Experiments in Computational Metaphysics: Gödel's Proof of God's Existence

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ABSTRACT

"Computer scientists prove the existence of God" --- variants of this headline appeared in the international press in autumn 2013. Unfortunately, many media reports had only moderate success in communicating to the wider public what had actually been achieved and what not. This article outlines the main findings of the authors' joint work in computational metaphysics. More precisely, the article focuses on their computer-supported analysis of variants and recent emendations of Kurt Gödel's modern ontological argument for the existence of God. In the conducted experiments, automated theorem provers discovered some interesting and relevant facts.

1. Introduction

In autumn 2013, headlines such as "Computer Scientist 'Prove' God Exists", "God's Existence Theorem Is Correct", "God Is Alive", etc. appeared in the media, first in Germany and Austria, and subsequently in the international press. Unfortunately, many of these media reports had only moderate success in communicating to the wider public what had actually been achieved and what not.
This paper provides some more detailed background information (in chronological order) on the factual research contributions that triggered these media reports. The results presented here were achieved in a close collaboration between the two authors. Both were introduced to each other after a presentation Benzmüller delivered in October 2012 to the Kurt Gödel Society in Vienna. In this presentation, he demonstrated how quantified modal logics (QML) [1][2] and other non-classical logics, can be elegantly embedded [3] in classical higher-order logic (HOL, Church's type theory [4][5]). This means that HOL can be used to

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> *Higher-oder logic (HOL) can emulate quantified modal logic (QML) and existing theorem provers for HOL can be employed for reasoning within QML.*

emulate QML, even though QML comes with additional logical connectives which are not (directly) available in HOL.[1] Moreover, by employing the embedding approach, reasoning tools for HOL become readily and effectively applicable for reasoning within QML [6][7]. At the end of his talk, Benzmüller pointed out that the outlined approach should be applicable to formalize and verify Gödel's modern version of the ontological argument [8][9], which is formulated in a higher-order QML (cf. Fig.1), with theorem provers for HOL. Benzmüller's proposal was partly inspired by Fitting's textbook [11], and he had previously attempted some formalization along Fitting's work, but at the time still, without final success. This was due not only to insufficient insistence, but also to the comparably ambitious and demanding logic settings employed by Fitting.

Axiom A1	Either a property or its negation is positive, but not both:	$\forall\phi[P(\neg\phi) \leftrightarrow \neg P(\phi)]$
Axiom A2	A property necessarily implied by a positive property is positive:	$\forall\phi\forall\psi[(P(\phi) \wedge \Box\forall x[\phi(x) \rightarrow \psi(x)]) \rightarrow P(\psi)]$
Thm. T1	Positive properties are possibly exemplified:	$\forall\phi[P(\phi) \rightarrow \Diamond\exists x\phi(x)]$
Def. D1	A *God-like* being possesses all positive properties:	$G(x) \leftrightarrow \forall\phi[P(\phi) \rightarrow \phi(x)]$
Axiom A3	The property of being God-like is positive:	$P(G)$
Cor. C	Possibly, God exists:	$\Diamond\exists xG(x)$
Axiom A4	Positive properties are necessarily positive:	$\forall\phi[P(\phi) \rightarrow \Box P(\phi)]$
Def. D2	An *essence* of an individual is a property possessed by it and necessarily implying any of its properties:	$\phi\ ess.\ x \leftrightarrow \phi(x) \wedge \forall\psi(\psi(x) \rightarrow \Box\forall y(\phi(y) \rightarrow \psi(y)))$
Thm. T2	Being God-like is an essence of any God-like being:	$\forall x[G(x) \rightarrow G\ ess.\ x]$
Def. D3	*Necessary existence* of an individual is the necessary exemplification of all its essences:	$NE(x) \leftrightarrow \forall\phi[\phi\ ess.\ x \rightarrow \Box\exists y\phi(y)]$
Axiom A5	Necessary existence is a positive property:	$P(NE)$
Thm. T3	Necessarily, God exists:	$\Box\exists xG(x)$

Figure. 1. Axioms, definitions and theorems from Gödel's ontological argument for the existence of God [8]; here the variant by Scott [9] is presented.

