Static load deflection experiment on a beam for damage detection using the Deformation Area Difference Method

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ABSTRACT: A reliable and safety infrastructure for both transport and traffic is becoming increasingly important today. The condition assessment of bridges remains difficult and new methods must be found to provide reliable information. A meaningful in-situ assessment of bridges requires very detailed investigations which cannot be guaranteed by commonly used methods.

It is known that the structural response to external loading is influenced by local damages. However, the detection of local damage depends on many factors such as environmental effects (e.g. temperature), construction layer (e.g. asphalt) and accuracy of the structural response measurement. Within the paper, a new so-called Deformation Area Difference (DAD) Method is presented. The DAD method is based on a load deflection experiment and does not require a reference measurement of initial condition. Therefore, the DAD method can be applied on existing bridges. Moreover, the DAD method uses the most modern technologies such as high precision measurement techniques and attempts to combine digital photogrammetry with drone applications.

The DAD method uses information given in the curvature course from a theoretical model of the structure and compares it to real measurements. The paper shows results from a laboratory load-deflection experiment with a steel beam which has been gradually damaged at distinct positions. The load size is chosen so that the maximum deflection does not exceed the serviceability limit state. With the data obtained by the laboratory experiment, the damage degree, which can still be detected by the DAD method, is described. Furthermore, the influence of measurement accuracy on damage detection is discussed.

1 INTRODUCTION

Among the civil buildings and engineering structures, the bridge structures are of extraordinary importance for strategy, economy and ecology. There are worldwide many of existing bridge structures, so the road bridges amount for USA over 600,000 bridges (U.S. Department of Transportation, 2016), for Germany almost 40,000 (Brückenstatistik, 2017), 2017) and for Japan approximately 700.000 (Kawano, et al., 2017). The age structure of the existing bridges for USA and Germany is shown in Figure 1. The tendency of the age structure shows that the number of new bridge constructions is increasing rapidly. Consequently, most of today's existing bridges reach their designed lifespan (National Bridge Inventory, 2015) (Figure 1).

For the safe use of the infrastructure and for a strategic cost management of bridge structures, periodical inspection and repair measures are indispensable (Shirato & Tamakoshi, 2013). As bridge inspections have to be done all fourth to sixth years depending on the national standards, the related big effort considering the high number of bridges becomes evident.

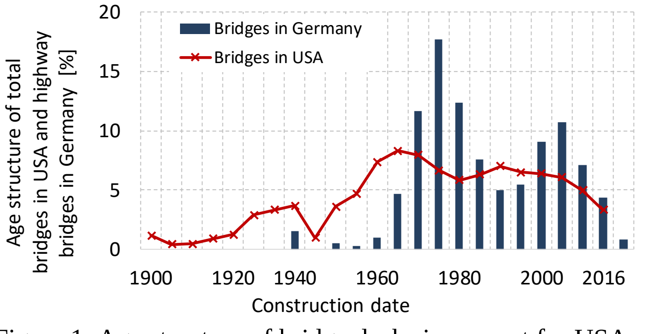

Figure 1. Age structure of bridge decks in percent for USA and Germany (U.S. Department of Transportation, 2016), ((bast), 2017)

Therefore, the development of modern, reliable and economical methods for condition assessment of bridge structures is of essential importance. Some of the current research projects prove that damages, which reduce the stiffness of the structures influence them load bearing capacity (Stöhr, et al, 2006) (He, 2017). Particularly, the curvature of a structure under load resulting from the structural response is in relation to the stiffness course along the structure (Stöhr, et al., 2006) (Wu & Law, 2004). Within the paper, a new method based on static load deflection experiments and precise measurement of the deflection line using recent measurement techniques is presented. The load deflection experiments on support structures have a long history. For example, on the bridge Reichsbrücke in Vienna (Figure 2) a loading test was carried out during the opening ceremony in 1937. The test with 84 trucks and 28 with stones loaded tramcars was performed to prove to the public the safety of the structure (Bolle, Schacht, & Marx, 2011). In order to investigate the shell structures more in detail a study of the load bearing capacity is carried out in 1931 by the company Dyckerhoff & Widmann AG (Figure 3). The structure with the thickness of only 1.50 cm loaded with 50 employees did not generate any cracking during the loading tests. The experience of the experimental test and the proof of the huge load bearing capacity paved the way for many further constructions in reinforced concrete shell structures (Bolle, Schacht, & Marx, 2011).

