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Beyond Tides - Determination of Core Properties from Superconducting Gravimeter Observations

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Abstract

The analysis of many long records of superconducting gravimeter observations at stations with a wide geographical distribution has led to the detection of the three translational modes of oscillation of Earth's solid inner core, both in European observations (Smylie et al., 1993), and in observations outside Europe (Courtier et al., 2000). Due to Earth's rotation, the modes are split with periods found at 3.5822 ± 0.0012, 3.7656 ± 0.0015, and 4.0150 ± 0.0010 hours. In this paper, we show how these observations can be used to determine the viscosity in the F-layer just outside the inner core and to confirm the inner core density of Earth model Cal8 of Bolt and Uhrhammer (Bullen and Bolt, 1985, Appendix).

1. Introduction

The Canadian Superconducting Gravimeter Installation (CSGI) at Cantley, Québec, has given Canadian geophysicists an exceptional observational resource. Nonetheless, to realize the full potential of these instruments, which are capable of observing gravity changes at the level of a few nanogals, it was recognized early on that, in addition to long unbroken records, it was necessary to use observations at more than one station to reduce the effects of local systematic errors, and this led to a proposal for a Global Geodynamics Project (GGP) (Aldridge et al., 1991); Crossley et al., 1999). Since that time, many long records have become available with a wide station geographical distibution. Their analysis has led to the detection of the three translational modes of oscillation of Earth's solid inner core, both in European observations (Smylie et al., 1993), and in observations outside Europe (Courtier et al., 2000). Due to Earth's rotation, the modes have split periods found at 3.5822 ± 0.0012, 3.7656 ± 0.0015, and 4.0150 ± 0.0010 hours.

The axial translational mode period is not much affected by rotation or viscosity while Coriolis splitting reduces the period of the retrograde equatorial mode and increases the period of the prograde equatorial mode, though not by as much as if the surrounding outer core fluid were inviscid. The reduction of the splitting of the two equatorial mode periods allows the inner core itself to be used as a kind of two-dimensional dynamic viscometer and a viscosity of 1.24×10^{11} $Pa \cdot s$ is recovered from the observed splitting. This value is close to that found theoretically for the bulk viscosity of a liquid with solid inclusions (Stevenson, 1983), and recently experimentally by Brazhkin and Lyapin (2000). It appears to confirm the semi-solid nature of the F-layer, long held to be the seat of the compositional convection driving the geodynamo. In addition, the mode periods are extremely sensitive to inner core density, increasing by 200 $minutes/gm \cdot cm^{-3}$. The recovered viscosity reproduces the observed equatorial mode periods to 6.48 s suggesting a density resolution of 4 parts in10^5. We use this density resolution to verify the inner core density profile of Earth model Cal8 of Bolt and Uhrhammer (Bullen and Bolt, 1985, Appendix). The agreement with the Cal8 Earth model is a beautiful confirmation of both theory and observation, and is the principal new contibution of the present study. As we shall see, the axial mode period for Cal8 is 27 s longer than the observed period and can be brought into coincidence by a reduction of the overall inner core density by only 2.25 $milligrams \cdot cm^{-3}$.

These results depend on a long history of development of gravimetric observation and study of solid Earth tides (Melchior, 1983). Modern expansions of the tide generating potential to more than three thousand terms (Xi, 1987, 1989) allow synthetic tidal signals to be generated by programmes such as G-Wave of J. B. Merriam (Merriam, 1992) with accuracies of a few nanogals. Only after the subtraction of the synthetic tidal signal and correction for atmospheric attraction and loading is made can the residual signal be analyzed for other geophysical effects.

Traditionally, seismic methods have been the principal source of information on Earth's deep interior. These methods are limited by their dependence on only combined effects of elasticity and density, and their lack of sensitivity to other Earth properties such as viscosity. Further, for the deepest parts of the Planet, the signal strengths are very weak so that even the detection of phases travelling through the solid inner core as S is controversial.

2. Inner Core Translational Modes and their Observation

The resonances associated with the translational modes were first identified visually in a product of four individual spectra from long superconducting gravimeter records taken in Europe amounting to a total of $111,000$ hourly observations (Smylie et al., 1992). The Product Spectrum is a very useful technique where non-simultaneous observations at a number of stations are to be combined to look for common features and to minimize local station systematic errors. A full statistical analysis of the Product Spectrum, together with a method of calculating confidence intervals for it, is given by Smylie et al (Smylie et al., 1993). Strict adherence to a splitting law derived from elementary dynamics assists identification of the triplet of resonances. Largely independent of Earth model, it can be shown that the central frequencies must fall on splitting curves. The observed periods obey the splitting law to about four significant figures and a computer based search for equally significant triplets of resonances failed to find others (Smylie et al., 1993).

