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Foreign Workers and the Wage Distribution: What Does the Influence Function Reveal?

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Abstract: This paper draws upon influence function regression methods to determine where foreign workers stand in the distribution of private sector wages in Luxembourg, and assess whether and how much their wages contribute to wage inequality. This is quantified by measuring the effect that a marginal increase in the proportion of foreign workers—foreign residents or cross-border workers—would have on selected quantiles and measures of inequality. Analysis of the 2006 Structure of Earnings Survey reveals that foreign workers have generally lower wages than natives and therefore tend to haul the overall wage distribution downwards. Yet, their influence on wage inequality reveals small and negative. All impacts are further muted when accounting for human capital and, especially, job characteristics. Not observing any large positive inequality contribution on the Luxembourg labour market is a striking result given the sheer size of the foreign workforce and its polarization at both ends of the skill distribution.

Keywords: immigrant wages; wage inequality; cross-border workers; influence function; RIF regression; Luxembourg

JEL Classification: J15; J31; J61

1. Introduction

While abundant research has documented the 'nativity wage gap'—the wage difference between foreign and native workers; see, among many others, Chiswick (1978); Borjas (1985, 1995); Adsera and Chiswick (2007)—much less evidence is available on how foreign workers actually 'fit in' and contribute to the shape of the wage distributions in host countries. The objective of this paper is to illustrate methods to examine the matter in question and describe how much foreign workers' wages contribute to private sector wage inequality in Luxembourg.

To be clear, our approach is descriptive or 'static': we are not attempting to identify causal or general equilibrium impacts of immigrants on the total wage distribution (or on native workers wages); see Blau and Kahn (2012); Card (2009) for reviews of this contentious literature.[1] Instead,

[1] The equilibrium impacts of immigration on the distribution of native workers wages remains a debated topic. They crucially depend on the degree of complementarity or substitutability between foreign and native labour—and this may vary by occupation and skill groups—so net impacts are not unambiguous. In the United States for instance, while Grossman (1982), Card (1990) or, more recently, Card (2009) and Ottaviano and Peri (2012) show that the impact of immigration on native wages is small or negligible, Borjas (1999, 2003) find that immigration lowers the wage of competing native workers. Manacorda et al. (2012) and Dustmann et al. (2013) show that the impact of immigration on wages in the UK is heterogeneous

we document where foreign workers' earnings stand in the earnings distribution and quantify their contribution to wage inequality. As we explain shortly, we do so by calculating the effect that a notional marginal substitution of native workers for foreign workers would have on the shape of the earnings distribution—on wage inequality indicators in particular. Checking if substituting native workers for (observationally equivalent) foreign workers has any impact on distributional statistics turns out to be an indirect but informative way to apprehend how foreign workers' wages fit in and contribute to the shape of the overall wage distribution.

The analysis builds upon (recentered) influence function (RIF) regression methods proposed in Firpo et al. (2009) which capture how marginal changes in the distribution of covariates impact on distributive statistics of interest. These statistics have typically been quantiles in recent applications, but the methods can be applied to any statistic summarizing particular features of a distribution. We consider both a series of quantiles (to analyse the overall shape of the distribution) and dispersion measures, namely the variance, the Gini coefficient and three percentile ratios (to examine wage inequality). Two key advantages of the RIF regression approach over, say, conventional inequality decomposition methods (Shorrocks 1984) is that RIF regressions (i) apply generally to any conventional statistic of interest and (ii) allow us to assess the distributive impact of foreign workers both unconditionally and conditionally, that is, holding covariates (such as human capital and job characteristics) that may account for wage differentials between native and foreign workers constant, as we describe in more details in Section 2.[2]

With a share of foreign workers in total employment above 70 percent, Luxembourg is the European Union country relying most on foreign labour to fuel its domestic economic activity. These foreign workers are composed of both immigrants (that is, foreign workers residing in Luxembourg) and cross-border workers (that is, foreign workers residing in neighbouring countries—Belgium, France or Germany—but who are employed and work in Luxembourg). As per official statistics from Statec, in the first quarter of 2013, Luxembourg natives represented just 29.2 percent of total employment, immigrants represented 26.8 percent and cross-border workers represented 43.9 percent of total employment.[3] In the fourth quarter of 2006—the period covered by our analysis, see *supra*—the respective proportions were at 31.2 percent, 26.4 percent and 42.3 percent. This atypical situation arises from the small size of the domestic population, comparatively high wages and the supply of labour from neighbouring regions (see, e.g., Annaert 2004; OECD 2012).

Foreign labour in Luxembourg is largely European but is nonetheless heterogeneous in skill and human capital and polarized on both low-skill and high-skill positions (Amétépé and Hartmann-Hirsch 2011; Fusco et al. 2014). Such polarization is not uncommon. Similar concentration on both tails has been documented for example in Switzerland (Müller and Ramirez 2009) or the United Kingdom (Dustmann et al. 2013). Because of this concentration in both ends of the occupation ladder and their sheer number, foreign workers are typically perceived as pushing earnings dispersion upwards. This paper confronts such a claim to data derived from a matched employer-employee dataset, the 2006 Luxembourg Structure of Earnings survey. We separate out the potential contributions of immigrants and cross-border workers as they differ in their characteristics—the latter being generally younger, better educated and with more recent and weaker attachment to the Luxembourg labour market. Cross-border workers are also less strongly polarized in skills and occupations than immigrants, exhibit lower within-group wage inequality and may therefore be expected to have smaller influence on overall wage dispersion.

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across the distribution: the overall effect on native wages is positive as a combination of a negative effect at lower percentiles of the distribution but a positive effect at higher percentiles. No or small positive impacts have been identified in Spain and Israel (Carrasco et al. 2008; Friedberg 2001).

2 See Van Kerm et al. (2017) for a study of the anatomy of wage differentials between natives and foreign workers using alternative approaches.

3 See http://www.statistiques.public.lu/stat/TableViewer/tableView.aspx?ReportId=12916 (accessed 2018-08-30).

Our first baseline finding is that, with only a few exceptions, foreign workers tend to drive the wage distribution down: most quantiles of the wage distribution would be reduced by an increase in the share of foreign workers. This is a direct implication of the lower wages generally paid to foreign workers compared to natives (Van Kerm et al. 2017). However, we also find that their impact on wage inequality is most often insignificant or even negative. All effects are further muted when human capital and job characteristics are taken into account (that is, when one considers substitutions of native workers for foreign workers with similar characteristics).

Section 2 formally defines the parameters of interest and details the RIF regression methodology. Section 3 describes the data used in the analysis. Section 4 presents our results and Section 5 concludes.

2. Methods

Our analysis of the distributive footprint of foreign workers rests on a simple extension of the influence function regression methods developed in Firpo et al. (2009). We assess the contribution of foreign workers to the overall distribution of wages by measuring the impact that a (marginal) substitution of native workers by foreign workers would have on several distributional statistics (holding the wages of different types of workers unchanged). This impact depends on the locations of wage distributions of native and foreign workers relative to each other and reflect the implications of this configuration for different distributional statistics of interest.

2.1. Gâteaux Derivatives for Changes in Covariate Distributions

The central concept is the *directional derivative* of statistical functionals known as the Gâteaux derivative (Dugger and Lambert 2014; Gâteaux 1913). Let F be the cumulative distribution function of the random variable studied—here, wages in Luxembourg—and $v(F)$ denote a functional of interest, such as a mean, median, variance, or some more complex measure such as the Gini coefficient. Denote by G some alternative distribution function (which we will further specify below). The Gâteaux derivative of v at F *in the direction of* G captures how v responds to an infinitesimal modification of F obtained by mixing the two distributions

$$\nabla v_{F\to G} := \lim_{\epsilon\downarrow 0} \frac{v(H_\epsilon^{F,G}) - v(F)}{\epsilon} \tag{1}$$

where

$$H_\epsilon^{F,G}(y) = (1-\epsilon)F(y) + \epsilon G(y)$$

(see Hampel et al. 1986; Huber 1981).

To study the contribution of foreign workers, let us construct G as follows. First, write the distribution F as a combination of wage distributions for different workers types weighted by their respective employment shares:

$$F(y) = \sum_{x\in\Omega} s_x\, F_{Y|X=x}(y)$$

where Ω denotes a set of K workers types, s_x is the proportion of workers of type x in total employment, and $F_{Y|X=x}$ denotes the wage distribution among workers of type x. We will here distinguish workers by their nationality and country of residence and form a partition across three types $\Omega = \{\text{National resident}, \text{Foreign resident}, \text{Cross-border worker}\}$ (see below for detailed definitions). Let us now define a distribution obtained by increasing the share of type k workers and reducing the share of type r workers (the 'reference' type) by the same amount, but holding the conditional distributions $F_{Y|X=x}$ constant:

$$G_k^1(y) := (s_k+t)\, F_{Y|X=k}(y) \;+\; (s_r-t)\, F_{Y|X=r}(y) \;+ \sum_{x\in\Omega\setminus\{k,r\}} s_x\, F_{Y|X=x}(y).$$

The distributions F and G_k^1 are obtained by 'swapping shares' and differ only in the relative proportions of workers k and r. With this definition, the Gâteaux derivative of $v(F)$ in the direction of G_k^1, $\nabla v_{F\to G_k^1}$, captures how v would respond to an exchange of type r workers for type k workers. We label this measure the *unconditional effect* (UE) of workers type k on v

$$\mathrm{UE}(v(F),k) := \nabla v_{F\to G_k^1}.$$

The sign and magnitude of $\mathrm{UE}(v(F),k)$ depend on differences in the conditional distributions of the different types of workers, their shares, and on the nature of v. For example, imagine type k workers were mainly found in both tails of the wage distribution (that is, if $F_{Y|X=k}$ had a bimodal distribution with modes at high and low wages) and type r workers were mainly located in the middle of the wage distribution. One would then expect $\mathrm{UE}(v(F),k)$ for central tendency measures (such as the mean or the median) to be about zero, while $\mathrm{UE}(v(F),k)$ for measures of inequality or dispersion would be positive since we would have more workers with wages in the tails of the distributions.

