ABCDE - Short Summer Tutorial

Default Entailment Course

Emil Weydert

University of Luxembourg, CSC/ICR

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In a nutshell

Human-level AI requires justifiable commonsense reasoning

$\rightarrow$ in particular: need for formal, normative accounts of

- **Defaults:** implications with exceptions
- **Default inference:** plausible reasoning with defaults

From A and *if A then normally B* plausibly infer B

NMR-Tutorial:

- Selected topics/lessons from 40 years of DR research
- Focus on theoretical/semantic issues

Contents

Abstract logic

A 2-valued logic $\mathcal{L} = (L, \vdash)$ is characterized by

- a language (type) L together with
- an inference relation $\vdash \subseteq 2^L \times L$, or
  an inference operator $C : 2^L \to 2^L$ with $C(\Phi) = \{\psi \mid \Phi \vdash \psi\}$

Classical: propositional/1st/2nd-order, modal/conditional, ...

Alternative: intuitionistic/constructive, resource-bounded, ...

Generalized: inductive/abductive, paraconsistent/ampliative, ...

Classical inference

In classical logic two standard ways to specify an inference rel. $\vdash$

- *Syntactic, proof-theoretic*: $\Phi \vdash_{\mathcal{R}} \psi$ ($\mathcal{R}$ rule base) iff there is an $\mathcal{R}$-derivation of ψ from Φ
- *Semantic, model-theoretic*: $\Phi \Vdash \psi$ iff every model satisfying Φ also satisfies ψ

Classical task: For a given semantic entailment $\Vdash$,

find a semi-decidable (ideally decidable) $\vdash_{\mathcal{R}} = \Vdash$

Tarskian inference

$\vdash_{\mathcal{R}}, \Vdash$ are Tarskian inference relations (finitary):

- **Inclusion:** $\Phi \vdash \psi$ for each $\psi \in \Phi$
- **Cut:** If $\Phi \vdash \varphi_1, ..., \varphi_n$, $\Phi \cup \{\vec{\varphi}\} \vdash \psi$, then $\Phi \vdash \psi$
- **Monotony:** If $\Phi \vdash \psi$, then $\Phi \cup \Psi \vdash \psi$

Default inference is not Tarskian: **Incl+Cut** ok, never **Mon**

Notation:

- Tarskian inference: $\vdash$ and Cn
- Generalized inference: $\mid\!\sim$ and C or $C_{\mid\sim}$

Nonmonotonic reasoning I

Real-world agents: must deal with incomplete, uncertain, erroneous, inconsistent, changing, and intractable info
$\rightarrow$ need for plausible guesses, withdrawable given new evidence/assumptions
$\rightarrow$ need for nonmonotonic reasoning: exploiting rules of thumb, heuristics, implicit assumptions, meta-level/self-reflective considerations ...

Goal: Enrich classical, monotonic core logic with reasonable formal accounts of nonmonotonic reasoning exploiting various concepts of rationality

Remind: Most reasoning outside of math is (also) nonmonotonic!

Nonmonotonic reasoning II

Nonmonotonic reasoning in practice:

- Inductive (prior/model/parameter choice, direct inference)
- Legal (norms ordered by recency, specificity, authority)
- Commonsense (cognitive heuristics, generics, implicatures)

To distinguish:

- Historically grounded, domain-specific reasoning conventions, e.g. in language and law
- Nonmonotonic reasoning concerned with underlying general theoretical and conceptual reasoning methods

Types of nonmonotonic reasoning

Generalized inference may violate any Tarskian principle:

- *Inconsistency repair:* **Cut**
  e.g. $\{\varphi, \neg\varphi, \psi\} \mid\!\sim_{inc} \psi$ but $\{\varphi, \neg\varphi, \psi, \neg\psi\} \not\mid\!\sim_{inc} \psi$
- *Resource-bounded reasoning:* **Incl**
- *Probabilistic threshold reasoning:* **Incl**
- *Inductive reasoning:* **Incl**
- *Default reasoning:* **Incl + Cut**

For NMR: alternative finer-grained principles (see later)

Informal defaults

Default: standard/generic assumption, overridable by more concrete information

e.g. *default/prototypical values in data bases, prima facie assumptions, legal conventions (presumption of innocence), implications/rules with exceptions, generic quantification*, ...

Three common, overlapping readings:

- *Plausibilistic/ontic*: plausible/normal implications
- *Auto-epistemic/context-dependent:* classical implications/rules with autoepistemic or defeasible assumptions
- *Normative*: prima facie norms, amendable laws

Our focus: epistemic/plausibilistic/ontic interpretations

Formal defaults

Default: A default over a base language L is an expression

$$\varphi \rightsquigarrow \psi \quad \text{read as} \quad \text{“}\textit{if } \varphi\textit{, then by default } \psi\text{”}$$

where $\varphi, \psi \in L$ and $\rightsquigarrow$ denotes a defeasible implication

Strict implication: necessary implications without exceptions

$$\varphi \twoheadrightarrow \psi \quad \text{read as} \quad \text{“}\varphi \textit{ strictly implies } \psi\text{”}$$

not to be confused with material implication $\varphi \to \psi$ over L

Conditional language:

$L(\rightsquigarrow, \twoheadrightarrow) = \{\varphi \rightsquigarrow \psi, \varphi \twoheadrightarrow \psi \mid \varphi, \psi \in L\}$

Note: Defaults typically encode contingent information

Propositional and first-order defaults

Propositional defaults:

- *Tweety is a bird plausibly implies that Tweety can fly*

  $Bird(Tweety) \leadsto Canfly(Tweety)$

First-order defaults:

- *Birds normally can fly*

  $Bird(x) \leadsto Canfly(x)$ (open/schematic defaults)

  $Bird(x) \leadsto_x Canfly(x)$ (default quantifier, more expressive)

Most work on DR is essentially propositional $\rightarrow$ *our focus*

Default inference

Default inference: defeasible consequence relation exploiting defaults and strict implication

Standard default inferential task: for $\Sigma \subseteq L$, $\Delta \subseteq L(\leadsto, \twoheadrightarrow)$

$$\Sigma \cup \Delta \;\mid\!\sim\; \psi \quad \text{or} \quad \Sigma \;\mid\!\sim_{\Delta} \psi$$

In addition one may also consider appropriate monotonic inference relations $\vdash \subset \mid\!\sim$ extending the basic logical inference on L:

$$\Sigma \cup \Delta \;\vdash\; \psi \quad \text{or} \quad \Sigma \;\vdash_{\Delta}\; \psi$$

Examples

A prototypical domain for benchmarks ...