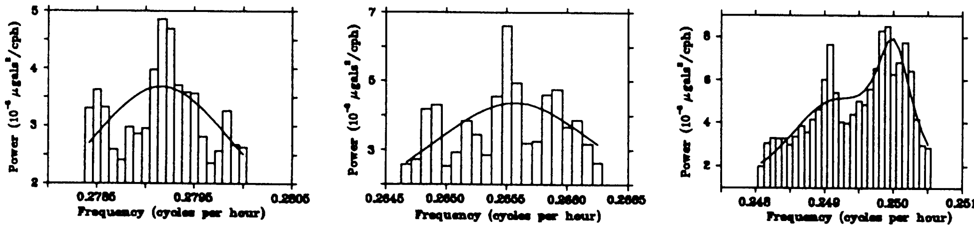

Fig. 1. Product Spectra of (from left to right) the retrograde, axial and prograde modes from observations at Bad Homburg, Brussels, Cantley and Strasbourg (Courtier et al., 2000). The prograde mode is near the large solar heating tide feature S_6 at exactly six cycles per solar day. A full statistical analysis of the Product Spectrum and a method of calculating confidence intervals for it have been given previously (Smylie et al., 1993).

Now almost 300,000 hours of superconducting gravimeter records have been analyzed, from both inside and outside Europe. In Figure 1 resonances from a Product Spectrum including observations at the Cantley, Québec station in Canada as well as those from Europe are shown.

No definitive answer has been given to the question of the source of excitation of the translational modes. Seismic excitation seems unlikely since most models of the seismic source are in force and moment equilibrium and cannot provide the required linear momentum transfer. The random motion in the outer core associated with the turbulent flow driving the geomagnetic dynamo is a more likely candidate. Only about a 1% imbalance in linear momentum in this motion would suffice to explain the observed excitation levels.

3. Ekman Boundary Layer Theory and Pressure and Viscous Drags on the Inner Core

The calculation of the viscous and pressure drags requires a model for the inviscid flow immediately outside the boundary layer surrounding the inner core. Since the effects of stratification and compressibility there are likely to be negligible, this flow field is given by solutions of the Poincaré equation. For a reference Earth frame rotating at mean rate Ω about a fixed spatial direction the angular frequency of the motion, ω, can be expressed by the dimensionless Coriolis frequency $\sigma = \omega/2\Omega$. Following Bryan (Bryan, 1889), the axis aligned with the rotation vector is stretched by the factor $1/\tau$ where $\tau^2 = 1 - 1/\sigma^2$. In the stretched 'auxillary' coordinates, the Poincaré equation becomes Laplace's equation.

Novel solutions of the Poincaré equation using the Legendre function of the second kind have been obtained (Smylie and McMillan, 1998), which allow the condition of continuity of the radial displacement to be satisfied exactly at the inner core boundary and asymptotically at the core-mantle boundary. Together with conservation of linear momentum between the inner core, outer core and shell, three equations are obtained whose solution in powers of $1/\sigma$ allow the pressure drag to be expressed in the form

$$4\Omega^2 \left(\alpha\sigma^2 + \beta\sigma + \gamma\right) U_I \quad (1)$$

with U_I representing the inner core displacement (Smylie and McMillan, 2000). It is found that

$$\alpha = M_I' \left(\frac{1}{2} + \frac{3}{2} \frac{M_I + (a/b)^3 M_S}{M_O + M_S \left(1 - (a/b)^3\right)} \right), \tag{2}$$

for both the axial and equatorial modes, $\beta = 0$ for the axial mode and

$$\beta = M_I' \left(\frac{1}{4} - \frac{3}{4} \frac{M_I + (a/b)^3 M_S}{M_O + M_S \left(1 - (a/b)^3\right)} \right) \tag{3}$$

for the equatorial modes. Similar, but more complicated expressions for γ can be found but they are not used and are omitted. a is the radius of the inner core, b is the radius of the core-mantle boundary, M_I is the mass of the inner core, M_O is the mass of the outer core, M_S is the mass of the shell and $M_I' = 4/3\pi a^3 \rho_0$ is the displaced mass with ρ_0 the density just outside the inner core. The displacements arising from solutions involving only Legendre functions of the first kind, widely used in rotating fluid dynamics (Greenspan, 1969), are pure translations in this case and play an essential role in the conservation of linear momentum.

