Handling Norms in Multi-Agent System by Means of Formal Argumentation

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Abstract

Formal argumentation is used to enrich and analyse normative multi-agent systems in various ways. In this chapter, we discuss three examples from the literature of handling norms by means of formal argumentation. First, we discuss how existing ways to resolve conflicts among norms using priorities can be represented in formal argumentation, by showing that the so-called Greedy and Reduction approaches can be represented using the weakest and the last link principles respectively. Based on such representation results, formal argumentation can be used to explain the detachment of obligations and permissions from hierarchical normative systems in a new way. Second, we discuss how formal argumentation can be used as a general theory for developing new approaches for normative reasoning, using a dynamic ASPIC-based legal argumentation

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theory. We show how existing logics of normative systems can be used to analyse such new argumentation systems. Third, we show how argumentation can be used to reason about other challenges in the area of normative multiagent systems as well, by discussing a model for arguing about legal interpretation. In particular, we show how fuzzy logic combined with formal argumentation can be used to reason about the adoption of graded categories and thus address the problem of open texture in normative interpretation. Our aim to discuss these three examples is to inspire new applications of formal argumentation to the challenges of normative reasoning in multiagent systems.

1 Introduction

Norms regulate our everyday life, and are used to assess conformance of behaviour with respect to regulations holding in multi-agent systems. Agents undertake discussions about norms to assess their validity or applicability subject to particular conditions, to derive the obligations and permissions to be enforced, or to claim that a certain normative conclusion cannot be derived from the existing regulations. Given the profound importance of norms in multi-agent systems, it is fundamental to understand, e.g., which norms are valid in certain environments, how to interpret them, and to determine the deontic conclusions of such norms. Some influential philosophers, such as Scott Shapiro [54], argue that the law has an inherent teleological nature and that norms are plans, and in most existing normative multiagent systems, norms are like plans which aim at achieving the social goals the members of a society have decided to share [12, 13]. However, it is not obvious that, for example, norms stating human rights can be considered as plans, and we therefore do not commit here to such philosophical claims.

Formal argumentation is typically based on logical arguments constructed from prioritised rules, and it is no surprise that the first applications of formal argumentation in the area of normative multiagent systems were concerned with the resolution of conflicting norms and norm compliance. Moreover, several frameworks have been proposed for normative and legal argumentation [10], but no comprehensive formal model of normative reasoning from arguments has been proposed yet. In this chapter we discuss three challenges to illustrate the variety of applications of formal argumentation techniques in the field of normative multi-agent systems.

- How can formal argumentation be used to explain existing approaches for reasoning about normative multi-agent systems?

- How can new argumentation systems for reasoning about norms be developed, and how can these new argumentation systems be analysed?

- Which issues in the area of normative multiagent systems can be modelled and analysed using formal argumentation, besides the resolution of conflicting norms and checking compliance of a system with a set of norms?

First, we discuss how existing detachment procedures for prioritized norms can be represented in argumentation, by showing how the so-called Greedy and Reduction approaches can be represented in argumentation by applying the weakest link and the last link principles respectively [35]. Based on such representation results, formal argumentation can be used to explain the detachment of obligations and permissions from hierarchical normative systems in a new way.

Second, we discuss an instance of ASPIC^{+} [40, 48, 59] capturing the inference schemes of arguments about norms like *legislative and interpretative* arguments. Moreover, we show how to adopt the input/output logic methodology [38] for the analysis of these new argumentation systems [59].

Third, we discuss the model of da Costa Pereira *et al.* [17], in which norm interpretation is a mechanism to deal with uncertainty, in contrast to existing models of norm interpretation in the context of Normative Multi-Agent Systems and AI&Law [12, 13, 66, 4, 39, 5]. This uncertainty reflects that, in legal theory, a definition of an empirical concept bounded in all now-foreseeable dimensions can break down in the face of unforeseen and unforeseeable events, and norms cannot anticipate all potential occurrences falling within the application scope of any legal norm [29, 37]. In other words, it reflects that the interpretation of legal rules is often uncertain: legal language is vague, the concepts used to describe a legal rule are not always precise, and the purpose of the rule may be differently perceived [30, 19, 36]. The model uses fuzzy logic to measure the uncertainty of legal concepts, and argumentation is used to handle the conflicts between different interpretations of norms. More precisely, a fuzzy argumentation system [56] to represent the interpretations, is combined with fuzzy labeling to evaluate the status of fuzzy arguments [18]. As in many logical analyses of legal reasoning, the model is not purely descriptive and it is rather meant to offer a rational reconstruction for explaining and checking the robustness of interpretive arguments. A formal model for legal impreciseness must be cognitively sound, in the sense that it works on reliable cognitive assumptions.

The remainder of the chapter is organised as follows. Second 2 introduces how prioritized norms can be represented in argumentation. In Section 3, we discuss the logical properties of the static legal argumentation system proposed by Prakken and Sartor, and we reformulate it in a normative perspective. Section 4 motivates our adoption of graded categories as a tool to tackle the problem of open texture in legal interpretation. Section 5 introduces a model of fuzzy argumentation and fuzzy labeling, and Section 6 interprets a norm with flexibiity and conducts a case study by using an example from medically assisted reproduction. Second 7 discusses related work and Section 8 concludes.

2 Argumentation semantics for hierarchical normative systems

Consider the following benchmark example introduced by Hansen [28], which we call here the *prioritised triangle* due to its graphical visualization in Figure 1.

Example 1 (Prioritised triangle [28])**.** *Imagine you have been invited to a party. Before the event, you receive several imperatives, which we consider as the following set of norms.*
- Your mother says: if you drink (p), then don't drive (¬x).
- Your best friend says: if you go to the party (a), then you'll drive (x) us.
- An acquaintance says: if you go to the party (a), then have a drink with me (p).

We assign numerical priorities to these norms, namely '3', '2' and '1' corresponding to the sources 'your mother', 'your best friend' and 'your acquaintance', respectively.

Let a*,* p *and* x *respectively denote the propositions that you go to the party; you drink; and you drive. In terms of a hierarchical normative systems [1], these norms are respectively represented as* $(a,p)_1$*,* $(p,\neg x)_3$ *and* $(a,x)_2$*. These three norms are visualized in Figure 1(a).*

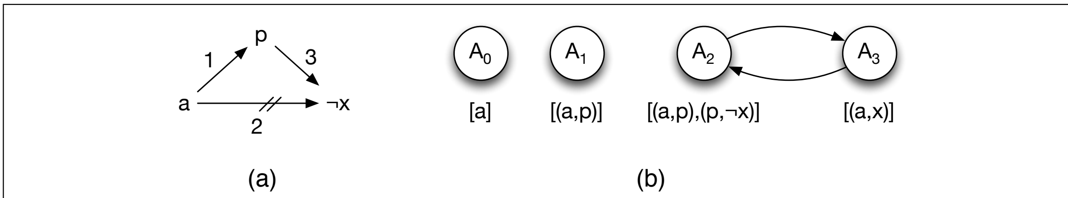

Figure 1: The prioritised normative system of the prioritised triangle example.

Consider the following two approaches resulting in different outcomes or *extensions* [15, 64, 35].

Greedy approach Based on the context, a set of propositions that are known to hold, this approach always applies the norm with the highest priority that does not introduce inconsistency to an extension and the context. Here we say that a norm is applicable when its body is in the context or has been produced by other norms and added to the extension. In this example, we begin with the context $\{a\}$, and (a,x) is first applied. Then (a,p) is applied. Finally, $(p,\neg x)$ cannot be applied as this would result in a conflict, and so, by using the Greedy approach, we obtain the extension $\{p,x\}$.

Reduction approach In this approach, a candidate extension is identified. All norms which are applicable according to this candidate extension are selected and transformed into unconditional or body-free norms (i.e., a norm (a, b) selected in this way is transformed to a norm $(\top, b)$). The modified normative system, with the transformed norms is evaluated using the Greedy approach. The candidate extension is selected as an extension by the Reduction approach if it is identified as an extension according to this application of the Greedy approach. In this example, selecting a candidate extension $\{p, \neg x\}$, we get a set of body-free norms $\{(\top, p), (\top, \neg x), (\top, x)\}$. The priorities assigned to these norms are carried through from the original normative system, and are therefore respectively 1, 3 and 2. After applying the Greedy approach, we get $\{p, \neg x\}$, which is thus an extension of the Reduction approach. If on the other hand we had selected the candidate extension $\{p, x\}$, this new extension would not appear in the greedy evaluation, because $(\top, x)$ has a lower priority than $(\top, \neg x)$. Consequently $\{p, x\}$ is *not* an extension of the Reduction approach.

We now consider the prioritised triangle example in formal argumentation. Given a normative system, we may construct an argumentation framework as illustrated in Figure 1(b), which is a directed graph in which nodes denote arguments, and edges denote attacks between arguments. An argument is represented as a path of a directed graph starting from a node in the context. In this simple example, there are four arguments A_0, A_1, A_2 and A_3, represented as $[a], [a, p], [a, p, \neg x]$ and $[a, x]$, respectively. Since the conclusions of A_2 and A_3 are inconsistent, A_2 attacks A_3 and vice versa. Priorities allow us to transform these attacks into *defeats* according to different principles.

