

A holding control strategy for diverging bus lines

G. Laskaris · O. Cats · E. Jenelius · M. Rinaldi · F. Viti

Abstract We introduce a holding criterion for network configurations with lines that operate jointly along a common corridor and then individually diverge. The proposed holding decision rule accounts for all different passengers groups in the overlapping segment and takes care of the transition to individual line operation. The holding rule is evaluated using simulation for different demand levels and segmentations and compared with other control schemes for a real-world network. Results show that gains in overall network performance as well as for specific passenger groups can be achieved under specific demand distributions.

Keywords: Transit line coordination · Fork network operations · Corridor management · Real time holding control

1 Introduction

Real-time control is essential for maintaining a high level of service in a transit network. Long travel times, bunching and unnecessary delays are some of the unwanted phenomena that occur daily due to the inherent variability of travel times and passenger demand. The effects of these phenomena can be limited by utilizing available Information and Communication Technology (ICT), which allows monitoring operations in real-time and reacting dynamically to tackle potential disruptions.

Depending on the source of stochasticity, operators focus on different parts of the network applying different types of control (Ibarra-Rojas et al., 2015). Considering

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control at the stop level, a stop can be skipped or the time at the bus stop can be extended beyond the minimum dwell time. This latter strategy, holding, is popular among operators and an extensively researched topic.

While holding has been thoroughly investigated for single line control, research has neglected the potential interactions between different lines due to network design. In modern urban networks there are well-defined corridors serving high demand areas. Such corridors are traversed by multiple lines to increase the frequency of the specific route segment and provide direct services with fewer transfers involved. Apart from network design theory, the first approach to improve the performance of shared transit corridors was made via tactical planning and timetable design (Ceder et al., 2001; Guihaire and Hao, 2010).

In the scientific literature as well as in practice, holding has mostly focused on regulating the operation of a single line. Lately, research on the topic has been extended to real-time control of the shared transit corridors. The study area is limited to the route segment where lines overlap. It is generally proven that cooperation between lines can improve the overall performance of the network. However, lines may have a set of individual stops (branch stops) prior of after the shared transit corridor, which are parts of the route, that have been out of the scope of research (and the resulting control policies). In addition, networks with a shared transit corridor consist of passenger groups with conflicting interests in the network: depending on their origin and destination, control decisions to regulate an individual line or jointly multiple lines will have different effects on each group.

The objective of this paper is to control the operation of bus lines that operate jointly on a shared corridor and then diverge to individually operated branches. We introduce a holding criterion for such network configurations which accounts for all different passenger groups in the overlapping segment and the transition to individual line operation. To our knowledge, this is the first work that explicitly accounts for the transition from joint to the individual operation and explores its effects to the network and to its passenger groups separately. We compare the holding criterion to single line control and we analyse the performance under different demand segmentations and levels to determine under which conditions coordinated control should be preferred over single line control. Results show under which demand segmentations, coordinated control can be effective in terms of network performance and regularity of the lines and when is recommended to control on a single line level.

The remainder of the paper is structured as follows: in Section 2 related work to this study are reported, followed by Section 3, where the methodology is presented. The experimental setup is described in Section 4 and the results are discussed in Section 5. Finally, in Section 6 conclusions are drawn.

2 Literature Review

2.1. Single Line Holding Control

According to the spatial classification of Eberlein et al. (2001), holding strategies belong to the family of station strategies, together with the stop skipping strategies. The main elements of holding control are the holding criterion and the stops where control should be applied (Cats et al., 2011). As far as the criterion is concerned, Zolfaghari et al., (2004) categorized the criteria based on the solution approach, differentiating between rule-based and optimization models. The choice of criterion depends on the characteristics of the line; the criterion may focus on reducing headway variability or minimizing passenger cost.

In the first category, vehicles in scheduled services cannot depart prior to the pre-set time (Oort et al., 2012). For frequency-based services, holding time was initially determined based on the headway between the current and its preceding vehicle. Abkowitz and Lepofsky (1990) instructed vehicles to be held until a certain threshold was reached. Fu and Yang, (2002) compared regulating the headway of a vehicle subject to its succeeding only and subject to its succeeding and preceding. They found the second strategy to be more effective, and concluded that vehicles should be held for a time between 0.6 and 0.8 times the planned headway. Daganzo (2009) formulated a dynamic holding control model based on the forward headway in order to maximize commercial speed. Xuan et al., (2011) proposed a set of holding strategies that incorporate both the headway from the preceding and the following vehicle. Bartholdi and Eisenstein, (2012) did not follow a predefined headway but let headways be self-coordinated accordingly to eliminate bunching and in large disturbances. Cats et al., (2011) combined the headway based on both the succeeding and the preceding vehicle with a term that limits the maximum allowed headway. In a simulation-based comparison, the strategy proved superior to other holding strategies.

The second category of holding strategies has as objective the minimization of passenger cost. The main component of passenger cost is the waiting time at stops. Barnett (1974) introduced a model to minimize waiting time of passengers at stops. The objective function of the passenger cost gradually included different components such as the in-vehicle delay or accounting for passengers that were denied from boarding either because of capacity constraints (Zolfaghari et al., 2004) or because of boarding limits (Delgado et al., 2009). Hickman, (2001) formulated an analytical holding model based on stochastic travel times. Zhao et al., (2003) used an agent-based approach for vehicles and stop in order to minimize passenger travel times. Sánchez-Martínez et al. (2016) included dynamic changes in running times and demand in their holding optimization model. Berrebi et al., (2015) minimised the sum of square headways to determine holding time managed to reduce the passenger waiting time.

2.2. Multiline Holding Control

Extending beyond a single line, the first category of holding rules that take into account vehicles originating from lines other than the controlled one is to regulate transfers at a single common stop. Abkowitz et al. (1987) defined four simple holding rules to regulate transfers on a single stop. Dessouky et al. (2003) introduced transferring time as a component of the total time subject to the minimization of which holding is calculated. Hadas and Ceder (2010) applied holding in order to maximize the number of direct transfers. Gavrilidou and Cats, (2018) formulated a controller which optimally calculates holding time trading off single line regularity and multiline synchronization based on the minimization of the generalized travel cost while considering different passengers information (passengers on board, crowding, capacity limitations).

Cooperation between lines on a shared transit corridor is proven to be beneficial for the operators by increasing their profit and the number of passengers served (Chen et al., 2010). Real-time control based on holding has recently been investigated. Hernández et al. (2015) applied holding control comparing different operations schemes. Argote-Cabanero et al. (2015) extended the work of Xuan et al. (2011) from single line holding control to multiline control. Fabian and Sánchez-Martínez (2017) compared schedule-based holding and headway-based holding strategies for a multi-branch light rail network, finding that holding based on the headway of the shared corridor outperforms schedule-based control.

