The Principle-Based Approach to Abstract Argumentation Semantics

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Abstract

The principle-based or axiomatic approach is a methodology to choose an argumentation semantics for a particular application, and to guide the search for new argumentation semantics. This article gives a complete classification of the fifteen main alternatives for argumentation semantics using the twenty-seven main principles discussed in the literature on abstract argumentation, extending Baroni and Giacomin's original classification with other semantics and principles proposed in the literature. It also lays the foundations for a study of representation and (im)possibility results for abstract argumentation, and for a principle-based approach for extended argumentation such as bipolar frameworks, preference-based frameworks, abstract dialectical frameworks, weighted frameworks, and input/output frameworks.

1 The principle-based approach

A considerable number of semantics exists in the argumentation literature. Whereas examining the behaviour of semantics on examples can certainly be insightful, a need for more systematic study and comparison of semantics has arisen. Baroni and Giacomin [2007] present a classification of argumentation semantics based on a set of principles. In this article, we extend their analysis with other principles and semantics proposed in the literature over the past decade.

The principle-based approach is a methodology that is also successfully applied in many other scientific disciplines. It can be used once a unique universal method is replaced by a variety of alternative methods, for example, once a variety of modal logics is used to represent knowledge instead of unique first order logic. The principle-based approach is also called the axiomatic approach, or the postulate based approach (for example in AGM theory change by Alchourrón *et al.* [1985]).

Maybe the best known example of the principle-based approach is concerned with the variety of voting rules, a core challenge in democratic societies, see, e.g., Tennenholtz and Zohar [2016]. It is difficult to find two countries that elect their governments in the same way, or two committees that decide using exactly the same procedure. Over the past two centuries many voting rules have been proposed, and researchers were wondering how we can know that the currently considered set of voting rules is sufficient or complete, and whether there is no better voting rule that has not been discovered yet. Voting theory addresses what we call the choice and search problems inherent to diversity:

Choice problem: If there are many voting rules, then how to choose one voting rule from this set of alternatives in a particular situation?

Search problem: How to guide the search for new and hopefully better voting rules?

In voting theory, the principle-based approach was introduced by Nobel prize winner Kenneth Arrow. The principle-based approach classifies existing approaches based on axiomatic principles, such that we can select a voting rule based on the set of requirements in an area. Moreover, there may be sets of principles for which no voting rule exist yet. Beyond voting theory, the principle-based approach has been applied in a large variety of domains, including abstract argumentation.

Formal argumentation theory, following the methodology in non-monotonic logic, logic programming and belief revision, defines a diversity of semantics. This immediately raises the same questions that were raised before for voting rules, and in many other areas. How do we know that the currently considered set of semantics is sufficient or complete? May there be a better semantics that has not been discovered yet? Moreover, the same choice and search problems of voting theory can be identified for argumentation theory as well:

Choice problem: If there are many semantics, then how to choose one semantics from this set of alternatives in a particular application?

Search problem: How to guide the search for new and hopefully better argumentation semantics?

The principle-based approach again addresses both problems. For example, if one needs to exclude the possibility of multiple extensions, one may choose the grounded or ideal semantics. If it is important that at least some extension is available, then stable semantics should not be used. As another common example, consider the admissibility principle that if an argument in an extension is attacked, then it is defended against this attack by another argument in the extension. If one needs a semantics that is admissible, then for example CF2 or stage2 cannot be chosen.

Principles have also been used to guide the search for new semantics. For example, the principle of resolution was defined by Baroni and Giacomin [2007], well before resolution based semantics were defined and studied by Baroni *et al.* [2011b]. Likewise it may be expected that the existing and new principles will guide the further search for suitable argumentation semantics. For example, consider the conflict-freeness principle that says that an extension does not contain arguments attacking each other. All semantics studied in this article satisfy this property. If one needs to define new argumentation semantics that are para-consistent in the sense that its extensions are not necessarily conflict free [Arieli, 2015], then one can still adopt other principles such as admissibility in the search for such para-consistent semantics.

The principle-based approach consists of three steps.

The first step in the principle-based approach is to define a general function, which will be the object of study. Kenneth Arrow defined social welfare functions from preference profiles to aggregated preference orders. For abstract argumentation, the obvious candidate is a function from graphs to sets of sets of nodes of the graph. Following Dung's terminology, we call the nodes of the graph arguments, we call sets of nodes extensions, we call the edges attacks, and we call the graphs themselves argumentation frameworks. Moreover, we call the function an argumentation semantics. Obviously nothing hinges on this terminology, and in principle the developed theory could be used for other applications of graph theory as well.

We call this function from argumentation frameworks to sets of extensions a two valued function, as a node is either in the extension, or not. Also multi valued functions are commonly used, in particular three valued functions conventionally called labelings. For three valued labelings, the values are usually called in, out and undecided. Other more general functions have been considered in abstract argumentation, for example in value based argumentation, bipolar argumentation, abstract dialectical frameworks, input/output frameworks, ranked semantics, and more. The principle-based approach can be applied to all of them, but in this article we will not consider such generalisations.

The second step of the principle-based approach is to define the principles. The central relation of the principle-based approach is the relation between semantics and principles. In abstract argumentation a two valued relation is used, such that every semantics either satisfies a given property or not. In this case, principles can be defined also as sets of semantics, and they can be represented by a constraint on the function from argumentation frameworks to sets of extensions. An alternative approach used in some other areas gives a numerical value to represent to which degree a semantics satisfies a principle.

The third step of the principle-based approach is to classify and study sets of principles. For example, a set of principles may imply another one, or a set of principles may be satisfiable in the sense that there is a semantics that satisfies all of them. A particular useful challenge is to find a set of principles that characterises a semantics, in the sense that the semantics is the only one that satisfies all the principles. Such characterisations are sometimes called representation theorems.

The principles used in a search problem are typically desirable, and desirable properties are sometimes called postulates. For the mathematical development of a principle-based theory, it may be less relevant whether principles are desirable or not.

Before we continue, we address two common misunderstandings about the principle-based approach, which are sometimes put forward as objections against it.

The first point is that not every function from argumentation frameworks to sets of extensions is an argumentation semantics. In other words, the objection is sometimes raised against the axiomatic approach that it allows for counterintuitive or even absurd argumentation semantics, just like the objection may be raised that not every function from preference profile to candidates is a voting rule. However, in the principle-based approach, such counterintuitive alternatives are excluded by the principles, they are not excluded a priori.

It may be observed that in formal argumentation, this objection is not restricted to principle-based abstract argumentation. A general framework for structured argumentation like ASPIC+ also allows for many counterintuitive or even absurd argumentation theories. However, from the perspective of the principle-based approach, the generality of the ASPIC+ approach can be used to study which combinations of definitions lead to argumentation theories satisfying desired principles [Caminada, forthcoming].

The second point is that a semantics is fundamentally different from a principle. In general a semantics is a function from argumentation frameworks to sets of extensions, and principles can be defined as sets of such functions and represented by a constraint on such functions. This misunderstanding arises because there are examples where a property can be represented as a semantics. For example, the completeness principle may be defined to state that each extension is complete, and the complete semantics may be defined such that the set of extensions of an argumentation framework are *all* its complete extensions. Likewise, some authors transform the admissibility principle into a “semantics” that associates with a framework all the admissible extensions. In this article we do not consider an admissibility semantics defined in this sense, only the admissibility principle.

Finally, we end this introduction with two methodological observations. First, we note that both argumentation semantics and argumentation principles can be organised and clustered in various ways. For example, sometimes a distinction is made between the set of admissibility based semantics and the set of naive based semantics, which are semantics satisfying the admissibility principle and the maximal conflict free principle respectively. In this article we have organised the semantics and principles in a way that seemed reasonable to us, but we did not use a systematic approach and we expect that some readers might have preferred an alternative organisation.

Second, while writing the article, several readers and reviewers have suggested additional semantics and principles to us. For example, we did not systematically study all resolution based semantics. The reason is pragmatic: this article has been growing while we were writing, and at some moment we needed to finish it. Moreover, we excluded several semantics proposed in the literature, such as AD1, AD2, CF1 introduced by Baroni *et al.* [2005], because they have not been further discussed or applied in the formal argumentation literature. However, if some of them will become more popular in the future, then the principle-based study in this article has to be extended to them as well. Finally, dynamic principles are studied by Baroni *et al.* [2014], Rienstra *et al.* [2015] and Baumann [forthcoming].

The layout of this article is as follows. Section 2 introduces the setting and notation, Section 3 introduces the argumentation semantics we study in the rest of the article, and Section 4 introduces the principles and presents the table detailing which principles are satisfied by each semantics.

2 Setting and notations

The current section introduces the setting and notations.

Definition 2.1 (Argumentation framework, [Dung, 1995])**.** *An* argumentation framework *is a couple $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ where $\mathcal{A}$ is a finite set and $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$. The elements of $\mathcal{A}$ are called* arguments *and $\mathcal{R}$ is called attack relation. We say that a* attacks *b if $(a, b) \in \mathcal{R}$; in that case we also write $a\mathcal{R}b$. For a set $S \subseteq \mathcal{A}$ and an argument $a \in \mathcal{A}$, we say that S attacks a if there exists $b \in S$ such that $b\mathcal{R}a$; we say that a attacks S if there exists $b \in S$ such that $a\mathcal{R}b$. We say that S attacks a set P if there exist $a \in S$, $b \in P$ such that a attacks b.*

We define $S^+ = \{a \in \mathcal{A} \mid S \text{ attacks } a\}$ and $S^- = \{a \in \mathcal{A} \mid a \text{ attacks } S\}$. Moreover, for an argument a, we define $a^+ = \{b \in \mathcal{A} \mid a \text{ attacks } b\}$ and

$a^- = \{b \in \mathcal{A} \mid b$ attacks $a\}$. We define $S^-_{out} = \{a \in \mathcal{A} \mid a \notin S$ and a attacks $S\}$. The set of all argumentation frameworks is denoted by $\mathcal{AF}$.

We can observe that an argumentation framework is just a finite graph. In the rest of the article, $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ stands for an argumentation framework.

Definition 2.2 (Projection, union, subset)**.** *For an argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and a set $S \subseteq \mathcal{A}$, we define $\mathcal{F}\downarrow_S = (S, \mathcal{R} \cap (S \times S))$. Let $\mathcal{F}_1 = (\mathcal{A}_1, \mathcal{R}_1)$ and $\mathcal{F}_2 = (\mathcal{A}_2, \mathcal{R}_2)$ be two argumentation frameworks. We define their union by $\mathcal{F}_1 \cup \mathcal{F}_2 = (\mathcal{A}_1 \cup \mathcal{A}_2, \mathcal{R}_1 \cup \mathcal{R}_2)$. We write $\mathcal{F}_1 \subseteq \mathcal{F}_2$ if and only if $\mathcal{A}_1 \subseteq \mathcal{A}_2$ and $\mathcal{R}_1 \subseteq \mathcal{R}_2$.*

For a set S, we denote its powerset by 2^S. Now we define the notion of semantics. It is a function that, given an argumentation framework $(\mathcal{A}, \mathcal{R})$, returns a set of subsets of $\mathcal{A}$.

Definition 2.3 (Semantics)**.** *An extension-based semantics is a function σ such that for every argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, we have $\sigma(\mathcal{F}) \in 2^{2^{\mathcal{A}}}$. The elements of $\sigma(\mathcal{F})$ are called* extensions*.*

Our definition requires a semantics to satisfy universal domain, i.e. to be defined for every argumentation framework. We could give a more general definition, thus allowing a semantics to be defined only for some argumentation frameworks. We do not do that in order to simplify the setting, since all the semantics of interest for our study are defined for all argumentation frameworks.

3 Semantics

This section introduces different argumentation semantics we study in the rest of the article. Note that most of the properties from the literature, which we study in Section 4, can appear in two variants: extension-based and labelling-based. In this article, we present their versions for extension-based approach.

We start by introducing the notions of conflict-freeness and admissibility.

Definition 3.1 (Conflict-freeness, admissibility, strong admissibility)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is* conflict-free *in $\mathcal{F}$ if and only if for every $a, b \in S$, $(a, b) \notin \mathcal{R}$.*

Argument $a \in \mathcal{A}$ is defended *by set S if and only if for every $b \in \mathcal{A}$ such that $b\mathcal{R}a$ there exists $c \in S$ such that $c\mathcal{R}b$. Argument $a \in \mathcal{A}$ is* strongly defended *by set Sif and only if for every $b \in \mathcal{A}$ such that $b\mathcal{R}a$ there exists $c \in S \setminus \{a\}$ such that $c\mathcal{R}b$ and c is strongly defended by $S \setminus \{a\}$. S is* admissible *in $\mathcal{F}$ if and only if it is* *conflict-free and it defends all its arguments. S is* strongly admissible *in $\mathcal{F}$ if and only if it is conflict-free and it strongly defends all its arguments.*

Stable, complete, preferred and grounded semantics were introduced by Dung [1995]:

Definition 3.2 (Complete, stable, grounded, preferred semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$.*

- *Set S is a* complete extension *of $\mathcal{F}$ if and only if it is conflict-free, it defends all its arguments and it contains all the arguments it defends.*
- *Set S is a* stable extension *of $\mathcal{F}$ if and only if it is conflict-free and it attacks all the arguments of $\mathcal{A} \setminus S$.*
- *S is the* grounded extension *of $\mathcal{F}$ if and only if it is a minimal with respect to set inclusion complete extension of $\mathcal{F}$.*
- *S is a* preferred extension *of $\mathcal{F}$ if and only if it is a maximal with respect to set inclusion admissible set of $\mathcal{F}$.*

Dung [1995] shows that each argumentation framework has a unique grounded extension. Stable extensions do not always exist, i.e. there exist argumentation frameworks whose set of stable extensions is empty. Semi-stable semantics [Verheij, 1996; Caminada, 2006b] guarantees that every argumentation framework has an extension. Furthermore, semi-stable semantics coincides with stable semantics on argumentation frameworks that have at least one stable extension.

Definition 3.3 (Semi-stable semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a semi-stable extension of $\mathcal{F}$ if and only if it is a complete extension and $S \cup S^+$ is maximal with respect to set inclusion among complete extensions, i.e. there exists no complete extension S_1 such that $S \cup S^+ \subset S_1 \cup S_1^+$.*

Ideal semantics [Dung *et al.*, 2007] is an alternative to grounded semantics. Like grounded semantics, ideal semantics always returns a unique extension, which is also a complete extension [Dung *et al.*, 2007]. From the definition of the grounded semantics, we conclude that the ideal extension is a superset of the grounded extension. Ideal semantics is thus less sceptical than grounded semantics.

Definition 3.4 (Ideal semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is the ideal extension of $\mathcal{F}$ if and only if it is a maximal with respect to set inclusion admissible subset of every preferred extension.*

We now introduce eager semantics [Caminada, 2007].

Definition 3.5 (Eager semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is the eager extension of $\mathcal{F}$ if and only if it is the maximal with respect to set inclusion admissible subset of every semi-stable extension.*

Caminada [2007] shows that each argumentation framework has a unique eager extension and that the eager extension is also a complete extension. Note that eager semantics is similar to ideal semantics: the ideal extension is the unique biggest admissible subset of every preferred extension; the eager extension is the unique biggest admissible subset of each semi-stable extension. Since each semi-stable extension is a preferred extension [Caminada, 2006], the eager extension is a superset of the ideal extension.