[1] Modal logic enriches classical propositional logic with the logical connectives ☐ and ◊. Here, ☐P is to be read as "necessarily P holds". ◊P in contrast expresses that "P possibly holds". Quantified modal logics additionally support quantification over individuals, quantification over propositions and even quantification over relations (sets) and functions.

Woltzenlogel Paleo, a Brazilian logician working in Vienna since 2006, travelled on holidays to Brazil in December 2012, and was diagnosed with a life-threatening illness soon after his arrival. He needed a lengthy and debilitating treatment, during which he and his family was assisted not only by an excellent medical team led by Dr. Ana Maria Lobo but also by Priest Edvaldo and his church in Piracicaba. In order to thank him for his support, Woltzenlogel Paleo decided to present him Gödel's proof. As he couldn't find any sufficiently rigorous, complete and convincing formalization of Gödel's proof, he started working on producing one on a natural deduction calculus for higher-order modal logics, which he created for this purpose. He stored his draft work in an open Github repository accessible to anyone interested in contributing and, remembering Benzmüller's talk and interest in this proof, informed him about the repository.

Initially, Benzmüller and Woltzenlogel Paleo worked largely independent, each one following his own approach. However, there was frequent and fruitful exchange of information (mainly by email).

2. Initial Experiments: Scott's Variant of Gödel's proof

> *First successes: the hand-made natural deduction proof was completed and the HOL theorem provers succeeded in proving some lemmas and theorems of Gödel's proof script fully automatically using the embedding approach.*

In late Spring 2013, two events happened independently and almost simultaneously: the hand-made natural deduction proof was completed, after two corrections proposed by Annika Siders[2]; and Benzmüller reported success in some preliminary experiments on proving the lemmas and theorems of the Gödel's proof automatically using the embedding approach. From that moment on, both authors tightly joined forces and continued the studies together. A major motivation for joining forces was the complementarity of both approaches: the hand-made natural deduction proof was human-readable, but tedious to check and hence less reliable; the automatic proofs were machine-generated and more reliable, but not human-readable. The initial focus of the joint efforts was on the embedding approach. Further work on

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[2] *Annika Siders became interested in the ontological argument in 2013, when she was preparing an introductory course on logic for philosophers and needed a logical argument that could show the relevance of formal logic and be of interest to a broad audience. She found the open Github repository and this led to her contributions to the natural deduction proof.*

the hand-made natural deduction proof was delayed, partly because the embedding approach proved very fruitful and partly due to a maternity leave of Siders. Nevertheless, the two quasi-orthogonal approaches were integrated later, when Woltzenlogel Paleo implemented a natural calculus for higher-order QML in Coq on top of the embedding approach. And the work on the hand-made natural deduction proof was resumed, and completed in 2015.

The initial series of experiments [12] aimed at thoroughly checking the correctness of Gödel's proof, and precisely identifying the weakest possible assumptions under which it holds. The HOL automated theorem proves LEO-II [13] and Satallax [14] and the HOL model finder Nitpick [14] were employed. The TPTP THF language [16] served as a concrete syntax format for encoding higher-order QML in HOL. The THF language is supported as common input syntax by the above reasoning tools (and several others). The initial experiments concentrated on Scott's version (cf. Fig.1) of Gödel's ontological argument. An essential difference between Gödel's and Scott's versions is that the latter adds a conjunct in the definition D2 of essential properties (this will be discussed further below). The findings from these experiments on Scott's variant were manifold (they were obtained on a standard MacBook):

- **i.** The axioms (and definitions) are consistent. This was confirmed by Nitpick, which presented a simple model within a few seconds.
- **ii.** Theorem T1 follows from Axioms A1 and A2 in modal logic K (and hence also in stronger modal logics such as KB, S4 and S5). [3] This was proved by LEO-II and Satallax in a few milliseconds. In fact, the left to right direction of the equivalence in A1 is sufficient to prove T1.
- **iii.** Corollary C follows from T1, D1 and A3, again already in modal logic K. This was proved by LEO-II and Satallax in a few milliseconds.
- **iv.** Theorem T2 follows from A1, D1, A4 and D2 in modal logic K. Again, the provers got this result quickly, Satallax within milliseconds and LEO-II within 20s.
- **v.** Theorem T3, necessary existence of a God-like entity, follows from D1, C, T2, D3 and A5. Again, this was proved by LEO-II and Satallax in a few