3 Methodology

3.1 Notation

Network related

c	index for the shared transit corridor;
b	index for the branches;
cb	index for the shared transit corridor to branch variables.

Stop related

N_c	number of corridor stops;
N_{bi}	number of branch stops of line i ;
N_i	number of stops of line i ;

Time related

$t_{ijk}^{arrival}$	arrival time at stop j of trip k of line i in [time units];
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t_{ijk}^{dwell}	dwelling time at stop j of trip k of line i in [time units];
t_{ijk}^{exit}	exit (departure) time at stop j of trip k of line i in [time units];
τ_{ijk}^{riding}	scheduled riding time between stops j-1 and j in [time units];
t_{ijk}^{hold}	holding time at stop j of trip k of line i in [time units];
t_{ijk}^{wait}	waiting time at stop j of trip k of line i in [time units];
$t_{ijk}^{in\ veh}$	in vehicle time at stop j of trip k of line i in [time units]; and
t_{ijk}^{travel}	travel time at stop j of trip k of line i in [time units].

Passenger related

λ_{ijk}	arrival rate of vehicle k at stop j of line i in [passengers per hour];
q_{ijk}	passengers on board of vehicle k at stop j of line I in [passengers].

3.2 Problem formulation

Objective Function

The holding criterion is based on the minimization of the additional time passengers experience when vehicles are instructed to hold. The decision variable is holding time. The travel time is expressed as the sum of waiting time and in-vehicle time that a passenger experiences on board (Equation 1). Waiting time is perceived as a greater disturbance for passengers, therefore its effects on the total travel time are considered more crucial than the in-vehicle time. This is given by adding a weight β^{wait} for the waiting time, which can be determined based on previous works such as the work of Wardman (2004).

$$t_{ijk}^{travel} = \beta^{wait} t_{ijk}^{wait} + t_{ijk}^{inveh} \quad (1)$$

Network Configuration

We consider a network which consists of a shared corridor with consecutive common stops among different lines until the splitting stop, after which lines split and serve different sets of stops. An example of this network type is illustrated in Figure 1. On the shared transit corridor, passengers that travel to branches seek for vehicles from the line that serves their final destination. Therefore, in this network type, no transfers between lines are necessary and transferring cost is not included in the formulation of the holding criterion. Transfers can occur to more complex networks with branches before the shared transit corridor, which is a subject of future research.



Fig 1 Schematic network configuration

Assumptions

The formulation is based on the following assumptions:

- Passengers do not perform transfers in this network configuration;
- Historical data for the demand of the lines are available
- The joint headway has been decided by tactical planning; and
- AVL data are available in real time.

Passenger groups

In this network configuration, there are three passenger groups that are taken into account: 1) passengers with origin and destination on the shared transit corridor; 2) passengers with origin on the shared transit corridor and destination on a branch (on either branch); and 3) passengers with origin and destination within the branch. The arrival rates of each group with origin m and destination n are denoted respectively as $\lambda_{c_{m,n}}$, $\lambda_{b_{m,n}}$ and $\lambda_{bc_{m,n}}$. The remaining demand from stop j of a line i operating in a network similar to Figure 1 with N stops expressed in arrival rates is expressed by the following formula:

$$\sum_{m=j}^N \sum_{n=m+1}^N \lambda_{m,n} = \sum_{m=j}^N \sum_{n=m+1}^{N_b} \lambda_{b_{m,n}} + \sum_{m=j}^{N_b} \sum_{n=m+1}^{N_c} \lambda_{bc_{m,n}} + \sum_{m=j}^{N_c} \sum_{n=m+1}^{N_c} \lambda_{c_{m,n}} \quad (2)$$

For the sake of simplicity, let:

$$\begin{aligned} \sum_{m=j}^N \sum_{n=m+1}^N \lambda_{m,n} &= \Lambda_j \\ \sum_{m=j}^{N_b} \sum_{n=m+1}^{N_b} \lambda_{b_{m,n}} &= \Lambda_{b_j} \\ \sum_{m=j}^{N_b} \sum_{n=m+1}^{N_c} \lambda_{bc_{m,n}} &= \Lambda_{bc_j} \\ \sum_{m=j}^{N_c} \sum_{n=m+1}^{N_c} \lambda_{c_{m,n}} &= \Lambda_{c_j} \end{aligned} \quad (3)$$

Where Λ expresses the sum of the arrival rates from a stop j until the end of the line and consists of all subgroups of the demand from the current stop until the end of the line. Given that, Equation 2 can be written as:

$$\Lambda_j = \Lambda c_j + \Lambda b c_j + \Lambda b_j \quad (4)$$

Waiting time

The number of passengers waiting at a given stop is estimated through the sum of the arrival rates generated at the stop multiplied by the actual headway. Passenger waiting time is assumed to be half the actual headway multiplied by the sum of arrival rates generated at the current stop. When a control action is triggered, passenger waiting time differs from the corresponding uncontrolled case. We calculate the passenger waiting time due to holding as the difference between waiting time with and without holding applied:

$$t_{ijk}^{wait} = t_{ijk}^{wait_H} - t_{ijk}^{wait_0} \quad (5)$$

We consider waiting time from the preceding vehicle p and the succeeding vehicle s. The waiting time from the succeeding and the preceding vehicle when no holding is applied are shown in the following formulas:

$$t_{ijk}^{wait_pH} = \frac{(t_{ijk}^{exit} - t_{ijk-1}^{exit})^2}{2} \Lambda_j \quad (6)$$

$$t_{ijk}^{wait_s0} = \frac{(t_{ijk+1}^{exit} - t_{ijk}^{exit})^2}{2} \Lambda_j \quad (7)$$

And the total waiting time without holding will be:

$$t_{ijk}^{wait_0} = t_{ijk}^{wait_p0} + t_{ijk}^{wait_s0} \quad (8)$$

Additionally, when a vehicle is instructed to hold then the waiting time from the preceding and the succeeding vehicles are expressed by Equations (9) and (10):

$$t_{ijk}^{wait_pH} = \frac{((t_{ijk}^{exit} + t_{ijk}^{hold}) - t_{ijk-1}^{exit})^2}{2} \Lambda_j \quad (9)$$

$$t_{ijk}^{wait_sH} = \frac{(t_{ijk+1}^{exit} - (t_{ijk}^{exit} + t_{ijk}^{hold}))^2}{2} \Lambda_j \quad (10)$$

By combining Equation (9) and (10), we get the total waiting time with holding time:

$$t_{ijk}^{wait_H} = t_{ijk}^{wait_pH} + t_{ijk}^{wait_sH} \quad (11)$$

Based on Equation (5), the difference in waiting time is expressed as a function of holding time:

$$t_{ijk}^{wait} (t_{ijk}^{hold}) = \Lambda_j (t_{ijk}^{hold})^2 + \left\{ \Lambda_j \left[(t_{ijk}^{exit} - t_{ijk-1}^{exit}) - (t_{ijk+1}^{exit} - t_{ijk}^{exit}) \right] \right\} t_{ijk}^{hold} \quad (12)$$