P,B,F for *Tweety is a penguin, a bird, can fly*
(P, B, F are assumed logically independent)

- $\{P, P \twoheadrightarrow B, B \rightsquigarrow F\} \mid\!\sim F$
- $\{P, P \twoheadrightarrow B, B \rightsquigarrow F\} \vdash P, B$
- $\{P, \neg F, P \twoheadrightarrow B, B \rightsquigarrow F\} \not\mid\!\sim \mathbf{F}$ (exception tolerance)
- $\{P, P \twoheadrightarrow B, B \rightsquigarrow F, P \rightsquigarrow \neg F\} \mid\!\sim \neg F$ (specificity principle)

Reiter's default rules

Reiter's Default Logic (RDL) 1980 an influential NM formalism

RDL is based on context-dependent rules with autoepistemic assumptions, e.g. expressed as

$B : F/F \sim$ *If Tweety is a bird, and it is epistemically possible/consistent that he can fly, then assume that he can fly*

Reiter's general default rules: over classical $(L, \vdash)$:

$$\varphi : \eta_1, \ldots, \eta_n/\psi \quad (\varphi, \eta_i, \psi \in L)$$

φ antecedent, η_i justifications, ψ consequent

"*If* φ *given and each* η_i *is consistent, then conclude* ψ"

Reasoning with Reiter's rules

Reiter's rules can be used to build defeasible proofs (arguments) producing maximal consistent speculative consequence sets - called *extensions* - closed under the base logic. There may be

- **Multiple:** $W \cup D = \{\varphi\} \cup \{\varphi : \psi/\psi, \varphi : \neg\psi/\neg\psi\}$

  Applying one rule blocks the other one
  $\rightarrow$ the application order is relevant!

  2 extensions: $E_1 = Cn(\{\varphi, \psi\}), E_2 = Cn(\{\varphi, \neg\psi\})$

- **One:** $W \cup D = \{\varphi, \neg\chi\} \cup \{\varphi : \psi/\psi, \psi : \chi/\chi\}$

  1 extension: $E = Cn(\{\varphi, \psi, \neg\psi\})$

- **None:** Consider the paradoxical rule: $D = \{\mathbf{T} : \neg\psi/\psi\}$

  If $\neg\psi$ is consistent with E, then $\psi \in E$
  If not, then ψ is not derivable, and $\psi \notin E$

Extension-based default reasoning

Extensions: acceptable speculative consequence sets $E = Cn(E)$, which can be defined in various ways (not restricted to RDL)

Extension-based NMR: $Ext : (W, D) \mapsto Ext(W, D) \subseteq 2^L$

Skeptical inference: $W \cup D \mid\!\sim^{Ext} \psi$ iff $\psi \in \bigcap Ext(W, D)$

Fixed point definition

Reiter's rules refer to the set of expected consequences S the justifications should be consistent with. Ideally, S should be the actual constructed extension E

Fixed point operator: links expected with actual consequences:

$F_{(D,W)}(S)$ is the smallest $S' = Cn(S')$ with $W \subseteq S'$ and closed under all default rules $\varphi : \eta_1, \ldots, \eta_n/\psi$ in D with $S \not\vdash \neg\eta_i$, i.e. whose justifications are consistent with S

Reiter's extensions: $Ext_{rdl}(D, W) = \{E \mid F_{(D,W)}(E) = E\}$

- Normal default theories ($\varphi : \psi/\psi$) always have extensions
- Prerequisite-free semi-normal ones ($\mathbf{T} : \eta \wedge \psi/\psi$) may have none
- Extensions are mutually inconsistent (no $E \subset E'$)

Some links

Strong links with logic programming and formal argumentation, which differ by their restricted languages and extension concepts

Clauses in logic programs: a_i, b, s_j ground literals

$$b \leftarrow a_1, \ldots, a_n, not(s_1), \ldots, not(s_m) \;\sim$$

$$a_1 \wedge \ldots \wedge a_n : \neg s_1, \ldots \neg s_m / b$$

Stable sets $\sim$ Reiter extensions restricted to ground atoms

Reiter's default inference

Reiter defaults: usually interpreted as normal default rules

Translation:

$\tau : \varphi \leadsto \psi \;\mapsto\; \varphi : \psi / \psi$ and $\tau : \varphi \twoheadrightarrow \psi \;\mapsto\; \varphi : \mathbf{T} / \psi$

$$\Sigma \cup \Delta \mathrel{|\!\sim}^{rdl} \psi \text{ iff } \psi \in \cap Ext_{rdl}(\Delta^{\tau}, \Sigma)$$

Existence of extensions is here guaranteed! ($E \vdash \mathbf{F}$ possible)

Alternative default implementations possible:

$\varphi \leadsto \psi \;\mapsto\; \mathbf{T} : \varphi \to \psi / \varphi \to \psi$ resp. $\mathbf{T} : \varphi \wedge \psi / \varphi \to \psi$

but no clear advantages - only some trade-offs

Digression - KLM principles

In the 80s/90s: proliferation of DR and NMR formalisms

→ e.g.: to repair perceived inadequacies of earlier proposals

→ but: iterating, no end in sight ...

→ seeking rationality principles to evaluate and classify the beasts

In fact: it is easier to discuss and analyze abstract principles than examples tainted by diverging implicit world knowledge!

→ Principles for nonmonotonic inference relations - on L, typically keeping defaults fixed [Gabbay 85, Kraus et al. 90, Makinson 94], with representation theorems based on possible worlds semantics

KLM-principles for RDL I

- **Supraclassicality (SC):** $\Sigma \vdash \psi$ implies $\Sigma \mid\!\sim_\Delta \psi$
- **Left Logical Equivalence LLE:**
  $\Sigma \dashv\vdash \Sigma'$ and $\Sigma \mid\!\sim_\Delta \psi$ implies $\Sigma' \mid\!\sim_\Delta \psi$
- **Right Weakening RW:**
  $\Sigma \mid\!\sim_\Delta \psi$ and $\psi \vdash \psi'$ implies $\Sigma \mid\!\sim_\Delta \psi'$
- **Right Conjunction AND:**
  $\Sigma \mid\!\sim_\Delta \psi$ and $\Sigma \mid\!\sim_\Delta \psi'$ implies $\Sigma \mid\!\sim_\Delta \psi \wedge \psi'$
- **Cautious Monotony CM:**
  $\Sigma \mid\!\sim_\Delta \varphi$ and $\Sigma \mid\!\sim_\Delta \psi$ implies $\Sigma \cup \{\varphi\} \mid\!\sim_\Delta \psi$
- **Cautious Transitivity CUT:**
  $\Sigma \mid\!\sim_\Delta \varphi$ and $\Sigma \cup \{\varphi\} \mid\!\sim_\Delta \psi$ implies $\Sigma \mid\!\sim_\Delta \psi$.

- **Reasoning by Cases OR:**
  $\Sigma \cup \{\varphi\} \mid\!\sim_\Delta \psi$ and $\Sigma \cup \{\varphi'\} \mid\!\sim_\Delta \psi$ implies $\Sigma \cup \{\varphi \vee \varphi'\} \mid\!\sim_\Delta \psi$
- **Rational Monotony RM:**
  $\Sigma \mid\!\sim_\Delta \psi$ and $\Sigma \not\mid\!\sim_\Delta \neg\varphi$ implies $\Sigma \cup \{\varphi\} \mid\!\sim_\Delta \psi$
- **Consistency Preservation CP:** $\Sigma \mid\!\sim_\Delta \mathbf{F}$ implies $\Sigma \vdash \mathbf{F}$.

