MECHANIZING *PRINCIPIA LOGICO-METAPHYSICA* IN FUNCTIONAL TYPE THEORY

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Abstract. *Principia Logico-Metaphysica* proposes a foundational logical theory for metaphysics, mathematics, and the sciences. It contains a canonical development of Abstract Object Theory [AOT], a metaphysical theory (inspired by ideas of Ernst Mally, formalized by Zalta) that differentiates between ordinary and abstract objects.

This article reports on recent work in which AOT has been successfully represented and partly automated in the proof assistant system Isabelle/HOL. Initial experiments within this framework reveal a crucial but overlooked fact: a deeply-rooted and known paradox is reintroduced in AOT when the logic of complex terms is simply adjoined to AOT's specially-formulated comprehension principle for relations. This result constitutes a new and important paradox, given how much expressive and analytic power is contributed by having the two kinds of complex terms in the system. Its discovery is the highlight of our joint project and provides strong evidence for a new kind of scientific practice in philosophy, namely, *computational metaphysics*.

Our results were made technically possible by a suitable adaptation of Benzmüller's metalogical approach to universal reasoning by semantically embedding theories in classical higher-order logic. This approach enables the fruitful reuse of state-of-the-art higher-order proof assistants, such as Isabelle/HOL, for mechanizing and experimentally exploring challenging logics and theories such as AOT. Our results also provide a fresh perspective on the question of whether relational type theory or functional type theory better serves as a foundation for logic and metaphysics.

§1. Abstract Summary. *Principia Logico-Metaphysica* (PLM) [13] aims at a foundational logical theory for metaphysics, mathematics and the sciences. It contains a canonical presentation of Abstract Object Theory (AOT) [14, 15], which distinguishes between abstract and ordinary objects, in the tradition of the work of Mally [6]. The theory, outlined in §2, systematizes two fundamental kinds of predication: classical exemplification for ordinary and abstract objects, and *encoding* for abstract objects. The latter is a new kind of predication that provides AOT with expressive power beyond that of quantified second-order modal logic, and this enables elegant formalizations of various metaphysical objects, including the objects presupposed by mathematics and the sciences. More generally, the system offers a universal logical theory that is capable of accurately representing the contents of human thought.

Independently, the use of *shallow semantical embeddings* (SSEs) of complex logical systems in classical higher-order logic (HOL) has shown great potential as a metalogical approach towards universal logical reasoning [1]. The SSE approach aims to unify logical reasoning by using HOL as a universal metalogic. Only the distinctive primitives of a target logic are represented in the metalogic using their semantical definitions (hence the *shallow* embedding), while the rest of the target system is captured by the existing infrastructure of HOL. For example, quantified modal logic can be encoded by representing propositions as sets of possible worlds and by representing the connectives, quantifiers, and modal operators as operations on those sets. This way the world-dependency of Kripke-style semantics can be elegantly represented in HOL. Utilizing the powerful options to handle and hide such definitions that are offered in modern proof assistants such as Isabelle/HOL [9], a human-friendly mechanization of even most challenging target logics, including the AOT, can thus be obtained.

AOT and the SSE approach are rather orthogonal. They have very different motivations and come with fundamentally different foundational assumptions. AOT uses a *hyperintensional second-order modal logic*, grounded on a *relational type theory*, as its foundation. It is in the tradition of Russell and Whitehead's *Principia Mathematica* [10, 7], which takes the notion of *relation* as primitive and defines the notion of *function* in terms of relations. The metalogic HOL in the SSE approach, by contrast, is fully extensional, and defined on top of a functional type theory in the tradition of the work of Frege [5] and Church [4]. It takes the notion of (fully extensional) *function* as primitive and defines the notion of *relation* in terms of functions. These fundamentally different and, to some extent, antagonistic roots in turn impose different requirements on the corresponding frameworks, in particular, with regard to the comprehension principles that assert the existence of relations and functions. Devising a mapping between the two formalisms has, unsurprisingly, been identified as a non-trivial, practical challenge by Oppenheimer and Zalta [11].

The work reported here tackles this challenge. Further details can be found in Kirchner's thesis [8], where the SSE approach is utilized to mechanize and analyze AOT in HOL. Kirchner constructed a shallow semantical embedding of the second-order modal fragment of AOT in HOL, and this embedding was subsequently represented in the proof assistant system Isabelle/HOL (see §4). The proof assistant system enabled us to conduct experiments in the spirit of a *computational metaphysics*, with fruitful results that have helped to advance the ideas of AOT.