Figure 2. Loading test of the Reichsbrücke in Vienna in 1937 (Bolle, Schacht, & Marx, 2011)

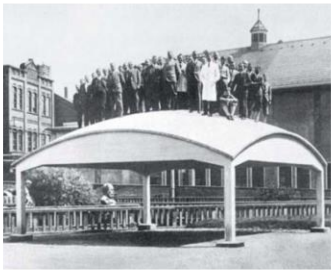

Figure 3. Experimental shell with human crowd (Bolle, Schacht, & Marx, 2011)

Currently there are several research studies for identification or assessment of damages based on static load or dynamic tests (Perera, et al., 2016) (Xu, et al., 2018). Several algorithms are developed by different projects. However, the authors describe common general problems often due to missing reference data particularly for existing old bridges, the choice of suitable measuring and loading techniques, global influences such as temperature variations or the difficult interpretation of the stiffness introduction by the asphalt layer etc (Boumechra, 2016) (He, 2017) (Sung, et a., 2016).

The proposed Deformation Area Difference Method uses new measurement techniques such as digital photogrammetry allowing a higher precise measurement of the deflection line along the longitudinal axis of the structure. The aim of the DAD method is the identification and localisation of local stiffness reducing damages for bridge structures. In accordance with the DAD method application, the measured deflection line will be derived multiple times in order to set it in relation to the stiffness. The reference system can be generated from a theoretical model of the structure for old bridges or initial measurements after the construction for new bridges. First, the paper will discuss the method itself, then the laboratory experiment using the theoretical as well as experimental results. The applied measurement technique for the application of the DAD method is photogrammetry (Baqersad, et al., 2017). The laboratory experiment consists of a steel beam HEA180 with the span length of 5.60 m. The laboratory test includes several steps of damage scenarios which are caused due to different slits on the flange. The loading is carried out path-controlled whereby the maximum deflection did not exceed the limit of the serviceability limit state. The successful application of the DAD method contributes to the state-of-the-art with a new non-destructive method for damage localization and assessment. Investigation of further measurement techniques is presented in Erdenebat, et al., 2016). The method is also investigated for a reinforced concrete beam (Erdenebat, et al., 2016) (Erdenebat, et al., 2017).

2 THE DAD-METHOD

The base of the Deformation Area Difference Method is the static load deflection experiment on real bridge structures including the higher precise measurement of the deflection line along the longitudinal axis. The main objective of the investigation is to identify and to localize the load bearing capacity influencing damages in bridge structures. Generally, local damages in structures influence the stiffness of the structure, load deflection behaviour, furthermore the inclination and curvature of the deflection line. Modern FEM based design software allows modelling of complex bridge structures close to reality. However, the comparison of the expected load deflection behaviour with the measured load deflection behaviour only does not lead to a reliable assessment. In fact, the load bearing behaviour of bridge structures is influenced by temperature effects and due to non-structural components such as the asphalt layer etc (Waldmann, et al., 2015). In addition, the lacking design documents and specific material characteristics for existing old bridges challenges the precise modelling of the structure. Therefore, the DAD method does not compare directly both deflection curves but investigates the area between the curves from the theoretical model and measurements. The decisive curves are the deflection lines from the theoretical modelling of the structure as reference line and the measured deflection line as curve with possible damages. The bending moment and curvature relation allows the consideration of the stiffness of the structure. Therefore, the stiffness influencing discontinuity respectively damage could be identified using that relation. The derivation of the deflection line enables the calculation of the inclination angle. Under the assumption that the maximum deflection is small, the double derivation of the deflection line corresponds to the curvature of the structure. The assumption is generally fulfilled for load deflection experiments within the serviceability limit state in order to develop a non-destructive method for damage detection. Within the method, particularly the area between the both curves, the measured curve and the reference curve, is considered. First, the total area is divided into several identical sections, whereby the length of the section depends on the density of the deflection measuring points and the mesh density of the theoretical model. The DAD-value is the area difference between the reference and measured curves squared section by section and divided by the square of the total area. Such normalization is done for example in order to compare the differently distributed random variables to each other (Batista, 2015) (Goss, et al., 2017). The calculation of the DAD values from the deflection line will be carried out according to equation (1), DAD values from inclination angle (2) and DAD values from curvature according to equation (3). The extensive derivation of the mathematical function is presented in (Erdenebat, et al., 2018).