In our article, we want to conduct an exhaustive investigation of properties of extension-based semantics. Thus, for the sake of completeness, we introduce even the semantics that are not very commonly used or studied in the literature, like stage semantics, naive semantics and prudent variants of grounded, complete, stable and preferred semantics.

Stage semantics [Verheij, 1996] was defined in a slightly different setting than ours; we provide an alternative but equivalent definition [Verheij, 1996; Baroni *et al.*, 2011a].

Definition 3.6 (Stage semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a stage extension of $\mathcal{F}$ if and only if S is a conflict-free set and $S \cup S^+$ is maximal with respect to set inclusion, i.e. S is conflict-free, and there exists no conflict-free set S_1 such that $S \cup S^+ \subset S_1 \cup S_1^+$.*

Note the difference between semi-stable and stage semantics: semi-stable extension is a complete extension whereas stage extension is a conflict-free set; stage extension is not necessarily an admissible set.

Definition 3.7 (Naive semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a naive extension of $\mathcal{F}$ if and only if S is a maximal conflict-free set.*

Prudent semantics [Coste-Marquis *et al.*, 2005] is based on the idea that an extension should not contain arguments a and b if a indirectly attacks b. An indirect attack is an odd length attack chain.

Definition 3.8 (Indirect conflict)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, $S \subseteq \mathcal{A}$ and $a, b \in \mathcal{A}$. We say that a indirectly attacks b if and only if there is an odd-length path from a to b with respect to the attack relation. We say that S is without indirect conflicts and we write $\mathtt{wic}(S)$ if and only if there exist no $x, y \in S$ such that x indirectly attacks y.*

The semantics introduced by Dung (grounded, complete, stable, preferred) is based on admissibility; prudent semantics is based on p-admissibility. Prudent semantics is called grounded prudent, complete prudent, stable prudent and preferred prudent by Coste-Marquis *et al.* [2005]. In order to make the names shorter, we call them p-grounded, p-complete, p-stable and p-preferred.

Definition 3.9 (p-admissible sets)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a p-admissible set in $\mathcal{F}$ if and only if every $a \in A$ is defended by S and S is without indirect conflicts.*

Definition 3.10 (p-complete semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a p-complete extension in $\mathcal{F}$ if and only if S is a p-admissible set and for every argument $a \in \mathcal{A}$ we have: if a defended by S and $S \cup \{a\}$ is without indirect conflicts, then $a \in S$.*

We now introduce p-characteristic function, which is needed to define p-grounded semantics. Note that grounded semantics can be defined using characteristic function, but we preferred to provide an alternative equivalent definition.

Definition 3.11 (p-characteristic function)**.** *The p-characteristic function of an argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ is defined as follows:*

- $\mathcal{CF}^p_{\mathcal{F}} : 2^{\mathcal{A}} \rightarrow 2^{\mathcal{A}}$
- $\mathcal{CF}^p_{\mathcal{F}}(S) = \{a \in \mathcal{A} \mid S \textit{ defends } a \textit{ and } \texttt{wic}(S \cup \{a\})\}$

Definition 3.12 (p-grounded semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$. Let j be the lowest integer such that*

$$\underbrace{\mathcal{CF}^p_{\mathcal{F}}(\mathcal{CF}^p_{\mathcal{F}}(\ldots \mathcal{CF}^p_{\mathcal{F}}}_{j \textit{ times}}(\emptyset)\ldots) = \underbrace{\mathcal{CF}^p_{\mathcal{F}}(\mathcal{CF}^p_{\mathcal{F}}(\ldots \mathcal{CF}^p_{\mathcal{F}}}_{j+1 \textit{ times}}(\emptyset)\ldots) = S.$$

The p-grounded extension is the set S.

The p-grounded extension is a p-complete extension [Coste-Marquis *et al.*, 2005]. Note that it is not the case in general that the p-grounded extension is included into every p-preferred extension [Coste-Marquis *et al.*, 2005].

Definition 3.13 (p-stable semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a p-stable extension in $\mathcal{F}$ if and only if S is without indirect conflicts and S attacks (in a direct way) each argument in $\mathcal{A} \setminus S$.*

Definition 3.14 (p-preferred semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $S \subseteq \mathcal{A}$. Set S is a p-preferred extension if and only if S is a maximal for set inclusion p-admissible set.*

Evert p-stable extension is a p-preferred extension [Coste-Marquis *et al.*, 2005].

We now introduce CF2 semantics [Baroni *et al.*, 2005]. For more explanations and examples, the reader is referred to the original paper. The definition of this semantics is complicated; we must introduce several auxiliary definitions in order to present it.

Let us first introduce the notion of strongly connected component (SCC) introduced by Baroni *et al.* [2005].

Definition 3.15 (Strongly Connected Component)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$. The binary relation of path-equivalence between nodes, denoted as $PE_{\mathcal{F}} \subseteq \mathcal{A} \times \mathcal{A}$, is defined as follows:*

- *for every $a \in \mathcal{A}$, $(a, a) \in PE_{\mathcal{F}}$*
- *given two distinct arguments $a, b \in \mathcal{A}$, we say that $(a, b) \in PE_{\mathcal{F}}$ if and only if and only if there is a path from a to b and a path from b to a.*

The strongly connected components of $\mathcal{F}$ are the equivalence classes of arguments under the relation of path-equivalence. The set of strongly connected components is denoted by $SCCS_{\mathcal{F}}$. Given an argument $a \in \mathcal{A}$, notation $SCC_{\mathcal{F}}(a)$ stands for the strongly connected component that contains a.

In the particular case when the argumentation framework is empty, i.e. $\mathcal{F} = (\emptyset, \emptyset)$, we assume that $SCCS_{\mathcal{F}} = \{\emptyset\}$. The choices in the antecedent strongly connected components determine a partition of the nodes of S into three subsets: defeated, provisionally defeated and undefeated. D stands for defeated, P for provisionally defeated and U for undefeated.

Definition 3.16 (D, P, U [Baroni *et al.*, 2005])**.** *Given an argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, a set $\mathcal{E} \subseteq \mathcal{A}$ and a strongly connected component $S \in SCCS_{\mathcal{F}}$, we define:*

- $D_{\mathcal{F}}(S, \mathcal{E}) = \{a \in S \mid (\mathcal{E} \cap S^-_{out})$ *attacks* $a\}$
- $P_{\mathcal{F}}(S, \mathcal{E}) = \{a \in S \mid (\mathcal{E} \cap S^-_{out})$ *does not attack a and* $\exists b \in (S^-_{out} \cap a^-)$ *such that $\mathcal{E}$ does not attack b*$\}$
- $U_{\mathcal{F}}(S, \mathcal{E}) = S \setminus (D_{\mathcal{F}}(S, \mathcal{E}) \cup D_{\mathcal{F}}(S, \mathcal{E}))$

We define $UP_{\mathcal{F}}(S, \mathcal{E}) = U_{\mathcal{F}}(S, \mathcal{E}) \cup P_{\mathcal{F}}(S, \mathcal{E})$.

Definition 3.17 (CF2 semantics)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $\mathcal{E} \subseteq \mathcal{A}$. Set $\mathcal{E}$ is an extension of CF2 semantics if and only if*

- *$\mathcal{E}$ is a naive extension of $\mathcal{F}$ if $|SCCS_{\mathcal{F}}| = 1$*
- *for every $S \in SCCS_{\mathcal{F}}$, $(\mathcal{E} \cap S)$ is a CF2 extension of $\mathcal{F}\downarrow_{UP_{\mathcal{F}}(S,\mathcal{E})}$ otherwise*

Observe that $\mathcal{F}\downarrow_{UP_{\mathcal{F}}(S,\mathcal{E})} = \{a \in S \mid \text{ there exists no } b \in \mathcal{E} \setminus S \text{ s.t. } (b,a) \in \mathcal{R}\}$.

We now introduce stage2 semantics [Dvorák and Gaggl, 2016].

Definition 3.18. *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $\mathcal{E} \subseteq \mathcal{A}$. Set $\mathcal{E}$ is a stage2 extension if and only if*

- *$\mathcal{E}$ is a stage extension of $\mathcal{F}$ if $|SCCS_{\mathcal{F}}| = 1$*
- *for every $S \in SCCS_{\mathcal{F}}$, $(\mathcal{E} \cap S)$ is a stage2 extension of $\mathcal{F}\downarrow_{UP_{\mathcal{F}}(S,\mathcal{E})}$ otherwise*

Dvorák and Gaggl [2016] showed that every stage2 extension is a CF2 extension and that every stable extension is a stage2 extension.

This ends the discussion on extension based semantics of abstract argumentaton. There exist additional proposals for argumentation semantics in the literature, such as for example resolution based semantics of Baroni *et al.* [2011b], but we do not consider them in this article.

In this article, we focus on the extension-based approach, which means that each semantics is defined by specifying the extensions it returns for a given argumentation framework. There exists an alternative, labelling-based approach. Instead of calculating extensions, it provides labellings, one labelling being a function that attaches to every argument a label *in*, *out* or *undec* (which stands for "undecided").

Definition 3.19 (Labelling-based semantics)**.** *Let $\Lambda = \{in, out, undec\}$. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. A labelling on $\mathcal{F}$ is a total function $\mathcal{L}ab : \mathcal{A} \to \Lambda$. A labelling-based semantics is a function λ defined for every element of $\mathcal{AF}$ such that for every argumentation framework $\mathcal{F}$, we have that $\lambda(\mathcal{F})$ is a set of labellings on $\mathcal{F}$.*

To illustrate, let us provide a labelling-based definition of complete semantics.

Definition 3.20 (Complete labelling)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $\mathcal{L}ab$ a labelling on $\mathcal{F}$. We say that $\mathcal{L}ab$ is a complete labelling if and only if for every $a \in \mathcal{A}$:*

- *if a is labelled in then all its attackers are labelled out*
- *if a is labelled out then none of its attackers is labelled in*
- *if a is labelled undec then not all its attackers are labelled out and none of its attackers is labelled in.*

We denote by $in(\mathcal{L}ab)$ *(resp.* $out(\mathcal{L}ab)$*,* $und(\mathcal{L}ab)$*) the set of arguments labelled in (resp. out, und).*

For every $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, the set of complete extensions under σ is exactly the set $\{in(\mathcal{L}ab) \mid \mathcal{L}ab$ is a complete labelling$\}$.

Moreover, there exists a general way that allows to obtain a labelling-based definition of a semantics given its extension-based definition, under the condition that the semantics returns conflict-free sets.

Definition 3.21 (Extension to labelling)**.** *Given an extension* $\mathcal{E}$*, labelling* $\mathcal{L}ab_{\mathcal{E}}$ *is defined as follows:* $\mathcal{L}ab_{\mathcal{E}}(a) = in$ *if* $a \in \mathcal{E}$*,* $\mathcal{L}ab_{\mathcal{E}}(a) = out$ *if* $a \in \mathcal{E}^{+}$*,* $\mathcal{L}ab_{\mathcal{E}}(a) = und$ *otherwise. Then, given a semantics* σ*, we say that* $\mathcal{L}ab$ *is a* σ *labelling of* $\mathcal{F}$ *if and only if there exists* $\mathcal{E} \in \sigma(\mathcal{F})$ *such that* $\mathcal{L}ab = \mathcal{L}ab_{\mathcal{E}}$*.*

Other ways to obtain a labelling from an extension are possible, for example we could say that an argument is *out* if it is attacked by an argument in the extension, or it attacks an argument in the extension. This would make the definition of *out* more symmetric and more in line with naive based semantics. However, it seems such alternatives have not been explored systematically in the literature. Moreover, even if extension and labelling based semantics are inter-translatable, it may affect other definitions such as equivalence of frameworks. Finally, using Definition 3.21, every principle defined in terms of extension based semantics can be translated into labelings and vice versa, though one of the definitions may be more compact or intuitive than the other.

We saw an intuitive way to define complete labellings in Definition 3.20. Intuitive labelling-based definitions of other semantics also exist in the literature. For example: a grounded labelling is a complete labelling such that the set of arguments labelled *in* is minimal with respect to set inclusion among all complete labellings; a stable labelling is a complete labelling such that the set of undecided arguments is empty; a preferred labelling is a complete labelling such that the set of arguments labelled *in* is maximal with respect to set inclusion among all complete labellings. The reader interested in more details about the labelling-based approach is referred to the paper by Baroni *et al.* [2011a].

4 List of Principles

This section presents the properties from the literature and reviews all the semantics with respect to the properties.

Definition 4.1 (Isomorphic argumentation frameworks)**.** *Two argumentation frameworks* $\mathcal{F}_1 = (\mathcal{A}_1, \mathcal{R}_1)$ *and* $\mathcal{F}_2 = (\mathcal{A}_2, \mathcal{R}_2)$ *are* isomorphic *if and only if there*

	Defence	Admiss.	Strong adm.	Naivety	Ind. CF	Reinst.	Weak reinst.	CF- -reinst.
complete	✓	✓	×	×	×	✓	✓	✓
grounded	✓	✓	✓	×	×	✓	✓	✓
preferred	✓	✓	×	×	×	✓	✓	✓
stable	✓	✓	×	✓	×	✓	✓	✓
semi-stable	✓	✓	×	×	×	✓	✓	✓
ideal	✓	✓	×	×	×	✓	✓	✓
eager	✓	✓	×	×	×	✓	✓	✓
p-complete	✓	✓	×	×	✓	×	×	×
p-grounded	✓	✓	✓	×	✓	×	×	×
p-preferred	✓	✓	×	×	✓	×	×	×
p-stable	✓	✓	×	✓	✓	✓	✓	✓
naive	×	×	×	✓	×	×	×	✓
CF2	×	×	×	✓	×	×	✓	✓
stage	×	×	×	✓	×	×	×	✓
stage2	×	×	×	✓	×	×	✓	✓

Table 1: Properties of semantics: basic properties, admissibility and reinstatement

exists a bijective function $m : \mathcal{A}_1 \to \mathcal{A}_2$*, such that* $(a, b) \in \mathcal{R}_1$ *if and only if* $(m(a), m(b)) \in \mathcal{R}_2$*. This is denoted by* $\mathcal{F}_1 \doteq_m \mathcal{F}_2$*.*

The first property, called "language independence" by Baroni and Giacomin [2007] is an obvious requirement for argumentation semantics. It is sometimes called abstraction [Amgoud and Besnard, 2013; Bonzon *et al.*, 2016a] or anonymity [Amgoud *et al.*, 2016].

Principle 1 (Language independence)**.** *A semantics σ satisfies the* language independence *principle if and only if for every two argumentation frameworks* $\mathcal{F}_1$ *and* $\mathcal{F}_2$*, if* $\mathcal{F}_1 \doteq_m \mathcal{F}_2$ *then* $\sigma(\mathcal{F}_2) = \{m(\mathcal{E}) \mid \mathcal{E} \in \sigma(\mathcal{F}_1)\}$*.*

It is immediate to see that all the semantics satisfy language independence, since the definitions of semantics take into account only the topology of the graph, and not the arguments' names.