Our definition of G_k^1 is a special case of the more general form of counterfactual distribution G^* examined in Firpo et al. (2009) which is obtained by changing the (joint) distribution of (potentially many) conditioning variables from F_X to G_X but holding the conditional distributions of wages $F_{Y|X=x}$ constant,

$$G^*(y) := \int_{\Omega_X} F_{Y|x}(y)dG_X(x).$$

Firpo et al. (2009, Theorem 1) demonstrate that for such a G^*, the Gâteaux derivative is obtained by integrating the conditional expectation of the *influence function* of $v(F)$ with respect to the conditioning variables X,[4]

$$\nabla v_{F\to G^*} = \int_{\Omega_X} \mathrm{E}\left[\mathrm{IF}(y;v,F)|X=x\right] d(G_X^* - F_X)(x). \tag{2}$$

(The shape of the influence function $\mathrm{IF}(y;v,F)$ is described in more detail below.) Firpo et al. (2009, Corollary 1) further show that in case distribution G_X^* is obtained by applying a 'location shift' to a continuous variable from X_j to $X_j + t$ then $\nabla v_{F\to G^*}$ is equal to the average partial derivative

$$\nabla v_{F\to G^*} = \int \frac{\partial \mathrm{E}\left[\mathrm{IF}(y;v,F)|X=x\right]}{\partial x} dF_X(x). \tag{3}$$

This motivates Firpo et al.'s (2009) application of (recentered) influence function regression to estimate $\nabla v_{F\to G^*}$ (see below).

A 'location shift' is inadequate for covariates following a multinomial distribution—such as our workers type variable—since the distribution of $X + t$ is undefined when X can only take on a set of fixed, discrete values. An analogous expression can be however derived for G^* based on 'shares swaps' rather than 'location shifts'. Consider first the case of a single covariate. The 'shares swap' described above to construct G_k^1 consists in exchanging a fixed fraction of type k data mass for type r data mass. We show in Appendix A that $\nabla v_{F\to G_k^1}$ based on such a 'shares swap' can be expressed as

$$\nabla v_{F\to G_k^1} = (\mathrm{E}\left[\mathrm{IF}(y;v,F)|X=k\right] - \mathrm{E}\left[\mathrm{IF}(y;v,F)|X=r\right]) \times t \tag{4}$$

where $\mathrm{E}[\mathrm{IF}(y;v,F)|X=x]$ is the expected value of $\mathrm{IF}(y;v,F)$ among workers of type x. This result is a extension of Firpo et al. (2007, Corollary 3) to the case of multinomial variables and is a discrete data variant of Equation (3). Note however the presence of the scaling factor t. While Firpo et al. (2009) set $t = 1$ for the 'location shift' definition of G^*, this would imply implausible subgroup shares $(s_r - t) \leq 1$

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[4] Please note that Theorem 1 in Firpo et al. (2009) integrates the *recentered* influence function defined as $\mathrm{RIF}(y;v,F) = v(F) + \mathrm{IF}(y;v,F)$. Our expression in terms of the influence function is equivalent since $v(F)$ in the recentered influence function expression of the theorem can be differenced away.

and $(s_k + t) \geq 1$ in our definition of G^*. We adopt instead $t = 0.10$ to keep counterfactual subgroup shares between 0 and 1.

Consider now the case of multiple covariates. One limitation of G_k^1 is that changing the shares of workers k and r may imply changing the distribution of other relevant characteristics that determine wages and are correlated with workers types. In this case, it would be unclear if $\mathrm{UE}(v(F), k)$ captures the effect of changes in workers type directly or through the implied change in other characteristics. Imagine foreign workers are low-skilled and primarily work in low-paid occupations. The UE on, say, mean wages of an increase in the proportion of foreign workers is then likely negative, since it increases disproportionately low-paid workers overall. In the presence of multiple conditioning variables, there is interest in constructing a 'shares swap' which transforms the distribution of variable X but leaves the distribution of other observable conditioning variables Z unchanged, as Firpo et al. (2009) do with location shift counterfactuals. We therefore consider a substitution between native and immigrant workers done conditionally on a set of human capital and/or job characteristics Z. A proportion t of workers are assumed substituted within all configurations of Z so as to marginally change the proportion of foreign workers but preserve the overall distribution of workers' characteristics Z in the population. This is obtained by defining

$$
\begin{aligned}
G_k^2(y) \quad := \quad & \int_{\Omega_Z} \Big((s_{k|Z=z} + t)\, F_{Y|X=k,Z=z}(y) \;+\; (s_{r|Z=z} - t)\, F_{Y|X=r,Z=z}(y) \\
& \quad + \sum_{x \in \Omega \setminus \{k,r\}} s_{x|Z=z}\, F_{Y|X=x,Z=z}(y) \Big)\, f_Z(z)dz
\end{aligned}
$$

and $F_{Y|X=x,Z=z}$ and $s_{x|Z=z}$ now denote respectively the conditional distribution of wage given worker type x and characteristics z and the share of workers of type x among workers with characteristics z. This substitution leaves the distribution of covariates Z unchanged. Appendix A shows that, by an immediate extension of Firpo et al. (2007, Corollary 3), $\nabla v_{F \to G_k^2}$ also takes the form of a discrete average partial effect

$$
\begin{aligned}
\mathrm{UPE}(v(F), k) \quad & := \quad \nabla v_{F \to G_k^2} \\
& = \quad \left(\int_{\Omega_Z} \mathrm{E}[\mathrm{IF}(y; v, F) | X = k, Z = z] - \mathrm{E}[\mathrm{IF}(y; v, F) | X = r, Z = z]\ f_Z(z)dz \right) t.
\end{aligned} \tag{5}
$$

This defines the second quantity of interest that we examine in the paper and that we label the 'unconditional *partial* effect' (UPE) of workers type k on v, $\mathrm{UPE}(v(F), k)$, as Firpo et al. (2009) do.[5]

2.2. Influence Functions

The influence function of a functional reflects its sensitivity to different areas of the distribution. Formally, the influence function is the Gâteaux derivative of F in the direction of a Dirac distribution Δ_y that has point mass at y:

$$
\mathrm{IF}(y; v, F) := \nabla v_{F \to \Delta_y} = \lim_{\epsilon \downarrow 0} \frac{v((1-\epsilon)F + \epsilon \Delta_y) - v(F)}{\epsilon}
$$

and $\Delta_y(s) = 0$ if $s < y$ and 1 otherwise (Hampel 1974). Expressions for influence functions have been derived for most functionals commonly used in distributive analysis (see, e.g., Essama-Nssah and Lambert 2012).

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[5] The unconditional partial effect is labelled a 'policy effect' in Rothe (2010) or a 'counterfactual effect' in Chernozhukov et al. (2013).

In our empirical application, we examine a variety of distributional functionals: (i) 19 ventiles are examined to show the general contribution of foreign workers to the overall wage distribution, and (ii) several dispersion measures are examined to capture their influence on wage inequality, namely the variance, the Gini coefficient, and the percentile ratios P90/P10, P90/P50 and P50/P10.[6] These functionals have well-known influence function. The IF for quantile τ is

$$\mathrm{IF}(y; q_\tau, F) = \frac{\tau - \mathbf{1}\left[y \le q_\tau\right]}{f(q_\tau)}$$

where $f(q_\tau)$ is the density function at the quantile τ (Firpo et al. 2009). The IF for ratios of quantiles is obtained by applying the derivation rules described in Deville (1999):

$$\mathrm{IF}\left(y; \mathrm{R}(q_h, q_l), F\right) = \frac{1}{q_l}\left(\mathrm{IF}(y; q_h, F) - \frac{q_h}{q_l}\mathrm{IF}(y; q_l, F)\right)$$

where $\mathrm{R}(q_h, q_l) = \frac{q_h}{q_l}$ denotes the ratio of two quantiles, corresponding here to the percentiles ratios P90/P10, P90/P50 and P50/P10. Finally, the IF for the variance and the Gini coefficient are, respectively,

$$\mathrm{IF}(y; \mathrm{Var}, F) = \mathrm{Var}(F) + (y - \mu(F))^2$$

and

$$\mathrm{IF}(y; \mathrm{GINI}, F) = -\frac{\mu(F) + y}{\mu(F)}\mathrm{GINI}(F) + 1 - \frac{y}{\mu(F)} + \frac{2}{\mu(F)}\int_0^y F(x)dx$$

(see, e.g., Essama-Nssah and Lambert 2012).

2.3. Estimation by Influence Function Regression

$\mathrm{UE}(v(F), k)$ and $\mathrm{UPE}(v(F), k)$ are functions of conditional expectations of influence functions evaluated at different values of covariates. Assuming a linear and additive relationship between x, z and $\mathrm{IF}(y; v, F)$ leads to an estimator for UPE or UPE called the RIF-OLS estimator by Firpo et al. (2009):

$$\mathrm{E}[\mathrm{IF}(y; v, F)|X = x, Z = z] = \alpha + z\gamma + \sum_{g \in \Omega \setminus \{r\}} \beta_g \mathbf{1}\left[g = x\right] \tag{6}$$

where $\mathbf{1}\left[g = x\right]$ is a dummy for worker type x and z is a vector of potential additional covariates. Inserting Equation (6) in Equations (4) and (5) shows that, under this specification, $t\beta_k$ equals $\mathrm{UE}(v(F), k)$ in the absence of any additional covariates z_i and equals $\mathrm{UPE}(v(F), k)$ if covariates z_i are included in the model. (Notice that the dummy for natives ($\mathbf{1}\left[g = r\right]$) is taken as omitted category.). An influence function regression therefore provides a straightforward way to estimate UE and UPE.