[3] Modal logic K ist he weakest logic we considered in our experiments. The modal logic KB is obtained from K by adding the axiom scheme B: $P \rightarrow \Box\Diamond P$, in words, if P holds (contingently) then it is necessarily possible that P holds; this axiom scheme corresponds to a symmetric accessibility relation in possible world semantics. Obviously, every theorem of logic K is also a theorem of logic KB. The opposite is not true: there are theorems in logic KB which do not hold in logic K. Logics S4 and S5 add further axiom schemes, that is, they are even stronger than logic KB (and K).

milliseconds. However, this time modal logic KB was required to obtain the result. KB strengthens modal logic K by postulating the B axiom scheme. In modal logic K, theorem T2 does not follow from the axioms and definitions. This was confirmed by Nitpick, which reported a counter model.

These results were announced in a short abstract uploaded to arXiv in August 2013.

The sufficiency of modal logic KB is philosophically profound. Our motivation to investigate the weakest modal logic sufficient for the ontological argument was our own perception that the modal logic S5 (which is usually assumed for ontological proofs) might be too strong, because it entails (perhaps counter-intuitively) that anything that is possibly necessary is necessary, $\Diamond\Box P \rightarrow \Box P$. Later, we found out that the sufficiency of KB had already been conjectured by Anderson [17] and acknowledged by Sobel [18], but no formal proof had ever been presented. Soon after the automatic proof using only KB was found, Annika Siders submitted an improvement of the hand-made natural deduction proof that relied only on KB.

> *Findings: The theorem provers found out that the comparatively weak logic KB is sufficient to prove the final result, that is, necessary existence of God. This contribution is philosophically profound."*

A few weeks later, the following two additional results were obtained and presented on the 1st of November 2013 at Freie Universität Berlin, in the first joint talk of Benzmüller and Woltzenlogel Paleo.

vi. The God-like entity, whose existence was proved in Step (5), is flawless in the sense that it may only exemplify positive properties. The provers got this quickly (in modal logic KB) from A1 and D1; Satallax within milliseconds and LEO-II within 20s.

vii. Moreover, this God-like entity is unique, i.e. monotheism is a consequence of Gödel's theory. Satallax proved this in milliseconds from D1 and flawlessness of God.

In philosophical pen and paper proofs, assumptions about the logical foundations are often not made explicit. This has also been the case for Gödel's proof script. In computer formalizations, however, the detailed settings of the employed logic have to be explicitly provided and concrete choices cannot be avoided. That is, the very logic settings become fully transparent. The results reported above were achieved in a setting with full comprehension (which is inherited in the embedding approach from the HOL meta-logic), rigid terms and 'possibilist' (constant domain) quantification over individuals.

Advantage: In computer formalizations the very logic settings become fully transparent.

In a second series of experiments, the encoding of the quantifiers for individuals has been varied to capture also 'actualist' (varying domain) quantification. All our previous results remained valid with this modification as well. Together, the embedding approach and all experimental results mentioned up to this point (and those from Sections 3 and 4 below) were presented in detail at the European Conference on Artificial Intelligence in 2014 [19].

Interesting alternative proofs were found by the theorem provers.

Automated theorem provers sometimes find interesting alternative proofs. This apparently was also the case in our experiments. By analyzing the proofs, one can see, for example, that the property of being self-identical, which is mentioned in Gödel's manuscript and in Scott's notes, can be avoided. In this particular case, however, the finding was not entirely new. The natural deduction proof of Theorem 1, constructed independently by Woltzenlogel Paleo, also did not use self-identity. Moreover, the possibility of a proof not relying on self-identity had already been pointed out by an anonymous referee to Anderson [17].

3. Possible and Necessary Truths and the Modal Collapse

Finding: Gödel's axioms and definitions are so powerful that they imply what is known as the modal collapse: contingent truth implies necessary truth, or in formal notation: $P \rightarrow \Box P$. The HOL provers were able to confirm this.

Anselm's ontological argument [10] does not properly differentiate between contingent, possible and necessary truths. In contrast, Gödel formalized his proof in a modal (higher-order) logic, which supports such discrimination. For example, Gödel's corollary C (cf. Fig.1) proves from preceding assumptions that it is *possible* for God to exist.

