



A Reduced Order Kalman Filter for Computational Fluid-Dynamics Applications

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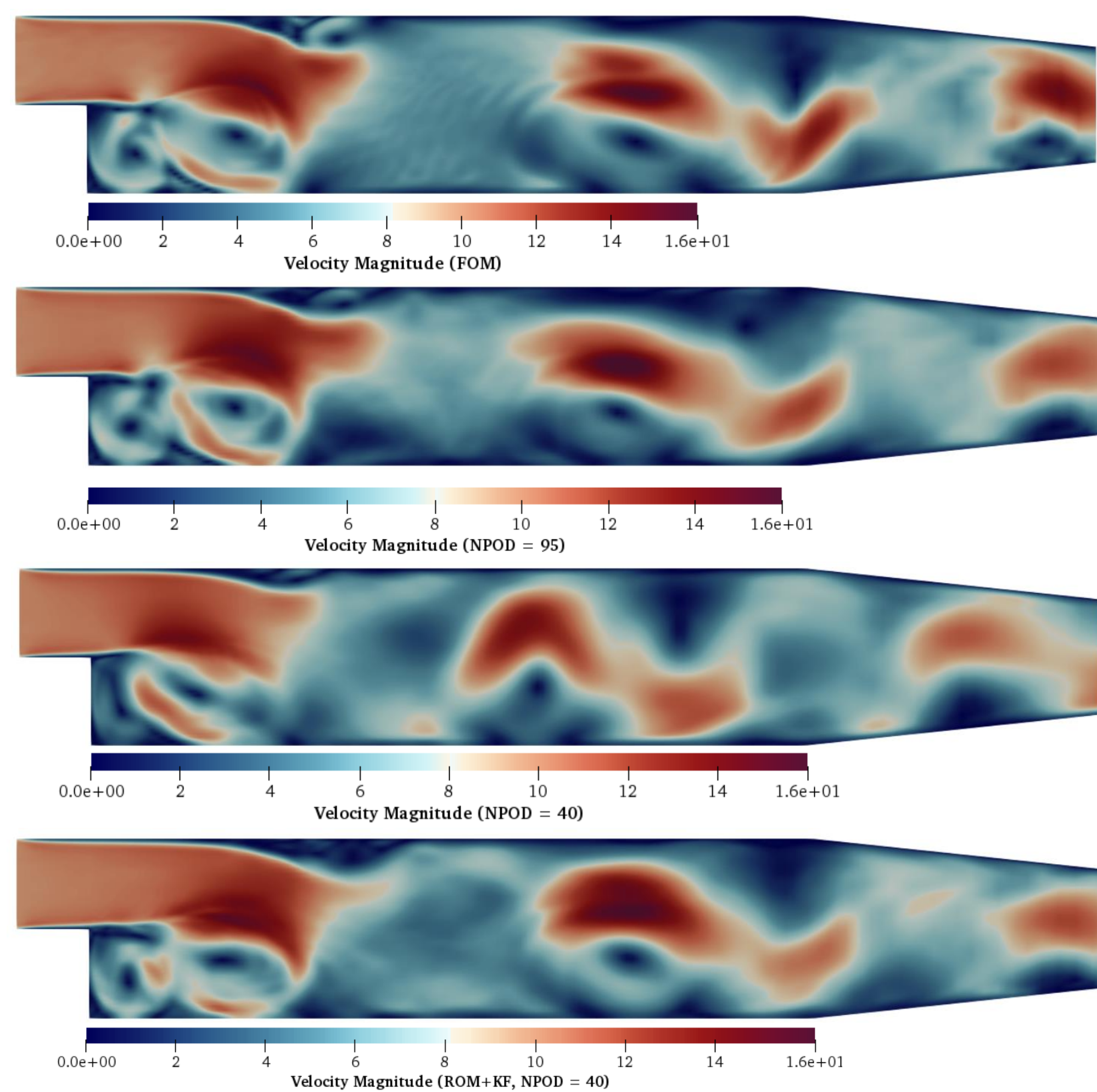
RATIONALE

- **Objective:** reducing the shortcomings of numerical simulations in the analysis of complex engineering systems, namely the **epistemic uncertainties** associated with the modelling procedure itself and the **computational requirements** of real, industrial cases.
- In the **nuclear power plants** field, the development and improvement of simulations tools for the **real-time control of the nuclear reactor** plays a fundamental role. Ideally, these tools should be **fast-running** (for real-time applications) but at the same time **detailed enough** to improve the knowledge of the state of the system.
- **Question:** it is possible to provide the control simulation tools with relevant **spatial information capabilities**, enhancing the level of detail without a strong computational burden?