The inspiration for Kirchner's embedding comes from the model for AOT proposed by Peter Aczel[1]. Kirchner also benefited from Benzmüller's initial attempts to embed AOT in Isabelle/HOL. An important goal of the research was to avoid *artifactual theorems*, i.e., theorems that (a) are derivable on the basis of special facts about the Aczel model that was used to embed AOT in Isabelle/HOL, but (b) aren't theorems of AOT. In previous applications of the SSE approach, this issue didn't arise. For example, in the context of the analysis of Gödel's modal ontological argument for the existence of God (cf. [2]), extensive results about the Kripke models were available *a priori*. But AOT is, in part, a body of theorems, and so care has been taken not to derive artifactual theorems about the Aczel model that are not theorems of AOT itself.

[1] An earlier model for the theory was proposed by Dana Scott. His model is equivalent to a special case of an Aczel model with only one *special urelement*.

This explains why the embedding of AOT in Isabelle/HOL involves several layers of abstraction. In the Aczel model of AOT that serves as a starting point, abstract objects are modeled as sets of properties, where properties are themselves modeled as sets of urelements. Once the axioms of AOT are derived from the shallow semantic embedding of AOT in HOL, a controlled and suitably constricted logic layer is defined: by reconstructing the inference principles of AOT in the system that derives the axioms of AOT, only the theorems of AOT become derivable. By utilizing Isabelle/HOL's sophisticated support tools for interactive and automated proof development (see §6) at this highest level of the embedding, it became straightforward to map the pen and paper proofs of PLM into corresponding, intuitive, and user-friendly proofs in Isabelle/HOL. In nearly all cases this mapping is roughly one-to-one, and in several cases the computer proofs are even shorter. In other words, the *de Bruijn factor* [12] of this work is close to 1. In addition, the layered construction of the embedding has enabled a detailed, experimental analysis in Isabelle/HOL of the underlying Aczel model and the semantical properties of AOT.

As an unexpected, but key result of this experimental study, it was discovered that if a classical logic for complex terms such as λ-expressions and definite descriptions is adjoined to AOT's specially-formulated comprehension principle for relations without taking any special precautions, a known paradox that had been successfully put to rest becomes reintroduced (see §5). Since the complex terms add significant expressive and analytic power to AOT, and play a role in many of its more interesting theorems, the re-emergence of the known paradox has become a new paradox that has to be addressed. In the ongoing attempts to find an elegant formulation of AOT that avoids the new paradox, the computational representation in Isabelle/HOL now provides a testing infrastructure and serves as an invaluable aid for analyzing various conjectures and hypothetical solutions to the problem. This illustrates the very idea of *computational metaphysics*: humans and machines team up and split the tedious work in proportion to their cognitive and computational strengths and competencies. And as intended, the results we achieved reconfirm the practical relevance of the SSE approach to universal logical reasoning.

Though the details of the embedding of AOT in Isabelle/HOL are developed in Kirchner [8], we discuss the core aspects of this work in the remainder of this article.

§2. The Theory of Abstract Objects. AOT draws two fundamental distinctions, between *abstract* and *ordinary* objects and between two modes of predication, namely, classical *exemplification* (Fx) and *encoding* (xF). Whereas ordinary objects are characterized only by the properties they exemplify, abstract objects may be characterized by both the properties they exemplify and the properties they encode. But only the latter play a role in their identity conditions: abstract objects x and y are identical if and only if they necessarily encode the same properties (formally, $\Box\forall F(xF \equiv yF) \rightarrow x{=}y$, for abstract objects x and y). The identity for ordinary objects on the other hand is classical: two ordinary objects x and y are identical if they necessarily exemplify the same properties (formally, $\Box\forall F(Fx \equiv Fy) \rightarrow x{=}y$, for ordinary objects x and y). It is axiomatic that ordinary objects don't encode properties, and so only abstract objects can be the subject of true encoding predications.

The key axiom of AOT is the comprehension principle for abstract objects. It asserts, for every expressible condition on properties (intuitively, for every expressible set of properties), that there exists an abstract object that encodes exactly the properties that satisfy the condition; formally, $\exists x(A!x \wedge \forall F(xF \equiv \phi))$, where ϕ is any condition on F in which x doesn't occur free. Therefore, abstract objects can be modeled as elements of the power set of properties: every abstract object uniquely corresponds to a specific set of properties.

Given this basic theory of abstract objects, AOT can elegantly define a wide variety of objects that have been postulated in philosophy or presupposed in the sciences, including Leibnizian concepts, Platonic forms, possible worlds, natural numbers, logically-defined sets, etc.

