

RESEARCH ARTICLE

Arguing About Constitutive and Regulative Norms

Gabriella Pigozzi^{a*} and Leendert van der Torre^b

^a*Université Paris-Dauphine, PSL Research University, CNRS, LAMSADE, 75016 Paris, France;*

^b*Computer Science and Communications, University of Luxembourg, Esch-sur-Alzette, Luxembourg*

(Received 00 Month 201X; final version received 00 Month 201X)

Formal arguments are often represented by (support, conclusion) pairs, but in this paper we consider normative arguments represented by sequences of (brute, institutional, deontic) triples, where constitutive norms derive institutional facts from brute facts, and regulative norms derive deontic facts like obligations and permissions from institutional facts. The institutional facts may be seen as the reasons explaining or warranting the deontic obligations and permissions, and therefore they can be attacked by other normative arguments too. We represent different aspects of normative reasoning by different kinds of consistency checks among these triples, and we use formal argumentation theory to resolve conflicts among such normative arguments. In particular, we introduce various requirements for arguing about norms concerning violations, contrary-to-duty obligations, dilemmas, conflict resolution, and different kinds of norms, and we introduce a formal argumentation theory satisfying the requirements. In order to illustrate our framework, we introduce a running example based on university regulations for prospective and actual students.

Keywords: normative reasoning; deontic logic; argumentation; input/output logic

1. Introduction

We formalise various aspects of normative reasoning as different kinds of conflicts, and we use formal argumentation to represent and reason about these normative conflicts (da Costa Pereira et al., 2017). In particular, we represent argumentation in terms of a graph of arguments with a binary attack relation among them, where the arguments are structured as logical formulas and the attack relations translate into consistency checks among logical formulas. This approach was introduced by Dung (1995), and we thus apply his “calculus of opposition” to various aspects of normative reasoning. Our argumentation theory satisfies the following five requirements of normative reasoning (Pigozzi & van der Torre, 2017).

The first requirement is *representing violations* by distinguishing “is” from “ought.” A violation is represented as a conflict between what is the case and what ought to be the case. David Hume introduced the so-called is-ought problem, which roughly means that there is a fundamental difference between positive statements and prescriptive or normative statements. The is-ought problem can be considered in two directions. First, what is the case cannot be the basis for what ought to be the case. This is related to G. E. Moore’s naturalistic fallacy using natural properties as the basis of moral properties. Second, what ought to be the case cannot be the basis for what is the case. This is related to the fallacy of wishful thinking: an agent may desire to win the lottery, but from that desire he should not deduce that he will win the lottery. Likewise, in a kind of deontic wishful thinking, an agent should not deduce from the mere fact

*Corresponding author. Email: gabriella.pigozzi@lamsade.dauphine.fr

that he is obliged to review a paper, that he will actually do it. We call agents *realistic* if they do not make such fallacious inferences. The fundamental distinction between “is” and “ought” is the main reason why deontic logic is normally formalised as a branch of modal logic. It distinguishes brute facts like p from deontic facts like obligations Op and permissions Pp , and it represents violations by mixed formulas like $p \wedge O\neg p$.

The second requirement is known as *contrary-to-duty reasoning* and concerns the representation of consequences of violations such as sanctions and reparations. A contrary-to-duty obligation expresses what one should do when obligations have been violated. In other words, contrary-to-duty obligations are triggered by conflicts between what is the case and what ought to be the case, and they may be seen as a way to resolve this conflict, if only partially. Of course, it is better to review a paper, than not doing the review and being sanctioned for that. Many deontic logic paradoxes contain contrary-to-duty obligations, such as the gentle murderer paradox: a person should not kill, but, if he kills, he should do it gently. Such scenarios should be represented in a consistent way, but in many deontic logics such formalisations are inconsistent, or they have counterintuitive consequences.

Reasoning about dilemmas is the third requirement of normative reasoning. Roughly, deontic dilemmas are conflicting obligations. So-called Standard Deontic Logic makes deontic dilemmas inconsistent by the deontic axiom $\neg(Op \wedge O\neg p)$, but many alternative logics allow the consistent representation of such dilemmas and thus reject this axiom.

The fourth requirement says that in hierarchical normative systems, *conflicts among norms can be resolved* by reference to the hierarchy, which can be based on the authority that promulgated the norm, but which can also refer to other information such as the time of the promulgation, or the specificity of the norm.

Finally, the fifth requirement says that constitutive norms must be distinguished from regulative ones, and that permissive norms must be represented as well.

Regulative norms, including permissive ones, indicate what is obligatory or permitted. In formal deontic logic, permissions are studied less frequently than obligation. For a long time, it was naively assumed that it could simply be taken as a dual of obligation, just as possibility is the dual of necessity in modal logic. However, Bulygin (1986) observed that an authoritative kind of permission must be used in the context of multiple authorities and updating normative systems: if a higher authority permits you to do something, a lower authority can no longer prohibit it. Deontic logic has been concerned mainly with regulative norms, but the logic of constitutive norms (Grossi & Jones, 2013) is a subject of study on its own.

Constitutive norms are rules that create the possibility of or define an activity. For example, according to Searle (1969), the activity of playing chess is constituted by action in accordance with these rules. The institutions of marriage, money, and promising are like the institutions of baseball and chess in that they are systems of such constitutive rules or conventions. As another example, a signature may count as a legal contract, and a legal contract may define a permission to use a resource and an obligation to pay. Searle points out that, unlike regulative norms, constitutive rules do not regulate actions but define new forms of behaviour. Constitutive norms link brute facts (like the signature of a contract) to institutional facts (a legal contract) and are usually represented as counts-as conditionals: X counts as Y in context C . Searle’s analysis insists on the contextual nature of constitutive norms: a signature counts-as a legal contract when posed on a paper stating the terms of such a contract. If, however, I exercise writing my signature on a white sheet, that does not constitute a legal contract. Constitutive norms have been identified as the key mechanism to normative reasoning in dynamic and uncertain environments, for example to realize agent communication, electronic contracting, dynamics of organizations, see, e.g., the work of Boella & van der Torre (2006a).

The traditional challenges of is-ought, contrary to duty, dilemmas, and conflict resolution become even more challenging when we distinguish among these different kinds of norms. In

particular, an important challenge in defining constitutive norms like counts-as conditionals is defining their *relation* with regulative norms like obligations and permissions. Boella & van der Torre (2006b) use the notion of a logical architecture combining several logics into a more complex logical system, called logical input/output nets (or lions). Here, we focus on the analysis of the types of conflicts that can arise between constitutive and regulative norms.

To apply consistency-based techniques from formal argumentation to normative reasoning in a uniform and general way, we adopt two techniques from normative systems. First, we follow Alchourrón & Bulygin (1981) and Makinson (1999) by departing from the main tradition to model normative reasoning as a branch of modal logic. So-called Standard Deontic Logic (SDL) is a normal propositional modal logic of type KD, which means that it extends the propositional tautologies with the axioms $K : O(p \rightarrow q) \rightarrow (Op \rightarrow Oq)$ and $D : \neg(Op \wedge O\neg p)$, and it is closed under the inference rules *modus ponens* $p, p \rightarrow q / q$ and *necessitation* p / Op . Prohibition and permission are defined by $Fp = O\neg p$ and $Pp = \neg O\neg p$. SDL is an unusually simple and elegant theory. Not surprisingly for such a highly simplified theory, there are many features of actual normative reasoning that SDL does not capture. For example, the Handbook of Deontic Logic and Normative Systems (Gabbay, Horty, Parent, van der Meyden, & van der Torre, 2013) explains in detail the so-called ‘paradoxes of deontic logic’, which are usually dismissed as consequences of the simplifications of SDL. E.g. Ross’s paradox (Ross, 1941), the counterintuitive derivation of “you ought to mail or burn the letter” from “you ought to mail the letter”, is typically viewed as a side effect of the interpretation of ‘or’ in natural language.

In this paper, we use an alternative representation without modal operators that still clearly distinguishes “is” from “ought,” based on a tradition in reasoning with normative systems. The distinction between norms and obligations was pinned down by Makinson (1999) and formally further developed in input/output logic (Makinson & van der Torre, 2000). To detach an obligation from a norm, there must be a context, and the norm must be conditional. Thus norms are just particular kinds of rules, and one may view a normative system simply as a set of rules. As Makinson explains, on the one hand, the absence of explicit modal operators in normative systems may be seen as a limitation, but on the other hand, it also facilitates the formal analysis. Makinson attributes the “liberating effect” of no longer explicitly representing the modal operator to Alchourrón & Bulygin (1981) :

An *unconditional normative code* is defined to be a pair $N = (A, B)$ where A, B are sets of purely boolean formulae. Intuitively they represent, respectively, the states of affairs that the code *explicitly requires* to come into effect, and those that it *explicitly permits* to do so.

There is thus a small, but immensely significant step compared to the sketch of Stenius (1963). Alchourrón and Bulygin appear to have been the first to realise the liberating effect of taking the set of promulgations of a normative code to be made up of purely boolean formulae. At the same time, they consider explicit permissions along with promulgations. (Makinson, 1999, p. 32-33)

Second, we depart from systems of formal argumentation representing arguments as (support, conclusion) pairs. Instead, we represent arguments by sequences of (brute, institutional, deontic) triples, where constitutive norms derive institutional facts from brute facts, and regulative norms derive deontic facts from institutional facts. For instance, a triple (signature, contract, payment) represents that there is an obligation for a payment due to a signature which led to a contract. The contract thus warrants or explains the obligation for the payment. Such expressions were recently introduced in a deontic logic of Sun & van der Torre (2014), and we therefore use one of their methods to combine constitutive and regulative norms. This alternative representation of triples allows us to define a wider range of consistency checks.

The layout of this paper is as follows. In Section 2 we introduce prioritized normative systems, normative detachments, our running example and the five requirements. In Section 3 we consider the resolution of conflicts among constitutive and regulative norms using formal argumentation. In Section 4 we consider issues for further research. The related work and conclusion sections end the paper.

2. Properties regarding conflicts between constitutive and regulative norms

We first introduce the formal definition of normative system we use in this paper, and a running example that we use throughout the paper to illustrate how different kinds of consistency checks can be used to represent different aspects of normative reasoning. Then we introduce the requirements we use in this paper to evaluate the formal argumentation systems proposed in Section 3.

2.1 Normative system and normative detachments

A normative system consists of defeasible constitutive (C), regulative (R) and permissive (P) norms, and a set of facts (F). Some authors write constitutive norms as “ X counts as Y in context C .” However, given the lack of consensus in the literature concerning the representation of the context, and for reasons of simplicity, we follow Lindahl & Odelstad (2003) and Boella & van der Torre (2004) and abstract away from the context. We thus consider constitutive norms as rules “ X counts as Y .”

