

# Real time multiline holding control for networks with shared transit corridor

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## Introduction

The inherent stochastic nature of public transport operations poses a continuous challenge for operators. Real-time control assists in limiting the propagation of phenomena that are interwoven with highly variable travel times and passengers demand. The chosen control criteria depend on the service and the characteristics of the transit line [1].

When vehicle holding at stations is used as a control strategy, the objectives vary from schedule adherence, to headway adherence or minimization of passenger cost. Based on the objective, rule-based methods or optimization can be used as a solution approach [2]. In the first category, the holding rules may refer to scheduled times [3] or to headway adherence [4]–[6]. In the second category, optimization focuses on the minimization of passenger cost such as in the works of [7], [8], [2], [9]. Extending holding beyond the single-line level, the first level of interaction between different transit lines is their synchronization at a transfer stop [10]–[12].

Control strategies for service networks that consist of multiple lines that intersect and overlap has been limited. Recent contributions by [13]–[15] consider the operations of a common corridor consisting of overlapping line segments while discarding passenger interchanges and line management before and after the corridor.

In this study we formulate a general holding criterion that can be applied to any segment (shared transit corridor and branches prior and after) of networks with overlapping segments and accounts for transfers where needed. The contributions of this study are twofold: (i) we apply a holding-based rule for networks that share a set of consecutive stops aiming for regularity and minimizing the perceived passenger cost; (ii) the proposed rule accounts for the travel cost of transfers occurring along the overlapping segment(s).

## Methodology

### Network configuration

We focus on networks which consist of multiple transit lines and have at least one set of common consecutive stops that is sufficiently large to consider it a shared transit corridor. Stop sets served by a single line are defined as branches. The boundaries between different stop sets are defined at specific stops (critical stops), where the number of lines operating upstream and downstream is different. Depending on the number of lines upstream and downstream, critical stops are denoted as merging and splitting stops.

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It should be noted that control actions are made for each vehicle based on the expected conditions along the current segment and up to the next critical downstream stop.

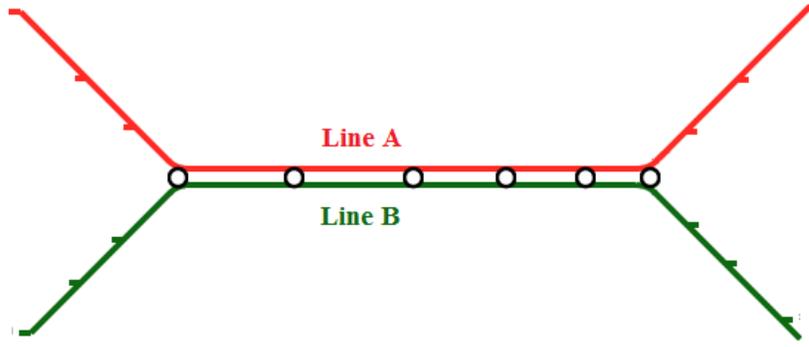


Figure 1 Schematic representation of the network

### Regularity criterion

The following equation is derived from a generalized passenger travel time function, consisting of waiting time and in-vehicle time. The general form of the holding criterion is given below:

Equation 1: General form of the criterion

$$t_{ijk}^{\text{hold,reg}} = \max \left\{ \begin{array}{l} \theta_1 \left[ \frac{(t_{jk+1}^{\text{exit}} - t_{ijk}^{\text{exit}}) - (t_{ijk}^{\text{exit}} - t_{jk-1}^{\text{exit}})}{2} \right] + \theta_2 \left[ \frac{(t_{ijk+1}^{\text{exit}} - t_{ijk}^{\text{exit}}) - (t_{ijk}^{\text{exit}} - t_{ijk-1}^{\text{exit}})}{2} \right] \\ + \theta_3 \left[ \frac{(\hat{t}_{ij^{\text{crit}}k+1}^{\text{exit}} - \hat{t}_{ij^{\text{crit}}k}^{\text{exit}}) - (\hat{t}_{ij^{\text{crit}}k}^{\text{exit}} - \hat{t}_{ij^{\text{crit}}k-1}^{\text{exit}})}{2} \right] - \frac{q_{ijk}}{4 \left( \sum_{o=j}^{N_{jc}} \sum_{d=o+1}^{N_{jc}} \lambda_{o,d} + \sum_{o=j}^{N_{jc}} \sum_{d=o+1}^{N_j} \lambda_{o,d} + \sum_{o=\text{crit}}^{N_j} \sum_{d=o+1}^{N_j} \lambda_{o,d} \right)}, 0 \end{array} \right\} \quad (1)$$

Where,

- $t_{ijk}^{\text{hold}}$  the holding time for trip k of line i at stop j in [time units];
- $t_{ijk}^{\text{exit}}$  the departure (exit) time in [time units];
- $\hat{t}_{ijk}^{\text{exit}}$  the expected departure time from the next critical stop in [time units];
- $q_{ijk}$  the occupancy of trip k of line i at stop j in [passengers];
- $\lambda_{ijk}^{\text{hold}}$  the arrival rate between an origin o and a destination d in [passengers/time unit];

As mentioned above, formula (1) sets the optimal holding time as a function of the stop set currently visited, passenger demand, and the next critical stop taking into account of each of the stops the lines involved.

The first two terms regularize the headway of the line and/or the shared transit corridor, subject to the passenger demand that is affected by the corresponding headway.

The third term smoothens the transition between different stop sets by estimating the expected departure time from the next critical stop downstream, and it is affected by the expected arrival of vehicles from the lines that will continue to interact downstream of the critical stop.

The fourth and final term is the ratio between the passengers on board and the sum of the arrival rates from the current and the remaining downstream stops until the end of the line. The passenger ratio is subtracted from the holding time calculated by the previous terms, to limit the effect of the additional time a vehicle is held at a stop on other passenger groups.

### Adjusting weights

Each of the terms in Eq. 1 is weighted by the ratio between the corresponding passenger segment and the total demand. Furthermore, the weights include a decay function based on the distance from the next critical stop to avoid controlling when relying on estimations with lower accuracy. A parameter  $\alpha$  is applied at the terms that regularize headways at the current stop and hence the same distance weight. The parameter is set based on the demand share of each term.

**Equation 2: Weights**

$$\begin{aligned}
\theta_1 &= \frac{\sum_{o=j}^{N_{j_c}} \sum_{d=o+1}^{N_{j_c}} \lambda_{o,d}}{\sum_{o=j}^{N_{j_c}} \sum_{d=o+1}^{N_{j_c}} \lambda_{o,d} + \sum_{o=j}^{N_{j_c}} \sum_{d=o+1}^{N_j} \lambda_{o,d} + \sum_{o=crit}^{N_j} \sum_{d=o+1}^{N_j} \lambda_{o,d}} + (\alpha) \left( 1 - \frac{1}{j^{crit} - j} \right) \\
\theta_2 &= \frac{\sum_{o=j}^{N_{j_c}} \sum_{d=o+1}^{N_j} \lambda_{o,d}}{\sum_{o=j}^{N_{j_c}} \sum_{d=o+1}^{N_{j_c}} \lambda_{o,d} + \sum_{o=j}^{N_{j_c}} \sum_{d=o+1}^{N_j} \lambda_{o,d} + \sum_{o=crit}^{N_j} \sum_{d=o+1}^{N_j} \lambda_{o,d}} + (1-\alpha) \left( 1 - \frac{1}{j^{crit} - j} \right) \\
\theta_3 &= \frac{\sum_{o=crit}^{N_j} \sum_{d=o+1}^{N_j} \lambda_{o,d}}{\sum_{o=j}^{N_{j_c}} \sum_{d=o+1}^{N_{j_c}} \lambda_{o,d} + \sum_{o=j}^{N_{j_c}} \sum_{d=o+1}^{N_j} \lambda_{o,d} + \sum_{o=crit}^{N_j} \sum_{d=o+1}^{N_j} \lambda_{o,d}} + \left( \frac{1}{j^{crit} - j} \right)
\end{aligned} \tag{2}$$