Another interesting aspect of the theory is its hyperintensionality. Relation identity is defined in terms of encoding rather than in terms of exemplification. Two properties F and G, for instance, are considered to be identical if they are necessarily *encoded* by the same abstract objects (formally, $\Box\forall x(xF \equiv xG) \rightarrow F\!=\!G$). However, the theory does not impose any restrictions on the properties encoded by a particular abstract object. For example, the fact that an abstract object encodes the property $[\lambda x\, Fx \;\&\; Gx]$ does not necessarily imply that it also encodes the property $[\lambda x\, Gx \;\&\; Fx]$ (which, although extensionally equivalent, is a distinct intensional entity).

Therefore, without additional axioms, pairs of materially equivalent properties (in the exemplification sense), and even necessarily equivalent properties, are not forced to be identical. This is a key aspect of the theory that makes it possible to represent the contents of human thought much more accurately than classical logic would allow. For instance, the properties *being a creature with a heart* and *being a creature with a kidney* may be regarded as distinct properties, although they are extensionally equivalent. And *being a barber who shaves all and only those persons who don't shave themselves* and *being a set of all those sets that aren't members of themselves* may be regarded as distinct properties, although they are necessarily equivalent (both necessarily fail to be exemplified).

A full description of the theory goes beyond the scope of this paper, but formal descriptions are available in two books [14, 15] and various papers by Zalta. A regularly updated, online monograph titled *Principia Logico-Metaphysica* ([13]) contains the latest formulation of the theory and serves to compile, in one location, both new theorems and theorems from many of the published books and papers.

The complexity and versatility of AOT, as well as its philosophical ambition, make it an ideal candidate to test the universality of the SSE approach. However, recent work [11] has posed a challenge for any embedding of AOT in functional type theory. In the next section, we briefly discuss this challenge.

§3. AOT in Functional Logic. Russell discovered the well-known paradox in naive set theory by considering the set of all sets that do not contain themselves and by asking whether this set contains itself. A similar construction ('the Clark-Boolos paradox') is possible in a naive AOT (cf. [3] for details about the paradox first described by Clark and reconstructed independently by Boolos): assume that the term $[\lambda x\ \exists F(xF \,\&\, \neg Fx)]$ (i.e., being an x that encodes a property x does not exemplify) denotes a valid property K. The comprehension axiom of abstract objects then ensures that there is an abstract object which encodes K and no other properties. The question, does this abstract object exemplify K, leads to paradox.

AOT undermines the paradox by restricting the matrix of λ-expressions to so-called *propositional formulas*, that is, to formulas without encoding subformulas. This way the term $[\lambda x\ \exists F(xF \,\&\, \neg Fx)]$ is no longer well-formed and the construction of the paradox fails. Thus, AOT contains formulas, e.g., $\exists F(xF \,\&\, \neg Fx)$, that may *not* be converted to terms by placing them within a λ-expression.

However, in functional type theory, it is assumed that every formula can be converted to a term. That is crucial to the analysis of the universal quantifier. The binding operator $\forall x$ in a formula of the form $\forall x\phi$ is represented as a function that maps the *property* $[\lambda x\ \phi]$ to a truth value, namely, the function that maps $[\lambda x\,\phi]$ to The True just in case every object y in the domain is such that $[\lambda x\,\phi](y)$ holds. So in order to represent quantified AOT formulas that contain encoding subformulas, such as $\forall x\exists F(xF \,\&\, \neg Fx)$, their matrices have to be convertible to terms. This would force $[\lambda x\ \exists F(xF \,\&\, \neg Fx)]$ to be well-formed and would lead to paradox.[2]

Thus, it is not trivial to devise a semantical embedding that supports AOT's distinction between formulas and propositional formulas, but at the same time preserves a general theory of quantification. Another challenge has been to accurately encode the hyperintensionality of AOT: while relations in AOT are maximally intensional, functions (and relations) in HOL are fully extensional, and can therefore not be used to represent the relations of AOT directly.