The leading order boundary layer equations (Moore, 1978) for the extra displacement components required to adjust the exterior flow to the no-slip condition at the inner core boundary can be solved and matched to the exterior flow (Smylie and McMillan, 1998). In these equations the local and Coriolis accelerations are balanced by viscous forces in the standard Ekman layer approximation. The ratio of the latter to the former is expressed by the dimensionless Ekman number

$$E_k = \frac{\eta}{\Omega a^2 \rho_0} \tag{4}$$

where η is the dynamic viscosity. Direct integration of the viscous stress acting on the inner core boundary yields the viscous drag forces.

The viscous drags can be written in terms of the corresponding pressure drags given by expressions (1) (2) and (3). For the axial mode with pressure drag D_p^a, the viscous drag is

$$D_v^a = \frac{1-i}{4} \sqrt{E_k} \left(D_p^a + 4\Omega^2 M_I' \sigma^2 U_I \right) f^a(\sigma), \tag{5}$$

where

$$f^a(\sigma) = \Big\{ 8 \left[(\sigma+1)^{3/2} + (\sigma-1)^{3/2} \right] - \frac{16}{5} \left[(\sigma+1)^{5/2} - (\sigma-1)^{5/2} \right] \Big\}. \tag{6}$$

For the equatorial modes with pressure drag D_p^e, the viscous drag is

$$D_v^e = \frac{1 \mp i}{8} \sqrt{E_k} \left(D_p^e + 4\Omega^2 M_I' \sigma (\sigma - 1) U_I \right) f^e(\sigma), \tag{7}$$

where $f^e(\sigma)$, shown plotted for both modes in Figure 2, is given by

$$f^e(\sigma) = \Big\{ \mp 24 (\pm\sigma \mp 1)^{1/2} - 16 (\pm\sigma \mp 1)^{3/2} - \frac{16}{5} \left[(\pm\sigma - 1)^{5/2} - (\pm\sigma + 1)^{5/2} \right] \Big\}. \tag{8}$$

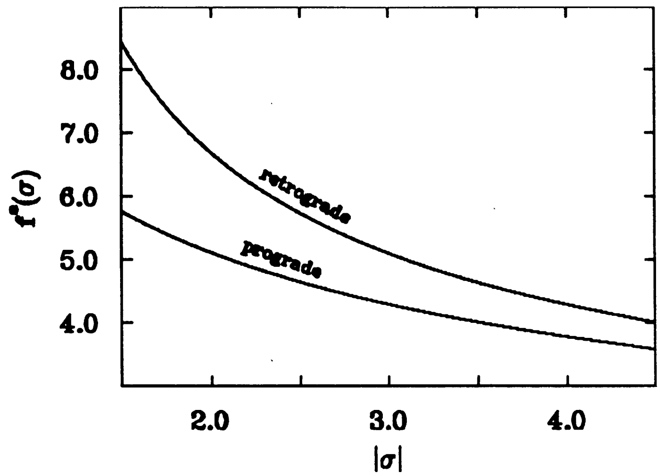

Fig. 2. Viscous drag factor $f^e(\sigma)$ plotted as a function of $|\sigma|$. σ is positive for the retrograde equatorial mode, negative for the prograde equatorial mode. The asymmetry of the drag factor provides a natural redundancy check on the viscosity recovered from the two equatorial translational modes of the inner core.

4. Splitting Laws and Viscosity Measurement

The pressure drag form (1), together with the viscous drags given by expressions (5) and (7), allow us to write complex equations of motion for the axial and equatorial translational oscillations. In general, the real parts of these equations can be expressed as splitting laws for the period in the form

$$\left(\frac{T}{T_0}\right)^2 + 2g^v \frac{T_0}{T_S}\left(\frac{T}{T_0}\right) - 1 = 0, \tag{9}$$

where T is the period, T_0 is the unsplit period, T_S is the length of the sidereal day and g^v is a dimensionless viscous splitting parameter. In this form, if we plot T as a function of T/T_0, only g^v remains as a viscosity-dependent free parameter. For the axial mode, the viscous splitting parameter is related to the inviscid splitting parameter by

$$g^v = g^i\left[1 + \frac{1}{4}\frac{M_I - M_I'}{M_I + \alpha}\sqrt{E_k} f^a(\sigma)\right], \tag{10}$$

and for the equatorial modes by

$$g^v = g^i\left[1 - \frac{1}{8}\left(\frac{M_I' - \beta}{M_I + \beta} + \frac{M_I' + \alpha}{M_I + \alpha}\right)\sqrt{E_k} f^e(\sigma)\right]. \tag{11}$$