As mentioned in the introduction, we follow Makinson’s departure from the classical approach to use explicit modal operators in the logical language to model normative concepts like obligation or permission. In his framework, a violation is a conflict between a deontic and a brute or institutional fact. A constitutive norm is represented by $(a, i)_c$, where a and i are propositional formulas, and c is a natural number. Note that we use i to remind that the consequence of the norm is an institutional fact, and we write c to remind that it concerns the priority of a constitutive norm. Likewise, a regulative norm is represented by $(a, d)^r$, where we use d to remind that the consequence of the norm is a deontic fact, and we write r to remind that it concerns the priority of a regulative norm. To stay as general as possible we do not impose restrictions on the different kinds of facts, so brute, institutional and deontic facts are based on the same set of propositional atoms.

Definition 1 (Normative system). *Let $P = \{p_0, p_1, \dots\}$ be a set of propositional letters, L be the language of classical propositional logic built upon P and N the set of natural numbers together with ∞ . We write $\phi \dashv\vdash \psi$ for logical equivalence in the logic L . A prioritised normative system $NS = (C, R, P, F)$ consists of:*

- *a set of prioritised constitutive rules or norms C , of the form $\{(a_1, i_1)_{c_1}, \dots, (a_k, i_k)_{c_k}\}$, containing ordered pairs of formulas $a, i \in L$, together with a natural number $c \in N$;*
- *two sets of prioritised regulative rules or norms R, P of the form $\{(a_1, d_1)^{r_1}, \dots, (a_k, d_k)^{r_k}\}$, containing ordered pairs of formulas $a, d \in L$, together with a natural number $r \in N$;*
- *a set F of facts, which is a consistent set of sentences from L .*

Normative reasoning is the detachment of institutional facts, obligations and permissions from the norms C, R, P and the brute facts F . In this paper, we represent these detachments by triples. Like many current detachment approaches to normative systems, we do not present a single set of detached obligations and permissions. Instead we call such sets an extension, and detachment is a process that associates sets of extensions with a normative system. This set of extensions represents the unresolved dilemmas in the normative system. A normative detachment is either an obligation (b, i, d) , read as “in situation b , because of i , it is obligatory that d ,” or a permission (b, i, d) , read as “in situation b , because of i , it is permitted that d .”

Obligations (b, i, d) can also be read as “in context b , because of i , it is obligatory that d ” (similarly for permissions). However, in order to avoid confusions with the fact that we abstract away from the context, we will use ‘situation’ rather than ‘context’. It should be noted that these are different notions of ‘context’: when Searle writes constitutive norms as X counts as Y in context C , the context is meant to represent an institution. However, in our case, a context is a fact, a situation, or an input.

Definition 2 (Detachment). *A normative detachment from a prioritised normative system is either an obligation $(b, i, d)_c^r$ or permission $\langle b, i, d \rangle_c^r$ with $b, i, d \in L$ and $c, r \in \mathbb{N}$ such that $b \wedge i \wedge d$ is consistent in L . A detachment extension is a pair (Ob, Pe) of detached obligations Ob and permissions Pe . We write $det(NS)$ for the set of all detachment extensions of NS .*

Normative systems and detachment are illustrated in Subsection 2.2. As a special case of normative detachments, we represent facts as $(b, \top, \top)_\infty^\infty$, and we represent institutional facts as $(b, i, \top)_c^\infty$. This unified representation facilitates the argumentation theory in Section 3. In that section, we define detachment functions $det(NS)$ relative to an argumentation semantics. In the remainder of this section, after having presented the running example, we motivate such an argumentation theory. Since priorities are used only to resolve conflicts, we do not represent them explicitly in the running example in this section.

2.2 Running example

As a running example we consider a university regulation of prospective and actual students. We use examples of norms that apply to students who submit applications to enrol at Prestigious University (P.U.) and to students who are accepted to enrol at P.U. Applications to P.U. are open (*CanApply*) to people who are less than 26 years old (*Less26Years*) and who hold at least a high-school degree (*HighDegree*). As a new student, the enrolment to P.U. involves two steps. One has first to submit an online pre-application (*OnlinePreApplication*). Among the documents to be sent within two weeks from the online applications are certificates of the exams passed the previous year (*SendExamsCertificates*) and a certified photocopy of the passport (*CertifiedPhotocopyPassport*). Once one has completed the online pre-application, he gains the status of prospective student (*ProspectiveStudent*). If the first step succeeds (*AcceptedApplication*), one becomes an eligible student (*EligibleStudent*) with the obligation for the self-funding students (*SelfFundingStudent*) to pay the tuition fees (*PayFee*).

We also consider rules that apply to students who are already enrolled at P.U. For example, students with a valid I.D. card by P.U. (*ValidStudentCard*) counts-as students of P.U. (*PUSStudent*) and have several obligations, like attending courses to which they enrolled (*AttendCourses*), taking the exams (*SitExam*), and not committing plagiarism ($\neg DoPlagiarism$), where *Plagiarism* consists of copying and using in a student's own essay another's work without crediting the source (*CopyNoCredit*), showing their student I.D. card when entering P.U. premises (one of the taken counter-terrorism measures, *ShowStudentCard*). If a student is caught in plagiarism, he encounters a discipline measure (*DisciplinaryMeasure*), that is, he is suspended by P.U. (*PlagiarismSuspension*) which means that he is not a student ($\neg PUSStudent$) for a period decided by a disciplinary committee. During that time he cannot sit an exam ($\neg SitExam$) and so he must take the second exam session (*ReSitExam*). However, a student is granted permission to appeal a disciplinary measure (*CanAppeal*). Finally, exams can take various forms (written essays, projects, oral or written exams etc.) and any of these (so, for example *SubmitEssay*) counts-as taking an exam (*SitExam*).

The following example illustrates the simplest way to combine constitutive and regulative norms into an obligation. If there is a constitutive norm that “ a counts as p ” and a regulative norms that “if p then obligatory x ,” then we can construct the obligation (a, p, x) .

Example 1. *The normative system $NS = (C, R, P, F)$ consists of:*

Constitutive norms $C = \{(OnlinePreApplication, ProspectiveStudent), (AcceptedApplication, EligibleStudent), (PayFee, SelfFundingStudent), (CopyNoCredit, Plagiarism), (ValidStudentCard, PUSStudent), (PlagiarismSuspension, DisciplinaryMeasure), (DisciplinaryMeasure, \neg PUSStudent), (SubmitEssay, SitExam)\};$

Regulative norms $R = \{(ProspectiveStudent, SendExamsCertificates), (ProspectiveStudent, CertifiedPhotocopyPassport), (EligibleStudent, PayFee),$

$(Plagiarism, ReSitExam), (PUStudent, AttendCourses), (PUStudent, \neg DoPlagiarism),$
 $(PUStudent, ShowStudentCard), (PUStudent, SitExam), (\neg PUStudent, \neg SitExam),$
 $(SitExam, \neg CopyNoCredit)\};$

Permissive norms $P = \{(Less26Years, CanApply), (HighDegree, CanApply),$
 $(DisciplinaryMeasure, CanAppeal)\};$

Facts $F = \{OnlinePreApplication, AcceptedApplication, PayFee, CopyNoCredit,$
 $ValidStudentCard, PlagiarismSuspension, DisciplinaryMeasure, SubmitEssay\}.$

An example of an obligation that we can build from this normative system is $(ValidStudentCard, PUStudent, ShowStudentCard)$: having a valid student card from P.U. counts-as being a student of that university and P.U. students have to show their student card when they enter the university's premises.

Some of the students at P.U. are Chinese. The administration of P.U. recently discovered that these applicants face a particular situation as producing a certified photocopy of the passport is forbidden by Chinese laws $(\neg CertifiedPhotocopyPassport)$. So having a Chinese passport $(ChinesePassport)$ counts as holding a Chinese citizenship $(ChineseCitizen)$ and Chinese citizens can apply to P.U. university $(CanApply)$ but cannot provide the required certified photocopy of the passport.

Example 2 (Continued). The normative system $NS' = (C \cup C', R \cup R', P \cup P', F \cup F')$ is NS extended with constitutive norm $C' = \{(ChinesePassport, ChineseCitizen)\}$ regulative norm $R' = \{(ChineseCitizen, \neg CertifiedPhotocopyPassport)\}$, permissive norms $P' = \{(ChineseCitizen, CanApply)\}$, and $F' = \{ChinesePassport\}$.

Examples of obligations and permissions are $(ChinesePassport, ChineseCitizen, \neg CertifiedPhotocopyPassport)$ and $(ChinesePassport, ChineseCitizen, CanApply)$.

	$C' = \{(PRCPass, PRCCit)\}$	$R' = \{(PRCCit, \neg CPP)\}$
$F' = \{PRCPass\}$	$(PRCPass, PRCCit, \neg CPP)$	

Table 1. In the figure we use the following abbreviations: *CertifiedPhotocopyPassport* (CPP), *ChinesePassport* (PRCPass), *ChineseCitizen* (PRCCit).

	$C' = \{(PRCPass, PRCCit)\}$	$P' = \{(PRCCit, Apply)\}$
$F' = \{PRCPass\}$	$(PRCPass, PRCCit, Apply)$	

Table 2. In the figure we use the following abbreviations: *ChinesePassport* (PRCPass), *ChineseCitizen* (PRCCit) and *CanApply* (Apply).

The following example illustrates a problem that can arise when the combination of constitutive and regulative norms is iterated.

Example 3. Let the normative system NS be the one of Example 1. When the application is accepted, the applicant acquires the status of eligible student and thus has the obligation to pay the tuition fee. This is expressed by the triple $(AcceptedApplication, EligibleStudent, PayFee)$. For simplicity, in Table 3 we represent only the relevant facts, constitutive and regulative norms (indicated by the asterisk). We also have that paying the tuition fees means that a student is self-funding, expressed by the constitutive norm $(PayFee, SelfFundingStudent)$.

	$C^* = \{(Accepted, Eligible)\}$	$R^* = \{(Eligible, Pay)\}$
$F^* = \{(Accepted)\}$	$(Accepted, Eligible, Pay)$	

Table 3. The table shows how the triple $(AcceptedApplication, EligibleStudent, PayFee)$ is obtained from the constitutive $(AcceptedApplication, EligibleStudent)$ and the regulative $(EligibleStudent, PayFee)$. For readability reasons, *AcceptedApplication* is shortened to *Accepted*, *EligibleStudent* to *Eligible* and *PayFee* to *Pay*.

Example 3 concerns the question on how to combine constitutive and regulative norms, and in particular an inference from “ a counts as p ” (*AcceptedApplication*, *EligibleStudent*), “if p then obligatory x ” (*EligibleStudent*, *PayFee*) and “ x counts as y ,” where x could be a brute fact or an institutional fact (*PayFee*, *SelfFundingStudent*). The question is whether one would like to derive “in context a , because of p , it is obligatory that y ”, in this example (*AcceptedApplication*, *EligibleStudent*, *SelfFundingStudent*), as in Table 4. In this case the answer seems to be negative: a student can be eligible without being self-funding, for example he may have received a grant.