Last link ranks an argument based on the strength of its last inference, if the last link principle is applied, then $[a, p, \neg x]$ defeats $[a, x]$. As result, the principle allows us to conclude $\{p, \neg x\}$.

Weakest link ranks an argument based on the strength of its weakest inference. If the weakest link principle is used instead, $[a, x]$ defeats $[a, p, \neg x]$, and concludes $\{p, x\}$.

In this example, the last link principle thus gives the same result as the Reduction approach, and weakest link gives the same result as the Greedy approach. Liao *et al.* [35] show that this is not a coincidence, but it holds for all totally ordered normative systems. This result addresses the challenge raised by Dung [20] aiming at representing nonmonotonic logics through formal argumentation. In particular, argumentation is a way to exchange and communicate viewpoints, thus having an argumentation theory representing a nonmonotonic logic is desirable for such a logic, in particular when the argumentation theory is simple and efficient. Note that it is not helpful for the development of nonmonotonic logics themselves, but it helps when we want to apply such logics in distributed and multiagent scenarios.

Based on such representation results, formal argumentation can be used to explain the detachment of obligations and permissions from hierarchical normative systems in a new way. Moreover, many other challenges in normative reasoning have been expressed as inconsistent sets of formulas that are intuitively consistent, traditionally called deontic paradoxes. The most well known are the so-called contrary-to-duty paradoxes, which are concerned with handling norm violations. Techniques from non-monotonic reasoning have been applied to handle contrary-to-duty reasoning, and formal argumentation techniques can be applied in the same way [44]. Finally, the most discussed practical problem in normative systems is norm conformance and compliance, which is a computational problem to check whether a business process is in accordance with a set of norms. Handling priorities among norms is again a central challenge for norm compliance, and formal argumentation techniques for resolving conflicts between norms can be extended with reasoning about business processes to reason about norm compliance [57].

3 New argumentation systems for normative reasoning

In the previous section, we used an argumentation system to explain the conclusions that are detached from a hierarchical normative system. The converse is done as well: new argumentation systems for normative reasoning have been developed for normative reasoning, for which detachment procedures have been defined to analyse these argumentation systems. We illustrate this by the argumentation system for legal reasoning proposed by Prakken and Sartor [48], which has been analyzed and extended by van der Torre and Villata [59].

Definition 1 (LAS-PS)**.** *A legal argumentation system or LAS is a tuple $\langle \mathcal{L}, -, \mathcal{R} \rangle$ where $\mathcal{L}$ is the legal language of all sentences α, $- : \mathcal{L} \to 2^{\mathcal{L}}$ is a function given by $-(P) = \{\neg P\}$, $-(\neg P) = \{P\}$ and $-(N) = \emptyset$, and $\mathcal{R}$ contains the Defeasible modus ponens (DMP), rule for each possible norm N of the form $\phi_1 \wedge \ldots \wedge \phi_n \rightsquigarrow \psi$.*

DMP: $\phi_1, \ldots, \phi_n, \phi_1 \wedge \ldots \wedge \phi_n \rightsquigarrow \psi \Rightarrow \psi$;

In order to illustrate the legal argumentation framework, the running example proposed by Prakken and Sartor [48] is adapted.

Example 2 (Smoking regulations)**.** *Consider propositional atoms $P ::= a|b|c|d|e|f$ where a: "people want to smoke in a closed space", b: "the public place has special* *secluded smoking areas", c: "people need to smoke cannabis on medical grounds", d: "people are forbidden from smoking cannabis and tobacco in public places", e: "cannabis is allowed for medical treatment", f: "people are permitted to smoke cannabis in recreational cannabis establishments". $\mathcal{R}$ contains expressions for inference rules DMP of the form, for example:* $a, a \rightsquigarrow b \Rightarrow b$ *and* $a, \neg c, a \wedge \neg c \rightsquigarrow d \Rightarrow d$.

Prakken and Sartor [48] follow Modgil and Prakken [40], and do not consider a model theoretic semantics for this language. Instead, they define a set of arguments.

Definition 2 (LAS PS arguments)**.** *A knowledge base K is a set of sentences of $\mathcal{L}$. The set of arguments A on the basis of a knowledge base K in a legal argumentation system LAS is called $Arg(LAS, K)$ and is the smallest set of expressions containing the literals in K and closed under the following rule:*

if $A_1, \ldots, A_n \subseteq Arg(LAS, K)$ *and* $concl(A_1), \ldots, concl(A_n) \Rightarrow L \in \mathcal{R}$ *then we have also* $(A_1, \ldots, A_n \Rightarrow L) \in Arg(LAS, K)$,
where $concl(A)$ *is defined by* $concl(L) = L$ *and* $concl(A_1, \ldots, A_n \Rightarrow L) = L$. *We may leave out the brackets if there is no risk of confusion.*

To study this notion of norm based argument, consequence is defined by considering only the conclusions of the arguments, in other words, by abstracting away the explicit arguments. Following input/output logic conventions, the consequence is called *Out*.

Definition 3 (Output PS)**.** $Out(LAS, K) = \{concl(A) \mid A \in Arg(LAS, K)\}$.

Example 3 (Continued)**.** *Consider the knowledge base of the smoking regulations* $K_1 = \{a, b, c, e, a \wedge b \rightsquigarrow \neg d, c \wedge \neg d \wedge e \rightsquigarrow f\}$ *where the norms state that*

- *if people want to smoke in a closed space and the public place has smoking special secluded areas, then people are not forbidden from smoking cannabis and tobacco in public places;*
- *if people need to smoke cannabis on medical grounds and it is not forbidden from smoking cannabis and tobacco in public places and cannabis is allowed for medical treatment, then people are permitted to smoke cannabis in recreational cannabis establishments;*

Arguments can be constructed combining DMP inference rules as follows:

- $A_1 : a, b, a \wedge b \rightsquigarrow \neg d \Rightarrow \neg d$;
- $A_2 : c, (a, b, a \wedge b \rightsquigarrow \neg d \Rightarrow \neg d), e, c \wedge \neg d \wedge e \rightsquigarrow f \Rightarrow f$.

Therefore, from arguments A_1, A_2, we have that $concl(A_1) = \neg d$ and $concl(A_2) = f$. We conclude that $Out(LAS_1, K_1) = \{a, b, c, \neg d, e, f\}$.

We now introduce a logical analysis. Van der Torre and Villata [59] use a proof system with expressions $K \therefore L$. The proof system contains four rules, called Identity (ID), Strengthening of the input (SI), Factual Detachment (FD), and Deontic Detachment (DD). The former is sometimes called Monotonicity (Mon), and the latter two are sometimes called Modus Ponens (MP) or Cumulative Transitivity (CT). The notion of consequence is called simple-minded reusable throughput or out_3^+ by Makinson and van der Torre [38].

Definition 4 (Derivations PS)**.** *$der(LAS)$ is the smallest set of expressions $K \therefore L$ closed under the following four rules.*

ID: *$\{L\} \therefore L$ for a literal L*

SI: *from $K \therefore L$ derive $K \cup K' \therefore L$*

FD: *$\{L_1, \ldots, L_n, L_1 \wedge \ldots \wedge L_n \rightsquigarrow L\} \therefore L$ for a norm $L_1 \wedge \ldots \wedge L_n \rightsquigarrow L$*

DD: *from $K \therefore L_i$ for $1 \leq i \leq n$ and $K \cup \{L_1 \ldots, L_n\} \therefore L$ derive $K \therefore L$*

The close relation between arguments and derivations in a deontic logic or a logic of normative systems is illustrated by the following property:
$K \therefore L \in der(LAS)$ iff $L \in Out(LAS, K)$.

This is not surprising, as the similarity is quite clear from the structure of arguments. However, making the relation precise by framing the legal argument system into an input/output logic highlights a drawback of the legal argumentation system of Prakken and Sartor: simple-minded reusable throughput is usually adopted for default logics and logic programs, not for the normative reasoning.

To establish the results with constrained input/output logic, only rebut is considered. Thus undercut is not considered. Moreover, they do not consider defeasible knowledge and undermining. So the only attack is the attack of an argument with an opposite literal. This is obviously a very simple notion of attack which is of little use in most applications, but it useful to establish the relation with logical approaches.

Definition 5 (Attack PS)**.** *The set of sub-arguments of B is the smallest set containing B, and closed under the rule: if $A_1, \ldots, A_n \Rightarrow L$ is a sub-argument of B, then $A_1, \ldots, A_n$ are also sub-arguments of B.*

A attacks B iff there is a sub-argument B' of B such that $concl(A) \in -(concl(B'))$. We write $attack(AS, K)$ for the set of all attacks among $Arg(AS, K)$.

A semantics associates sets of extensions with an argumentation framework, where each extension consists of a set of arguments. For each extension, the output consists of the set of conclusions of the arguments, as for *Out* before. A semantics thus gives us a set of sets of conclusions, which is called an *Outfamily*.