A more flexible specification can allow for interactions between X and Z in the estimation of UPE and still provide straightforward estimation

$$\mathrm{E}[\mathrm{IF}(y; v, F)|X = x, Z = z] = \alpha + z\gamma + \sum_{g \in \Omega \setminus \{r\}} \left(\beta_g + z\gamma_g\right)\mathbf{1}\left[g = x\right]. \tag{7}$$

Inserting this specification in Equation (5) gives

$$\mathrm{UPE}(v(F), k) = t\left(\beta_k + \int_{\Omega_Z} z\gamma_k\, f_Z(z)dz\right), \tag{8}$$

[6] Estimates based on several other relative inequality measures were also examined (quantile group shares ratios, the standard deviation of log wage, generalized entropy measures) and lead to similar conclusions. They are not reported here but are available on request.

that is, UPE$(v(F), k)$ is the discrete partial effect of the workers type dummy variables on IF$(y; v, F)$ averaged over all additional covariates distributions.

Note that the dependent variable in the RIF-OLS regressions (IF$(y_i; v, F)$) is a function of F which is unknown but itself derived from one's sample. First, the value of IF$(y_i; v, \hat{F})$ is computed for all sample observations i for the statistic of interest v. Second, IF$(y_i; v, \hat{F})$ is regressed by OLS on worker type dummies x_i and, for UPE$(v(F), k)$, on additional covariates z_i. This two-stage procedure therefore results in complex sample dependence between observations which can be taken into account by resorting to bootstrap resampling for inference (Firpo et al. 2009).

3. Data

Our analysis exploits data from the 2006 Luxembourg Structure of Earnings Survey. The survey is collected in all European Union countries on the basis of common variable definitions and sampling design defined in European Community regulations. It aims to provide detailed information on earnings in the European Union. The Luxembourg SES is collected by STATEC—Institut national de la statistique et des études économiques, the national statistical institute.

The SES is a nationally representative matched employer-employee survey covering, in 2006, non-profit and private sector firms (NACE C–K and M–O) employing at least 10 workers. This sampling frame covers 79 percent of salaried workers in Luxembourg at the time of the survey (STATEC 2009).[7] The distinctive feature of the SES in the context of Luxembourg is that—since it is based on a sampling frame of firms—it collects information on both resident and cross-border workers.

The survey has a two-stage design. A sample of firms (stratified by firm size) was drawn in a first stage. A sample of workers from the selected firms was drawn in a second stage. In total, the 2006 Luxembourg SES dataset covers 1856 firms and 31,329 workers (STATEC 2009). Information is available on both employers (sector of activity, size, collective agreement coverage) and employees (earnings plus basic demographic information (including educational achievements) and occupation and job characteristics).

Table 1 describes the distribution of gross hourly wage in our sample for three distinct groups of workers: Luxembourg nationals, immigrants (foreign residents) and cross-border worker. Hourly wage is calculated as the earnings received in the month of reference of the survey (October 2006) divided by the number of paid hours worked in the month. We limit the sample to workers aged 18 to 65. Luxembourg workers have much higher wages than foreign workers. Mean wage of Luxembourg workers is 29% higher than mean wage of immigrant workers (€23.23 versus €18.02) and 30% higher than mean wage for cross-border workers (€23.23 versus €17.83). Median wages differ across the groups in even stronger proportions with Luxembourg workers having 46% and 36% higher median wage than immigrant workers and cross-border workers respectively. While cross-border workers and immigrant workers have similar mean and median wage, wage dispersion—whether measured by the Gini coefficient, the variance of wages or percentile ratios—is much higher among immigrant workers. In turn, native workers exhibit about the same degree of wage inequality as immigrant workers, but at much higher levels of wage. Differences in the distribution of wages across the three groups are illustrated further in Figure 1 which shows the density function of the overall wage distribution (on a logarithmic scale) along with the distributions for the three subgroups scaled by their employment shares. The three groups clearly exhibit different distributions of wage both in location and in spread. Given this complex configuration, the contribution of foreign workers to overall inequality is far from obvious and is difficult to deduce from subgroup summary indicators.

[7] Most noticeably, civil servants and agricultural sector workers are excluded from the sampling frame. These sectors employ only few foreign workers (in particular cross-border workers).

Table 1. Employment share and hourly wage distribution statistics by worker type (Luxembourg nationals, immigrants and cross-border workers).

	Luxembourg Nationals	**Immigrant Workers**	**Cross-Border Workers**
Employment share	0.25	0.27	0.49
Mean and selected percentiles (€)			
Mean	23.2	18.0	17.8
10th percentile (P10)	10.7	9.1	10.2
25th percentile (P25)	14.5	10.9	12.0
Median (P50)	20.3	13.9	14.9
75th percentile (P75)	27.9	20.1	20.4
90th percentile (P90)	37.0	31.6	28.8
Measures of dispersion and inequality			
Standard deviation	15.3	13.2	10.3
Gini coefficient	0.284	0.303	0.251
P90/P10 ratio	3.5	3.5	2.8
P50/P10 ratio	1.9	1.5	1.5
P90/P50 ratio	1.8	2.3	1.9

Notes: Based on the 2006 Luxembourg Structure of Earnings Survey. Sample weights applied.

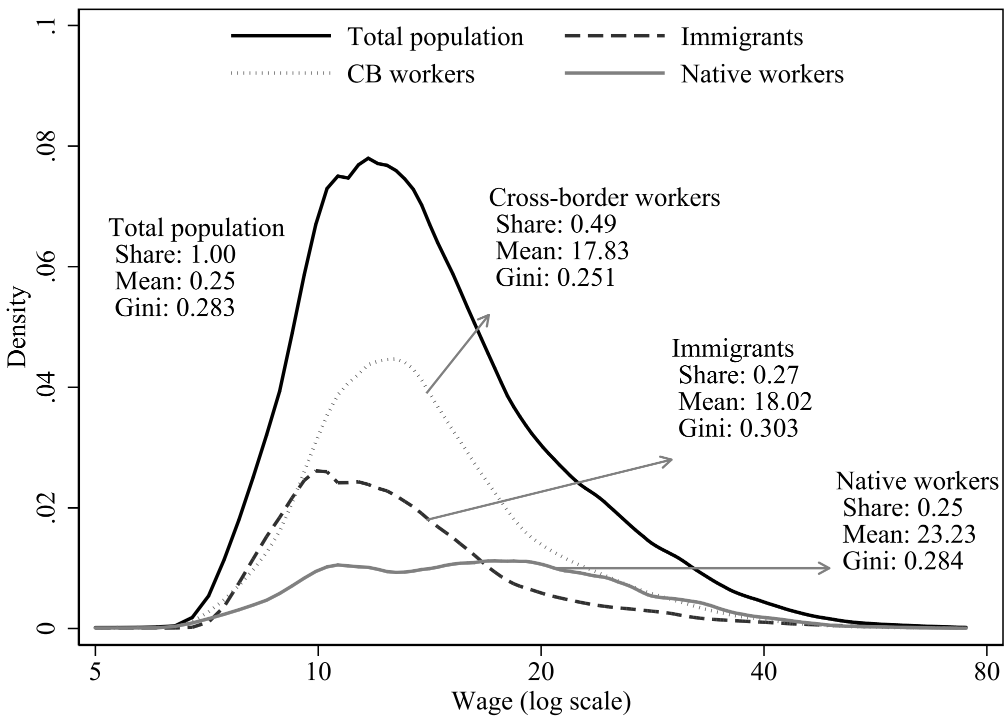

Figure 1. Hourly wage distribution: density function estimates for total population and by worker type. Note: Subgroup densities are scaled by subgroup employment share. Source: Structure of Earnings Survey for Luxembourg 2006.

Additionally, the three subgroups differ a lot in job and productivity-related characteristics; see Table 2. This partly accounts for the wage differences between natives and foreign workers. For instance, native workers are more likely to work in large firms, have much longer job tenure or more likely to hold supervisory positions. Immigrants are more likely to work in the construction sector while cross-border workers are more likely to work in the real estate sector. Worth noting is also the polarized distribution of educational achievements of immigrants—with a higher fraction of both primary and tertiary education workers. Cross-border workers generally have higher educational achievements, but are also younger and have the lowest job tenure. Careful qualification of the contribution of foreign workers to wage inequality ought to account for these differences.

Table 2. Detailed nationality, human capital and job characteristics by worker type (Luxembourg nationals, immigrants and cross-border workers).

	Luxembourg Nationals	Immigrant Workers	Cross-Border Workers
Luxembourg	1.00	–	–
Belgian	–	0.08	0.22
French	–	0.13	0.50
German	–	0.05	0.22
Portuguese	–	0.47	0.01
Other EU	–	0.18	0.04
Non-EU	–	0.10	0.01
Female	0.39	0.38	0.32
Age	39.90	37.63	37.20
Primary educ. or less (ref.)	0.11	0.24	0.08
Secondary education	0.80	0.62	0.80
Tertiary education	0.08	0.14	0.12
Years at current employer	11.82	6.32	5.59
Manager	0.17	0.14	0.14
10–49 employees in firm	0.24	0.32	0.27
50–249 employees in firm	0.24	0.30	0.35
250–499 employees in firm (ref.)	0.11	0.13	0.14
500–999 employees in firm	0.08	0.11	0.11
1000+ employees in firm	0.33	0.14	0.13
Part time contract	0.18	0.15	0.13
Industry/Manufacture	0.17	0.10	0.18
Construction	0.05	0.21	0.14
Wholesale	0.12	0.10	0.13
Hotel/Restaurant	0.01	0.06	0.03
Trans/Comm	0.16	0.07	0.09
Finance	0.17	0.16	0.17
Real estate	0.08	0.18	0.19
Education, Health & Other not-for-profit (ref.)	0.24	0.11	0.08
Managerial	0.07	0.06	0.04
Professional	0.10	0.09	0.12
Associate professional	0.23	0.13	0.18
Clerk	0.23	0.11	0.15
Service worker	0.09	0.10	0.11
Craft and trade worker	0.13	0.21	0.20
Manufacturers	0.08	0.09	0.13
Low skilled and laborer (ref.)	0.08	0.20	0.07
Number of observations	7537	8367	15105

Notes: Based on the 2006 Luxembourg Structure of Earnings Survey. Sample weights applied.

4. Results

How do foreign workers contribute to the shape of the private sector wage distributions in Luxembourg?

