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A DISCRETE APPROACH TO DESCRIBE THE ELASTIC-PLASTIC BEHAVIOUR OF SNOW

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Abstract. Snow is composed of small ice particles and, therefore, behaves as a granular material with a variety of sizes and shapes. Understanding the mechanical behaviour of snow is important in areas such as natural hazards e.g. snow avalanches, or traction characteristics of tires with a snow covered road. Therefore, the objective of the current contribution is to present an advanced discrete approach to predict the elastic-plastic behaviour of snow. For this purpose, snow is described by a finite number of discrete ice grains similar to the Discrete-Element Method (DEM) with ice bonds as a link between individual grains and including a creep law for ice. Thus, the mechanical behaviour of an ensemble of ice grains is determined by the impact between ice grains and the mechanical load of the bonds. The latter may rupture under excessive load or may be formed during a contact between ice grains. Hence, the integral behaviour of snow is represented by the combined properties of impact and bonds depending on strain rate, density and temperature.

The collision model is based on the linear hysteretic model developed by Walton and Braun [], which accounts for the effect of plasticity. For this behaviour the impact between grains is distinguished into a loading and detaching phase represented by a loading and unloading stiffness constant. Additionally, friction behaviour into the tangential direction at the point of impact and dissipation i.e. coefficient of restitution is taken into account.

A bond between two ice grains is represented by a cylinder that is allowed to undergo tension, shear, torsion about its axis and bending. These translational and rotational displacements of a bonding cylinder lead to appropriate strains that yield corresponding stresses with a constitutive model including Youngs modulus E and the Poisson ratio ν. The stresses acting on a bond are converted to forces and moments that determine

in conjunction with Newtons 2nd law the kinematics of the ice grains and the overall mechanical behaviour of the ensemble.

This approach was employed to predict the mechanical behaviour of snow under compression for different strain rates, and thus, covering the length scales of individual ice grains to the global dimensions of the specimen. Thus, both brittle and ductile behaviour of snow were represented and good agreement between experimental data and predictions was obtained.

1 INTRODUCTION

Smow regarded as a material exhibits a complex mechanical i.e. static and dynamic behaviour that has been subject to intensive research both experimentally and numerically. According to Bartelt and Lening [1], Brun et al. [2] and Jordan [3] modelling as a complementary effort to experiments is required to gain a deeper insight into the physical behaviour of snow. Therefore, models for snow have to address the following important characteristics of snow:

- micro-structure
- multi-component porous media
- strength, large deformations and fracture
- temperature

The above-mentioned aspects for snow modelling were largely covered by a continuous approach e.g. finite element method, whereas a discrete method has rarely been applied. First efforts to describe snow by FEM go back to Smith [4] and Lang and Sommerfeld [5].

Meanwhile, modelling of snow has progressed significantly to more sophisticated approaches as described by Creseri et al. [6]. They included an elastic-viscous-plastic law and a constitutive model that accounts for the sintering of snow and were able predict the ductile regime of snow accurately. Constitutive modelling was also employed by von Moos [7] to describe the ductile behaviour of snow for which the elastic and viscous material laws cummulated in a complex Burgers law. The latter was already applied by Shapiro et al. [8] who supported the Burgers model as a very versatile model description, however at the expense of a large number of parameters, that have to be determined mainly through validation by experiments. Similarly, Schweizer [9] described snow as a material with linear viscous property and investigated into skier induced load on layers of snow. Stoffel and Bartelt [10] and Stoffel [11] and Bartelt et al. [12] applied an elastic-viscous law within a 2-dimensional finite element code and predicted creep of snow and its mechanical load on snow defence structures. The interacion between snow and a plate aiming at a wheel-snow contact were predicted by Haehnel and Shoop [13] by a capped

Drucker-Prager model. In addition to an elastic-plastic material law Gaume et al. [14] included a strain-softening correlation due to shear and thus, studied the impact of weak layers and snow heterogeneity on the release angle for avalanches. The capabilities of elastic-viscous approaches were further developed by Nicot [15, 16] who introduced statistical micro-scale characteristics for snow by directional bonds that were allowed breaking and thus, accounting for fracture.