Conflict-freeness is one of the basic principles. Introduced by Dung [1995] and stated as a principle by Baroni and Giacomin [2007], it is satisfied by all argumentation semantics studied in this article. Note that one can define a non conflict-free semantics [Arieli, 2015]. As another example of relaxing conflict-freeness consider the work by Dunne *et al.* [2011], who introduce a framework where each attack is associated a weight; given an inconsistency budget β, they accept to disregard the set of attacks up to total weight of β.

Principle 2 (Conflict-freeness)**.** *A semantics σ satisfies the* conflict-freeness *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$, $\mathcal{E}$ is conflict-free set in $\mathcal{F}$.*

Defence is a well-known property introduced by Dung [1995].

Principle 3 (Defence)**.** *A semantics σ satisfies the* defence *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$, for every $a \in \mathcal{E}$, $\mathcal{E}$ defends a.*

Baroni and Giacomin [2007] show that complete, grounded, preferred, stable, semi-stable, ideal, p-complete, p-grounded, p-preferred, p-stable satisfy defence and that CF2 does not satisfy defence. Let us consider the four remaining semantics: stage, stage2, eager and naive. The argumentation framework from Figure 1 shows that stage, stage2 and naive semantics violate defence since they all return three extensions: $\{a\}$, $\{b\}$ and $\{c\}$. Eager semantics satisfies defence (this follows directly from its definition).

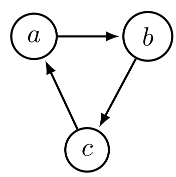

Figure 1: Stage, stage2, naive and CF2 semantics violate admissibility, defence and reinstatement, since they return three extensions: $\{a\}$, $\{b\}$ and $\{c\}$.

Baroni and Giacomin [2007] suppose that every extension is conflict-free. Thus an extension defends all it arguments if and only if it is admissible. However, if conflict-freeness is seen as an optional criterion, we can distinguish between the principles of admissibility and defence.

Principle 4 (Admissibility)**.** *A semantics σ satisfies the* admissibility *principle if and only if for every argumentation framework $\mathcal{F}$, every $\mathcal{E} \in \sigma(\mathcal{F})$ is admissible in $\mathcal{F}$.*

Observation 1. *If a semantics σ satisfies admissibility it also satisfies conflict-freeness and defence.*

We now study the notion of strong admissibility [Baroni and Giacomin, 2007].

Principle 5 (Strong admissibility)**.** *A semantics σ satisfies the* strong admissibility *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$ it holds that $a \in \mathcal{E}$ implies that $\mathcal{E}$ strongly defends a.*

Observation 2. *If a semantics σ satisfies strong admissibility then it satisfies admissibility.*

To understand the notion of strong admissibility, consider the example from Figure 2. Set $\{a, d\}$ is admissible but is not strongly admissible. Informally speaking, this is because a is defended by d whereas d is defended by a. The intuition behind strong admissibility is that this kind of defence is not strong enough because it is cyclic, i.e. arguments defend each other. However, argument e is not attacked, thus $\{e\}$ is strongly admissible. Furthermore, $\{e\}$ strongly defends a, so $\{a, e\}$ is strongly admissible. Also, $\{a, e\}$ strongly defends d. Thus $\{a, d, e\}$ is strongly admissible.

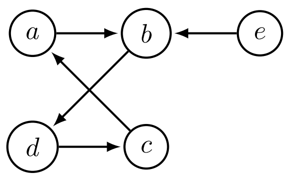

Figure 2: Set $\{a, d\}$ is admissible but is not strongly admissible. Set $\{a, d, e\}$ is admissible and strongly admissible.

Baroni and Giacomin [2007] show that grounded and p-grounded semantics satisfy strong admissibility and that complete, preferred, stable, semi-stable, ideal, p-complete, p-preferred, p-stable and CF2 do not satisfy this principle. Let us consider stage, stage2, eager and naive semantics. Since stage, stage2 and naive semantics violate admissibility, they also violate strong admissibility. To see that eager semantics violates strong admissibility too, consider the example from Figure 3, suggested by Caminada [2007]. The eager extension is $\{b, d\}$; this set is not strongly admissible since it does not strongly defend b.

Another principle, which we call *naivety*, says that every extension under semantics σ is a naive extension.

Principle 6 (Naivety)**.** *A semantics σ satisfies the* naivety *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$, $\mathcal{E}$ is maximal for set inclusion conflict-free set in $\mathcal{F}$.*

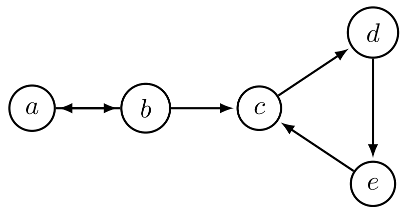

Figure 3: Eager semantics violates strong admissibility because eager extension $\{b, d\}$ does not strongly defend b. The same example shows that eager semantics violates directionality. Observe that $U = \{a, b\}$ is an unattacked set. Denote the whole framework by $\mathcal{F} = (\mathcal{A}, \mathcal{R})$. The eager extension of $\mathcal{F}$ is the set $\{b, d\}$ whereas the eager extension of $\mathcal{F} \downarrow_U$ is the empty set.

We see directly from the definitions of stable, stage, naive, p-stable and CF2 semantics that they satisfy naivety. Since every stage2 extension is also a CF2 extension [Dvorák and Gaggl, 2016], naivety is also satisfied by stage2 semantics. It is easy to see that the other semantics violate this principle.

Coste-Marquis *et al.* [2005] introduced prudent semantics, which are based on the notion of indirect conflict-freeness.

Principle 7 (Indirect conflict-freeness)**.** *A semantics σ satisfies the* indirect conflict-freeness *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$, $\mathcal{E}$ is without indirect conflicts in $\mathcal{F}$.*

Observation 3. *If a semantics σ satisfies indirect conflict-freeness then it satisfies conflict-freeness.*

By examining the definitions of prudent semantics, we see that they all satisfy indirect conflict-freeness, since this concept is built in through the use of p-admissibility and p-characteristic function.

The other semantics do not satisfy indirect conflict-freeness. To show this, consider the argumentation framework depicted in Figure 4, suggested by [Coste-Marquis *et al.*, 2005]. All the semantics except prudent ones have an extension containing both a and e. Hence, they violate indirect conflict-freeness since e indirectly attacks a.

Defence says that an extension must defend all the arguments it contains. Reinstatement can be seen as its counterpart, since it says that an extension must contain all the arguments it defends. This principle was first studied in a systematic way by Baroni and Giacomin [2007].

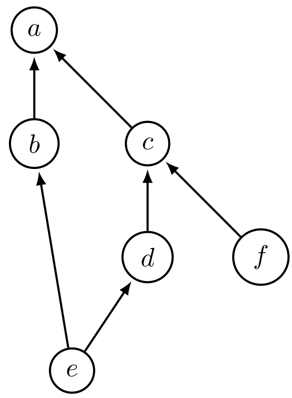

Figure 4: All semantics except prudent semantics violate indirect conflict-freeness. They all yield an extension containing both a and e, even if e indirectly attacks a.

Principle 8 (Reinstatement)**.** *A semantics σ satisfies the* reinstatement *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$, for every $a \in \mathcal{A}$ it holds that if $\mathcal{E}$ defends a then $a \in \mathcal{E}$.*

The results in Table 1 concerning complete, grounded, preferred, stable, semi-stable, ideal, p-complete, p-grounded, p-preferred, p-stable and CF2 semantics were proved by Baroni and Giacomin [2007]. To summarise, all the semantics they study satisfy reinstatement except p-grounded, p-complete, p-preferred and CF2. Let us consider eager, stage, stage2 and naive semantics.

Regarding eager semantics, suppose that $\mathcal{E}$ is an eager extension and that a is defended by $\mathcal{E}$. The eager extension is a complete extension [Caminada, 2007], and complete semantics satisfies reinstatement. Thus, $a \in \mathcal{E}$, which means that eager semantics satisfies reinstatement.

Stage, stage2 and naive semantics violate reinstatement, as proved by [Dvorák and Gaggl, 2016]. Another way to see this is to consider the counter-example from Figure 1.

Baroni and Giacomin [2007] study another property called weak reinstatement.

Principle 9 (Weak reinstatement)**.** *A semantics σ satisfies the* weak reinstatement *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$ it holds that*

$$\mathcal{E} \textit{ strongly defends } a \textit{ implies } a \in \mathcal{E}.$$

Observation 4. *If a semantics σ satisfies reinstatement then it satisfies weak reinstatement.*

The results in Table 1 concerning complete, grounded, preferred, stable, semi-stable, ideal, p-complete, p-grounded, p-preferred, p-stable and CF2 semantics were proved by Baroni and Giacomin [2007]. From Observation 4 we conclude that eager semantics satisfies weak reinstatement.

Stage and naive semantics violate weak reinstatement as can be seen from Figure 5. This was also shown by Dvorák and Gaggl [2016]. Namely, $\{b\}$ is a stage and a naive extension that strongly defends a but does not contain it. Stage2 semantics does satisfy weak reinstatement [Dvorák and Gaggl, 2016].

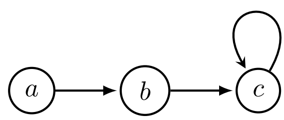

Figure 5: Stage and naive semantics violate weak reinstatement, since $\mathcal{E} = \{b\}$ is an extension that strongly defends a, but $\mathcal{E}$ does not contain a.

The reinstatement principle makes sure that as soon as an argument a is defended by an extension $\mathcal{E}$, a should belong to $\mathcal{E}$—without specifying that a is not in conflict with arguments of $\mathcal{E}$. To take this into account, another principle was defined by Baroni and Giacomin [2007].

Principle 10 ($\mathcal{CF}$-reinstatement)**.** *A semantics σ satisfies the* $\mathcal{CF}$-reinstatement *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E} \in \sigma(\mathcal{F})$, for every $a \in \mathcal{A}$ it holds that if $\mathcal{E}$ defends a and $\mathcal{E} \cup \{a\}$ is conflict-free then $a \in \mathcal{E}$.*

Observation 5. *If a semantics σ satisfies reinstatement then it satisfies $\mathcal{CF}$-reinstatement.*

The results in Table 1 concerning complete, grounded, preferred, stable, semi-stable, ideal, p-complete, p-grounded, p-preferred, p-stable and CF2 semantics were proved by Baroni and Giacomin [2007].

If $\mathcal{E}$ is a naive extension and a an argument such that $\mathcal{E}$ defends a and $\mathcal{E} \cup \{a\}$ is conflict-free, then $a \in \mathcal{E}$ since $\mathcal{E}$ is a maximal conflict-free set. This means that naive semantics satisfies $\mathcal{CF}$-reinstatement.

Observation 5 implies that eager semantics satisfies $\mathcal{CF}$-reinstatement.

	I-max.	Allowing abstention	Crash resistance	Non--interference	Direct.	Weak--direct.	Semi--direct.
complete	×	✓	✓	✓	✓	✓	✓
grounded	✓	✓	✓	✓	✓	✓	✓
preferred	✓	×	✓	✓	✓	✓	✓
stable	✓	×	×	×	×	✓	×
semi-stable	✓	×	✓	✓	×	×	×
ideal	✓	✓	✓	✓	✓	✓	✓
eager	✓	✓	✓	✓	✓	✓	✓
p-complete	×	✓	✓	✓	×	×	✓
p-grounded	✓	✓	✓	✓	✓	✓	✓
p-preferred	✓	×	✓	✓	×	×	✓
p-stable	✓	×	×	×	×	✓	×
naive	✓	×	✓	✓	×	×	✓
CF2	✓	×	✓	✓	✓	✓	✓
stage	✓	×	✓	✓	×	×	×
stage2	✓	×	✓	✓	✓	✓	✓

Table 2: Properties of semantics, part 2

Stage and stage2 semantics satisfy $\mathcal{CF}$-reinstatement, as shown by Dvorák and Gaggl [2016].

The next principle was first considered by Baroni and Giacomin [2007]. It says that an extension cannot contain another extension.

Principle 11 (I-maximality)**.** *A semantics σ satisfies the* I-maximality *principle if and only if for every argumentation framework $\mathcal{F}$, for every $\mathcal{E}_1, \mathcal{E}_2 \in \sigma(\mathcal{F})$, if $\mathcal{E}_1 \subseteq \mathcal{E}_2$ then $\mathcal{E}_1 = \mathcal{E}_2$.*

I-maximality is trivially satisfied by single extension semantics. It is thus satisfied by eager semantics. We see directly from the definitions of naive and stage semantics that they satisfy I-maximality. Dvorák and Gaggl [2016] show that stage2 semantics satisfies I-maximality. Baroni and Giacomin [2007] show that I-maximality is satisfied by all other semantics except complete and p-complete semantics.

Baroni *et al.* [2011a] define a principle called rejection, which says that if an argument a is labelled in and a attacks b, then b should be labelled out. If we use the translation from extension to a labelling we mentioned in Definition 3.21, we see that all the labellings satisfy this property. However, it would be possible to be more general by defining a labelling-based semantics that does not satisfy this property. Let us define a semantics σ that always returns a unique labelling such that an argument is labelled in if it is not attacked, it is labelled undec if it is attacked by exactly one argument and it is labelled out otherwise. Consider the example from

Figure 5: argument a will be labelled in, argument b undec and argument c out, which violates the rejection principle.

We next consider the allowing abstention principle [Baroni *et al.*, 2011a].

Principle 12 (Allowing abstention). *A semantics σ satisfies the* allowing abstention *principle if and only if for every argumentation framework $\mathcal{F}$, for every $a \in \mathcal{A}$, if there exist two extensions $\mathcal{E}_1, \mathcal{E}_2 \in \sigma(\mathcal{F})$ such that $a \in \mathcal{E}_1$ and $a \in \mathcal{E}_2^+$ then there exists an extension $\mathcal{E}_3 \in \sigma(\mathcal{F})$ such that $a \notin (\mathcal{E}_3 \cup \mathcal{E}_3^+)$.*

Baroni *et al.* [2011a] show that complete semantics satisfies the previous principle and that preferred, stable, semi-stable, stage and CF2 semantics falsify it. Observe that unique status semantics trivially satisfy this principle. Allowing abstention is thus satisfied by grounded, ideal, eager and p-grounded semantics.[1]

Let us now consider the remaining semantics, namely: naive, p-stable, p-preferred, p-complete and stage2 semantics.

We first prove that p-complete semantics satisfies allowing abstention. We start with a lemma.

