Abstract and Concrete Decision Graphs for Choosing Extensions of Argumentation Frameworks - Technical Report

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Abstract. Most argumentation semantics allow for multiple extensions, which raises the question of how to choose among extensions. We propose to study this question as a decision problem. Inspired by decision trees commonly used in economics, we introduce the notion of a *decision graph* for deciding between the multiple extensions of a given AF in a given semantics. We distinguish between *abstract decision graphs* and concrete instantiations thereof. Inspired by the principle-based approach to argumentation, we formulate two principles that mappings from argumentation frameworks to decision graphs should satisfy, the principle of decision-graph directionality and the one of directional decision-making. We then propose a concrete instantiation of decision graphs, which satisfies one of these principles. Finally, we discuss the potential for further research based on this novel methodology.

Keywords. abstract argumentation, argumentation semantics, extension selection

1. Introduction

Given that many argumentation semantics have been proposed in the literature [3] and that most argumentation semantics allow for multiple extensions [17], applications of abstract argumentation theory are faced with two decision problems: First, how to choose among the various argumentation semantics? Second, given an argumentation semantics, how to choose an extension?

An important methodology to support rational decisions concerning the first problem is the principle-based approach [5,10,17]. In this paper, we propose a novel methodology to support decision-making concerning the second problem, i.e. concerning the selection of one among many extensions of a given AF in a given semantics.

Sometimes the need to choose an extension is circumvened by merging all extensions into a single justification status for each argument [18,3]. For example,

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an argument is said to be *strongly accepted* iff it is in all extensions. However, this approach gives up the desirable properties of extensions that have been built into the chosen semantics. For example, the set of strongly accepted arguments in preferred semantics may not be admissible. This problem can be avoided by choosing one extension rather than merging all extensions into a single justification status. But this makes the question of how to choose among multiple extensions a very pressing question.

In this paper, we do not favor one particular method for choosing an extension, but instead propose a methodology for studying this problem as a decision problem. Inspired by decision trees commonly used in economics, we introduce the notion of a *decision graph* for deciding among the multiple extensions of a given AF in a given semantics. The edges of a decision graph represent specific decision steps that bring one closer to the final choice of a single extension. We distinguish between *abstract decision graphs*, where the only content present in the nodes is extension-labels on the leaves, and instantiations of these with *concrete decision graphs* that give a particular meaning to every node of the decision graph. Just like the distinction that Dung [11] introduced between abstract argumentation frameworks and structured instantions thereof, this distinction helps to distill the features of decision graphs that come from the decision graph structure alone and study these separately.

Note that in this paper we do not propose to extend Dung's notion of argumentation frameworks. Dung has been criticized for its abstract nature and therefore Dung's formalism has been generalized in many ways, for example with structured frameworks, ADFs, etc. Such extensions are outside the scope of this paper, but some interesting possibilities suggested by our work are discussed in the future work section of this paper. Instead, in this paper we give a new perspective on existing abstract argumentation semantics in terms of decision graphs.

Furthermore, note that our choice to base decision graphs on the traditional extension-based approach to abstract argumentation semantics rather than on the labeling-based approach [2,3] is merely due to the fact that this simplifies the exposition of our ideas. All the ideas developed in this paper could also be developed with respect to the labelling-approach, and actually this would give rise to more fine-grained decision graphs, which allow for more flexibility in the decision-making process. We leave the exploration of this adaptation of our ideas to future work.

The layout of this paper is as follows. In Section 2 we introduce *abstract decision graphs* as well as the notion of *decision mappings* that map each AF to an abstract decision graph. Inspired by the principle-based approach to argumentation theory, we additionally define in this section two principles of decision mappings that seem desirable, the principle of decision-graph directionality and the one of directional decision-making. In Section 3 we define a first concrete instantiation of decision graphs, namely *most fine-grained decision graphs*, whose corresponding decision mapping satisfies one of the two principles from Section 2 and does not satisfy the other. In Section 4 we introduce an alternative instantiation of decision graphs called *SCC-directional decision graphs* that is based on the well-known SCC-recursive scheme and that satisfies both principles from Section 2. In Section 5 we discuss related work, and in Section 6 we conclude and discuss topics for further research.