$$DAD_{w,i}(x) = \frac{\Delta A^2_{w,i}}{\sum_{i=1}^{n} \Delta A^2_{w,i}} = \frac{\left[w_d(x_i) - w_t(x_i) - w_t(x_{i-1}) + w_d(x_{i-1})\right]^2}{\sum_{i=1}^{n}\left[w_d(x_i) - w_t(x_i) - w_t(x_{i-1}) + w_d(x_{i-1})\right]^2} \quad (1)$$

$$DAD_{\varphi,i}(x) = \frac{\Delta A^2_{\varphi,i}}{\sum_{i=1}^{n} \Delta A^2_{\varphi,i}} = \frac{\left[w_d(x_i) - w_t(x_i) + w_t(x_{i-1}) - w_d(x_{i-1})\right]^2}{\sum_{i=1}^{n}\left[w_d(x_i) - w_t(x_i) + w_t(x_{i-1}) - w_d(x_{i-1})\right]^2} \quad (2)$$

$$DAD_{\kappa,i}(x) = \frac{\Delta A^2_{\kappa,i}}{\sum_{i=1}^{n} \Delta A^2_{\kappa,i}} = \frac{\left[\varphi_d(x_i) - \varphi_t(x_i) + \varphi_t(x_{i-1}) - \varphi_d(x_{i-1})\right]^2}{\sum_{i=1}^{n}\left[\varphi_d(x_i) - \varphi_t(x_i) + \varphi_t(x_{i-1}) - \varphi_d(x_{i-1})\right]^2} \quad (3)$$

DAD: Value of the Deformation Area Difference method

$w_t(x)$: Value of the theoretical (reference) deflection at the position x

$w_d(x)$: Value of the measured (damaged) deflection at the position x

$\varphi_t(x)$: Value of the theoretical (reference) inclination angle, first derivation of the theoretical deflection

$\varphi_d(x)$: Value of the measured (damaged) inclination angle, first derivation of the measured deflection

$\kappa_t(x)$: Value of the theoretical (reference) curvature, second derivation of the theoretical deflection

$\kappa_d(x)$: Value of the measured (damaged) curvature, second derivation of the measured deflection

In case of local damages, the maximum deflection of the structure tends to the direction of the local damage. However, the position of the maximum deflection does not correspond to the damage position (Erdenebat, et al., 2018). First, only the course of the inclination angle shows discontinuities at the position of stiffness changes or at the local damage. As the DAD method considers the area between the curves, the integration of the curve functions are needed to calculate the area difference. Therefore, the calculation of the DAD values from curvature will be calculated by using the inclination values (equation (3)) and the DAD values from the inclination obtained with the deflection values (equation (2)).

Figure 4 shows exemplary a section of a beam with three measuring points. The measuring points have a defined distance of Δx to each other. The distance Δx corresponds to the precision of the damage localisation. In other words, the smaller the section Δx, the more precisely the damage can be localised. The height difference between the two points depends on the horizontal distance Δx. The smaller the distance Δx is, the smaller the difference of the height between two points. The smaller the measurable height difference the bigger the influence of the measurement accuracy. The continuous course of the deflection line requires a highly accurate measurement of the differences between the measuring points.

The inclination angle φ_I (red in Figure 4) is calculated from point pi to the next point p_{i+1}. In this case, the considered distance between the measuring points amounts to one Δx. In comparison, the inclination φ_{II} (blue in Figure 4) is calculated from one measuring point pi to the second next one p_{i+2} with the considering distance of two Δx. so, the inclination angle difference between two measuring points depends on the considered distances between the measuring points. In summary, on the one hand, the distance between the measuring points should be large enough in order to have measurable deflection differences between the measuring points without the influence of the measurement accuracy. However, on the other hand, the measuring points should be as close as possible so that the damage can be precisely identified within a limited area. The optimum has to be investigated case by case.