While the above-mentioned investigations relied on a macroscopic description of snow, the microscopic properties of snow moved into the focus of researchers. The micro-structure of snow was first considered by Schneebeli [17] who applied X-ray microtomography to re-construct the 3-dimensional micro-structure of a snow sample. X-ray microtomography was also used by Hagenmuller [18] to build the micro-structure and a linear elastic material law in conjunction with a fracture criterion represented the snow behaviour. A similar approach was employed by Theile et al. [19] in his finite element approach, however, extended by Glens law for secondary creep to describe the anisotropy of snow.

These modelling efforts based largely on an elastic-viscous description of snow in conjunction with finite elements contributed to a more comprehensive understanding of snow, however, large deformations as frequently encountered with snow are still difficult to address due to the continuous formulation of the finite element method. In addition, snow considered as ice grains suggests a discrete numerical technique to describe the behaviour of snow. Thus, Johnson and Hopkins [20] developed the so-called μ-model with the discrete element method as a first discrete approach. Bonds acted as joints between discrete elements that were allowed breaking and re-building i.e. refreezing. Michael [21] and Michael et al. [22, 23] applied also the discrete element method for snow modelling and was able to describe both the brittle and ductile behaviour of snow including the transition between the two regimes. Hagenmuller [24] developed a rather simple linear elastic model including a fracture criteria to describe a snow pack under deformation. However, no validation studies were carried out that could prove the quality of their approach. The material point method (MPM) was applied by Stomakhin et al. [25] as a representative of a hybrid approach to describe the behaviour of snow which was represented by an empirical correlation. A background grid is employed to account for changes in the topology, while snow particles were tracked by the Lagrangian method. However, their modeling efforts were motivated by nice appearances for movies rather than scientific investigations.

2 NUMERICAL APPROACH

Due to the discrete nature of ice grains, the Extended Discrete Element Method (XDEM), derived from the classical Discrete Element Method (DEM), was employed. It is an advanced approach on a micro-scale to describe the mechanical behaviour of snow. Hence, snow is build from individual ice grains that interact via forces and moments during a contact between two or several grains. The afore-mentioned forces and moments result from various interactions between grains:

- impact between grains
- bond between grains
- deformation of a bond
- fracture/generation of a bond

Besides grain collision, bonds between grains account for the the complex mechanical behaviour of snow. This behaviour is described by advanced physical models that represent the following inter-granular properties:

- elastic-plastic grain collision
- inter-granular friction
- bond growth due to creep of ice
- elastic viscous-plastic deformation of bonds
- fracture of bonds

The above-mentioned properties are formulated as to be dependent on temperature, pressure and loading rates. A validation of these advanced predictive capabilities of snow behaviour was carried out by a comparison between predicted results and sintering measurements of [26] and on creep models of ice developed by [27]. For a more detailed description, the reader is referred to Michael [22].

3 RESULTS

According to Kinisota [28], Narita [29] and Fukue [30], the mechanical behaviour of snow may be separated into two distinct regimes referred to as brittle and ductile. Between the brittle and ductile regimes exists a third regime that is characterised by a transitional behaviour and therefore, is called transitional regime. The most influential variable identified to distinguish into the different regimes is the strain rate $\dot{\varepsilon}$. Based on the experimental data of Kinisota [28] and Fukue [30] the transitional regime occurs at a strain rate of $\sim\ 5 \cdot 10^{-4}\ 1/s$ and thus, acts as a separation parameter between the brittle and ductile regimes. Hence, snow deforms as a ductile material at strain rates below the critical strain rate of $\sim\ 5 \cdot 10^{-4}\ 1/s$ and behaves as a brittle material above the critical strain rate.

In order to predict the above-mentioned behaviour, cylindrical snow samples were generated and compressed under unconfined conditions. The strain rates chosen were according to the two regimes namely ductile and brittle presented in the following sections. The predicted stress-strain relationship was compared to measurements and and good agreement was achieved.