2. Abstract decision graphs

In this section, we introduce *abstract decision graphs*, where the only content present in the nodes is extension-labels on the leaves. We do however have a few requirements on the graph: It should be a directed acyclic graph, with a single root from which all other nodes are reachable, to represent our starting point in the decision-making process. Also, we require that each node connect to a distinct set of reachable endpoints, since we are interested in the processes where some extensions are discarded at every step as we traverse the graph. To formally define it, we first need an auxiliary notion:

Definition 2.1. Given a directed acyclic graph (D, R) and a node $d \in D$, we define *reachable-leaves*$(d) = \{d' \in D \mid d'$ is a leaf of (D, R) and d' is R-reachable from $d\}$.

Definition 2.2. For a given AF $F = \langle A, \rightarrow \rangle$ and a semantics *sem*, we say that a tuple $DG = (D, R, L)$ is an *abstract decision graph* of F with respect to *sem* iff the following conditions are satisfied:

1. (D, R) is a directed acyclic graph;
2. L is a bijection from the leaves of (D, R) to *sem*(F);
3. $\exists i \in D$ such that every $d \in D \setminus \{i\}$ is R-reachable from i;
4. for all distinct $d, d' \in D$, *reachable-leaves*$(d) \neq$ *reachable-leaves*(d').

The elements of D are called *decision points.*

Example 2.1. Consider the argumentation framework depicted in Fig. 1.(a). A possible abstract decision graph with respect to preferred semantics is the one depicted in Fig. 1.(b).

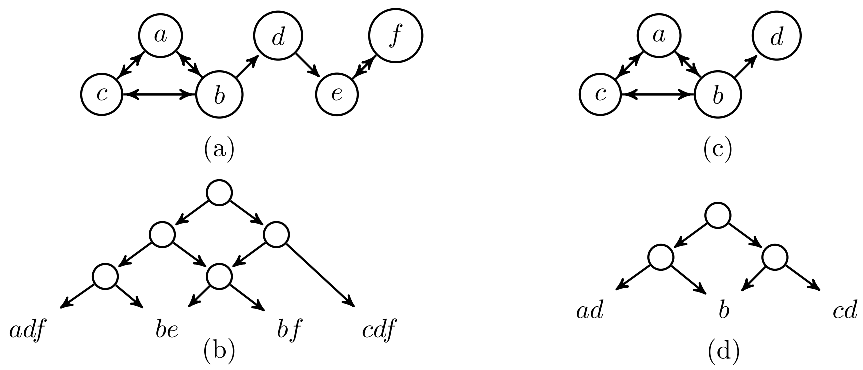

Figure 1. (a) Example argumentation framework $F_1 = \langle A_1, \rightarrow_1 \rangle$. (b) A possible abstract decision graph DG of F_1 with respect to preferred semantics. (c) An initial sub-framework $F_1' = \langle A_1', \rightarrow_1' \rangle$ of F_1, where $A_1' = \{a, b, c, d\}$. (d) The restriction $DG \downarrow_{A_1'}$.

We now wish to examine some properties of functions which return abstract decision graphs for given pairs of AF and semantics, which we call *decision mappings.*

Let $\mathbb{F}$ be the class of all argumentation frameworks, $\mathbb{S}$ be the class of all argumentation semantics and $\mathbb{D}$ the class of all abstract decision graphs.

Definition 2.3. We say that a function $g : \mathbb{F} \times \mathbb{S} \mapsto \mathbb{D}$ is a *decision mapping* iff for any argumentation framework F and semantics *sem*, it returns an abstract decision graph of F with respect to *sem*.

One important principle studied in the principle-based approach to argumentation theory is the Principle of Directionality, which was introduced by Baroni and Giacomin [5], and which has been extensively studied for abstract argumentation semantics [17]. We now propose a way to translate this principle to a similar principle for decision mappings.