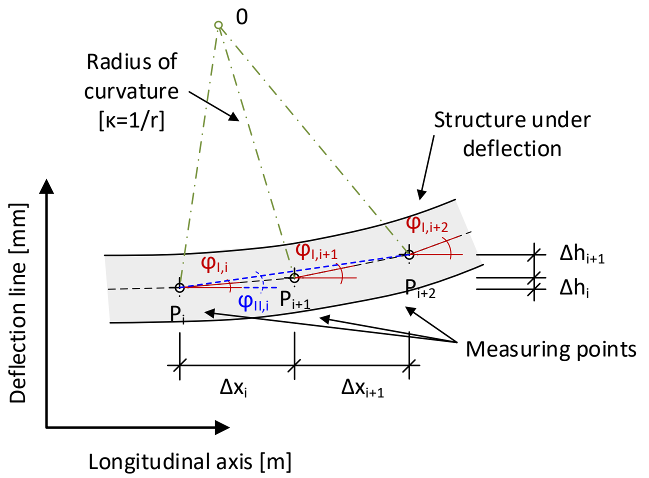

Figure 4. Relation of deflection, inclination, curvature as well as the deflection difference between several measuring points

Pi: Number of the measuring point
Δxi: Distance between two measuring points along the beam
Δhi: Height difference between two measuring points
φI,i: Inclination angle from one measuring point to next one
φII,i: Inclination angle from point to next second point
κ=1/r: Curvature

The application of the method will be presented in the following sections on a laboratory experiment using a steel beam (Figure 5).

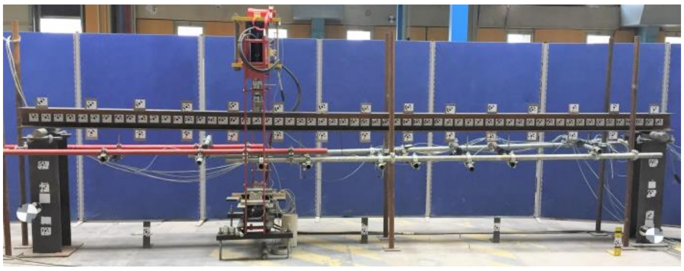

Figure 5. Test specimen: HEA180 S235 steel beam

3 THE LABORATORY EXPERIMENT

3.1 *Applied measurement techniques*

The combination of the calibrated full-frame SLR camera Nikon D800 with the surveying software Elcovision 10 enables a highly precise measurement of the deflection line. Several coded targets are attached along the beam for the photogrammetric analysis. The software automatically identifies the coded targets during the evaluation process and generates the point cloud for the deflection measurement. The used lens 50 mm allowed for captures from a distance of about 5.0 m a minimum pixel size of about 0.40 mm.
Furthermore, the total station Leica TS30 was used for two purposes. Firstly, in order to enable the calibration of the camera, a calibration wall with several measured targets is needed. Secondly, in order to scale the measured targets on the beam, reference points are needed which have highly precise coordinates. So the total station is used to measure the targets on the calibration wall and to measure the reference points.

3.2 *Experimental setup*

The laboratory experiment consists of a steel beam with the cross-section HEA 180 in a S235 steel quality. The total length of the beam amounts 6.00 m whereby the span is 5.60 m. The loading of the beam is applied at 2.00 m from the left end of the beam (Figure 6). On the web of the I-profile 59 coded photogrammetry targets are stuck in a distance of 10 cm to each other. The rest the of targets on the bottom and on the steel pillars are reference targets to scale the measuring targets. The reference targets are independently measured by the total station as already mentioned. The damaging of structure is provided at three different positions. Position Nr. 1 is at 3.60 m, Nr. 2 at 4.80 m and Nr. 3 at 1.20 m from the left end of the beam. The damaging of the cross-section is carried out by slitting the bottom flange stepwise.