3.1 Brittle Deformation

As above-mentioned, the brittle deformation behaviour was predicted for cylindrical snow samples with a density $\rho = 408\ kg/m^3$ and strain rates of $4 \cdot \dot{\varepsilon} = 10^{-3}\ 1/s$ and $4 \cdot \dot{\varepsilon} = 10^{-2}\ 1/s$ under unconfined conditions which refers unambiguous to the brittle regime. The integral axial stress dependent on the strain was obtained as a result and is depicted in fig. 1.

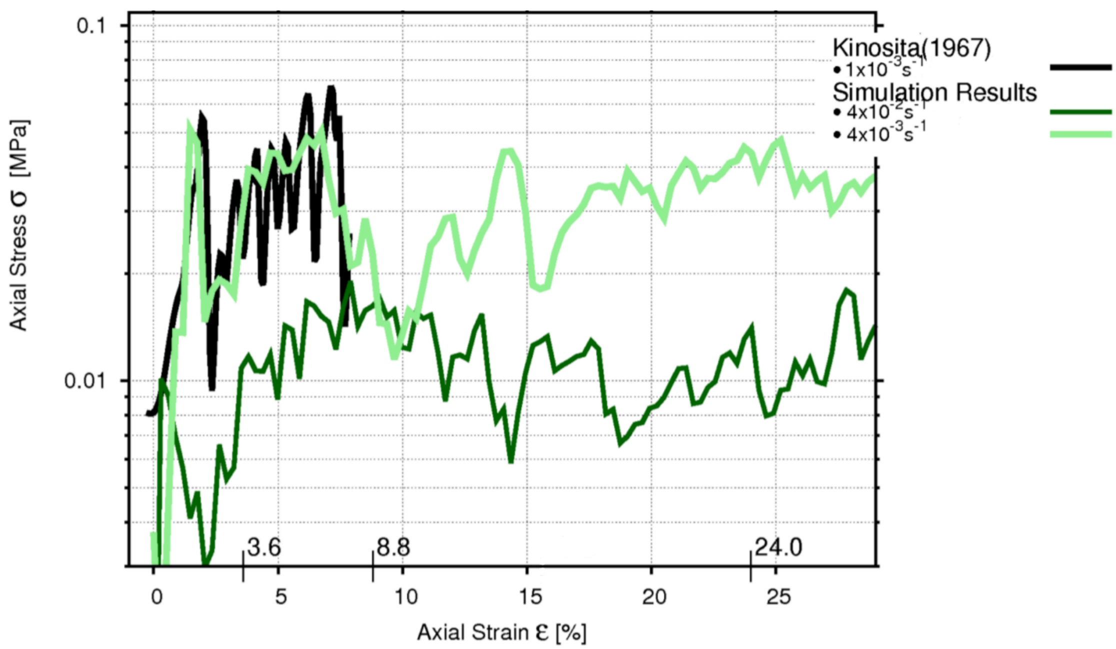
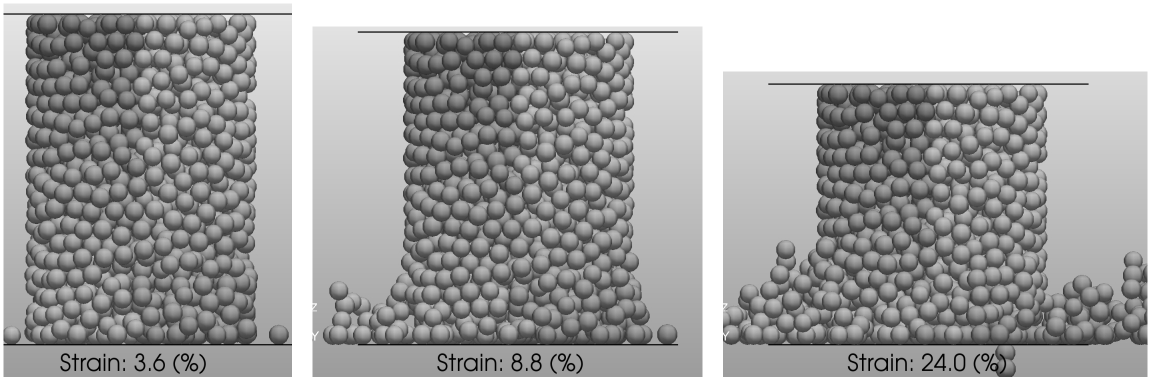


Figure 1: Axial stress versus strain during an unconfined compression test of a cylindrical snow sample ($\rho = 408\ kg/m^3$) in the brittle regime at strain rates of $\dot{\varepsilon} = 4 \cdot 10^{-3}\ 1/s$ and $\dot{\varepsilon} = 4 \cdot 10^{-2}\ 1/s$.