We start with the notion of *initial sub-framework*, which is a sub-framework such that no argument outside of it attacks an argument inside of it. In terms of directionality, these are sub-frameworks that one should be able to evaluate locally, i.e. without having to take into account the rest of the framework.

Definition 2.4. We say that $F' = \langle A', \rightarrow' \rangle$ is an *initial sub-framework* of $F = \langle A, \rightarrow \rangle$ iff $A' \subseteq A$, $\rightarrow' = \rightarrow \cap A' \times A'$ and $\nexists a \in A \setminus A', b \in A'.\ a \rightarrow b$.

We then wish to be able to contract a decision graph so that it only represents decisions made on a sub-framework of the original one, but while ensuring the conditions defined for an abstract decision graph are still respected. For this, we first define a notion of equivalence between decision points based on whether their reachable endpoints are equal with respect to the sub-framework of interest.

Definition 2.5. Given an abstract decision graph $DG = (D, R, L)$ of an AF $F = \langle A, \rightarrow \rangle$ with respect to a semantics *sem*, a subset of arguments $A' \subseteq A$ and two decision points d_1, d_2, we say that d_1 and d_2 are decisional-equivalent with respect to A' (denoted as $d_1 \simeq_{A'} d_2$) iff $\{E \cap A' \mid E \in \textit{reachable-leaves}(d_1)\} = \{E \cap A' \mid E \in \textit{reachable-leaves}(d_2)\}$.

Definition 2.6. Given an abstract decision graph $DG = (D, R, L)$ of an AF $F = \langle A, \rightarrow \rangle$ with respect to a semantics *sem* and a subset of arguments $A' \subseteq A$, we define the *restriction* of DG to A' as $DG \downarrow_{A'} = (D', R', L')$, where:

1. D' is the set of equivalent classes of $\simeq_{A'}$ in DG;
2. $(d, d') \in R'$ iff $d \neq d'$ and $\exists d_1 \in d, d_2 \in d'$ such that $(d_1, d_2) \in R$;
3. L' is a bijection from the leaves of (D', R') to $\textit{sem}(\langle A', \rightarrow \cap A' \times A' \rangle)$ such that $\forall d \in D'$, if $d_1 \in d$, then $L'(d) = L(d_1) \cap A'$.

Lemma 2.1. *Given an abstract decision graph DG of an AF $\langle A, \rightarrow \rangle$ and a set $A' \subseteq A$, the restriction $DG \downarrow_{A'}$ is also an abstract decision graph.*

Example 2.2. Fig. 1.(c) depicts an initial sub-framework $F' = \langle A', \rightarrow' \rangle$ of the framework F depicted in Fig. 1.(a). The restriction of the decision graph in 1.(b) to A' is depicted in Fig. 1.(d).

We can now define our principle of directionality for decision mappings:

Definition 2.7. We say that a decision mapping g satisfies *decision-graph directionality with respect to a semantics sem* iff for any argumentation frameworks $F = \langle A, \rightarrow\rangle$ and $F_1 = \langle A_1, \rightarrow_1\rangle$ such that F' is an initial sub-framework of F, $g(F', \mathit{sem}) = g(F, \mathit{sem}) \downarrow_{F'}$.

An interesting principle can be derived from the notions defined above, also on the topic of directionality, but this time while also incorporating the ideas of decision-making. The idea here is that the decision should follow the directionality of the graph, so that if an argument a can reach another argument b but not vice-versa, then decisions about the status of a should come no later than decisions about b. For this we first define what it means for the status of an arguments to be decided at a given decision point.

Definition 2.8. Given a decision graph $DG = (D, R, L)$ of an AF $F = \langle A, \rightarrow\rangle$ and a decision point $d \in D$, we define the set A_d of arguments whose status is decided in d to be $\{a \in A \mid \text{either } \forall d' \in \mathit{reachable\text{-}leaves}(d).\ a \in L(d'), \text{ or } \forall d' \in \mathit{reachable\text{-}leaves}(d).\ a \notin L(d')\}$.

We then define a notion of initial arguments within the ones whose status is decided.

