A performance evaluation of weight-constrained conditioned portfolio optimization

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Outline

1. Portfolio optimization with conditioning information

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2. General formulation of the problem

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Problem context

- Discrete-time optimization
- Minimize portfolio variance for a given expected portfolio mean
- Postulate that there exists some relationship $\mu(s)$ between a signal s and each asset return r observed at the end of the investment interval:

$$r_t = \mu(s_{t-1}) + \epsilon_t,$$

with $E[\epsilon_t | s_{t-1}] = 0$.

- How do we optimally use this information in an otherwise classical (unconditional mean / unconditional variance) portfolio optimization process?

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Problem history

- Ferson and Siegel (2001): closed-form solution of unconstrained mean-variance problem using unconditional moments
- Chiang (2008): closed-form solutions to the benchmark tracking variant of the Ferson-Siegel problem
- Basu et al. (2006), Luo et al. (2008): empirical studies covering conditioned optima of portfolios of trading strategies

Possible signals

Taken from a continuous scale ranging from purely macroeconomic indices to investor sentiment indicators. Indicators taking into account investor attitude may be based on some model or calculated in an ad-hoc fashion.

Examples include

- short-term treasury bill rates (Fama and Schwert 1977);
- CBOE Market Volatility Index (VIX) (Whaley 1993) or its European equivalents (VDAX etc.);
- risk aversion indices using averaging and normalisation (UBS Investor Sentiment Index 2003) or PCA reduction (Coudert and Gex 2007) of several macroeconomic indicators;
- global risk aversion indices (GRAI) (Kumar and Persaud 2004) based on a measure of rank correlation between current returns and previous risks;
- option-based risk aversion indices (Tarashev et al. 2003);
- sentiment indicators directly obtained from surveys (e.g. University of Michigan Consumer Sentiment Index)

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   - Description
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Unconditioned expected return and variance given conditioning information

These are obtained as expectation integrals over the signal domain. If a risk-free asset with return r_t is available,

$$E(P) = E\left[u'(s)(\mu(s) - r_f 1)\right] = E\left[I_1(u, s)\right]$$

and

$$\sigma^2(P) = E\Big[u'(s)\big[(\mu(s) - r_t 1)(\mu(s) - r_t 1)' + \sigma_\epsilon^2\big]u(s)\Big] - \mu_P^2$$

$$= E\left[I_2(u, s)\right] - \mu_P^2$$

for an expected unconditional return of μ_P and a conditional covariance matrix σ_ϵ^2.

Optimal control formulation

$$
\begin{aligned}
\text{Minimize} \quad & J_{[s^-,s^+]}(x,u) = \int_{s^-}^{s^+} l_2(u,s)p_s(s)ds \text{ as } s^- \to -\infty, s^+ \to +\infty \\
\text{subject to} \quad & \dot{x}(s) = l_1(u,s)p_s(s) \; \forall s \in [s^-, s^+], \text{ with} \\
& \lim_{s \to -\infty} x(s) = x_-, \lim_{s \to +\infty} x(s) = x_+, \\
\text{and} \quad & u(s) \in U, \; \forall s \in [s^-, s^+]
\end{aligned}
$$

where $U \subseteq \mathbb{R}^n$, $x(s) \in \mathbb{R}^m$ and L as well as f are continuous and differentiable in both x and u.

Since the signal s is not necessarily bounded, the resulting control problem involves expectation integrals with infinite boundaries in the general case.

Necessity and sufficiency results

- The Pontryagin Minimum Principle (PMP) and Mangasarian sufficiency theorem are shown to continue holding if the control problem domain corresponds to the full real axis: the corresponding optimal control problems are well-posed.
- The PMP is then used to show that the given optimal control formulation of the conditioned mean-variance problem generalizes classical (Ferson and Siegel; Markowitz) problem expressions

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Aim of the study

Carry out backtests executing constrained-weight conditioned optimization strategies with different settings.

Data set

- 11 years of daily data, from January 1999 to February 2010 (2891 samples)
- Risky assets: 10 different EUR-based funds commercialized in Luxembourg chosen across asset categories (equity, fixed income) and across Morningstar style criteria
- Risk-free proxy: EURIBOR with 1 week tenor
- Signals: VDAX, volatility of bond index, PCA-based indices built using both 2 and 4 factors and estimation window sizes of 50, 100 and 200 points, Kumar and Persaud currency-based GRAI obtained using 1 month and 3 month forward rates