The predicted results were compared to measurements of Kinosita [28], who carried out experiments at a temperature of $T = -2C$ and a strain rate of $\dot{\varepsilon} = 10^{-3}\ 1/s$. In particular, the predicted results of fig. 1 at a strain rate of $\dot{\varepsilon} = 4 \cdot 10^{-3}\ 1/s$ agree well with the experimental data of Kinosita [28]. Both lines depict a similar saw tooth-like behaviour that is a characteristic property of the brittle deformation regime: A certain critical stress is reached during a period of almost linear increase of the stress versus strain representing a saw tooth. Fracture within the snow sample occurs at peak stress values and thus, releases abruptly the stress to lower values. The abrupt release of stress is accompanied by a significant plastic deformation of the sample as shown in fig. 2 for the lower line of fig. 1.

Fig. 2 shows that fracture occurs at the bottom part of the snow sample where also plastic deformation takes place. After rupture has taken place in the snow sample at a critical stress level, stress increases almost linearly with strain again repeating the next



(a) $\varepsilon = 3.6\%$ (b) $\varepsilon = 8.8\%$ (c) $\varepsilon = 24.0\%$

Figure 2: Brittle behaviour of a cylindrical snow sample ($\rho = 408\ kg/m^3$) during a strain rate of $4 \cdot 10^{-2}\ 1/s$.

saw tooth until fracture occurs at almost the same peak stress level. The maximum peak stress is referred to as the yield stress of a sample in the brittle deformation regime.

3.2 Ductile Deformation

Similar to the previous section, ductile deformation of a snow sample was predicted for a strain rates of $\dot{\varepsilon} = 4 \cdot 10^{-6}\ 1/s$, however different bonding properties. The latter include the number of bonding neighbours N_b and the ratio of bond to grain radius r_b/r_g. Furthermore, predicted results were compared to experimental data of Scapozza and Bartelt [31], Von Moos [32] and Chandel et al. [33] at respective strain rates and temperatures and are depicted in fig. 3.

The stress-strain curves in fig. 3 initially increase almost linearly with a high gradient and flatten out by approaching the yield stress σ_y. During a further increase of strain, only insignificant variation in stress occur, so that the sample remains at an almost constant stress level. The latter depends strongly on the bonding properties namely bonding neighbours N_b and the ratio of bond to grain radius r_b/r_g and is confirmed by experiments of Fukue [30]. He identified also the influence of the micro-structure as the main influence on the sample stiffness, that increases with longer sintering periods under the same density and is well captured with the predictions in fig. 3.

After the yield stress is reached, measurements show also a significant change in gradient, however, follow the characteristics of work-hardening behaviour and was observed by Fukue [30], Scapozza and Bartelt and [31], Von Moos [32]. This behaviour is not represented by the predicted results indicating that creep of bonds subjected to stress is over-predicted.

The deformation behaviour is also shown in fig. 4 for different states referrring to a strain of $\varepsilon = 4.4\%$, $\varepsilon = 16.0\%$ and $\varepsilon = 20.0\%$.

Contrary to the previous section on brittle deformation, the bonds experience almost