Definition 6 (Outfamily PS). *An extension is a set of arguments, and an argumentation semantics $sem(arg, attack)$ is a function that takes as input a set of arguments and a binary attack relation among the arguments, and as output a set of extensions.*

$$\mathit{Outfamily}(K, sem) = \{\{concl(A) \mid A \in S\} \mid S \in sem(arg(AS, K), attack(AS, K))\}.$$

Constrained output in the input/output logic framework is defined as follows, being inspired by maximal consistent set constructions in belief revision and non-monotonic reasoning. *Maxf* takes the maximal sets of norms of K such that the output of K is consistent, and *Outf* takes the output of these maximal norm sets.

Definition 7 (Outf). *Let $K = K^L \cup K^N$ consist of literals K^L and norms K^N.*
$\mathit{Conf}(K) = \{N \subseteq K^N \mid \mathit{Out}(K^L \cup N)\ \textit{consistent}\}$
$\mathit{Maxf}(K) = \{N \subseteq K^N \mid N\ \textit{maximal w.r.t}\ \subseteq\ \textit{in}\ \mathit{Conf}(K)\}$
$\mathit{Outf}(K) = \{\mathit{Out}(K^L \cup N) \mid N \in \mathit{Maxf}(K)\}$

Theorem 1 (Characterization PS). *$\mathit{Outfamily}(KB, sem) = \mathit{Outf}(K)$ for sem is stable or preferred.*

Van der Torre and Villata [59] add an additional modal operator O to the language. All norms are of the form $L_1 \wedge \ldots \wedge L_n \rightsquigarrow L$, as before, or $L_1 \wedge \ldots \wedge L_n \rightsquigarrow OL$. The body contains simple literals and the head contains either a literal or an obligation. They redefine the concepts or *LAS*, *Out*, *der*, etc. As there is no risk for confusion, we refer to them with the same names as in the previous sections.

Definition 8 (LAS O). *Given a set of propositional atoms. The literals, norms and legal language $\mathcal{L}$are given by the following BNF.*
$L ::= P \mid \neg P$ *with P in propositional atoms*
$M ::= L \mid OL$
$N ::= L \wedge \ldots \wedge L \rightsquigarrow M$
$\alpha ::= L \mid N$
A legal argumentation system with obligations or LAS is as defined before, where the $-$ function is extended to obligations.

The definition of arguments is adapted in the obvious way. In the output, they consider only the obligatory propositions.

Definition 9 (Output O). $Out(LAS, K) = \{L \mid A \in Arg(LAS, K), concl(A) = OL\}$.

Example 4. *We consider a revised version of the running example about smoking regulations. We have that the LAS_2 is based on propositional atoms $P ::= a|b|c|d|e$ where a: "the person wants to smoke in a closed space", b: "the person is in a private space", c: "the person needs to smoke on medical grounds", d: "the person is forbidden from smoking", e: "use electronic cigarettes". and $\mathcal{R}$contains expressions for inference rules of the form:*

- $a, a \rightsquigarrow b \Rightarrow b$;
- $a, \neg c, a \wedge \neg c \rightsquigarrow d \Rightarrow Od$;

Consider now the extended knowledge base of the smoking regulations represented by $K_2 = \{a, \neg b, \neg c, a \wedge \neg c \rightsquigarrow d, a \wedge b \rightsquigarrow \neg d, c \rightsquigarrow \neg d, a \wedge d \rightsquigarrow Oe\}$ where the norms state that

- *if the person is in a closed space and she does not need to smoke on medical grounds, then the person is forbidden from smoking;*
- *if the person wants to smoke in a closed space and she is in a private space, then the person is not forbidden from smoking;*
- *if the person needs to smoke on medical grounds, then she is not forbidden from smoking;*
- *if the person wants to smoke in a closed space and she is forbidden from smoking, then it is obligatory to use electronic cigarettes;*

We can construct the following arguments:

- $A_1 : a, \neg c, a \wedge \neg c \rightsquigarrow d \Rightarrow d$;
- $A_2 : a, (a, \neg c, a \wedge \neg c \rightsquigarrow d \Rightarrow d), a \wedge d \rightsquigarrow Oe \Rightarrow Oe$;

We have that $concl(A_1) = \{d\}$ and $concl(A_2) = \{Oe\}$, and we can thus conclude $Out(LAS_2, K_2) = \{e\}$ i.e., the conclusion is an obligation to use electronic cigarettes.

The constrained version can be defined analogously.

The proof system contains two rules, Strengthening of the Input (SI) and Factual Detachment (FD). The notion of consequence is called simple-minded output or out_1 by Makinson and van der Torre [38].

Definition 10 (Derivations O)**.** *$der(LAS)$ is the smallest set of expressions $K \therefore L$ closed under the following two rules.*

SI: *from $K \therefore L$ derive $K \cup K' \therefore L$*

FD: $\{L_1, \ldots, L_n, L_1 \wedge \ldots \wedge L_n \rightsquigarrow L\} \therefore L$ *for a norm* $L_1 \wedge \ldots \wedge L_n \rightsquigarrow OL$

Again we have $K \therefore L \in der(LAS)$ iff $L \in Out(LAS, K)$. The system does not satisfy deontic detachment, e.g. from $K = \{a, a \rightsquigarrow Ob, b \rightsquigarrow Oc\}$ we cannot derive Oc. This is reflected in the proof system by the lack of the DD rule.

Finally, van der Torre and Villata show how can to redefine the concepts of LAS, Out, der, etc., to re-introducing deontic detachment. This illustrates how the formal analysis can inspire the development of new argumentation systems.

4 From Open Texture to Graded Categories

4.1 Flexible legal interpretation based on graded categories

Legal systems are the product of human mind and are written in natural language. This implies that the basic processes of human cognition have to be taken into account when interpreting norms, and that, as natural languages are inherently vague and imprecise, so are norms.

The application of laws to a new situation is a metaphorical process: the new situation is mapped on to a situation in which applying law is obvious, by analogy. Here, by metaphor we mean using a well understood, prototypical situation to represent and reason about a less understood, novel situation. Metaphors are one of the basic building blocks of human cognition [34].

Norms are written with references to categories. As pointed out by Lakoff [33], "Categorization is not a matter to be taken lightly. There is nothing more basic than categorization to our thought, perception, action, and speech." The "classical theory" that categories are defined by common properties is not entirely wrong, but it is only a small part of the story. It is now clear that categories may be based on prototypes. Some categories are vague or imprecise; some do not have gradation of membership, while others do. The category "US Senator" is well defined, but categories like "rich person" or "tall man" are graded, simply because there are different degrees of richness and tallness. However, it is important to notice that these degrees of membership depend both on the the context in which the norm will be applied and on the goal associated to the norm. To be considered tall in the Netherlands is not the same as to be considered tall in Portugal, for example. We have thus first to consider the context and then the goal associated to the norm.

We explore the use of fuzzy logic as a suitable technical tool to capture the imprecision related to categories. More precisely, a category may be represented as a fuzzy set: the membership of an element to a category is a graded notion.

As a result, we get that a norm may apply to a given situation only to a certain extent and different norms may apply to different extents to the same situation.

4.1.1 Fuzzy Logic

Fuzzy logic was initiated by Lotfi Zadeh [65] with his seminal work on fuzzy sets. Fuzzy set theory provides a mathematical framework for representing and treating vagueness, imprecision, lack of information, and partial truth. Fuzzy logic is based on the notion of fuzzy set, a generalization of classical sets obtained by replacing the characteristic function of a set A, χ_A which takes up values in $\{0, 1\}$, i.e. $\chi_A(x) = 1$ iff $x \in A$, $\chi_A(x) = 0$ otherwise, with a *membership function* μ_A, which can take up any value in $[0, 1]$. The value $\mu_A(x)$ is the membership degree of element x in A, i.e., the degree to which x belongs in A. A fuzzy set is completely defined by its membership function. In fact, we can say that a fuzzy set *is* its membership function.

Operation on Fuzzy Sets The usual set-theoretic operations of union, intersection, and complement can be defined as a generalization of their counterparts on classical sets by introducing two families of operators, called triangular norms and triangular co-norms [52, 53, 42]. A triangular norm (or t-norm) is a binary operation $T : [0, 1] \times [0, 1] \to [0, 1]$ satisfying the following conditions for x, y, $z \in [0, 1]$:

- $T(x, y) = T(y, x)$ (commutativity);
- $T(x, T(y, z)) = T(T(x, y), z)$ (associativity):
- $y \leq z \Rightarrow T(x, y) \leq T(x, z)$ (monotonicity);
- $T(x, 1) = x$ (neutral element 1).

A well-known property about t-norms is:

$$T(x, y) \leq \min(x, y). \tag{1}$$

A triangular conorm (or t-conorm or s-norm), dual to a triangular norm, is a binary operation $S : [0, 1] \times [0, 1] \to [0, 1]$, whose neutral element is 0 instead of 1, with all other conditions identical to those of a t-norm:

- $S(x, y) = S(y, x)$ (commutativity);

- $S(x, S(y, z)) = S(S(x, y), z)$ (associativity):
- $y \leq z \Rightarrow S(x, y) \leq S(x, z)$ (monotonicity);
- $S(x, 0) = x$ (neutral element 0).

