Real-time Error Control for Surgical Simulation: Application to Percutaneous Interventions

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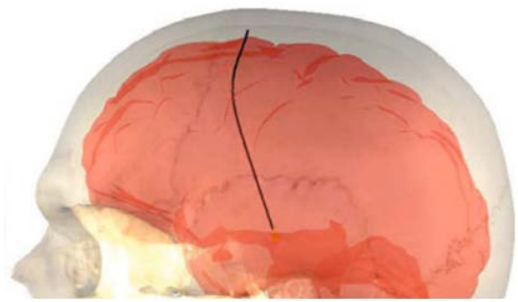
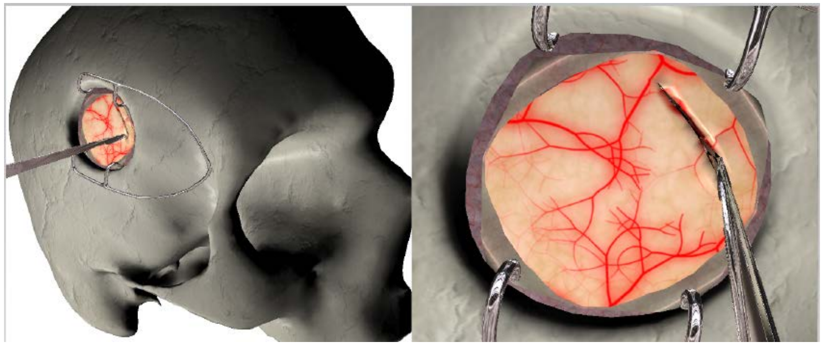
Introduction

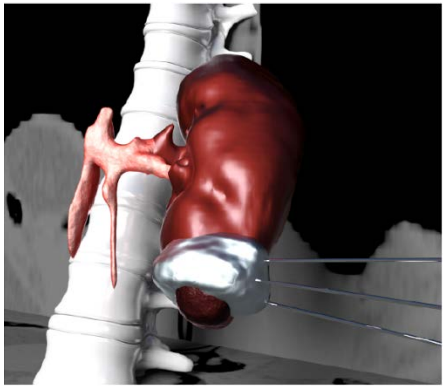
- Real-time simulations



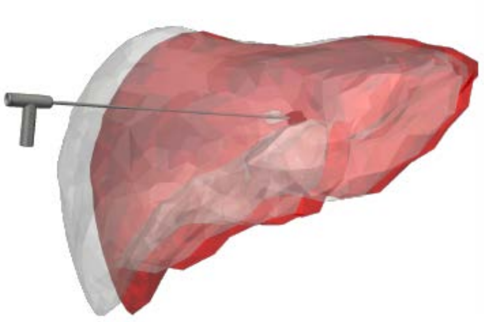
Bilger et al, MICCAI, 2011



Courtecuisse et al, Med. Image Anal., 2014



Talbot et al, SIGGRAPH, 2015



Hamzé et al, Comput. Med. Imag. Grap. 2015

What makes real-time simulation feasible?

- Coarse meshes
- Advanced solvers (GPU-based, parallel computing, asynchronous solver (Courtecuisse et al), etc.)
- Simplified models
- Numerical techniques to treat condition numbers, stability, etc.
- Model order reduction

⇒ Introduction of error!

Modeling and Simulation: Error sources

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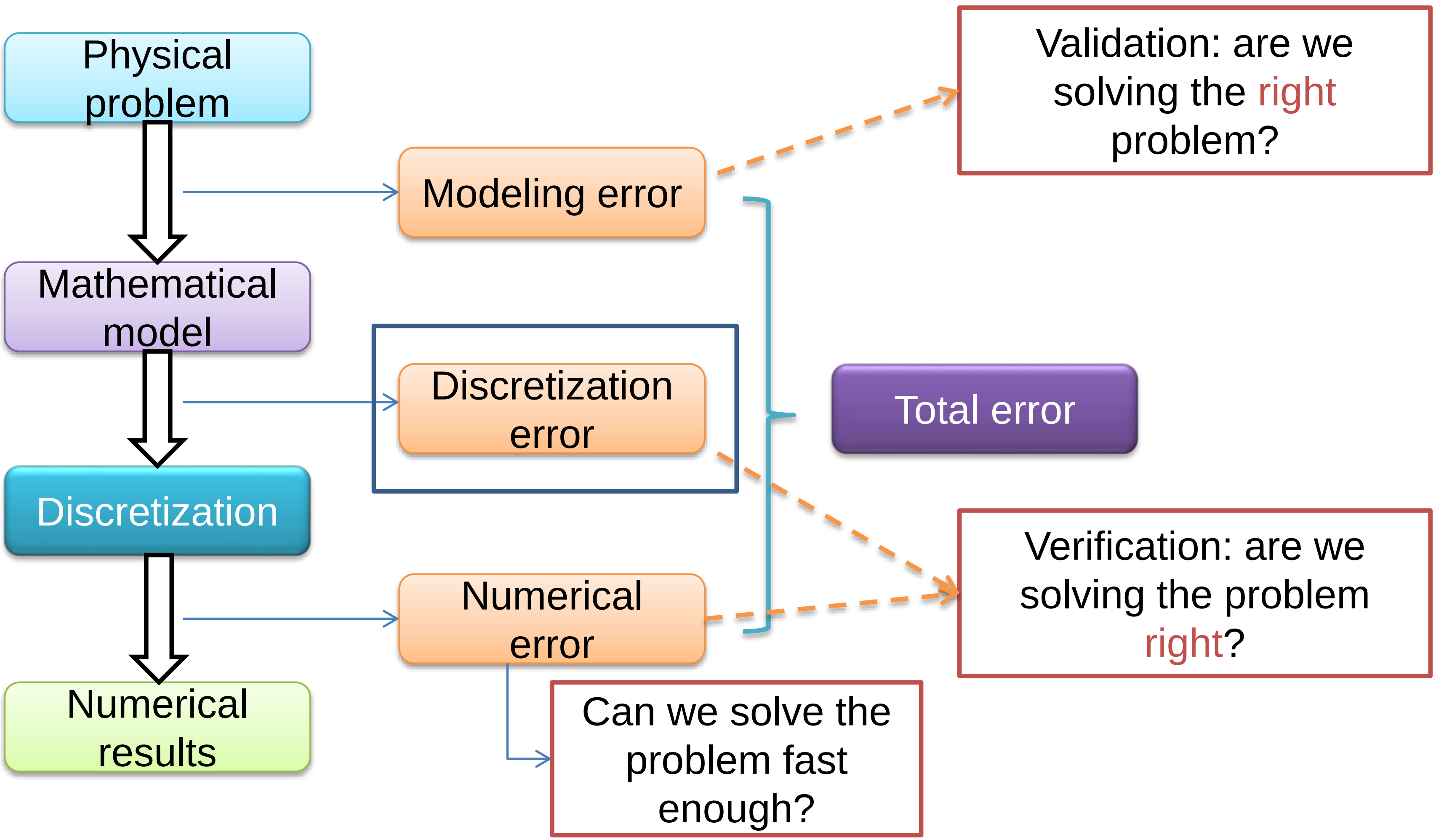
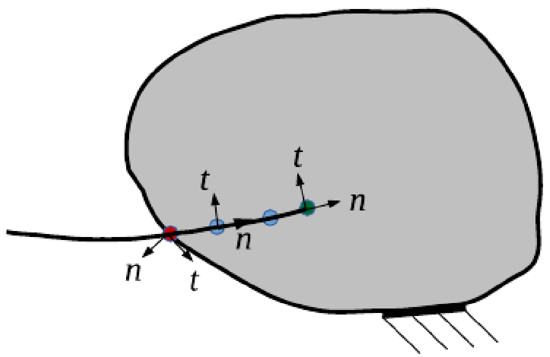