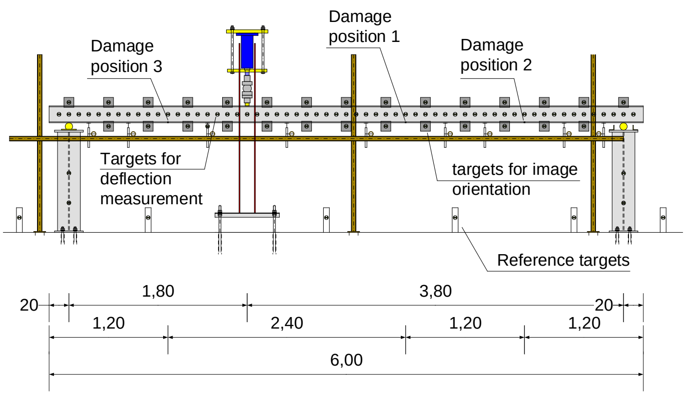

Figure 6. Experimental setup

The top and bottom flange of the cross-section are equipped with small steel plates of 10x10 cm (Figure 5 and Figure 6). These steel plates are needed for additional targets to improve the orientation and to guarantee an overlapping of several images. Thus, the measurement accuracy could be increased.

3.3 *Experimental procedure*

As already mentioned, the beam is damaged on three different positions over the length of the beam. The damaging of the cross-section is created by slitting the bottom flange stepwise. For the first damage level a slitting length of 10 mm has been chosen (Table 1). The corresponding reduction of the stiffness at this position amounts to 2.75 %. In order not to exceed the serviceability limit state of the beam, the defined maximum deflection has been fixed to 5600/250= 22.4 mm. The planned force for the test amounts 30 kN and the expected deflections are given in Table 1.

The damage degree increases depending on the damage level up to 71.45 %. The damaging procedure starts at position 1 from damage level 1 to 5, subsequently at position 2 from damage level 3 to 5 and at position 3 from damage level to 3 to 5. At the procedure number 13 (Table 1), the bottom flange is completely slotted whereby the reduction of stiffness amounts 71.45 %. The degree of the damage is calculated for the section where the flange is slotted and describes a local damage for the beam.

Table 1. Damage levels and theoretical deflections under 30 kN load

Procedure Nr.	Damage position	Damage level	Slit length in the flange	Damage degree	Deflection
	-	-	mm	%	mm
1	0	0	0	0	17.79
2	1	1	10	2.75	17.79
3	1	2	20	5.16	17.80
4	1	3	40	10.79	17.80
5	1	4	80	23.75	17.82
6	1	5	140	48.97	17.91
7	2	3	40	10.79	17.91
8	2	4	80	23.75	17.91
9	2	5	140	48.97	17.93
10	3	3	40	10.79	17.93
11	3	4	80	23.75	17.94
12	3	5	140	48.97	17.99
13	1	6	180	71.45	19.70

Figure 7 illustrates exemplary the damaging of the cross-section for the HEA180 profile at the procedure numbers 4 and 5 respectively damage level 3 and 4 (Table 1). The slitting occurred using circular saw with the width of about 5 mm (Figure 8).

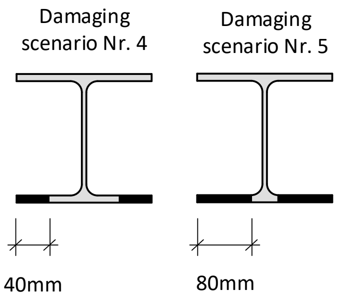

Figure 7. Damaging scenario exemplary for Nr. 4 and Nr. 5

Figure 8. Damaging of the cross-section at the damage position 1 damage level 4 due to slitting the bottom flange

3.4 *Theoretical results of the experimental beam*

The application of the DAD method requires a reference system for damage localisation. The reference system can be generated by the deflection data provided by a numerical model of the structure or in case of a new built bridge structure an initial deflection measurement after construction. In the current study a numerical model is used (Figure 9), to serve as reference system. It also allows to introduce and to explain the DAD method based on a numerical calculation without measurement noise.