Benchmark problem

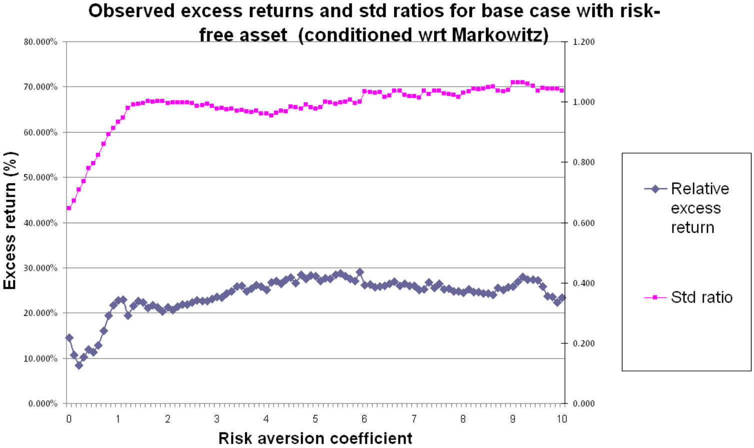
- Take VDAX index as signal, with 60 point estimation window and weights constrained to allow for long investment only
- Rebalance Markowitz-optimal portfolio alongside conditioned optimal portfolio, both with and without the availability of a risk-free proxy asset, over the 11-year period
- Assume lagged relationship $\mu(s)$ between signal and return can be represented by a linear regression
- Use kernel density estimates for signal densities
- Estimate the above using a given rolling window size (15 to 120 points)
- Use direct collocation discretisation method for numerical problem solutions

Benchmark problem (2)

- Vary the parameters to check both for robustness of strategy results and whether results can be further improved while staying with a linear regression model for the relationship between signals and returns
- Obtain efficient frontier for every date and choose portfolio based on quadratic utility functions with risk aversion coefficients between 0 and 10
- Compare sharp ratios (ex ante), additive observed returns (ex post), observed standard deviations (ex post) of both strategies
- Try different window sizes, different signal lags, weight averages over different signal points, different signals