Models: Needle Insertion

Needle insertion problem



$$\operatorname{div}\boldsymbol{\sigma} + \mathbf{b} + \boldsymbol{\lambda} = \rho\ddot{\mathbf{u}} \qquad \text{in } \Omega$$

$$\boldsymbol{\epsilon} = \frac{1}{2}\left(\operatorname{grad}\mathbf{u} + (\operatorname{grad}\mathbf{u})^T\right)$$

$$\boldsymbol{\sigma} = f(\boldsymbol{\epsilon}, \boldsymbol{\nu})$$

$$\boldsymbol{\sigma}\cdot\mathbf{n} = \bar{\mathbf{t}} \text{ on } \Gamma_t;$$

$$\mathbf{u} = \bar{\mathbf{u}} \text{ on } \Gamma_u,$$

$\lambda_n^{ts} > \lambda_{p_0},$ (penetrate)

$$\lambda_t^{ts} < \mu\lambda_n^{ts} \quad \text{(stick)}; \qquad \lambda_t^{ts} = \mu\lambda_n^{ts} \quad \text{(slip)},$$

$$\lambda_n^{nt} < \mu\lambda_t^{nt} + \lambda_{c_0} \text{ (stick)}; \qquad \lambda_n^{nt} \geq \mu\lambda_t^{nt} + \lambda_{c_0} \text{ (cut and slip)},$$

$$\lambda_n^{ns} < \mu\lambda_t^{ns} \text{ (stick)}; \qquad \lambda_n^{ns} = \mu\lambda_t^{ns} \text{ (slide)}.$$

FEM discretization

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- System equation

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}^{ext} + \mathbf{H}^T\boldsymbol{\lambda},$$

- Implicit Euler scheme

$$\dot{\mathbf{u}}_{t+\tau} = \dot{\mathbf{u}}_t + \tau\ddot{\mathbf{u}}_{t+\tau}; \quad \mathbf{u}_{t+\tau} = \mathbf{u}_t + \tau\dot{\mathbf{u}}_{t+\tau},$$

- Final discrete system

$$\underbrace{(\mathbf{M} - \tau\mathbf{C} - \tau^2\mathbf{K})}_{\mathbf{A}} d\mathbf{v} = \underbrace{\tau\mathbf{f}(\mathbf{x}^t, \mathbf{v}^t) + \tau^2\mathbf{K}\mathbf{v}^t}_{\mathbf{b}} + \mathbf{H}^T\boldsymbol{\lambda}$$

- Update velocity and position

$$\mathbf{v}_{t+\tau} = d\mathbf{v} + \mathbf{v}_t; \quad \mathbf{x}_{t+\tau} = \mathbf{x}_t + \tau\mathbf{v}_{t+\tau}.$$

➢ Hexahedral elements for tissue
➢ Beam elements for needle

Corotational formulation

- Classical FEM

$$\mathbf{f}_e(\boldsymbol{\sigma}) = \int_{\Omega_e} \mathbf{B}_e^T \boldsymbol{\sigma} \, \mathrm{d}\Omega = \int_{\Omega_e} \mathbf{B}_e^T \mathbf{E} \mathbf{B}_e \mathbf{u}_e \, \mathrm{d}\Omega$$
$$= \mathbf{K}_e \cdot \mathbf{u}_e = \mathbf{K}_e \cdot (\mathbf{x}_e - \mathbf{x_0}_e)$$

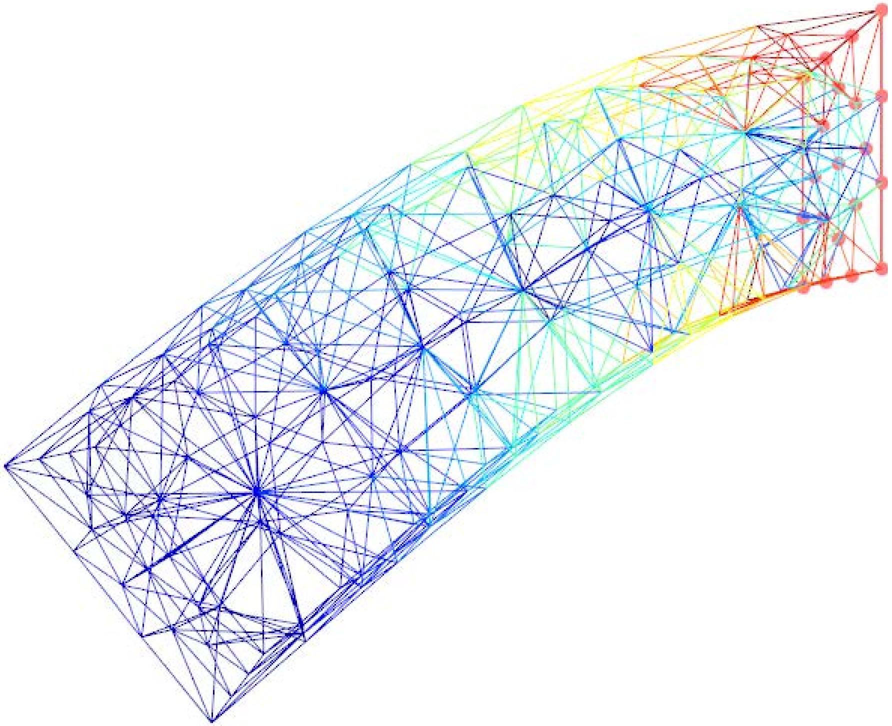
- Corotational FEM

$$\mathbf{f}_e = \mathbf{R}_e \mathbf{K}_e (\mathbf{R}_e^T \mathbf{x}_e - \mathbf{x_0}_e)$$