A well-known property about t-conorms is:

$$S(x, y) \geq \max(x, y). \tag{2}$$

If T is a t-norm, then $S(x, y) \equiv 1 - T(1 - x, 1 - y)$ is a t-conorm and *vice versa*: T and S in this case form a *dual pair* of a t-norm and a t-conorm. Noteworthy examples of such dual pairs are:

- $T_M(x, y) = \min\{x, y\}$, $S_M(x, y) = \max\{x, y\}$ (minimum t-norm and maximum t-conorm or Gödel t-norm and t-conorm);
- $T_P(x, y) = xy$, $S_P(x, y) = x + y - xy$ (product t-norm and t-conorm or probabilistic product and sum);
- $T_L(x, y) = \max\{x + y - 1, 0\}$, $S_L(x, y) = \min\{x + y, 1\}$ (Lukasiewicz t-norm and t-conorm or bounded sum);

For a given choice of a dual pair of a t-norm and a t-conorm (T, S), given two fuzzy sets A and B and an element x, the set-theoretic operations of union, intersection, and complement are thus defined as follows:

$$\begin{aligned} \mu_{A \cup B}(x) &= S(\mu_A(x), \mu_B(x)); && (3) \\ \mu_{A \cap B}(x) &= T(\mu_A(x), \mu_B(x)); && (4) \\ \mu_{\bar{A}}(x) &= 1 - \mu_A(x). && (5) \end{aligned}$$

4.2 Representing Norms

A norm r may be represented as a rule $b_1, \ldots, b_n \Rightarrow l$ such that l is the legal effect of r, such as an obligation linked to the norm [50]. A norm then has a conditional structure such as $b_1, \ldots, b_n \Rightarrow l$ (if $b_1, \ldots, b_n$ hold, then l ought to be the case). An agent is compliant with respect to this norm if l is obtained whenever $b_1, \ldots, b_n$ is derived. Often, logical models of legal reasoning assume that conditions of norms give a complete description of their applicability [50].

However, this assumption is too strong, due to the complexity and dynamics of the world. Norms cannot take into account all the possible conditions where they should or should not be applied, giving rise to the so called “penumbra”: a core of cases which can clearly be classified as belonging to the concept. By a penumbra of hard cases, membership of the concept can be disputed. Moreover, not only does the world change as also pointed out in [36], giving rise to circumstances unexpected to the legislator who introduced the norm, but even the ontology of reality can change with respect to the one constructed by the law to describe the applicability conditions of norms. See, e.g., the problems concerning the application of existing laws to privacy, intellectual property or technological innovations in healthcare. To cope with unforeseen circumstances, the judicial system, at the moment in which a case concerning a violation is discussed in court, is empowered to interpret, i.e., to change norms, under some restrictions not to go beyond the purpose from which the norms stem.

The clauses of a norm often refer to imprecise concepts, which can take up different meanings depending on the purpose of the norm. The case for using fuzzy categories to account for such imprecise concepts has been made by da Costa Pereira *et al.* [17]: those imprecise concepts are a product of the human mind and, more precisely, of a categorization process. According to prototype theory, which is one of the most prominent and influential accounts of the cognitive processes of categorization, each category is defined by one or more prototypes [60], which are typical exemplars of it. A prototype may be regarded as being represented by a property list which has salient properties of the objects that are classified into the concept.

We may formalize these notions in a way that is compatible with an underlying knowledge representation standard and technical infrastructure like the ones provided by the W3C for the Semantic Web, i.e. OWL based on description logics for the terminological part and RDF for the assertional part. This would allow a practical implementation of our proposal using state-of-the-art knowledge engineering technologies. Nevertheless, we keep our formalization abstract for the sake of clarity.

Definition 11 (Language)**.** *Given a knowledge base* K*, an* atom *is a unary or binary predicate of the form $C(s)$, $R(s_1, s_2)$, where the predicate symbol C is a concept name in* K *and R is a role name in* K*, s, s_1, s_2 are* terms*. A* term *is either a variable (denoted by x, y, z) or a constant (denoted by a, b, c) standing for an individual name or data value.*

According to this formalisation, an individual object o is described by all the facts of the form $C(o)$, $R(o, y)$ and $R(y, o)$ such that $\mathrm{K} \models C(o)$, $\mathrm{K} \models R(o, y)$ and $\mathrm{K} \models R(y, o)$, where $\models$ stands for entailment. We call these facts the *properties* of o.

Definition 12 (Graded Category)**.** *A* graded category *$\tilde{C}$ is described by a non-empty set of prototypes* $\mathrm{Prot}(\tilde{C}) = \{o_1, o_2, \ldots, o_n\}$*, where each $o_i \in \mathrm{Prot}(\tilde{C})$ is an individual name in* K.

We can consider that the choice of the actual (more plausible) category with respect to a prototype may be seen as if the prototype represented a kind of generalisation, which applied deductively, will allow to "classify" (categorise) new "problems" (instances) [7].

The membership of an instance to a category depends on its similarity to its prototype(s). Using a similarity measure with values in $[0, 1]$ allows us to represent graded categories as fuzzy sets. A similarity measure of that kind may be defined. Here, we adapt the contrast model of similarity proposed by Tversky [58]. In such a model, an object is represented by means of a set of features and the similarity between two objects is defined as an increasing function of the features in common to the two objects, *common features*, and as a decreasing function of the features that are present in one object but not in the other, *distinctive features*.

Definition 13 (Number of Common Features)**.** *Given two objects or individuals* a, b *in* K*, the number of their common features* $\mathrm{c}(a, b)$ *is defined as*

$$\begin{array}{rcl} \mathrm{c}(a,b) & = & \|\{C : \mathrm{K} \models C(a) \wedge C(b)\}\| \\ & + & \|\{\langle R, c\rangle : \mathrm{K} \models R(a,c) \wedge R(b,c)\}\| \\ & + & \|\{\langle c, R\rangle : \mathrm{K} \models R(c,a) \wedge R(c,b)\}\|, \end{array}$$

where $\wedge$ *represents the* and *logical connective.*

Definition 14 (Number of Distinctive Features)**.** *Given two objects or individuals* a, b *in* K*, the number of their distinctive features* $\mathrm{dis}(a, b)$ *is defined as*

$$\begin{array}{rcl} \mathrm{dis}(a,b) & = & \|\{C : \mathrm{K} \models C(a) \oplus C(b)\}\| \\ & + & \|\{\langle R, c\rangle : \mathrm{K} \models R(a,c) \oplus R(b,c)\}\| \\ & + & \|\{\langle c, R\rangle : \mathrm{K} \models R(c,a) \oplus R(c,b)\}\|, \end{array}$$

where $\oplus$ *represents the* exclusive or *logical connective.*

It might be the case, in a given application, that some features are more important than others. This might be taken into account by defining different weights for each feature, depending on the application. Let w : Predicates $\to \mathbb{R}^+$ be a function associating a weight to each concept and role name in the language. The two functions c and dis might then be redefined as follows:

$$\begin{array}{rcl} \mathrm{c}(a,b) & = & \sum_{C:\mathrm{K}\models C(a)\wedge C(b)} w(C) \\ & + & \sum_R w(R) \cdot \|\{c : \mathrm{K} \models R(a,c) \wedge R(b,c)\}\| \\ & + & \sum_R w(R) \cdot \|\{c : \mathrm{K} \models R(c,a) \wedge R(c,b)\}\|; \\ \mathrm{dis}(a,b) & = & \sum_{C:\mathrm{K}\models C(a)\oplus C(b)} w(C) \\ & + & \sum_R w(R) \cdot \|\{c : \mathrm{K} \models R(a,c) \oplus R(b,c)\}\| \\ & + & \sum_R w(R) \cdot \|\{c, : \mathrm{K} \models R(c,a) \oplus R(c,b)\}\|. \end{array}$$

These boil down to Definitions 13 and 14 when $w(C) = 1$ for all C and $w(R) = 1$ for all R.

Definition 15 (Object Similarity)**.** *Given two objects or individuals a, b in* K*, their* similarity *is defined as*

$$s(a, b) = \frac{\mathrm{c}(a, b)}{\mathrm{c}(a, b) + \mathrm{dis}(a, b)}.$$

This similarity function satisfies a number of desirable properties. For all individuals a, b,

- $0 \leq s(a, b) \leq 1$;
- $s(a, b) = 1$ if and only if $a = b$;
- $s(a, b) = s(b, a)$;

We may now define the notion of membership degree of an object o in a graded category.

Definition 16. *Given a graded category $\tilde{C}$ and an arbitrary individual name o, the* degree of membership *of o in $\tilde{C}$ is given by*

$$\mu_{\tilde{C}}(o) = \mathop{S}_{p \in \mathrm{Prot}(\tilde{C})} s(o, p).$$

Since the category of an item in the left-hand-side of a rule may be vague or imprecise, the degrees of truth of such an item with respect to the actual situation may be partial. This implies that a rule can be partially activated, i.e., the state of affairs to be reached thanks to the compliance to that rule can be uncertain.