Definition 2.9. Given a decision graph $DG = (D, R, L)$ of an AF $F = \langle A, \rightarrow\rangle$ and a decision $d \in D$, we define the set of $\mathit{initial}(d, F)$ to be $\{a \in A_d \mid \nexists b \in A \setminus A_d$ such that there is a $\rightarrow$-path from b to a but not vice-versa $\}$.

Definition 2.10. We say that a decision mapping g satisfies *directional decision-making with respect to a semantics sem* iff for any argumentation framework $F = \langle A, \rightarrow\rangle$, if $g(F, \mathit{sem}) = (D, R, L)$, then for all $d \in D$, $\{e \in \mathit{sem}(F) \mid \exists d' \in \mathit{reachable\text{-}leaves}(d) \text{ such that } (L(d') \cap \mathit{initial}(d, F)) \subseteq e\} = \{L(d') \mid d' \in \mathit{reachable\text{-}leaves}(d)\}$

3. Most fine-grained decision graphs

In this section, we see an example of a decision mapping producing *concrete decision graphs*, where we now have labels on the intermediary points too.

We first introduce the notion of a *partial extension*, which allows to represent intermediate steps in the decision process about which arguments to include in the extension and which arguments to exclude. Given an argument a, we denote the information that a has been chosen to be in the extension by $+a$, and the information that a has been chosen to not be in the extension by $-a$. This motivates the following definition:

Definition 3.1. Given set of arguments A, we define a *partial extension* for A to be a subset Γ of $\{+, -\} \times A$ such that for no argument $a \in A$, $+a \in \Gamma$ and $-a \in \Gamma$. We denote the elements of a partial extension by $+a$ and $-a$ rather than by $(+, a)$ and $(-, a)$. The set of all partial extensions for A is denoted by $\mathbb{P}_A$.

When neither $+a$ nor $-a$ is in a given partial extension, this means that the status of argument a has not been determined yet (not to be confused with the *undecided* label from the labeling-based approach). When the status of all arguments has been determined, a *total extension* is reached:

Definition 3.2. A partial extension Γ of A is called a *total extension* iff for every $a \in A$, either $+a \in \Gamma$ or $-a \in \Gamma$.

Note that there is a direct correspondence between the classical notion of an extension as a subset of the sets of arguments, and the notion of a total extension defined here: Having $+a$ in the total extension corresponds to a being in the corresponding extension, and having $-a$ in the total extension corresponds to a not being in the corresponding extension. This motivates the following definition:

Definition 3.3. Given a set of arguments A and a total extension Γ for A, we define $\varepsilon(\Gamma) := \{a \in A \mid +a \in \Gamma\}$.

Note that ε is a bijection between the total extension for A and subsets of A. So when A is specified, we can also refer to its inverse ε^{-1}, defined by $\varepsilon^{-1}(E) := \{+a \mid a \in E\} \cup \{-a \mid a \in A \setminus E\}$.

The following notion allows us to refer to the set of arguments whose status has already been determined:

Definition 3.4. Let A be a set of arguments and $\Gamma \in \mathbb{P}_A$ be a partial extension. We define the *coverage* of Γ to be $\bar{\Gamma} := \{a \in A \mid +a \in \Gamma \vee -a \in \Gamma\}$.

We are now interested in fixing an argumentation framework and a semantics, and focusing on the decision structure of how the initial partial extension, from which all total extensions are reachable, leads to each one of these total extensions.

Definition 3.5. Given an argumentation framework $F = \langle A, \rightarrow \rangle$, a semantics *sem* and an abstract decision graph (D, R, L) of F with respect to *sem*, we say that (D, R, L) is a *concrete decision graph* (CDG) of F with respect to *sem* iff the following conditions hold:

1. $D \subseteq \mathbb{P}_A$
2. for all leaves $d \in D$, $L(d) = \varepsilon(d)$;
3. $sem(F) = \{\varepsilon(d) \mid d \in D$ and d is a total extension$\}$;
4. if $(d, d') \in R$, then $d \subset d'$.