Corollary C is then used further to prove T3, that *necessarily* God exists. This discrimination of possible and necessary truths via modal operators enabled Gödel to address a relevant critique, studied by Leibniz, about St. Anselm's original work on the ontological argument: Anselm's argument assumes that it is possible for a God-like being to exist, and without this assumption the argument fails. At first sight, it thus appears that Gödel's argument very convincingly addresses these (and other) issues. However, as Sobel [18][10] showed, Gödel's axioms and definitions are so powerful that they imply what is known as the *modal collapse*: contingent truth implies necessary truth, or in formal notation: $P \rightarrow \Box P$. From this, we also get that possible truth implies necessary truth and vice versa. In other words, there are no unnecessary contingent truths. One may even see modal collapse as a result against free will.

The theorem provers were in fact able to confirm the modal collapse within a few seconds. Moreover, the provers also showed that the result is independent of using possibilist or actualist quantifiers (for individuals).

What does this mean for Gödel's ontological argument? Is it doomed to fail? Well, not necessarily. On the one hand, the modal collapse, being derivable from the assumptions of the ontological argument, may actually serve those philosophical views well which support forms of determinism. Kovác [20] goes as far to argue that modal collapse may actually conform with Gödel's own philosophical viewpoints. On the other hand, modal collapse has recently incited several philosophers to develop emendations of Gödel's argument in order to remedy the situation.

In collaboration with Leon Weber, we have meanwhile extended our computer-supported analysis to several of these emendations [21]. The findings of these more recent studied will be addressed in Sec. 5 below.

4. A Subtle Difference in the Notion of Essence

The above results apply to Scott's variant [9] of Gödel's proof which slightly differs from the version that was found in Gödel's Nachlass' [8]. One difference is to be found in the notion of essence. In Scott's version, "*an essence of an individual is a property possessed by it and necessarily implying any of its properties*". For Gödel, in contrast, "*an essence of an individual is a property that is necessarily implying any of its properties*". Gödel omits the conjunct "*possessed by it*". So what happens if this conjunct in the definition of D2 (cf. Fig.1) is left out?

Finding: The theorem prover LEO-II showed that the axioms and definitions in Gödel's original proof script are inconsistent. This result was new to us.

To study the consequences, we have replayed the experiments as reported above, but this time for the varied definition D2. Interestingly, the model finder Nitpick failed to report a model. To assess the situation, we subsequently tried to use the HOL theorem provers to prove the inconsistency of the modified set of axioms and definitions. To our surprise, the prover LEO-II indeed succeeded (in about 30 seconds) in doing so. We have both not been aware of this inconsistency. In fact, related comments in philosophy papers often classify Scott's modification only as a 'cosmetic' change to what is often addressed as a minor oversight by Gödel. So what causes this inconsistency and how is the argument working? Unfortunately, the technical, machine-oriented proof object that was returned by LEO-II has been so inaccessible that even Benzmüller, the developer of the tool, failed for a long time to extract a persuasive human-level argument from it.

5. Repeating the Experiments in Isabelle/HOL and Coq

The automated theorem provers LEO-II and Satallax do not offer small and trusted kernels on which the correctness of their proofs can rest assured. Moreover, their low-level, machine-oriented proof calculi are making it particularly hard for humans to follow the very technical chain of proof steps they report. Though their background theory is sound, their actual reasoning could be flawed due to potential, yet undetected bugs in their implementations.

Our experiments were repeated with the highly trustful proof assistants Isabelle/HOL and Coq. This provided additional assurance.

To address this issue and to add another layer of trust to our results, we therefore decided to repeat and verify all previous experiments by using the prominent proof assistants Isabelle/HOL [22] and Coq [23] which do provide a smaller trusted kernel.

Isabelle/HOL, in particular, provides strong internal proof automation facilities, and it integrates (or links to) external automated theorem provers, including LEO-II and Satallax. Moreover, means to reconstruct external proofs within Isabelle/HOL's highly trusted kernel have been developed in recent years [24][25]. By using these facilities, we quickly succeeded in replaying the experiments within Isabelle/HOL, which reassured our previous findings [26]. Unfortunately, however, automatic proof reconstruction in Isabelle/HOL failed for one of the reported results, namely LEO-II's inconsistency result discussed above.