Let us consider the following rule r: $b_1, \ldots, b_n \Rightarrow l$, where the clauses b_i have the form "o_i is $\tilde{C}_i$" and let $\tilde{C}_1, \ldots, \tilde{C}_n$ be the categories of $b_1, \ldots, b_n$, respectively. A clause b_i of a norm involving a graded category may thus be true only to a degree. The premise of the norm may be partially true and a norm may thus apply only to some extent.

If the membership of an instance in a category depends on its similarity to the prototype of the category and also on the purpose of the norm, then we must conclude that both the prototype of a category and the similarity measure used to compute the membership might vary as a function of the purpose. While it may be hard to see how the similarity measure could change as a function of purpose, it is reasonable to assume that the legislators may have different prototypes in mind for a category with the same name when they write norms for different purposes.

This amounts to assuming that, given a graded category $\tilde{C}$, its set of prototypes may vary as a function of the purpose or goal G of the norm. We write $\mathrm{Prot}(\tilde{C} \mid G)$ to denote the set of the prototypes of category $\tilde{C}$ when the purpose of a norm is G.

The degree of truth α_{iG} of clause b_i = "o_i is $\tilde{C}_i$", given that the purpose of the norm is G, may be computed as

$$\alpha_{iG} = \mu_{\tilde{C}_i}(o_i \mid G) = \underset{p \in \mathrm{Prot}(\tilde{C}|G)}{S} s(o_i, p). \tag{6}$$

Definition 17. *The* degree to which the premise $b_1, \ldots, b_n$ *of rule of the form* $b_1, \ldots, b_n \Rightarrow l$ *is satisfied, given that the purpose of* r *is* G*, is given by*

$$\mathrm{Deg}(b_1, \ldots, b_n \Rightarrow l \mid G) = \underset{i=1,\ldots,n}{T} \alpha_{iG}.$$

The state of affairs which is reached thanks to the compliance of r will be associated with the truth degree of $\mathrm{Deg}(r \mid G)$ — this is also the degree associated to l after the activation of r.

5 Fuzzy Argumentation and Fuzzy Labeling

In recent years, several research efforts have attempted to combine formal argumentation and fuzzy logic, in such a way that the uncertainty of arguments can be measured by their fuzzy degrees, while the conflicts between arguments can be properly handled by Dung's argumentation semantics. Among them, Tamani and Croitoru [56] proposed a quantitative preference based argumentation system, called F-ASPIC. Based on ASPIC and fuzzy set theory, it can be used to model structured argumentation with fuzzy concepts. However, it is not clear how the status of a fuzzy argument is evaluated. Meanwhile, da Costa Perira *et al.* [18] introduce a labeling-based approach to evaluate the status of fuzzy arguments. Therefore, these two approaches are combined to lay a foundation for legal interpretation.

5.1 Fuzzy Argumentation System

A fuzzy argumentation system based on Tamani and Croitoru's F-ASPIC is proposed, with some adaptations to make it fit our framework, and with the addition of the fuzzy labeling algorithm proposed by [18].

The main differences between our framework and F-ASPIC [56] are as follows.

In our framework, we do not need to represent rules with different degrees of importance, as Tamani and Croitoru do. Unlike in F-ASPIC, the antecedent of a rule may be partially satisfied, if it involves graded categories. As a consequence, the consequent of that rule will have a partial truth degree and an argument depending on that rule has a partial membership in the set $\mathcal{A}$ of "active" arguments in the senese of da Costa Pereira *et al.*. So, although from a semantical point of view these gradual notions of partial truth or satisfaction are quite different from Tamani and Croitoru's notion of *importance* and *strength*, they lead to a mathematical treatment which is formally identical. Our main adaptation of F-ASPIC is therefore to replace, in the wording and in the formalism, these notions.

For the sake of simplicity, we assume that every element of the language and every rule are fallible. Hence, we do not differentiate between strict rules and defeasible rules, as ASPIC+ does, but we assume that we only have defeasible rules. This assumption makes the rationality postulates [2] trivially satisfied. However, it does not make things technically simpler (partial truth is basically preserved via strict rules, since they encode indisputable inferences). As a matter of fact, since strict rules satisfy contraposition (i.e., $P \Rightarrow Q$ is equivalent to $\neg Q \Rightarrow \neg P$), while defeasible rules do not have to, such behavior, when required, has to be explicitly simulated.

Definition 18 (Fuzzy argumentation system)**.** *A fuzzy argumentation system, denoted as FAS, is a tuple* $(\mathcal{L}, cf, \mathcal{R}, n, \mathrm{Deg})$ *where*

- *$\mathcal{L}$ is a logical language.*
- *cf is a contrariness function (in this chapter, we only consider the classical negation $\neg$),*
- *$\mathcal{R}$ is the set of (defeasible) inference rules of the form $\phi_1, \ldots, \phi_m \Rightarrow \phi$ (where $\phi_i, \phi \in \mathcal{L}$) .*
- *$n : \mathcal{R} \mapsto \mathcal{L}$ is a naming convention for rules.*
- $\mathrm{Deg} : \mathcal{R} \to [0, 1]$ *is a function returning the degree of activation of a rule, given a grounding of the formulas occurring in it. Intuitively,* $\mathrm{Deg}(r)$ *represents the degree of truth of the antecedent of r.*

In the original F-ASPIC system, fuzzy arguments are then constructed with respect to a fuzzy knowledge base $\mathcal{K}$, assigning a degree of importance $\mu_{\mathcal{K}}(p)$ to each proposition $p \in \mathcal{L}$. In our framework, however, we do not attach a degree of importance to propositions of formulas *per se*, but we need to evaluate a degree of truth of their grounding with respect to graded categories. To be more precise, the atomic propositions that are liable to have a partial degree of truth are those of the form "x is C", where C is a graded category. Given a substitution of variable x with an individual object o, the truth value of the grounding "o is C" will be given, as suggested in the previous section, by the similarity measure $s(o, p)$ of o to one of the prototypes p of C (i.e., one p in the set $\mathrm{Prot}(C)$). To this aim, we keep the same symbol $\mathcal{K}$, but we regard it as a fuzzy valuation function.

Definition 19 (Fuzzy Valuation Function)**.** *A fuzzy valuation function in a* $FAS = (\mathcal{L}, cf, \mathcal{R}, n, \mathrm{Deg})$ *is a fuzzy set* $\mathcal{K} : \mathcal{L}_{\mathrm{ground}} \to [0, 1]$ *such that:*

- *if* $\phi \in \mathcal{L}_{\mathrm{ground}}$ *is a ground atomic proposition of the form "o is C", with* C *a graded category,*
$$\mathcal{K}(o \textit{ is } C) = \underset{p \in \mathrm{Prot}(C)}{S} s(o, p); \tag{7}$$

- *if* $\phi \in \mathcal{L}_{\mathrm{ground}}$ *is a ground atomic proposition not involving graded categories,* $\mathcal{K}(\phi) \in \{0, 1\}$*;*

- *if* $\phi, \psi \in \mathcal{L}_{\mathrm{ground}}$*,*
$$\begin{array}{rcl} \mathcal{K}(\neg\phi) & = & 1 - \mathcal{K}(\phi), \\ \mathcal{K}(\phi \wedge \psi) & = & T(\mathcal{K}(\phi), \mathcal{K}(\psi)) \\ \mathcal{K}(\phi \vee \psi) & = & S(\mathcal{K}(\phi), \mathcal{K}(\psi)) \end{array}$$

  where T *represents a triangular norm and* S *an associated triangular co-norm.*

Let $r : b_1, \ldots, b_n \Rightarrow l$ be a rule. In a very simple case, the degree of activation Deg of r simply corresponds to the value returned by the Fuzzy Valuation Function $\mathcal{K}(\bigwedge_{1 \leq k \leq n} b_k)$.

Definition 20 (Fuzzy argument)**.** *A fuzzy argument* A *on the basis of an argumentation theory with fuzzy valuation function* $\mathcal{K}$ *and a fuzzy argumentation system is*

- ϕ *if* $\phi \in \mathcal{L}$ *with:* $Prem(A) = \{\phi\}$, $Conc(A) = \phi$, $Sub(A) = \{A\}$, $Rules(A) = \emptyset$.

- $A_1, \ldots, A_m \Rightarrow \phi$ *if* $A_1, \ldots, A_m$ *are arguments such that there exists a rule* $Conc(A_1), \ldots, Conc(A_m) \Rightarrow \psi$ *in* $\mathcal{R}$*. In this case,* $Prem(A) = Prem(A_1) \cup \cdots \cup Prem(A_m)$, $Conc(A) = \psi$, $Sub(A) = Sub(A_1) \cup \cdots \cup Sub(A_m) \cup \{A\}$, $Rules(A) = Rules(A_1) \cup \cdots \cup Rules(A_m) \cup \{Conc(A_1), \ldots, Conc(A_m) \Rightarrow \psi\}$.

Given an argument A, $Conc(A)$ denotes the conclusion of A, $Prem(A)$ the set of the premises of A, $Sub(A)$ the set of the sub-arguments of A (including A itself), and $Rules(A)$ the set of rules involved in A.