4.1. Unconditional Impacts: UE Estimates

Figure 2 shows our first baseline results: the UEs on a set of 19 quantiles from the 5th percentile to the 95th percentile for both immigrants and cross-border workers.[8] The quantile UEs are negative for both groups of workers: given the configuration of each group's wage distribution relative to each other, a marginal increase in the share of foreign workers would haul the overall wage distribution downwards. The quantile UEs for immigrants are lower than for cross-border workers until the 70th percentile. This is a reflection of immigrants' comparatively lower wages and their relatively larger concentration in the bottom part of the wage distribution. Beyond the 70th percentile, the quantile UEs for cross-border workers continue to decline while they start increasing for immigrants. This finding brings empirical support to claims that immigrants in Luxembourg include both top earners that contribute to high wages as much as natives do (yet not *more*) and low skill migrants that drive bottom quantiles down (Amétépé and Hartmann-Hirsch 2011). The pattern does not hold true however for cross-border workers.

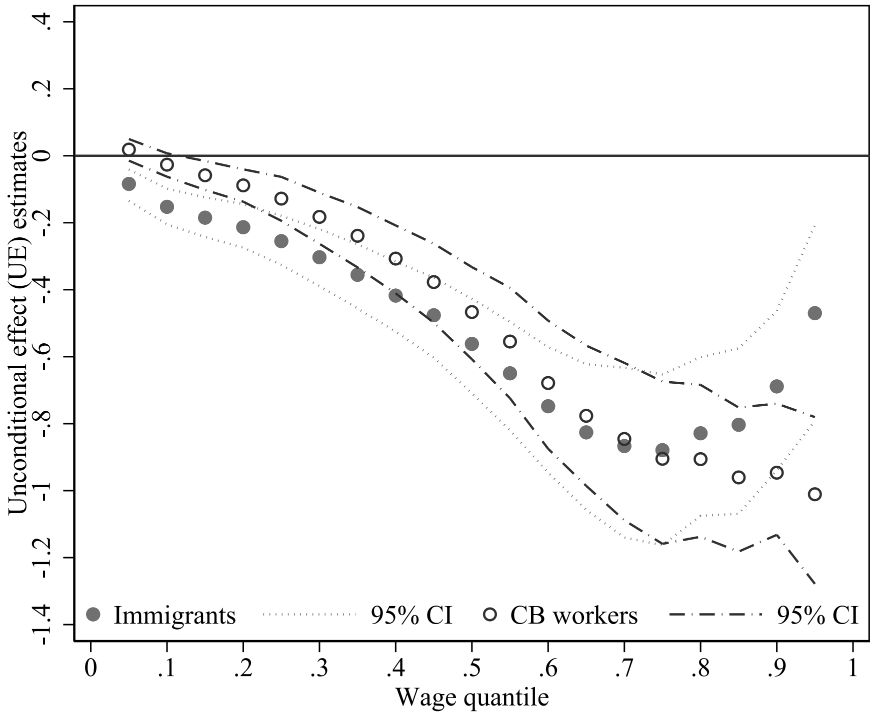

Figure 2. Estimates of unconditional effects (UE) on quantiles of the wage distribution for immigrant and cross-border workers.

What these patterns imply for wage dispersion and inequality can be read from estimates of UEs on inequality measures reported in Table 3 under Model 1. All estimates (but one) for both immigrant and cross-border workers are negative. The only exception is the contribution to the P90/P50 ratio of immigrant workers. Overall, wages of foreign workers tend to *reduce* inequality; moving towards a distribution composed of 10 percentage point less Luxembourg workers and 10 percentage points more foreign workers (holding the wage distributions within each group constant) would lead to a

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8 RIF regression calculations were done with the statistical software Stata (version 14.2) (StataCorp 2015) and the user-written package for Stata `rifreg` available from Nicole Fortin at http://faculty.arts.ubc.ca/nfortin/datahead.html. Bootstrap confidence intervals for the UE and UPE estimates were constructed on the basis of 1,000 replications from a repeated half-sample bootstrap resampling scheme (Saigo et al. 2001) and account for the two-stage design of the survey (see Section 3). We use the *rhsbsample* Stata user-written package for generating the replication weights (Van Kerm 2013). Pointwise confidence intervals are based on the bias-corrected percentile method (Efron 1981).

reduction of inequality. Magnitudes are very small however. They are close to zero and not statistically different thereof for immigrant workers. UE estimates are different from zero for cross-border workers, but they remain small. For example, using these estimates to approximate the change in the Gini suggests a reduction of the Gini of only 0.005 against a baseline Gini of 0.283.[9] This finding contrasts what could be conjectured from the polarization of foreign workers in both tails of the skill and pay distributions and the evidence on the quantile UEs from Figure 2. The wage distribution of foreign workers does not appear so polarized (compared to the distribution of native workers) as to drive inequality upwards.

The only exception to the negative UEs is the contribution of immigrant workers to the P90/P50 ratio. This ratio differs somewhat from the other indices in that it captures 'upper half inequality'. The rebound of the quantile UEs for immigrant workers at the top of the distribution shown in Figure 2 explains this exception. All quantiles would be dragged down by "moving towards" more immigrant workers but the decline is stronger at the median than at the top of the distribution. This suggests that a share of foreign workers are located at high wages and do contribute to upper-end inequality.

Table 3. Estimates of UEs (Model 1) and UPEs (Model 2 and Model 3) on the variance of hourly wages and indicators of relative inequality.

	UE	**UPE**			
	Model 1	**Model 2a**	**Model 2b**	**Model 3a**	**Model 3b**
			Variance		
Immigrant worker	−7.58	−6.97	−7.92	−1.91	−1.16
	(6.75)	(7.85)	(8.59)	(4.83)	(3.53)
Cross-border worker	−14.17	−13.52	−14.57	−6.76	−5.31
	(6.20)	(7.29)	(7.85)	(4.30)	(2.59)
			Gini coefficient		
Immigrant worker	−0.0003	−0.0004	−0.0007	0.0002	0.0006
	(0.0016)	(0.0017)	(0.0019)	(0.0012)	(0.0011)
Cross-border worker	−0.0054	−0.0053	−0.0053	−0.0030	−0.0021
	(0.0015)	(0.0015)	(0.0017)	(0.0010)	(0.0008)
			Percentile ratio P90/P10		
Immigrant worker	−0.022	−0.005	−0.009	0.003	0.010
	(0.014)	(0.013)	(0.013)	(0.011)	(0.012)
Cross-border worker	−0.091	−0.071	−0.068	−0.044	−0.026
	(0.013)	(0.011)	(0.011)	(0.010)	(0.010)
			Percentile ratio P50/P10		
Immigrant worker	−0.032	−0.023	−0.024	−0.012	−0.012
	(0.005)	(0.005)	(0.004)	(0.003)	(0.003)
Cross-border worker	−0.043	−0.033	−0.032	−0.022	−0.020
	(0.005)	(0.005)	(0.004)	(0.004)	(0.003)

9 Formally, the approximation is the leading term of a von Mises expansion of functional differences (Fernholz 1983; Hampel 1974):

$$
\begin{aligned}
v(G) - v(F) &= \int_0^1 \mathrm{IF}(y; v, F)\, dG(y) + r(v, F, G) \\
&\approx \int_0^1 \mathrm{IF}(y; v, F)\, dG(y).
\end{aligned}
$$

The accuracy of the approximation depends on how close G is to F. The connection to the Gâteaux derivative becomes clearer if one sets $\epsilon = 1$ in the definition Equation (1): the von Mises approximation consists in linearly projecting the Gâteaux derivative (defined for infinitesimal mixing $\epsilon \to 0$) all the way from F to G (corresponding to $\epsilon = 1$). This is unlikely to be a good approximation for F and G far apart and therefore provides another argument for setting t relatively small in the defnition of G.

Table 3. *Cont.*

	UE	UPE			
	Model 1	**Model 2a**	**Model 2b**	**Model 3a**	**Model 3b**
		Percentile ratio P90/P50			
Immigrant worker	0.028	0.027	0.026	0.017	0.021
	(0.011)	(0.009)	(0.009)	(0.007)	(0.007)
Cross-border worker	−0.002	−0.002	−0.001	0.001	0.009
	(0.011)	(0.010)	(0.009)	(0.007)	(0.007)

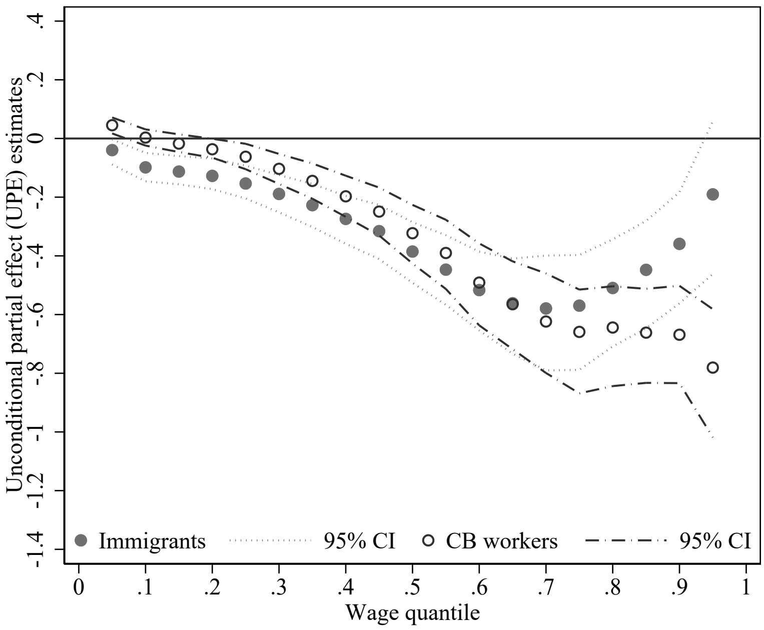
Notes: Based on the 2006 Luxembourg Structure of Earnings Survey data. Model 1 does not include covariates (and therefore estimates UEs), Model 2 includes human capital covariates, Model 3 includes both human capital and job characteristics. Models 2a and 3a are based on a simple additive RIF-OLS regression models, models 2b and 3b allow interaction between nationality groups and all covariates in the RIF-OLS regression. Bootstrap standard errors in brackets.