We consider two different waiting terms at each stop. The first term takes into account all vehicles that serve the current stop and the second only of the vehicles of the same line with the current vehicle. In this network configuration, on the shared transit corridor two passenger groups coexist and have different objectives: passengers travelling within the corridor can be satisfied by all lines and passengers travelling to the branches wait for a vehicle from the line that serves their final destination. Thus, the first group is affected by regularizing the joint headway on the corridor while the second by the headway of the desired line. For that reason, we introduce two terms derived from the waiting time term, each depending on the aforementioned headway. The first term calculates the passenger waiting time regardless of the line, while the second is subject to vehicles from the same line i with the current vehicle. The two terms are given in Equations (13) and (14) respectively:

$$t_{jk}^{\text{wait}}(t_{ijk}^{\text{hold}}) = \Lambda c_j (t_{ijk}^{\text{hold}})^2 + \left\{ \Lambda c_j \left[(t_{ijk}^{\text{exit}} - t_{jk-1}^{\text{exit}}) - (t_{jk+1}^{\text{exit}} - t_{ijk}^{\text{exit}}) \right] \right\} t_{ijk}^{\text{hold}} \quad (13)$$

$$t_{ijk}^{\text{wait}}(t_{ijk}^{\text{hold}}) = \Lambda c b_j (t_{ijk}^{\text{hold}})^2 + \left\{ \Lambda c b_j \left[(t_{ijk}^{\text{exit}} - t_{ijk-1}^{\text{exit}}) - (t_{ijk+1}^{\text{exit}} - t_{ijk}^{\text{exit}}) \right] \right\} t_{ijk}^{\text{hold}} \quad (14)$$

Projection to the final common stop

Similarly, we regulate the expected departure from the last common stop, where the transition to the individual operation is made. The arrival of all vehicles of the same line is projected to the final common stop. Projection to the last common stop is done by summing the scheduled riding times between the last recorded stop that a vehicle has visited with the departure time from this stop, as formulated in Equation (15). After vehicle trajectories are projected and the preceding and the succeeding vehicles with respect to the current one are determined, the expected departure time from the last common stop is regulated based on the expected waiting times at the splitting stop. The passengers that are affected by this term are those travelling on the branch, expressed by Λb_i served by line i . The expected waiting time at the splitting stop expressed as a function of holding time is given by Equation (16).

$$\tilde{t}_{i,j^{\text{split}},k}^{\text{exit}} = t_{ijk}^{\text{arrival}} + t_{ijk}^{\text{dwell}} + \sum_{l=j}^{j^{\text{split}}} \tau_{l,l+1}^{\text{riding}} \quad (15)$$

$$\tilde{t}_{i,j^{\text{split}},k}^{\text{wait}}(t_{ijk}^{\text{hold}}) = \Lambda b_j (t_{ijk}^{\text{hold}})^2 + \left\{ \Lambda b_j \left[(\tilde{t}_{i,j^{\text{split}},k}^{\text{exit}} - \tilde{t}_{i,j^{\text{split}},k-1}^{\text{exit}}) - (\tilde{t}_{i,j^{\text{split}},k+1}^{\text{exit}} - \tilde{t}_{i,j^{\text{split}},k}^{\text{exit}}) \right] \right\} t_{ijk}^{\text{hold}} \quad (16)$$

In-vehicle time

In-vehicle time due to holding is the product of holding time and the number of passengers on board:

$$t_{ijk}^{\text{inveh}} = q_{ijk} t_{ijk}^{\text{hold}} \quad (17)$$

Total travel time

Total travel time consists of all three components of waiting time for the different passenger groups and the in-vehicle time:

$$\begin{aligned}
t_{ijk}^{travel}(t_{ijk}^{hold}) &= \beta^{wait} t_{ijk}^{wait}(t_{ijk}^{hold}) + t_{ijk}^{inveh}(t_{ijk}^{hold}) = \\
&\beta^{wait} (\Lambda c_j + \Lambda cb_j + \Lambda b_j) (t_{ijk}^{hold})^2 + \\
&t_{ijk}^{hold} \beta^{wait} \left\{ \Lambda c_j \left[(t_{ijk}^{exit} - t_{jk-1}^{exit}) - (t_{jk+1}^{exit} - t_{ijk}^{exit}) \right] \right. \\
&+ \Lambda cb_j \left[(t_{ijk}^{exit} - t_{jk-1}^{exit}) - (t_{jk+1}^{exit} - t_{ijk}^{exit}) \right] + \Lambda b_j \left[(\tilde{t}_{i,j^{split},k}^{exit} - \tilde{t}_{i,j^{split},k-1}^{exit}) - (\tilde{t}_{i,j^{split},k+1}^{exit} - \tilde{t}_{i,j^{split},k}^{exit}) \right] \left. \right\} + \\
&q_{ijk}^{hold} t_{ijk}^{hold} \\
&= \beta^{wait} \Lambda_j (t_{ijk}^{hold})^2 + \\
&t_{ijk}^{hold} \beta^{wait} \left\{ \Lambda c_j \left[(t_{ijk}^{exit} - t_{jk-1}^{exit}) - (t_{jk+1}^{exit} - t_{ijk}^{exit}) \right] + \Lambda cb_j \left[(t_{ijk}^{exit} - t_{jk-1}^{exit}) - (t_{jk+1}^{exit} - t_{ijk}^{exit}) \right] + \right. \\
&\left. \Lambda b_j \left[(\tilde{t}_{i,j^{split},k}^{exit} - \tilde{t}_{i,j^{split},k-1}^{exit}) - (\tilde{t}_{i,j^{split},k+1}^{exit} - \tilde{t}_{i,j^{split},k}^{exit}) \right] \right\} + q_{ijk}^{hold} t_{ijk}^{hold}
\end{aligned} \tag{18}$$

The optimal holding time is then calculated by taking the first derivative subject to holding time and setting it equal to zero, and solving the resulting equation with respect to holding time t_{ijk}^{hold} with the constraint that $t_{ijk}^{hold} \geq 0$:

$$\begin{aligned}
t_{ijk}^{hold} &= \frac{\Lambda c_j \left[(t_{ijk}^{exit} - t_{jk-1}^{exit}) - (t_{jk+1}^{exit} - t_{ijk}^{exit}) \right]}{\Lambda_j} + \frac{\Lambda cb_j \left[(t_{ijk}^{exit} - t_{jk-1}^{exit}) - (t_{jk+1}^{exit} - t_{ijk}^{exit}) \right]}{\Lambda_j} + \\
&+ \frac{\Lambda b_j \left[(\tilde{t}_{i,j^{split},k}^{exit} - \tilde{t}_{i,j^{split},k-1}^{exit}) - (\tilde{t}_{i,j^{split},k+1}^{exit} - \tilde{t}_{i,j^{split},k}^{exit}) \right]}{\Lambda_j} - \frac{q_{ijk}}{2\beta^{wait}\Lambda_j}
\end{aligned} \tag{19}$$

The first two terms regulate the departure from the current stop: the first one considers all vehicles that interact in the shared transit corridor, regardless of the line they serve, while the second regulates the departure subject to departures of consecutive vehicles of the same line i with the current vehicle. The third term regulates the expected departures at line level from the splitting stop, to ensure that the lines will continue to their branch stops with low headway variability. For the third term, the expected departure time from the splitting stop j^{split} is estimated by summing the scheduled riding times between the current stop of each vehicle and the splitting stop. Finally, the holding time calculated is adjusted to the ratio of the passengers on board and the remaining passengers downstream expressed by the corresponding arrival rates.