Lemma 4.2. *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an argumentation framework, $\mathtt{GE}_p$ its p-grounded extension and $\mathcal{E} \subseteq \mathcal{A}$ be a set that defends all its arguments. Then, $\mathcal{E}$ does not attack $\mathtt{GE}_p$.*

Proof. Let $\mathcal{CF}^p$ be the p-characteristic function. Denote ${\mathtt{GE}_p}^0 = \emptyset$, ${\mathtt{GE}_p}^1 = \mathcal{CF}^p(\emptyset)$, ${\mathtt{GE}_p}^2 = \mathcal{CF}^p(\mathcal{CF}^p(\emptyset))$, ... and denote by $\mathtt{GE}_p$ the p-grounded extension of $\mathcal{F}$. Let $\mathcal{E}$ be a set that defends all its arguments. By means of contradiction, suppose that there exist $x \in \mathcal{E}$, $y \in \mathtt{GE}_p$ such that $x\mathcal{R}y$. Let $k \in \mathbb{N}$ be the minimal number such that $y \in {\mathtt{GE}_p}^k$. From the definition of function $\mathcal{CF}^p$, there exists $l < k$ such that there exists $y_1 \in {\mathtt{GE}_p}^l$ such that $y_1\mathcal{R}x$. Since $\mathcal{E}$ defends all its arguments, there exists $x_1 \in \mathcal{E}$ such that $x_1\mathcal{R}y_1$. Again, there exists $m < l$ such that there exists $y_2 \in {\mathtt{GE}_p}^m$ such that $y_2\mathcal{R}x_1$. By continuing this process, we conclude that there exists $y_s \in {\mathtt{GE}_p}^1$ such that there exists $x_s \in \mathcal{E}$ such that $x_S\mathcal{R}y_s$. This is impossible, since the arguments of ${\mathtt{GE}_p}^1$ are not attacked. Contradiction. □

Proposition 4.3. *p-complete semantics satisfies allowing abstention.*

Proof. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, let $a, b \in \mathcal{A}$, let $b\mathcal{R}a$ and let $\mathcal{E}_1$ and $\mathcal{E}_2$ be p-complete extensions such that $a \in \mathcal{E}_1$ and $b \in \mathcal{E}_2$. Denote by $\mathtt{GE}_p$ the p-grounded extension of $\mathcal{F}$. Let us prove that $a \notin \mathtt{GE}_p$ and that $\mathtt{GE}_p$ does not attack a. First, since $b\mathcal{R}a$

[1]Note that Table 2 by Baroni *et al.* [2011a] specifies that grounded semantics does not satisfy dilemma abstaining. The reason is that Baroni et al. consider the property as being “non-applicable” to unique status semantics (personal communication, 2016).

and b belongs to a p-complete extension (and every p-complete extension defends all its arguments), Lemma 4.2 implies that $a \notin \mathtt{GE}_p$. Let us now show that $\mathtt{GE}_p$ does not attack a. By means of contradiction, suppose the contrary. Let $b \in \mathtt{GE}_p$ be an argument such that $b\mathcal{R}a$. Since $a \in \mathcal{E}_1$, and $\mathcal{E}_1$ defends all its arguments, then there exists $c \in \mathcal{E}_1$ such that $c\mathcal{R}b$. Contradiction with Lemma 4.2. Thus, it must be that $\mathtt{GE}_p$ does not attack a. It is known that the p-grounded extension is a p-complete extension [Coste-Marquis *et al.*, 2005]. Thus, we showed that there exists a p-complete extension that neither contains nor attacks argument a. □

To see why naive, p-stable, p-preferred and stage2 semantics violate allowing abstention, consider the argumentation framework depicted in Figure 6. The principle is violated since all those semantics return two extensions, $\{a\}$ and $\{b\}$.

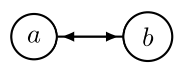

Figure 6: Several semantics violate allowing abstention principle.

To define crash resistance [Caminada *et al.*, 2012], we first need to introduce the following two definitions.

Definition 4.4 (Disjoint argumentation frameworks). *Two argumentation frameworks $\mathcal{F}_1 = (\mathcal{A}_1, \mathcal{R}_1)$ and $\mathcal{F}_2 = (\mathcal{A}_2, \mathcal{R}_2)$ are disjoint if and only if $\mathcal{A}_1 \cap \mathcal{A}_2 = \emptyset$.*

A framework $\mathcal{F}^\star$ is contaminating if joining $\mathcal{F}^\star$ with an arbitrary disjoint framework $\mathcal{F}$ results in a framework $\mathcal{F} \cup \mathcal{F}^\star$ having the same extensions as $\mathcal{F}^\star$. The intuition behind this definition is that $\mathcal{F}^\star$ contaminates every framework.

Definition 4.5 (Contaminating). *An argumentation framework $\mathcal{F}^\star$ is contaminating for a semantics σ if and only if for every argumentation framework $\mathcal{F}$ disjoint from $\mathcal{F}^\star$ it holds that $\sigma(\mathcal{F}^\star \cup \mathcal{F}) = \sigma(\mathcal{F}^\star)$.*

A semantics is crash resistant if and only if there are no contaminating frameworks. The intuition behind this name is that a contaminating framework causes the system to crash.

Principle 13 (Crash resistance). *A semantics σ satisfies the* crash resistance *principle if and only if there are no contaminating argumentation frameworks for σ.*

Crash resistance forbids only the most extreme form of interferences between disjoint subgraphs. A stronger property, non-interference, was defined by Caminada *et al.* [2012]. We first need to define a notion of isolated set, i.e. a set that neither attacks outside arguments nor is attacked by them.

Definition 4.6 (Isolated set of arguments)**.** *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an argumentation framework. A set $S \subseteq \mathcal{A}$ is isolated in $\mathcal{F}$ if and only if*

$$((S \times (\mathcal{A} \setminus S)) \cup ((\mathcal{A} \setminus S) \times S)) \cap \mathcal{R} = \emptyset.$$

A semantics satisfies non-interference principle if for every isolated set S, the intersections of the extensions with set S coincide with the extensions of the restriction of the framework on S.

Principle 14 (Non-interference)**.** *A semantics σ satisfies the* non-interference *principle if and only if for every argumentation framework $\mathcal{F}$, for every set of arguments S isolated in $\mathcal{F}$ it holds that $\sigma(\mathcal{F} \downarrow_S) = \{\mathcal{E} \cap S \mid \mathcal{E} \in \sigma(\mathcal{F})\}$.*

The previous principle can be made even stronger by considering the case when the set S is not attacked by the rest of the framework, but can attack the rest of the framework. Let us formalize the notion of an unattacked set.

Definition 4.7 (Unattacked arguments)**.** *Given an argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, a set U is* unattacked *if and only if there exists no $a \in \mathcal{A} \setminus U$ such that a attacks U. The set of unattacked sets in $\mathcal{F}$ is denoted $\mathcal{US}(\mathcal{F})$.*

We can now define the principle of directionality, introduced by Baroni and Giacomin [2007].

Principle 15 (Directionality)**.** *A semantics σ satisfies the* directionality *principle if and only if for every argumentation framework $\mathcal{F}$, for every $U \in \mathcal{US}(\mathcal{F})$, it holds that $\sigma(\mathcal{F} \downarrow_U) = \{\mathcal{E} \cap U \mid \mathcal{E} \in \sigma(\mathcal{F})\}$.*

Baroni *et al.* [2011a] show the following dependencies between directionality, interference and crash resistance.

Observation 6. *Directionality implies non interference, and non interference implies crash resistance.*

Let us see which semantics satisfy directionality. Baroni and Giacomin [2007] proved that complete, grounded, preferred, ideal, p-grounded and CF2 semantics satisfy directionality. They also showed that stable, semi-stable, p-complete, p-stable and p-preferred semantics violate this principle. Baroni *et al.* [2011a] show that stage semantics does not satisfy directionality; however, Dvorák and Gaggl [2016] show that stage2 semantics does satisfy directionality.

The only remaining semantics are eager and naive. The argumentation framework from Figure 7 shows that naive semantics does not satisfy directionality. The

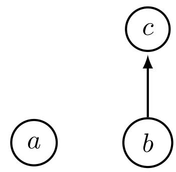

Figure 7: Naive semantics violates directionality and weak directionality. Denote the whole framework by $\mathcal{F} = (\mathcal{A}, \mathcal{R})$. Let $U = \{a, b\}$. Observe that $\{a, c\}$ is a naive extension of $\mathcal{F}$ but that $\{a\}$ is not a naive extension of $\mathcal{F} \downarrow_U$.

argumentation framework from Figure 3 shows that eager semantics does not satisfy directionality.

Let us now consider non-interference. Baroni *et al.* [2011a] showed that non-interference is satisfied by complete, grounded, preferred, semi-stable, ideal, stage and CF2 semantics. Eager semantics satisfies non-interference since it satisfies directionality. From the definition of non-interference we see that this principle is satisfied by naive semantics. Since p-grounded semantics satisfies directionality, it also satisfies non-interference.

Proposition 4.8. *p-complete, p-preferred semantics satisfy non-interference.*

Proof. We present the proof for p-complete semantics, the one for p-preferred semantics is similar. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and $\mathcal{A}' \subseteq \mathcal{A}$ be an isolated set in $\mathcal{F}$. Denote by $\mathcal{F}' = (\mathcal{A}', \mathcal{R}')$ the restriction of $\mathcal{F}$ on $\mathcal{A}'$. Let us first suppose that $\mathcal{E}$ is a complete prudent extension of $\mathcal{F}$. Denote $\mathcal{E}' = \mathcal{E} \cap \mathcal{A}'$. We have $icf(\mathcal{E}')$. It is easy to see that every $\alpha \in \mathcal{E}'$ is defended by $\mathcal{E}'$ from all attacks from $\mathcal{A}'$. Also, for an $\alpha \in \mathcal{A}' \setminus \mathcal{E}'$, we can easily see that either $\mathcal{E}' \cup \{\alpha\}$ is not without indirect conflicts or α is attacked by some argument and not defended by $\mathcal{E}'$. Suppose now that $\mathcal{E}'$ is a complete prudent extension of $\mathcal{F}'$. Then $\mathcal{E}'$ is p-admissible in $\mathcal{F}$, so there must be a complete prudent extension $\mathcal{E}''$ of $\mathcal{F}$ such that $\mathcal{E}' \subseteq \mathcal{E}''$. $\square$

Stage2 semantics satisfies non-interference since it satisfies directionality. Finally, p-stable semantics violates non-interference. Indeed, as we will soon see, p-stable semantics violates crash resistance. Since non-interference implies crash resistance, we conclude that p-stable semantics violates non-interference.

Let us now consider crash resistance. Baroni *et al.* [2011a] showed that non-interference is satisfied by complete, grounded, preferred, semi-stable, ideal, stage and CF2 semantics. Eager, naive, p-grounded, p-complete, p-preferred and stage2 semantics satisfy crash resistance since they satisfy non-interference. To see that stable semantics and p-stable semantics violate crash resistance, consider the framework $\mathcal{F}^* = (\{a\}, \{(a, a)\})$. We see that $\mathcal{F}^*$ is contaminating for stable and p-stable semantics. Thus, they both violate crash resistance.

Let us now consider two variants of directionality, called weak directionality and semi-directionality suggested by M. Giacomin (personal communication, 2016).

Principle 16 (Weak directionality)**.** *A semantics σ satisfies the* weak directionality *principle if and only if for every argumentation framework $\mathcal{F}$, for every $U \in \mathcal{US}(\mathcal{F})$, it holds that $\sigma(\mathcal{F}\downarrow_U) \supseteq \{\mathcal{E} \cap U \mid \mathcal{E} \in \sigma(\mathcal{F})\}$.*

Principle 17 (Semi-directionality)**.** *A semantics σ satisfies the* semi-directionality *principle if and only if for every argumentation framework $\mathcal{F}$, for every $U \in \mathcal{US}(\mathcal{F})$, it holds that $\sigma(\mathcal{F}\downarrow_U) \subseteq \{\mathcal{E} \cap U \mid \mathcal{E} \in \sigma(\mathcal{F})\}$.*

Observation 7. *A semantics σ satisfies directionality if and only if σ satisfies both weak directionality and semi-directionality.*

Thus, grounded, complete, preferred, ideal, eager, p-grounded, stage2 and CF2 semantics satisfy both weak directionality and semi-directionality. It is immediate from the definition that stable semantics satisfies weak directionality. Since stable semantics does not satisfy directionality, it does not satisfy semi-directionality.

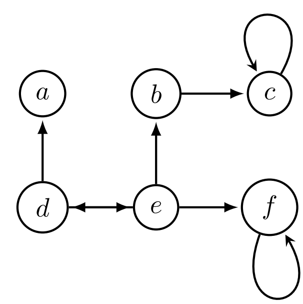

Figure 8: Semi-stable and stage semantics violate weak directionality. Let $U = \{d, e, f\}$. Set $\{b, d\}$ is an extension of this argumentation framework, but $\{b\}$ is not an extension of the restriction of this framework on U.

Example from Figure 8 shows that semi-stable semantics does not satisfy weak directionality. To see that semi-stable semantics does not satisfy semi-directionality, consider the example from Figure 9, suggested by M. Giacomin. Stage semantics violates weak directionality, the same counter-example as for semi-stable semantics

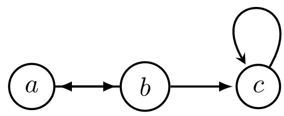

Figure 9: Semi-stable and stage semantics violate semi-directionality. Let $U = \{a, b\}$. Set $\{a\}$ is an extension of the restriction of the framework on U, but there is no extension $\mathcal{E}$ of the whole framework such that $\mathcal{E} \cap U = \{a\}$.

(Figure 8) can be used. Stage semantics also violates semi-directionality, and we can again use the same counter-example as for semi-stable semantics (Figure 9).

Directly from the definition of naive semantics we see that it satisfies semi-directionality. Since it does not satisfy directionality, we conclude from Observation 7 that it does not satisfy weak directionality.

Proposition 4.9. *p-complete and p-preferred semantics satisfy semi-directionality.*

Proof. We present the proof for p-complete semantics, the proof for p-preferred semantics is similar. Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an argumentation framework, $U \subseteq \mathcal{A}$ an unattacked set and $\mathcal{F}' = \mathcal{F} \downarrow_U$ the restriction of $\mathcal{F}$ on U. Let $\mathcal{E}'$ be a p-complete extension of $\mathcal{F}'$. Then $\mathcal{E}'$ is without indirect conflicts and is p-admissible in $\mathcal{F}'$. It is immediate to see that $\mathcal{E}'$ is also p-admissible in $\mathcal{F}$. It is clear that there exists no $x \in U \setminus \mathcal{E}'$ such that x is defended by $\mathcal{E}'$ and $\mathcal{E}' \cup \{x\}$ is without indirect conflicts. Thus, there exists a (possibly empty) set $\mathcal{E} \subset (\mathcal{A} \setminus U)$ such that $\mathcal{E} \cup \mathcal{E}'$ is a p-complete extension. $\square$

Since both p-complete and p-preferred semantics violate directionality, the previous proposition and Observation 7 imply that they both violate weak directionality.

Directly from the definition of p-stable semantics, we see that this semantics satisfies weak directionality. From Observation 7 we conclude that it does not satisfy semi-directionality.

We now consider the six properties related to skepticism and resolution adequacy [Baroni and Giacomin, 2007].

The first definition says that a set of extensions $\texttt{Ext}_1$ is more skeptical than $\texttt{Ext}_2$ if the set of skeptically accepted arguments with respect to $\texttt{Ext}_1$ is a subset of the set of skeptically accepted arguments with respect to $\texttt{Ext}_2$.