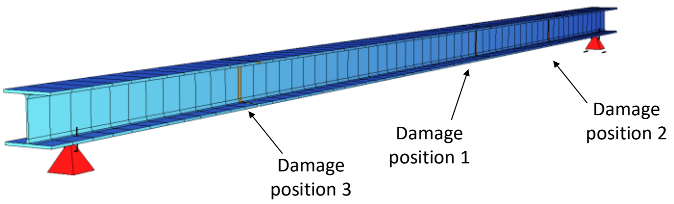

Figure 9. Model of the experimental beam with the damaged elements

Figure 10 to Figure 14 show the results from numerical calculation of the beam for different damage scenarios. The horizontal axis of the figures shows with 6 m the length of the beam, and the vertical axis shows the curvature DAD-values calculated according to equation (3). The red marks above the diagram indicate the position of the damages number 1, 2 and 3 (compare to Figure 6). The label next to the dashed red line, such as 1/2 in Figure 11, represents the damage position 1 and damage level 2. As already mentioned in Table 1, the degree of the damage at level 1 amounts only to 2.75 %. Because of the non-destructive load deflection experiment at serviceability limit state, the maximum deflection amounts to 17.79 mm. The small deflection and the small degree of damage face the challenge of damage localization (Figure 10). The results of the theoretical calculation show already a clear localization of the damage at damage level 2 (Figure 11). Figure 13 and Figure 14 illustrate the results for damages at three different positions, namely at position 1 (damage level 5), at position 2 (damage level 5) and at position 3 damage level 3. Figure 13 shows the procedure number 10 according to Table 1. The degree of the damage amounts 10.79 % at damage position 3 in comparison to 48.97 % at damage position 1. Therefore, the localization of the damage at position 3 becomes only visible when the scale of the vertical axis in the diagram is modified (zoomed) (Figure 14).

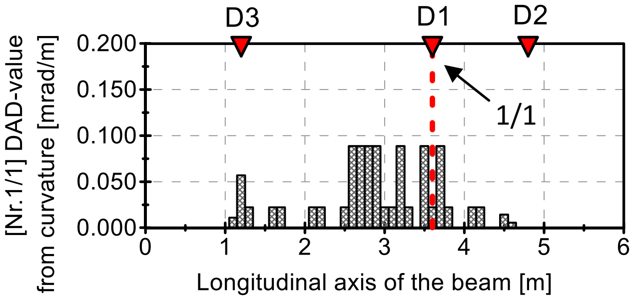

Figure 10. Curvature DAD value for damage position 1 and damage level 1 from the numerical calculation

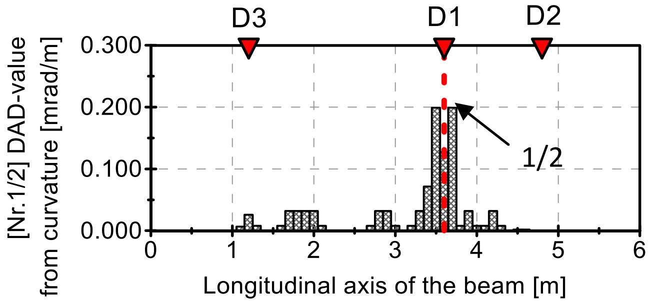

Figure 11. Curvature DAD value for damage position 1 and damage level 2 from numerical calculation

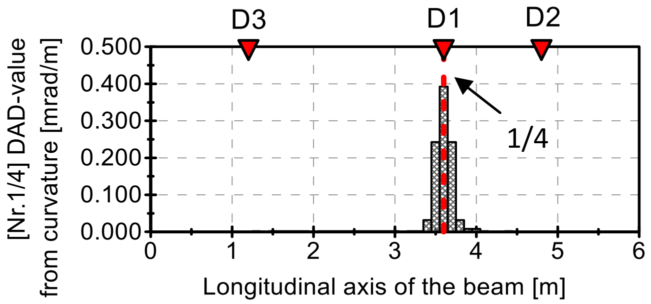

Figure 12. Curvature DAD value for the damage position 1 and damage level 4 from numerical calculation

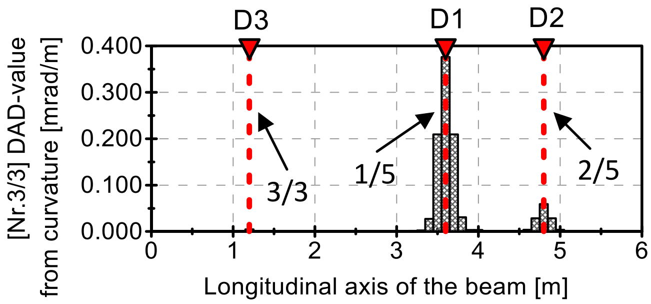

Figure 13. Curvature DAD value for the damage position 3 and damage level 3 from numerical calculation