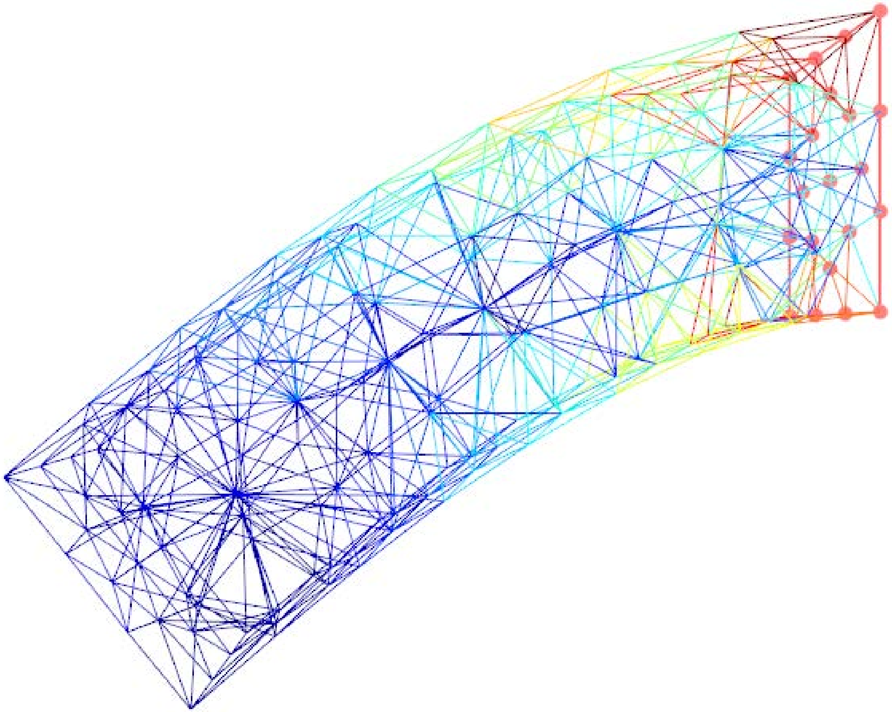
  - Polar decomposition

$$\boldsymbol{F_e} = \boldsymbol{R_e}.\boldsymbol{U_e}$$

Corotational vs. linear



Linear



Corotational

ERROR ESTIMATE

A posteriori error estimate

- Residual-based (Babuska et al, 1978)
- Equilibrated fluxes (Ladevéze et al, 1983)
- Smoothing of the stress field (Zienkiewicz and Zhu, 1992)

  (Superconvergent Patch Recovery -- SPR)

Error estimate

- Energy norm

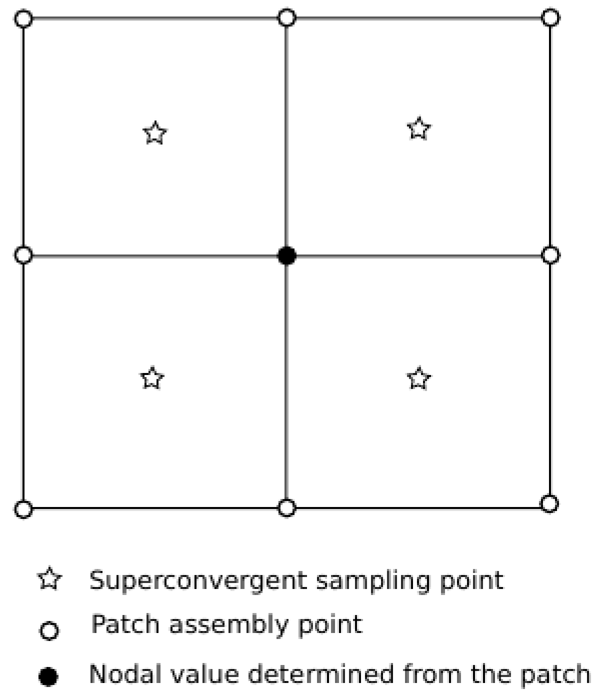
$$\eta_e = \sqrt{\int_{\Omega^e} (\boldsymbol{\epsilon}^h - \boldsymbol{\epsilon}^s)^T (\boldsymbol{\sigma}^h - \boldsymbol{\sigma}^s) \mathrm{d}\Omega},$$

- Patch recovery

$$\tilde{\sigma}_j^s = \mathbf{P}\mathbf{a}_j = [1\; x\; y\; z\; xy\; yz\; zx\; xyz][a_1\; a_2\; a_3\; a_4\; a_5\; a_6\; a_7\; a_8]_j^T$$

Minimize:

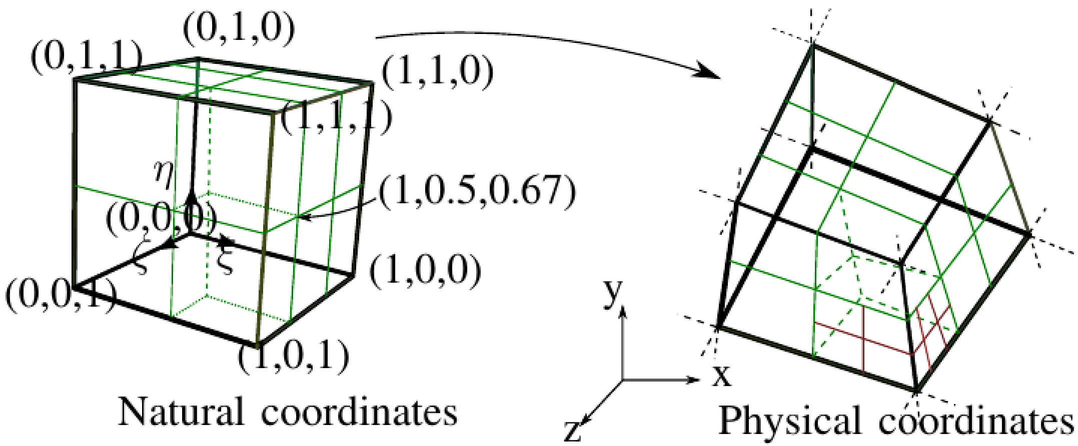
$$\Pi = \sum_{k=1}^{8} \left[\sigma_j^h(x_k, y_k, z_k) - \mathbf{P}_k \mathbf{a}_{j_k}\right]$$