Definition 4.10 ($\preceq^E_\cap$)**.** *Let* $\mathtt{Ext}_1$ *and* $\mathtt{Ext}_2$ *be two sets of sets of arguments. We say that* $\mathtt{Ext}_1 \preceq^E_\cap \mathtt{Ext}_2$ *if and only if*

$$\bigcap_{\mathcal{E}_1 \in \mathtt{Ext}_1} \mathcal{E}_1 \subseteq \bigcap_{\mathcal{E}_2 \in \mathtt{Ext}_2} \mathcal{E}_2.$$

The previous definition compares only the intersections of extensions. A finer criterion was introduced by Baroni *et al.* [2004].

Definition 4.11 ($\preceq^E_W$)**.** *Let* $\mathtt{Ext}_1$ *and* $\mathtt{Ext}_2$ *be two sets of sets of arguments. We say that* $\mathtt{Ext}_1 \preceq^E_W \mathtt{Ext}_2$ *if and only if*

$$\textit{for every } \mathcal{E}_2 \in \mathtt{Ext}_2, \textit{ there exists } \mathcal{E}_1 \in \mathtt{Ext}_1 \textit{ such that } \mathcal{E}_1 \subseteq \mathcal{E}_2.$$

Baroni and Giacomin [2007] refine the previous relation by introducing the following definition.

Definition 4.12 ($\preceq^E_S$)**.** *Let* $\mathtt{Ext}_1$ *and* $\mathtt{Ext}_2$ *be two sets of sets of arguments. We say that* $\mathtt{Ext}_1 \preceq^E_S \mathtt{Ext}_2$ *if and only if* $\mathtt{Ext}_1 \preceq^E_W \mathtt{Ext}_2$ *and*

$$\textit{for every } \mathcal{E}_1 \in \mathtt{Ext}_1, \textit{ there exists } \mathcal{E}_2 \in \mathtt{Ext}_2 \textit{ such that } \mathcal{E}_1 \subseteq \mathcal{E}_2.$$

Letters W and S in the previous definitions stand for *weak* and *strong.* Baroni and Giacomin [2007] showed that the three relations are reflexive and transitive and that they are also in strict order of implication. Namely, given two sets of sets of arguments $\mathtt{Ext}_1$ and $\mathtt{Ext}_2$, we have

Observation 8.

$$\mathtt{Ext}_1 \preceq^E_S \mathtt{Ext}_2 \textit{ implies } \mathtt{Ext}_1 \preceq^E_W \mathtt{Ext}_2$$

$$\mathtt{Ext}_1 \preceq^E_W \mathtt{Ext}_2 \textit{ implies } \mathtt{Ext}_1 \preceq^E_\cap \mathtt{Ext}_2$$

We now define a skepticism relation $\preceq^A$ between argumentation frameworks. It says that $\mathcal{F}_1 \preceq^A \mathcal{F}_2$ if $\mathcal{F}_1$ may have some symmetric attacks where $\mathcal{F}_2$ has a directed attack.

Definition 4.13 ($\preceq^A$)**.** *Given an argumentation framework* $\mathcal{F} = (\mathcal{A}, \mathcal{R})$*, the conflict set is defined as* $\mathtt{CONF}(\mathcal{F}) = \{(a,b) \in \mathcal{A} \times \mathcal{A} \mid (a,b) \in \mathcal{R} \textit{ or } (b,a) \in \mathcal{R}\}$*. Given two argumentation frameworks* $\mathcal{F}_1 = (\mathcal{A}_1, \mathcal{R}_1)$ *and* $\mathcal{F}_2 = (\mathcal{A}_2, \mathcal{R}_2)$*, we say that* $\mathcal{F}_1 \preceq^A \mathcal{F}_2$ *if and only if* $\mathtt{CONF}(\mathcal{F}_1) = \mathtt{CONF}(\mathcal{F}_2)$ *and* $\mathcal{R}_2 \subseteq \mathcal{R}_1$*.*

Observe that $\preceq^A$ is a partial order, as it consists of an equality and a set inclusion relation [Baroni and Giacomin, 2007]. Note that within the set of argumentation frameworks comparable with a given argumentation framework $\mathcal{F}$, there might be several maximal elements with respect to $\preceq^A$, since there might be several ways to replace all symmetric attacks by asymmetric ones.

We can now introduce the skepticism adequacy principle. Its idea is that if $\mathcal{F}$ is more skeptical than $\mathcal{F}'$ then the set of extensions of $\mathcal{F}$ is more skeptical than that of $\mathcal{F}'$.

Principle 18 (Skepticism adequacy)**.** *Given a skepticism relation $\prec^E$ between sets of sets of arguments, a semantics σ satisfies the $\preceq^E$-*skepticism adequacy *principle if and only if for every two argumentation frameworks $\mathcal{F}$ and $\mathcal{F}'$ such that $\mathcal{F} \preceq^A \mathcal{F}'$ it holds that $\sigma(\mathcal{F}) \preceq^E \sigma(\mathcal{F}')$.*

For example if $\mathcal{F}$ consists of two arguments a and b attacking each other and $\mathcal{F}'$ has only an attack from a to b, then the intersection of the extensions of $\mathcal{F}$ ($\emptyset$ for all semantics) is a subset of extensions of $\mathcal{F}'$, typically $\{a\}$. Roughly speaking: the more symmetric attacks we replace, the more we know, but we do not loose any accepted arguments.

Observation 9.

- *If σ satisfies $\preceq^E_S$-sk. adequacy then it satisfies $\preceq^E_W$-sk. adequacy*
- *If σ satisfies $\preceq^E_W$-sk. adequacy then it satisfies $\preceq^E_\cap$-sk. adequacy*

Let us see which semantics satisfy skepticism adequacy. Baroni and Giacomin [2007] proved all the results for grounded, complete, stable, preferred, semi-stable, ideal, all four prudent and CF2 semantics.

Eager semantics does not satisfy $\preceq^E_\cap$-skepticism adequacy, as illustrated by the example depicted in Figure 10. From Observation 9, we conclude that eager semantics violates $\preceq^E_W$-skepticism adequacy and $\preceq^E_S$-skepticism adequacy.

Naive semantics satisfies all three variants of skepticism adequacy since $\texttt{CONF}(\mathcal{F}_1) = \texttt{CONF}(\mathcal{F}_2)$ implies $\sigma(\mathcal{F}_1) = \sigma(\mathcal{F}_2)$.

Stage semantics does not satisfy $\preceq^E_\cap$-skepticism adequacy, as illustrated by the example from Figure 11. From Observation 9, we conclude that stage semantics violates $\preceq^E_W$-skepticism adequacy and $\preceq^E_S$-skepticism adequacy.

Finally, stage2 semantics does not satisfy $\preceq^E_\cap$-skepticism adequacy, as illustrated by the example from Figure 12. From Observation 9, we conclude that stage2 semantics violates $\preceq^E_W$-skepticism adequacy and $\preceq^E_S$-skepticism adequacy.

Let us now consider resolution adequacy [Baroni and Giacomin, 2007].

	$\preceq^E_\cap$-sk. ad.	$\preceq^E_W$-sk. ad.	$\preceq^E_S$-sk. ad.	$\preceq^E_\cap$-res. ad.	$\preceq^E_W$-res. ad.	$\preceq^E_S$-res. ad.
complete	✓	✓	×	×	×	×
grounded	✓	✓	✓	×	×	×
preferred	×	×	×	✓	✓	✓
stable	✓	✓	×	✓	✓	✓
semi-stable	×	×	×	✓	✓	×
ideal	×	×	×	×	×	×
eager	×	×	×	×	×	×
p-complete	×	×	×	×	×	×
p-grounded	×	×	×	✓	×	×
p-preferred	×	×	×	×	×	×
p-stable	×	×	×	✓	✓	×
naive	✓	✓	✓	✓	✓	✓
CF2	✓	✓	×	×	×	×
stage	×	×	×	✓	✓	×
stage2	×	×	×	×	×	×

Table 3: Properties of semantics, skepticism and resolution adequacy

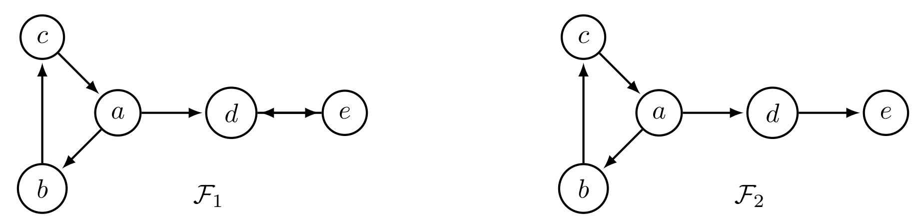

Figure 10: Eager semantics does not satisfy $\preceq^E_\cap$-skepticism adequacy. We have $\mathcal{F}_1 \preceq^A \mathcal{F}_2$. The eager extension of $\mathcal{F}_1$ is $\{e\}$ and the eager extension of $\mathcal{F}_2$ is $\emptyset$. Thus the set of skeptically accepted arguments of $\mathcal{F}_1$ equals $\{e\}$ is not a subset of the set of skeptically accepted arguments of $\mathcal{F}_2$.

Definition 4.14 (RES)**.** *We denote by $RES(\mathcal{F})$ the set of all argumentation frameworks comparable with $\mathcal{F}$ and maximal with respect to $\preceq^A$.*

Definition 4.15 (UR)**.** *Given an argumentation framework $\mathcal{F}$ and a semantics σ, we define $UR(\mathcal{F}, \sigma) = \bigcup_{\mathcal{F}' \in RES(\mathcal{F})} \sigma(\mathcal{F}')$.*

Principle 19 (Resolution adequacy, [Baroni and Giacomin, 2007])**.**
*Given a skepticism relation $\preceq^E$ between sets of sets of arguments, a semantics σ satisfies the $\preceq^E$-*resolution adequacy *principle if and only if for every argumentation framework $\mathcal{F}$ we have $UR(\mathcal{F}, \sigma) \preceq^E \sigma(\mathcal{F})$.*

Figure 11: Stage semantics does not satisfy $\preceq^E_\cap$-skepticism adequacy. We have $\mathcal{F}_1 \preceq^A \mathcal{F}_2$. Framework $\mathcal{F}_1$ has a unique stage extension $\{a\}$ and framework $\mathcal{F}_2$ has two stage extensions $\{a\}$ and $\{b\}$. Thus the set of skeptically accepted arguments of $\mathcal{F}_1$ equals $\{a\}$ is not a subset of the set of skeptically accepted arguments of $\mathcal{F}_2$, which is the empty set.

Figure 12: Stage2 semantics does not satisfy $\preceq^E_\cap$-skepticism adequacy. We have $\mathcal{F}_1 \preceq^A \mathcal{F}_2$. Framework $\mathcal{F}_1$ has a unique stage2 extension $\{a\}$ and framework $\mathcal{F}_2$ has three stage2 extensions $\{a\}$, $\{b\}$ and $\{c\}$. Thus the set of skeptically accepted arguments of $\mathcal{F}_1$ equals $\{a\}$ is not a subset of the set of skeptically accepted arguments of $\mathcal{F}_2$, which is the empty set.

We consider three variants of the resolution adequacy principle: $\preceq^E_\cap$-resolution adequacy, $\preceq^E_W$-resolution adequacy and $\preceq^E_S$-resolution adequacy.

Observation 10.

- *If σ satisfies $\preceq^E_S$-res. adequacy then it satisfies $\preceq^E_W$-res. adequacy*
- *If σ satisfies $\preceq^E_W$-res. adequacy then it satisfies $\preceq^E_\cap$-res. adequacy*

The results regarding grounded, complete, stable, preferred, semi-stable, ideal, all four prudent and CF2 semantics were shown by Baroni and Giacomin [2007].

Eager semantics violates $\preceq^{E}_{\cap}$-resolution adequacy, as illustrated by the example from Figure 13. Consequently, it does not satisfy the other two forms of resolution adequacy. Consider naive semantics; from its definition we see that for every argumentation framework $\mathcal{F}$, for every $\mathcal{F}' \in RES(\mathcal{F})$, we have $\sigma(\mathcal{F}) = \sigma(\mathcal{F}')$. Thus, naive semantics satisfies all three forms of resolution adequacy.

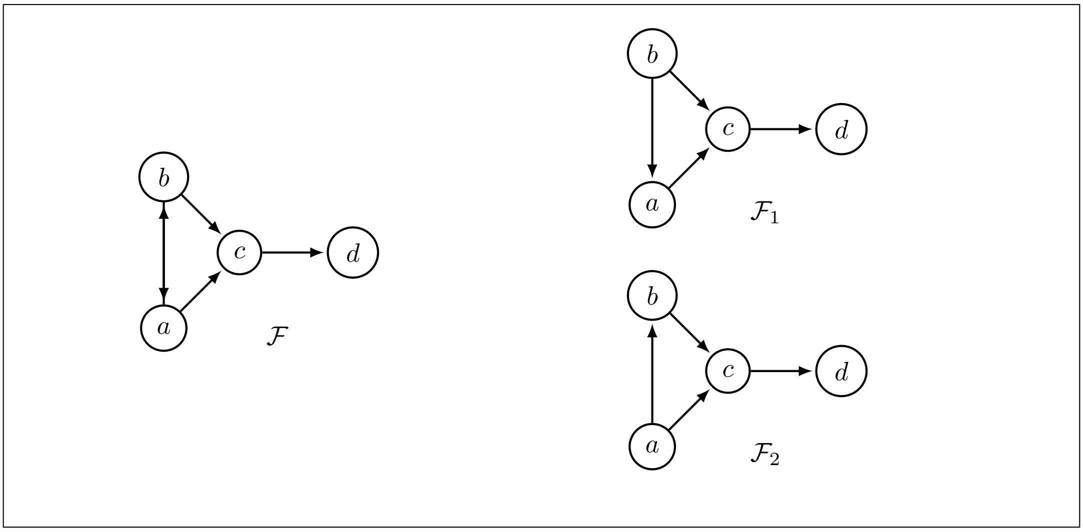

Figure 13: Eager semantics does not satisfy $\preceq^{E}_{\cap}$-resolution adequacy. We have $RES(\mathcal{F}) = \{\mathcal{F}_1, \mathcal{F}_2\}$. Namely, the eager extension of $\mathcal{F}_1$ is $\{b, d\}$ and the eager extension of $\mathcal{F}_2$ is $\{a, d\}$. Since the eager extension of $\mathcal{F}$ is the empty set, and $\{a, d\} \cap \{b, d\} = \{d\} \not\subseteq \emptyset$, the criterion is not satisfied.

Proposition 4.16. *Stage semantics satisfies $\preceq^{E}_{W}$-resolution adequacy.*

Proof. To show this, it is sufficient to show the following claim: for every argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, for every stage extension $\mathcal{E}$ of $\mathcal{F}$, there exists $\mathcal{F}' \in RES(\mathcal{F})$ such that $\mathcal{E}$ is a stage extension of $\mathcal{F}'$. Let $\mathcal{E}$ be a stage extension of $\mathcal{F}$. Let $\mathcal{F}' = (\mathcal{A}, \mathcal{R}') \in RES(\mathcal{F})$ be such that for every $a, b \in \mathcal{A}$ if $a \in \mathcal{E}$ then $(a, b) \in \mathcal{R}'$. (In other words, all attacks from $\mathcal{E}$ are preserved.) $\mathcal{E}$ is conflict-free in $\mathcal{F}'$, and all the attacks from $\mathcal{E}$ are preserved. Observe that the set of conflict-free sets of $\mathcal{F}$ and the set of conflict-free sets of $\mathcal{F}'$ coincide. Also, no conflict-free set attacks more arguments in $\mathcal{F}'$ than it attacks in $\mathcal{F}$. Thus, since $\mathcal{E}$ is a stage extension in $\mathcal{F}$, it is also a stage extension in $\mathcal{F}'$.