(<i>Accepted</i> , <i>Eligible</i> , <i>Pay</i>)	(<i>Accepted</i> , <i>Eligible</i> , <i>Self</i>)
$C = \{(Pay, Self)\}$	

Table 4. Inference of (*AcceptedApplication*, *EligibleStudent*, *SelfFundingStudent*) from (*AcceptedApplication*, *EligibleStudent*, *PayFee*) and the constitutive (*PayFee*, *SelfFundingStudent*). For readability reasons, *AcceptedApplication* is shortened to *Accepted*, *EligibleStudent* to *Eligible*, *PayFee* to *Pay* and *SelfFundingStudent* to *Self*.

The problem of combining constitutive and regulative norms is to allow for such iterations of them. For example, in simple combinations, we can apply first the constitutive norms and thereafter the regulative norms, but thereafter no longer the constitutive norms. A more sophisticated way to combine the two kinds of norms is introduced by Sun & van der Torre (2014), and is used in Section 3.

The fact that constitutive and regulative norms may be used in any sequence, as illustrated by the example, may suggest a formal similarity between on the one hand the handling of constitutive and regulative norms, and on the other hand strict and defeasible rules in ASPIC+ (Modgil & Prakken, 2013). In particular, this similarity holds when we consider the notion of realism in Section 2.3 below. However, the setting here is different because constitutive norms also have a priority, and our solution is very different. Moreover, the example may suggest a similarity between the handling of belief and obligation rules in agent logics (Governatori & Rotolo, 2008). We leave this to further research.

2.3 Realism and the fallacy of normative wishful thinking

Most importantly for arguing about constitutive and regulative norms, we have to be careful how we resolve conflicts between obligations and facts. In particular, as discussed in the introduction, we have the possibility of normative wishful thinking. Suppose there is an obligation (b, i, d) and a second obligation ($b', i', -i$), and assume that our argumentation theory is such that we want to rule out such conflicts. On the one hand i is an institutional fact used to derive the obligation for d , but on the other hand this institutional fact is a violation. To resolve this conflict by accepting ($b', i', -i$) is a kind of normative wishful thinking. How can it happen that a norm changes a brute or an institutional fact? An obligation cannot change anything. At worst we can have a violation.

At first sight it may seem strange to exclude such violations from an extension. However, this is often done, for example to focus only on obligations which can still be achieved, or simply to prevent counterintuitive conclusions. This is called an input/output constraint in constrained input/output, see (Makinson & van der Torre, 2001) for a further discussion.

To formalise this intuition, we assume that a tautology \top is always obligatory as well as permitted. This refers a well known dilemma in deontic logic. On the one hand, the obligatory tautology has been criticised by von Wright as well as by some more recent authors, because there does not seem to be a conceptual reason to assume that tautologies are obligatory. On the other hand, the obligatory tautology is seen by many as a harmless borderline case playing no role in any of the deontic paradoxes. Moreover, it often makes the formal definitions easier, for example, in modal logic it makes it possible to make use of the necessitation inference rule. In our case, with the assumption of obligatory tautologies, detached institutional facts

are represented by detachments (b, i, \top) , where \top is any tautology. Using this assumption, a detachment function satisfies the realism property if the addition of regulative and permissive norms does not change the brute and institutional facts (b, i, \top) .

Definition 3 (Realism). Let $det_i(NS) = \{(Ob_i \subseteq Ob, Pe_i \subseteq Pe) \mid (Ob, Pe) \in det(NS), Ob_i = \{(b, i, \top) \in Ob\}, Pe_i = \{(b, i, \top) \in Pe\}\}$.

A detachment function det satisfies the realism property iff for all C, R, R', P, P' and F we have $det_i(C, R, P, F) = det_i(C, R \cup R', P \cup P', F)$.

Example 4. Consider the regulative and permissive norms that in Example 2 we added to the original NS : $R' = \{(ChineseCitizen, \neg CertifiedPhotocopyPassport)\}$, and $P' = \{(ChineseCitizen, CanApply)\}$. Clearly, the addition of these norms to the normative system NS of Example 1 does not change the brute and institutional facts of NS . So we get that $det_i(NS) = det_i(C, R, P, F) = det_i(C, R \cup R', P \cup P', F)$.

2.4 Conflict-free

A basic requirement of extensions is that they must be conflict-free, and a general definition of conflict-freeness refers to consistency. In normative systems, there are several notions of consistency and thus of conflict-freeness. As usual, in a consistency check we may consider several obligations, but only one permission. Two permissions can be consistent. For example, if a door is permitted to be open and permitted to be closed, it is not a conflict.

Definition 4 (Conflict-free). Let X be a nonempty subset of $\{b, i, d\}$, $Ob_b = \{b \mid (b, i, d) \in Ob\}$, $Ob_i = \{i \mid (b, i, d) \in Ob\}$, $Ob_d = \{d \mid (b, i, d) \in Ob\}$ and $Ob_X = \cup_{x \in X} Ob_x$.

A detachment function satisfies X conflict-free iff for all $(Ob, Pe) \in det(NS)$ we have that for every $\langle b, i, d \rangle \in Pe$, we have that $Ob_X \cup \{(b, i, d)\}$ is consistent.

We illustrate different kinds of conflicts between triples (a, p, x) and (b, q, y) , where a, b are brute facts, p, q are institutional facts and x, y are deontic or permissive facts. We distinguish three types of conflicts between two norms, which are the three basic requirements of all deontic logics, as discussed in the introduction: violations, institutional conflicts and dilemmas.

The table below summarises the different types of conflicts with the references to the corresponding examples. We assume that facts are consistent, so there cannot be ontological conflicts, i.e., conflicts between brute facts. We also assume that brute and institutional facts do not conflict.

	Brute	Institutional	Deontic
Brute	No ontological conflicts	No conflicts	Violations * (a, p, x) and $(b, q, \neg a)$ Example 5
Institutional	No conflicts	Conflicts (a, p, x) and $(b, \neg p, y)$ Example 7	Violations ** (a, p, x) and $(b, q, \neg p)$ Example 6
Deontic	See *	See **	Dilemmas (a, p, x) and $(b, q, \neg x)$ Example 8

Table 5. Different types of conflicts between triples (a, p, x) and (b, q, y) , where a, b are brute facts, p, q are institutional facts and x, y are deontic or permissive facts.

2.5 Violations and contrary-to-duty reasoning

Deontic logic paradoxes often are concerned with violations and their consequences. Maybe the most discussed principle in deontic logic is so-called deontic detachment: from an obligation for p , and a conditional obligation for q if p , can we derive the unconditional obligation for q ? In many examples deontic detachment seems necessary to derive the right cues for action. For example, in Chisholm’s famous scenario, assume that a man is obliged to go the assistance of his neighbours tomorrow, and that he is obliged today to tell them that he is coming, if he will go. From these two obligations, it seems that we need to derive the unconditional obligation to tell today that he is coming tomorrow. Surely, if he does not tell them today he is coming, the ideal state is no longer reachable, and thus some violation is going to occur. For this reason, preference-based possible worlds semantics known as the Danielsson-Hansson-Lewis semantics (Danielsson, 1968; Hansson, 1969; Lewis, 1973) of standard deontic logic, will make this inference. However, there are also problems with deontic detachment. (Broome, 2013, §7.4) gives the following example: From “You ought to exercise hard everyday,” and “If you exercise hard everyday, you ought to eat heartily,” should we derive “You ought to eat heartily”? Intuitively, the obligation to eat heartily no longer holds if you take no exercise. This example was used also by Parent and van der Torre to motivate so-called aggregative input/output, which we use in Section 3.

In our context, we can have a violation when there is a conflict between an obligation and a brute fact (a, p, x) and $(b, q, \neg a)$ or an institutional fact (a, p, x) and $(b, q, \neg p)$.

Example 5. Take the normative system NS of Example 1 where we have (*CopyNoCredit*, *Plagiarism*, *ReSitExam*) which states that copying and using in his own work (an essay, for example) another’s work without giving credit to the source is an act of plagiarism and this makes obligatory retaking the exam in which the fraud took place. Exams can take different forms (written essays, projects, oral or written exams etc.). When students enrol at P.U., they accept its Behaviour Code which (among other things) forbids them from plagiarism. So the triple (*SubmitEssay*, *SitExam*, \neg *CopyNoCredit*) states that submitting an essay amounts to taking an exam and in an exam one cannot do plagiarism. In this case we would have a violation of an obligation: a student had the obligation not to copy someone else’s work without giving credit, but in fact he did. This is illustrated in the table below, where for simplicity we represent only the facts, constitutive and regulative norms that are relevant to the example (the subset of relevant facts, regulative and constitutive norms is denoted by an asterisk).

	$C^* = \{(Copy, Plag), (Essay, Sit)\}$	$R^* = \{(Plag, ReSit), (Sit, \neg Copy)\}$
$F^* = \{(Copy, Essay)\}$		$(Copy, Plag, ReSit)$ $(Essay, Sit, \neg Copy)$

Table 6. Example of violation, where the conflict is between an obligation and a brute fact (a, p, x) and $(b, q, \neg a)$. In the table we use the following abbreviations: *CopyNoCredit* (*Copy*), *Plagiarism* (*Plag*), *SubmitEssay* (*Essay*), *SitExam* (*Sit*), and *ReSitExam* (*ReSit*).

Example 6. We can construct an example of a conflict between a deontic and an institutional fact by considering the normative system $N'' = (C \cup C'', R \cup R'', P, F)$, obtained by extending NS with constitutive norm $C'' = \{(GrantRecipient, ExonerateFee)\}$ regulative norms $R'' = \{(SelfFundingStudent, FinancialProof), (ExonerateFee, \neg SelfFundingStudent)\}$, and $F'' = \{GrantRecipient\}$. The regulative norm $(SelfFundingStudent, FinancialProof)$ expresses the fact that a student who is self-funding must provide a proof that he is financially capable to afford living in the expensive state of P.U. On the other hand, the regulative norm $(ExonerateFee, \neg SelfFundingStudent)$ says that a student who has been exonerated from paying the fee cannot be a self-funding student. The conflict between a deontic and an institutional fact is illustrated in Table 7.

	$C'' = \{(Grant, Exon)\}$	$R'' = \{(Self, Proof), (Exon, \neg Self)\}$
$F'' = \{(Grant)\}$	$(Pay, Self, Proof)$ $(Grant, Exon, \neg Self)$	

Table 7. Example of a violation, where the a conflict is between an obligation and an institutional fact (a, p, x) and $(b, q, \neg p)$. In the table we use the following abbreviations: *PayFee* (*Pay*), *SelfFundingStudent* (*Self*), *GrantRecipient* (*Grant*), *ExonerateFee* (*Exon*), and *FinancialProof* (*Proof*).

2.6 Institutional and ontological conflicts

We assume that the facts are consistent, so there cannot be conflicts between two brute facts (ontological conflict), as in (a, p, x) and $(\neg a, q, y)$. We also assume that brute and institutional facts do not conflict. In contrast to the case of dilemmas with deontic facts, in the deontic logic literature there is not much discussion on whether institutional facts can conflict or not, as in (a, p, x) and $(b, \neg p, y)$. The following example illustrates such a conflict.