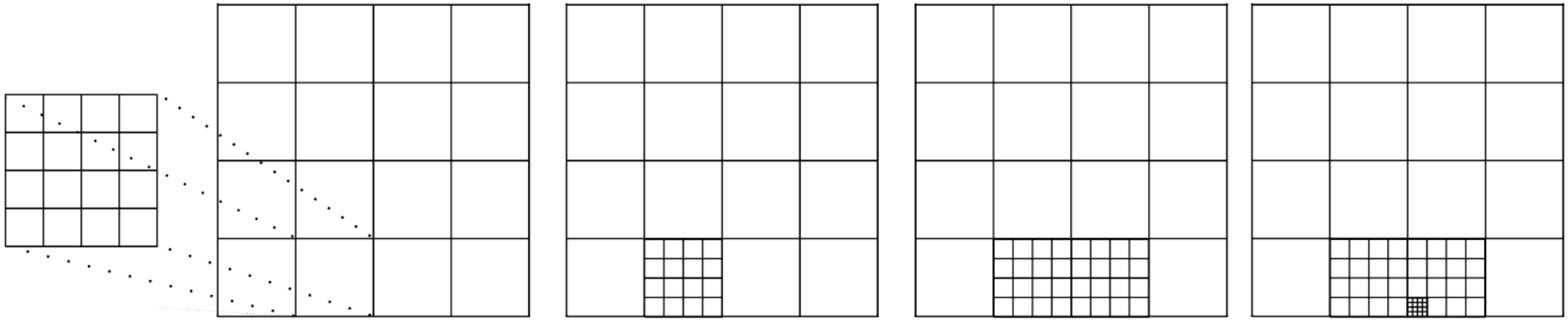
Zienkiewicz-Zhu smoothing procedure
(Zienkiewicz and Zhu, 1992)

Adaptive refinement

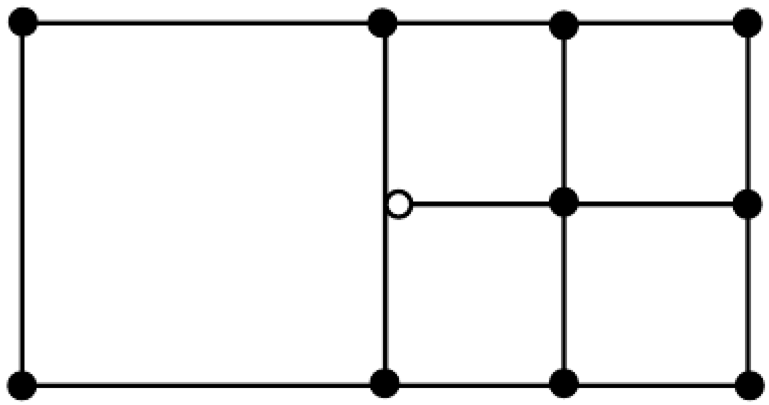
Criterion (maximum strategy): $\eta_e \geq \theta \eta_M, \quad 0 < \theta < 1, \text{ where } \eta_M = \max_e \eta_e.$



Template-based refinement



Handling hanging nodes



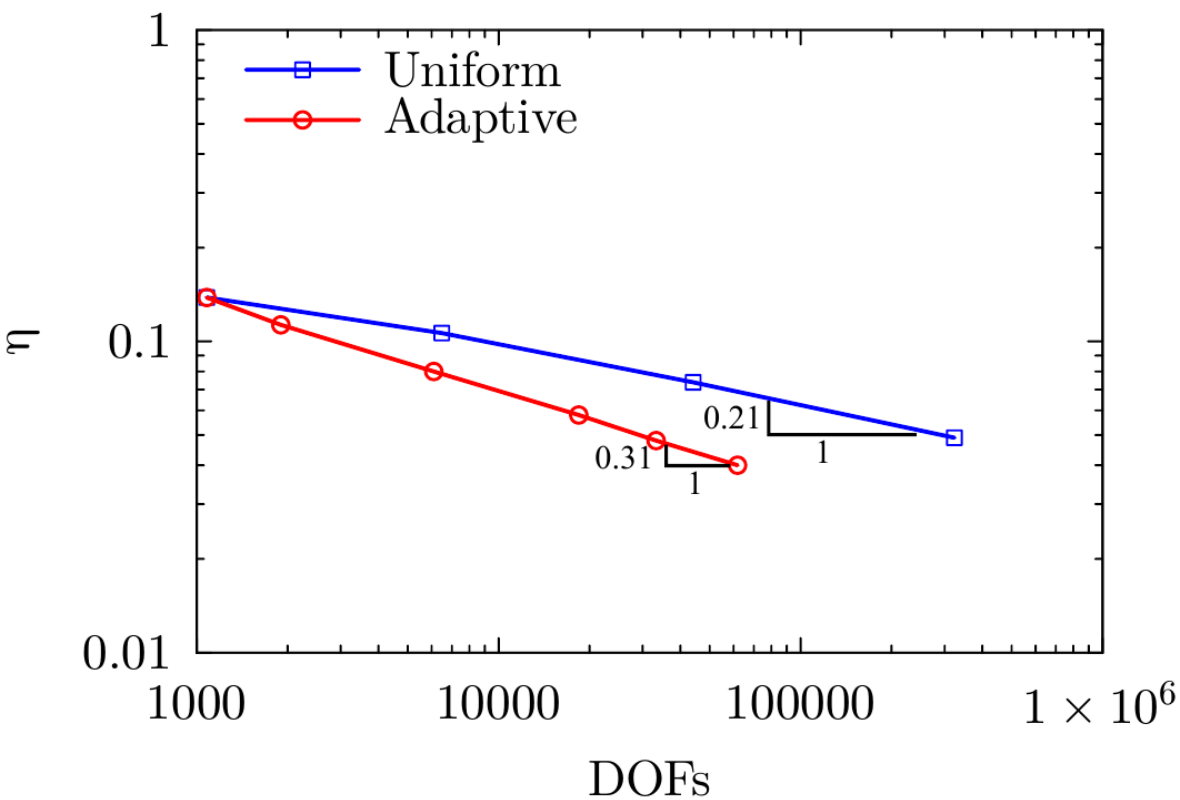
Using Lagrange multipliers:

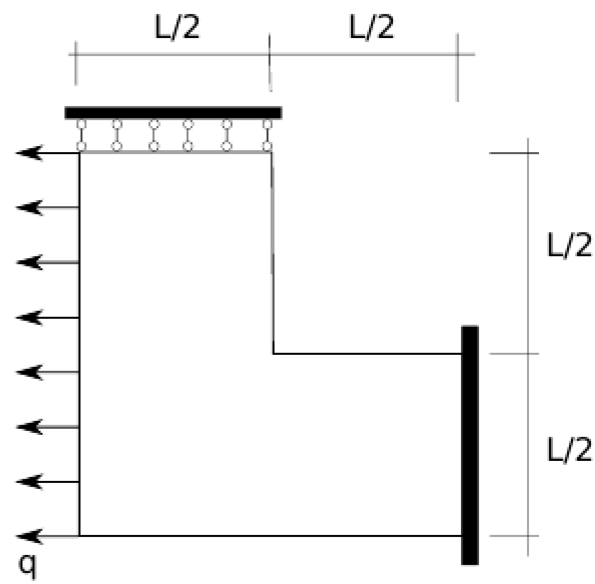
$$\begin{pmatrix} \mathbf{A} & \mathbf{J}^T \\ \mathbf{J} & \mathbf{0} \end{pmatrix} \begin{Bmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{b} \\ \mathbf{0} \end{Bmatrix}$$