Weights

As shown in Equation (19), the contribution of each term is weighted based on the demand. We also introduce a weighting factor based on the distance to ensure a smoother transition from joint operation to single line operation. The distance term is based on the current stop's distance from the last common stop, j^{split} . The first two weights regulate the headways of vehicles within the corridor, therefore they share the same distance weight multiplied by a parameter $\alpha=0.5$, ensuring that the two terms are equally important when calculating holding time. The first term affects the

passengers travelling within the corridor that are indifferent towards the different lines; the second affects the passengers travelling from the shared transit corridor to a specific branch and therefore wait for a specific line that serves their final destination. Finally, the passengers after the shared transit corridor waiting only for a specific line are included.

$$\begin{aligned}\theta_1 &= \frac{\Lambda_j^c}{\Lambda_j} + (\alpha) \left(1 - \frac{1}{j^{\text{split}} - j} \right) \\ \theta_2 &= \frac{\Lambda_j^{\text{cb}}}{\Lambda_j} + (1-\alpha) \left(1 - \frac{1}{j^{\text{split}} - j} \right) \\ \theta_3 &= \frac{\Lambda_j^b}{\Lambda_j} + \left(\frac{1}{j^{\text{split}} - j} \right)\end{aligned}\quad (20)$$

The final holding criterion on the shared corridor is given in Equation (21):

$$\begin{aligned}t_{ijk}^{\text{hold}} = \max \left\{ \theta_1 \frac{[(t_{ijk+1}^{\text{exit}} - t_{ijk}^{\text{exit}}) - (t_{ijk}^{\text{exit}} - t_{ijk-1}^{\text{exit}})]}{2} + \theta_2 \frac{[(t_{ijk+1}^{\text{exit}} - t_{ijk}^{\text{exit}}) - (t_{ijk}^{\text{exit}} - t_{ijk-1}^{\text{exit}})]}{2} \right. \\ \left. + \theta_3 \frac{[(\tilde{t}_{i,j^{\text{split}},k}^{\text{exit}} - \tilde{t}_{i,j^{\text{split}},k-1}^{\text{exit}}) - (\tilde{t}_{i,j^{\text{split}},k+1}^{\text{exit}} - \tilde{t}_{i,j^{\text{split}},k}^{\text{exit}})]}{2} - \frac{q_{ijk}}{2\beta^{\text{wait}} \Lambda_j}, 0 \right\}\end{aligned}\quad (21)$$

Diverging Branch criterion

After exiting the shared transit corridor, a single line criterion is used to maintain control on each branch, derived from the shared corridor holding criterion in Equation (21) considering neither the remaining demand downstream nor the existence of a further downstream splitting stop. The single line criterion was introduced by (Laskaris et al. 2016) and for every stop j which belongs to the branch ($j > j^{\text{split}}$) is given by Equation (22):

$$t_{ijk}^{\text{hold}} = \max \left\{ \frac{[(t_{ijk+1}^{\text{exit}} - t_{ijk}^{\text{exit}}) - (t_{ijk}^{\text{exit}} - t_{ijk-1}^{\text{exit}})]}{2} - \frac{q_{ijk}}{2\beta^{\text{wait}} \Lambda_j}, 0 \right\}\quad (22)$$

4 Case Study

4.1. Study area

The routes of lines 176 and 177 of the bus network of the city of Stockholm, Sweden, are structured in fork network configuration, consisting of a common set of stops and two branches, each one served by one of two lines (Figure 2). The lines operate between Mörby and the Ekerö community, to Solbacka and Skärvik. There are 24 common stops all located in the district of Solna, providing connections with the commuter train, light rail and subway. Line 176 has 19 branch stops and line 177 has 12 branch stops. Because of their layout, they provide an ideal ground to evaluate the

proposed holding rule for the westbound direction. The frequency of the lines is set to 10 min with a joint frequency of 5 min at the shared transit corridor and the vehicles depart alternately from the common terminal at Mörby.

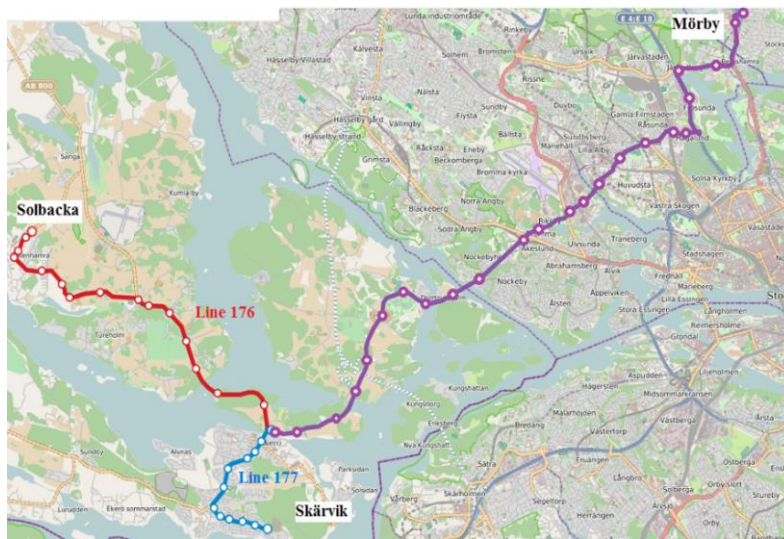


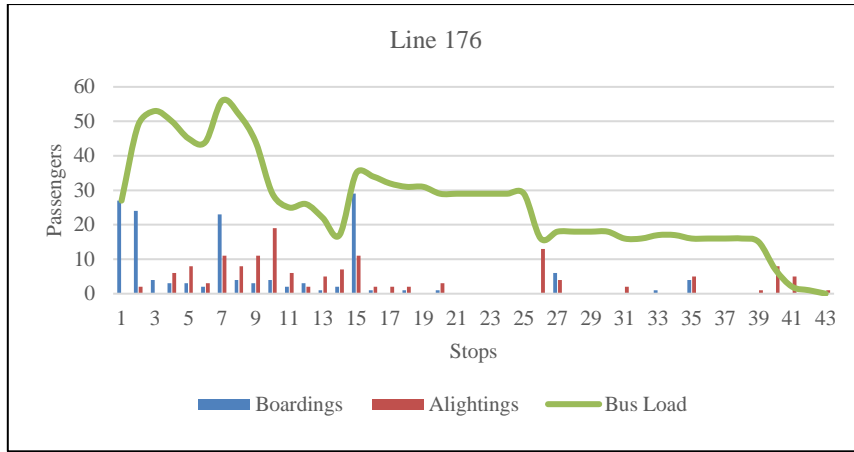
Fig 2 Lines 176 and 177 in Stockholm, Sweden

The demand of lines 176 and 177 are depicted in Figure 3 and the segmentation of the demand is given in Table 1.