We wish to define a decision mapping which constructs a concrete decision graph with as much granularity as possible. For this purpose, we start by observing the graph resulting from the subset relation on the partial extensions.

Definition 3.6. Given an argumentation framework $F = \langle A, \rightarrow \rangle$ and a semantics *sem*, we define the *most exhaustive update* of F with respect to *sem* to be $meu(F, sem) := (S, U)$, where $S := \{s \in \mathbb{P}_A \mid \exists E \in sem(F)$ such that $s \subseteq \varepsilon^{-1}(E)\}$ and $U := \{(s, s') \in U \mid s \subset s'\}$.

We now distinguish between two kinds of edges in the most exhaustive update: the edges that relate two partial extensions that both lead to the same final total extensions, and the ones where the set of reachable total extensions becomes smaller. This corresponds to the idea that in some steps, no new information is gained, no decisions are made, and thus only reasoning is performed, while in other cases, the range of possible extensions is reduced and thus decisions are made.

Definition 3.7. Let A be a set of arguments, let (S, U) be a directed graph (e.g. a most exhaustive update) such that $S \subseteq \mathbb{P}_A$. We say that $(s, s') \in U$ is a *reasoning step* iff *reachable-leaves*(s) = *reachable-leaves*(s'). Otherwise, we say that (s, s') is a *decision step*. We denote the set of all reasoning steps in (S, U) by $\mathbb{R}((S, U))$.

We define the most fine-grained decision graphs by focusing on the decision steps in the most exhaustive update. For this, we need to condense the most exhaustive update such that reasoning is made automatically. This is akin to approaches in epistemic logic in which knowledge is assumed to be logically closed, i.e. in which reasoning is assumed to be instantaneously completed. We also identify the decision points in the decision graphs, which are the nodes where no more reasoning can be made and taking a decision cannot be avoided.

Definition 3.8. Let A be a set of arguments, let (S, U) be a directed graph such that $S \subseteq \mathbb{P}_A$ and let $s \in S$. We say that $s' \in S$ is the *reasoning completion of s in* (S, U) iff s' is a maximal with respect to $\subseteq$ partial extension such that either there is an $\mathbb{R}((S, U))$-path from s to s', or $s = s'$. If $s \in S$ is the reasoning completion of itself, we say that s is a *decision point in* (S, U). We denote the set of all decision points in (S, U) by $\mathcal{D}((S, U))$. We call the graph (E, V), where $E := \mathcal{D}((S, U))$ and $V := U \cap (E \times E)$, the *decision contraction* of (S, U) and denote it by $dc((S, U))$.

We want to define a maximally fine-grained decision graph, so our intention is that no decisions are skipped, however small they may be. Thus we filter out the edges which relate two nodes already connected with more fine-grained paths.

Definition 3.9. Let $G = (E, V)$ be a directed acyclic graph. We define the *fine-grained filtering* of G as $fgf(G) := (E, V')$, where $V' = \{(e, e') \in V \mid \text{there is no } V\text{-path from } e \text{ to } e' \text{ of length } > 1\}$.

Now we are ready to define the most fine-grained decision graph of an AF with respect to a semantics:

Definition 3.10. Given an argumentation framework $F = \langle A, \rightarrow \rangle$ and a semantics *sem*, we define the *most fine-grained decision graph* of F with respect to *sem* to be $mfg(F, sem) := fgf((E, V))$, where $E := \mathcal{D}(meu(F, sem))$ and $V := \{(e, e') \in E \times E \mid e \subseteq e'\}$.