NUMERICAL RESULTS

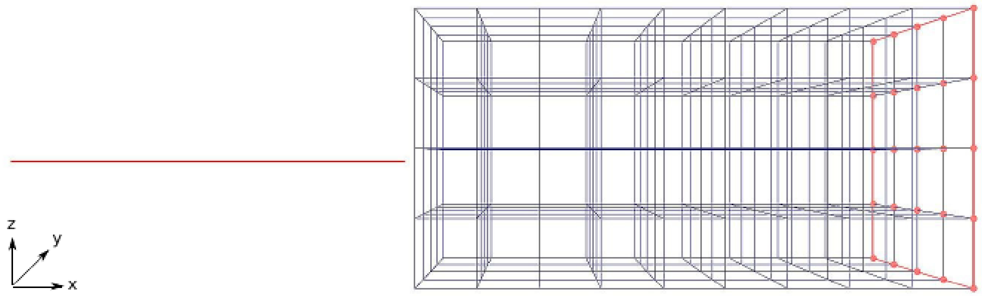
Convergence study

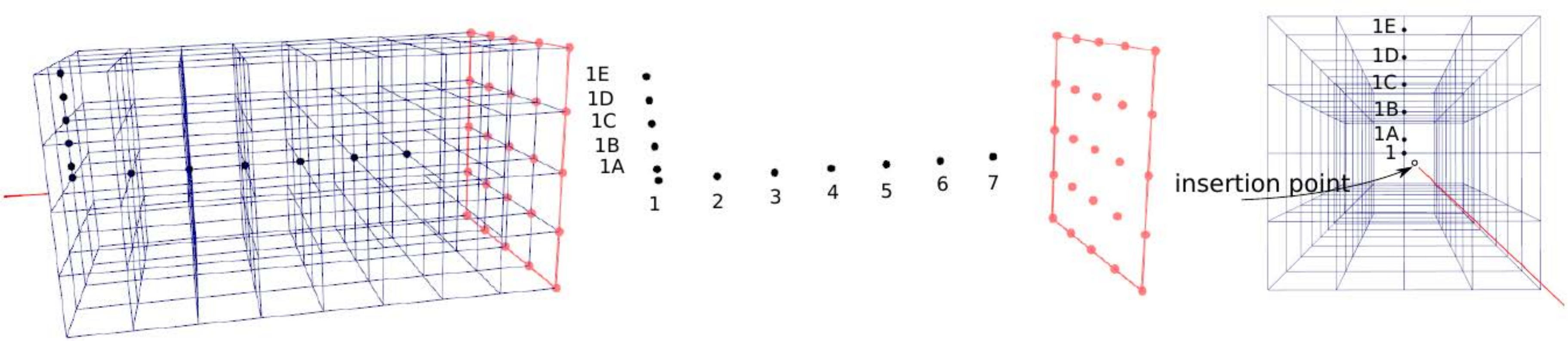
Theoretical slope for singular problems in 3D: 2/9 ≈ 0.22





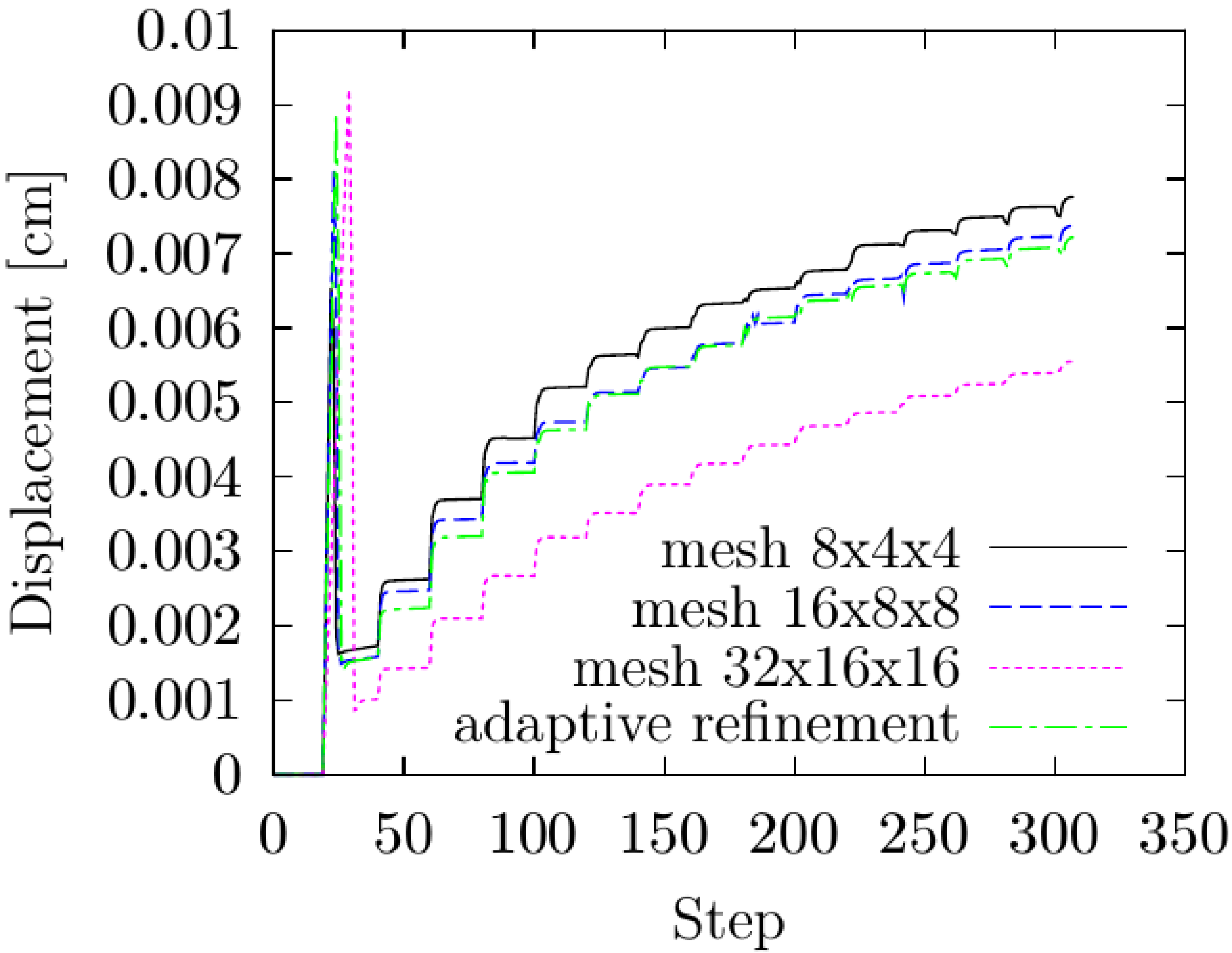
Needle insertion simulation





Needle insertion

Displacement of Point 1C



DOFs:

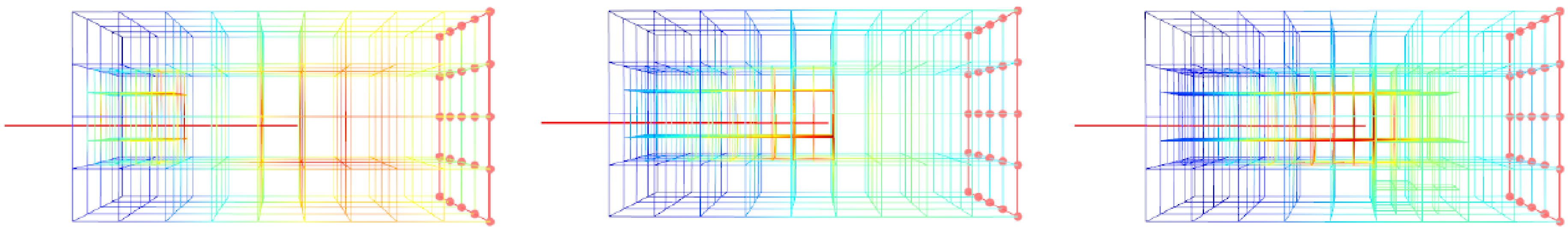
675

4 131

28 611

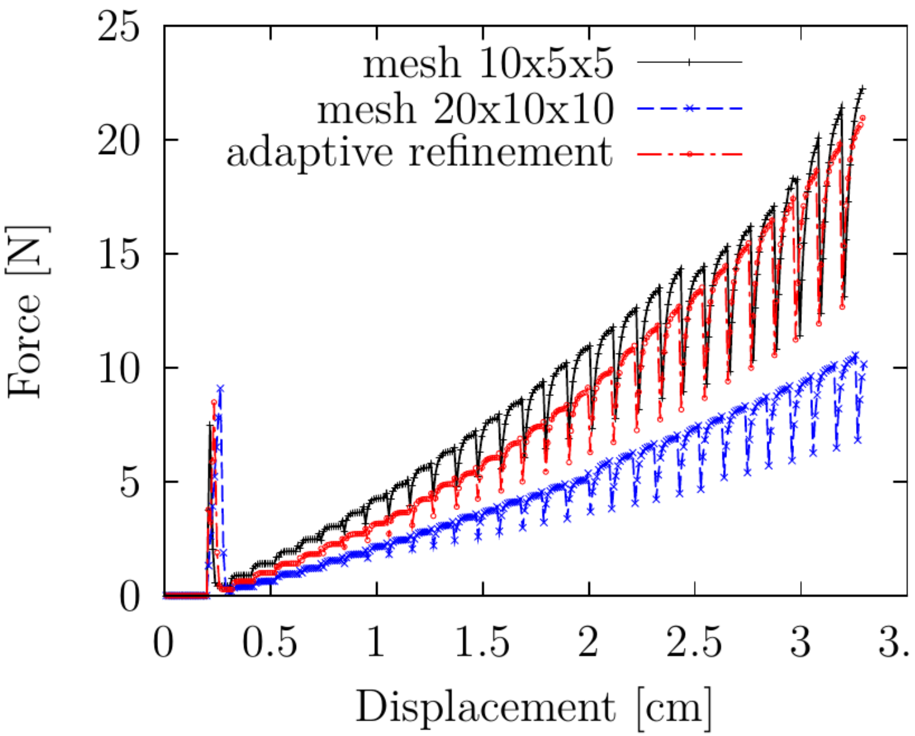
1 461

Influence of frictional coefficient

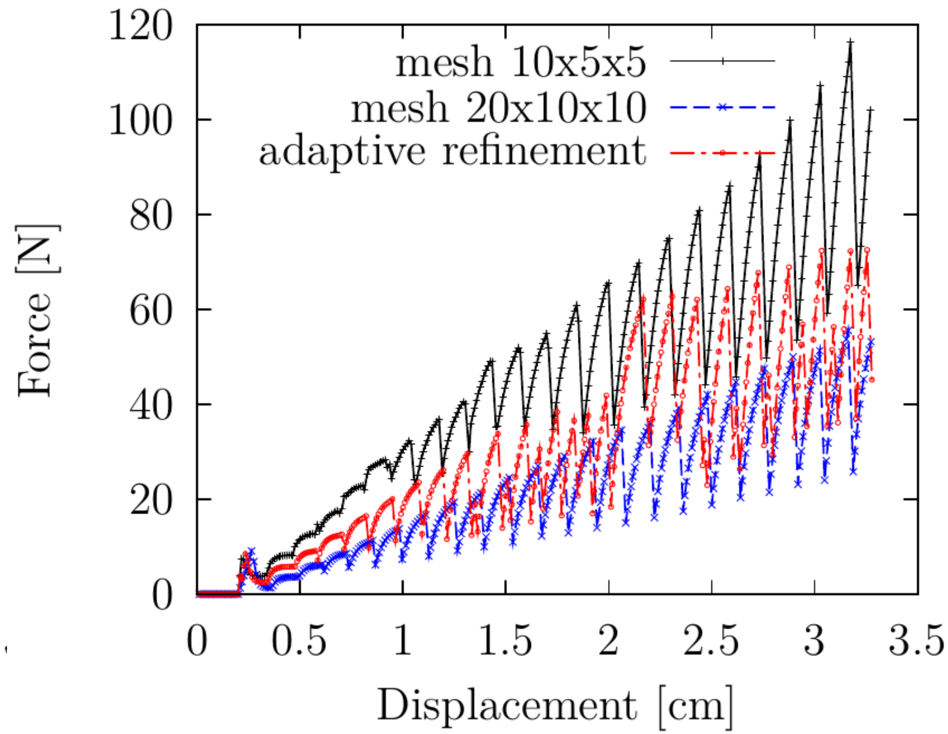


(a) $\mu = 0.1$ (b) $\mu = 0.3$ (c) $\mu = 0.5$

Force-displacement

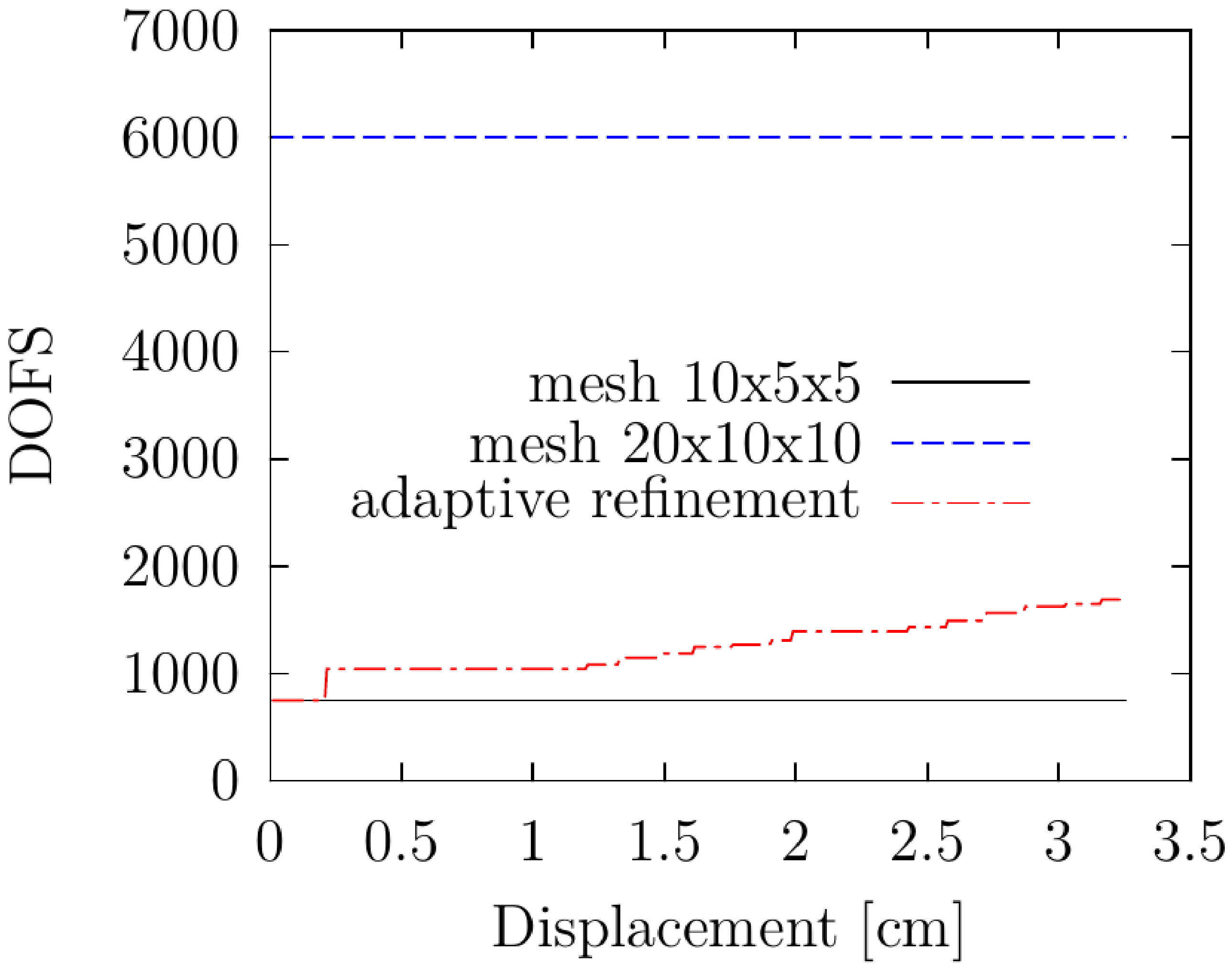


$$\mu = 0.1$$



$$\mu = 0.5$$

Number of DOFs



