On quasitrivial and associative operations

Abstract: We characterize the class of binary quasitrivial and associative operations on a given set. Here quasitriviality (also known as conservativeness) means that the operation always outputs one of its input values. We also provide several characterizations of the subclass of binary quasitrivial and associative operations that are symmetric and/or nondecreasing. As we will see, quasitrivial, associative, and nondecreasing operations are characterized in terms of weak orderings through the so-called single-peakedness property introduced in social choice theory by Duncan Black. Finally, we address a number of counting issues: we enumerate all binary quasitrivial and associative operations on a given finite set as well as those operations that are symmetric, are nondecreasing, have neutral and/or annihilator elements. As we will see, these considerations lead to several, previously unknown, integer sequences.