KLM-principles for RDL

$\mid\!\sim_{rdl}$ satisfies **SC, LLE, RW, AND, CUT**

Note that **CUT** is a prerequisite for incremental reasoning

$\mid\!\sim_{rdl}$ violates **Cautious monotony:**

Let $\Delta = \{\varphi \rightsquigarrow \psi, \psi \rightsquigarrow \chi, \chi \rightsquigarrow \neg\psi\}$, then

$$\{\varphi\} \cup \Delta \mid\!\sim^{rdl} \psi, \chi \quad \text{but} \quad \{\chi, \varphi\} \cup \Delta \not\mid\!\sim^{rdl} \psi$$

because $Cn(\{\varphi, \chi, \neg\psi\})$ is an extension

$\mid\!\sim_{rdl}$ also violates **OR:**

$\{\varphi \vee \neg\varphi\} \cup \{\varphi \rightsquigarrow \psi, \neg\varphi \rightsquigarrow \psi\} \not\mid\!\sim \psi$ (no triggering)

Nonmonotonic modal logics

Nomonotonic modal logics: represent $W \cup D$ in a modal logic AEL, GK, ... [McDermott,Doyle 80, Moore 83, Lin Shoham 92]

$\rightarrow$ more expressivity, flexibility, transparency

[Tru 91]: translate Reiter defaults using a knowledge modality K:

- $\varphi \in L \;\mapsto\; K(\varphi)$
- $\varphi : \eta/\psi \;\mapsto\; K(\varphi) \wedge K(\neg K(\neg\eta)) \rightarrow K(\psi)$

Extension concept Ext^X for any modal logic X ($\Phi \subseteq L(K)$):

$$Ext^X(\Phi) = \{E \subseteq L(K) \mid E = \{\psi \in L(K) \mid \Phi \cup \neg K(L(K) - E) \vdash_X \psi\}$$

$Ext_{rdl}(\Phi) = Ext^X(\Phi)$ for $T \subseteq X \subseteq S4$ restricted to L [Tru 91]

Logic of defaults: allows to prove the equivalence of default bases

Specificity principle

Intuition: if defaults conflict, prefer the most specific one

If φ subsumes φ' and ψ, ψ' conflict

i.e. $\varphi \vdash \varphi'$ or $\varphi \twoheadrightarrow \varphi' \in \Delta$, and $\psi \vdash \neg\psi'$:

$$\varphi' \leadsto \psi' \preceq_{spec} \varphi \leadsto \psi \text{ (... is at least as specific as ...)}$$

Also desirable for defeasible subsumption: $\varphi \leadsto \varphi' \in \Delta$

Student, adults, jobs: $\{s, s \leadsto a, a \leadsto j, s \leadsto \neg j\} \mid\!\sim a, \neg j$

But: how to prioritize given longer conflicting chains of defaults?

Early try: theories of inheritance nets [Touretzky 86, Horty 94]

However: low expressivity, purely syntactic, clash of intuitions

The specificity issue for RDL

No specificity in RDL: it fails in its simplest form

$\{p, p \twoheadrightarrow b, b \leadsto f, p \leadsto \neg f\} \not\mid\!\sim^{rdl} f, \neg f$

Two extensions $Cn(\{p, b, f\})$, $Cn(\{p, b, \neg f\})$

Repair by encoding specificity with semi-normal rules? But:

- semi-normal default theories may have no extensions
- very cumbersome, possible side-effects
- $\rightarrow$ maybe better with explicit preferences

Defeasible specificity hard to characterize, but we may try ...

Preferences for default reasoning

Which preferences? What's their meaning? How to exploit them?

Extrinsic preferences: external attributions

- authority of the source (e.g. for default norms)
- utility, usefulness (for practical reasoning)
- application order (procedural, execution needs)

Our topic: Intrinsic preferences: fixed by the defaults

- strict/defeasible specificity
- reliability, strength
- aggregated preference structure

Default reasoning with preferences

Simplest: meta-level preferences over proper defaults $\Delta \cap L(\rightsquigarrow)$

$\rightarrow$ preferences guide the default inference process: many ways

Standard default inference $\mid\!\sim$ parametrized by a transitive $\prec$:

$$\Sigma \cup \Delta \ \mid\!\sim_{\prec} \ \psi$$

Several approaches based on Reiter's account: e.g. [Brewka 94]

(NMR tradition: $\delta < \delta'$ means "δ *preferred to* δ'")

Prioritized default logics

Prioritized default theory: $(W, D, <)$, where

- (W, D) is a default theory
- $D = D_s \cup D_n$ collects strict resp. normal default rules
- $< \subseteq D_n^2$ is a strict, well-founded partial order
  i.e., every subset has $<$-minimal element(s) (true for finite $<$)

Why partiality? $<_{spec}$ may be partial, or total + partially known

Why well-foundedness? Bottom-up construction of extensions

Outcome: handles transparent inheritance/specificity scenarios

Prioritized extensions - an example

Greedy quasi-inductive definition with priorities:

E is a prioritized extension of $(W, D, <)$, or $E \in Ext_{pdl}$, iff

there is a well-ordering $\prec$ of D_n extending $<$ $(< \subseteq \prec)$ s.t.

$E = \bigcup E_\alpha^\prec$ where $E_0^\prec = Cn_{D_s}(W)$,

$E_{\alpha+1}^\prec = Cn_{D_s}(E_\alpha^\prec \cup \{cons(\delta)\})$,
if there is a $\prec$-minimal default $\delta \in D$ active in $E_\alpha^\prec$
($\varphi : \psi/\psi$ is active in X iff $X \vdash \varphi$ and $X \not\vdash \neg\psi, \psi$)

otherwise $E_{\alpha+1}^\prec = E_\alpha^\prec$

Some problems

- **Risk of incoherence:** by complex, meaning-blind interactions between defaults, preferences and the logical structure (especially for intrinsic preferences) ignoring each other
- **Complex specificity:** Specificity orderings may achieve logical coherence, but viable transparent notions of defeasible specificity are elusive as the theory of inheritance nets has shown
- **RDL legacy:** PDL inherits several deficiencies of RDL
- **Greedy approach:** possible tensions between the application order and the chosen preference order

e.g. $a \leadsto b < \mathbf{T} \leadsto \neg b < \mathbf{T} \leadsto a : Cn(\{a, \neg b\})$ or $Cn(\{a, b\}\})$?

Alternative definitions: either similar issues, or no extensions

Qualitative plausibility models

Default conditionals:

$$\varphi \rightsquigarrow \psi \;:\; \varphi \textit{ plausibly/normally implies } \psi$$

Idea: Defaults seen as constraints over epistemic orders

Models: Preferred model structures over $\mathcal{L} = (L, \models)$:

- $(W, \preceq, w_0)$ with $w_0 \in W \subseteq [\![\mathbf{T}]\!]_{\mathcal{L}}$, and
- $\preceq$ a preorder over W $(\prec \,=\, \preceq \cap \not\succeq)$
  ($v \preceq w$: v is at least as preferred/plausible as w)

Satisfaction relation: $\models_{pr}$ for $L \cup L(\twoheadrightarrow) \cup L(\rightsquigarrow)$.

