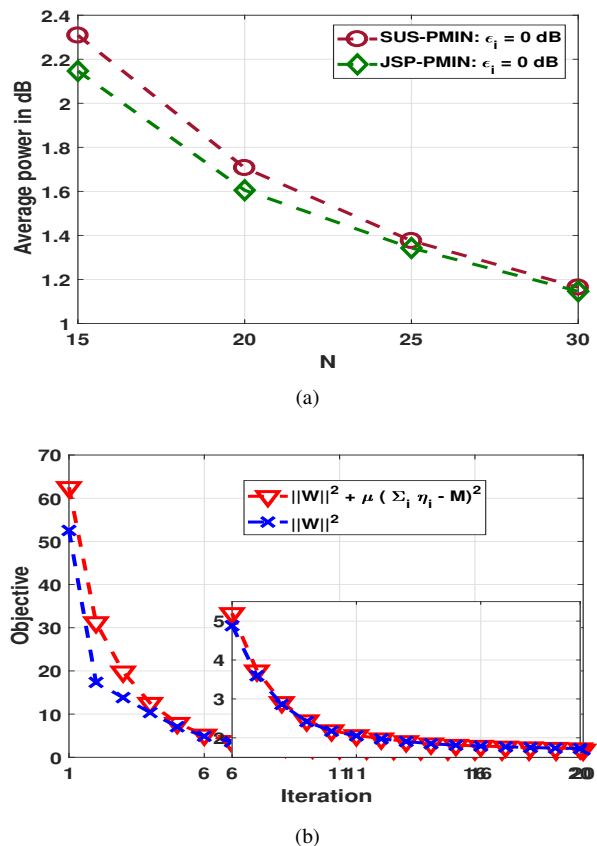


Fig. 6: Comparison of different PMIN optimization approaches for $M = 10$, $P_T = 10$ dB and $\{\eta_i = 0 \text{ dB}\}_{i=1}^N$ (a) N varying from 10 to 30 in steps of 5 (b) convergence of the JSP-PMIN (with penalty) and convergence of the objective to binary for $N = 15$.

VII. CONCLUSIONS

In this paper, the joint scheduling and precoding problem was considered for multiuser MISO downlink channels for

three performance optimization criteria: weighted sum rate maximization, maximization of minimum SINR and power minimization. Unlike the existing works, the design is formulated in a way that is amenable to the joint update of scheduling and precoding. Observing that the original optimization to be an instance of the MINLP problem for the three considered criteria, the paper proposed efficient reformulations and relaxations to transform these into structured DC programming problems. Subsequently, the paper proposed joint scheduling and precoding CCP based algorithms (JSP-WSR, JSP-MMSINR, and JSP-PMIN) which are guaranteed to converge to a stationary point for the aforementioned DC problems. Finally, the paper proposed a low-complexity procedure to obtain a good feasible initial point, critical to the implementation of CCP based algorithms. Through simulations, the paper established the efficacy of the proposed joint techniques with respect to the decoupled benchmark solutions.