§4. Embedding AOT in Isabelle/HOL. The embedding of AOT in Isabelle/HOL overcomes these issues by constructing a modal, hyperintensional variant of the Aczel-model of the theory. Modality is represented by introducing a dependency on primitive possible worlds in the manner of Kripke semantics of modal logic. Hyperintensionality is achieved by an additional dependency on a separate domain of primitive *states*. Consequently, propositions are represented as Boolean-valued functions acting on possible worlds and states. The model also includes a domain partitioned into ordinary and *special* urelements. Properties are represented as functions mapping urelements to propositions. Whereas the ordinary objects of AOT can be represented by the ordinary urelements, the abstract objects of AOT are represented as sets of properties and these sets are non-injectively assigned a proxy among the *special* urelements. Now if x is an

[2] A reader who is familiar with Isabelle/HOL might find this notation confusing. In AOT, the symbol ϕ is a metavariable that ranges over formulas that may contain free occurrences of x that can be bound by a binding operator. In Isabelle/HOL, however, ϕ would be represented as a function from individuals to truth-values, and the quantified formula would be written as $\forall x.\,\phi\,x$. Such a formula is true, if $\phi\ x$, i.e. the function application of ϕ to x, holds for all x in the domain. In this scenario it holds that $\phi = (\lambda x.\,\phi x)$. Consequently the primitive, functional λ-expressions of Isabelle/HOL cannot be used to represent the λ-expressions of AOT, since the λ-expressions of Isabelle/HOL cannot simultaneously exclude non-propositional formulas and allow quantified formulas with encoding subformulas.

ordinary object, then the truth conditions of an exemplification formula Px are captured by the proposition that is the result of applying the function representing the property P to the ordinary urelement representing x. If x is an abstract object, then the truth conditions of an exemplification formula Px are captured by the proposition that is the result of applying the function representing the property P to the *special* urelement that serves as the proxy of x. An encoding formula xP, by contrast, is true just in case x is an abstract object and the property P is an element of the set of properties representing x. This latter feature of the model validates the comprehension axiom of abstract objects: for every set of properties there exists a unique abstract object, that encodes exactly those properties.

Since well-formed λ-expressions in AOT are required to have a propositional matrix, they correspond to functions acting on urelements. Given that encoding subformulas are excluded from these expressions in AOT, the only formulas that can occur in the matrix of a λ-expression are those built up from exemplification formulas. The truth conditions of these formulas are determined solely by the properties and relations of the urelements in the model.[3]

Consequently, the λ-expressions of AOT are not represented using the unrestricted primitive λ-expressions of HOL, but have a more complex semantic representation. Non-well-formed λ-expressions of AOT, which can't be syntactically excluded from the SSE representation, are given a non-standard semantics and this avoids the paradox that would otherwise occur in a naive representation. As a result, β-conversion only holds in general for terms that are syntactically well-formed in AOT.

The model structure we've just described can represent all the terms of the target logic and retains the desired level of intensionality. Moreover, the axiom system and inference rules of AOT become derivable. Thus, the embedding can abstract away from the model by introducing a novel, layered approach. A representation of the formal semantics of PLM is implemented as a first layer of abstraction and this sits on top of the model structure. On the basis of that representation, the axiom system and the fundamental inference rules of PLM are derived. Using this layered approach it is possible to reason directly in the target logic but independently of the underlying model structure. This avoids the derivation of artifactual theorems, despite the fact that the model structure might validate formulas that aren't theorems of AOT. And, just as importantly, the model guarantees that the system of AOT is sound.

Furthermore, the layered construction of the embedding and the support from the automation infrastructure provided by Isabelle/HOL have other advantages. It is straightforward to convert statements derived within Isabelle/HOL into traditional pen and paper proofs for AOT. Thus, the approach facilitates experimental studies within the computational implementation and informs discussions about them. Moreover, the approach is suitable for conducting a deeper analysis

[3]One exception to this rule will be briefly discussed in the next section, when we describe how the Clark-Boolos paradox can be reintroduced.

of AOT and its model structure. This analysis led to the discovery of how a previously known paradox could easily resurface if care isn't taken in the formulation of PLM. This paradox will be sketched in the next section.

§5. Discovery of a Paradox. The idea behind the representation of AOT's λ-expressions in the embedding is to allow the formation of λ-expressions with any matrix whatsoever, but in such a way that requires only those expressions conforming to PLM's syntactic restrictions to have a standard semantics. In particular, β-conversion does not hold in general, but does hold for λ-expressions with a propositional matrix.

When Kirchner checked the accuracy of this representation, it became apparent that certain λ-expressions involving definite descriptions, which were allowed in the latest formulation of AOT in PLM, did not exhibit the desired behavior in the embedding; in particular, β-conversion could not be derived for those expressions. Using the sophisticated infrastructure provided by Isabelle/HOL, it became possible to show that β-conversion for these terms does not hold in any Aczel-model, and this suggested that there might be some problem with these expressions.