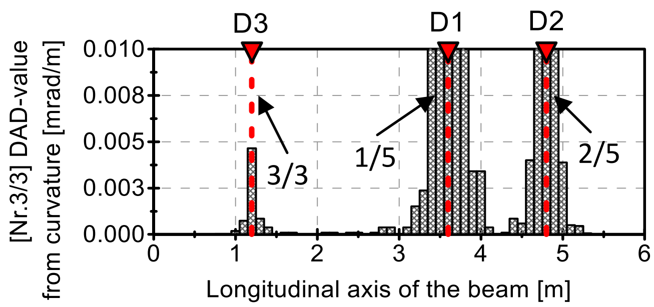

Figure 14. Curvature DAD value for the damage position 3 and damage level 3 from numerical calculation (vertical axis scaled)

3.5 *Experimental results*

As already presented, the basis of the DAD method application is the measurement of the deflection line along the longitudinal axis of the structure as precisely as possible. The highly precise measurement by photogrammetry with a calibrated camera allowed measurement with minimum standard deviation of about 0,04 mm. However, the measured deflection line particularly the derivation of the deflection line is influenced considerably by measurement noise. Therefore, the reliable localisation of damage remains challenging. Figure 15 to Figure 19 show a part of the experimental results respectively the DAD-values from curvature. Figure 15 contains the DAD values for the damage position 1 and damage level 2 calculated from the experiment. Although the course of the DAD-values identify some discontinuities in the range of the damage 1/2 in Figure 15, the diagram contains other DAD-values due to noise. At this low damage level of only 5.16 % the impact of noise on the results is higher as real damage.

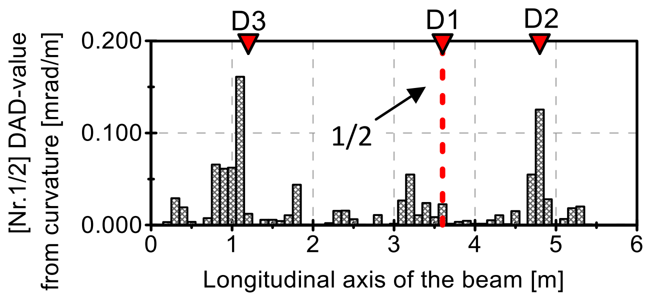

Figure 15. Curvature DAD value for damage position 1 and damage level 2 from the experimental deflection measurements

Figure 16 presents the results of the experiment with damages on 2 positions. According to Table 1 and damage number 8, at the considered stage the degree of damage amounts 48.97 % at damage position 1 whereas at damage position 2 the damage degree is 23.75 %. By comparison with Figure 15, the degree of the damage at damage position 1 increases from 5.16 % to 48.97 % and as a consequence the discontinuity at 3.60 m (damage position 1) shows a significant increase according to the increased damage degree (Figure 16). At damage position 2 a rise of DAD-value can be identified.

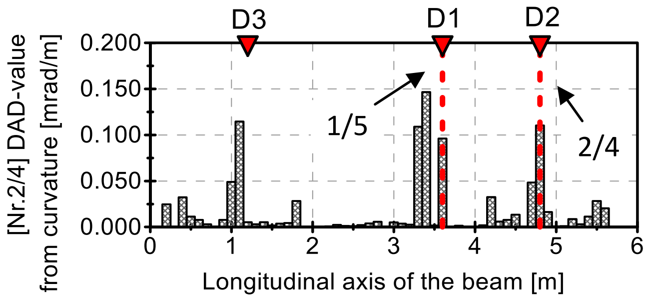

Figure 16. Curvature DAD value for the damage position 2 and damage level 4 from the experimental deflection measurements

However the measurement accuracy and the effect of noise make a reliable localisation of damage position. According to Figure 17, the big damage degree in the mid-range of the span (1/5 Figure 17) still identifies the damaged position. However, there are disturbing peaks resulting from measurement noise.