From the fact that for every argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, for every stage extension $\mathcal{E}$ of $\mathcal{F}$, there exists $\mathcal{F}' \in RES(\mathcal{F})$ such that $\mathcal{E}$ is a stage extension of $\mathcal{F}'$, we conclude that stage semantics satisfies $\preceq^{E}_{W}$-resolution adequacy. $\square$

Since stage semantics satisfies $\preceq^E_W$-resolution adequacy, then it satisfies $\preceq^E_\cap$-resolution adequacy. The example from Figure 14 shows that stage semantics does not satisfy $\preceq^E_S$-resolution adequacy.

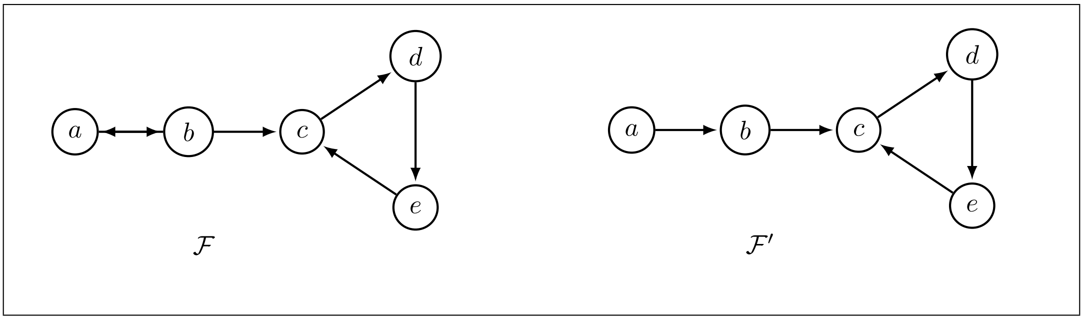

Figure 14: Stage semantics does not satisfy $\preceq^E_S$-resolution adequacy. We have $\mathcal{F}' \in RES(\mathcal{F})$, set $\mathcal{E}' = \{a, c\}$ is a stage extension of $\mathcal{F}'$, but there exists no stage extension $\mathcal{E}$ of $\mathcal{F}$ such that $\mathcal{E}' \subseteq \mathcal{E}$.

Stage2 semantics violates $\preceq^E_\cap$-resolution adequacy, as illustrated by the example from Figure 15. Consequently, it does not satisfy the other two forms of resolution adequacy.

	Succinctness	Tightness	Conflict- -sensitiveness	Com- -closure	SCC- -recursiveness	Cardinality
complete	×	×	×	✓	✓	1+
grounded	×	✓	✓	✓	✓	1
preferred	×	×	✓	✓	✓	1+
stable	×	✓	✓	✓	✓	0+
semi-stable	×	×	✓	✓	×	1+
ideal	×	✓	✓	✓	×	1
eager	×	✓	✓	✓	×	1+
p-complete	×	×	×	×	×	1+
p-grounded	×	✓	✓	✓	×	1
p-preferred	×	✓	✓	✓	×	1+
p-stable	×	✓	✓	✓	×	0+
naive	×	✓	✓	✓	×	1+
CF2	✓	✓	✓	✓	✓	1+
stage	×	✓	✓	✓	×	1+
stage2	✓	✓	✓	✓	✓	1+

Table 4: Properties of semantics, part 4

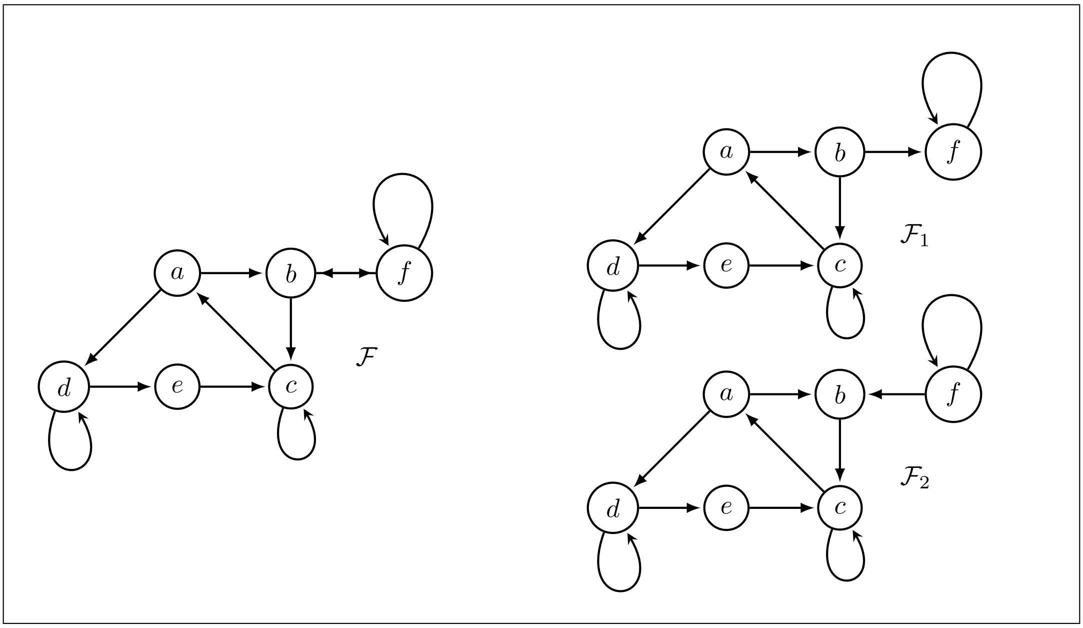

Figure 15: (Example provided by Wolfgang Dvorak, personal communication) Stage2 semantics does not satisfy $\preceq^E_\cap$-resolution adequacy. We have $RES(\mathcal{F}) = \{\mathcal{F}_1, \mathcal{F}_2\}$. Namely, the stage2 extensions of $\mathcal{F}$ are $\{a, e\}$ and $\{b, e\}$, and the stage2 extension of $\mathcal{F}_1$ and $\mathcal{F}_2$ is $\{a, e\}$. Since $\{a, e\} \not\subseteq \{a, e\} \cap \{b, e\} = \{e\}$, the criterion is not satisfied. The intuitive reason for the different behaviour from stage is that resolutions can break up a SCC into several SCCS and arguments that are not in the same SCC are not considered for range maximality.

Baroni *et al.* [2011b] introduce resolution-based family of semantics, which are developed to satisfy the resolution properties.

Let us now consider the last group of properties listed in Table 4. We first need to define the notion of strong equivalence [Oikarinen and Woltran, 2010]. Two frameworks $\mathcal{F}_1$ and $\mathcal{F}_2$ are strongly equivalent if for every argumentation framework $\mathcal{F}_3$, we have that $\mathcal{F}_1 \cup \mathcal{F}_3$ has the same extensions as $\mathcal{F}_2 \cup \mathcal{F}_3$.

Definition 4.17 (Strong equivalence)**.** *Two argumentation frameworks $\mathcal{F}_1$ and $\mathcal{F}_2$ are strongly equivalent with respect to semantics σ, in symbols $\mathcal{F}_1 \equiv^\sigma_s \mathcal{F}_2$ if and only if for each argumentation framework $\mathcal{F}_3$, $\sigma(\mathcal{F}_1 \cup \mathcal{F}_3) = \sigma(\mathcal{F}_2 \cup \mathcal{F}_3)$.*

An attack is redundant in $\mathcal{F}$ if removing it does not change the extensions of any $\mathcal{F}'$ that contains $\mathcal{F}$.

Definition 4.18 (Redundant attack). *Let $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ be an argumentation framework and σ and semantics. Attack $(a, b) \in \mathcal{R}$ is said to be redundant in $\mathcal{F}$ with respect to σ if and only if for all argumentation frameworks $\mathcal{F}'$ such that $\mathcal{F} \subseteq \mathcal{F}'$ we have $\sigma(\mathcal{F}') = \sigma(\mathcal{F}' \setminus (a, b))$.*

We can now define the succinctness principle [Gaggl and Woltran, 2013].

Principle 20 (Succinctness). *A semantics σ satisfies the* succinctness *principle if and only if no argumentation framework contains a redundant attack with respect to σ.*

Gaggl and Woltran [2013] show that a semantics σ satisfies succinctness if and only if for every two argumentation frameworks $\mathcal{F}_1$ and $\mathcal{F}_2$ strong equivalence under σ coincides with $\mathcal{F}_1 = \mathcal{F}_2$.

Only CF2 and stage2 semantics satisfy succinctness. Namely, Oikarinen and Woltran [2010] showed that the notions of strong equivalence and syntactic equivalence do not coincide under complete, grounded, preferred, stable, semi-stable and ideal semantics. Gaggl and Woltran [2013] show that strong equivalence and syntactic equivalence do not coincide under stage and naive semantics. They also show that strong equivalence coincides with syntactic equivalence under CF2 semantics. Dvorák and Gaggl [2016] show that the same is true under stage2 semantics, which means that it also satisfies succinctness.

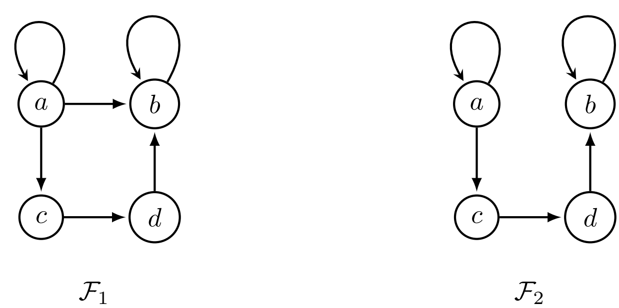

Figure 16: Several semantics violate succinctness

Consider eager semantics. Using Theorem 2 by Oikarinen and Woltran [2010], we can see that $\mathcal{F}_1$ and $\mathcal{F}_2$ from Figure 16 are strongly equivalent under semi-stable semantics. Since the eager semantics is uniquely determined by the set of semi-stable extensions, this means that $\mathcal{F}_1$ and $\mathcal{F}_2$ are strongly equivalent under eager semantics. Hence, eager semantics does not satisfy succinctness. Let us now show that all four prudent semantics violate succinctness.

Let $\mathcal{F}_1 = (\mathcal{A}, \mathcal{R}_1)$ and $\mathcal{F}_2 = (\mathcal{A}, \mathcal{R}_2)$ be the two argumentation frameworks from Figure 16. Let $\mathcal{F} = (\mathcal{A}', \mathcal{R}')$ be an arbitrary argumentation framework. Denote $\mathcal{F}_1' = \mathcal{F}_1 \cup \mathcal{F}$ and $\mathcal{F}_2' = \mathcal{F}_2 \cup \mathcal{F}$. Let us prove that the sets without indirect conflicts of $\mathcal{F}_1'$ and $\mathcal{F}_2'$ coincide. It is immediate that if $\mathcal{E} \subseteq \mathcal{A} \cup \mathcal{A}'$ is not without indirect conflicts in $\mathcal{F}_2'$, it is also not without indirect conflicts in $\mathcal{F}_1'$, since $\mathcal{R}_2 \subseteq \mathcal{R}_1$. Let $\mathcal{E} \subseteq \mathcal{A} \cup \mathcal{A}'$ and let us prove that if $\mathcal{E}$ is not without indirect conflicts in $\mathcal{F}_1'$ then it is not without indirect conflicts in $\mathcal{F}_2'$. Let $\{(x_1, x_2), (x_2, x_3), \ldots, (x_{n-1}, x_n)\} \subseteq \mathcal{R}_1 \cup \mathcal{R}'$ with n being even and $x_1, x_n \in \mathcal{E}$. If $\{(x_1, x_2), (x_2, x_3), \ldots, (x_{n-1}, x_n)\} \subseteq \mathcal{R}_2 \cup \mathcal{R}'$ then $\mathcal{E}$ clearly has an indirect conflict in $\mathcal{F}_2'$. Otherwise, it must be that for some $i \in \{1, \ldots, n-1\}$ we have $x_i = a$ and $x_{i+1} = b$. Then $\{(x_1, x_2), \ldots, (x_i, c), (c, d), (d, x_{i+1}), \ldots, (x_{n-1}, x_n)\} \subseteq \mathcal{R}_2 \cup \mathcal{R}'$, thus $\mathcal{E}$ is not without indirect conflicts in $\mathcal{F}_2'$. Hence, the sets without indirect conflicts of $\mathcal{F}_1'$ and $\mathcal{F}_2'$ coincide. It is immediate to see that $\mathcal{E} \subseteq \mathcal{A} \cup \mathcal{A}'$ defends all it arguments in $\mathcal{F}_1'$ if and only if it defends all its arguments in $\mathcal{F}_2'$. Thus, the sets of p-complete extensions of $\mathcal{F}_1'$ and $\mathcal{F}_2'$ coincide. Also, the p-grounded extension of $\mathcal{F}_1'$ is exactly the p-grounded extension of $\mathcal{F}_2'$. Since every $\mathcal{E}$ without indirect conflicts attacks an argument x in $\mathcal{F}_1'$ if and only if $\mathcal{E}$ attacks x in $\mathcal{F}_2'$, p-stable extensions of $\mathcal{F}_1'$ and $\mathcal{F}_2'$ coincide. Since the sets without indirect conflicts coincide, then maximal sets without indirect conflict coincide. Thus, p-preferred extensions of $\mathcal{F}_1'$ and $\mathcal{F}_2'$ coincide. We conclude that all variants of prudent semantics violate succinctness.

The next principle we consider is tightness. Let us first define the notion of pairs. A couple (a, b) is in $\mathcal{P}airs$ if there is an extension containing both a and b.

Definition 4.19 ($\mathcal{P}airs$)**.** *Given a set of extensions $\mathcal{S} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$, we define*

$$\mathcal{P}airs(\mathcal{S}) = \{(a, b) \mid \text{ there exists } \mathcal{E}_i \in \mathcal{S} \text{ such that } \{a, b\} \subseteq \mathcal{E}_i\}.$$

Tightness was introduced by Dunne *et al.* [2015]. Roughly speaking, it says that if argument a does not belong to extension $\mathcal{E}$, then there must be argument $b \in \mathcal{E}$ which is somehow incompatible with a.

Principle 21 (Tightness)**.** *A set of extensions $\mathcal{S} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$ is tight if and only if for every extension $\mathcal{E}_i$ and for every $a \in \mathcal{A}$ that appears in at least one extension from $\mathcal{S}$ it holds that if $\mathcal{E}_i \cup \{a\} \notin \mathcal{S}$ then there exists $b \in \mathcal{E}_i$ such that $(a, b) \notin \mathcal{P}airs(\mathcal{S})$.*

A semantics σ satisfies the tightness *principle if and only if for every argumentation framework $\mathcal{F}$, $\sigma(\mathcal{F})$ is tight.*

Dunne *et al.* [2015] show that stable, stage and naive semantics satisfy tightness. Example 4 from their paper shows an argumentation framework $\mathcal{F}$ such that

$\sigma(\mathcal{F}) = \{\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3\}$ with $\mathcal{E}_1 = \{a, b\}$, $\mathcal{E}_2 = \{a, d, e\}$, $\mathcal{E}_3 = \{b, c, e\}$, under preferred and semi-stable semantics. This example shows that those two semantics violate tightness since $\{a, b, e\}$ is not an extension.