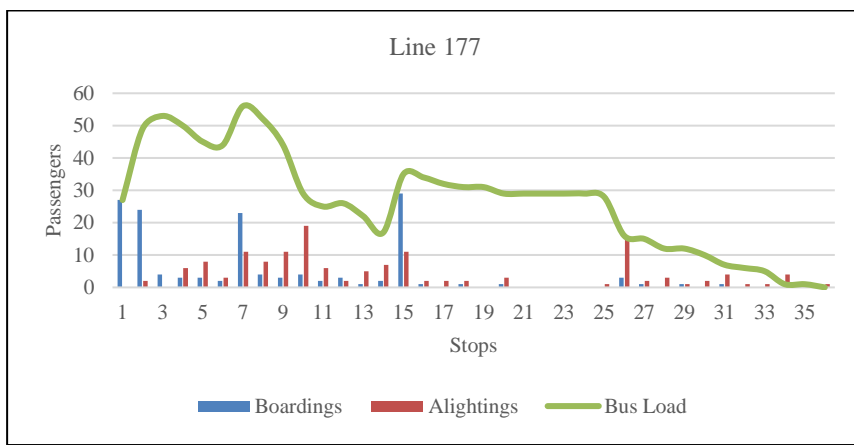
Table 1 Demand Segmentation for Lines 176 and 177

	Line 176		Line 177	
	Passenger s	% demand	Passenger s	% demand
Total Demand	148	100	143	100
Demand Generated on Corridor	137	92.57	137	95.8
Demand Generated on Branch	11	7.43	6	4.2
Demand within Corridor	108	72.97	108	75.52
Demand Corridor to Branch	29	19.59	29	20.28
Demand within Branch	11	7.43	6	4.2

It can be observed that the majority of the demand travels within the common part, followed by the passengers that travel from the shared transit corridor to the branch and the smallest share of the demand of each line travels within the branch stops.



(a)



(b)

Fig 3 Demand profiles of lines 176 (a) and 177 (b) (Eastbound)

4.2. Scenarios

The newly developed holding criterion is compared with a no-control scheme and a single line control strategy, dubbed Even Headway. The latter strategy regulates the departure of a vehicle from a stop based on consecutive departures of the vehicles from the same stops and at the same time limits the maximum time a vehicle can be held by a specific share of the planned headway of the line. The selected single line strategy has proven to be the most effective compared to other holding strategies in previous studies (Cats et al., 2011, 2012). Moreover, the effect of the demand segmentation to the criterion is explored by increasing the demand travelling from the shared transit corridor to the branch and the demand within the branch in two additional scenarios. The alternative segmentations are given in Table 2. A portion of

the demand travelling within the shared transit corridor is moved to the passengers travelling from corridor to branch or within branch. Finally, the scenarios are tested for two levels of demand, the base demand and an increased demand by 50%.

Table 2 Alternative demand segmentation

Demand Segmentation	Share of Demand		
	Passengers travelling within the corridor	Passengers travelling from corridor to branch	Passengers travelling within the branch
1	73%	20%	7%
2	66%	27%	7%
3	66%	20%	14%

For the remainder of this work, the scenarios will be denoted by CS_x_y. CS stands for the control scheme used in each scenario, varying between NC for no control, EH for Even Headway (single line strategy) and CPC for the Cooperative Passenger Cost strategy. Index x is for the different demand segmentations as reported in Table 2 and index y for the demand level with 100 for the base demand and 150 for the increased demand.

Simulation Tool: Busmezzo is a transit simulator embedded in the mesoscopic traffic simulator Mezzo (Burghout et al., 2005; Toledo et al., 2010). Busmezzo has been used to simulate bus operation and different holding strategies. Passenger behaviour in the network can be evaluated individually since the simulator treats passengers as agents. Simulation includes a warm-up period, where passengers start to be generated after there are vehicles within the network and a cool-down period until the end of the simulation (Cats and Hartl, 2016). Due to the stochastic nature of the simulator, each scenario is repeated for a sufficient number of replications to ensure a low statistical error. Passenger travel time is chosen as the reference measurement with a desired standard deviation of 1.5%. The number of replications needed is given by the following formula.

$$N' \geq t_{\frac{\alpha}{2}, N-1}^2 \frac{X_s^2}{X_d^2}$$

where,

- N' sample size;
- $t_{\frac{\alpha}{2}, N-1}^2$ student -t value for reliability α and a sample N;
- X_d standard deviation of the chosen indicator for the sample N;
- X_s accepted standard deviation.

For a statistical error of 5% for 25 replications the value from t-student distribution is 2.06389857. The minimum number of replications is 21, making 25 a sufficient number of replications.

5 Results

5.1. Shared transit corridor

The results of the performance of each control scheme per scenario are summarized in Table 3.

Table 3 Performance Indicators of the Shared Transit Corridor

Shared Transit Corridor		CV of Headway	Level of Bunching	Waiting time per passenger [sec]	In vehicle time per passenger [sec]	Weighted travel time per passenger [sec]
1_100	NC	0.507	0.397	155.4	194.3	505.2
	EH	0.421	0.307	153.6	196.1	503.3
	CPC	0.391	0.283	151.7	194.7	498.2
1_150	NC	0.512	0.378	150.5	214.1	515.1
	EH	0.468	0.341	149.1	215.8	513.9
	CPC	0.424	0.315	148.9	214.4	512.1
2_100	NC	0.410	0.306	156.1	186.6	498.7
	EH	0.320	0.205	150.8	188.3	489.8
	CPC	0.352	0.245	153.0	187.1	493.1
2_150	NC	0.521	0.399	153.2	198.6	505.0
	EH	0.391	0.281	150.4	199.0	499.9
	CPC	0.471	0.347	151.4	200.8	503.6
3_100	NC	0.398	0.279	153.9	185.1	492.9
	EH	0.325	0.219	152.8	187.0	492.5
	CPC	0.312	0.205	152.1	186.2	490.5
3_150	NC	0.513	0.388	155.1	195.8	506.1
	EH	0.409	0.291	153.1	197.4	503.6
	CPC	0.397	0.279	152.8	196.8	502.4

