11th Seminar
Ordered Structures in Games and Decisions
(OSGAD 2017)

Abstracts

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Université Paris I — Panthéon-Sorbonne

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http://ces.univ-paris1.fr/membre/seminaire/OSGAD/OSGAD.htm

PROGRAM

♦ **9:00 Welcome to participants**

❏ 9:20-10:10 : Tamás SOLYMOSI

Redundant coalitions for core-related solutions of cooperative games

❏ 10:10-11:00 : Anna BOGOMOLNAIA

Random voting under dichotomous preferences

♦ **11:00-11:20 Coffee break**

❏ 11:20-12:10 : Stéphane GONZALEZ

Axiomatic Foundations of a Unifying Core Concept for Games in Effectiveness Form

❏ 12:10-13:00 : Marc PIRLOT

The simplest decision model with criteria interactions

♦ **13:00-14:30 Lunch**

❏ 14:30-15:20 : Jean-Luc MARICHAL

Associative and quasitrivial operations on finite sets

❏ 15:20-16:10 : Antoine BILLOT

Utilitarian Aggregation with Reasonably Heterogeneous Beliefs

♦ **16:10-16:30 Coffee break**

❏ 16:30-17:20 : Alexandre SKODA

Inheritance of convexity for restricted games associated with general partitions

1. **Tamás SOLYMOSI (Department of Operations Research and Actuarial Sciences, Corvinus University of Budapest, Hungary)**

   Redundant coalitions for core-related solutions of cooperative games

   The various solutions of transferable utility games take into account the cooperative possibilities of all coalitions of players in one way or another. Although their definitions formally involve each of the exponentially many coalitional values, many of the excess-based solutions are actually determined by a smaller family of coalitions. Disregarding redundant coalitions can make the analysis and computation of solutions significantly easier.

   In the talk we focus on the core, the (per-capita) least core, the (per-capita) nucleolus, and identify smaller (in some cases the smallest) families of coalitions which completely determine the given solution. We present several old and some new results, and demonstrate the usefulness of such simplification possibilities on various classes of games related to optimization problems.

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2. **Anna BOGOMOLNAIA (Adam Smith Business School, University of Glasgow, United Kingdom)**

   Random voting under dichotomous preferences

   What is a democratic way to strike a fair compromise between mutually exclusive public outcomes, when the compromise can take the form of probabilistic shares, time shares, budget shares, etc.?

   We revisit probabilistic voting under dichotomous preferences where agents simply report that they *like/dislike* every outcome. We focus on two often conflicting normative concerns. Minorities should not be crushed by the opposing majority; also, the size of the support for a given outcome should increase its weight: numbers matter. The *Unanimous Fair Shares* property adresses both concerns by giving to any group of like-minded agents an influence proportional to its size. *Individual Fair Shares* weakens UFS by only guaranteeing to each agent a $1/n$-th influence on the outcome (where n is the total number of agents).

   No Efficient and Strategyproof rule can guarantee Unanimous, or even Individual, Fair Shares. We propose three second best mechanisms achieving two of these three design goals.

   The *Conditional Utilitarian rule*, a simple variant of the classic "random dictator", is Strategyproof and guarantees Unanimous Fair Shares. It is much easier to compute and strictly more efficient than the familiar *Random Priority* rule. In numerical simulations its inefficiency is consistently low.

   The efficient *Egalitarian rule* guarantees Individual (but not Unanimous) Fair Shares and is *Excludable Strategyproof*: this property weakens Strategyproofness by ensuring that an agent is excluded from consuming those public outcomes she reportedly dislikes. But this rule ignore numbers: it treats a unanimous group of agents exactly as if it contains a single agent.

The efficient *Nash rule* (maximizing the product of utilities) offers much stronger welfare guarantees than both rules above: it picks a core stable mixture when any group of agents can enforce any outcome with a probability proportional to its size. But the Nash rule fails even the excludable form of Strategyproofness.

We also uncover several challenging open questions.