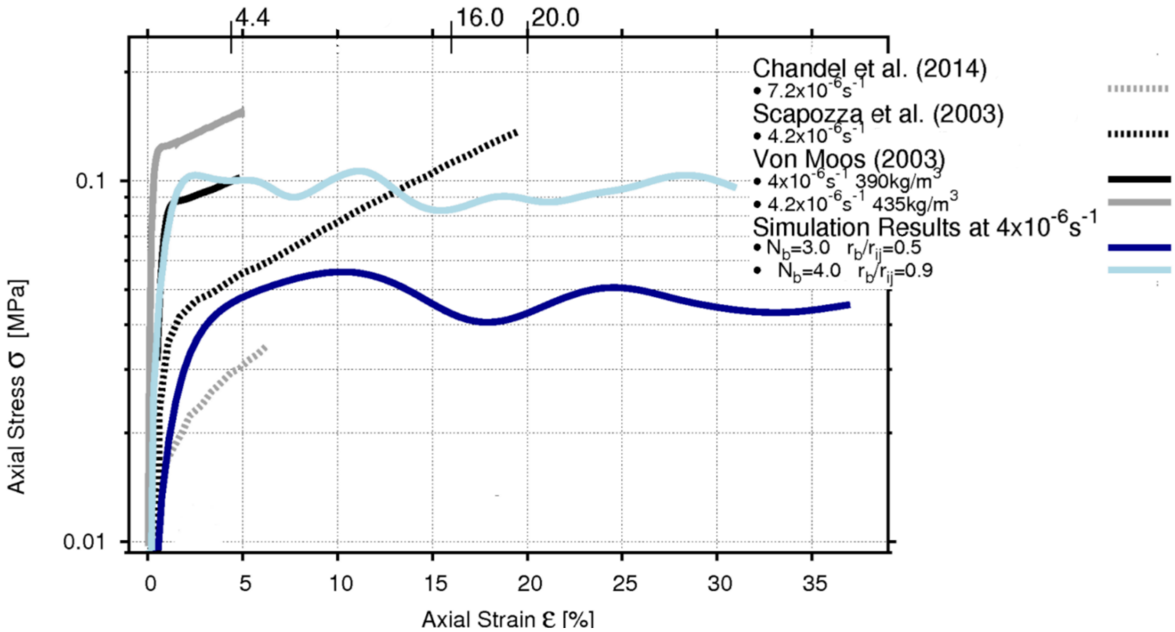
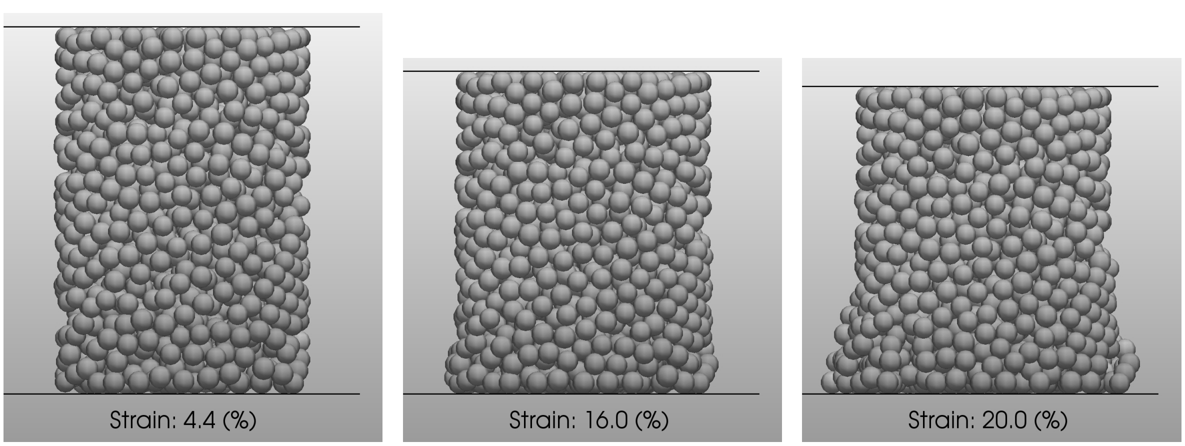


Figure 3: Axial stress versus strain during an unconfined compression test of a cylindrical snow sample ($\rho = 408\ kg/m^3$) in the ductle regime at strain rate of $\dot{\varepsilon} = 4 \cdot 10^{-6}\ 1/s$.