Then, the degree of activation of each argument is measured by a fuzzy degree, called *strength of argument* in F-ASPIC, which can also be interpreted as a degree of membership in the set of active arguments, defined as follows.

Definition 21 (Strength of argument). *Given a fuzzy argument A, its strength, denoted $\mathcal{A}(A)$, is defined as follows:*

- *if A is of the form ϕ, then $\mathcal{A}(A) = \mathcal{K}(\phi)$;*
- *otherwise,*

$$\mathcal{A}(A) = \underset{r \in Rules(A)}{S} T\left(\text{Deg}(r), \underset{\phi \in Prem(A)}{T} \mathcal{K}(\phi)\right). \tag{8}$$

Then, with respect to the notions of rebut, undercut and defeat in ASPIC, the counterparts in the setting of fuzzy argumentation are defined as follows.

Unlike F-ASPIC, our framework does not require the definition of a fuzzy counterpart of the rebut, undercut, and defeat relation. We rely on the usual crisp relations, defined as follows.

Definition 22 (Attacks). *A attacks B iff A undercuts, rebuts or undermines B, where the function n is a naming convention for rules, which maps each rule to a well-formed formula in $\mathcal{L}$ [41], and*

- *A undercuts B (on B') iff $Conc(A) = \neg n(r)$ for some $B' \in Sub(B)$.*
- *A rebuts B (on B') iff $Conc(A) = \neg\phi$ for some $\exists B' \in Sub(B)$ of the form $B''_1, \ldots, B''_m \Rightarrow \phi$.*
- *A undermines B (on B') iff $Conc(A) = \neg\phi$ for some $B' = \phi$, $\phi \in Prem(B)$.*

Definition 23 (Defeat). *A defeats B iff A undercuts B on B', or A rebuts (undermines) B on B' and $\mathcal{A}(A) \not< \mathcal{A}(B')$.*

We use $\mathcal{A}$ and $\mathcal{D}$ to denote, respectively, the fuzzy set of active arguments (whose membership is their strength) and the defeat relation between them. Then, a fuzzy argumentation framework is represented as $\mathcal{F} = (\mathcal{A}, \mathcal{D})$.

This fuzzification of $\mathcal{A}$ provides a natural way of associating strengths to arguments, and suggests rethinking the labeling of an argumentation framework in terms of fuzzy degrees of argument acceptability [18]. The status of arguments can thus be evaluated by means of Fuzzy AF-labeling.

Definition 24 (Fuzzy AF-labeling)**.** *Let* $(\mathcal{A}, \mathcal{D})$ *be a fuzzy argumentation framework. A fuzzy AF-labeling is a total function* $\alpha\colon \mathcal{A} \mapsto [0,1]$*.*

Definition 25 (Fuzzy Reinstatement labeling)**.** *Let* $(\mathcal{A}, \mathcal{D})$ *be a fuzzy argumentation framework, and* α *be a fuzzy AF-labeling. We say that* α *is a fuzzy reinstatement labeling iff, for all argument* A*,*

$$\alpha(A) = \min\{\mathcal{A}(A), 1 - \max_{B:(B,A)\in\mathcal{D}}\alpha(B)\} \tag{9}$$

Da Costa Periera *et al.* [18] made clear that given a fuzzy argumentation framework, its fuzzy reinstatement labeling may be computed by solving a system of n non-linear equations, where $n = \|\mathrm{supp}(\mathcal{A})\|$, i.e., the number of arguments belonging to some non-zero degree in the fuzzy argumentation framework, of the same form as Equation 9, in n unknown variables, namely, the labels $\alpha(A)$ for all $A \in \mathrm{supp}(\mathcal{A})$.

This can be done quite efficiently using an iterative method as follows: we start with an all-in labeling (a labeling in which every argument is labeled with the degree it belongs to $\mathcal{A}$). We denote by $\alpha_0 = \mathcal{A}$ this initial labeling, and by α_t the labeling obtained after the t^{th} iteration of the labeling algorithm.

Definition 26. *Let* α_t *be a fuzzy labeling. An iteration in* α_t *is carried out by computing a new labeling* α_{t+1} *for all arguments* A *as follows:*

$$\alpha_{t+1}(A) = \frac{1}{2}\alpha_t(A) + \frac{1}{2}\min\{\mathcal{A}(A), 1 - \max_{B:(B,A)\in\mathcal{D}}\alpha_t(B)\}. \tag{10}$$

Note that Equation 10 guarantees that $\alpha_t(A) \leq \mathcal{A}(A)$ for all arguments A and for each step of the algorithm.

The above definition actually defines a sequence $\{\alpha_t\}_{t=0,1,\ldots}$ of labelings, whose convergence has been proven [18]. We may now define the fuzzy labeling of a fuzzy argumentation framework as the limit of $\{\alpha_t\}_{t=0,1,\ldots}$.

Definition 27. *Let* $\langle\mathcal{A}, \mathcal{D}\rangle$ *be a fuzzy argumentation framework. A fuzzy reinstatement labeling for such argumentation framework is, for all arguments* A*,*

$$\alpha(A) = \lim_{t\to\infty}\alpha_t(A). \tag{11}$$

Once this fuzzy reinstatement labeling has been computed, $\alpha(A)$ gives the degree to which each argument A in the framework is accepted; this degree may be used to compute the corresponding degree to which the purpose of a norm is G:

$$\alpha(G) = \max_{A:Conc(A)=G}\alpha(A). \tag{12}$$

As it is clear from the above definitions, an argument may be accepted partially and thus the purpose of a norm may be uncertain. Now, different strategies may be used to deal with such an uncertainty. One possibility is to consider the purpose G for which $\alpha(G)$ is maximal. Another is to evaluate the norm with respect to all purposes such that $\alpha(G) > 0$ and then combine the results weighted by ther corresponding $\alpha(G)$.

6 Interpreting a Norm with Flexibility

In addition to taking graded categories into account, any norm is always associated with a purpose: that is what is called the *purpose* of the norm. The idea is then to capture the fact that, when a legislator states a norm, she has in mind a state of affairs to be reached through compliance with that norm. With that in mind, the degree to which a concept in the rule belongs to a category would also depend on the purpose associated with the rule. In other words, given a norm like $b_1, \ldots, b_n \Rightarrow l$, the degree associated to l depends on the degrees of truth of conditions b_i. These degrees depend in turn on the purpose associated to the norm: for example, the greater the extent to which the prohibition to smoke in public spaces promotes the goal *public health*, the greater is the degree of applicability of a rule like Public_Space ⇒ No_Smoking assuming the fuzziness of the concept Public_Space. However, the actual purpose of the legislator can be controversial [36]: for example, not enough evidence or factual information might be available which could help discover what the legislator was intending when writing a norm. Note that the historical purpose could be obsolete due to social, economic or political change, and the legislator has not reacted in a timely manner or at all. Here, as done in legal theory [43, 50], we adopt an objective teleological approach to interpretation, which means that the purpose of a norm is the one that any rational interpreter would assign to it. Hence, we use an argumentative system which will determine which purpose, with respect to the current knowledge, is the most plausible purpose of a norm.

The case study in our chapter is the application of the Italian Legislative Act n. 40/2004 on “Medically Assisted Reproduction.” Before the declaration of uncostitutionality ruled by the Constitutional Court (opinion n. 96/2015), the statute included section 4, par. 1: “The recourse to medically assisted reproduction techniques is allowed only [...] in the cases of sterility or infertility [...].” The purpose of the discussion is to see whether this provision can be interpreted so that non-sterile or fertile couples, in which one or both spouses are immune carriers of a serious genetic anomaly, could access those techniques.

These couples are able to conceive and bear a child, though the probability that the baby will contract the disease is high. These diseases are normally severely disabling, provoke physical dysfunctions, often prevent the full psychological development of the baby, and can cause premature death. The mentioned medical techniques can detect the illness in advance and consequently let the parents take aware decisions about the pregnancy.

The legislative act does not explicitly define 'sterility' and 'infertility.' On the basis of art. 7 l. 40/2004, every three years, the Ministry of Health is required to promulgate a decree containing the updated guidelines for the application of the law. According to these guidelines, the terms 'sterility' and 'infertility' are considered synonyms and refer to the lack of conception, in addition to those cases of certified pathology, after 12/24 months of regular sexual relations in a heterosexual couple.

In civil law systems, when it comes to statutory interpretation, one option is teleological interpretation, according to which, when interpreting a provision, judges often take into account what explicit or implicit purposes can be ascribed to the norm [43, 36].

As for the purposes, law n. 40/2004 states as follows:

Art. 1, on "Purposes". Par.1: In order to favour the solution of reproduction problems caused by human sterility or infertility, it is allowed the recourse to medically assisted reproduction techniques, according to the conditions and the modalities provided for by the present law, which guarantees the respect of the rights of all the subjects involved, included the conceived baby.

Let us also consider the following norm from art. 4 of L. n. 40/2004:

> The use of techniques of medically assisted procreation is [...] confined to the cases with issue of infertility or [...] sterility certified by a medical procedure.