Example 3.1. The concept of most fine-grained decision graph is illustrated in Fig. 2 and Fig. 3. In Fig. 2, one can see that if we did not differentiate partial extensions such as $-a$ from $\emptyset$, we would get a decision graph with one less layer

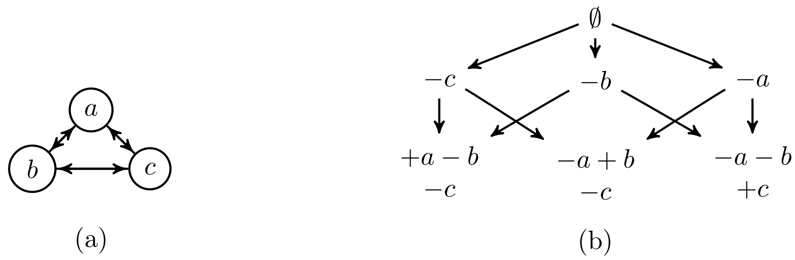

Figure 2. (a) Example argumentation framework F_2. (b) Most fine-grained decision graph of F_2 with respect to complete semantics, $mfg(F_2, complete)$.

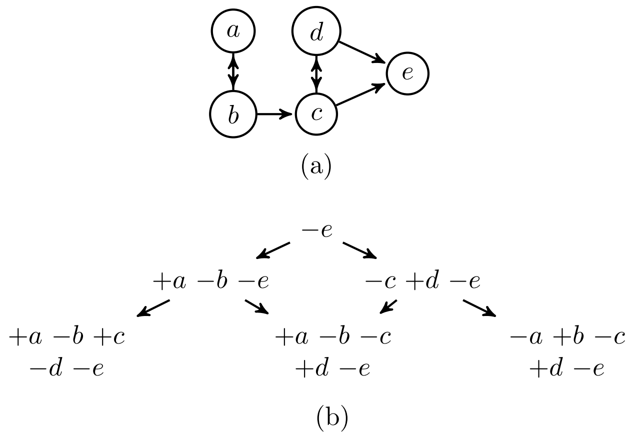

Figure 3. (a) Example argumentation framework F_3. (b) Most fine-grained decision graph of F_3 with respect to preferred semantics, $mfg(F_3, preferred)$.

of granularity. In Fig. 3, observe that the root is $-e$ instead of $\emptyset$, since $-e$ is an element of all total extensions. Also, one can see that the decision mapping mfg does not satisfy the principle of directional decision-making with respect to the preferred semantics, since it is also possible to initially make a decision about the status of the arguments c, d, e, even though there is a $\rightarrow$-path from b to all of these arguments, but not vice-versa.

We now state some properties of most fine-grained decision graphs:

Theorem 3.1. *For any AF F and semantics sem, $mfg(F, sem)$ is a concrete decision graph.*

Corollary 3.1. *mfg is a decision mapping.*

Theorem 3.2. *mfg satisfies the principle of decision-graph directionality with respect to any semantics that satisfies the principle of directionality defined in [5].*

Theorem 3.3. *mfg does not satisfy the principle of directional decision-making with respect to any of complete, preferred, semi-stable, naive, stage, CF2 or stage2 semantics.*

4. SCC-directional decision graphs

In the previous section, we have seen that the most fine-grained decision graphs do not satisfy the principle of directional decision making. An important notion in connection with the directionality of the attack relation is the SCC-recursive schema [6], which has been used in algorithms [16,15,7] and in the definition of new semantics (such as the CF2 [4] and stage 2 semantics [13]).

In this section, we focus on how the SCC-recursive schema can be used as an additional layer to restrict the relation in the decision graphs. We define the SCC-directional decision graphs, and then prove some properties about the relation between the canonical semantics and the recursive semantics.

We first provide preliminary notions of SCCs from the literature [6].

Definition 4.1. Let $F = \langle A, D\rangle$ be an AF. We say that $S \subseteq A$ is a *strongly connected component (SCC)* iff S is a maximal set with respect to $\subseteq$ such that for all distinct $a, b \in S$, there is a path from a to b in D. We denote the set of all SCCs in F by $SCCS_F$.

Definition 4.2. Given an argumentation framework $F = \langle A, D\rangle$ and an SCC $S \in SCCS_F$, we define

$$sccparents_F(S) := \{P \in SCCS_F \mid P \neq S \text{ and } \exists a \in P, \exists b \in S, (a, b) \in D\}$$

and recursively define

$$sccanc_F(S) := sccparents_F(S) \cup \bigcup_{P \in sccparents_F(S)} sccanc_F(P)$$

We now impose a restriction on decision graphs that only allows partial extensions to specify the status of an argument if it also specifies the status of all its SCC-ancestors (given by the $sccanc_F$ function).