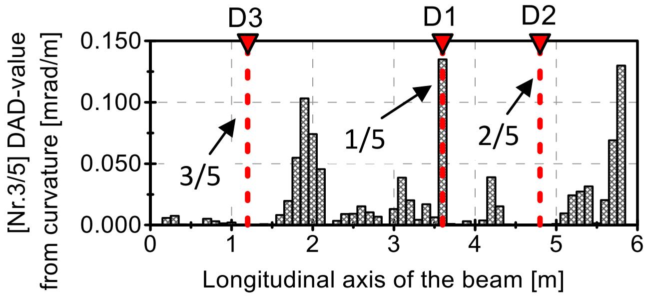

Figure 17. Curvature DAD value for damage position 3 and damage level 5 from the experimental deflection measurements

At the last step of the experiment, the bottom flange of the steel cross-section is completely slotted and during loading, the yield strength of the steel at the slotted place is reached Figure 18. The localisation of the failure position was clearly possible using the DAD-values. The Figure 19 shows the identification of the failure at 3.60 m respectively at the last step number 13 (Table 1).

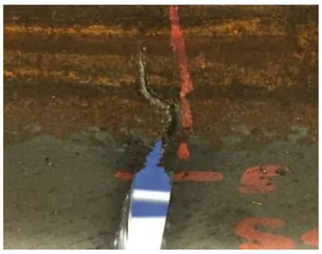

Figure 18. Failure of the beam at position 1

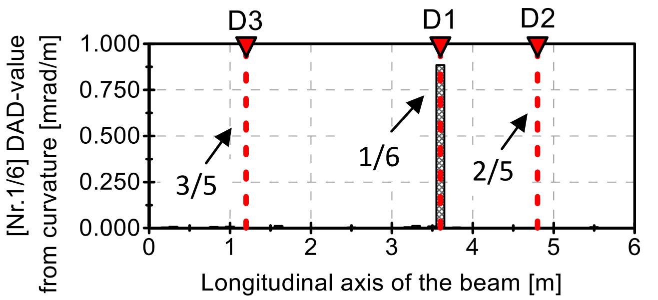

Figure 19. Curvature DAD value for damage position 1 and damage level 6 from the experimental deflection measurement

4 CONCLUSION AND OUTLOOK

As already mentioned in the introduction, the experimental loading tests have a long tradition. However, technological developments and technical advances open up new possibilities, particularly the modern software as well as high-precision measurement techniques. The DAD method presented within this paper exploits the combination of the different innovative prospects in order to contribute to the state-of-the-art for condition assessment of bridge structures. The proposed method is based on static load deflection measurements with the aim to localise damage in bridges. The procedure of the method application is as follows:

- Modelling of the structure exemplary with the finite element method. The model should consider each planned discontinuities due to stiffness changes along the structure and generates the reference system.
- Realisation of a load deflection experiment on a bridge structure and highly-precise measurement of the deflection line. The compliance of the maximum deflection under serviceability limit state in order to allow a non-destructive bridge inspection.
- Application of the DAD-method in order to identify discontinuities resulting from a local stiffness reduction induced by damage. Calculation of the inclination angle and curvature from the measured and calculated deflection line. Consideration of the area between the measured and calculated curves. Normalisation of the each area section between the measuring points by the total area between the both curves.
- Eventually, analysis of the measurement noise, of measuring point distance variations and of external influences such as environmental effects.
- Statement of the damage localisation probability. Identification of discontinuities and stiffness changes along the longitudinal axis of the structure.

Within the study, a laboratory experiment with a steel beam type HEA180. The steel beam is locally and stepwise damaged by slitting the bottom flange. During the loading of the beam, the deflection curve is measured by close-range photogrammetry with the high resolution camera Nikon D800. The paper includes the presentation of the method based on theoretical numerical and experimental results. Numerical results showed the capacity of the method for detection of damage without influence of measurement noise and environmental influences. The method allows a clear and reliable identification of damages within the serviceability limit state based on theoretical values. The photogrammetry enabled the highly precise measurements of the deflection line with minimum effect of noise. However, the multiple derivation of the deflection line for calculation of the inclination angle and curvature leads to increased noise effects. The course of the DAD values resulting from the curvature showed peaks and discontinuities in the area of manually generated damages. Nevertheless, the reliable and clear localisation of the damage is disturbed by the relatively high proportion of noise effects. For the future work, the effect of noise resulting from the measurement accuracy should be investigated in order to increase the reliability of damage localisation.

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