(joint work with Hervé Moulin and Haris Aziz)

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3. **Stéphane GONZALEZ (Université de Saint Etienne, GATE Lyon Saint Etienne)**

   Axiomatic Foundations of a Unifying Core Concept for Games in Effectiveness Form

   This paper provides two axiomatic characterizations of the core of games in effectiveness form (Rosenthal, 1972) that unifies a wide variety of solution concepts prominent in the literature on social choice theory and game theory. Both characterizations use the non-emptiness axiom saying that the solution is nonempty on the domain with a nonempty core. The first characterization relies on coalitional unanimity and Maskin monotonicity properties together with an independence principle with respect to leaving players. The second characterization invokes a strict dominant set property and an independence principle with respect to irrelevant states. These results give new insights on characterization of well-known solution concepts ((strong) Nash equilibrium, Condorcet winner...).

   (joint work with Aymeric Lardon)

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4. **Marc PIRLOT (Université de Mons, Belgium)**

   The simplest decision model with criteria interactions

   A positive Boolean function can be viewed as an ordered classification model assigning objects or alternatives described by n-dimensional Boolean vectors to either the *Good* (coded by 1) or the *Bad* (coded by 0) category. A Boolean function is positive when it is nondecreasing in its arguments, which implies that the classification respects the monotonicity property. This model is closely related to the Noncompensatory Sorting Model (proposed and characterized by Bouyssou and Marchant [1,2]), for assigning alternatives evaluated on multiple criteria to ordered categories.

   Some positive Boolean functions are “threshold”, which means that the “good” objects can be separated from the “bad” ones by an affine function. This can be interpreted as assigning a weight to each dimension or criterion and deciding that an object is "good" if and only if the sum of the weights of the criteria on which its value is 1 reaches some minimal threshold value.

Some (many, in general) positive Boolean functions are not "threshold". It is not possible to separate the good objects from the bad ones by a linear function. Instead, a k-additive capacity is needed, for some $k > 1$, or, in other terms, the good objects can be separated from the bad ones by a multilinear polynomial of degree k. In the $k = 2$ case, one usually says that criteria interact in pairs. These interactions are interpreted as positive or negative synergies between criteria. When higher values of k are needed to separate the good from the bad objects, there are more complex interactions between criteria, involving up to k-tuples of criteria.

In order to try to better understand the notion of interaction, we have enumerated and listed all positive Boolean functions up to $n = 6$. For each of them, we determined the minimal value of k for which the good and the bad objects can be separated by a k-additive capacity [3]. The number of positive Boolean functions of n variables, the Dedekind number D_n, grows very rapidly with n and there is no known close-form formula for computing it. These numbers are known only up to $n = 8$. Recently, Stephen and Yusun [4] have enumerated all *non equivalent* Boolean functions up to $n = 7$. We used their method and determined, for each positive Boolean function, up to $n = 6$, the minimal value of k for which the good and bad objects can be separated by a k-additive capacity.

We analyze a few interesting examples taken from the exhaustive lists generated and discuss the interpretations in terms of positive or negative synergies that could be used in the course of an elicitation process with a decision maker.

[1] D. Bouyssou and T. Marchant. An axiomatic approach to noncompensatory sorting methods in MCDM, I: The case of two categories. *European Journal of Operational Research*, 178(1):217–245, 2007.

[2] D. Bouyssou and T. Marchant. An axiomatic approach to noncompensatory sorting methods in MCDM, II: More than two categories. *European Journal of Operational Research*, 178(1):246–276, 2007.

[3] E. Ersek Uyanık, O. Sobrie, V. Mousseau, and M. Pirlot. Enumerating and categorizing positive boolean functions separable by a k-additive capacity. *Discrete Applied Mathematics*, 229:17–30, 2017.

[4] T. Stephen and T. Yusun. Counting inequivalent monotone boolean functions. *Discrete Applied Mathematics*, 167:15 – 24, 2014.

