

Implementation of a Modern Dyadic Deontic Logic in Isabelle/HOL

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Abstract

A shallow semantical embedding of a dyadic deontic logic (by Carmo and Jones) in Isabelle/HOL is presented. First experiments provide evidence that this logic *implementation* fruitfully enables interactive and automated reasoning at the meta-level the object-level.

1 Introduction

Normative notions such as obligation and permission are the subject of deontic logics [5, 7], and conditional obligations are addressed in so called dyadic deontic logic.

A particular dyadic deontic logic has recently been proposed by Carmo and Jones [4]. This dyadic deontic logic comes with a neighborhood semantics and a weakly complete axiomatization over the class of finite models. Their framework is immune to some well known contrary-to-duty issues which can still be found in many other, related approaches.

As a first contribution we present an “*implementation*” of the Carmo-Jones-Logic (CJL) in Isabelle/HOL. This implementation utilizes the shallow semantical embedding approach that has been put forward by Benzmüller as a pragmatical solution towards universal logic reasoning (see [3, 1]). This approach uses classical higher-order logic as (universal) meta-logic to specify, in a shallow way, the syntax and semantics of various object logics, in our case CJL. Because of its neighborhood semantics and since it provides and combines modal and conditional operators, CJL constitutes a non-trivial object logic to implement in the shallow semantical embedding approach.

As a second contribution we employ our implementation to study some meta-logical properties of CJL in Isabelle/HOL. This also includes questions about the relationship of dyadic deontic fragment of CJL to other deontic logics, for example, Input/Output logic [6]. For this, we analysis a list of normative inference patterns that have been suggested by Parent and van der Torre [8]. A particular focus of our experiments is on nested dyadic obligations and we show

that nested dyadic obligations in CJL can be eliminated. Moreover, we investigate the relationship of this dyadic obligation operator to standard conditional operators.

As third contribution we illustrate how our implementation supports the reasoning at object-level. More precisely, we show how classical deontic reasoning examples from the literature can now be represented in Isabelle/HOL and we examine how our implementation performs when being applied to these examples.

Future work includes a formal proof of the faithfulness of the shallow semantical embedding along the lines of related proofs for standard conditional logics [2] and quantified modal logics [1]. Moreover, the proposed embedding may provide a clue to turn CJL weak completeness theorem into a strong one.

References

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