## Mandelbrot and the Smile

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## **Abstract**

It is a well-documented empirical fact that index option prices systematically differ from Black-Scholes prices. However, previous research provides inconclusive results whether the observed volatility smile could be explained by a discrete-time dynamic model of stock returns with skewed, leptokurtic innovations. The improvements in pricing errors are particularly pronounced for out-of-the money put options, while the models partly underperform a Gaussian alternative for near-the-money options. Motivated by theses empirical evidence, I develop a new GARCH option-pricing model with a more flexible innovation structure. In an application of the model to DAX index options, I test the relative performance of the approach against a standard nested GARCH specification and the well-known practitioners Black-Scholes model. I show that the performance of the truncated Lévy GARCH option pricing model is superior to existing approaches.

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**Keywords**: GARCH, option pricing, out-of-sample, truncated Levy distribution.

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