4.2. Accounting for Human Capital and Job Characteristics: UPE Estimates

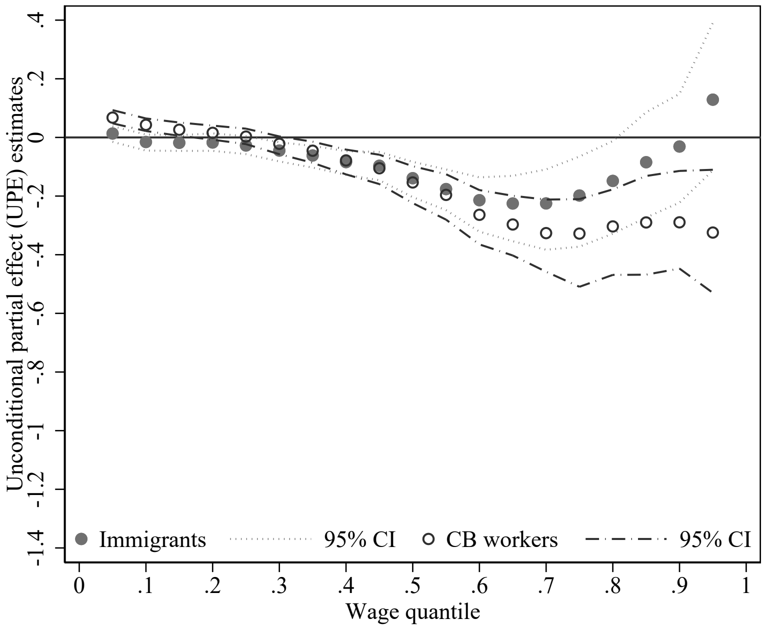
Cross-border workers and immigrants have markedly different characteristics from native workers. It is therefore useful to consider UPEs that capture the locations of each subgroup's wage distributions relative to each other conditionally on covariates. This is where the interest of the methodology materializes since traditional inequality decomposition analysis does not lean itself to examining conditional wage distributions with a rich set of covariates. UPE captures the direction of a change in quantiles and inequality measures for a change in the share of foreign and national workers that would keep constant the overall distribution of human capital and job characteristics.

UPEs on 19 quantiles are shown in Figure 3: the top panel accounts for individual characteristics only (age, gender and level of education), the bottom panel accounts for both individual and job characteristics. (Full regression coefficient estimates for three selected quantiles are reported in Appendix B.) Adjusting for individual characteristics reduces only moderately the absolute impact of foreign workers. However, further adjusting for job characteristics markedly reduces this impact. Quantile UPEs remain generally negative (except at the very bottom for cross-border workers and at the very top for immigrants), but they are much smaller in absolute value and globally significantly below zero only between the 35th and 80th percentiles. UPEs are, overall, very small throughout the bottom half of the distribution. UPEs for cross-border workers become more markedly negative for quantiles above the median and UPEs for immigrants display again a U-shape with declining values until the 65th percentile and an increase up to about zero for the highest quantile.

The impacts of foreign workers on inequality measures are further muted. Table 3 under Model 2 and Model 3 reports estimates of inequality UPEs. Estimates under Model 2 control for human capital characteristics only; estimates under Model 3 also control for job characteristics. Two sets of estimates are reported. Estimates under Model 2a and 3a are based on the basic additive IF-OLS model of Equation (6)—also used in Figure 3. Estimates under Model 2b and 3b are based on the more flexible specification (7) allowing for interactions. All UPE estimates have the same sign as the UE estimates. They remain negative and mostly significantly different from zero for cross-border workers, while immigrant workers' impacts on wage dispersion and inequality remain negative, small and not significantly different from zero, with the exception of the P90/P50 ratio. The size of the coefficients in Model 2 is comparable to UE estimates (Model 1). The coefficients fall in (absolute) size in Model 3 as part of the wage differences is driven by job characteristics, yet the coefficients tend to remain significantly different from zero where the UEs were already significant.



(**a**)



(**b**)

Figure 3. Estimates of unconditional partial effects (UPE) on quantiles of the wage distribution for immigrant and cross-border (CB) workers. (**a**) Conditional on age, education level and gender; (**b**) Conditional on age, education level, gender and job characteristics.

4.3. Impacts by Disaggregate Nationality Groups

Disaggregation of foreign workers types by less coarsely defined nationality groups leads to more homogeneous subgroups and sheds some light on the patterns just observed. Cross-border workers from France, Belgium and Germany form a relatively homogeneous labour force in terms of skill composition. There is much more heterogeneity across immigrant groups. Portuguese immigrants with generally low educational achievements form the largest share of immigrants (47 percent in our sample). Belgian, French and German immigrants taken together represent 26 percent of our sample, other EU immigrants, 18 percent and non-EU immigrants, 10 percent.

Figures 4 and 5 show disaggregated quantile UEs and UPEs by country of residence (for cross-border workers) and by broad nationality groups (for immigrants). The coefficients are for marginal impacts of substituting workers from one of these groups against native residents. Unsurprisingly, the impacts of the three cross-border groups are similar, with the largest negative impact attributed to Belgian residents. There is much more heterogeneity across immigrant groups. Portuguese immigrants are consistently found to depress all quantiles: they are paid relatively low wages. However the impact largely disappears (even at high quantiles) after controlling for individual and job characteristics. At the other end of the spectrum, Belgian, German and French residents appear to have positive UEs, especially at top quantiles: they are paid higher wages than natives and drive up the top quantiles.[10] Non-EU and other EU immigrants have parallel profiles although at different levels. UEs and UPEs for both groups exhibit a markedly U-shape; this suggests that a fraction of the population in these groups tend to receive low pay (and therefore quickly reduce quantiles in the bottom of the distribution) while a fraction of the groups is highly paid and increases top quantiles. This pattern is particularly strong for non-EU immigrants that—after accounting for human capital and job characteristics—is the group that depresses most bottom quantiles and increases most top quantiles.

Table 4 shows disaggregated UPE estimates for the Gini and the variance and Table 5 shows UPE estimates for the percentile ratios. The P90/P50 ratio apart, the only group that appears to contribute positively and significantly to inequality is the non-EU immigrants. By contrast, wages of French and Belgian cross-border workers are consistently found to significantly reduce inequality (with similar orders of magnitude). All other groups generally have no systematically significant impacts after controlling for human capital and job characteristics. The UPEs of Portuguese residents is negative and significant on the variance but the effects disappear on the Gini coefficient.

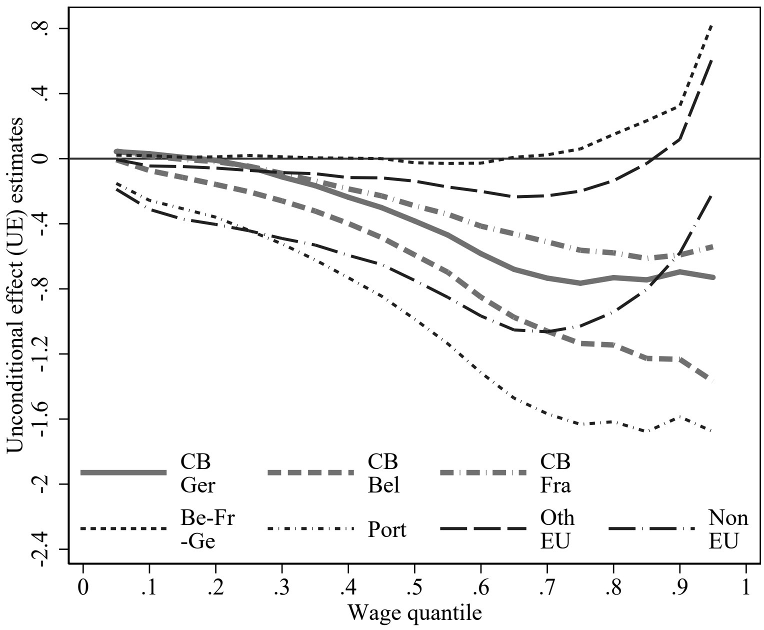
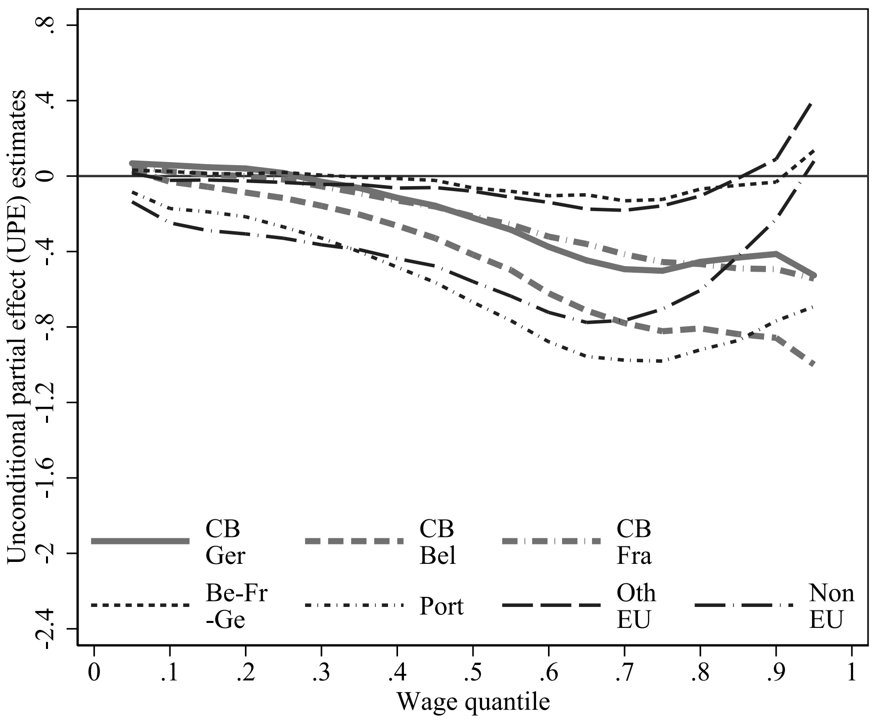
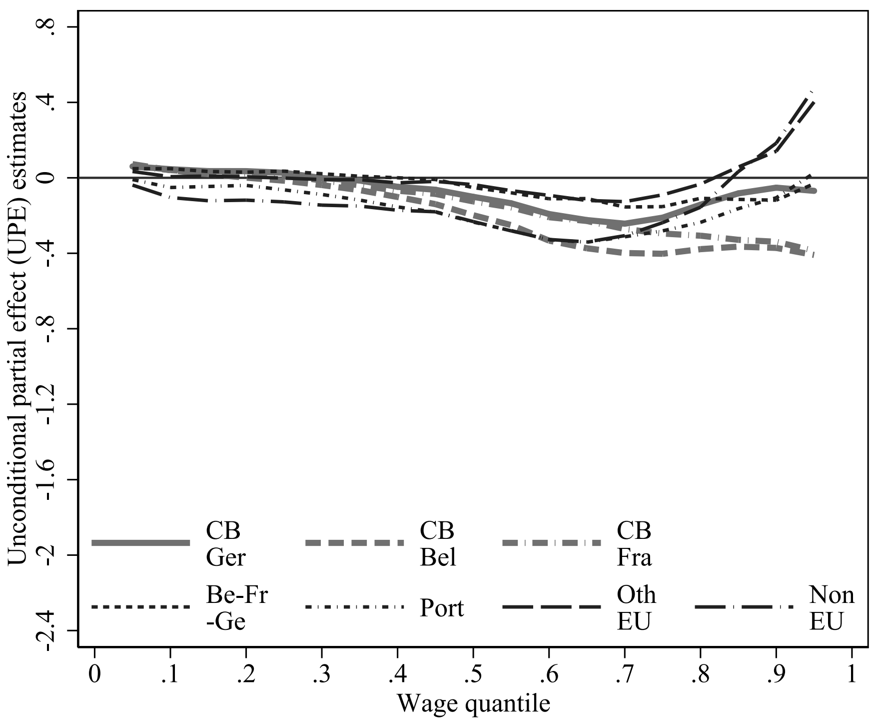