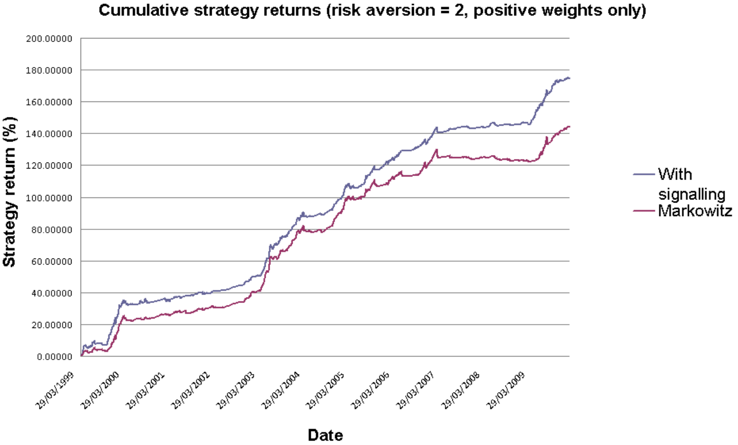
With risk-free asset

Ex post observed relative excess additive returns, standard deviation ratios



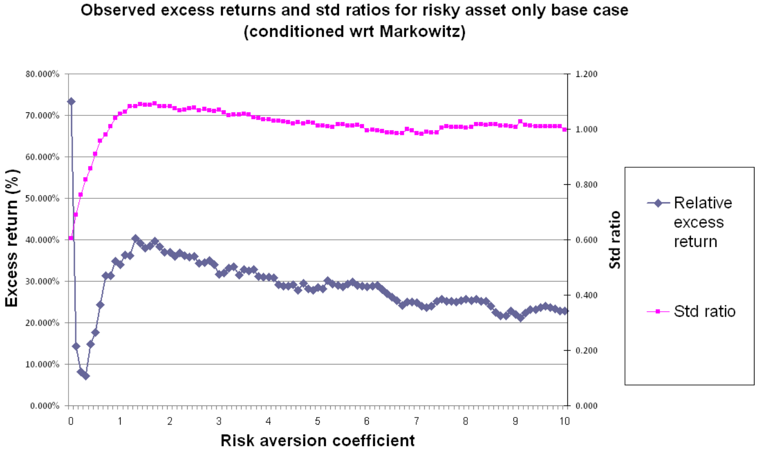
With risk-free asset

Time path of additive strategy returns for $\lambda = 2$



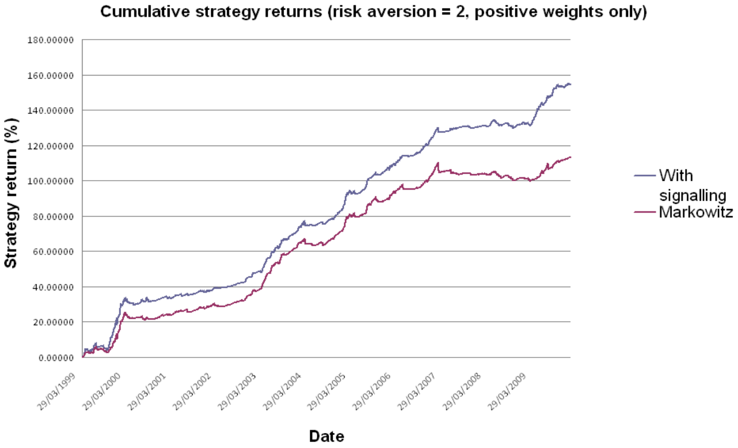
With risky assets only

Ex post observed relative excess additive returns, standard deviation ratios

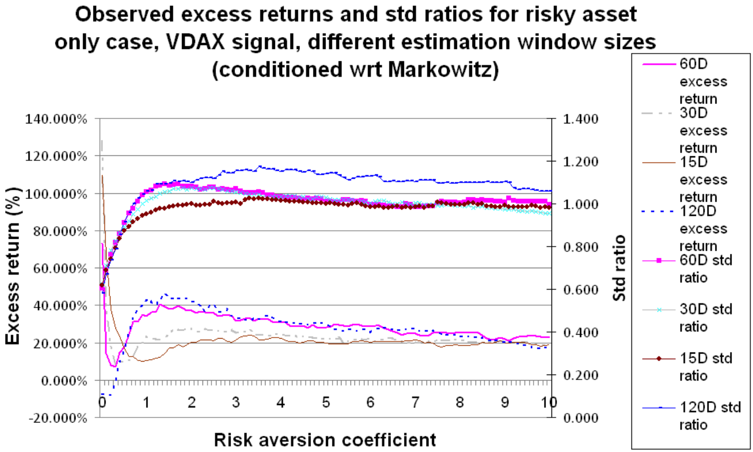


With risky assets only

Time path of additive strategy returns for $\lambda = 2$

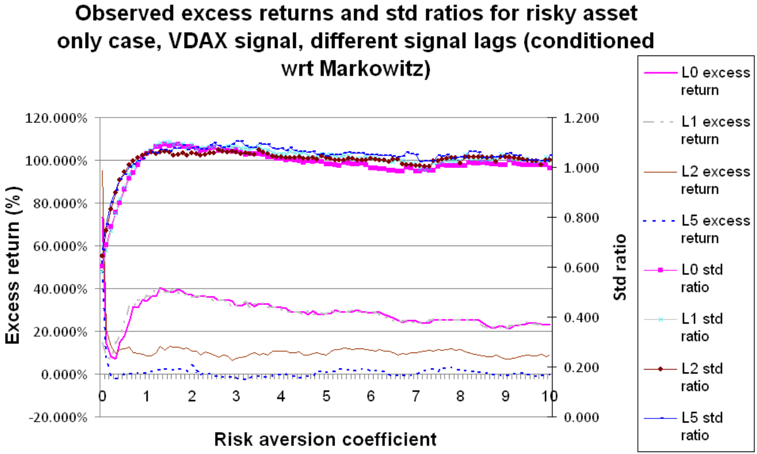


Ex post results for different estimation window sizes



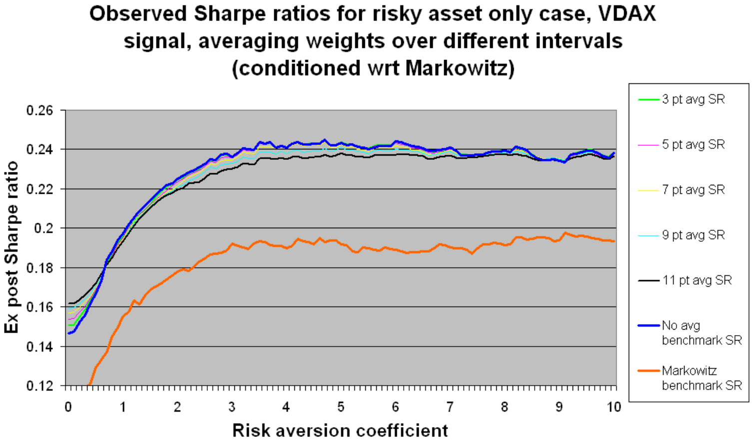
- Excess returns (and standard deviations) larger as window sizes increase
- Trade-off between statistical quality of estimates and impact of conditional nonstationarities

Ex post results for different signal lags



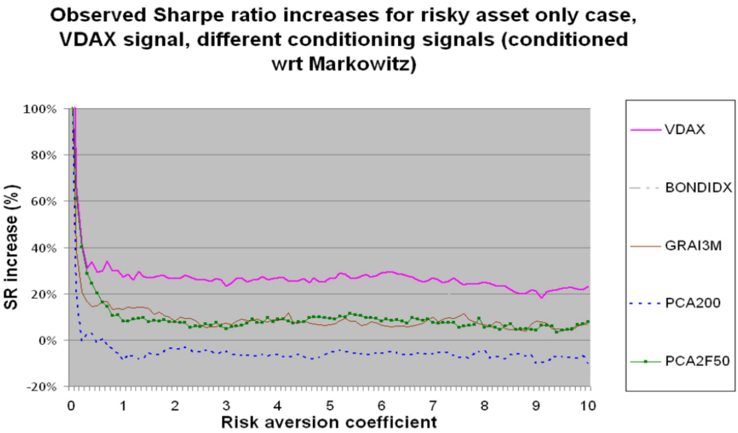
- Excess returns larger and standard deviations smaller as lag size increases
- Trade-off between statistical quality of estimators and easier modelling

Ex post results for weight averages over different number of signal points



- Negligible changes in excess returns, slight changes in standard deviations: little risk attached to signal observations

Ex post results for different signals



- Best results seen for baseline VDAX signal, averaging seems to distract from signal power