(a) $\varepsilon = 4.4\%$ (b) $\varepsilon = 16.0\%$ (c) $\varepsilon = 20.0\%$

Figure 4: Ductile behaviour of a cylindrical snow sample ($\rho = 408\ kg/m^3$) during a strain rate of $4 \cdot 10^{-6}\ 1/s$.

no fracture, so that the initial bonding structure of the sample remains intact. Thus, the bonds sustain the stress applied and only undergo deformation and displacement. Fig. 4a depicts the sample at the verge of yielding, for which only compaction of the sample without any significant change in shape took place. However, with an increased strain of $\varepsilon = 20.0\%$, the sample deforms radially at the bottom area. This behaviour is accompanied by an inhomogeneous stress distribution, whereby higher stress values are found at the bottom of the sample. In addition, bonds experience tension due to radial deformation, however, in general are able to sustain the stress applied without failure.

4 CONCLUSIONS

Within this study a discrete approach derived from the classical Discrete Element Method (DEM) was employed to describe the behaviour of snow in both the brittle and ductile deformation regime. The numerical approach included elastic-plastic grain collision, inter-granular friction, bond growth due to creep of ice, elastic viscous-plastic deformation of bonds and fracture of bonds. The predicted results were obtained from unconfined compression tests of cylindrical snow samples wit a density of $\rho = 408\ kg/m^3$. In order to predict brittle and ductile deformation strain rates of $4 \cdot \dot{\varepsilon} = 10^{-3}\ 1/s$, $4 \cdot \dot{\varepsilon} = 10^{-2}\ 1/s$ and $\dot{\varepsilon} = 4 \cdot 10^{-6}\ 1/s$ were applied. These values fall well into the brittle and ductile deformation regime that are separated by a transitional regime at a strain rate of $\sim\ 5 \cdot 10^{-4}\ 1/s$.

Predicted results were compared to experimental data of Kinosita [28] Scapozza and Bartelt [31], Von Moos [32] and Chandel et al. [33] that allowed correlating respective stress-strain curves. Under conditions applied good agreement between measurements and predictions was achieved. However, predicted results in the ductile regime after work hardening had occurred deviated from the measured data, and thus directing to an area for improved model development. Furthermore, results suggest that sinterig age e.g. the history a snow sample has experienced is among the parameters having the most significant impact on the mechanical behaviour of a snow sample.

REFERENCES

[1] P. Bartelt and M. Lehning. A physical SNOWPACK model for the Swiss avalanche warning: Part I: numerical model. *Cold Regions Science and Technology*, 35(3):123–145, 2002.

[2] E. Brun, E. Martin, V. Simon, C. Gendre, and C. Coleou. An energy and mass model of snow cover suitable for operational avalanche forecasting. *J. of Glaciol.*, 35(121):333342, 1989.

[3] Jordan. A one-dimensional temperature model for a snow cover. technical documentation for sntherm. Technical report, CRREL:9116, 1991.

[4] F. W. Smith. Elastic stresses in layered snow packs. *Journal of Glaciology*, 11(63):407–414, 1972.

[5] T. Lang and R. Sommerfeld. The modeling and measurements of the deformation of a sloping snow pack. *Journal of Glaciology*, 19(81):153–163, 1977.

[6] S Cresseri, F. Genna, and C. Jommi. Numerical integration of an elasticviscoplastic constitutive model for dry metamorphosed snow. *International Journal for Numerical and Analytical Methods in Geomechanics*, 34(12):1271–1296, 2010.

[7] M. von Moos. *Untersuchungen über das visko-elastische verhalten von Schnee auf der Grundlage von triaxialen Kriechversuchen*. PhD thesis, ETH Zürich, 2001.

[8] L. H. Shapiro, J. B. Johnson, M. Sturm, and G. L. Blaisdell. Snow Mechanics - Review of the State of Knowledge and Applications. Technical report, US-Army CRREL, 1997.

[9] J. Schweizer. The influence of the layered character of snow cover on the triggering of slab avalanches. *Annals of Glaciology*, 18:193–198, 1993.

[10] M. Stoffel and P. Bartelt. Modelling snow slab release using a temperature-dependent viscoelastic finite element model with weak layers. *Survey in Geophysics*, 24:417–430, 2003.

[11] M. Stoffel. *Numerical Modelling of Snow using Finite Elements*. PhD thesis, Swiss Federal Institute of Technology Zurich, 2005.