Figure 4. Estimates of unconditional effects (UE) on quantiles of the wage distribution for immigrant and cross-border workers disaggregated by nationality groups.

[10] Endogenous selection is likely at play here with high wage workers from Belgium, France or Germany affording the potential costs of migrating into Luxembourg.



(**a**)



(**b**)

Figure 5. Estimates of unconditional partial effects (UPE) on quantiles of the wage distribution for immigrant and cross-border workers disaggregated by nationality groups. (**a**) Conditional on age, education level and gender; (**b**) Conditional on age, education level, gender and job characteristics.

Examination of percentile ratios shows again how the U-shape of quantile UEs imply that foreign workers tend to increase dispersion in the upper half of the distribution and reduce dispersion in the bottom half. This is mostly marked for non-EU residents.

Table 4. Estimates of UEs (Model 1) and UPEs (Model 2 and Model 3) on the variance of hourly wages and the Gini coefficient for disaggregated nationality groups.

	UE	UPE			
	Model 1	**Model 2a**	**Model 2b**	**Model 3a**	**Model 3b**
			Variance		
Be-Fr-Ge resident	3.91	−6.84	−3.91	−6.37	−4.74
	(9.31)	(11.77)	(10.57)	(9.33)	(5.34)
Portuguese resident	−19.42	−10.56	−20.31	−1.41	−13.45
	(7.13)	(6.95)	(8.83)	(3.03)	(3.79)
Other EU resident	−0.32	−4.95	−7.33	−2.66	−4.42
	(8.02)	(9.42)	(9.25)	(6.52)	(4.23)
Non-EU resident	5.42	5.98	0.63	11.51	3.58
	(5.33)	(5.09)	(6.85)	(5.60)	(8.46)
German CB	−10.72	−10.41	−11.22	−3.62	−1.77
	(6.63)	(7.76)	(8.29)	(5.40)	(4.47)
French CB	−17.66	−15.51	−17.82	−7.27	−8.71
	(6.62)	(7.56)	(8.34)	(4.05)	(2.95)
Belgian CB	−10.20	−11.92	−12.07	−8.23	−4.32
	(5.24)	(6.57)	(6.68)	(4.08)	(2.22)
			Gini coefficient		
Be-Fr-Ge resident	0.0029	−0.0019	−0.0006	−0.0026	−0.0016
	(0.0019)	(0.0023)	(0.0021)	(0.0018)	(0.0012)
Portuguese resident	−0.0046	−0.0012	−0.0063	0.0007	−0.0061
	(0.0019)	(0.0018)	(0.0019)	(0.0010)	(0.0010)
Other EU resident	0.0027	0.0006	−0.0003	0.0007	−0.0003
	(0.0022)	(0.0024)	(0.0022)	(0.0018)	(0.0013)
Non-EU resident	0.0060	0.0058	0.0047	0.0057	0.0050
	(0.0021)	(0.0020)	(0.0026)	(0.0018)	(0.0028)
German CB	−0.0044	−0.0043	−0.0037	−0.0013	−0.0001
	(0.0016)	(0.0017)	(0.0019)	(0.0014)	(0.0016)
French CB	−0.0066	−0.0059	−0.0067	−0.0034	−0.0039
	(0.0016)	(0.0016)	(0.0018)	(0.0010)	(0.0008)
Belgian CB	−0.0040	−0.0047	−0.0046	−0.0036	−0.0019
	(0.0014)	(0.0015)	(0.0015)	(0.0010)	(0.0007)

Notes: Based on the 2006 Luxembourg Structure of Earnings Survey data. Model 1 does not include covariates (and therefore estimates UEs), Model 2 includes human capital covariates, Model 3 includes both human capital and job characteristics. Models 2a and 3a are based on a simple additive RIF-OLS regression models, models 2b and 3b allow interaction between nationality groups and all covariates in the RIF-OLS regression. Bootstrap standard errors in brackets.

Table 5. Estimates of UEs (Model 1) and UPEs (Model 2 and Model 3) on three percentile ratios (P90/P10, P50/P10 and P90/P50) for disaggregated nationality groups.

	UE	UPE			
	Model 1	**Model 2a**	**Model 2b**	**Model 3a**	**Model 3b**
		Percentile ratio P90/P10			
Be-Fr-Ge resident	0.028	−0.011	0.001	−0.029	−0.011
	(0.016)	(0.013)	(0.013)	(0.013)	(0.012)
Portuguese resident	−0.082	−0.023	−0.091	0.007	−0.082
	(0.019)	(0.018)	(0.017)	(0.013)	(0.015)
Other EU resident	0.027	0.017	0.013	0.013	0.009
	(0.021)	(0.019)	(0.018)	(0.018)	(0.016)
Non-EU resident	0.042	0.058	0.042	0.055	0.067
	(0.026)	(0.023)	(0.028)	(0.020)	(0.028)
German CB	−0.083	−0.063	−0.046	−0.021	−0.004
	(0.015)	(0.013)	(0.017)	(0.014)	(0.021)
French CB	−0.106	−0.081	−0.084	−0.052	−0.048
	(0.014)	(0.012)	(0.012)	(0.010)	(0.011)
Belgian CB	−0.068	−0.059	−0.059	−0.048	−0.026
	(0.013)	(0.011)	(0.011)	(0.011)	(0.010)
		Percentile ratio P50/P10			
Be-Fr-Ge resident	−0.005	−0.010	−0.009	−0.013	−0.009
	(0.004)	(0.004)	(0.004)	(0.004)	(0.005)
Portuguese resident	−0.059	−0.040	−0.048	−0.015	−0.023
	(0.008)	(0.007)	(0.008)	(0.005)	(0.007)
Other EU resident	−0.007	−0.005	−0.004	−0.005	−0.002
	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)
Non-EU resident	−0.026	−0.017	−0.024	−0.007	−0.011
	(0.009)	(0.008)	(0.008)	(0.007)	(0.006)
German CB	−0.044	−0.032	−0.030	−0.018	−0.022
	(0.007)	(0.006)	(0.006)	(0.004)	(0.005)
French CB	−0.048	−0.038	−0.036	−0.027	−0.022
	(0.006)	(0.005)	(0.005)	(0.004)	(0.004)
Belgian CB	−0.032	−0.025	−0.027	−0.019	−0.021
	(0.005)	(0.004)	(0.004)	(0.004)	(0.004)
		Percentile ratio P90/P50			
Be-Fr-Ge resident	0.025	0.006	0.012	−0.001	0.004
	(0.009)	(0.008)	(0.008)	(0.008)	(0.008)
Portuguese resident	0.024	0.037	0.004	0.025	−0.023
	(0.017)	(0.014)	(0.015)	(0.009)	(0.012)
Other EU resident	0.026	0.017	0.013	0.014	0.008
	(0.012)	(0.011)	(0.010)	(0.011)	(0.010)
Non-EU resident	0.060	0.059	0.058	0.043	0.057
	(0.014)	(0.012)	(0.015)	(0.011)	(0.016)
German CB	0.004	0.002	0.011	0.010	0.026
	(0.012)	(0.010)	(0.012)	(0.008)	(0.014)
French CB	−0.004	−0.002	−0.006	0.002	−0.002
	(0.013)	(0.011)	(0.010)	(0.008)	(0.008)
Belgian CB	−0.001	−0.004	−0.002	−0.006	0.010
	(0.009)	(0.009)	(0.009)	(0.007)	(0.007)

Notes: Based on the 2006 Luxembourg Structure of Earnings Survey data. Model 1 does not include covariates (and therefore estimates UEs), Model 2 includes human capital covariates, Model 3 includes both human capital and job characteristics. Models 2a and 3a are based on a simple additive RIF-OLS regression models, models 2b and 3b allow interaction between nationality groups and all covariates in the RIF-OLS regression. Bootstrap standard errors in brackets.