Example 7. Consider the following scenario in our running example: A student with a valid ID card of P.U. (*a*) counts as a student of that university (*p*) and must attend the courses to which he enrolled (*x*). But a student who has been suspended for plagiarism (*b*) is not a student of that university ($\neg p$) and cannot pass exams while suspended (*y*). We call it institutional conflict because we would obtain both (*ValidStudentCard*, *PUS*student, *AttendCourses*) and (*DisciplinaryMeasure*, \neg *PUS*student, \neg *SitExam*).

As before, in the table we list only the relevant facts, constitutive and regulative norms.

	$C^* = \{(Card, PU), (Discip, \neg PU)\}$	$R^* = \{(PU, Courses), (\neg PU, \neg Sit)\}$
$F^* = \{(Card, Discip)\}$	$(Card, PU, Courses)$ $(Discip, \neg PU, \neg Sit)$	

Table 8. Example of institutional conflict between (a, p, x) and $(b, \neg p, y)$. The following abbreviations are used: *ValidStudentCard* (*Card*), *PUS*student (*PU*), *AttendCourses* (*Courses*), *SitExam* (*Sit*), and *DisciplinaryMeasure* (*Discip*).

At first sight it seems to us that the situation with institutional facts is similar to the one for deontic dilemmas, that is, one can take the stance that they must be ruled out, or one can take the stance that they can be represented together in one extension. A priori there is no logical reason to allow or disallow the consistent representation of institutional conflicts, just like there is no logical reason to allow or disallow the consistent representation of deontic dilemmas. There are deontic logics that represent them consistently, and there are deontic logics that make them inconsistent. This mirrors different philosophical positions, like Kant’s theory of norms in which dilemmas are inconsistent, and Sartre’s theory in which they are consistent.

One may observe that there are other formalisms with triples, such as Reiter default rules $a : b/c$. With such so-called justifications *b*, it may well happen that there are two applicable rules with contradictory justifications. We leave a comparison to such formalisms to further research.

2.7 Dilemmas

Consider norms which on the one hand require you to leave the room, while on the other hand requiring you not to leave the room at the same time. There are essentially two views on normative conflicts: in the one view, they do not exist. In the other view, conflicts and dilemmas are ubiquitous.

According to the view that normative conflicts are ubiquitous, we may become the addressees of conflicting normative demands at any time. In such cases, we are inclined to say that there is something wrong with the normative system. This intuition is captured by one of the axioms of Standard Deontic Logic $D : \neg(Ox \wedge O\neg x)$ that states that there cannot be co-existing obligations

to bring about x and to bring about $\neg x$. Using the standard cross-definitions of the deontic modalities: x cannot be both, obligatory and forbidden, or: if x is obligatory then it is also permitted. Any logic about norms must take into account possible conflicts.

The opposite view, that normative conflicts do not exist, appeals to the very notion of obligation: it is essential for the function of norms to direct human behavior that the subject of the norms is capable of following them. To state two norms which cannot both be fulfilled is confusing the subject, not giving him or her directions. To say that a subject has two conflicting obligations is therefore a misuse of the term ‘obligation.’ So there cannot be conflicting obligations, and if things appear differently, a careful inspection of the normative situation is required that resolves the dilemma in favor of the one or other of what only appeared both to be obligations. In particular, this inspection may reveal a priority ordering of the apparent obligations that helps resolve the conflict.

Example 8. We recall that the extended normative system NS' of Example 2 represents the situation in which the admission committee receives the online pre-application of some Chinese students but they discover that according to People’s Republic of China law, certified photocopies of passports are not authorized. Yet, these documents are necessary for applicants to pass to the second step of the application process. Since nothing forbids Chinese citizens to apply to P.U. There is even the permissive norm $\langle \text{ChinesePassport}, \text{ChineseCitizen}, \text{CanApply} \rangle$. The committee realizes they face a dilemma: $(\text{OnlinePreApplication}, \text{ProspectiveStudent}, \text{CertifiedPhotocopyPassport})$ and $(\text{ChinesePassport}, \text{ChineseCitizen}, \neg \text{CertifiedPhotocopyPassport})$.

In the table we list only the relevant facts, constitutive and regulative norms of NS' .

	$C^* = \{(\text{Pre}, \text{Persp}), (\text{PRCPass}, \text{PRCCit})\}$	$R^* = \{(\text{Persp}, \text{CPP}), (\text{PRCCit}, \neg \text{CPP})\}$
$F^* = \{(\text{Pre}, \text{PRCPass})\}$	$(\text{Pre}, \text{Persp}, \text{CPP})$ $(\text{PRCPass}, \text{PRCCit}, \neg \text{CPP})$	

Table 9. Example of a dilemma, as in (a, p, x) and $(b, q, \neg x)$. In the table we use the following abbreviations: *OnlinePreApplication* (*Pre*), *ChinesePassport* (*PRCPass*), *ProspectiveStudent* (*Persp*), *ChineseCitizen* (*PRCCit*), and *CertifiedPhotocopyPassport* (*CPP*).

2.8 Resolving conflicts

Reinstatement plays a central role in the semantics in abstract argumentation, see also the criticism of Horty (2001) in default reasoning, but it has received very little attention in the deontic logic literature. As a rare exception, two kinds of reinstatement in normative systems have been discussed by van der Torre & Tan (2001), who distinguish two kinds of reinstatement, called RI and RIO respectively, and who argue that this kind of reinstatement sometimes should be accepted (namely for so-called prima facie obligations) and sometimes should be rejected (for obligations under uncertainty). In both kinds of reinstatement, there is a general norm and a more specific norm, and it is assumed that the more specific norm overrides the more general norm. We can represent this overriding by using the priorities, so more specific norm has a higher priority than the more general norm. In the properties below we do not consider specificity at all.

We distinguish two kinds of properties. Factual reinstatement considers the situation in which the norm with the higher priority is blocked by a fact. If factual reinstatement holds, then the least prioritized norm will be in force again. For example, consider the general norm $(\top, p)^1$ and the more prioritized norm $(r, \neg p \wedge \neg q)^2$. If the facts are $F = \{r, q\}$, then we derive the obligation $(r \wedge q, \top, p) \in \cap det(NS)$. Norm reinstatement considers the situation in which the more prioritized norm is itself overridden by a third norm. A detachment function satisfies norm

reinstatement if in that case, the first norm is active again. For example, if we have regulative norms $R = \{(\top, p)^1, (r, \neg p \wedge \neg q)^2, (r \wedge s, q)^3\}$ and facts $F = \{r, s\}$, then we can detach the obligation $(r \wedge s, \top, p) \in \cap det(NS)$.

Definition 5 (Reinstatement). *A detachment function satisfies factual reinstatement iff for the normative system $NS = (C, R, P, F)$, $C = \emptyset$, $R = \{(\phi_1, \psi_1)^{r_1}, (\phi_2, \psi_2)^{r_2}\}$ with $r_2 > r_1$, $\phi_1 \wedge \psi_1 \wedge \phi_2$ consistent, $\phi_1 \wedge \phi_2 \wedge \psi_2$ consistent, $\phi_1 \wedge \psi_1 \wedge \phi_2 \wedge \psi_2$ inconsistent, $P = \emptyset$, $F = \{\phi_1, \phi_2, \neg \psi_2\}$ we derive the obligation $(\phi_1, \top, \psi_1) \in \cap det(NS)$.*

A detachment function satisfies norm reinstatement iff for the normative system $C = \emptyset$, $R = \{(\phi_1, \psi_1)^{r_1}, (\phi_2, \psi_2)^{r_2}, (\phi_3, \psi_3)^{r_3}\}$ with $r_3 > r_2 > r_1$, $\phi_1 \wedge \psi_1 \wedge \phi_2$ consistent, $\phi_1 \wedge \phi_2 \wedge \psi_2$ consistent, $\phi_1 \wedge \psi_1 \wedge \phi_2 \wedge \psi_2$ inconsistent, $\phi_2 \wedge \psi_2 \wedge \phi_3$ consistent, $\phi_2 \wedge \phi_3 \wedge \psi_3$ consistent, $\phi_2 \wedge \psi_2 \wedge \phi_3 \wedge \psi_3$ inconsistent, $P = \emptyset$, $F = \{\phi_1, \phi_2, \phi_3\}$, we derive the obligation $(\phi_1 \wedge \phi_2 \wedge \phi_3, \psi_1) \in \cap det(NS)$.

Example 9. *The admission committee at P.U. is engaged to equally treat all applications. This is represented as the norm $(\top, TreatApplicationsEqually)^1$. However, in light of the recent terrorist events, if they discover that an applicant has committed hate speech (for example on his Facebook page, HateSpeech), then they do not have to treat his applications equally and they should refuse the status of prospective student to that applicant: $(HateSpeech, \neg TreatApplicationsEqually \wedge \neg ProspectiveStudent)^2$. This unless the applicant has already passed on justice for that crime (PassedJustice): $(HateSpeech \wedge PassedJustice, ProspectiveStudent)^3$. When the admission committee treats the application of a person who committed hate speech but has already been judged for that crime, the committee must treat his application equally. Consequently, the obligation $(HateSpeech \wedge PassedJustice, \top, TreatApplicationsEqually)$ can be detached.*

3. Resolving conflicts using priorities and formal argumentation

In this section, we introduce an argumentation theory for prioritised norms. Given a prioritised normative system, we first define obligation and permission arguments, then the attack and defeat relations among them, and finally we use Dung's theory (Dung, 1995) to compute extensions of arguments and to obtain conclusions from these arguments.

There is no consensus on the right or best way to detach normative statements from a normative system. In the deontic logic literature, it is therefore common to define various approaches, and to study the behaviour of each of them. For example, Makinson and van der Torre define seven distinct unconstrained input/output logics, and various ways to impose constraints on them. Likewise, there is a variety of non-monotonic logics, as well as a variety of formal argumentation semantics. However, this does not mean that everything goes. In the previous section, we already defined five requirements for arguing about prioritized normative systems. Moreover, various kinds of requirements have been defined in the formal argumentation literature, often referred to as *rationality postulates*.

Our argumentation theory builds on ideas from structured argumentation, such as ASPIC⁺ (Modgil & Prakken, 2013), in the following sense. One of the requirements of formal argumentation is that the conclusions of the arguments of an extension are consistent, which is based on the idea that a set of arguments is conflict-free. However, a set of propositions like $\{p, q, \neg(p \wedge q)\}$ is inconsistent, whereas each pair of propositions is consistent. In ASPIC⁺, the solution is that there is an argument for $q \wedge \neg p$ not attacked by the argument for p . This solution is based on the property that an extension is closed under conjunction, and thus contains also $\{p \wedge q, p \wedge \neg q, q \wedge \neg p\}$, and the property that arguments do not attack the conclusion of arguments derived by aggregating two arguments. This idea is further refined for the use of priorities. For more details about ASPIC⁺, see Section 5 and the original papers by Prakken and co-authors. In this paper, we apply the same solution.