It is clear from the results that demand distribution affects the performance on the shared transit corridor. With CPC, the holding criterion gradually starts regulating the operation of a single line on the shared transit corridor. When the demand share of the passengers that travel to the branch increases compared to that of the corridor demand, the results of CPC are comparable to a single line strategy. Analytically, for the actual demand profile of the line, CPC outperforms EH in both regularity

indicators for both demand levels. However, when the demand increases at the branch (third demand segmentation scenario) the results of both control strategies are at the same level. CPC is shown to have poor performance when the share of passengers that are generated in the shared transit corridor and travel to the branch is dominant. In such scenarios, CPC is required to switch from regulating joint operation to single line too abruptly, reducing its performance on this network part and making the single line control strategy more efficient, since it does not switch objectives along the route. This behaviour is also reflected in the travel times per passenger.

5.2. Line Results

Applying single line control is obviously more effective at line level compared to cooperative control. As expected, the results summarized in Table 4 show how for both lines, EH outperforms CPC at every scenario.

Table 4 Performance Indicators at line level

		Line 176					Line 177				
		CV of Headway	Bunching	Waiting time per passenger	In vehicle time per passenger	Weighted Travel Time	CV of Headway	Bunching	Waiting time per passenger	In vehicle time per passenger	Weighted Travel Time
1_100	NC	0.27	0.10	314.7	148.0	777.4	0.33	0.14	320.9	157.1	799.0
	EH	0.17	0.01	310.8	148.8	770.4	0.17	0.01	312.5	158.6	783.6
	CPC	0.21	0.03	311.1	148.3	770.5	0.21	0.03	314.4	158.0	786.7
1_150	NC	0.38	0.21	320.5	160.6	801.5	0.40	0.25	333.3	172.0	838.6
	EH	0.20	0.03	314.2	161.5	789.8	0.24	0.07	321.3	173.4	816.0
	CPC	0.25	0.07	314.9	160.8	790.6	0.34	0.16	326.5	173.5	826.6
2_100	NC	0.26	0.09	312.5	143.8	768.8	0.25	0.10	316.8	152.4	785.9
	EH	0.14	0.00	306.4	145.0	757.7	0.13	0.00	307.6	153.3	768.4
	CPC	0.19	0.02	309.9	144.0	763.8	0.21	0.03	312.2	153.5	777.9
2_150	NC	0.32	0.15	313.2	151.9	778.2	0.34	0.18	322.0	162.6	806.5
	EH	0.18	0.01	311.9	154.0	777.8	0.19	0.02	311.3	163.4	785.9
	CPC	0.25	0.08	314.0	153.1	781.1	0.27	0.09	316.5	163.0	796.0
3_100	NC	0.26	0.08	317.5	143.7	778.6	0.21	0.04	309.9	151.9	771.6
	EH	0.14	0.00	306.2	145.0	757.3	0.13	0.00	307.5	153.0	767.9
	CPC	0.15	0.01	305.8	143.9	755.5	0.16	0.01	311.6	152.9	776.2
3_150	NC	0.38	0.21	328.8	151.8	809.5	0.35	0.20	328.8	162.3	819.9
	EH	0.18	0.01	311.0	152.7	774.7	0.20	0.03	314.4	163.5	792.3
	CPC	0.25	0.05	314.9	152.5	782.3	0.25	0.07	318.9	163.1	800.8

The results with applied control are better than any scenario without control. It is observed that CPC has marginal differences with EH in the case of increased branch demand for line 176. The most significant differences between controlled scenarios are observed for increased corridor to branch demand, where the poorest performance of CPC compared to EH is observed. It is also noted that line 177 does not benefit from CPC with marginal gains compared to NC especially at the second demand segmentation.

Comparing the progression of the variability of headway along the stops of the lines, the effect of the demand can be clearly observed. Applying a single line strategy has better results in regulating the headways of the line, since this is its main objective, compared to the cooperative holding strategy, which includes also the regularization of the joint headway at the shared transit corridor. CPC's performance is comparable to the performance of EH for the first and the third demand segmentation for both lines. There is a transition period for the holding criterion from the shared transit corridor criterion to the single line criterion. This can be observed at the demand level 150 scenario set, where with CPC headway variation increases at the end of the set of common stops and for the first stops of the branch until it changes to regularization of the single line at the last stops of the route. This benefits the longer line, which manages to adjust. On the contrary, line 177 has to make the transition faster without having enough time to recover the performance loss resulting from the transition itself.