REFERENCES

- [1] A. Bandi, M. R. B. Shakar, S. Maleki, C. Symeon, and B. Ottersten, "A Novel Approach to Joint User Selection and Precoding for Multiuser MISO Downlink Channels," in *Proc. GlobSIP*, November 2018.
- [2] H. Weingarten, Y. Steinberg, and S. S. Shamai, "The Capacity Region of the Gaussian Multiple-Input Multiple-Output Broadcast Channel," *IEEE Trans. Inf. Theory*, vol. 52, no. 9, pp. 3936–3964, Sept 2006.
- [3] H. Viswanathan, S. Venkatesan, and H. Huang, "Downlink capacity evaluation of cellular networks with known-interference cancellation," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 5, pp. 802–811, June 2003.
- [4] P. Zetterberg and B. Ottersten, "The spectrum efficiency of a base station antenna array system for spatially selective transmission," *IEEE Trans. Veh. Technol.*, vol. 44, no. 3, pp. 651–660, Aug 1995.
- [5] S. Anderson, M. Millnert, M. Viberg, and B. Wahlberg, "An adaptive array for mobile communication systems," *IEEE Trans. Veh. Technol.*, vol. 40, no. 1, pp. 230–236, Feb 1991.
- [6] A. Wiesel, Y. C. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed mimo receivers," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161–176, Jan 2006.
- [7] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in *Handbook of Antennas in Wireless Communications*. CRC Press, 2001, pp. 18–1–18–33, qC 20111107.
- [8] G. Dimic and N. D. Sidiropoulos, "On downlink beamforming with greedy user selection: performance analysis and a simple new algorithm," *IEEE Trans. Signal Process.*, vol. 53, no. 10, pp. 3857–3868, Oct 2005.
- [9] T. Yoo and A. Goldsmith, "On the optimality of multiantenna broadcast scheduling using zero-forcing beamforming," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 3, pp. 528–541, March 2006.
- [10] E. Castaeda, A. Silva, A. Gameiro, and M. Kountouris, "An Overview on Resource Allocation Techniques for Multi-User MIMO Systems," *IEEE Commun. Surveys Tuts.*, vol. 19, no. 1, pp. 239–284, Firstquarter 2017.
- [11] T. Yoo, N. Jindal, and A. Goldsmith, "Multi-Antenna Downlink Channels with Limited Feedback and User Selection," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1478–1491, September 2007.
- [12] G. Lee and Y. Sung, "A New Approach to User Scheduling in Massive Multi-User MIMO Broadcast Channels," *IEEE Trans. Commun.*, vol. 66, no. 4, pp. 1481–1495, April 2018.
- [13] B. Song, Y. Lin, and R. L. Cruz, "Weighted max-min fair beamforming, power control, and scheduling for a MISO downlink," *IEEE Trans. Wireless Commun.*, vol. 7, no. 2, pp. 464–469, February 2008.
- [14] W. Yu, T. Kwon, and C. Shin, "Multicell Coordination via Joint Scheduling, Beamforming, and Power Spectrum Adaptation," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 1–14, July 2013.
- [15] E. Matakani, N. D. Sidiropoulos, Z. q. Luo, and L. Tassiulas, "Convex approximation techniques for joint multiuser downlink beamforming and admission control," *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2682–2693, July 2008.
- [16] M. Li, I. B. Collings, S. V. Hanly, C. Liu, and P. Whiting, "Multicell Coordinated Scheduling With Multiuser Zero-Forcing Beamforming," *IEEE Trans. Wireless Commun.*, vol. 15, no. 2, pp. 827–842, Feb 2016.
- [17] M. Kountouris, D. Gesbert, and T. Sizer, "Enhanced multiuser random beamforming: dealing with the not so large number of users case," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 8, pp. 1536–1545, October 2008.
- [18] M. L. Ku, L. C. Wang, and Y. L. Liu, "Joint Antenna Beamforming, Multiuser Scheduling, and Power Allocation for Hierarchical Cellular Systems," *IEEE J. Sel. Areas Commun.*, vol. 33, no. 5, pp. 896–909, May 2015.
- [19] L. Yu, E. Karipidis, and E. G. Larsson, "Coordinated scheduling and beamforming for multicell spectrum sharing networks using branch and bound," in *Proc. EUSIPCO*, Aug 2012, pp. 819–823.