- $(W, \preceq, w_0) \models_{pr} \varphi$ iff $w_0 \models \varphi$
- $(W, \preceq, w_0) \models_{pr} \varphi \twoheadrightarrow \psi$ iff $[\![\varphi]\!] \cap W \subseteq [\![\psi]\!]$

Preferential conditional semantics

Naively: $(W, \preceq, w_0) \models_{min} \varphi \rightsquigarrow \psi$ iff $Min_{\preceq}([\![\varphi]\!]) \subseteq [\![\psi]\!]$

But: There may be no minima - and imposing them artificially (stopperedness, smoothness) is neither natural nor necessary

Example: $W = \{w, w_1, w_2, ...\}$ with $w \models \neg\varphi$, $w_i \models \varphi$, and

let $\preceq$ be an infinite descending chain $\ldots \prec w_3 \prec w_2 \prec w_1$. Then

$(W, \preceq, w_0) \models_{min} \mathbf{T} \rightsquigarrow \neg\varphi$ - despite arbitrarily preferred φ-worlds

Generalized semantics: $(W, \preceq, w_0) \models_{min} \varphi \rightsquigarrow \psi$ iff

for each $w \models \varphi$, there is $w \succeq v \models \varphi$ s.t. for all $v \succeq v' \models \varphi$, $v' \models \psi$

Now: $(W, \preceq, w) \models_{min} \varphi \twoheadrightarrow \psi$ iff $(W, \preceq, w) \models_{min} \varphi \wedge \neg\psi \rightsquigarrow \mathbf{F}$

Preferential conditional logic I

$\models_{min}$ defines a monotonic entailment relation $\vdash_{pcl}$:

$$\Sigma \cup \Delta \vdash_{pcl} \gamma \;\text{ iff }\; [\![\Sigma \cup \Delta]\!]_{pcl} \subseteq [\![\gamma]\!]_{pcl} \text{ (for } \gamma \in L \cup L(\rightsquigarrow, \twoheadrightarrow))$$

Axioms of preferential conditional logic $\vdash_{pcl}$:

- $\varphi, \varphi \rightarrow \psi / \psi$ (*Modus Ponens rule*)
- $\varphi \rightsquigarrow \varphi$ (*Reflexivity*)
- If $\vdash \varphi \leftrightarrow \varphi'$ then $\varphi \rightsquigarrow \psi \;/\; \varphi' \rightsquigarrow \psi$ (*Left logical equivalence*)
- If $\vdash \psi \rightarrow \psi'$ then $\varphi \rightsquigarrow \psi \;/\; \varphi \rightsquigarrow \psi'$ (*Right weakening*)
- $\varphi \rightsquigarrow \psi,\; \varphi \rightsquigarrow \psi' \;/\; \varphi \rightsquigarrow \psi \wedge \psi'$ (*Right conjunction*)

Preferential conditional logic II

- $\varphi \rightsquigarrow \psi,\ \varphi' \rightsquigarrow \psi \ / \ \varphi \vee \varphi' \rightsquigarrow \psi$ (*Reasoning by cases*)
- $\varphi \rightsquigarrow \varphi',\ \varphi \rightsquigarrow \psi \ / \ \varphi \wedge \varphi' \rightsquigarrow \psi$ (*Cautious monotony*)
- $\varphi \rightsquigarrow \varphi',\ \varphi \wedge \varphi' \rightsquigarrow \psi \ / \ \varphi \rightsquigarrow \psi$ (*Cautious transitivity*)
- $\varphi \rightsquigarrow \mathbf{F} \ / \ \neg\varphi$ (*Necessity*)
- $\varphi \twoheadrightarrow \psi$ if and only if $\varphi \wedge \neg\psi \rightsquigarrow \mathbf{F}$ (*Strict implication*)

Object-level modus ponens fails: $\varphi, \varphi \rightsquigarrow \psi \not\vdash_{pcl} \psi$

because the actual world can be exceptional!

Preferential entailment

How to specify nonmonotonic reasoning with defalt conditionals?

Simplest approach: $\mid\!\sim^p$ (*Preferential entailment/System P*)

$\{\varphi_1, \ldots, \varphi_n\} \cup \Delta \mid\!\sim^p \psi$ iff $\Delta \vdash_{pcl} \varphi_1 \wedge \ldots \wedge \varphi_n \leadsto \psi$

Basic defeasible modus ponens: $\{\varphi\} \cup \{\varphi \leadsto \psi\} \cup \Delta \mid\!\sim^p \psi$

$\{\varphi, \neg\psi\} \cup \{\varphi \leadsto \psi\} \cup \Delta \not\mid\!\sim^p \psi$ if $\{\varphi \leadsto \psi\} \cup \Delta \not\vdash \varphi \twoheadrightarrow \psi$

Simple specificity: $\{s, s \leadsto a, a \leadsto j, s \leadsto \neg j\} \mid\!\sim^p a, \neg j$

Defeasible monotony fails: $\varphi \leadsto \psi \not\vdash_{pcl} \varphi \wedge \chi \leadsto \psi$

hence $\{\varphi, \chi\} \cup \{\varphi \leadsto \psi\} \not\mid\!\sim^p \psi$ (only if $\varphi \wedge \chi \not\vdash \psi$)

System Z I

In System P, irrelevant generic info χ can block an inference!

Idea: Inference based on plausibility maximization

Rational closure [Lehmann, Magidor 92], System Z [Pearl 90]

Z-algorithm: (our variant) for finite $\Sigma \cup \Delta$:

1. Translate $\varphi \twoheadrightarrow \psi$ into $\varphi \wedge \neg\psi \rightsquigarrow \mathbf{F}$

2. Construct by induction $(\Delta_{\geq i} \mid 0 \leq i)$ and $(\rho_i \mid 0 \leq i)$ s.t.

- $\rho_0 = \mathbf{T}$, $\Delta_{\geq i} = \{\varphi \rightsquigarrow \psi \in \Delta \mid \varphi \vdash \rho_i\}$
- $\rho_{i+1} = \bigvee\{\varphi \wedge \neg\psi \mid (\varphi \rightsquigarrow \psi) \in \Delta_{\geq i}\}$

For $i < j$ we have $\rho_j \vdash \rho_i$ and $\Delta_{\geq j} \subseteq \Delta_{\geq i}$.

For finite Δ, there is a smallest N s.t. $\Delta_{\geq N} = \Delta_{\geq N+1}$, $\rho_N \dashv\vdash \rho_{N+1}$

System Z II

Z-rank $\sim$ degree of exceptionality

Z-rank of defaults: $Z(\delta)$ = maximal n s.t. $\delta \in \Delta_{\geq n}$

Z-rank of worlds: $Z(w) = max\{n \mid w \models \rho_n\}$,
i.e. largest $Z(\delta)$ s.t. w violates δ ($Z(\varphi \leadsto \mathbf{F}) = \infty$)

There is a canonical ranked model $(W^Z_\Delta, \preceq^Z_\Delta)$ with

- $W^Z_\Delta = [\![\neg\rho_\infty]\!] = \{w \in [\![\mathbf{T}]\!]_\mathcal{L} \mid Z(w) < \infty\}$
- $v \preceq^Z_\Delta w$ iff $Z(v) \leq Z(w)$

Z-entailment (System Z):

$\{\varphi_1, \ldots, \varphi_n\} \cup \Delta \mathrel{|\!\sim^z} \psi$ iff $(W^Z_\Delta, \preceq^Z_\Delta) \models_{min} \wedge\varphi_i \leadsto \psi$

Properties of System Z

$\mid\!\sim^z_\Delta$ verifies all the KLM-principles:

SC, LLE, RW, AND, OR, CUM (= CUT+CM), RM

Defeasible transitivity:

$\{s, s \rightsquigarrow a, a \rightsquigarrow j\} \mid\!\sim^z a, j$

$\{s, \neg j, s \rightsquigarrow a, a \rightsquigarrow j\} \mid\!\sim^z a, \neg j$