3.1 Arguments as sequences of detachments

We define arguments as sequences of normative detachments. There are various ways in which these sequences can be defined, and more research is needed to determine the best one. In this paper we use maybe the simplest approach, where all institutional facts are aggregated in the second parameter of the normative detachments, and the deontic facts are aggregated in the third parameter of the normative detachments. Likewise, for the priorities, there are several options discussed in the argumentation literature, and there is no consensus on the best approach. In this paper we use the last link principle, which gives the normative detachment the priority of the last applied norm. However, we could have used the weakest link principle as well, which would give the normative detachment the minimum of the current value and the priority of the last applied norm, or any other way (Liao, Oren, van der Torre, & Villata, 2016, 2018).

Definition 6 (Obligation argument). *Let $NS = (C, R, P, F)$ be a prioritised normative system. An obligation argument is a sequence of normative detachments $(b, i, d)_c^r$ such that every element of the sequence is either: $(f, \top, \top)_\infty^\infty$ where $f = \wedge F' \subseteq F$, or it is derived from previous elements in the sequence using the following rules equivalence (EQ), aggregative cumulative transitivity for the intermediate or output (ACTI and ACTO) or conjunction (AND):*

- from $(a, p, x)_c^r$ to $(b, q, y)_c^r$ if $a \dashv\vdash b$, $p \dashv\vdash q$ and $x \dashv\vdash y$, EQ
- from $(a, p, x)_c^r$ to $(a, p \wedge q, x)_c^r$ for some $(b, q)_c \in C$ with $a \wedge p \wedge x \vdash b$, ACTI
- from $(a, p, x)_c^r$ to $(a, p, x \wedge y)_c^r$ for some $(b, y)^r \in R$ with $a \wedge p \wedge x \vdash b$. ACTO
- from $(a, p, x)_c^r$ and $(b, q, y)_d^s$ to $(a \wedge b, p \wedge q, x \wedge y)_{\min(c,d)}^{\min(r,s)}$ AND

We write $OA(C, R, F)$ for the set of all such obligation arguments, we call the last element of the argument the conclusion of the argument, and argument A is a sub-argument of argument B if and only if A is an initial sequence of B.

Example 10. *Once the admission committee of Example 8 realizes that they have a problem with prospective Chinese students as Chinese citizens cannot have certified photocopy of their passport ($ChineseCitizen, \neg CertifiedPhotocopyPassport$), the committee considers putting in place a special procedure for students in this situation. If someone owns only a Chinese passport, a special procedure must be activated: ($OnlyChinesePassport, SpecialProcedure$). This guarantees that the special procedure is activated only for those who hold a Chinese passport and that is the only passport they possess.*

We now make explicit the priorities of the rules we encountered in Example 2. Let $F' = \{ChinesePassport\}$, $C' = \{(ChinesePassport, ChineseCitizen)_2\}$ and $R'' = \{(ChineseCitizen, \neg CertifiedPhotocopyPassport)^3, (OnlyChinesePassport, SpecialProcedure)^5\}$. In $OA(C', R'', F')$ we have the obligation argument

- 1 $(ChinesePassport, \top, \top)_\infty^\infty$
- 2 $(ChinesePassport, ChineseCitizen, \top)_2^\infty$
- 3 $(ChinesePassport, ChineseCitizen, \neg CertifiedPhotocopyPassport)_2^3$

From $(ChinesePassport, ChineseCitizen, \neg CertifiedPhotocopyPassport)_2^3$ and $(OnlyChinesePassport, SpecialProcedure)^5$ we can derive $(ChinesePassport, ChineseCitizen, \neg CertifiedPhotocopyPassport \wedge SpecialProcedure)_2^5$ using aggregative cumulative transitivity for the output (ACTO).

Permissions can be defined in various ways. Here we use so-called positive permission, that is based on the use of a single permissive norm. It reuses the operation to define obligation arguments also for permission arguments, which explain why this is a strong notion of permission. See Makinson & van der Torre (2003) for further details, and comparison with other notions of permission. Makinson and van der Torre note that a proof system for obligation can be extended to a proof system for strong permission if the input/output logic validates the so-called non-iteration property: every formula that can be derived, can be derived using a detachment in which each leaf occurs at most once. Aggregative input/output logic satisfies this property,

suggesting that this is a suitable definition of strong permission.

Definition 7 (Permission argument). *The set of permission derivations and arguments are defined by: $PA(C, R, P, F) = \cup \{OA(C, R \cup \{(a, x)^r\}, F) \mid (a, x)^r \in P\}$.*

3.2 Resolving conflicts among obligation and permission arguments

First, facts must override obligations due to the Realism property, so the priority of facts must be higher than the priority of obligations or permissions. We therefore give facts the priority ∞ . Moreover, we combine priorities based on the last link principle. To distinguish the priorities of the constitutive norms from the priorities of the regulative norms, each detached obligation gets two priorities. This is used in the definition of attack below.

We now turn to the attack relation among the arguments. Roughly, a normative detachment $(b_1, i_1, d_1)_{c_1}^{r_1}$ attacks another normative detachment $(b_2, i_2, d_2)_{c_2}^{r_2}$ if $b_1 \wedge i_1 \wedge d_1 \wedge b_2 \wedge i_2 \wedge d_2$ is inconsistent. Our main technical challenge introduced in Section 2 is to make sure that the detachment function satisfies the Realism property. We ensure this by the way we use the priorities. In particular, we always consider the priorities of the regulative norms first, and only when they are equal, we consider the priorities of the constitutive norms. Since the brute and institutional facts have priority ∞ , we can show in Proposition 1 below that the detachment function satisfies the Realism property. Thus, a normative detachment $(b_1, i_1, d_1)_{c_1}^{r_1}$ attacks another normative detachment $(b_2, i_2, d_2)_{c_2}^{r_2}$ if $b_1 \wedge i_1 \wedge d_1 \wedge b_2 \wedge i_2 \wedge d_2$ is inconsistent, and either $r_1 \geq r_2$, or $r_1 = r_2$ and $c_1 \geq c_2$.

Moreover, an additional complication to deal with in formal argumentation is which normative detachments of the arguments we should consider. Once an argument is rejected, all arguments building on this argument must be rejected too. For this reason, when we consider an attack from argument A to argument B , we consider the conclusion of argument A , but we may consider any of the normative detachments of argument B . Finally, two permission arguments are allowed to conflict, so we check that at least one of the arguments is an obligation argument. All together, this amounts to the following definition of the attack relation.

Definition 8 (Obligation attack). *Argument A with conclusion $(b_1, i_1, d_1)_{c_1}^{r_1}$ obligation attacks argument B iff B contains a sub-argument C with conclusion $(b_2, i_2, d_2)_{c_2}^{r_2}$ not derived by AND such that*

- (1) $b_1 \wedge i_1 \wedge d_1 \wedge b_2 \wedge i_2 \wedge d_2$ inconsistent,
- (2) $r_1 \geq r_2$, or $r_1 = r_2$ and $c_1 \geq c_2$, and
- (3) at least one of the arguments is an obligation argument.

The priorities among the regulative norms decide which argument is accepted. One may argue that, if $r_1 = r_2$, it is not intuitive that the conflict is resolved based on the priority of the constitutive norms. In order to show that this is reasonable, consider the following example.

Example 11. *Consider again Example 8. Moreover, assume that the priorities are $r_1 = r_2 = 4$. The constitutive and regulative norms are $C = \{(OnlinePreApplication, ProspectiveStudent)_2, (ChinesePassport, ChineseCitizen)_3\}$, $R = \{(ProspectiveStudent, CertifiedPhotocopyPassport)^4, (ChineseCitizen, \neg CertifiedPhotocopyPassport)^4\}$. We have two conclusions of arguments: $(OnlinePreApplication, ProspectiveStudent, CertifiedPhotocopyPassport)_2^4$ and $(ChinesePassport, ChineseCitizen, \neg CertifiedPhotocopyPassport)_3^4$. Since $r_1 = r_2$, then it is the priority on c_1 and c_2 that decide. The constitutive norm that someone with a Chinese passport counts as a Chinese citizen should have a higher priority than the norm that says that someone who submits an online pre-application counts as a prospective student. Thus, in that situation, the university should accommodate its own procedure because $(ChinesePassport, ChineseCitizen)$ has a higher priority than $(ProspectiveStudent, CertifiedPhotocopyPassport)$.*

We use Dung's theory to define extensions of obligation and permission arguments. See

Dung’s paper for further explanation and examples.

Definition 9 (Abstract argumentation semantics (Dung, 1995)). *An argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ is a set of arguments \mathcal{A} together with a binary attack relation \mathcal{R} . For a set of arguments $B \subseteq \mathcal{A}$, B is conflict-free iff $\nexists \alpha, \beta \in B$ such that $(\alpha, \beta) \in \mathcal{R}$. B defends an argument α iff $\forall (\beta, \alpha) \in \mathcal{R}, \exists \gamma \in B$ such that $(\gamma, \beta) \in \mathcal{R}$. The set of arguments defended by B in \mathcal{F} is denoted as $\mathcal{D}_{\mathcal{F}}(B)$.*

A set of B is a complete extension of \mathcal{F} , iff B is conflict-free and $B = \mathcal{D}_{\mathcal{F}}(B)$. B is a preferred (grounded) extension iff B is a maximal (resp. minimal) complete extension. B is a stable extension, iff B is conflict-free, and $\forall \alpha \in \mathcal{A} \setminus B, \exists \beta \in B$ s.t. $(\beta, \alpha) \in \mathcal{R}$. We use $sem \in \{cmp, prf, grd, stb\}$ to denote complete, preferred, grounded, or stable semantics. The set of argument extensions of $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ is denoted as $sem(\mathcal{F})$.

The normative detachments are the conclusions of these accepted arguments.

Definition 10 (Argumentation framework of a normative system). *Given a normative system, its argumentation framework $AF(NS) = (\mathcal{A}, \mathcal{R})$ consists of the obligation and permission arguments \mathcal{A} together with the attack relation among these arguments \mathcal{R} . The conclusion extensions (Ob, Pe) consists of the conclusions of the obligation and permission arguments,*

$$det(NS) = \{concl(E) \mid E \in sem(AF(NS))\},$$

where the conclusions of an extension are the conclusions of the arguments in the extension, $concl((E) = \{concl(A) \mid A \in E\}$, and these conclusions are grouped by obligations Ob and permissions Pe .

We now discuss the properties of this new argumentation theory. The first requirement of normative reasoning is the representation of violations, and in particular the prevention of is-ought problems such as the fallacy of deontic wishful thinking. Definition 3 formalizes realism by requiring that institutional facts cannot be overridden by obligations. The following proposition shows that our argumentation theory satisfies the Realism property.