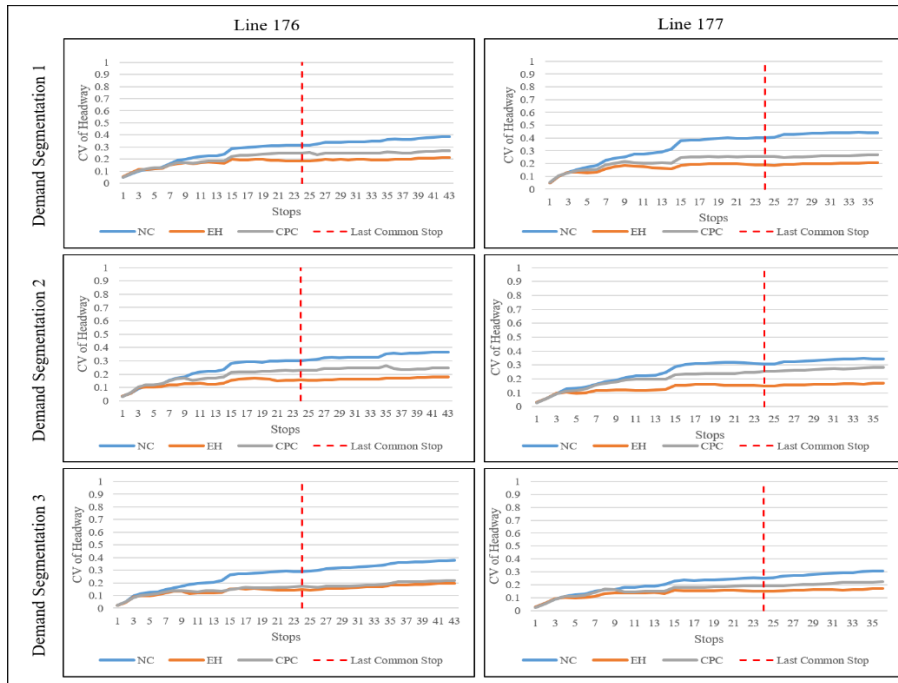


Fig 4 CV of headway per stop for the two lines for demand level 1

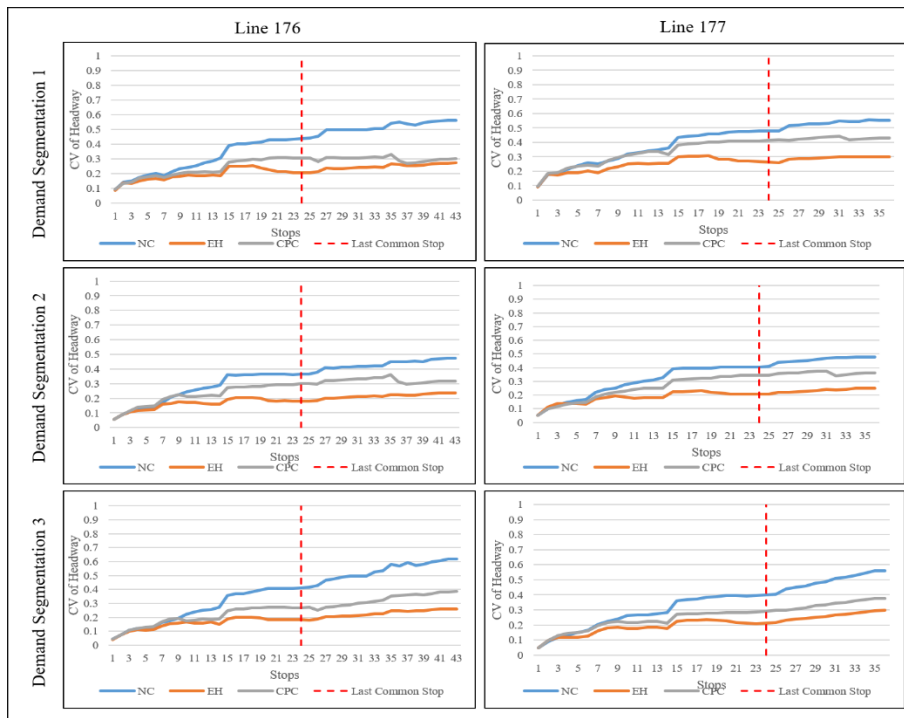


Fig 5 CV of headway per stop for the two lines for demand level 2

5.3. Travel Times

The 90th percentile of travel times and its variability is used as a measure of performance of the controller. The histograms of travel times for the actual demand and the peak demand for all passenger segmentations are depicted in Figure 6 and Figure 7 respectively. It is noted from the results that CPC has less variable travel times for the actual demand profile and it yields better performances for the longer line (line 176). With CPC, travel times of line 176, in the actual demand scenario provide a less variable travel time distribution with also shorter travel time on average. Under standard demand conditions (100), CPC outperforms EH. When demand increases (scenario 150), variability affects all control scenarios (NC, EH, CPC), and EH becomes the better alternative. Based on the results, vehicle scheduling with CPC has no high fleet requirements at demand level 100 due to low variability in travel time. However, this is not the case for the scenario set with demand level 150, where variability increases especially for line 177 and may require additional vehicles to be dispatched to serve the line.

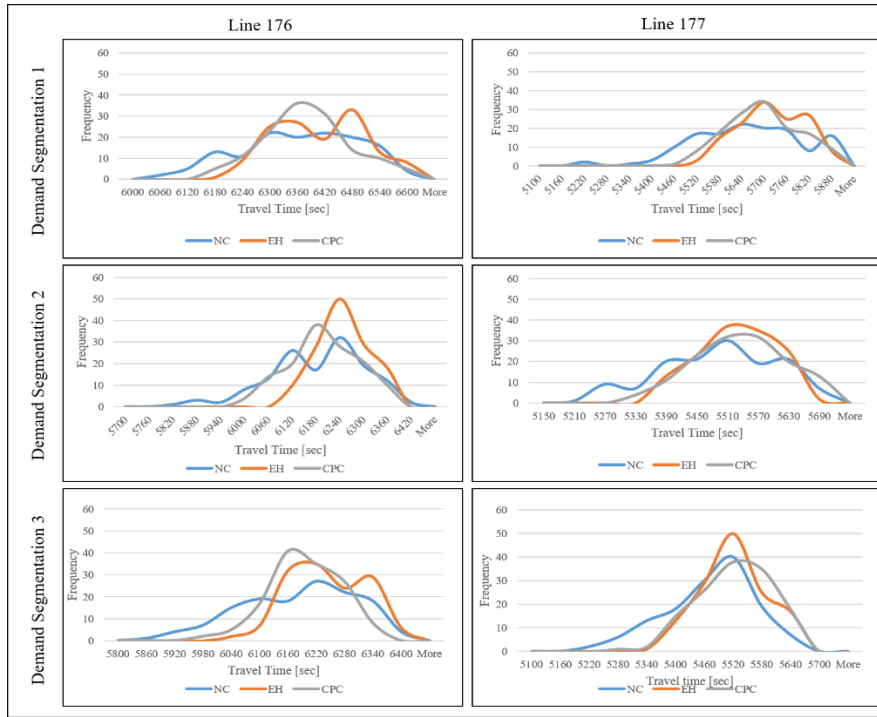


Fig 6 Travel time distribution for lines 176 and 177 for the actual demand scenarios



Fig 7 Travel time distribution for lines 176 and 177 for the increased demand scenarios

5.4. Passenger Travel Times

Network Performance: Figure 8 shows the differences in passenger travel time compared to no control with each strategy for the two components of the passenger cost and as a sum of the weighted travel time. For the actual demand distribution at the actual demand level CPC performs better mostly because of the greater reduction in in-vehicle time. The performance is marginally better in the increased demand scenario. With EH, higher reductions are achieved in waiting times in the majority of the scenarios, while CPC reductions are observed in in-vehicle time. For the high demand level scenario, there are substantial gains compared to EH for demand segmentation 1 and marginal gains for demand segmentation 3.