- [20] A. Douik, H. Dahrouj, T. Y. Al-Naffouri, and M. S. Alouini, "Coordinated Scheduling and Power Control in Cloud-Radio Access Networks," *IEEE Trans. Wireless Commun.*, vol. 15, no. 4, pp. 2523–2536, April 2016.
- [21] O. Goussevskaia, R. Wattenhofer, M. M. Halldorsson, and E. Welzl, "Capacity of arbitrary wireless networks," in *IEEE INFOCOM 2009*, April 2009, pp. 1872–1880.
- [22] B. Dai and W. Yu, "Sparse Beamforming and User-Centric Clustering for Downlink Cloud Radio Access Network," *IEEE Access*, vol. 2, pp. 1326–1339, 2014.
- [23] M. Tao, E. Chen, H. Zhou, and W. Yu, "Content-Centric Sparse Multicast Beamforming for Cache-Enabled Cloud RAN," *IEEE Trans. Wireless Commun.*, vol. 15, no. 9, pp. 6118–6131, Sept 2016.
- [24] S. He, J. Wang, Y. Huang, B. Ottersten, and W. Hong, "Codebook-Based Hybrid Precoding for Millimeter Wave Multiuser Systems," *IEEE Trans. Signal Process.*, vol. 65, no. 20, pp. 5289–5304, Oct 2017.
- [25] A. H. Phan, H. D. Tuan, H. H. Kha, and H. H. Nguyen, "Beamforming Optimization in Multi-User Amplify-and-Forward Wireless Relay Networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 4, pp. 1510–1520, April 2012.
- [26] U. Rashid, H. D. Tuan, and H. H. Nguyen, "Relay Beamforming Designs in Multi-User Wireless Relay Networks Based on Throughput Maximin Optimization," *IEEE Trans. Commun.*, vol. 61, no. 5, pp. 1739–1749, May 2013.
- [27] G. R. Lanckriet and B. K. Sriperumbudur, "On the Convergence of the Concave-Convex Procedure," in *Advances in Neural Information Processing Systems 22*, 2009, pp. 1759–1767.
- [28] H. Shi, R. V. Prasad, E. Onur, and I. G. M. M. Niemegeers, "Fairness in wireless networks: issues, measures and challenges," *IEEE Commun. Surveys Tuts.*, vol. 16, no. 1, pp. 5–24, First 2014.
- [29] D. Christopoulos, S. Chatzinotas, and B. Ottersten, "Multicast multi-group precoding and user scheduling for frame-based satellite communications," *IEEE Trans. Wireless Commun.*, vol. 14, no. 9, pp. 4695–4707, Sept 2015.
- [30] I. Mitliagkas, N. D. Sidiropoulos, and A. Swami, "Joint Power and Admission Control for Ad-Hoc and Cognitive Underlay Networks: Convex Approximation and Distributed Implementation," *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 4110–4121, December 2011.
- [31] Y. Cheng and M. Pesavento, "Joint Discrete Rate Adaptation and Downlink Beamforming Using Mixed Integer Conic Programming," *IEEE Trans. Signal Process.*, vol. 63, no. 7, pp. 1750–1764, April 2015.
- [32] J. Rubio, A. Pascual-Iserte, D. P. Palomar, and A. Goldsmith, "Joint Optimization of Power and Data Transfer in Multiuser MIMO Systems," *IEEE Trans. Signal Process.*, vol. 65, no. 1, pp. 212–227, Jan 2017.
- [33] C. T. K. Ng and H. Huang, "Linear Precoding in Cooperative MIMO Cellular Networks with Limited Coordination Clusters," *IEEE J. Sel. Areas Commun.*, vol. 28, no. 9, pp. 1446–1454, December 2010.
- [34] S. You, L. Chen, and Y. E. Liu, "Convex-concave procedure for weighted sum-rate maximization in a MIMO interference network," in *Proc. Globecom*, Dec 2014, pp. 4060–4065.
- [35] A. L. Yuille and A. Rangarajan, "The concave-convex procedure (CCCP)," in *NIPS*, 2001.
- [36] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox Users Guide*. USA: MathWorks., 1995.
- [37] D. W. H. Cai, T. Q. S. Quek, and C. W. Tan, "A Unified Analysis of Max-Min Weighted SINR for MIMO Downlink System," *IEEE Trans. Signal Process.*, vol. 59, no. 8, pp. 3850–3862, Aug 2011.
- [38] L. Zheng, Y. P. Hong, C. W. Tan, C. Hsieh, and C. Lee, "Wireless MaxMin Utility Fairness With General Monotonic Constraints by Perron-Frobenius Theory," *IEEE Trans. Inf. Theory*, vol. 62, no. 12, pp. 7283–7298, Dec 2016.