### Transfer criterion

At the shared segment, passengers can also transfer between lines. We treat the stops of the networks examined as shared transfer stops as characterized by Hadas and Ceder, (2010), assuming that passengers do not walk to a nearby connecting stop and their transferring time is equal to the walking time between vehicles.

We apply a transfer criterion as presented by [12]. The authors applied the following criterion for transferring passengers on a single stop, given different levels of information on the passenger demand. In order to be in line with the formulation of the regularity criterion, we assume that passenger information is based on historical data for boarding, alighting and transferring passengers.

The holding time needed for synchronization is equal to the difference between current time and the expected arrival of the next vehicle of the transferring line and is given by the following formula:

**Equation 3: Holding time for transfer**

$$t_i^{\text{hold, sync}} = \left( \tilde{t}_{i+1, j}^{\text{arrival}} - t^{\text{current}} \right) + \tau^{\text{transfer}} \quad (3)$$

Where

$\tilde{t}_{i+1, j}^{\text{arrival}}$  the expected arrival time of the following vehicle of the connecting line  $i+1$  at stop  $j$  in [time units]

$t^{\text{current}}$  current time in [time units]; and

$\tau^{\text{transfer}}$  minimum transferring time between vehicles in [time units].

At each of the shared transit corridor stops, holding time aims at providing the minimum cost for the passengers. Therefore, the decision to hold for regularity (Equation 1) or for synchronization (Equation 3) is based on the minimum passenger cost:

**Equation 4: Passenger cost**

$$\text{Pax\_Cost} = \beta_{\text{wait}} c^{\text{wait}} + \beta_{\text{transfer}} c^{\text{transfer}} + \beta_{\text{held}} c^{\text{held}} \quad (4)$$

Passenger cost consists of all different components of passenger travel time. Waiting time cost  $c^{\text{wait}}$  is the product of the half of the headway between consecutive arrivals and the arrival rate of the passengers at the current and the downstream stops of the rolling horizon. Transfer cost  $c^{\text{transfer}}$  (if current vehicle will not be held for synchronization) is the time transferring passengers (fraction of alighting passengers given that OD data is available) have to wait until the next arrival of the desired line. Finally, the cost of held passengers  $c^{\text{held}}$  is the product of the passengers on board and the additional time of the control action, they experience. All components are weighted according to results of previous studies for a given rolling horizon. The rolling horizon for the cost of the waiting passengers is set to the number of remaining downstream common stops. We assume that passengers can perform a transfer at any common stop, given that their choice for alighting at a stop minimizes their travel time.

**Equation 5: Criterion for holding for regularity or synchronization**

$$t^{\text{hold}} = \left\{ t^{\text{hold, reg}}, t^{\text{hold, sync}} \right\} \text{ s.t. } \min \left\{ \text{Pax\_Cost}^{\text{reg}}, \text{Pax\_Cost}^{\text{sync}} \right\} \quad (5)$$

## Application and Outlook

We apply the proposed control logic on a simple artificial network with two lines that merge, operate jointly and diverge as depicted in Figure 1 within a simulation model environment. The controller is compared with the combined criterion by [12] for single line regularity and transfer synchronization. The controllers will be then evaluated based on the perceived passenger times for each passenger group, regularity and transfer indices and the frequency of the regularity and the synchronization criterion. Their performance will be tested for scenarios with different demand

distributions, including different transfer shares and demand levels. Furthermore, the stops where the passengers performed transfers at the overlapping segment will be discussed.

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