Computational time

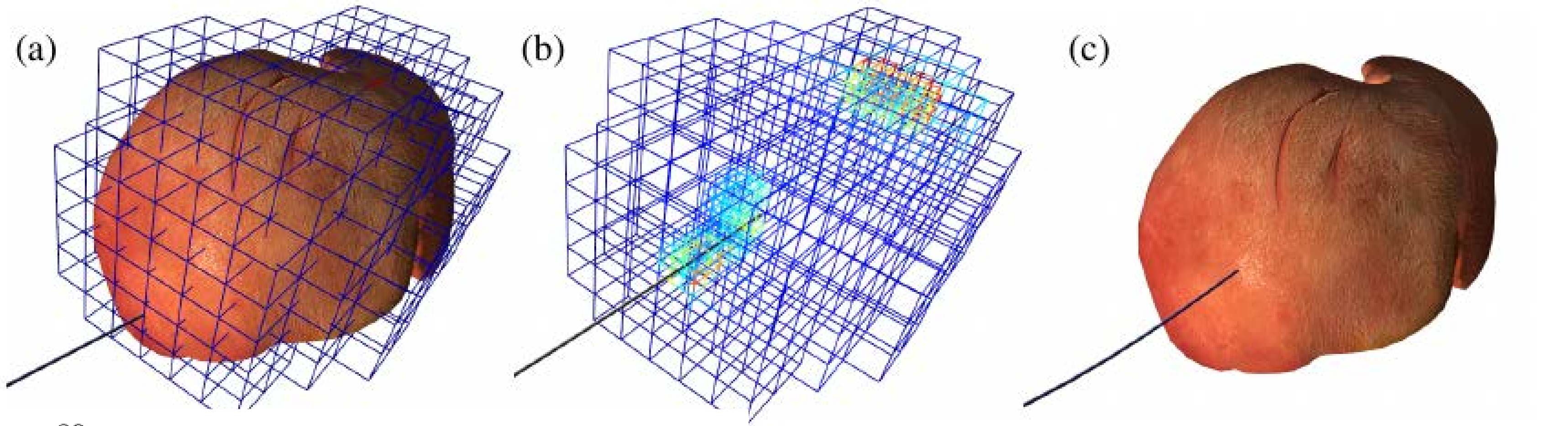
- Error = 5%

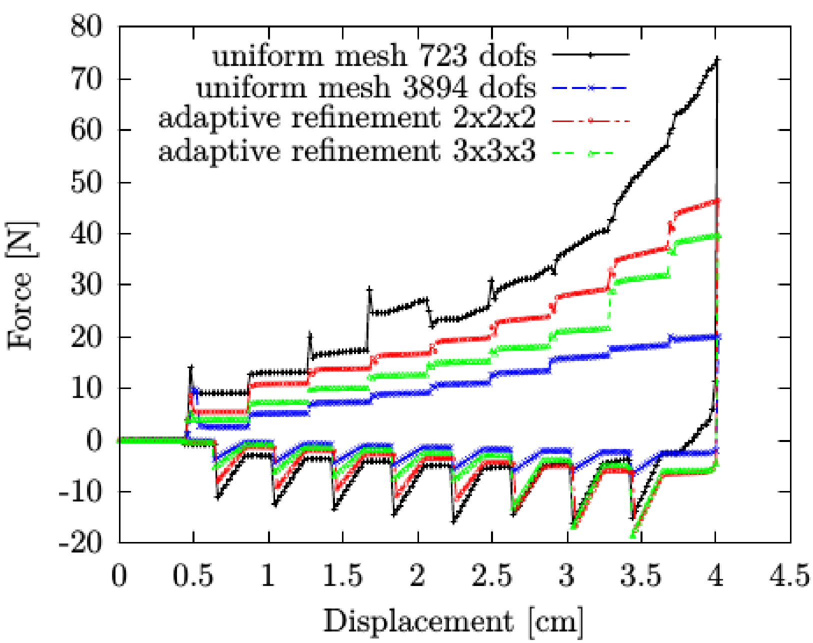
Computational time (10^{-3}s), TC: topological changes, MS: Matrix solving

	DOFs	Time		Total time
		TC	MS	
Full refinement	44064	x	12140.8	12140.8
Local refinement	7473	319.24	417.17	736.41

⇨ Speed up by a factor of 16!