[12] P. Bartelt, M. Christen, and M. Wittwer. Progam hafeleli - two dimensional numerical simulation of the creeping deformatino and temperature distribution in a phase changing snow pack. In *Snow Engineering: Recent Advances and Developments*, 2000.

[13] R. B. Haehnel and S. A. Shoop. A macroscale model for low density snow subjected to rapid loading. *Cold Regions Science and Technology*, 40(3):193–211, 2004.

[14] J. Gaume, G. Chambon, M. Naaim, and N. Eckert. Influence of the weak layers heterogeneity on slab avalanches releases using a finite element method. In Bonelli S., Dascalu C., and Nicot F., editors, *Advances in Bifurcation and Degradation in Geomaterials*, volume 11 of *Springer Series in Geomachanics and Geoengineering*, pages 261–266, 2011.

[15] F. Nicot. Constitutive modelling of snow as a cohesive-granular material. *Granular Matter*, 6:47–60, 2004.

[16] F. Nicot. From constitutive modelling of a snow cover to the design of flexible protective structures Part 1 – Mechanical modelling. *International Journal of Solids and Structures*, 41:3317–3337, 2004.

[17] M Schneebeli. Numerical simulation of elastic stress in the microstructure of snow. *Annals Of Glaciology*, 38(1):339–342, 2004.

[18] P. Hagenmuller. Experiments and simulation of the brittle failure of snow. Master's thesis, WSL Institute for Snow and Avalanche Research SLF, 2011.

[19] T. Theile, H. Loewe, T. C. Theile, and M. Schneebeli. Simulating creep of snow based on microstructure and the anisotropic deformation of ice. *Acta Materialia*, 2011.

[20] J. B. Johnson and M. A. Hopkins. Identifying microstructural deformation mechanics in snow using discrete-element modeling. *Journal of Glaciology*, 51, 2005.

[21] M. Michael. *A discrete apporach to describe the kinetics between snow and a tire tread*. PhD thesis, UNiverity of Luxembourg, 2014.

[22] M. Michael, B. Peters, and F. Nicot. Advanced micromechanical description of snow behaviour part i mechanical and numerical modelling. *International Journal of Solids and Structures*, submitted.

[23] M. Michael, B. Peters, and F. Nicot. Advanced micromechanical description of snow behaviour part ii model validation. *International Journal of Solids and Structures*, submitted.

[24] P. Hagenmuller, G. Chambon, and M. Naaim. Microstructure-based modeling of snow mechanics: a discrete element approach. *The Cryosphere Discuss*, 9:1425–1460, 2015.

[25] A. Stomakhin, C. Schroeder, L. Chai, J. Teran, and A. Selle. A material point method for snow simulation. In *Proceedings of Siggraph 2013*, 2013.

[26] D. Szabo and M. Schneebeli. Subsecond sintering of ice. *Applied Physics letters*, 90, 2007.

[27] P. Barnes, D. Tabor, and J. Walker. The Friction and Creep of Polycristaline Ice. In Barnes, editor, *Proceedings of the Royal Society of London*, volume 324 of *A*, 1971.

[28] Seiiti Kinosita. Compression of Snow at Constant Speed. *Physics of Snow and Ice*, 1(2):911–927, 1967.

[29] Hideki Narita. An experimental study on tensile fracture of snow. *Contributions from the Institute of Low Temperature Science*, A32:1–37, 1984.

[30] Masaharu Fukue. *Mechanical performance of snow under loading.* PhD thesis, Department of Civil Engineering and Applied Mechanics, McGill, 1977.

[31] C Scapozza and P Bartelt. Triaxial tests on snow at low strain rate. Part II. Constitutivebehaviour. *Journal Of Glaciology*, 49(164):91–101, 2003.

[32] Markus von Moos. *Untersuchungen "uber das visko-elastische verhalten von Schnee auf der Grundlage von triaxialen Kriechversuchen.* PhD thesis, ETH Z"urich, 2001.

[33] Chaman Chandel, Praveen K. Srivastava, and P. Mahajan. Micromechanical analysis of deformation of snow using x-ray tomography. *Cold Regions Science and Technology*, (0):–, 2014.