5. Summary and Conclusions

This paper assesses how foreign workers fit in and contribute to the shape of the wage distribution in Luxembourg, a high immigration country. We empirically confront claims that foreign workers wages inflate overall wage inequality. We do this indirectly. Following Firpo et al. (2009) and Rothe (2010), we define distributional parameters of interest that quantify how various distributional statistics would respond to a notional marginal substitution of native workers for foreign workers. This is estimated both unconditionally and conditionally on workers' human capital and job characteristics using influence function regression.

We remark first that quantiles of the wage distribution are generally driven *down* by foreign workers. Exceptions are only found for top quantiles which are driven up by immigrants from neighbouring countries and other EU countries (Portugal excluded) if we do *not* condition on covariates, and by non-EU immigrants and EU immigrants (Portugal and neighbouring countries excluded) if we condition on human capital and job characteristics. This is consistent with the fact that foreign workers are polarized and 'sandwich' the distribution at both high skill and low skill positions. The implication of these patterns for wage inequality is very limited however. There is hardly any indication that immigrants wages inflate the variance, the Gini coefficient or percentile ratios. The only significant exception is for non-EU immigrants—not more than 10 percent of immigrants—that appear to contribute positively to wage dispersion. All other immigrants affect inequality downwards if at all, while cross-border workers significantly drive inequality down.

Influence function regression reveals well-suited to the type of analysis conducted in this paper, and it could easily be expanded to additional distributional statistics, such as measures of earnings polarization or low pay. Of course, resulting estimates of marginal impacts must not be mis-interpreted as long-term general equilibrium effects of migration. We do not estimate longer-term equilibrium effects of such a change in employment composition or, to put it differently, we assume wages of employed workers to be unaffected by a marginal increase in foreign workers in total employment.[11] Our results instead provide descriptive evidence on how the structure of wages of foreign workers contribute to the overall wage distribution and in particular to wage inequality.

Author Contributions: Both authors contributed equally to the paper.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. Proofs

Assume for clarity, and without loss of generality, that the multinomial variable of workers type X takes on values $0, 1, \ldots, K$ and its cumulative distribution F_X can be written

$$F_X(x) = \sum_{j=0}^{K} s_j \mathbf{1}\left[x \geq j\right]$$

[11] Please note that estimates of general equilibrium effects of immigration available for other countries are in fact generally small (Blau and Kahn 2012; Card 2009), although of course these findings may not necessarily apply to the Luxembourg case.

(where $\mathbf{1}\left[\text{cond}\right]$ is 1 if cond is true and 0 otherwise). The distribution of workers type after shifting shares from the reference worker type r to worker type k, $G_X^{1,k}$, is then

$$G_X^{1,k}(x) = \sum_{j=0}^{K} s_j^* \mathbf{1}\left[x \geq j\right]$$

where $s_r^* = s_r - t$, $s_k^* = s_k + t$, and $s_j^* = s_j$ otherwise. The implied difference in the probability distributions $d(G_X^{1,k} - F_X)$ is

$$\begin{aligned} d(G_X^{1,k} - F_X)(x) &= t\left(\mathbf{1}\left[x = k\right] - \mathbf{1}\left[x = r\right]\right) \\ &= \begin{cases} t & \text{if } x = k \\ -t & \text{if } x = r \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Using Theorem 1 from Firpo et al. (2009) and inserting $d(G_X^{1,k} - F_X)(x)$ in Equation (2) we have our resulting expression

$$\text{UE}(v(F), k) = \left(\text{E}\left[\text{IF}(y; v, F)|X = k\right] - \text{E}\left[\text{IF}(y; v, F)|X = r\right]\right) t.$$

Derivation of the expression for $\text{UPE}(v(F), k)$ follows the same logic, after conditioning on covariates Z. Let us first write the joint distribution of covariates X and Z

$$\begin{aligned} F_{X,Z}(x,z) &= F_{X|Z\leq z}(x)\, F_Z(z) \\ &= \left(\sum_{j=0}^{K} s_{j|Z\leq z} \mathbf{1}\left[x \geq j\right]\right) F_Z(z) \end{aligned}$$

and $G_{X,Z}^{2,k}$ as

$$G_{X,Z}^{2,k}(x,z) = \left(\sum_{j=0}^{K} s^*_{j|Z\leq z} \mathbf{1}\left[x \geq j\right]\right) F_Z(z)$$

where the shares of workers types conditional on Z are $s^*_{r|Z\leq z} = s_{r|Z\leq z} - t$, $s^*_{k|Z\leq z} = s_{k|Z\leq z} + t$, and $s^*_{j|Z\leq z} = s_{j|Z\leq z}$ otherwise. We now have

$$d(G_{X,Z}^{2,k} - F_{X,Z})(x,z) = \begin{cases} t\, dF_Z(z) & \text{if } x = k \\ -t\, dF_Z(z) & \text{if } x = r \\ 0 & \text{otherwise} \end{cases}$$

Using Theorem 1 from Firpo et al. (2009) again, now expanded to the full set of covariates X and Z, namely

$$\nabla v_{F\to G^*} = \int_{\Omega_Z} \int_{\Omega_X} \text{E}\left[\text{IF}(y; v, F)|X = x, Z = z\right]\, d(G^*_{X,Z} - F_{X,Z})(x,z) \tag{A1}$$

and substituting $d(G_{X,Z}^{2,k} - F_{X,Z})(x,z)$ in Equation (A1) yields our second main expression

$$\begin{aligned} \text{UPE}(v(F), k) &= \nabla v_{F\to G_k^2} \\ &= \left(\int_{\Omega_Z} \left(\text{E}\left[\text{IF}(y; v, F)|X = k, Z = z\right] - \text{E}\left[\text{IF}(y; v, F)|X = r, Z = z\right]\right)\, dF_Z(z)\right) t. \end{aligned}$$

Appendix B. Detailed Influence Function Regression Results

Table A1. Coefficient estimates of influence function regressions - P10.

	Aggregate Nationality Groups			Disaggregate Nationality Groups		
	Model 1	**Model 2**	**Model 3**	**Model 1**	**Model 2**	**Model 3**
Immigrant worker	−0.154 *	−0.102 *	−0.018 †			
Cross-border worker	−0.027 ‡	0.002	0.042 *			
Be-Fr-Ge resident				0.018 ‡	0.024 †	0.049 *
Portuguese resident				−0.258 *	−0.178 *	−0.056 *
Other EU resident				−0.045 *	−0.023 ‡	0.006
Non-EU resident				−0.314 *	−0.250 *	−0.107 *
German CB				0.031 †	0.059 *	0.047 *
French CB				−0.073 *	−0.031 †	0.039 *
Belgian CB				0.016	0.021 †	0.039 *
Female		−0.187 *	−0.121 *		−0.185 *	−0.119 *
Age		0.434 *	0.385 *		0.425 *	0.384 *
Age squared/100		−0.005 *	−0.004 *		−0.005 *	−0.004 *
Secondary education		0.130 *	0.008		0.100 *	0.002
Tertiary education		0.304 *	0.066 †		0.245 *	0.053 ‡
Job tenure		0.186 *	0.148 *		0.191 *	0.152 *
Job tenure squared/100		−0.004 *	−0.003 *		−0.004 *	−0.003 *
Manager			0.008			0.006
10–49 employees in firm			0.033 ‡			0.035 ‡
50–249 employees in firm			0.041 †			0.042 †
500–999 employees in firm			−0.070 ‡			−0.069 ‡
1000+ employees in firm			0.021			0.023
Part time contract			−0.074 *			−0.074 *
Industry/Manufacture			−0.086 *			−0.087 *
Construction			−0.069 *			−0.060 †
Wholesale			−0.268 *			−0.267 *
Hotel/Restaurant			−0.374 *			−0.372 *
Trans/Comm			−0.050 ‡			−0.050 ‡
Finance			−0.064 *			−0.067 *
Real estate			−0.159 *			−0.157 *
Managerial			0.452 *			0.435 *
Professional			0.488 *			0.471 *
Associate professional			0.505 *			0.488 *
Clerk			0.493 *			0.477 *
Service worker			0.301 *			0.289 *
Craft and trade worker			0.380 *			0.371 *
Manufacturers			0.386 *			0.376 *
Constant	1.042 *	0.005	−0.132	1.042 *	0.057	−0.111

Notes: Based on 2006 Luxembourg Structure of Earnings Survey data. *, † and ‡ indicate statistical significance at 1, 5 and 10 percent levels respectively (repeated half-sample percentile bootstrap confidence intervals not covering zero at the corresponding confidence levels). Model 1 does not include covariates, Model 2 includes human capital covariates, Model 3 includes both human capital and job characteristics. Binary or multinomial covariates scaled by $t = 0.1$.

Table A2. Coefficient estimates of influence function regressions - P50 (median).