Plots of the splitting laws of the form (9) for all three modes are shown in Figure 3 for the parameters of Earth model Cal8 of Bolt and Uhrhammer (Bullen and Bolt, 1985, Appendix).

As illustrated in Figure 3, we have used the parameters of Earth model Cal8 to recover the viscosity although the result is not strongly dependent on Earth model. Recovered viscosities and periods are listed in Table 1 for both Cal8 and CORE11 Earth models.

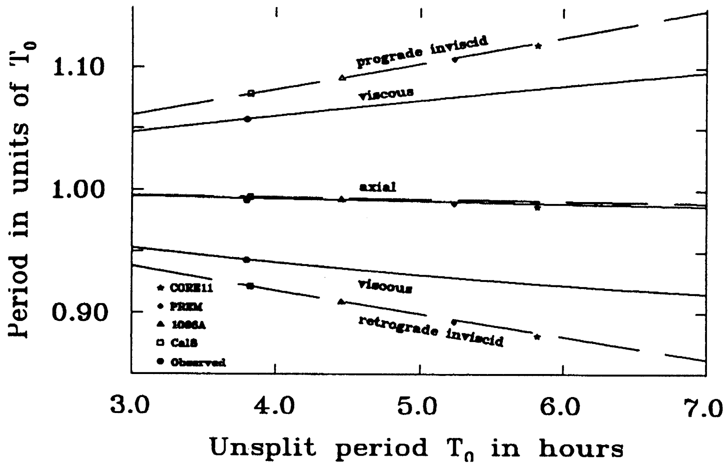

Fig. 3. Splitting curves for the three translational modes. The inviscid curves for the three modes are shown dashed based on Earth model Cal8 (open squares). Inviscid periods are overplotted for Earth models Core11 (open stars) (Widmer et al., 1988), PREM (open diamonds) (Dziewonski and Anderson, 1981) and 1066A (open triangles) (Gilbert and Dziewonski, 1975). Solid viscous splitting curves are for a single viscosity of 1.243×10^{11} $Pa \cdot s$.

5. Resolution of Inner Core Density

Unlike seismic and regular free oscillation methods, the translational modes of the inner core have periods which respond principally to inner core density and the viscosity of the surrounding fluid. As shown in Figure 3, the axial mode suffers little rotational and viscous splitting. Its observed period then becomes a very strict constraint on inner core density. The density profiles for the four Earth models considered in this study are shown in Figure 4.

More detailed plots of inner core density for the four Earth models are given in Figure 5. Each Earth model is shown with its corresponding unsplit period T_0. These range from just over 5.8 hours for Core11 to just over 3.8 hours in the case of Cal8. Core11 has a central density near 13 $gm \cdot cm^{-3}$ while that for Cal8 is near 13.6 $gm \cdot cm^{-3}$. The sensitivity of the axial mode period is then about 200 $minutes/gm \cdot cm^{-3}$.

6. Discussion

Since their first observation in the Product Spectrum of four long superconducting gravimeter records (Smylie, 1992), the identification of the translational triplet has met many challenges on both observational and theoretical grounds. First, the accurate computation of the mode periods is difficult using conventional vector spherical harmonic representations in the fluid outer core, because of strong Coriolis coupling and the resulting poor convergence of these global functions. Although these periods are now routinely accurately