Defeasible specificity:

$\{s, s \rightsquigarrow \neg j, s \rightsquigarrow a, a \rightsquigarrow j\} \mid\!\sim^z a, \neg j$ (also $\{...\} \not\mid\!\sim^z j$)

LLE for defaults: $\Delta \dashv\vdash_{pcl} \Delta'$ implies $\mid\!\sim^z_\Delta = \mid\!\sim^z_{\Delta'}$

Problems for System Z

Simple exceptional inheritance fails:

$\{dutch, \neg tall, dutch \rightsquigarrow tall, dutch \rightsquigarrow loud\} \not\mid\!\sim^{z} loud$

The Z-model of Δ is given by: $dtl \prec d\neg tl \sim dt\neg l \sim d\neg t\neg l$

i.e. $Z(dtl) = 0$ and $Z(d\neg tl) = Z(dt\neg l) = Z(d\neg t\neg l) = 1$

Hence $dutch \wedge \neg tall \not\mid\!\sim^{z}_{\Delta} loud$: no exceptional inheritance

Replacing *dutch* by $\mathbf{T}$, we can also falsify the *Irrelevance Principle*

IRR: If $\Sigma \cup \Delta$ and $\Sigma' \cup \Delta' \not\vdash_{pcl} \mathbf{F}$ have disjoint vocabularies,

then $\Sigma \cup \Delta \mid\!\sim \psi$ iff $\Sigma \cup \Sigma' \cup \Delta \cup \Delta' \mid\!\sim \psi$

System LEX I

Idea: compare not just the highest-ranked violated defaults but also lower-ranked ones, as well as their number at the different ranks

Lexicographic entailment [Lehmann 1995]: $\mid\!\sim^{lex}$

Violation sequences for worlds: $lex_\Delta(w) = (z_i(w) \mid i \leq \infty)$

where $z_i(w) = |\{\delta \in \Delta \mid w \models \varphi_\delta \wedge \neg\psi_\delta, Z(\delta) = i\}|$

Δ_{dutch}: $lex(d\neg tl) = (1, 0, \ldots 0)$, $lex(d\neg t\neg l) = (2, 0, \ldots 0)$

$\Delta_{imp} = \{p \leadsto \mathbf{F}\}$: $lex(p) = (0, 0, \ldots 1)$, $lex(\neg p) = (0, 0, \ldots 0)$

System LEX II

Lex-ordering: $v \preceq^{lex} w$ iff $z_\infty(w) \neq 0$, or

$z_\infty(w) = 0$ and for the highest i with $z_i(v) \neq z_i(w)$, $z_i(v) \leq z_i(w)$

LEX: $\Sigma \cup \Delta \mid\!\sim^{lex} \psi$ iff $(W_\Delta^{lex}, \preceq_\Delta^{lex}) \models_{min} \wedge\varphi_i \rightsquigarrow \psi$

Winged birds example: $\Delta = \{p \twoheadrightarrow b, b \rightsquigarrow f, p \rightsquigarrow \neg f, b \rightsquigarrow w\}$

$\{p\} \cup \Delta \mid\!\sim^{lex} \neg f, w$ because

$lex(pb\neg f\neg w) = (2, 0, ..0), lex(pbf\neg w) = (1, 1, ..0),$
$lex(pb\neg fw) = (1, 0, ..0), lex(pbfw) = (0, 1, ..0)$

$pb\neg fw$ is obviously the most plausible world

Properties and problems

LEX extends Z: $\mid\!\sim^{z} \subset \mid\!\sim^{lex}$, hence it is more speculative

Static priorities: Z-ranks of defaults are pre-computed, no inductive prioritization considering e.g. the fine-grained preference status of default antecedents

Radical ad hoc prioritization: Violating a more specific conflicting defaults has automatically more weight than violating two independent less specific defaults - which is probabilistically unsound and in conflict with irrelevance considerations.

Beyond plausibility orders

Drawbacks of qualitative plausibility orders:

- For $w \prec w'$ and $v \prec v'$, the relative plausibility of w' w.r.t. w cannot be compared to that of v' w.r.t. v
- No proper conditional independence notion
- Insufficient expressiveness/granularity
- Translation between/aggregation of plausibility contexts unclear
- Expected utility hard to model

Idea: Use plausibility valuations from world sets to an additive structure of ordered values

Fine-grained example: probability measures $P : Prop \to [0,1]$)

Plausibility valuations

General plausibility val. [Friedman, Halpern 96]:

$Pl : B \to (V, \bot, \top, \prec)$ with

$Pl(\emptyset) = \bot, Pl(W) = \top$ and $Pl(A) \preceq Pl(B)$ if $A \subseteq B$

Desiderata

- Conditional plausibility + reasonable independence concept
- $\prec$ total order (partial order = set of total ones)

Simplest solution: Ranking measures [Spohn 88,12, Wey 95]

Ranking measures generalize

- Spohn's rk-functions measuring the implausibility/surprise of propositions, used to model revisable graded plain belief
- Real-valued multiplicative possibility [Dubois, Prade 94]

Real-valued ranking measures

Real-valued ranking measures (integers not enough!)

Let $\mathbb{B}_{\mathcal{L}}$ be the boolean algebra of $\mathcal{L}$-propositions

$R : \mathbb{B} \to ([0, \infty], +, \leq)$ is a real-valued ranking measure (rkm) iff

- $([0, \infty], 0, +, \geq)$: ordered additive stucture of pos. reals with ∞
- $R(W) = 0$, $R(\emptyset) = \infty$ (expresses impossibility)
- $R(A \cup B) = \min_{\leq}\{R(A), R(B)\}$ for all $A, B \in \mathbb{B}$

Conditional ranking measure:
$R(B|A) = R(A \cap B) - R(A)$ for $R(A) \neq \infty$, else $R(B|A) = \infty$.

R_0 is the uniform rkm with $R_0(A) = 0$ for $A \neq \emptyset$. $R(\varphi) := R([\![\varphi]\!])$

Ranking epistemology

Ranking measure values $\sim$ degrees of implausibility/surprise

Ranking measures may model belief states (Spohn):

Belief strength in φ is r iff $R(\neg\varphi) = r$

Conventional threshold: $Bel(\varphi)$ iff $R(\neg\varphi) \geq 1$ $(R(\varphi) = 0)$

Belief is closed under conjunction (plain belief) because

$R(\neg(\varphi \wedge \psi)) = R(\neg\varphi \vee \neg\psi) = min\{R(\neg\varphi), R(\neg\psi)\} \geq 1$

$R(\neg\varphi) = \infty$: $\neg\varphi$ is epistemically impossible, i.e. φ a dogm

Probabilistic link: $R(A) = r \sim P(A) = O(\varepsilon^r)$,
for infinitesimals $0 < \varepsilon \ll 1$

Ranking measure semantics

Rkm semantics for default $\rightsquigarrow$ and strict implication $\twoheadrightarrow$:

$R \models_{rk} \varphi \rightsquigarrow \psi$ iff $R(\varphi \wedge \psi) + 1 \leq R(\varphi \wedge \neg\psi)$ iff $R(\neg\psi|\varphi) \geq 1$

$R \models_{rk} \varphi \twoheadrightarrow \psi$ iff $R(\varphi \wedge \psi) + \infty \leq R(\varphi \wedge \neg\psi)$ iff $R(\varphi \wedge \neg\psi) = \infty$

