An n-ary generalization of the concept of distance

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Generalizations of the concept of distance in which $n \geq 3$ elements are considered have been investigated by several authors (see [1, Chapter 3] and the references therein). The general idea is to provide some functions that measure a degree of dispersion among n points. In this talk, we consider the class of n-distances, which are defined as follows.

Definition 1. *Let X be a nonempty set and $n \geq 2$. A map $d\colon X^n \to [0, +\infty[$ is an* n-distance *on X if it satisfies*

(i) $d(x_1, \ldots, x_n) = 0$ *if and only if* $x_1 = \cdots = x_n$,
(ii) $d(x_1, \ldots, x_n) = d(x_{\pi(1)}, \ldots, x_{\pi(n)})$ *for all* $x_1, \ldots, x_n \in X$ *and all* $\pi \in S_n$,
(ii) $d(x_1, \ldots, x_n) \leq \sum_{i=1}^{n} d(x_1, \ldots, x_n)_i^z$ *for all* $x_1, \ldots, x_n, z \in X$,

where we denote by $d(x_1, \ldots, x_n)_i^z$ the function obtained from $d(x_1, \ldots, x_n)$ by setting its ith variable to z

For an n-distance $d\colon X^n \to [0, +\infty[$, the set of the reals K of $]0, 1]$ for which the condition

$$d(x_1, \ldots, x_n) \;\leq\; K \sum_{i=1}^{n} d(x_1, \ldots, x_n)_i^z\,, \qquad x_1, \ldots, x_n, z \in X,$$

holds has an infimum K^*, called the *best constant associated* with d. The purpose of the talk is to provide natural examples of n-distances based on the Fermat point and geometric constructions, and to provide their best constants. We will also provide examples of n-distances that are not the n-ary part of multidistances as defined in [2].

The results presented in this talk can be found in [3].

References

1. M. M. Deza and E. Deza. *Encyclopedia of distances*, third edition. Springer, 2014.
2. J. Martín and G. Mayor. Multi-argument distances. *Fuzzy Sets and Systems* 167:92–100, 2011.
3. G. Kiss, J.-L. Marichal and B. Teheux A generalization of the concept of distance based on the simplex inequality. *Contributions to Algebra and Geometry*, https://doi.org/10.1007/s13366-018-0379-5