	Aggregate Nationality Groups			Disaggregate Nationality Groups		
	Model 1	**Model 2**	**Model 3**	**Model 1**	**Model 2**	**Model 3**
Immigrant worker	−0.564 *	−0.390 *	−0.142 *			
Cross-border worker	−0.468 *	−0.325 *	−0.155 *			
Be-Fr-Ge resident				−0.025	−0.064 †	−0.052 †
Portuguese resident				−0.989 *	−0.677 *	−0.241 *
Other EU resident				−0.138 *	−0.081 †	−0.037
Non-EU resident				−0.751 *	−0.562 *	−0.234 *
German CB				−0.384 *	−0.222 *	−0.100 *
French CB				−0.594 *	−0.419 *	−0.201 *
Belgian CB				−0.290 *	−0.219 *	−0.128 *
Female		−0.020	−0.134 *		−0.018	−0.129 *
Age		0.851 *	0.564 *		0.820 *	0.559 *
Age squared/100		−0.010 *	−0.006 *		−0.010 *	−0.006 *
Secondary education		0.429 *	0.033		0.331 *	0.016
Tertiary education		1.285 *	0.194 *		1.092 *	0.165 *
Job tenure		0.415 *	0.286 *		0.427 *	0.295 *
Job tenure squared/100		−0.004 *	−0.004 *		−0.005 *	−0.004 *
Manager			0.216 *			0.212 *
10-49 employees in firm			−0.054			−0.054
50-249 employees in firm			−0.026			−0.026
500-999 employees in firm			−0.001			0.003
1000+ employees in firm			0.160 ‡			0.165 ‡
Part time contract			0.071 *			0.069 *
Industry/Manufacture			−0.223 *			−0.222 *
Construction			−0.522 *			−0.502 *
Wholesale			−0.527 *			−0.521 *
Hotel/Restaurant			−0.519 *			−0.509 *
Trans/Comm			−0.137 ‡			−0.141 ‡
Finance			0.031			0.024
Real estate			−0.324 *			−0.310 *
Managerial			1.017 *			0.972 *
Professional			1.153 *			1.112 *
Associate professional			1.033 *			0.992 *
Clerk			0.644 *			0.607 *
Service worker			0.258 *			0.233 *
Craft and trade worker			0.328 *			0.309 *
Manufacturers			0.265 *			0.239 *
Constant	1.939 *	−0.589 *	−0.066	1.939 *	−0.417 *	−0.009

Notes: Based on 2006 Luxembourg Structure of Earnings Survey data. *, † and ‡ indicate statistical significance at 1, 5 and 10 percent levels respectively (repeated half-sample percentile bootstrap confidence intervals not covering zero at the corresponding confidence levels). Model 1 does not include covariates, Model 2 includes human capital covariates, Model 3 includes both human capital and job characteristics. Binary or multinomial covariates scaled by $t = 0.1$.

Table A3. Coefficient estimates of influence function regressions - P90.

	Aggregate Nationality Groups			Disaggregate Nationality Groups		
	Model 1	**Model 2**	**Model 3**	**Model 1**	**Model 2**	**Model 3**
Immigrant worker	−0.718 *	−0.378 *	−0.031			
Cross-border worker	−0.986 *	−0.699 *	−0.301 *			
Be-Fr-Ge resident				0.338 †	−0.034	−0.123
Portuguese resident				−1.652 *	−0.809 *	−0.108
Other EU resident				0.124	0.093	0.147
Non-EU resident				−0.604 *	−0.239	0.193 ‡
German CB				−0.725 *	−0.430 *	−0.055
French CB				−1.284 *	−0.895 *	−0.386 *
Belgian CB				−0.616 *	−0.519 *	−0.352 *
Female		−0.268 *	−0.380 *		−0.262 *	−0.377 *
Age		0.543 *	−0.108		0.485 *	−0.130
Age squared/100		−0.000	0.006 *		0.000	0.006 *
Secondary education		1.029 *	0.297 *		0.890 *	0.284 *
Tertiary education		4.034 *	1.220 *		3.768 *	1.202 *
Job tenure		0.654 *	0.302 *		0.679 *	0.325 *
Job tenure squared/100		−0.006	0.002		−0.007	0.002
Manager			0.787 *			0.793 *
10-49 employees in firm			−0.147			−0.153 ‡
50-249 employees in firm			−0.034			−0.030
500-999 employees in firm			0.213 ‡			0.227 ‡
1000+ employees in firm			0.063			0.081
Part time contract			0.507 *			0.503 *
Industry/Manufacture			−0.888 *			−0.881 *
Construction			−0.896 *			−0.883 *
Wholesale			−0.704 *			−0.685 *
Hotel/Restaurant			−0.826 *			−0.809 *
Trans/Comm			−0.278			−0.301
Finance			−0.228			−0.231
Real estate			−1.087 *			−1.051 *
Managerial			4.910 *			4.891 *
Professional			2.221 *			2.208 *
Associate professional			1.024 *			1.001 *
Clerk			0.054			0.035
Service worker			0.165 †			0.158 †
Craft and trade worker			0.129			0.119
Manufacturers			−0.046			−0.059
Constant	3.879 *	0.116	2.136 *	3.879 *	0.384	2.195 *

Notes: Based on 2006 Luxembourg Structure of Earnings Survey data. *, † and ‡ indicate statistical significance at 1, 5 and 10 percent levels respectively (repeated half-sample percentile bootstrap confidence intervals not covering zero at the corresponding confidence levels). Model 1 does not include covariates, Model 2 includes human capital covariates, Model 3 includes both human capital and job characteristics. Binary or multinomial covariates scaled by $t = 0.1$.

Table A4. Coefficient estimates of influence function regressions - Variance.

	Aggregate Nationality Groups			Disaggregate Nationality Groups		
	Model 1	**Model 2**	**Model 3**	**Model 1**	**Model 2**	**Model 3**
Immigrant worker	−7.58	−6.97	−1.91			
Cross-border worker	−14.17 *	−13.52 *	−6.76 †			
Be-Fr-Ge resident				3.91	−6.84	−6.37
Portuguese resident				−19.42 *	−10.56 *	−1.41
Other EU resident				−0.32	−4.95	−2.66
Non-EU resident				5.42	5.98	11.51 †
German CB				−10.72 *	−10.41 †	−3.62
French CB				−17.66 *	−15.51 *	−7.27 *
Belgian CB				−10.20 *	−11.92 *	−8.23 *
Female		−5.25 *	−10.38 ‡		−5.20 *	−10.42 ‡
Age		−36.81 *	−42.57 *		−37.43 *	−42.97 *
Age squared/100		0.60 *	0.61 *		0.60 *	0.62 *
Secondary education		10.50 *	2.55 ‡		9.65 *	2.67 †
Tertiary education		63.31 *	31.23 *		61.75 *	31.52 *
Job tenure		10.92 *	7.59 †		11.27 *	7.80 †
Job tenure squared/100		−0.42 *	−0.25 ‡		−0.42 *	−0.25 †
Manager			2.43			2.67
10–49 employees in firm			0.31			0.13
50–249 employees in firm			12.25 †			12.34 †
500–999 employees in firm			1.31			1.49
1000+ employees in firm			−1.79			−1.61
Part time contract			20.26 *			20.21 *
Industry/Manufacture			−22.51 †			−22.34 †
Construction			−23.64 *			−23.79 *
Wholesale			−14.48			−14.22
Hotel/Restaurant			−16.43 †			−16.51 †
Trans/Comm			−15.91			−16.27
Finance			−21.29			−21.07
Real estate			−25.67 †			−25.33 †
Managerial			103.07 *			104.00 *
Professional			18.75 ‡			19.77 ‡
Associate professional			7.13			7.93
Clerk			1.31			2.10
Service worker			−0.39			0.11
Craft and trade worker			2.85			3.29
Manufacturers			0.59			1.16
Constant	41.24 *	72.65 *	101.35 *	41.24 *	74.77 *	101.11 *

Notes: Based on 2006 Luxembourg Structure of Earnings Survey data. *, † and ‡ indicate statistical significance at 1, 5 and 10 percent levels respectively (repeated half-sample percentile bootstrap confidence intervals not covering zero at the corresponding confidence levels). Model 1 does not include covariates, Model 2 includes human capital covariates, Model 3 includes both human capital and job characteristics. Binary or multinomial covariates scaled by $t = 0.1$.

Table A5. Coefficient estimates of influence function regressions - Gini.

	Aggregate Nationality Groups			Disaggregate Nationality Groups		
	Model 1	**Model 2**	**Model 3**	**Model 1**	**Model 2**	**Model 3**
Immigrant worker	−0.0003	−0.0004	0.0002			
Cross-border worker	−0.0054 *	−0.0053 *	−0.0030 *			
Be-Fr-Ge resident				0.0029 ‡	−0.0019	−0.0026 ‡
Portuguese resident				−0.0046 *	−0.0012	0.0007
Other EU resident				0.0027 ‡	0.0006	0.0007
Non-EU resident				0.0060 *	0.0058 *	0.0057 *
German CB				−0.0044 *	−0.0043 *	−0.0013
French CB				−0.0066 *	−0.0059 *	−0.0034 *
Belgian CB				−0.0040 *	−0.0047 *	−0.0036 *
Female		−0.0006	−0.0023 †		−0.0006	−0.0023 †
Age		−0.0192 *	−0.0213 *		−0.0194 *	−0.0215 *
Age squared/100		0.0003 *	0.0003 *		0.0003 *	0.0003 *
Secondary education		0.0033 *	0.0012 ‡		0.0032 *	0.0013 ‡
Tertiary education		0.0254 *	0.0126 *		0.0253 *	0.0129 *
Job tenure		0.0012 ‡	−0.0002		0.0014 ‡	−0.0001
Job tenure squared/100		−0.0001 ‡	0.0000		−0.0001 ‡	0.0000
Manager			0.0028 *			0.0029 *
10–49 employees in firm			−0.0003			−0.0004
50–249 employees in firm			0.0022			0.0022
500–999 employees in firm			0.0017			0.0018 ‡
1000+ employees in firm			−0.0009			−0.0008
Part time contract			0.0067 *			0.0067 *
Industry/Manufacture			−0.0060 †			−0.0060 †
Construction			−0.0056 †			−0.0058 †
Wholesale			−0.0003			−0.0002
Hotel/Restaurant			−0.0007			−0.0007
Trans/Comm			−0.0028			−0.0029
Finance			−0.0043			−0.0042
Real estate			−0.0073 *			−0.0071 *
Managerial			0.0398 *			0.0403 *
Professional			0.0018			0.0023
Associate professional			−0.0051 †			−0.0047 †
Clerk			−0.0085 *			−0.0081 *
Service worker			−0.0034 *			−0.0031 *
Craft and trade worker			−0.0054 *			−0.0052 *
Manufacturers			−0.0072 *			−0.0069 *
Constant	0.0311 *	0.0534 *	0.0679 *	0.0311 *	0.0538 *	0.0677 *

Notes: Based on 2006 Luxembourg Structure of Earnings Survey data. *, † and ‡ indicate statistical significance at 1, 5 and 10 percent levels respectively (repeated half-sample percentile bootstrap confidence intervals not covering zero at the corresponding confidence levels). Model 1 does not include covariates, Model 2 includes human capital covariates, Model 3 includes both human capital and job characteristics. Binary or multinomial covariates scaled by $t = 0.1$.

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