In addition to Isabelle/HOL, we also replayed the experiments in the Coq proof assistant. However, the main motivation now was to demonstrate that the embedding approach not only serves proof automation well but also enables user interaction. Woltzenlogel Paleo quickly succeeded in reconstructing the findings interactively within Coq. As part of this work, we also demonstrated how a direct natural deduction calculus [27] for higher-modal logic can be implemented within the embedding approach as tactics in Coq [28]. This offers interesting perspectives for future work to integrate proof search in the direct and embedding approaches.

6. Further Experiments: Emendations of Gödel's Proof

The success of the experiments, particularly the observed nearly perfect match between the argumentation granularity in Gödel's ontological argument and the proof automation capabilities of the HOL provers in the embedding approach was by no means expected. A very relevant question thus came up, namely whether the approach would scale also for the verification of other research papers in this area. We, therefore, decided to look at more recent emendations on the ontological argument attempting to remedy the modal collapse. The correctness of the emendations and of several meta-remarks about them is much more controversial, and this served as an additional motivation for the use of a computer-assisted approach.

Finding: We observed a nearly perfect match between the argumentation granularity in the papers on Gödel's ontological argument and the proof automation capabilities of the HOL provers.

Hájek [29][30] proposed in his emendation the use of cautious instead of full comprehension principles, and Fitting [11] took greater care of the semantics of higher-order quantifiers in the presence of modalities. That is, both authors suggested to change the specific logic settings. Others, such as Anderson [17], Hájek [31] and Bjørdal [32], proposed emendations of Gödel's axioms and definitions. They require neither comprehension restrictions nor more complex semantics. Therefore, they are technically simpler to analyze within the embedding approach.

We have formalized and studied those using Isabelle/HOL in combination with the automated HOL reasoners. The approach again performed very well. Like in the previous experiments, the HOL reasoners quickly responded to the formalized argumentation steps, either by automatically confirming them as valid, or by refuting them, in which case counter models were presented.

Interestingly, the HOL provers not only confirmed many claimed results, but also exposed a few mistakes and produced novel insights. In particular, the provers were able to settle a long standing debate on the redundancy of axioms A4 and A5 in different settings that was going on among between Magari, Anderson and Hájek; we presented these results at the First World Congress on Logic and Religion [21].

> *Finding: The HOL theorem provers were even able to settle a long standing debate between two philosophers.*
> *This pretty much matches Leibniz vision known as "Calculemus!"*

These additional experiments strikingly demonstrate the potential benefits of a computational metaphysics as exemplified here in which humans and computer programs join forces in order to settle philosophical disputes. This pretty much matches Leibniz vision known as "*Calculemus!*".

7. Leo-II's Inconsistency Proof

Inspired by a discussion of the inconsistency issue with Chad Brown, Benzmüller recently succeeded in reconstructing and verifying LEO-II's argument by hand within Isabelle/HOL. Once revealed and understood, the argument is in fact surprisingly comprehensible:

a. The argument starts from axiom A1 ("*Either a property or its negation is positive, but not both*") and axiom A2 ("*A property necessarily implied by a positive property is positive*"). These two assumptions imply theorem T1 ("*Positive properties are possibly exemplified*"), as we already know.
b. Now take the modified definition of D2 ("*An essence of an individual is a property that is necessarily implying any of its properties*") and consider the empty property, or alternatively the property of being self-different, as a candidate. Then Lemma 1 is derivable: "*The empty property (or self-difference) is an essence of every entity*".
c. From the definition of necessary existence ("*Necessary existence of an individual is the necessary exemplification of all its essences*"), modal

axiom B and Lemma 1 follows Lemma 2: "*Exemplification of necessary existence is not possible*".

d. Axiom A5 "*Necessary existence is a positive property*", theorem T1 and Lemma 2 now imply falsehood.

LEO-II's inconsistency result on Gödel's original proof script has meanwhile been reconstructed and verified in Isabelle/HOL.

LEO-II's proof object actually contains the crucial property instantiation performed in Step b. However, this key step gets lost in the technical noise of the proof. It is important to remark that this instantiation has not been synthesized by LEO-II, for example, by employing higher-order (pre-)unification. Instead, it has been guessed during proof search using the blindly guessing primitive substitution rule. As experts know, this rule unfortunately cannot be fully abandoned in HOL automated theorem proving (without loosing Henkin-completeness) [5]. This point to an interesting aspect: attempts to repeat this successful inconsistency proof of LEO-II with first-order theorem provers will likely be doomed to fail, since the only source to come up with this crucial instantiation of the empty property (or self-difference) appears to be the comprehension schemes of set theory.