AVAILABLE TOOLS

To reduce the computational burden	To improve the computational model accuracy
REDUCED ORDER MODELS (ROM)	DATA-DRIVEN ALGORITHMS (DDA)
<ul style="list-style-type: none">• Replace the high-fidelity (accurate) problem by one featuring much lower complexity• Input-output relationships have to be preserved• Must be stable, sufficiently accurate and within scope of the analysis and design tools• Computationally efficient	<ul style="list-style-type: none">• Real-time integration of experimental data within the numerical model, thus improving the efficiency of the latter• Observations offers a local (spatial) but accurate information on the state of the system• Feedback on the accuracy of both the model prediction and the experimental data itself

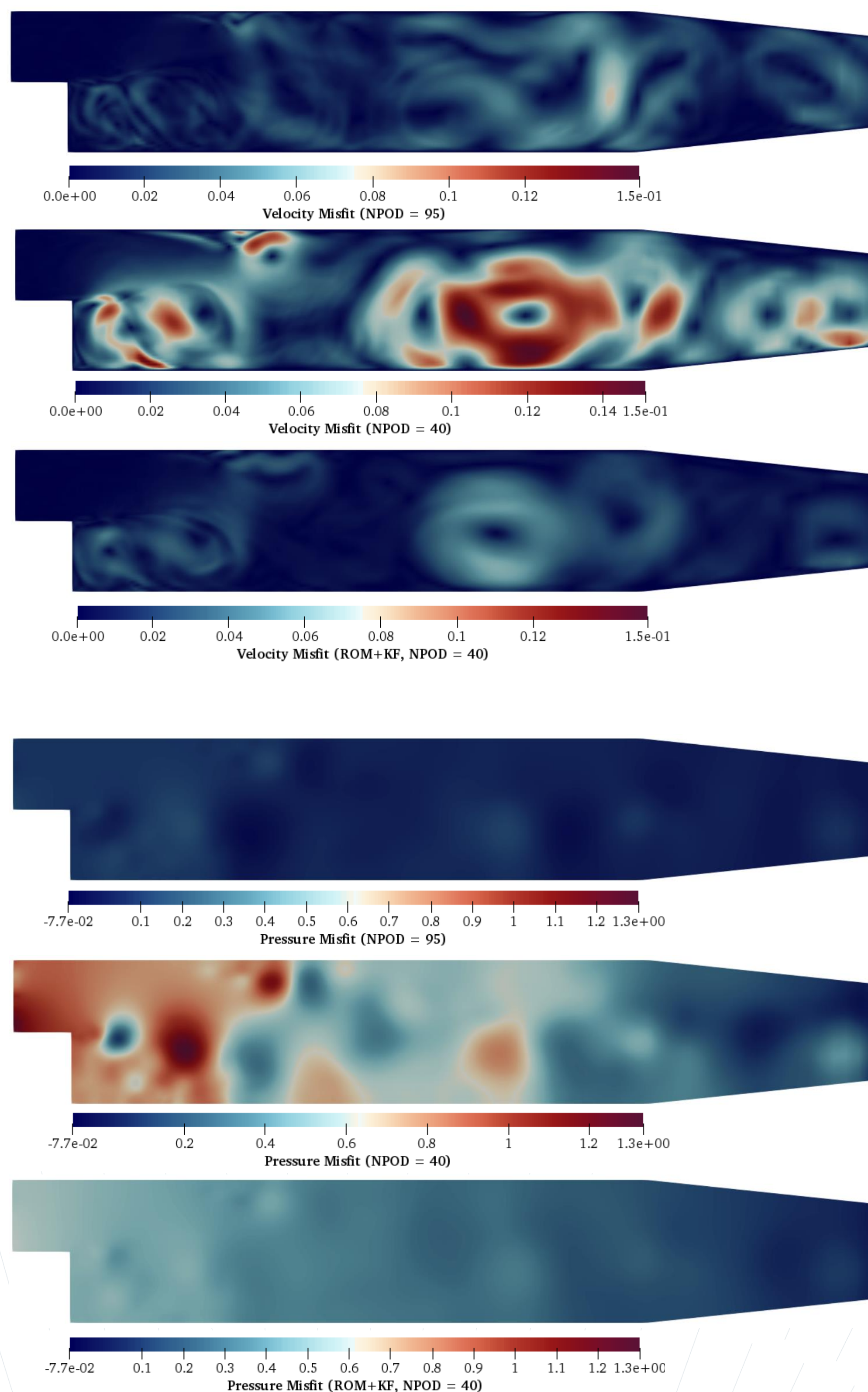
TEST CASE – BACKWARD FACING STEP



Comparison of the velocity magnitude [m/s] at a given time step for the various cases. From top to bottom: full-order model, reduced model with 95 basis, reduced model with 40 basis, and combined reduced model – Kalman filter with 40 basis.