$[\![\Delta]\!]_{rk} = \{R \mid R \models_{rk} \delta \text{ for all } \delta \in \Delta\}$

We sometimes need a weaker satisfaction $\models_{rk}^{>0}$ using $R(\neg\psi|\varphi) > 0$

Monotonic rk-entailment: $\Delta \vdash_{rk} \delta$ iff $[\![\Delta]\!]_{rk} \subseteq [\![\delta]\!]_{rk}$

$\vdash_{rk}$ satisfies the rules for preferential conditional logic

Rkm-based default entailment

Goal: A rkm-based framework to specify default inference

Idea: Rkm semantics + preferred model theory for conditionals

Preferred rkm choice function: $\mathcal{I} : \Delta \mapsto \mathcal{I}(\Delta) \subseteq [\![\Delta]\!]_{rk}$

Rkm-based default entailment w.r.t. $\mathcal{I}$: $\mid\!\sim^{\mathcal{I}}$:

$\Sigma \cup \Delta \mid\!\sim^{\mathcal{I}} \psi$ iff for all $R \in \mathcal{I}(\Delta)$ $R \models_{rk}^{>0} \wedge\Sigma \rightsquigarrow \psi$

Defeasible modus ponens:

$\{\varphi\} \cup \{\varphi \rightsquigarrow \psi\} \cup \Delta \mid\!\sim^{\mathcal{I}} \psi$ $(\{\varphi \rightsquigarrow \psi\} \cup \Delta \vdash_{rk} \varphi \rightsquigarrow \psi)$

Preferentiality: $\mid\!\sim^{\mathcal{I}}_{\Delta}$ verifies SC, LLE, RW, AND, CUT, CM, OR

Rkm-based reconstructions

Preferential entailment: $\mid\!\sim^{p} = \mid\!\sim^{\mathcal{I}_p}$ where $\mathcal{I}_p(\Delta) = [\![\Delta]\!]_{rk}$

System P is the weakest $\mid\!\sim^{\mathcal{I}}$ because $\mathcal{I}_p(\Delta)$ is maximal

Conditional monotonicity: $\Sigma \cup \Delta \mid\!\sim^{p} \psi$ implies $\Sigma \cup \Delta \cup \Delta' \mid\!\sim^{p} \psi$

For finite $\Delta \not\models_{rk} \mathbf{F}$, there is a unique rkm-mode R^z_Δ which attributes the lowest possible rkm-values to each proposition:

$$R^z_\Delta(A) = inf_{\leq}\{R(A) \mid R \models_{rk} \Delta\}$$

System Z: $\mid\!\sim^{z} = \mid\!\sim^{\mathcal{I}_z}$ with $\mathcal{I}_z(\Delta) = \{R^z_\Delta\}$

LLE **for defaults:** for Systems P, Z we have

$[\![\Delta]\!]_{rk} = [\![\Delta']\!]_{rk}$ implies $\mid\!\sim^{\mathcal{I}}_{\Delta} = \mid\!\sim^{\mathcal{I}}_{\Delta'}$

Reconstruction of System LEX

Semantic problems with System LEX:

Non-standard: Reconstructing $\mid\!\sim^{lex}$ requires infinite rkm-values:

$1 < 1{+}1 < \ldots N < \ldots N{+}N < \ldots N^2 < N^2{+}1 < \ldots N^2{+}N <$
$\ldots N^2 + N^2 < \ldots N^3 < \ldots < \infty$

$(1, 0, \ldots 0) < (2, 0, \ldots 0) < \ldots (0, 1, \ldots 0) < \ldots (0, 2, \ldots 0) < ...\infty$

$\rightarrow$ LEX imposes excessively high implausibility values

Context dependence: The relative rkm-values may depend on the rkm-value of a generic context X:

$$R^{lex,N}_{\{\mathbf{T}\leadsto a\}}(\neg a) = 1 \;\text{ but }\; R^{lex,N}_{\{X\leadsto a, \mathbf{T}\leadsto\neg X\}}(\neg a \mid X) = N$$

Are there less extreme rkm-inference notions supporting inheritance?

Ranking measure constructions

How to find reasonable $R \models_{rk} \Delta$ respecting the structure of Δ?

Idea: Focus on those R obtained by iterated Spohn-style revision on the uniform rkm prior R_0 with $\varphi \to \psi$ for $\varphi \rightsquigarrow \psi \in \Delta$

Informally: Specify ranking models by adding context-dependent penalties ≥ 0 for each default a world violates

Minimal change when strengthening belief in φ: make $\neg\varphi$-worlds uniformly more implausible

Ranking construction models: Let $\Delta = \{\varphi_i \rightsquigarrow \psi_i \mid i \leq n\}$

$Constr(\Delta) = \{R \models_{rk} \Delta \mid R = R_0 + \sum_{i \leq n} a_i[\varphi_i \wedge \neg\psi_i], a_i \in [0.\infty]\}$

Fact: If $\Delta \not\models \mathbf{F}$, then $Constr(\Delta) \neq \emptyset$

Construction-based default entailment I

Strategy: Default inference based on preferred rkm-constructions,

i.e. $\mid\!\sim^{\mathcal{I}}$ with $\mathcal{I}(\Delta) \subseteq Constr(\Delta)$

System J: $\mid\!\sim^{j} = \mid\!\sim^{\mathcal{I}^j}$ where $\mathcal{I}^j(\Delta) = Constr(\Delta)$

Exceptional inheritance: for logically independent a, b,

$\{\neg a, \mathbf{T} \rightsquigarrow a, \mathbf{T} \rightsquigarrow b\} \mid\!\sim^{j} b$

$R \models_{rk} \neg a \rightsquigarrow b$ if $R = R_0 + x(\mathbf{T} \wedge \neg a) + y(\mathbf{T} \wedge \neg b)$ for $1 \leq x, y$

because then $R(\neg a \wedge b) + 1 = x + 1 \leq x + y = R(\neg a \wedge \neg b)$

Construction-based default entailment II

Advantages: Simplicity, robustness, intuitive behaviour

System J^+: $\mathcal{I}^{j^+}(\Delta)$ = rk-construction models with $a_i \geq 1$

- $\{\neg a\} \cup \{\mathbf{T} \rightsquigarrow a, \mathbf{T} \rightsquigarrow a \vee b\} \not\mid\!\sim^{j} b$, but
- $\{\neg a\} \cup \{\mathbf{T} \rightsquigarrow a, \mathbf{T} \rightsquigarrow a \vee b\} \mid\!\sim^{j^+} b$

Systems J, J^+ may be too cautious by not fully exploiting the idea of plausibility maximization

Minimal rkm-constructions

Idea: Maximizing plausibility by minimizing shifting

JM: $\mathcal{I}^{jm}(\Delta)$ = set of $R \in Constr(\Delta)$ with pointwise minimal shifting vectors $\vec{a_i}$

Non-uniqueness:

$\mathcal{I}^{jm}(\{\mathbf{T} \leadsto a, \mathbf{T} \leadsto b, \mathbf{T} \leadsto a \wedge b\})$ is uncountable

$\mathcal{I}^{jm}(\Delta) = \{R_0 + x[\neg a] + x[\neg b] + y[\neg a \vee \neg b] \mid x + y = 1\}$

Minimal rkm-constructions have not been born equal ...