Definition 4.3. Let F be an argumentation framework and Γ a partial extension of F. We say that Γ satisfies *SCC-directionality in F* iff for all $S \in SCCs_F$, if $S \cap \bar{\Gamma} \neq \emptyset$, then for all $S' \in sccan_F(S)$, $S' \subseteq \bar{\Gamma}$.

Definition 4.4. Given an argumentation framework $F = \langle A, \rightarrow\rangle$ and a semantics *sem* such that $meu(F, sem) = (S, U)$, we define the *SCC-directional update* of F with respect to *sem* to be $sdu(F, sem) := (S', U')$, where $S' := \{\Gamma \in \mathbb{P}_A \mid \Gamma \in S$ and Γ satisfies SCC-directionality in $F\}$ and $U' := U \cap (S' \times S')$.

Definition 4.5. Given an argumentation framework F and a semantics *sem*, we define the *SCC-directional decision graph* of F with respect to *sem* to be $sddg(F, sem) := fgf(dc(sdu(F, sem)))$.

Example 4.1. Consider the argumentation framework F_3 from Fig. 3. We have its SCC-directional decision graph with respect to preferred semantics in Fig. 4. Notice that the right-hand path containing $-a$ $+d$ now directly leads to a total extension, and the total extension containing $+a$ $+d$ is now only reachable by first choosing $+a$ $-b$. Also notice how in this graph the root is $\emptyset$ instead of $-e$,

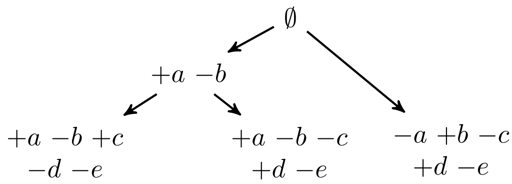

Figure 4. SCC-directional decision graph of the argumentation framework F_3 depicted in Fig. 3 with respect to preferred semantics, $sddg(F_3, preferred)$.

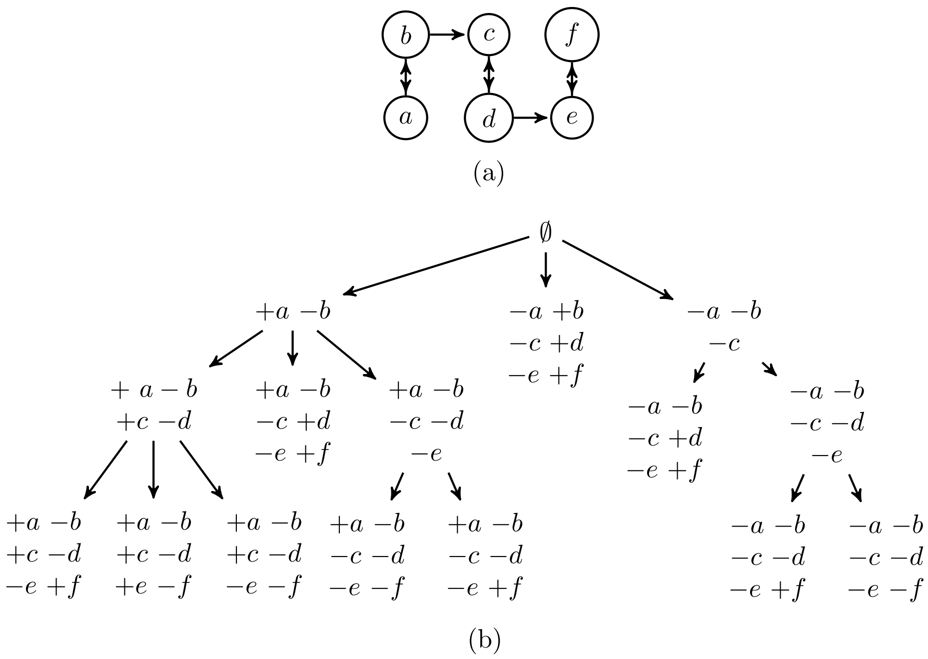

Figure 5. (a) Example argumentation framework F_4. (b) SCC-directional decision graph of F_4 with respect to complete semantics, $sddg(F_4, complete)$.

since even though $-e$ already follows as a reasoning step, due to e being part of a later SCC, its status is left unspecified until the one of the other arguments is determined.