8. Discussion

Readers of the public media reports that were triggered by our work overwhelmingly seem to reject the ontological argument.

Ontological arguments in the tradition of Anselm's, since their first revelation, have fascinated generations of philosophers. In fact, they usually trigger strong reactions, against or in favor of them. In philosophical circles, the debate is not yet settled and the allurement of ontological arguments seems far from fading. Readers of the public media reports that were triggered by our work, however, overwhelmingly seem to reject them. This becomes apparent from hundreds of blog entries and comments linked at the respective media websites. Such blog entries and comments have recently been statistically analysed by Fuhrmann [33]. Clearly, without a certain level of education in metaphysics, modern logic and the axiomatic method, Gödel's modal ontological argument, appears largely inaccessible. This may provide some level of explanation. Generally, a certain amount of philosophical education seems required for not being irritated by the ontological argument in the first place.

However, the media writers are also to be blamed, because of their apparent interest in creating 'headline stories', and in copying, nitpicking and obfuscating text passages from each other instead of presenting unbiased, properly investigated and individually prepared information. For example, in an early interview on the topic to the German edition of Spiegel Online, Benzmüller mentioned that 'the initial experiments were conducted on his MacBook'. While this has been true, he should have better used the neutral term 'notebook', since the experiments can (and have been) repeated also with other computer technology. The original Spiegel Online article mentioned 'MacBook', however, it did not prominently overemphasize this point (and a good technology writer could have clarified the independence of the experimental results from the hardware anyway). However, when the news subsequently made its way to the US, some intentionally (and very naively) obfuscated headlines appeared such as "Researchers say they used MacBook to prove Gödel's God theorem" or "God exists, say Apple fanboy scientists". Such headline stories were written even by 'award winning creative directors' such as Chris Matyszczyk, who in fact never talked to us directly. One may argue, therefore, that these articles say a lot more about media quality and media standards in the US than about the quality of the actual research content they address. And it is little surprise that such intentionally obfuscated and jaundiced media reports trigger negative attitudes of bloggers towards the ontological argument.

Finally, we think that philosophical tradition has to be blamed to a certain extent, due to the low availability of authoritative texts trying to appropriately communicate the ontological argument to a wider audience. Both of us (we have begun our studies on the topic only recently) have experienced that a large proportion of the existing texts are targeting a rather exclusive circle of readers sharing a significant background expertise, in particular, on the detailed historical development in the area. This in turn excludes, and presumably negatively affects, those readers not willing to clear their backlog before commenting on the topic. It seems that other academic disciplines have in fact achieved a higher level of historical and contextual independence in their modern literature. (But maybe this is not really a worthwhile option for philosophy?)

Belief in a (God-like) supreme being is not trivially irrational.

But what is now our position on the ontological argument? Well, we both share the opinion with proponents of Gödel's work that prominent objections to his proof, including Gaunilo-like repectively Oppy-like parodies, are currently not on a par with Gödel's work, and its recent emendations with respect to technical precision and persuasive power. Investigating and eventually either confirming their correctness or unveiling flaws in them, with the assistance of our technology, remains an exciting direction for future work. Moreover, there clearly are theologically and metaphysically relevant objections, including the modal collapse, which are not yet fully settled. However, as a conclusion one may say that the ontological argument succeeds at least in the following sense: it shows that *belief in a (God-like) supreme being is not trivially irrational*. There are consistent axiomatizations that non-trivially entail the necessary existence of a God-like being. As for any axiomatization, and not only those with a religious theme, it often remains a 'matter of faith' to believe in the truth of the proposed axioms in the actual universe.

Our core contribution is a technological approach and machinery that, as has been well demonstrated here, can fruitfully support further logical investigations in this area. This machinery may eventually even be helpful for settling some of the open questions. In particular, our technology seems ready to be used with the aim of minimizing logic related causes of defect in this area. As an expedient, this machinery should (at least in the long run) be able to significantly ease the technically involved practical work in metaphysics.

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