Law n. 40/2004 is connected to other statutes of the legal system. In particular, the Italian Legislative Act n.194/1978 on "Social Protection of Maternity and Abortion" provides for the possibility of a therapeutic abortion if, during pregnancy, a pathological condition is ascertained, including those relating to significant anomalies or malformations of the baby, that put at risk the physical or psychic health of the woman." Severe genetic diseases are thus included. Moreover, along law n. 194/1978, the chance of a serious danger for the life of the woman is seen as a reason to proceed to abortion. This second legislative act is thus meant to promote the right to health both of the mother and of the child.

In light of the previous remarks, we can outline a list of interpretive arguments supporting different interpretations. Our main target is to see what interpretation better promotes the purposes that can be ascribed to the norm, if a purpose can be considered prominent, and what attacks can occur.

In what follows we present a plausible set of rules representing norms and interpretive legal arguments about such norms [49]. In both cases, fuzzy argumentation is related to the promotion of legal purposes.

In particular, the following (defeasible) rules can identify the basic the interpretive arguments arg_1, arg_2, arg_3, respectively, at stake:

$$\begin{aligned}
r_1 :\quad & \neg\mathrm{Ste}(x), \mathrm{Rsn_Exp_Life}(x) \Rightarrow \neg\mathrm{Med_Rpr}(x) \\
r_2 :\quad & \mathrm{Med_Rpr}(x), \mathrm{Genetic_Dis}(x), \mathrm{Well_Being}(x) \Rightarrow \\
& \mathrm{Sol_Rep_Prob}(x) \\
r_3 :\quad & \neg\mathrm{Sol_Rep_Prob}(x), \mathrm{Genetic_Dis}(x) \Rightarrow \\
& \neg\mathrm{Rsn_Exp_Life}(x) \\
r_4 :\quad & \mathrm{Gener_Child}(x) \Rightarrow \neg\mathrm{Ste}(x)
\end{aligned}$$

where

- Ste(x) = "x is sterile",
- Med_Rpr(x) = "x can access to medically assisted reproduction techniques",
- Rsn_Exp_Life(x) ="x grants a reasonably expected life",
- Genetic_Dis(x) ="x is affected by a serious genetic disease",
- Well_Being(x) ="x enjoys psychological well-being",
- Sol_Rep_Prob(x) ="legally solved for x the reproduction problems",
- Gener_Child(x) = "x can generate children".

Consider the case mentioned above: a couple is actually able to conceive and generate children (Gener_Child(CP)), but they are both carriers of a serious genetic disease (Genetic_Dis(CP)), which does not allow children to live for more than a few years. Then according to the above rules, we have the following arguments:

$$
\begin{aligned}
arg_1 = \quad & \neg\mathrm{Sol\_Rep\_Prob(CP)}, \mathrm{Genetic\_Dis(CP)} \Rightarrow \\
& \neg\mathrm{Rsn\_Exp\_Life(CP)} \\
arg_2 = \quad & \mathrm{Gener\_Child(CP)} \Rightarrow \neg\mathrm{Ste(CP)} \Rightarrow \\
& \mathrm{Rsn\_Exp\_Life(CP)}, \neg\mathrm{Ste(CP)} \Rightarrow \neg\mathrm{Med\_Rpr(CP)} \\
arg_3 = \quad & \mathrm{Med\_Rpr(CP)}, \mathrm{Genetic\_Dis(CP)}, \\
& \mathrm{Well\_Being(CP)} \Rightarrow \mathrm{Sol\_Rep\_Prob(CP)}.
\end{aligned}
$$

The attack relation between arguments are: arg_1 attacks arg_2, arg_2 attacks arg_3, and arg_3 attacks arg_1. Then, we have the following argumentation framework:

$$
\begin{tikzcd}
arg_1 \arrow[r] & arg_2 \arrow[r] & arg_3 \arrow[ll, bend left]
\end{tikzcd}
$$

Figure 2: An argumentation framework

Let us consider these purposes:

- **Hlth_Of_MnC** =“purpose: the right to health both of the mother and the child”; this purpose is associated to rule r_2, i.e., we assume that r_2 promotes purpose **Hlth_Of_MnC**;

- **No_Eugenic** =“purpose: no eugenic selection”; this purpose is associated to rules r_1 and r_4, i.e., we assume that r_1 and r_4 promote purpose **No_Eugenic**.

For the sake of illustration, let us also assume that only two concepts are fuzzy: Gener_Child and Well_Being. Hence, if we consider, for example, r_4, this means that fuzziness depends only on the fact that rule r_4 makes the degree of ¬Ste(CP) as dependent on the degree of capability of generating children by CP. No other source of vagueness are considered for r_4. Analogous considerations apply to rule r_2 in regard to Well_Being.

Given these purposes, we can measure the *degrees to which the premise* of rules r_2 and r_4 are satisfied by CP.

- **Rule r_4:** Let us assume that only one prototype p_1 is associated to Gener_Child and **No_Eugenic** (for example, a standard fertile couple statistically identified in the population of couples) in which, among others, the expected life of children is greater than 50 years and the incidence of genetic diseases is less than 20%. Clearly, these are distinctive features that differentiates p_1 with respect to CP: suppose that the overall distinctive features are $d_1, \ldots, d_6$, while the common features are $c_1, \ldots, c_4$.

If we apply Definition 15, then $s(\text{CP}, p_1) = \frac{4}{4+6} = \frac{4}{10} = 0.4$. Since p is the unique prototype for Gener_Child with respect to **No_Eugenic** and that G for r_4 is {**No_Eugenic**}, then it is easy to check that (see, in particular, Definitions 16 and 19)

$$\mu_{\text{Gener\tilde{\_}Child}}(\text{CP}) = \text{Deg}(r_4 \mid G) = \mathcal{K}(\text{Gener\_Child}(\text{CP})) = 0.4.$$

- **Rule r_2:** Let us assume that only one prototype p_2 is associated to Well_Being and **Hlth_Of_MnC** and that the overall distinctive features are $d'_1, \ldots, d'_16$, while the common features are $c'_1, \ldots, c'_4$. For the same reason, given that $A(r_2)$ stands for Med_Rpr(CP) ∧ Genetic_Dis(CP) ∧ Well_Being(CP),

$$s(\text{CP}, p_2) = \mu_{\text{Well\tilde{\_}Being}}(\text{CP}) = \text{Deg}(r_2 \mid G') = \mathcal{K}(A(r_2)) = 0.2.$$

Given these degrees of activation of rules, the following table illustrates how to apply the machinery of fuzzy labeling to this scenario, given the above degrees of activation of the rules that determine the strength of arguments. As we noted, we defined the fuzzy labeling of a fuzzy argumentation framework as the limit of $\{\alpha_t\}_{t=0,1,\ldots}$. The convergence is obtained quickly: a small number of iterations is enough to get close to the limit.

t	$\alpha_\mathbf{t}$(**arg1**)	$\alpha_\mathbf{t}$(**arg2**)	$\alpha_\mathbf{t}$(**arg3**)
0	1	0.4	0.2
1	0.9	0.2	0.2
2	0.85	0.15	0.2
3	0.825	0.15	0.2
4	0.8125	0.1625	**0.2**
5	**0.8**	0.175	↓
6	↓	**0.2**	

Table 1: Fuzzy labeling

Therefore, arg_1 is accepted to degree 0.8 while arg_2 and arg_3 are given a much lower acceptance degree, namely 0.2. In other words, arg_1 is much more acceptable than arg_2 and arg_3. Its important to observe that these degrees just represent an order of plausibility, as if saying that arg_1 is four times as plausible as arg_2 or arg_3.

7 Related work

Young *et al.* [64] endowed Brewka's prioritized default logic (PDL) with argumentation semantics using the ASPIC$^+$ framework for structured argumentation [41]. More precisely, their goal is to define a preference ordering over arguments $\succsim$, based on the strict total order over defeasible rules defined to instantiate $ASPIC^+$ to PDL, so as to ensure that an extension within PDL corresponds to the justified conclusions of its $ASPIC^+$ instantiation. Several options are investigated, and they demonstrate that the standard $ASPIC^+$ *elitist* ordering cannot be used to calculate $\succsim$ as there is no correspondence between the argumentation-defined inferences and PDL, and the same holds for a disjoint elitist preference ordering. The authors come up with a new argument preference ordering definition which captures both preferences over arguments and also *when* defeasible rules become applicable in the arguments' construction, leading to the definition of a strict total order on defeasible rules and corresponding non-strict arguments. Their representation theorem shows that a correspondence always exists between the inferences made in PDL and the conclusions of justified arguments in the $ASPIC^+$ instantiation under stable semantics.

Brewka and Eiter [15] consider programs supplied with priority information, which is given by a supplementary strict partial ordering of the rules. This additional information is used to solve potential conflicts. Moreover, their idea is that conclusions should be only those literals that are contained in at least one answer set. They propose to use preferences on rules for selecting a subset of the answer sets, called the *preferred answer sets.* In their approach, a rule is applied unless it is defeated via its assumptions by rules of higher priorities.