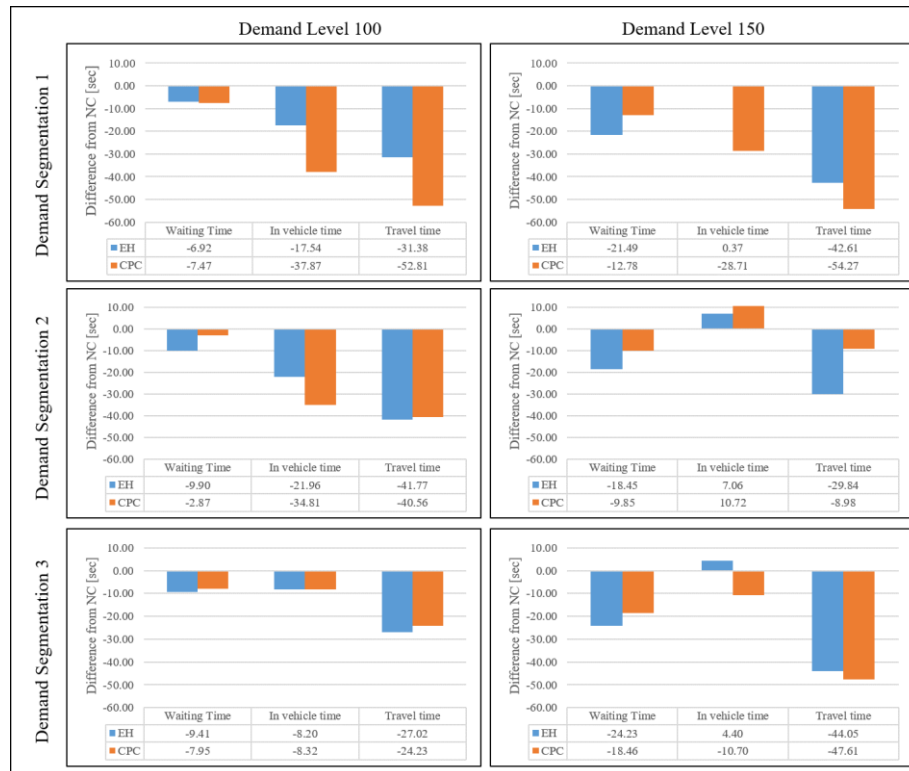


Fig 8 Network performance with control compared to NC

Passenger Cost per Passenger Group: We also explore the direct effect of each control scheme on each passenger group for the two demand levels. The difference in performance per passenger group for the two lines is shown in Table 5. For demand segmentation 1 in both demand levels examined, CPC manages to have greater reductions to travel time for the passengers travelling within the corridor and for the passengers travelling from the corridor to the branch. The same applies for demand segmentation 3, with the increased branch demand, where better performance is also observed in passenger group that travels within the branch. With demand

segmentation 2, single line control benefits more all passengers groups with CPC to be marginally better in some cases. It should be noted that line 176 is benefited more from the application of CPC than line 177.

6. Conclusions

We introduce a (multiline) holding criterion for diverging fork networks based on the minimization of the additional passenger travel times due to holding. The criterion regularise the joint headway and the line headway at the shared transit corridor, while also regulates the expected departure from the last common stop accounting for all different passenger groups and adjusting holding time to the number of passengers that experience the control action. We evaluated the criterion using simulation on a case study of two lines of city of Stockholm, Sweden under different demand levels and compositions.

The proposed holding criterion can regulate the operation of the network and result to higher gains than single line control under certain demand distributions. Demand distribution has a significant effect on the holding criterion. When the majority of the demand is on the passengers groups that do not interact CPC can have marginal gains or outperform single line holding. On the other hand, a high number of traversing passengers reduces the effectiveness of the criterion, in which case single line control is recommended.

Furthermore, a transition period for the criterion to shift from joint control and single line control which yields a loss of performance around the last common stop. If the line does not have a considerably long branch there is no time to mitigate the effect and ends up with poorer performance.

Further research will involve additional tests on different demand profiles and route lengths. Moreover, the criterion will be tested in networks with branches prior to the shared transit corridor, where vehicles initiate their joint operation already with propagated variability. Finally at the shared transit corridor, a transfer criterion will be included in order to allow synchronization over regularity based on the difference of passenger cost of the two criteria.

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Table 5 Performance difference between NC and EH, CPC for the different passenger groups for the actual demand scenario

		Line 176									Line 177								
		Branch			Corridor to Branch			Corridor			Branch			Corridor to Branch			Corridor		
		Waiting Time %	In vehicle time %	Travel Time %	Waiting Time %	In vehicle time %	Travel Time %	Waiting Time %	In vehicle time %	Travel Time %	Waiting Time %	In vehicle time %	Travel Time %	Waiting Time %	In vehicle time %	Travel Time %	Waiting Time %	In vehicle time %	Travel Time %
Demand Level 100																			
1_100	EH	-9.3	-0.8	-4.8	-4.7	1.0	-0.4	-5.6	1.2	-0.2	-7.9	1.3	-4.9	-8.5	0.2	-2.5	-0.8	0.7	0.4
	CPC	-4.8	-1.2	-2.9	-4.2	-0.3	-1.3	-5.9	0.0	-1.2	-10.8	3.8	-6.0	-10.1	-0.1	-3.2	-1.8	-0.6	-0.8
2_100	EH	-6.1	-1.9	-4.0	-2.2	1.1	0.5	-3.1	-1.4	-1.8	-11.5	-0.2	-7.0	-6.4	-0.3	-1.6	-5.5	0.6	-0.7
	CPC	-3.1	-3.8	-3.5	-1.7	0.1	-0.3	-0.6	-0.3	-0.4	-8.2	-1.2	-5.4	-2.2	-0.5	-0.9	-3.2	-0.3	-1.0
3_100	EH	-6.5	3.2	-2.8	-5.0	1.1	-0.3	-3.4	-0.5	-1.1	-3.4	0.0	-2.6	-0.8	1.1	0.6	-5.0	1.2	-0.2
	CPC	-4.5	-0.9	-3.1	-3.8	-0.7	-1.4	-4.7	0.5	-0.7	-1.3	4.9	0.2	-0.8	-0.5	-0.6	-3.1	-0.4	-1.0
Demand Level 150																			
1_150	EH	-11.6	-1.4	-6.3	-12.4	0.2	-2.8	-1.3	0.5	-0.7	-14.5	0.4	-9.8	-10.0	0.1	-3.1	-4.8	0.1	-1.1
	CPC	-8.7	-0.7	-4.5	-10.1	-0.2	-3.1	-3.8	-0.5	-1.3	-6.0	3.5	-3.0	-9.8	-1.3	-4.0	-5.2	-1.5	-2.5
2_150	EH	-0.4	4.4	2.0	-6.2	1.1	-0.3	-3.8	-0.2	-1.0	-14.1	0.0	-8.5	-9.2	0.3	-1.7	-9.8	1.3	-1.2
	CPC	-0.9	3.0	1.1	-5.4	0.3	-0.9	-1.5	0.6	0.1	-3.9	2.0	-1.5	-3.5	-0.1	-0.8	-5.2	1.2	-0.3
3_150	EH	-19.2	-3.2	-13.3	-4.8	0.3	-0.8	-3.3	-0.4	-1.0	-13.3	-2.5	-10.7	-6.3	-0.7	-2.0	-5.2	-0.3	-1.4
	CPC	-14.7	-2.6	-10.2	-4.0	-0.3	-1.1	-1.2	-2.2	-2.0	-10.1	-0.5	-7.8	-3.6	-0.8	-1.4	-5.1	-0.1	-1.3

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