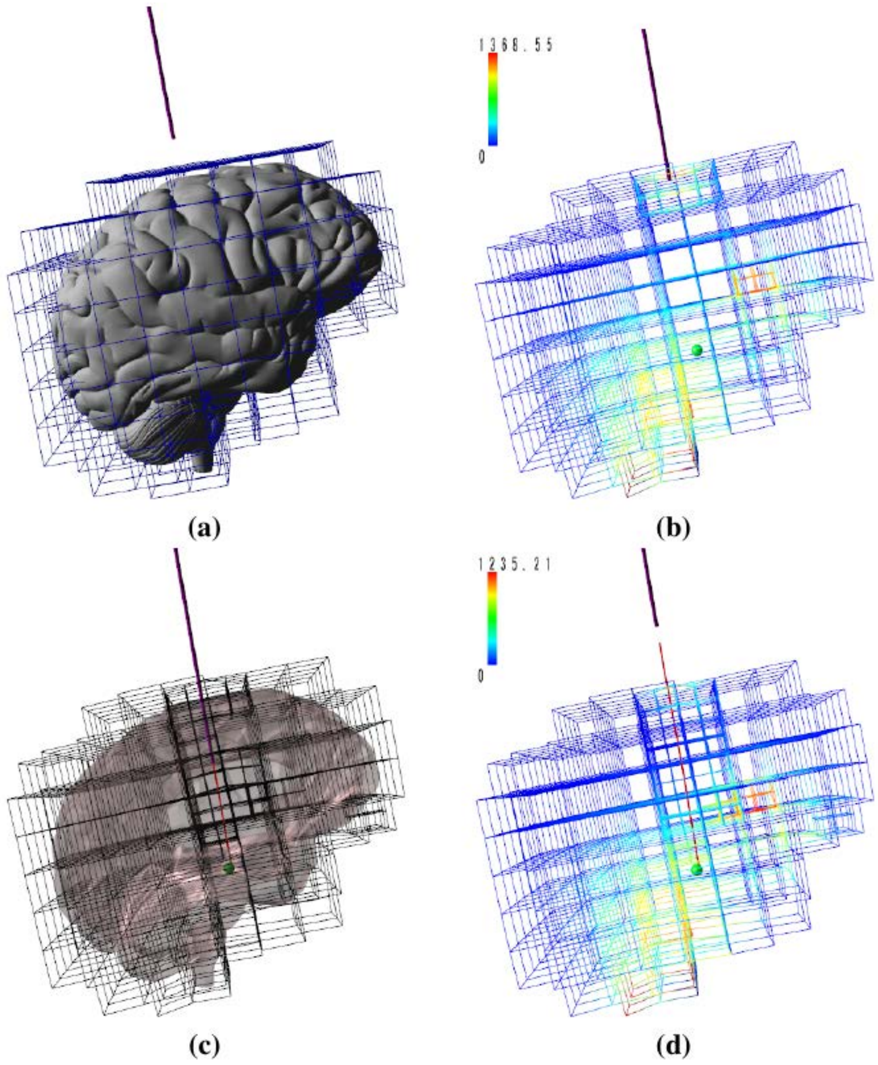
Liver



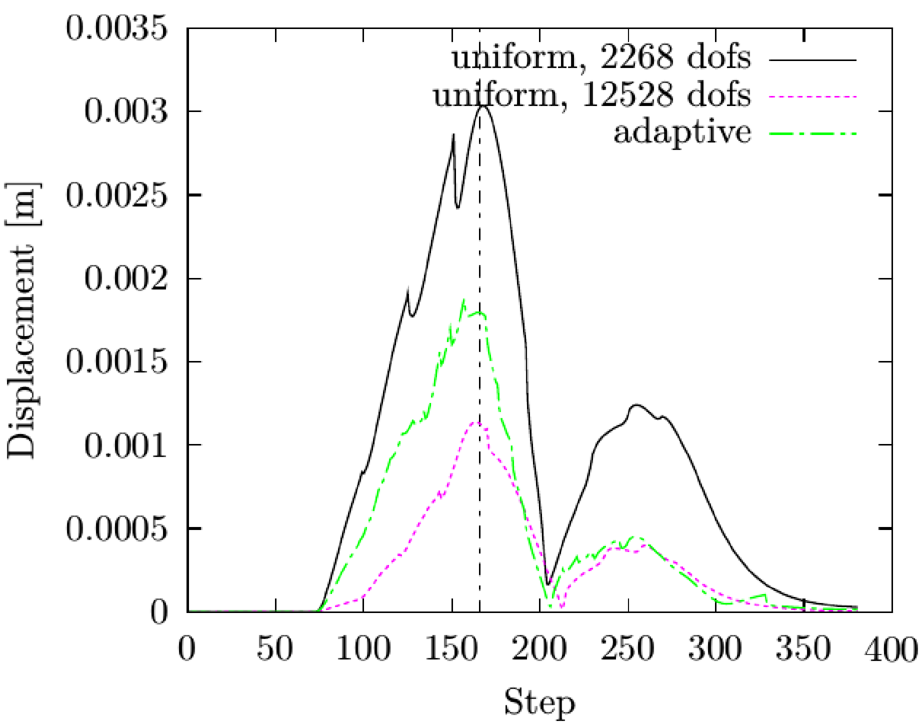


HP Bui et al, *IEEE Trans. Biomed. Eng.*, 2016.

Electrode implantation for DBS



Displacement of subthalamic nucleus (STN)



Maximum DOFs for adaptive scheme: 3 135

HP Bui et al, arXiv:1704.07636 [cs.CE]

Electrode implantation for DBS

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HP Bui et al, arXiv:1704.07636 [cs.CE]

Conclusions

- A posteriori error estimate to automatically drive local mesh refinement
- Saving computational time, make it feasible for real-time simulations
- Control the error by providing a suitable refinement criterion
- Perspectives:
  - Patient-specific simulations
  - Uncertainty quantification

Acknowledgements

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- Legato team (Luxembourg) and MIMESIS team (Strasbourg)
- SOFA community

- *Real-time Error Control for Surgical Simulation*, HP Bui et al, *IEEE Trans. Biomed. Eng.*, 2016.
- *Controlling the Error on Target Motion through Real-time Mesh Adaptation: Applications to Deep Brain Stimulation*, HP Bui et al, *arXiv:1704.07636 [cs.CE]*

Thank you!

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