Kirchner then investigated the consequences of assuming β-conversion holds for such terms in Isabelle/HOL. This assumption turned out to be inconsistent; the layered structure of the embedding made it possible to construct a proof of the inconsistency using object level reasoning at the highest level of abstraction. This way, a human-friendly proof of the paradox was reconstructed and quickly confirmed by Zalta. The paradox is based on the fact that the logic of λ-expressions that contain certain definite descriptions would effectively circumvent the restriction that their matrices not have encoding subformulas. Indeed, the paradox turned out to be one that was previously known (the Clark-Boolos paradox mentioned earlier), but which had re-emerged through the *backdoor* (see the discussion below). This new route to a previously known paradox constituted a new paradox.

More specifically, the new paradox is due in part to the precise definition of *subformula*. The matrix of a λ-expression in AOT is allowed to contain encoding formulas as long as they are *nested within a definite description*. Encoding formulas so nested are not considered subformulas of the matrix and so such matrices are still considered propositional formulas. Therefore, the term $[\lambda x\, G\iota z\psi]$ is considered well-formed, even if ψ contains encoding subformulas. Choosing G to be a property that is universally true (e.g. $[\lambda y\, \forall p(p \rightarrow p)]$) and ψ as $z = x \wedge \exists F\, (xF \wedge \neg Fx)$ results in a property that turns out to be (actually) extensionally equivalent to the property K described above in Section 3. This is sufficient to reconstruct the Clark-Boolos paradox.

The confirmation of the paradox initiated fruitful discussions about the best option for closing this backdoor route to paradox. These considerations led Zalta to discover not only a way to avoid the paradox but also a way to simplify the system with a defined notion of *logical existence* for the terms. AOT now uses that notion instead of identity in the formulation of the axioms of free logic.

§6. Automation. The complexity of the target system and the use of multiple abstraction layers presents a challenge for the development and use of automated reasoning tools. One option for automating proofs is to use Isabelle/HOL's inbuilt reasoning tools (e.g. Sledgehammer and Nitpick) to unfold the semantical embedding of AOT in HOL and to reason with the resulting statements about the model. A better option is to directly automate the proof theory of PLM at an abstract layer, i.e., without unfolding the semantical embedding. We adopted this latter option since it allows for the interactive construction of complex, but human-friendly, proofs for PLM. To simplify the implementation of this option, we used the *Eisbach package* of Isabelle to define powerful proof methods for the system PLM, including a resolution prover that can automatically derive the classical propositional tautologies directly in AOT.

§7. Relations vs. Functions. One problem that initially motivated our project was the question of whether relational type theory or functional type theory provides a better framework for developing a fundamental metaphysical system. Oppenheimer and Zalta argue in [11] that it is more difficult to represent AOT's reasoning system in functional type theory than in relational type theory. They conclude from this that relations and relational type theory are more fundamental than functions and functional type theory. But though the SSE embedding of AOT in Isabelle/HOL doesn't challenge this conclusion directly, it does show that the functional setting of HOL can offer a reasonably accurate representation of the reasoning that can be done in AOT. Whether this approach undermines Oppenheimer and Zalta's claim remains to be seen. However, it provides several new insights and points toward important further work. Although the representation of AOT in the functional setting is complex, our works shows that the representation of its second-order fragment is indeed feasible using a complex semantical structure. Furthermore, the key to the development of a sound axiomatization of the complex relation terms of AOT is to be found in the study of, and solution to, both the challenges we encountered in constructing the SSE and the paradox we discovered.

With a paradox-free emendation of PLM, future research should include an extended analysis of the faithfulness of the embedding approach we used; this would shed further light on the debate about relational and functional type theory. This study should be complemented by an analysis of the reverse direction, i.e. an embedding of the fundamental logic of HOL in the (relational) type-theoretic version of AOT. Both studies should then be carefully assessed.

§8. Conclusion. The semantical embedding approach has been fruitfully employed to encode the logic of Zalta's Principia Logico-Metaphysica in Isabelle/HOL. By devising and utilizing a multi-layered approach (which at the most abstract level directly mechanizes the proof-theoretic system of Principia Logico-Metaphysica), the issues arising for an embedding in classical higher-order logic are not too difficult to overcome. A highly complex target system based on a fundamentally different tradition of logical reasoning (relational instead of functional logic) has been represented and analyzed using the approach of shallow semantical embeddings. The power of this approach has been demonstrated by the discovery of a previously unnoticed paradox that was latent in Principia Logico-Metaphysica. The novel ideas of layered abstraction levels and customized proof methods for these levels have shown great potential and may serve as a valuable first step for future research. Furthermore, the work contributes to the philosophical debate about the tension between functional type theory and relational type theory and their inter-representability, and it clearly demonstrates the merits of *shallow semantical embeddings* as a means towards universal logical reasoning.

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