Dung [21] presents an approach to deal with contradictory conclusions in defeasible reasoning with priorities. More precisely, he starts from the observation that often, the proposed approaches to defeasible reasoning with priorities (e.g., [14, 51, 40]) sanction contradictory conclusions, as exemplified by *ASPIC*$^+$ using the weakest link principle together with the elitist ordering which returns contradictory conclusions with respect to its other three attack relations, and the conclusions reached with the well known approach of Brewka and Eiter [15]. Dung shows then that the semantics for any complex interpretation of default preferences can be characterized by a subset of the set of stable extensions with respect to the normal attack relation assignments, i.e., a normal form for ordinary attack relation assignments. Dung's *normal attack relation* satisfies some desirable properties (Credulous cumulativity and Attack monotonicity) that cannot be satisfied by the *ASPIC*$^+$ semantics [21], i.e., the semantics of structured argumentation with respect to a given ordering of structured arguments (elitist or democratic pre-order) in *ASPIC*$^+$. In the setting of this paper, this notion could be defined as follows. Let $\alpha = (a_1, \ldots, a_n)$ and $\beta = (b_1, \ldots, b_m)$ be arguments constructed from a hierarchical abstract normative system. Since we have no Pollock style undercutting argument (as in $ASPIC^+$) and each norm is assumed to be defeasible, α is said to normally attack argument β if and only if β has a sub-argument β' such that $concl(\alpha) = \overline{concl(\beta')}$, and $r((a_{n-1}, a_n)) \geq r((b_{m-1}, b_m))$. According to the weakest link principle and Definition 23, the normal defeat relation is equivalent to the defeat relation using the last link principle in this paper.

Kakas *et al.* [32] present a logic of arguments called *argumentation logic*, where the foundations of classical logical reasoning are represented from an argumentation perspective. More precisely, their goal is to integrate into the single argumentative representation framework both classical reasoning, as in propositional logic, and defeasible reasoning.

You *et al.* [63] define a prioritized argumentative characterization of non-monotonic reasoning, by casting default reasoning as a form of prioritized argumentation. They illustrate how the parameterized formulation of priority may be used to allow various extensions and modifications to default reasoning.

We, and all these approaches, share the idea that an argumentative characterization of NMR formalisms, like prioritized default logic in Young's case and hierarchical abstract normative systems in our approach, contributes to make the inference process more transparent to humans. However, the targeted NMR formalism is different, leading to different challenges in the representation results. To the best of our knowledge, no other approach addressed the challenge of an argumentative characterization of prioritized normative reasoning.

Prakken and Sartor [48] proposed to define a dynamic argumentation system as a tuple $S = \langle \mathcal{L}, -, \mathcal{R}, n \rangle$ where $\mathcal{L}$ is a logical language including symbols for predicates, functions, constants and variables, $=$ for equality, $\neg$ for negation and $\rightsquigarrow$ for normative conditionals, and the universal quantifier $\forall$, $\mathcal{R}$ is the set of inference rules, and n is the naming convention. A norm has the form $\forall(L_1 \wedge \ldots \wedge L_n \rightsquigarrow L)$, where $L_1, \ldots, L_n$ are literals. In particular, they define inference schemes for validity ($Valid(N(\phi)) \rightarrow \phi$), and applicability (i.e., undercutting, $\neg Applicable(w) \rightarrow \neg DMP(w)$). As future direction, the authors foster the extension of the framework by enriching the logical language with a formal account of modalities such as obligation. This is the issue we addressed in this chapter.

Van der Torre and Villata [59] extend their dynamic legal argumentation framework with deontic modalities, and they propose an general framework for legal reasoning based on ASPIC-like argumentation and input/output logic. The framework allows to reason over normative concepts like factual and deontic detachment, and to assess norms' equivalence. The properties of our logical framework are proved. All new concepts are illustrated by a running example. Our main technical contribution is to give a formal analysis of legal argumentation, and a bridge to standard formalisms for normative systems like input/output logic. Compared to other input/output logics, van der Torre and Villata do not have weakening of the output or aggregation of obligations due to the clausal language. For a comparison with other deontic logics in the recent handbook on deontic logic and normative systems we can define the inference relation in terms of consequence sets as usual (e.g., $KB \models \phi$ iff $\phi \in Out(KB)$).

A framework for legal interpretation capable of taking graded, purpose-dependent institutional facts into account has been proposed by da Costa Periera *et al.* [17]. Such a framework uses argumentation to handle conflicts between different interpretations of legal concepts. The originality of this proposal lies in the use of argumentation to identify the most likely purpose of a norm, which in turn circumscribes the interpretation of the categories (institutional facts, legal concepts) referred to by the norm. The idea of using many-valued logics in argumentation theory is not new. Just to name a few, [16] define a notion of gradual acceptability such that a numerical value is assigned to each argument on the basis of its attackers; Janssen *et al.* [31] propose a fuzzy approach enriching the expressive power of classical argumentation, whose originality lies in the fact that the framework allows to represent the relative strength of the attacks; Grossi and Modgil [26] propose a graded generalization of argumentation semantics in which the origin of the justification degrees is supposed to be exclusively endogenous, i.e., based exclusively on the topology of the attack relation. Qualitative approaches to arguments' acceptability have been proposed in preference-based argumentation frameworks (PAF) [3], value-based argumentation frameworks (VAF) [9], and weighted argumentation frameworks (WAF) [22]. These approaches do not define graded semantics: *(i)* PAFs take into account preference orderings in the selection of acceptable conflicting arguments; *(ii)* VAFs are based on the assumption that some arguments can be stronger than others with respect to a certain value they advance, and this affects the success of an attack; and *(iii)* in WAFs, the weights are used for deciding which attacks can be ignored when computing the extensions. In these approaches, however, preference, values, and weights are provided only as input for the computation of extensions; they do not return an acceptability degree for arguments as output. Finally, Gabbay [23] proposes an equational approach which returns multiple (graded) solutions, and thus several rankings for one argumentation framework.

Other frameworks for legal argumentation are listed below, but all of them concentrate on specific problems of reasoning with legal arguments, whilst the aim of our framework, as well as of Prakken and Sartor, is to integrate various aspects so far addressed separately towards a logic comprehensive model of dynamic legal argumentation. The combination of inferences establishing the validity of norms with inferences using valid norms has been proposed by Yoshino [62]. The view that valid norms are defeasible reasons for legal conclusions was at the core of reason based logic by Hage [27]. Arguments about applicability and inapplicability of norms are discussed by Gordon, Prakken and colleagues [24, 47]. Modeling reasoning with norms through argumentation schemes has been formalized by Verheij [61]. Further connections between norms and argumentation include, among others, case based reasoning [6], arguing in rule based systems [45, 47], dialogues and dialectics [24], argument schemes [25, 11].

Several works in the literature of AI & Law have considered the role of purposes in the legal interpretation. Indeed, this idea is standard in legal theory and the purpose of legal rules is recognised by jurists as decisive in clarifying the scope of the legal concepts that qualify the applicability conditions for those rules [8, 46, 55, 27]. [8, 46] use purposes/goals and values in frameworks of case based reasoning for modeling precedents mainly in a common law context. [55] analyse a number of legal arguments even in statutory law, which include cases close to the ones discussed here. Hage [27] addresses, among others, the problem of reconstructing extensive and restrictive interpretation. This is done in Reason-Based Logic, a logical formalism that can deal with rules and reasons: the idea is that the satisfaction of rules' applicability conditions is usually a reason for application of these rules, but there can also be other (and possibly competing) reasons, among which we have the goals that led the legislator to make the rules. More recently, various work [12, 13, 66] proposed formal models for teleological interpretation in statutory law. All these approaches in AI & Law highlight the importance of rule purposes/goals. However, it seems that no work so far has attempted to couple this view with fuzzy logic and argumentation. In this perspective, we believe that this chapter may contribute to fill a gap in the literature.

8 Conclusions

In this article, we discuss three examples from the literature of handling norms by means of formal argumentation. First, we discuss how the so-called Greedy and Reduction approaches can be represented using the weakest and the last link principles respectively [35]. Based on such representation results, formal argumentation can be used to explain the detachment of obligations and permissions from hierarchical normative systems in a new way. Second, we discuss a dynamic ASPIC-based legal argumentation theory [48], and we discuss how existing logics of normative systems can be used to analyse such new argumentation systems [59]. Third, we show how argumentation can be used to reason about other challenges in normative systems as well, by discussing a model for arguing about legal interpretation [17]. In particular, we show how fuzzy logic combined with formal argumentation can be used to reason about the adoption of graded categories and thus address the problem of open texture in normative interpretation. We refer to the original papers for further details.

Our aim to discuss these three examples is to inspire new applications of formal argumentation to the challenges of normative reasoning in multiagent systems. We do not assume that the possible interactions between normative reasoning and formal argumentation is restricted to the three examples we discuss in this article. Besides resolving conflicting norms, norm compliance, norm dynamics and norm interpretation, it has been used also to argue about enforced obligations and permissions, and to establish norms' validity by deriving their conclusions. Moreover, other central challenge in normative multi-agent system are discussed in the article of Pigozzi and van der Torre, and we believe that formal argumentation is also applicable to various other challenges. For example, agents can argue about the creation or emerging of norms from the mental states of individual agents, or how normative systems can be merged.

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