Big Birds Hammer

Big Birds Hammer: *Birds are normally small, birds can normally fly, exceptional birds (small or unable to fly) normally cannot fly. What about the flying abilities of big birds?*

$$\{b, \neg s\} \cup \{b \leadsto f, b \leadsto s, b \wedge \neg(s \wedge f) \leadsto \neg f\} \mathrel{|\!\!\sim} \neg f \ ?$$

By specificity one may expect $\{b, \neg s\} \cup \Delta \mathrel{|\!\!\sim} \neg f$

(which holds for System Z, minimal information entailment)

But $\mathcal{I}^{jm}(\Delta) = \{(2-x)[b \wedge \neg s \wedge f] + x[b \wedge \neg s] + 1[b \wedge \neg f] \mid x \in [0, 2]\}$,

$R^1(b \wedge \neg s \wedge f) = 2 = R^1(b \wedge \neg s \wedge \neg f)$, hence $\{b, \neg s\} \cup \Delta \not\mathrel{|\!\!\sim}^{jm} \neg f$

The best fitting solution: $R^0 = 2[b \wedge \neg s \wedge f] + 1[b \wedge \neg f]$

Justifiable constructibility

Idea: Ranking constraints should not be over-satisfied

Justifiable constructibility:

$R = \Sigma_{i \leq n} a_i[\varphi_i \wedge \neg\psi_i]$ is justifiably constructible model of Δ iff proper shifting of $[\![\varphi_i \wedge \neg\psi_i]\!]$, i.e. $a_i > 0$, implies satisfaction as an equality constraint: $R(\varphi_i \wedge \psi_i) + 1 = R(\varphi_i \wedge \neg\psi_i)$

System JJ: $\mathcal{I}^{jj}(\Delta)$ = justifiably constructible rk-models of Δ

Fact: $\mathcal{I}^{jj}(\Delta) \subseteq \mathcal{I}^{jm}(\Delta)$

Big Birds Hammer: JJ provides the unique correct solution:

$\mathcal{I}^{jj}(\Delta_{bbh}) = \{2[b \wedge \neg s \wedge f] + 1[b \wedge \neg f]\}$

Non-uniqueness: If $\Delta = \{\mathbf{T} \rightsquigarrow a, \mathbf{T} \rightsquigarrow b, \mathbf{T} \rightsquigarrow a \wedge b\}$, then $\mathcal{I}^{jj}(\Delta) = \mathcal{I}^{jm}(\Delta)$ is again uncountable

Canonical preferred ranking models

Goal: Specify for each Δ a canonical preferred rkm model improving on Systems Z/LEX. Two main strategies:

- **Ranking measure fusion:** "take the average"
- **Canonical rkm construction:** liike for System Z/LEX

Ranking measure fusion: For each rk-choice $\mathcal{I}$, let $\hat{\mathcal{I}}$ be s.t.

$$\hat{\mathcal{I}}(\Delta) = \{R^*_\Delta\} \text{ with } R^*_\Delta(A) = Inf_\leq(\{R(A) \mid R \in \mathcal{I}(\Delta)\})$$

R^*_Δ is the most plausible lower bound of the $\mathcal{I}$-preferred Δ-models

Example: for System Z, $\mathcal{I}^z(\Delta) = \hat{\mathcal{I}}^p = [\![\Delta]\!]_{rk}$

Fact: $R^*_\Delta \in Mod_{rk}(\Delta)$, but $R^*_\Delta \notin Constr(\Delta)$ is possible

System JJR

JJR: $\mathcal{I}^{jjr}(\Delta) = \hat{\mathcal{I}^{jj}}(\Delta) = \{R^{jjr}_{\Delta}\}$ - the best of both worlds?

If the justifiably constructible model is unique, it is the JJR-model

For $\Delta = \{\mathbf{T} \rightsquigarrow a, \mathbf{T} \rightsquigarrow b, \mathbf{T} \rightsquigarrow a \wedge b\}$, $R^{jjr}_{\Delta} = 1[\neg a \vee \neg b]$
(it is the Z-model of Δ and justifiably constructible)

Constructibility counterexample: Nested crossing

$\Delta_{nc} = \{\mathbf{T} \rightsquigarrow a, \mathbf{T} \rightsquigarrow b, \mathbf{T} \rightsquigarrow r, \mathbf{T} \rightsquigarrow s, \mathbf{T} \twoheadrightarrow (a \wedge b \leftrightarrow r \wedge s)\}$

$\mathcal{I}^{jj}(\Delta_{nc}) = \{x[\neg a] + x[\neg b] + (1-x)[\neg r] + (1-x)[\neg s] \mid x \in [0,1]\}$

$R^{jjr}_{\Delta_{nc}} = 1[\neg(a \wedge b)] + 1[\neg(a \vee b \vee r \vee s)] + \infty[\neg(a \wedge b \leftrightarrow r \wedge s)]$
$\notin Constr(\Delta_{nc})$

Canonical preferred rk-constructions

Goal: Incremental construction of a canonical rkm-model R^*_Δ of Δ in the spirit of System Z

Examples: the only rk-constructible ones I am aware of ...

System JZ, JLZ [Wey 98, 03]

Philosophy: Minimize the rk-construction efforts everywhere

Let $\Delta = \{\varphi_i \rightsquigarrow \psi_i \models i \leq n\}$

We seek a "minimally constructed" $R^*_\Delta = \Sigma_i a_i[\varphi_i \wedge \neg\psi_i]$

JZ-construction

Guiding principles of the JZ-construction

- *Justifiable constructibility*: no superfluous shifting
- *No default redundancy*: equivalent defaults considered once w.l.o.g.: $([\![\varphi_i \wedge \neg\psi_i]\!], [\![\varphi_i]\!]) = ([\![\varphi_j \wedge \neg\psi_j]\!], [\![\varphi_j]\!])$ implies $i = j$
- *Bottom-up plausibility maximization*: first construct the most plausible layers, ignoring the necessarily less plausible ones
- *Local shifting minimization*: when constructing a layer, realize the not-yet-settled defaults by lexicographically minimizing the longer shifts first

Note: Here the priorities are dynamic, not as for System Z, LEX

System JZ

System JZ: flagship proposal for rkm-based default inference

JZ-idea: Proceed rank by rank, trying to locally approximate ranking minimization (system Z) by local ranking constructions lexicographically minimizing the shifting efforts for each target rank

Relative plausibility maximization: $PM(R, \Delta)$

An auxiliary notion generalizing $R^z_\Delta = PM(R_0, \Delta)$

The most plausible rkm-model of Δ above R for $R(\wedge\Delta^{\rightarrow}) \neq \infty$

$PM(R, \Delta)(A) = inf_{\leq}\{R'(A) \mid R \leq R', R' \models_{rk} \Delta\}$

JZ-algorithm I

Induction: We jointly construct sequences $R_i, R_i^*, \Delta_i, \Delta_i'$,

Start: $i = 0$: $R_0 = R_0$, $R_0^* = PM(R_0, \Delta) = R_\Delta^z$, $\Delta_0 = \emptyset$, $s_0 = 0$

Step: $i \to i+1$: R_i preceding partial ranking construction,
$R_i^* = PM(R_i, \Delta)$, Δ_i collection of settled defaults at stage i

s_{i+1} smallest $s > s_i$ of the form $s = R_i^*(\varphi_j \wedge \neg\psi_j)$ for $\delta_j \in \Delta - \Delta_i$