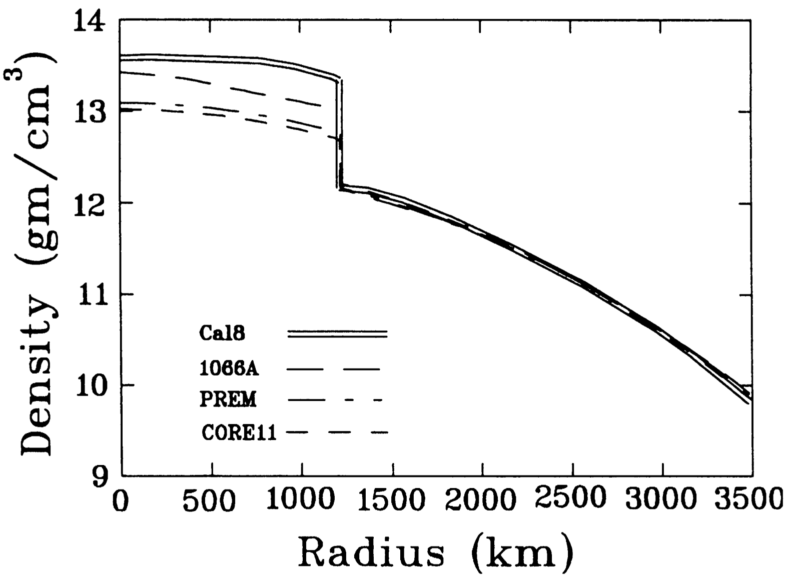

Fig. 4. Density profiles in the inner and outer cores for Earth models Cal8, 1066A, PREM and Core11. They show little difference in the outer core but range over about 4.4% in inner core density.

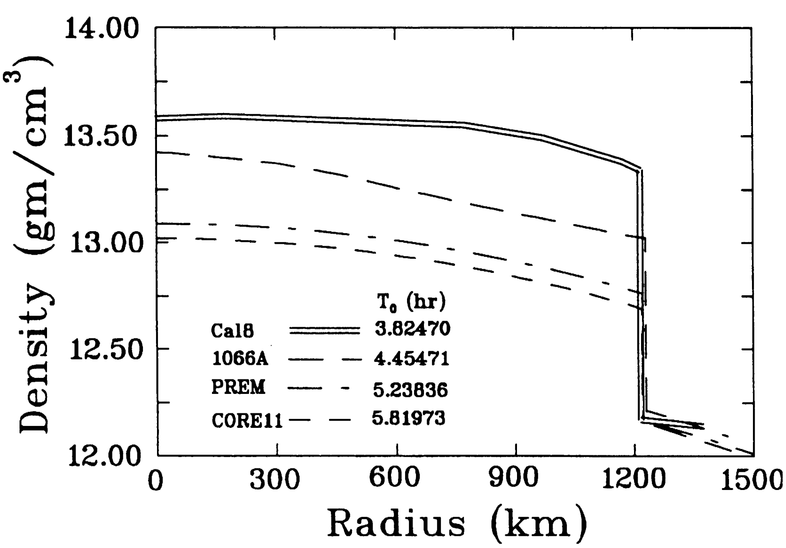

Fig. 5. Detailed density profiles in the inner core for Earth models Cal8, 1066A, PREM and Core11. The 0.6 $gm \cdot cm^{-3}$ density range causes nearly a 2 h difference in the unsplit period T_0.

Table 1. Recovered viscosities and viscous periods.

Periods	Retrograde (hours)	Axial (hours)	Prograde (hours)
Observed Periods	3.5822 ±0.0012	3.7656 ±0.0015	4.0150 ±0.0010
CORE11 Viscous Periods ($\nu = 9.58 \times 10^6\ m^2\ s^{-1}$, $\eta = 1.17 \times 10^{11}\ Pa \cdot s$)	3.5793	3.7647	4.0121
Cal8 Viscous Periods ($\nu = 10.21 \times 10^6\ m^2\ s^{-1}$, $\eta = 1.243 \times 10^{11}\ Pa \cdot s$)	3.5840	3.7731	4.0168
Cal8 Inviscid Periods	3.5168	3.7926	4.1118

computed by local bicubic spline representations (Smylie, Jiang, Brennan and Sato, 1992; Smylie and Jiang, 1993) there still persist in the literature examples where they are incorrectly given (see Crossley et al., 1999, where the inviscid periods for PREM are incorrectly quoted as 4.77, 5.31 and 5.98 hours, while the correct periods are 4.6776, 5.1814 and 5.7991 hours). A very stringent test of observed translational mode periods is given by the splitting law based on the Lagrangian of the motion. This test has been used (Smylie et al., 1993) to search the whole spectrum for correctly split modes. Only the originally identified modes are found at a very high level of signifigence (see Figure 12 of that paper) and other claimed periods are found to have no observational support (see Figures 13 and 14).

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