Proposition 1 (Realism). *The detachment function det satisfies the realism property for complete, grounded, and preferred semantics.*

Proof. We use the following variant of the directionality property (Baroni & Giacomin, 2007). Assume an argumentation framework (A, R) that can be written as $(A_1 \cup A_2, R_1 \cup R_2 \cup R_3)$ such that $R_1 \subseteq A_1 \times A_1, R_2 \subseteq A_1 \times A_2$, and $R_3 \subseteq A_2 \times A_2$. This means that (A, R) can be split into two sub-frameworks (A_1, R_1) and (A_2, R_3) such that there may be attacks from the first framework to the second, but not vice versa. An argumentation semantics satisfies the directionality property iff the extensions of (A_1, R_1) are exactly the intersection of the extensions of (A, R) intersected with A_1 , i.e. $sem(A_1, R_1) = \{E \cap A_1 \mid E \in sem(A, R)\}$. It is well known that complete, grounded and preferred semantics satisfy directionality, whereas stable semantics does not.

There are no attacks on institutional arguments from non-institutional arguments, because they have priority ∞ . So let U be all arguments with conclusions of the form (b, i, \top) . For $NS = (C, R, P, F)$ and $NS' = (C, R \cup R', P \cup P', F)$, let (A_1, R_1) be the framework of NS , and let (A, R) be the argumentation framework of NS' . We have $sem(A_1, R_1) = \{E \cap A_1 \mid E \in sem(A, R)\}$, and thus $det_i(NS) = det_i(NS')$. Consequently, each semantics satisfying directionality satisfies realism.

Example 12. *Let us consider examples with conflicts involving both constitutive and regulative norms:*

(Obligation attack) Take $C = \{(\top, a)_{c_1}, (\top, b)_{c_2}\}$ and $R = \{(a, c)^{r_1}, (b, \neg c)^{r_2}\}$. There are two relevant arguments with conclusions $(\top, a, c)_{c_1}^{r_1}$ and $(\top, b, \neg c)_{c_2}^{r_2}$ respectively. It is the priorities

among the regulative norms that decide which argument is accepted between $(\top, a, c)_{c_1}^{r_1}$ and $(\top, b, \neg c)_{c_2}^{r_2}$. See Example 11.

(Violation) Take $C = \{(\top, a)_c\}$ and $R = \{(\top, \neg a)^r\}$. There are two relevant arguments: A with conclusion $(\top, a, \top)_c^\infty$ and B with conclusion $(\top, \top, \neg a)_\infty^r$. Since the priorities of the regulative norms are considered first, argument A attacks argument B but not the other way around. This is essentially Proposition 1, and as expected. In Example 6 we have a conflict between a deontic and an institutional fact between $(\text{PayFee}, \text{SelfFundingStudent}, \text{FinancialProof})$ and $(\text{GrantRecipient}, \text{ExonerateFee}, \neg \text{SelfFundingStudent})$. If we now add priorities of the regulative norms, for instance $(\text{SelfFundingStudent}, \text{FinancialProof})^5$ is greater than $(\text{ExonerateFee}, \neg \text{SelfFundingStudent})^2$, we have that the argument with conclusion $(\text{PayFee}, \text{SelfFundingStudent}, \text{FinancialProof})^5$ attacks the argument with conclusion $(\text{GrantRecipient}, \text{ExonerateFee}, \neg \text{SelfFundingStudent})^2$. This provokes the ‘disappearance’ of the obligation $\neg \text{SelfFundingStudent}$, which can be explained by the fact that only students who come from a disadvantaged economic background can receive a grant. If a stronger argument attacks $(\text{GrantRecipient}, \text{ExonerateFee}, \neg \text{SelfFundingStudent})$, then the obligation $\neg \text{SelfFundingStudent}$ must be dropped. The fact that an obligation for p is removed once there is a fact $\neg p$ is a positive feature in our approach, due to the Realism property. However, as argued in Parent & van der Torre (2017), we also have the intuition that an obligation cannot be defeated by violating it. After the violation of the obligation to do p , p is still in force. Even if you evade taxes, you are still obliged to truthfully declare you assets.

(Violation.) Suppose $C = \{(\top, a)_{c_1}, (\top, b)_{c_2}\}$ and $R = \{(a, \neg b)^r\}$. We have three relevant arguments with conclusions $(\top, a, \top)_{c_1}^\infty$, $(\top, a, \neg b)_{c_1}^r$ and $(\top, b, \top)_{c_2}^\infty$. The argument with conclusion $(\top, a, \neg b)_{c_1}^r$ will be defeated, as to Proposition 1.

(Institutional attack) Take $C = \{(a, \neg b)_{c_1}, (\top, b)_{c_2}\}$ and $R = \{(\top, a)^r\}$. The three relevant arguments have conclusions $(\top, \top, a)_\infty^r$, $(\top, \neg b, a)_{c_1}^r$ and $(\top, b, \top)_{c_2}^\infty$. The argument with conclusion $(\top, \neg b, a)_{c_1}^r$ will be defeated by the argument with conclusion $(\top, b, \top)_{c_2}^\infty$, based on no wishful thinking, as expected. See Example 16.

(Institutional attack) Take $C = \{(a, c)_{c_1}, (b, \neg c)_{c_2}\}$ and $R = \{(\top, a)^{r_1}, (\top, b)^{r_2}\}$. The two relevant arguments have conclusions $(\top, c, a)_{c_1}^{r_1}$ and $(\top, \neg c, b)_{c_2}^{r_2}$ respectively. The conflict is decided by the priorities on the norms.

We can now show that each standard semantics satisfies the reinstatement properties.

Proposition 2 (Reinstatement). *A detachment function det satisfies the factual and norm reinstatement property for complete, grounded, and preferred semantics.*

Proof. Consider the normative system $NS = (C, R, P, F)$ with $C = \emptyset$, $R = \{(\phi_1, \psi_1)^{r_1}, (\phi_2, \psi_2)^{r_2}\}$ with $r_2 > r_1$, $\phi_1 \wedge \psi_1 \wedge \phi_2$ consistent, $\phi_1 \wedge \phi_2 \wedge \psi_2$ consistent, $\phi_1 \wedge \psi_2 \wedge \phi_2 \wedge \psi_2$ inconsistent, $P = \emptyset$, $F = \{\phi_1, \phi_2, \neg \psi_2\}$. There are only three relevant arguments, argument A with conclusion $(\phi_1 \wedge \phi_2 \wedge \neg \psi_2, \top, \top)_\infty^\infty$, argument B with conclusion $(\phi_1 \wedge \phi_2 \wedge \neg \psi_2, \top, \psi_1)_\infty^1$ and argument C with conclusion $(\phi_1 \wedge \phi_2 \wedge \neg \psi_2, \psi_2)_\infty^2$. We have A attacking C and C attacking B, and thus for all standard semantics, due to reinstatement, there is only one extension, and this extension accepts argument A and B. We thus derive the obligation $(\phi_1, \top, \psi_1) \in \cap det(NS)$. Consequently, the detachment function satisfies factual reinstatement.

Consider the normative system $C = \emptyset$, $R = \{(\phi_1, \psi_1)^{r_1}, (\phi_2, \phi_2)^{r_2}, (\phi_3, \phi_3)^{r_3}\}$ with $r_3 > r_2 > r_1$, $\phi_1 \wedge \psi_1 \wedge \phi_2$ consistent, $\phi_1 \wedge \phi_2 \wedge \psi_2$ consistent, $\phi_1 \wedge \psi_2 \wedge \phi_2 \wedge \psi_2$ inconsistent, $P = \emptyset$, $F = \{\phi_1, \phi_2, \phi_3\}$. The three relevant arguments are argument A with conclusion $(\phi_1, \top, \psi_1)_\infty^{r_1}$, argument B with conclusion $(\phi_2, \top, \phi_2)_\infty^{r_2}$, and argument C with conclusion $(\phi_3, \top, \phi_3)_\infty^{r_3}$. We have C attacking B and B attacking A, and thus for all standard semantics there is only one extension, and this extension accepts argument C and A. The obligation $(\phi_1 \wedge \phi_2, \psi_1) \in \cap det(NS)$. Consequently, the detachment function satisfies norm reinstatement.

Finally, we show that the argumentation system is well behaved by proving the two properties of closure under sub-arguments and closure under conjunction. The following proposition

shows that the argumentation theory satisfies closure under sub-arguments.

Proposition 3 (Sub-argument closure). *For every extension, if an argument is in the extension, then also all its sub-arguments are in the extension.*

Proof. This follows directly from the definition of attack, because if an argument A attacks a sub-argument C of an argument B , then it also attacks argument B itself.

Proposition 4 (Closure under conjunction). *Let a set S of detachments be closed under conjunctions iff $(b_1, i_1, d_1), (b_2, i_2, d_2) \in S$ implies $(b_1 \wedge b_2, i_1 \wedge i_2, d_1 \wedge d_2) \in S$. The set of conclusions of an extension are closed under conjunction.*

Proof. Let A and B be two arguments in an extension. Let AB be the argument that contains first all the normative detachments of argument A , then all the normative detachments of argument B , and finally apply the AND rule to the conclusion of Argument A and the conclusion of argument B . Since the argument AB cannot be attacked on its conclusion, the attackers of AB are exactly the union of the attackers of A and the attackers of B . Since A and B are defended by the extension against all their attackers, also AB is defended. Due to completeness, argument AB is part of the extension as well.

Proposition 5 (Consistency). *Let the conjunction of a normative detachment (b, i, d) be the formula $b \wedge i \wedge d$, a normative detachment (b, i, d) be consistent iff its conjunction is consistent in propositional logic, and a set of detachments S be inconsistent if $\{b \wedge i \wedge d \mid (b, i, d) \in S\}$ is consistent. The set of the conclusions of an extension is consistent.*

Proof. We use proof by contradiction. Assume there is an extension such that the set of its conclusions is inconsistent. Let S be a minimal set of arguments whose set of conclusions is inconsistent. Let A be an argument of S whose priority is minimal, and let B be the conjunction of all elements of $S \setminus \{A\}$. Then B attacks A . However, extensions are conflict-free, so we reach a contradiction. Thus the assumption must be false, i.e. there cannot be an extension whose set of conclusions is inconsistent.

4. Further research

The literature on reasoning with constitutive and regulative norms is relatively small. There are many variants of the framework introduced in this paper which must be studied as well. Two issues to be studied are the use of alternative combinations of constitutive and regulative norms, and other attack relations. Moreover, some other topics for further research are mentioned as well.

4.1 Combining constitutive and regulative norms

Besides choosing a logic for constitutive norms and a logic for regulative norms, we need to choose a way to combine them. Sun & van der Torre (2014) present three ways to combine the constitutive and regulative norms. According to the first (called *simple minded*), the output of the constitutive norms are intermediate facts used as input for the regulative norms, formally: $\odot_1(C, R, A) = \circ(R, \circ(C, A))$. The second combination (called *throughput*) adds the input of the constitutive norms to the intermediate facts, formally: $\odot_1^+(C, R, A) = \circ(R, A \cup \circ(C, A))$. Moreover, they give abstract and detailed combinations for each way.

In this paper, we used the third combination (called *throughput reusable*). We define obligation arguments combining both constitutive and regulative norms, using the *throughput reusable* approach proposed by Sun and van der Torre as visualised in Figure 1. C and R are the

set of constitutive and regulative norms respectively. A is the set of formulas representing the facts, I is the set of intermediate concepts derived by A and C . Finally, O is the output.