	FOM	ROM (NPOD = 95)	ROM (NPOD = 40)	ROM + KF (NPOD = 40)
Offline	1260 s			
Online		428.57 s	75.00 s	230.70 s

Computational times for the various cases, evaluated as the CPU time needed in order to perform 5000 iterations with a constant time step equal to 0.00005 using the OpenFOAM software for CFD analysis and the PISO algorithm. A personal computer with 4GB RAM and CPU Intel Core i7-65000 @ 2.50 GHz

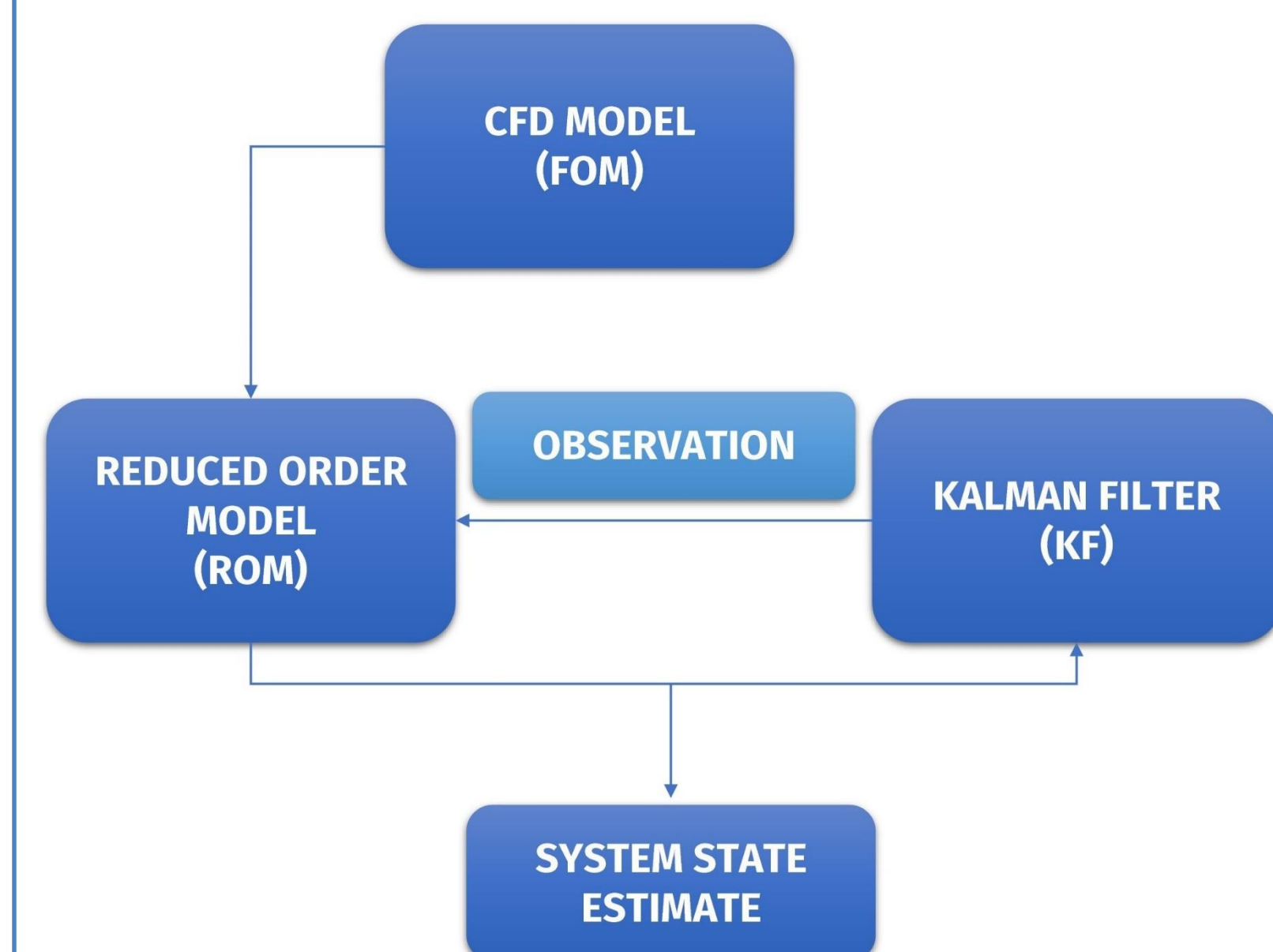


Comparison of the velocity (top) and pressure (bottom) relative error at a given time, computed as the normalised L2 norm between the full order model solution (taken as reference) and the reduced order ones. From top to bottom: error between FOM and ROM with 95 basis, error between FOM and ROM with 40 basis and error between FOM and ROM with Kalman filter with 40 basis.

1D Modelling Lumped parameter approach	VS	3D Modelling CFD simulations
Control-oriented		Design-oriented
Main features Fast-running ODE-based Integral information		Main features High-detailed PDE-based Spatial information
Main drawback Lacks predictive capabilities		Main drawback Too expensive for most available analysis tools

PROPOSED SOLUTION

Combine the reduced order model (POD-Galerkin) and the data-driven algorithm (Kalman filter) in order to develop an **online control system with feedback from real-time experimental data**



- The filter acts on the reduced variable (POD coefficients)
- Size of the error covariance matrix P equal to the number of reduced basis instead of the number of elements of the numerical mesh
- Sensible saving with respect to the Full-Order Model is expected

CONCLUSIONS

- Given the same accuracy of the reconstruction (namely, the same number of basis), the integration between ROM and Kalman filter allows for **better results, comparable to those obtained by a ROM with a greater number of basis** (namely, greater accuracy)
- The increase in computational time due to the implementation of the Kalman filter is not negligible, however it **remains lower than the one required by the more accurate ROM**