Directly from the definition of tightness, we conclude that unique status semantics satisfy this principle.

Observation 11. *If σ is a semantics that returns exactly one extension for every argumentation framework then σ satisfies tightness.*

Hence, grounded, p-grounded, ideal and eager semantics satisfy tightness. The example from Figure 17 shows that complete and p-complete semantics violate tightness.

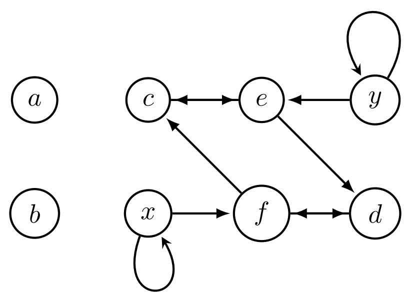

Figure 17: Complete and p-complete semantics violate tightness. There are two extensions $\mathcal{E}_1 = \{a, b\}$, $\mathcal{E}_2 = \{a, b, c, d\}$. Tightness is not satisfied since set $\mathcal{E}_1 \cup \{c\}$ is not an extension.

From Proposition 1 by Dunne *et al.* [2015], we have that the set of naive extensions is tight for every argumentation framework. Note that when σ is naive semantics and $\mathcal{F}$ an argumentation framework, all the elements of $\sigma(\mathcal{F})$ are pairwise incomparable with respect to $\subseteq$ (i.e. for each S, S', $S \subseteq S'$ implies $S = S'$). Hence, we can apply Lemma 2 by Dunne *et al.* [2015] and obtain

Observation 12. *If every extension under σ is a maximal conflict-free set, σ satisfies tightness.*

As an immediate consequence, p-stable, CF2 and stage2 semantics satisfy tightness. We now show that p-preferred semantics also satisfies this principle.

Proposition 4.20. *p-preferred semantics satisfies tightness.*

Proof. We use the proof by reductio ad absurdum. Let $\mathcal{E}$ be a p-preferred extension and let a be a credulously accepted argument such that

$$\text{for every } b \in \mathcal{E} \text{ there is a preferred p-extension } \mathcal{E}'' \text{ s.t. } \{a, b\} \subseteq \mathcal{E}'' \tag{1}$$

By means of contradiction, let us suppose that $\mathcal{E}' = \mathcal{E} \cup \{a\}$ is not a p-preferred extension. From (1), we conclude that $\mathcal{E}'$ is without indirect conflicts. Set $\mathcal{E}'$ is not p-admissible, since that would mean that $\mathcal{E}$ is not a maximal p-admissible set. Since $\mathcal{E}'$ is without indirect conflicts and $\mathcal{E}$ is p-admissible, there exists an argument b_1 such that $b_1 \mathcal{R} a$ and there is no $b' \in \mathcal{E}'$ such that $b' \mathcal{R} b_1$. Denote $B_1 = \{b \mid b \mathcal{R} a\}$.

Note that $\mathcal{E} \neq \emptyset$, since $\mathcal{E} = \emptyset$ would imply that there are no other p-preferred extensions and, consequently, a would not be credulously accepted. Thus, $\mathcal{E} \neq \emptyset$. Let $b \in \mathcal{E}$. From (1), there exists a p-preferred extension $\mathcal{E}_1$ such that $b \in \mathcal{E}_1$ and $a \in \mathcal{E}_1$. Since $a \in \mathcal{E}_1$ then for every $b_1^i \in B_1$ there exists $b_2^i \in \mathcal{E}_1$ such that $b_2^i \mathcal{R}$. Let $B_2 = \{b' \in \mathcal{E}_1 \mid \text{ there exists } b'' \in B_1 \text{ s.t. } b' \mathcal{R} b''\}$. In words, B_2 is the set of arguments from $\mathcal{E}_1$ that attack B_1 (they defend a from B_1).

Let us show that $\mathcal{E} \cup B_2$ is without indirect conflicts. By means of contradiction, suppose $\mathcal{E}$ indirectly attacks B_2. Then $\mathcal{E}$ indirectly attacks a, contradiction. Suppose now that B_2 indirectly attacks $\mathcal{E}$. Since $\mathcal{E}$ is p-admissible, then $\mathcal{E}$ attacks B_2, and thus (like in the previous case) $\mathcal{E}$ indirectly attacks a. Contradiction. So it must be that $\mathcal{E} \cup B_2$ is without indirect conflicts. Note also that since $B_2 \subseteq \mathcal{E}_1$ and $a \in \mathcal{E}_1$, we have that $\mathcal{E}_2 = \mathcal{E} \cup \{a\} \cup B_2$ is without indirect conflicts.

Note that $\mathcal{E}_2$ is not p-admissible, since it is a strict superset of a p-preferred extension. Set $\mathcal{E}$ is p-admissible and B_2 defends a so it must be that some argument(s) of B_2 are not defended by $\mathcal{E}_2$.

Let $B_3 = \{b \mid b \mathcal{R} B_2\}$. It must be that $B_3 \setminus B_2 \neq \emptyset$. Since $B_2 \subseteq \mathcal{E}_1$, and $\mathcal{E}_1$ is p-admissible, there exists $B_4 \subseteq \mathcal{E}_1$ such that B_4 defends B_2. Let $B_4 = \{b' \in \mathcal{E}_1 \mid \text{ there exists } b'' \in B_3 \text{ such that } b' \mathcal{R} b''\}$.

Note that $\mathcal{E}_4 = \mathcal{E} \cup \{a\} \cup B_2 \cup B_4$ is without indirect conflicts. By using the similar reasoning as in the case of $\mathcal{E}_2$, we conclude that $\mathcal{E}_4$ is not p-admissible. Let $B_5 = \{b \mid b \mathcal{R} B_4\}$. We have $B_5 \setminus (B_1 \cup B_3) \neq \emptyset$. By continuing this process, we construct an infinite sequence of different arguments $(b_1, b_3, \ldots, b_{i+1}, \ldots)$ such that $b_1 \in B_1$, $b_3 \in B_3 \setminus B_1$, $\ldots$, $b_{i+1} \in B_{i+1} \setminus (B_1 \cup \ldots \cup B_{i-1})$, $\ldots$, which is impossible, since the set of arguments is finite. $\square$

We now study the notion of conflict-sensitiveness [Dunne *et al.*, 2015]. Note that an equivalent principle was called adm-closure in some papers.

Principle 22 (Conflict-sensitiveness)**.** *A set of extensions $\mathcal{S} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$ is conflict-sensitive if and only if for every two extensions $\mathcal{E}_i$, $\mathcal{E}_j$ such that $\mathcal{E}_i \cup \mathcal{E}_j \notin \mathcal{S}$ it holds that there exist $a, b \in \mathcal{E}_i \cup \mathcal{E}_j$ such that $(a, b) \notin \mathcal{P}airs_{\mathcal{S}}$.*

A semantics σ satisfies the conflict-sensitiveness *principle if and only if for every argumentation framework $\mathcal{F}$, $\sigma(\mathcal{F})$ is conflict-sensitive.*

This principle checks whether the fact that $\mathcal{E}_i \cup \mathcal{E}_j$ is not an extension is justified by existence of $a \in \mathcal{E}_i$ and $b \in \mathcal{E}_j$ that cannot be taken together. Dunne *et al.* [2015] show that every tight set is also conflict-sensitive. Thus, grounded, stable, ideal, stage, eager, naive, p-grounded, p-stable, p-preferred, stage2 and CF2 semantics satisfy conflict-sensitiveness. Proposition 2 by Dunne *et al.* [2015] shows that preferred and semi-stable semantics satisfy conflict-sensitiveness. Our example from Figure 18 shows that complete and p-complete semantics violate this principle. As for tightness, it does not seem that violating this principle is a necessarily a bad thing. It can be rational to ask for both a and b in order to defend e. There is no conflict between a and e, it is just that e needs to be defended from both c and d.

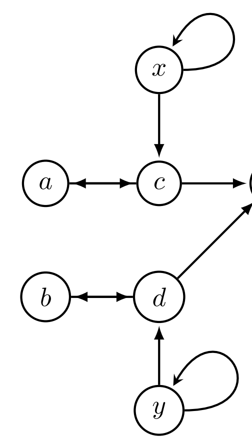

Figure 18: Complete and p-complete semantics violate conflict-sensitiveness. There are four extensions $\mathcal{E}_1 = \emptyset$, $\mathcal{E}_2 = \{a\}$, $\mathcal{E}_3 = \{b\}$, $\mathcal{E}_4 = \{a, b, e\}$. Conflict-sensitiveness is not satisfied since set $\{a, b\}$ is not an extension.

Let us now turn to com-closure [Dunne *et al.*, 2015]. To define this principle, we first need to introduce the notion of completion set. Completion sets are the smallest extensions that contain a given set.

Definition 4.21 (Completion set)**.** *Given a set of extensions $\mathcal{S} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$ and a set of arguments $\mathcal{E}$, set $\mathcal{E}'$ is a completion set of $\mathcal{E}$ in $\mathcal{S}$ if and only if $\mathcal{E}'$ is a minimal for $\subseteq$ set such that $\mathcal{E}' \in \mathcal{S}$ and $\mathcal{E} \subseteq \mathcal{E}'$.*

Roughly speaking, com-closure says that, given a set of extensions $\mathcal{S}$, if for every $\mathcal{T} \subseteq \mathcal{S}$ each two arguments from sets of $\mathcal{T}$ appear in some extension of $\mathcal{S}$, then $\mathcal{T}$ can be extended to an extension in a unique way.

Principle 23 (Com-closure)**.** *A set of extensions $\mathcal{S} = \{\mathcal{E}_1, \ldots, \mathcal{E}_n\}$ is com-closed if and only if for every $\mathcal{T} \subseteq \mathcal{S}$ the following holds: if $(a, b) \in \mathcal{P}airs_{\mathcal{S}}$ for each $a, b \in \cup_{\mathcal{E}_i \in \mathcal{T}} \mathcal{E}_i$, then $\cup_{\mathcal{E}_i \in \mathcal{T}} \mathcal{E}_i$ has a unique completion set in $\mathcal{S}$.*

A semantics σ satisfies the com-closure *principle if and only if for every argumentation framework $\mathcal{F}$, $\sigma(\mathcal{F})$ is com-closed.*

Dunne *et al.* [2015] show that each conflict-sensitive set of extensions is com-closed. Thus, all the semantics that satisfy conflict-sensitiveness also satisfy com-closure. Their Proposition 4 shows that complete semantics is com-closed. To see that p-complete semantics does not satisfy com-closure, consider the graph from Figure 19.

We now study the notion of SCC-recursiveness, which was introduced by Baroni *et al.* [2005].

Principle 24 (SCC-recursiveness)**.** *A semantics σ satisfies the* SCC-recursiveness *principle if and only if for every argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ we have $\sigma(\mathcal{F}) = \mathcal{GF}(\mathcal{F}, \mathcal{A})$, where for every $\mathcal{F} = (\mathcal{A}, \mathcal{R})$ and for every set $C \subseteq \mathcal{A}$, the function $\mathcal{GF}(\mathcal{F}, C) \subseteq 2^{\mathcal{A}}$ is defined as follows: for every $\mathcal{E} \subseteq \mathcal{A}$, $\mathcal{E} \in \mathcal{GF}(\mathcal{F}, C)$ if and only if*

- *in case $|SCCS_{\mathcal{F}}| = 1$, $\mathcal{E} \in \mathcal{BF}_S(\mathcal{F}, C)$,*
- *otherwise, $\forall S \in SCCS_{\mathcal{F}}, (\mathcal{E} \cap S) \in \mathcal{GF}(\mathcal{F} \downarrow_{UP_{\mathcal{F}}(S,\mathcal{E})}, U_{\mathcal{F}}(S, \mathcal{E}) \cap C)$,*

where $\mathcal{BF}_S(\mathcal{F}, C)$ is a function, called base function*, that, given an argumentation framework $\mathcal{F} = (\mathcal{A}, \mathcal{R})$, such that $|SCCS(\mathcal{F})| = 1$ and a set $C \subseteq \mathcal{A}$, gives a subset of $2^{\mathcal{A}}$.*

Baroni *et al.* [2005] proved that grounded, complete, stable and preferred semantics satisfy SCC-recursiveness. CF2 and stage2 semantics also satisfy this principle, since they are defined by using SCC recursive schema. None of the remaining semantics satisfies SCC-recursiveness. To show that ideal, semi-stable, stage and eager semantics does not satisfy SCC-recursiveness, consider the examples from Figures 20 and 21, which are both due to M. Giacomin (personal communication, 2016). Naive semantics does not satisfy SCC-recursiveness since it ignores the direction of attacks. Consider the example from Figure 22. All four prudent semantics violate SCC-recursiveness. Consider the argumentation framework from Figure 4. Let σ be any of the four prudent semantics. In this example, every argument forms an SCC.

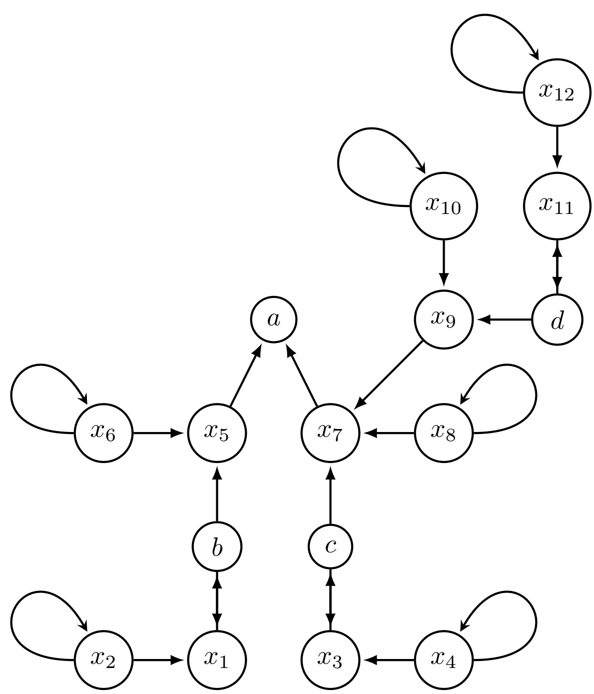

Figure 19: p-complete semantics is not com-closed. There are eight p-complete extensions: $\mathcal{E}_1 = \emptyset$, $\mathcal{E}_2 = \{b\}$, $\mathcal{E}_3 = \{c\}$, $\mathcal{E}_4 = \{d\}$, $\mathcal{E}_5 = \{b, d\}$, $\mathcal{E}_6 = \{c, d\}$, $\mathcal{E}_7 = \{b, c, d\}$, $\mathcal{E}_8 = \{b, c, a\}$. Let $\mathcal{T} = \{\mathcal{E}_2, \mathcal{E}_3\}$. Com-closure is not satisfied since set $\{b, c\}$ has two competition sets, namely $\mathcal{E}_7$ and $\mathcal{E}_8$.