$\Delta'_{i+1} = \{\delta_j \in \Delta - \Delta_i \mid R_i^*(\varphi_j \wedge \neg\psi_j) = s_{i+1}\}$

JZ-algorithm II

$R_{i+1} = R_i + \Sigma_{\delta_j \in \Delta'_{i+1}} a_i[\varphi_j \wedge \neg\psi_j]$,

where $\vec{a}$ is the lex-length-minimal tuple $\vec{x}$ with, for all $h \leq n$,

$(R_i + \Sigma_{\delta_j \in \Delta'_{i+1}} x_j[\varphi_j \wedge \neg\psi_j]+$
$\Sigma_{\delta_j \notin \Delta_i \cup \Delta'_{i+1}} \infty[\varphi_j \wedge \neg\psi_j])(\varphi_h \wedge \neg\psi_h)$
$\geq s_{i+1}$

i.e. reaching s_{i+1} while ignoring all the shiftable propositions $\varphi_h \wedge \neg\psi_h$ which verify $R^*_i(\varphi_h \wedge \neg\psi_h) > s_{i+1}$

$R^*_{i+1} = PM(R_{i+1}, \Delta)$ and $\Delta_{i+1} = \Delta_i \cup \Delta'_{i+1}$

Stop: If s_{i+1} does not exist, then $R^{jz}_{\Delta} = R_{i+1} = R_i$.

Examples I

Nested crossing:

$\Delta_{nc} = \{\mathbf{T} \rightsquigarrow a, \mathbf{T} \rightsquigarrow b, \mathbf{T} \rightsquigarrow r, \mathbf{T} \rightsquigarrow s, \mathbf{T} \twoheadrightarrow (a \wedge b \leftrightarrow r \wedge s)\}$

$R_0^* = PM(R_0, \Delta) = 1[\neg a \vee \neg b] + \infty[\neg(a \wedge b \leftrightarrow r \wedge s)]$

$R_0^*(\neg a) = ... = R_0^*(\neg s) = 1 < \infty = R_0^*(\neg(a \wedge b \leftrightarrow r \wedge s))$

hence $s_1 = 1$

The lex-length-minimal coeff. a_i s.t. for $R_1 = a_1[\neg a] + ... + a_4[\neg s]$

we have $R_1 + \infty[..](\neg a), ..., R_1 + \infty[..](\neg s) \geq s_1 = 1$ are $a_i = 1/2$

Examples II

$\Delta_1 = \Delta_1' = \{\mathbf{T} \leadsto a, \mathbf{T} \leadsto b, \mathbf{T} \leadsto r, \mathbf{T} \leadsto s\}$

$R_1^* = PM(R_1, \Delta) = R_1 + \infty[\neg(a \wedge b \leftrightarrow r \wedge s)]$, now $s_2 = \infty$

The lex-length-minimal coefficient for $\neg(a \wedge b \leftrightarrow r \wedge s)$ is ∞

$\Delta_2 = \Delta$, hence induction stops here and

$$R^{jz}_{\Delta_{nc}} = 1/2[a] + ... + 1/2[s] + \infty[\neg(a \wedge b \leftrightarrow r \wedge s)]$$

is justifiably constructible.

Observe: The symmetries of Δ_{nc}, justifiable constructibility, and canonicity are enough to fix the result

Note: $R^{jz}_{\Delta_{nc}}(\neg a \wedge \neg b) = 3/2$ - thus we need rational rkm-values!

Examples III

Big Birds Hammer light: *Birds are normally small, birds can normally fly, big birds are normally unable to fly*

$$\Delta_{bbl} = \{b \rightsquigarrow f, b \rightsquigarrow s, b \wedge \neg s \rightsquigarrow \neg f\}, \;\; \Sigma_{bbl} = \{b, \neg s\}$$

$R^*_0 = R^z_{\Delta_{bbl}} = 1[b \wedge \neg f] + 2[b \wedge f \wedge \neg s]$ (just to describe it).

Hence, in the first round we will only shift $b \wedge \neg f$ and $b \wedge \neg s$, ignoring the less plausible part $b \wedge \neg s \wedge f$. We have $s_1 = 1$ and $\Delta_1 = \{b \rightsquigarrow f, b \rightsquigarrow s\}$.

Shifting $b \wedge \neg s$ is then redundant and $R_1 = 1[b \wedge \neg f] + 0[b \wedge \neg s]$. R^*_1 is now just $R_1 + 2[b \wedge f \wedge \neg s]$, which puts $b \wedge f \wedge \neg s$ to 2, the next target rank is thus $s_2 = 2$. Hence $R_2 = R_1 + 2[b \wedge f \wedge \neg s]$. Because $\Delta_2 = \Delta_{bbl}$, we stop and $R^{jz}_{\Delta_{bbl}} = 1[b \wedge \neg f] + 2[b \wedge f \wedge \neg s]$

While $\{b, \neg f\} \cup \{b \leadsto f, b \leadsto s\} \mathrel{|\!\!\sim}^{jz} s$,

we have $\{b, \neg f\} \cup \{b \leadsto f, b \leadsto s, b \wedge \neg s \leadsto \neg f\} \mathrel{|\!\!\not\sim}^{jz} s, \neg s$

Properties and principles

System JZ verifies:

- all KLM postulates
- Irrelevance principle (and exceptional inheritance) (IRR)
- Representation/Language Independence (RI)
- Local default equivalence

System JZ violates:

- LLE for defaults

But this is actually unavoidable if we insist on IRR and RI

Desirable inference

Let us call a default inference notion *desirable* iff:

- **Supraclassicality for $\mathrel{|\!\sim}$ w.r.t. $\vdash$:** $\vdash \subseteq \mathrel{|\!\sim}_\Delta$
- **Basic nontriviality:** $\{\neg\varphi, \mathbf{T} \rightsquigarrow \varphi\} \mathrel{|\!\not\sim} \varphi$ if $\varphi \not\vdash \mathbf{F}$, $\neg\varphi \not\vdash \mathbf{F}$
- **Representation invariance for $\mathrel{|\!\sim}$:**
  for semantically invariant boolean isomorphisms $f : L \to L$
  $\Gamma(\vec{\varphi}) \mathrel{|\!\sim} \psi$ iff $\Gamma(f(\vec{\varphi})) \mathrel{|\!\sim} f(\psi)$
- **LLE for defaults:** $\mathrel{|\!\sim}_\Delta = \mathrel{|\!\sim}_{\Delta'}$ if $\Delta \dashv\vdash_{rk} \Delta'$
- **Exceptional inheritance:**
  $\{\neg a\} \cup \{\mathbf{T} \rightsquigarrow a, \mathbf{T} \rightsquigarrow b\} \mathrel{|\!\sim} b$ for logically independent a, b
  (follows from Irrelevance and and Representation invariance)

Exceptional inheritance paradox

Theorem: There are no desirable default inference notions!

What can we do? What to violate?

- Supraclassicality/nontriviality: untouchable cornerstones of default reasoning
- Representation invariance: conditio sine qua non for semantic-based approaches
- LLE for defaults: only very weak conditional logics for defaults
- Exceptional inheritance: only very weak inheritance patterns: e.g. System Z