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5. **Jean-Luc MARICHAL (Mathematics Research Unit, University of Luxembourg, Luxembourg)**

   Associative and quasitrivial operations on finite sets

   We investigate the class of binary associative and quasitrivial operations on a given finite set. Here the quasitriviality property (also known as conservativeness) means that the operation always outputs one of its input values. We also examine the special situations where the operations are commutative and nondecreasing, in which cases the operations reduce to discrete uninorms (which are discrete fuzzy connectives playing an important role in fuzzy logic).

   Interestingly, associative and quasitrivial operations that are nondecreasing are characterized in terms of total and weak orderings through the so-called single-peakedness property introduced in social choice theory by Duncan Black.

   We also address and solve a number of enumeration issues: we count the number of binary associative and quasitrivial operations on a given finite set as well as the number of those operations that are commutative and/or nondecreasing.

   (joint work with Miguel Couceiro and Jimmy Devillet)

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   Paper available at `https://arxiv.org/pdf/1709.09162.pdf.`

6. **Antoine BILLOT (Université Paris II)**

   Utilitarian Aggregation with Reasonably Heterogeneous Beliefs

   The utilitarian aggregation rule requires social utility and beliefs to be respectively the affine aggregation of individual utilities and beliefs. Since, in case of belief heterogeneity, the standard Pareto condition is incompatible with such a separate aggregation, a new condition, called the belief-proof Pareto condition, is proposed to alleviate occurrences of spurious agreement by restricting unanimity to beliefs that can be considered 'reasonable' by society. Then, we show, in the Anscombe-Aumann framework (Theorems 1 and 2) as well as in Savage's (Theorems 3 and 4), that the belief-proof Pareto condition is equivalent to separate aggregation under complete or incomplete information of society on individual beliefs.

   (joint work with Xiangyu Qu)

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7. **Alexandre SKODA (Université Paris I, Centre d'Economie de la Sorbonne)**

Inheritance of convexity for restricted games associated with general partitions

We consider on a given finite set N an arbitrary correspondence $\mathcal{P}$ which associates to every subset $A \subseteq N$ a partition $\mathcal{P}(A)$ of A. Then, for every cooperative game (N, v), we define the restricted game $(N, \overline{v})$ associated with $\mathcal{P}$ by:

$$\overline{v}(A) = \sum_{F \in \mathcal{P}(A)} v(F), \text{ for all } A \subseteq N.$$

We refer to this game as the $\mathcal{P}$-restricted game. Through many concrete choices for the correspondence $\mathcal{P}$, the new game $(N, \overline{v})$ can take into account many combinatorial structures and different aspects of cooperation restrictions. A classical example in the context of communication games is the correspondence associating to each coalition its partition into connected components. In this case, $(N, \overline{v})$ corresponds to the well-known Myerson's restricted game.

In a first part, we present necessary and sufficient conditions on a correspondence $\mathcal{P}$ to have inheritance of convexity from (N, v) to the $\mathcal{P}$-restricted game $(N, \overline{v})$. The main condition is a cyclic intersecting sequence free condition. As a consequence, we only need to verify inheritance of convexity for unanimity games and for a small class of extremal convex games. In particular, for Myerson's restricted game, inheritance of convexity can be verified by this way.

In a second part we investigate inheritance of convexity for some specific correspondences associated with communication games. In particular, we consider the correspondence $\mathcal{P}_{\min}$ defined for weighted communication graphs and obtained by deleting the minimum-weight edges in each coalition. For this correspondence, inheritance of convexity for unanimity games already implies inheritance of convexity. We establish necessary and sufficient conditions on the edge-weights of specific subgraphs of the communication graph as paths, stars, and cycles to have inheritance of convexity. They imply strong restrictions on edge-weights. In particular, the edge-weights can have at most three different values. We also highlight several nice links between the $\mathcal{P}_{\min}$-restricted game and Myerson's restricted game. In particular inheritance of convexity for Myerson's game is equivalent to inheritance of a weaker condition than convexity called $\mathcal{F}$-convexity for the $\mathcal{P}_{\min}$-restricted game. We also provide a characterization of inheritance of $\mathcal{F}$-convexity.

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