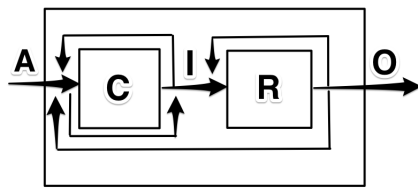


Figure 1. Reusable combination

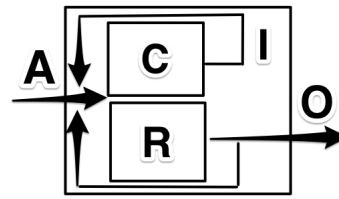


Figure 2. Serial visualization

As illustrated by the arrow from O to A in Figure 1, throughput reusable combination is the extension of throughput combination allowing the output of regulative norms to be reused as input for constitutive norms. In this case the input of C has three sources: the arrow A , the arrow from I to A , and the arrow from O to A . The input of R has exactly the same resource. Therefore we can change Figure 1 to Figure 2 such that C and R have the same input.¹

However, other combinations of constitutive and regulative norms besides the three combination methods of Sun and van der Torre must be studied. For example, if we derive from the constitutive norm (b, i) and the regulative norm (\top, b) the obligation detachment $(\top, \top, b \wedge i)$ rather than (\top, i, b) as in Sun and van der Torre’s combination method used in this paper. particular, as d is obligatory and d counts as e , it may be argued that e is obligatory as well.

4.2 Different kinds of attacks among obligation and permission arguments

In this section, we define another definition of the attack relation. Motivated independently, its meaning is slightly different from the one in the previous section.

Following our earlier analysis, we break down the definition of attack into obligation, violation and institutional attacks. In the tradition in much of preference-based argumentation (Amgoud & Cayrol, 1997; Modgil & Prakken, 2013), *defeat* is the relation among arguments on which the semantics is based, whereas *attack* is used for a relation among arguments which does not take the preferences among arguments into account. In particular, argument A defeats argument B iff A attacks B , and B is not preferred to A . Since we use a preorder in this paper, this is equivalent to saying that A is at least as preferred as B . In this paper, we do not distinguish defeat from attack.

All attacks are based on a consistency check. Finally, the institutional and obligation attack refer to the relevant priorities.

The first attack we consider is the obligation attack, reflecting dilemmas. Each obligation can be fulfilled individually, but they cannot be fulfilled jointly. In such a case, the priorities on the regulative norms decide the direction of the attack.

Definition 11 (Obligation attack (dilemma)). *Argument A with conclusion $(b_1, i_1, d_1)_{c_1}^{r_1}$ obligation attacks argument B iff B contains a sub-argument C with conclusion $(b_2, i_2, d_2)_{c_2}^{r_2}$ such that*

- (1) $b_1 \wedge i_1 \wedge d_1 \wedge b_2 \wedge i_2 \wedge d_2$ inconsistent,
- (2) $b_1 \wedge i_1 \wedge d_1 \wedge b_2 \wedge i_2$ and $b_1 \wedge i_1 \wedge b_2 \wedge i_2 \wedge d_2$ consistent, and
- (3) $r_1 \geq r_2$.

Example 13. *From Example 11 we recall that the selection committee faces a dilemma as the argument with conclusion*

¹Such detachments are used to axiomatize so-called reusable combination of constitutive and regulative norms, see the paper of Sun and van der Torre for further details.

$(OnlinePreApplication, ProspectiveStudents, CertifiedPhotocopyPassport)_2^4$ attacks the triple $(ChinesePassport, ChineseCitizen, \neg CertifiedPhotocopyPassport)_3^4$. Such an attack is decided by the values of the regulative rules, so the result is the same as in Example 11.

In our framework we do not have specificity built in, we only make use of priorities. The following example illustrates the difference.¹

Example 14. Let $(r, p)_1$ be a constitutive norm, $(p, q)^2$ and $(r, \neg q)^2$ the regulative norms, and let r be a brute fact. We can construct the arguments $A = (r, p, q)_1^2$ and $B = (r, \top, \neg q)_\infty^2$. In our framework arguments A and B attack each other. However, one may argue that A is the more specific norm here, as it is triggered in the exceptional setting where r counts as p . So, if we consider specificity, A should attack B but not vice versa.

The second attack we consider is the violation attack.

Definition 12 (Violation attack). Argument A with conclusion $(b_1, i_1, d_1)_{c_1}^{r_1}$ violation attacks argument B iff B contains a sub-detachment C with conclusion $(b_2, i_2, d_2)_{c_2}^{r_2}$ such that

- (1) $b_1 \wedge i_1 \wedge b_2 \wedge i_2 \wedge d_2$ inconsistent, and
- (2) $b_1 \wedge b_2 \wedge i_1 \wedge i_2$ consistent.

Example 15. Example 5 illustrated a conflict between a brute fact and an obligation. According to Definition 12, we have the argument with conclusion $(CopyNoCredit, Plagiarism, ReSitExam)$ attacking the argument with conclusion $(SubmitEssay, SitExam, \neg CopyNoCredit)$. We do not explicit the priority values of constitutive and regulative norms because in a violation attack they do not play a role.

We have also a violation attack when there is a conflict between a deontic and an institutional fact as in Example 6, where the argument with conclusion $(PayFee, SelfFundingStudent, FinancialProof)$ attacks the argument with conclusion $(GrantRecipient, ExonerateFee, \neg SelfFundingStudent)$.

Finally, the third attack is institutional attack. We do not have to consider the brute facts, because the brute facts are always consistent.

Definition 13 (Institutional attack). Argument A with conclusion $(b_1, i_1, d_1)_{c_1}^{r_1}$ institutionally attacks argument B iff B contains a sub-argument C with conclusion $(b_2, i_2, d_2)_{c_2}^{r_2}$ such that

- (1) $b_1 \wedge i_1 \wedge b_2 \wedge i_2$ inconsistent,
- (2) $c_1 \geq c_2$.

Example 16. In our running example (Example 7) we have that $(DisciplinaryMeasure, \neg PUStudent, \neg SitExam)_3^5$ institutionally attacks $(ValidStudentCard, PUStudent, AttendCourses)_2^2$. Remember that we assume that facts are consistent.

4.3 Other issues

More properties can be proven for the argumentation theory introduced in this paper. In particular, Dung introduced his abstract argumentation theory to represent existing non-monotonic logics and logic programming, and Liao et al. (2016) gave various representation results for prioritised normative systems. Extending these representation results to the argumentation theory introduced in this paper is another challenge for further research.

Moreover, we did not introduce formal distinctions between brute and institutional facts. In particular, we did not impose that brute and institutional facts are based on disjoint sets of propositional atoms, even though we acknowledge that their difference. We leave for future research the investigation of possible conflicts between brute and institutional facts. It may be

¹This example has been suggested by one of the anonymous reviewers, whom we thank.

interesting to consider cases in which brute and institutional facts can refer to the same concept (for example, *vehicle*), but this same concept can have different interpretations. For instance, a vehicle is understood to be a bicycle, a car, a camion, etc. But there could be a legal text in which ‘vehicle’ cannot be interpreted as a bicycle (even though a bicycle can be interpreted as a vehicle as a brute fact).

We consider only the setting where all conflicts are resolved. We leave the other cases, where some of the dilemmas are not resolved, for future research. Third, it has been argued that constitutive norms play a central role in norm change. How to argue about norm change (Governatori & Rotolo, 2010)?

5. Related work

In this paper we show how to construct a proof theory for our ternary arguments. In a normative system there may be a clear distinction between brute and institutional facts, because they are based on distinct sets of propositions. However, such a distinction also limits the expressive power. For example, existing proof systems such as the ones proposed by Sun & van der Torre (2014) allow to freely move between the input, intermediate and output, for example using the so-called proof rule “aggregative cumulative transitivity.” In this paper we therefore also allow this flexibility in the proof system and we do not assume that brute and institutional facts are disjoint.

We define arguments as derivations in aggregative input/output logic of Parent & van der Torre (2014), see also Ambrossio, Parent, & van der Torre (2016). We do not aim for the full ASPIC+ generality here, for example we do not consider strict rules, we represent the priority by a natural number (i.e. we assume that the ordering on norms is a total preorder) and we consider only the last link principle when lifting priorities on norm to priorities on arguments. Aggregative input/output logics have as their core detachment pattern so-called aggregative cumulative transitivity, that derives $(a, x \wedge y)$ from (a, x) and $(a \wedge x, y)$. It can be contrasted with a more common rule called cumulative transitivity, that derives (a, y) from the same premises.

It was observed by an anonymous reviewer that our approach is non-standard in the following sense. In argumentation formalisms with restricted rebuttal, arguments can only be attacked in defeasible conclusions. In formalisms with unrestricted rebuttal, arguments can be attacked in any of their conclusions, unless no defeasible rule whatsoever was used in the construction of the argument. The present solution mimics restricted rebuttal in the sense that arguments obtained via AND cannot be attacked in their conclusion. Still, and contrary to restricted rebuttal formalisms, arguments obtained via other ‘strict’ rules such as EQ can be attacked in their conclusion, which is more in line with the unrestricted rebuttal solution. According to the reviewer, all this means that it is hard to see how standard results for frameworks such as ASPIC carry over to the present framework, and that more work is needed in exploring these relations.

We extend aggregative input/output logic with priorities, such that each argument gets the priority of the norm that has been applied last. Parent and van der Torre do not include (\top, \top) in their logic, but Sun & van der Torre (2014) do. The reason is that it makes the semantics easier. This is analogous to the use of $O(\top)$ in modal deontic logic, which is accepted in so-called standard deontic logic, whereas von Wright did not allow it in his so-called Old System. For more about this, see (Gabbay et al., 2013). Technically, for our argumentation theory, this property is relatively harmless as such arguments cannot attack other arguments.

At first sight, it may seem surprising that at our level of abstraction, the difference between constitutive and regulative norms is represented only in the attack relation, that is, in the way conflicts among constitutive and regulative norms are resolved. A similar abstract analysis was given by Broersen, Dastani, Hulstijn, Huang, & van der Torre (2001) to relate various mental attitudes.

Various papers consider arguments among regulative and permissive norms, and several

frameworks have been proposed for legal argumentation (Bench-Capon, Prakken, & Sartor, 2010; Prakken & Sartor, 2013; van der Torre & Villata, 2014). Recently, Beirlaen & Straßer (2016) presented an argumentation system for detaching conditional obligations based on Dung’s grounded semantics. They define two ways in which deontic arguments may attack one another and present some mechanisms for conflict-resolution. Our work is related to Beirlaen and Strasser, though our aim here is to extend this line of work by exploring how conflicts among constitutive and regulative norms can be resolved.

An argumentation analysis of legal ontology is the object of many papers in the field of AI and Law. See, for instance, the work on “Popov v. Hayashi”, where the issue is to determine the notion of “possession”. The terminology of such legal ontologies is different from the one based on institutional facts and constitutive norms used in this paper. For example, they may express “a rose by any other name would smell as sweet”. Moreover, there is work using rule based system for constitutive and regulative rules (and some of them have an argumentation like flavour, including the work of Boella, Governatori, Rotolo, & van der Torre (2010a,b)).