Thus, each extension must contain both e and f. Furthermore, no extension can contain neither of b, c, d, since they are all attacked by e of f. Finally, if σ satisfied SCC-recursiveness, each extension would contain a, which is not the case.

The results considering cardinality are easy to obtain.

We do not include several properties that are not satisfied by any of the studied semantics. Let us mention three such properties. Downward closure [Dunne *et al.*, 2015] basically says that each subset of each extension is an extension. Non-triviality [Dunne *et al.*, 2012] says that it is not the case that $\sigma(\mathcal{F}) = \{\emptyset\}$; in words, the empty set is not the only extension. Decisiveness [Dunne *et al.*, 2012] is a stronger principle that asks that every framework has exactly one extension $\mathcal{E}$ and that $\mathcal{E}$ is not empty.

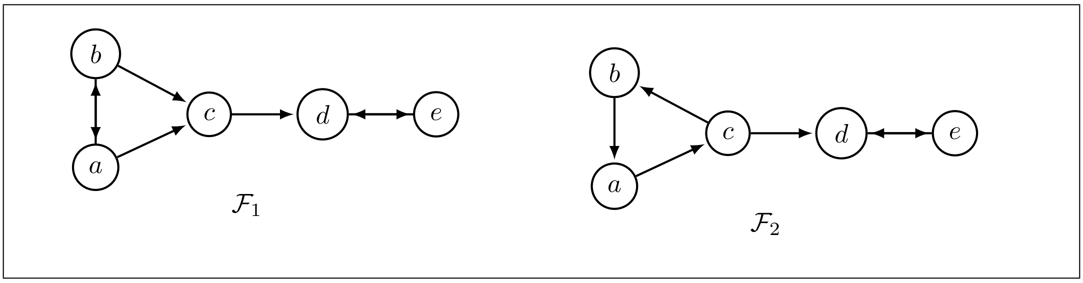

Figure 20: Ideal semantics is not SCC-recursive. Both in $\mathcal{F}_1$ and in $\mathcal{F}_2$, there are two SCCs: $S_1 = \{a, b, c\}$ and $S_2 = \{d, e\}$. Suppose ideal semantics is SCC-recursive. Then, we can calculate the ideal extension of an argumentation framework by starting from S_1 and then continuing to S_2. Denote by $\mathcal{F}_1^1$ the restriction of $\mathcal{F}_1$ on S_1 and by $\mathcal{F}_2^1$ the restriction of $\mathcal{F}_2$ on S_1. The ideal extension of $\mathcal{F}_1^1$ is the empty set. The ideal extension of $\mathcal{F}_2^1$ is also the empty set. So the exact same information is transferred to the next SCC, S_2. The second SCC, S_2 is the same for both frameworks, so given the same information from S_1, both frameworks should have the same ideal extension. However, $\sigma(\mathcal{F}_1) = \emptyset$ whereas $\sigma(\mathcal{F}_2) = \{e\}$. Thus, ideal semantics does not satisfy SCC-recursiveness.

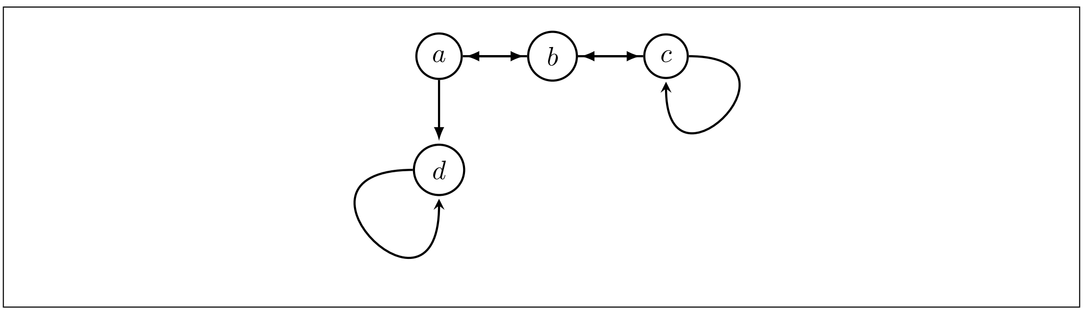

Figure 21: Semi-stable, stage and eager semantics violate SCC-recursiveness. Let σ be stage, semi-stable or eager semantics. Consider the first SCC, $S_1 = \{a, b, c\}$. If we restrict the argumentation framework to S_1, the only extension under σ is $\{b\}$. If σ satisfied SCC-recursiveness, each extension of this framework would contain b, which is not the case, since $\{a\}$ is an extension of this framework under σ.

5 Summary and outlook

The principle-based approach has developed over the past ten years into a cornerstone of formal argumentation theory, because it allows for a more systematic study and comparison of argumentation semantics. In this article we give a com-

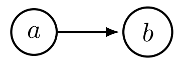

Figure 22: Naive semantics does not satisfy SCC-recursiveness. Note that the first SCC is $S_1 = \{a\}$. If naive semantics satisfied SCC-recursiveness, every naive extension of the whole framework would contain a, which is not the case since $\{b\}$ is a naive extension of this framework too.

plete analysis of the fifteen main alternatives for argumentation semantics using the twenty-seven main principles discussed in the literature on abstract argumentation. Moreover, Caminada [forthcoming] discusses the principles used in structured argumentation, which he calls rationality postulates, and Dung [2016] analyses *prioritised* argumentation using a principle-based or axiomatic approach.

The principle-based approach has also been used to provide a more systematic study and analysis of the semantics of extended argumentation frameworks, of the aggregation of argumentation frameworks, and of the dynamics of argumentation frameworks. For example, principles of ranking-based semantics have been proposed [Amgoud and Ben-Naim, 2016; Amgoud *et al.*, 2017; Bonzon *et al.*, 2016b], where the output is not a set of extensions but a ranking on the set of arguments, and principles have been developed for bipolar argumentation [Cayrol and Lagasquie-Schiex, 2015]. Likewise we expect a further systematic study of weighted argumentation frameworks, preference-based argumentation frameworks, input/output frameworks, abstract dialectical frameworks, and so on.

It may be expected that the principle-based approach will play an even more prominent role in the future of formal argumentation, as the number of alternatives for argumentation semantics increases, new argumentation principles are introduced, and more requirements of actual applications are expressed in terms of such principles. Moreover, in the future applications and principles concerned with infinite frameworks may become more prominent. For example, when the set of arguments becomes infinite, it may be that there are no semi-stable extensions. However, Baumann [forthcoming] illustrates how a meaningful version of eager semantics can be defined, which no longer has the property that it always returns exactly one extension.

Finally, the principle-based approach to formal argumentation may lead to the study of impossibility and possibility results, as well as the development of representation theorems characterising sets of argumentation semantics. The use of the principle-based approach in other areas of reasoning, such as voting theory or AGM theory change, may inspire such further formal investigations.

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References

[Alchourrón *et al.*, 1985] Carlos E. Alchourrón, Peter Gärdenfors, and David Makinson. On the logic of theory change: Partial meet contraction and revision functions. *J. Symb. Log.*, 50(2):510–530, 1985.

[Amgoud and Ben-Naim, 2016] Leila Amgoud and Jonathan Ben-Naim. Evaluation of arguments from support relations: Axioms and semantics. In Subbarao Kambhampati, editor, *Proceedings of the Twenty-Fifth International Joint Conference on Artificial Intelligence, IJCAI 2016, New York, NY, USA, 9-15 July 2016*, pages 900–906. IJCAI/AAAI Press, 2016.

[Amgoud and Besnard, 2013] Leila Amgoud and Philippe Besnard. Logical limits of abstract argumentation frameworks. *Journal of Applied Non-Classical Logics*, 23(3):229–267, 2013.

[Amgoud *et al.*, 2016] Leila Amgoud, Jonathan Ben-Naim, Dragan Doder, and Srdjan Vesic. Ranking arguments with compensation-based semantics. In *Proceedings of the 15th International Conference on Principles of Knowledge Representation and Reasoning, (KR'16)*, pages 12–21, 2016.

[Amgoud *et al.*, 2017] Leila Amgoud, Jonathan Ben-Naim, Dragan Doder, and Srdjan Vesic. Acceptability semantics for weighted argumentation frameworks. In *Proceedings of the Twenty-Sixth International Joint Conference on Artificial Intelligence (IJCAI'2017)*, 2017.

[Arieli, 2015] Ofer Arieli. Conflict-free and conflict-tolerant semantics for constrained argumentation frameworks. *J. Applied Logic*, 13(4):582–604, 2015.

[Baroni and Giacomin, 2007] P. Baroni and M. Giacomin. On principle-based evaluation of extension-based argumentation semantics. *Artificial Intelligence*, 171:675–700, 2007.

[Baroni *et al.*, 2004] Pietro Baroni, Massimiliano Giacomin, and Giovanni Guida. Towards a formalization of skepticism in extension-based argumentation semantics. In *Proceedings of the 4th Workshop on Computational Models of Natural Argument (CMNA'04)*, pages 47–52, 2004.

[Baroni *et al.*, 2005] Pietro Baroni, Massimiliano Giacomin, and Giovanni Guida. SCC-recursiveness: a general schema for argumentation semantics. *Artificial Intelligence*, 168:162–210, 2005.

[Baroni *et al.*, 2011a] Pietro Baroni, Martin Caminada, and Massimiliano Giacomin. An introduction to argumentation semantics. *Knowledge Eng. Review*, 26(4):365–410, 2011.

[Baroni *et al.*, 2011b] Pietro Baroni, Paul E. Dunne, and Massimiliano Giacomin. On the resolution-based family of abstract argumentation semantics and its grounded instance. *Artificial Intelligence*, 175(3-4):791–813, 2011.

[Baroni *et al.*, 2014] Pietro Baroni, Guido Boella, Federico Cerutti, Massimiliano Giacomin, Leendert van der Torre, and Serena Villata. On the input/output behavior of argumentation frameworks. *Artificial Intelligence*, 217:144–197, 2014.

[Baumann, forthcoming] Ringo Baumann. On the nature of argumentation semantics: Existence and uniqueness, expressibility, and replaceability. In Pietro Baroni, Dov Gabbay, Massimilano Giacomin, and Leendert van der Torre, editors, *Handbook of formal argumentation.* College Publications, forthcoming.

[Bonzon *et al.*, 2016a] Elise Bonzon, Jérôme Delobelle, Sébastien Konieczny, and Nicolas Maudet. A comparative study of ranking-based semantics for abstract argumentation. In *AAAI'2016*, 2016.

[Bonzon *et al.*, 2016b] Elise Bonzon, Jérôme Delobelle, Sébastien Konieczny, and Nicolas Maudet. A comparative study of ranking-based semantics for abstract argumentation. In Dale Schuurmans and Michael P. Wellman, editors, *Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence, February 12-17, 2016, Phoenix, Arizona, USA.*, pages 914–920. AAAI Press, 2016.

[Caminada *et al.*, 2012] Martin Caminada, Walter Carnielli, and Paul Dunne. Semi-stable semantics. *Journal of Logic and Computation*, 22(5):1207–1254, 2012.

[Caminada, 2006] Martin Caminada. On the issue of reinstatement in argumentation. In *Proceedings of the 10th European Conference on Logics in Artificial Intelligence (JELIA'06)*, pages 111–123. Springer, 2006.

[Caminada, 2007] Martin Caminada. Comparing two unique extension semantics for formal argumentation: Ideal and eager. In *The 19th Belgian-Dutch Conference on Artificial Intelligence, BNAIC'07*, pages 81–87, 2007.

[Caminada, forthcoming] Martin Caminada. Rationality postulates: applying argumentation theory for non-monotonic reasoning. In Pietro Baroni, Dov Gabbay, Massimilano Giacomin, and Leendert van der Torre, editors, *Handbook of formal argumentation.* College Publications, forthcoming.

[Coste-Marquis *et al.*, 2005] Sylvie Coste-Marquis, Caroline Devred, and Pierre Marquis. Prudent semantics for argumentation frameworks. In *Proceedings of the 17th International Conference on Tools with Artificial Intelligence (ICTAI'05)*, pages 568–572. IEEE, 2005.

[Dung *et al.*, 2007] Phan Minh Dung, Paolo Mancarella, and Francesca Toni. Computing ideal skeptical argumentation. *Artificial Intelligence*, 171:642–674, 2007.

[Dung, 1995] Phan Minh Dung. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence*, 77:321–357, 1995.

[Dung, 2016] Phan Minh Dung. An axiomatic analysis of structured argumentation with priorities. *Artificial Intelligence*, 231:107–150, 2016.

[Dunne *et al.*, 2011] Paul E. Dunne, Anthony Hunter, Peter McBurney, Simon Parsons, and Michael Wooldridge. Weighted argument systems: Basic definitions, algorithms, and complexity results. *Artificial Intelligence*, 175(2):457–486, 2011.

[Dunne *et al.*, 2012] Paul Dunne, Pierre Marquis, and Michael Wooldridge. Argument aggregation: Basic axioms and complexity results. In *4th International Conference on Computational Models of Argument (COMMA'12)*, pages 129–140, 2012.

[Dunne *et al.*, 2015] Paul E. Dunne, Wolfgang Dvorák, Thomas Linsbichler, and Stefan Woltran. Characteristics of multiple viewpoints in abstract argumentation. *Artificial Intelligence*, 228:153–178, 2015.

[Dvorák and Gaggl, 2016] Wolfgang Dvorák and Sarah Alice Gaggl. Stage semantics and the SCC-recursive schema for argumentation semantics. *Journal of Logic and Computation*, 26(4):1149–1202, 2016.

[Gaggl and Woltran, 2013] Sarah Alice Gaggl and Stefan Woltran. The CF2 argumentation semantics revisited. *Journal of Logic and Comutation*, 23(5):925–949, 2013.

[Oikarinen and Woltran, 2010] E. Oikarinen and S. Woltran. Characterizing strong equivalence for argumentation frameworks. In *Proceedings of the 12th International Conference on Principles of Knowledge Representation and Reasoning, (KR'10)*, pages 123–133, 2010.

[Rienstra *et al.*, 2015] Tjitze Rienstra, Chiaki Sakama, and Leendert van der Torre. Persistence and monotony properties of argumentation semantics. In *Theory and Applications of Formal Argumentation - Third International Workshop, TAFA 2015, Buenos Aires, Argentina, July 25-26, 2015, Revised Selected Papers*, pages 211–225, 2015.

[Tennenholtz and Zohar, 2016] Moshe Tennenholtz and Aviv Zohar. The axiomatic approach and the internet. In Felix Brandt, Vincent Conitzer, Ulle Endriss, Jérôme Lang, and Ariel D. Procaccia, editors, *Handbook of Computational Social Choice*, pages 427–452. Cambridge University Press, 2016.

[Verheij, 1996] Bart Verheij. Two approaches to dialectical argumentation: admissible sets and argumentation stages. In *Proceedings of the Eighth Dutch Conference on Artificial Intelligence (NAIC 1996)*, pages 357âĂŞ–368, 1996.

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