Example 4.2. Consider the argumentation framework F_4 with its corresponding SCC-directional decision graph in Fig. 5. Notice how the decision of choosing $-a$ $+b$ in the initial SCC $\{a, b\}$ directly leads to a total extension, since $-c$ $+d$ $-e$ $+f$ immediately follow as reasoning steps, while some other paths might have more granularity since they lead to a greater number of total extensions.

We now state some properties of SCC-direction decision graphs:

Theorem 4.1. *For any AF F and semantics sem, $sddg(F, sem)$ is a concrete decision graph.*

Corollary 4.1. *sddg is a decision mapping.*

Theorem 4.2. *sddg satisfies the principle of decision-graph directionality with respect to any semantics that satisfies the principle of directionality defined in [5].*

Theorem 4.3. *mfg satisfies the principle of directional decision-making with respect to any semantics.*

5. Related research

There is substantial work of applying formal argumentation theory to support decision-making [14,9,1]. In these papers, argumentation is used to support making decisions about other things than argumentation. That is quite different from our approach, in which we study how to theoretically study the making of decisions about which extension to choose among multiple extensions of an AF. It remains an open problem whether our decision graphs can be extended such that they can be applied to support decision-making as well.

Moreover, there is work on decision procedures. For example, Dvorak et al. [12] study the complexity of evaluations of AFs by exploiting decision procedures for problems of lower complexity whenever possible. Whether and how our general update semantics methodology can be applicable to the systematic study of algorithms for computing extensions, also has to be left to future research.

6. Conclusion and further research

In this paper, we have proposed a methodologically novel approach to choosing extensions of argumentation frameworks by studying abstract and concrete decision graphs that correspond to step-wise decision-making processes about the choice of extension. Inspired by the principle-based approach to abstract argumentation, we have studied two principles that mappings from AFs to decision graphs should satisfy.

We believe that there are many potential applications of our decision graph methodology. We briefly sketch some of them.

Apart from the two types of concrete decision graphs defined in this paper, there are many other types of concrete decision graphs that could be studied. For example, any algorithm for computing all extensions of a given AF with respect to a fixed semantics gives rise to a concrete decision graph with respect to that semantics, namely by reducing the search tree of the algorithm to its decision points, similarly as we have reduced the most exhaustive update to the fine-grained decision graph. Studying the properties of these decision trees could give novel insights into the study of algorithms for computing extensions.

Further principles of decision graphs can be defined and studied. This will help to differentiate better between different semantics as well as the different decision graphs that they give rise to.

Furthermore, one can study properties of the different decision-paths (paths through the decision graph) that a given decision graph gives rise to. Here as well a principle-based approach can make sense: These principles could help to choose an extension in a systematic way, and could thus be very relevant to application of abstract argumentation in which a unique extension has to be chosen from the set of all extensions.

Though this is not our intention in this paper, decision graphs can also be used to generalize Dung's semantic framework, in the following sense. Instead of associating a set of extensions with a framework, we can associate a set of decision graphs with a framework. These decision graphs can be our most fine-grained decision graphs, our SCC-recursive decision graphs, or some other kind of decision graphs. We have made a contribution in this direction in a paper accepted for the second Chinese Conference on Logic and Argumentation [8].

In this paper we have only studied the decision graph methodology with respect to Dung's AFs, but the methodology could also be applied to extensions of Dung's formalisms such as bipolar AFs, ADFs, higher-order AFs, value-based AFs etc.

A further interesting line of future research is to study whether and how our methodology could be applied outside abstract argumentation, e.g. to structured argumentation, logic programming, answer set programming, Reiter's default logic, causal theories, social choice theory etc.

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