6. Conclusion

In this paper we introduce an argumentation theory for normative arguments represented by (brute, institutional, deontic) triples, where constitutive norms derive institutional facts from brute facts, and regulative norms derive deontic facts like obligations and permissions from institutional facts. The institutional facts may be seen as the reasons explaining or warranting the deontic obligations and permissions, and therefore they can be attacked by other normative arguments too. Moreover, we introduce a running example based on university regulations for prospective and actual students.

We represent different aspects of normative reasoning by different kinds of consistency checks among these triples, and we use formal argumentation theory to resolve conflicts among such normative arguments. The main issue concerning the is-ought problem studied in this paper is the absence of normative wishful thinking, that is, we study how to prevent that obligations override brute and institutional facts. We introduce a property to formalise no normative wishful thinking, and we implement this property by giving facts the highest possible priority, and by using a lexicographic ordering which first checks the priority associated with the obligations and permissions (derived by regulative and permissive norms), and only when these priorities are the same, it considers the priority of the institutional facts (derived by constitutive norms). We distinguish dilemmas and institutional conflicts, which both lead to multiple extensions of obligation and permission arguments. We show that semantics satisfying reinstatement, also lead to factual and norm reinstatement for normative detachment.

Acknowledgments

We thank two anonymous referees for valuable comments. The contribution of Gabriella Pigozzi was supported by the project AMANDE ANR-13-BS02-0004 of the French National Research Agency (ANR). Leendert van der Torre has received funding from the European Union’s H2020 research and innovation programme under the Marie Curie grant agreement No. 690974 for the project MIREL: MIning and REasoning with Legal texts.

References

Alchourrón, C. E., & Bulygin, E. (1981). The expressive conception of norms. In R. Hilpinen (Ed.), *New Studies in Deontic Logic* (pp. 95–124). Dordrecht: D. Reidel.

- Ambrossio, D. A., Parent, X., & van der Torre, L. (2016). Cumulative aggregation. In O. Roy, A. Tamminga, & M. Willer (Eds.), *Deontic Logic and Normative Systems. 13th International Conference, DEON 2016* (p. 1-15). College Publications, UK.
- Amgoud, L., & Cayrol, C. (1997). Integrating preference orderings into argument-based reasoning. In *Qualitative and Quantitative Practical Reasoning: First International Joint Conference on Qualitative and Quantitative Practical Reasoning, ECSQARU-FAPR* (p. 159-170).
- Baroni, P., & Giacomin, M. (2007). On principle-based evaluation of extension-based argumentation semantics. *Artificial Intelligence Journal*, 171, 675–700.
- Beirlaen, M., & Straßer, C. (2016). A structured argumentation framework for detaching conditional obligations. In O. Roy, A. Tamminga, & M. Willer (Eds.), *Deontic Logic and Normative Systems. 13th International Conference, DEON 2016* (pp. 32–48). College Publications, UK.
- Bench-Capon, T., Prakken, H., & Sartor, G. (2010). Argumentation in legal reasoning. In I. Rahwan & G. Simari (Eds.), *Argumentation in Artificial Intelligence*. Springer.
- Boella, G., Governatori, G., Rotolo, A., & van der Torre, L. (2010a). Lex minus dixit quam voluit, lex magis dixit quam voluit: A formal study on legal compliance and interpretation. In G. A. P. Casanovas U. Pagallo & G. Sartor (Eds.), *AI Approaches to the Complexity of Legal Systems* (p. 162-183). Berlin.
- Boella, G., Governatori, G., Rotolo, A., & van der Torre, L. (2010b). A logical understanding of legal interpretation. In *Proceedings of the Twelfth International Conference on the Principles of Knowledge Representation and Reasoning (KR)* (p. 563-565).
- Boella, G., & van der Torre, L. (2004). Regulative and constitutive norms in normative multi-agent systems. In D. Dubois, C. A. Welty, & M.-A. Williams (Eds.), *Principles of Knowledge Representation and Reasoning: Proceedings of the Ninth International Conference (KR 2004)* (p. 255-266). Whistler, Canada: AAAI Press.
- Boella, G., & van der Torre, L. (2006a). Constitutive norms in the design of normative multi-agent systems. In *Computational Logic in Multi-Agent Systems, 6th International Workshop, CLIMA VI* (p. 303-319). Springer.
- Boella, G., & van der Torre, L. (2006b). A logical architecture of a normative system. In L. Goble & J.-J. C. Meyer (Eds.), *Deontic Logic and Artificial Normative Systems* (Vol. 4048, p. 24-35). Utrecht, The Netherlands: Springer.
- Broersen, J. M., Dastani, M., Hulstijn, J., Huang, Z., & van der Torre, L. W. N. (2001). The BOID architecture: conflicts between beliefs, obligations, intentions and desires. In *Proceedings of the fifth international conference on autonomous agents, AGENTS 2001* (pp. 9–16).
- Broome, J. (2013). *Rationality through reasoning*. West Sussex, UK: Wiley-Blackwell.
- Bulygin, E. (1986). *Permissive Norms and Normative Concepts*. Amsterdam: North Holland.
- da Costa Pereira, C., Liao, B., Malerba, A., Rotolo, A., Tettamanzi, A. G. B., van der Torre, L., & Villata, S. (2017). Handling norms in multi-agent systems by means of formal argumentation. *Ifcolog journal of logics and their applications*, 4(9), 3039-3074. (To appear in Chopra, A., van der Torre, L., Verhagen, H. and Villata, S. (eds):*Handbook of Normative Multiagent Systems*.)
- Danielsson, S. (1968). *Preference and Obligation: Studies in the Logic of Ethics*. Uppsala: Filosofiska foereningen.
- Dung, P. M. (1995). On the acceptability of arguments and its fundamental role in non-monotonic reasoning, logic programming and n-person games. *Artificial Intelligence*, 77(2), 321-358.
- Gabbay, D., Horty, J., Parent, X., van der Meyden, R., & van der Torre, L. (Eds.). (2013). *Handbook of Deontic Logic and Normative Systems*. London: College Publications.
- Governatori, G., & Rotolo, A. (2008, August). BIO logical agents: Norms, beliefs, intentions in defeasible logic. *Autonomous Agents and Multi-Agent Systems*, 17(1), 36–69.
- Governatori, G., & Rotolo, A. (2010). Changing legal systems: legal abrogations and annul-

- ments in defeasible logic. *Logic Journal of the IGPL*, 18(1), 157–194.
- Grossi, D., & Jones, A. (2013). Constitutive norms and counts-as conditionals. In J. Horty, D. Gabbay, X. Parent, R. van der Meyden, & L. van der Torre (Eds.), *Handbook of Deontic Logic and Normative Systems*. College Publications.
- Hansson, B. (1969). An analysis of some deontic logics. *Noûs*, 3(4), 373-398.
- Horty, J. (2001). Argument construction and reinstatement in logics for defeasible reasoning. *Artif. Intell. Law*, 9(1), 1–28.
- Lewis, D. (1973). *Counterfactuals*. Oxford: Blackwell.
- Liao, B., Oren, N., van der Torre, L., & Villata, S. (2016). Prioritized norms and defaults in formal argumentation. In O. Roy, A. Tamminga, & M. Willer (Eds.), *Deontic Logic and Normative Systems. 13th International Conference, DEON 2016* (p. 139-154). College Publications, UK.
- Liao, B., Oren, N., van der Torre, L., & Villata, S. (2018). Prioritized norms in formal argumentation. *Journal of Logic and Computation*, exy009. Retrieved from <http://dx.doi.org/10.1093/logcom/exy009> doi: 10.1093/logcom/exy009
- Lindahl, L., & Odelstad, J. (2003). Normative systems and their revision: An algebraic approach. *Artificial Intelligence and Law*, 11(2-3), 81-104.
- Makinson, D. (1999). On a fundamental problem of deontic logic. In P. McNamara & H. Prakken (Eds.), *Norms, Logics and Information Systems. New Studies on Deontic Logic and Computer Science* (p. 29-54). IOS Press.
- Makinson, D., & van der Torre, L. (2000). Input-output logics. *Journal of Philosophical Logic*, 29(4), 383–408.
- Makinson, D., & van der Torre, L. (2001). Constraints for input-output logics. *Journal of Philosophical Logic*, 30(2), 155-185.
- Makinson, D., & van der Torre, L. (2003). Permission from an input/output perspective. *Journal of Philosophical Logic*, 32, 391-416.
- Modgil, S., & Prakken, H. (2013). A general account of argumentation with preferences. *Artificial Intelligence*, 195, 361–397.
- Parent, X., & van der Torre, L. (2014). “Sing and Dance!”. In F. Cariani, D. Grossi, J. Meheus, & X. Parent (Eds.), *Deontic Logic and Normative Systems* (Vol. 8554, p. 149-165). Springer.
- Parent, X., & van der Torre, L. (2017). Detachment in normative systems: Examples, inference patterns, properties. *Ifcolog journal of logics and their applications*, 4(9), 2995-3038. (To appear in Chopra, A., van der Torre, L., Verhagen, H. and Villata, S. (eds):*Handbook of Normative Multiagent Systems*.)
- Pigozzi, G., & van der Torre, L. (2017). Multiagent deontic logic and its challenges from a normative systems perspective. *Ifcolog journal of logics and their applications*, 4(9), 2929-2994. (To appear in Chopra, A., van der Torre, L., Verhagen, H. and Villata, S. (eds):*Handbook of Normative Multiagent Systems*.)
- Prakken, H., & Sartor, G. (2013). Formalising arguments about norms. In *Legal Knowledge and Informations Systems. JURIX 2013* (pp. 121–130).
- Ross, A. (1941). Imperatives and logic. *Theoria*, 7, 53-71. (Reprinted in *Philosophy of Science* 11:30–46, 1944.)
- Searle, J. (1969). *Speech Acts. An Essay in the Philosophy of Language*. Cambridge University Press, Cambridge.
- Stenius, E. (1963). Principles of a logic of normative systems. *Acta Philosophica Fennica*, 16, 247-260.
- Sun, X., & van der Torre, L. (2014). Combining constitutive and regulative norms in input/output logic. In F. Cariani, D. Grossi, J. Meheus, & X. Parent (Eds.), *Deontic Logic and Normative Systems* (Vol. 8554, p. 241-257). Springer.
- van der Torre, L., & Tan, Y. (2001). Dynamic normative reasoning under uncertainty: How to distinguish between obligations under uncertainty and prima facie obligations. In D. Gabbay & P. Smets (Eds.), *Handbook of Defeasible Reasoning and Uncertainty Management*

Systems, Vol 6: Agents, Reasoning and Dynamics (p. 267-297). Kluwer.
van der Torre, L., & Villata, S. (2014). An ASPIC-based legal argumentation framework for deontic reasoning. In *Computational Models of Argument - Proceedings of COMMA 2014* (pp. 421–432).