The pragmatic oddity in norm-based deontic logics

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ABSTRACT

The ideal worlds of a possible worlds semantics may satisfy both a primary obligation and an associated secondary obligation, for example the obligation to keep a promise and the obligation to apologise for not keeping it. This is known as the pragmatic oddity introduced by Prakken and Sergot. We argue that an adequate treatment of the pragmatic oddity within a norm-based semantics can be obtained, by not allowing primary and secondary obligations to aggregate, because they are obligations of a different kind. On the basis of this conceptual analysis, we introduce two logics, depending on the stance taken on the representation of normative conflicts, and we present sound and complete proof systems for these logics. We then give a formal analysis, discuss extensions, and highlight various topics for further research.

CCS CONCEPTS

•Theory of computation → Logic; •Computing methodologies → Knowledge representation and reasoning; •Applied computing → Law;

KEYWORDS

Normative reasoning, norm-based deontic logic, deontic logic, paradox

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1 INTRODUCTION

Deontic logic has proved useful to the study of legal reasoning. Modelling so-called contrary-to-duty (CTD) reasoning remains one of its main challenges. Roughly speaking, this is the problem of how to reason about norm violation. There is a large literature on this issue, with roughly two groups of approaches. Those in the first group use a possible worlds semantics[2, 3, 8, 15, 30, 31, 34]. Those in the second use what Hansen [14] calls a "norm-based semantics". The core idea is to explain the laws of deontic logic not by some set of possible worlds among which some are ideal or at least better than others, but with reference to an explicit set of norms or an existing (legal, moral, etc) standard. The semantics is based on the notion of detachment, and draws on techniques developed for non-monotonic reasoning. The use of non-monotonic techniques goes back to at least Horty [18], and it has been studied in greater depth in Nute [27] and Horty [19]. Norm-based semantics goes back to Makinson [23], and it has been further developed by Makinson and van der Torre [24, 25] and Hansen [11–13] among others. Makinson [23] contrasts two traditions of research, one on deontic logic, and the other (which has roots in Alchourrón and Bulygin [1]) on normative systems. Norm-based semantics aims at unifying the two into a single formalism.

There is widespread agreement in the literature that an adequate deontic logic should be able to handle CTD reasoning. Jones and Sergot [21] have pointed out that structures of the "contrary-to-duty" type, in which the legal consequences of the violation of some primary obligation are specified, are quite common in the law. Should a deontic logic not able to model them, it would, of course, fail in its attempt at formalising legal reasoning. Although full-blooded, sophisticated accounts of CTDs within the possible world semantics tradition are now available [2, 3, 8, 31], not much work has been carried out in order to assess how well norm-based semantics fare when it comes to CTDs. With this question in mind, we focus on the input/output (I/O) logic initially developed by Makinson and van der Torre [24], and we will look at the question of how it can handle what Prakken and Sergot [30] call the "pragmatic oddity". They introduce and discuss it with reference to frameworks with a possible world semantics. We argue that norm-based semantics in general, and I/O logic in particular, is faced with a similar problem. Our own diagnosis is that we should not aggregate primary and secondary obligations, because they are of different kind. On the basis of this conceptual analysis, we introduce two logics, depending on the stance taken on the representation of normative conflicts, and we present sound and complete proof theories for these logics. In the aforementioned paper Jones and Sergot have shown how some of the most subtle and difficult issues currently being investigated in the literature on deontic logic could arise naturally even in apparently mundane examples of law. They have shown it, by taking the example of CTDs. The notion of normative conflict, we believe, provides another good illustration of this.

This paper is organised as follows. In Section 2, we explain and discuss the pragmatic oddity. In Section 3, we present our logics. In Section 4, we discuss possible extensions, and highlight various topics for further research. Section 5 gives a summary of the paper. We include the proofs of the theorems in the main text because they are not very long nor very complex, and they will give the reader a good insight in the formal machinery.

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2 PRAGMATIC ODDITY

2.1 The problem

To our knowledge, the term "pragmatic oddity" first occurred in Prakken and Sergot [30]'s paper on CTDs. In there they study the proper representation of contrary-to-duty structures, situations in which there is a primary obligation and what they call a secondary obligation, which comes into effect when the primary obligation is violated. The term "pragmatic oddity" is introduced with reference to the formalisation in SDL [4] of a common type of CTD structures, like the following one:

Example 2.1.

(1) You should keep your promise, $\bigcirc k$.
(2) If you haven't kept your promise, you should apologise, $\neg k \to \bigcirc a$.
(3) You haven't kept your promise, $\neg k$.

They point out that in SDL both $\bigcirc k$ and $\bigcirc a$ hold, and go on to say that:

> "It is a bit odd to say that in all ideal versions of this world you keep your promise and you apologise for not keeping it. This oddity—we might call it a 'pragmatic oddity'—seems to be absent from the natural language version, which means that the SDL representation is not fully adequate." [30, p. 95]

Prakken and Sergot also point out that for examples containing a temporal element a solution is available, in the form of temporal deontic logics. They argue that the temporal solution is not always available, since sometimes the primary and CTD rule pertain to the same point in time. To support their claim, they give the example of a set of holiday cottage regulations on keeping dogs:

Example 2.2.

(1) There should be no dog, $\bigcirc\neg d$;
(2) If there is a dog, then there ought to be a warning sign, $d \to \bigcirc s$;
(3) There is a dog, d.

They make the same observation:

> "Surely, it is strange to say that in all ideal worlds there is no dog and also a warning sign that there is no dog." [30, p. 96]

This second formulation makes it clear that "odd" means "counter-intuitive". Furthermore, the explicit reference to the semantics of SDL ("in all ideal worlds ...") may sound like as if the pragmatic oddity is a semantical problem, that arises only when taking into account the specific model-theoretic meaning given to the formulas in question in SDL. It is difficult to say if Prakken and Sergot think so. Be that as it may, they develop their own solution, which consists in modifying the representation of a CTD obligation. Thus, (2) in example 2.2 is rendered as $d \Rightarrow \bigcirc_d s$, where $\Rightarrow$ denotes a suitably defined conditional operator satisfying factual detachment. The expression $\bigcirc_d s$ is intended to be read as "there is a secondary obligation that s, presupposing the sub-ideal context d". The pragmatic oddity is avoided because, once detached, $\bigcirc_d s$ does not transport up to $\bigcirc s$. The authors motivate their approach by invoking the fact that "primary and CTD obligations are obligations of a different kind: a CTD obligation pertains to, or presupposes, a certain context in which a primary obligation is already violated". [30, p. 91]. They also stress that "there is no meaningful sense [...] in which the obligation $\bigcirc s$ can be detached from the expression $\bigcirc_{\neg d} s$". [30, p. 100]

2.2 Non-monotonic methods

The main emphasis in the literature on contrary-to-duty examples is on cases where, instead of a pragmatic oddity, we have a case of conflicting obligations which has to be avoided. The sentences are intuitively consistent. Yet the logic makes them inconsistent. The problem of such intuitively coherent examples of contrary to duty structures is that it is hard to find a consistent and also otherwise acceptable formalisation. Prakken and Sergot consider the following example.

Example 2.3.

(1) There must be no fence, $\bigcirc\neg f$;
(2) If there is a fence then it must be a white fence, $f \to \bigcirc w$;
(3) There is a fence, f.

In this connection, Prakken and Sergot discuss the idea, defended by McCarty [26] among others, that CTD reasoning is just an instance of defeasible reasoning. This question arises naturally from the above example, since the main problem was how to deal with conflicting primary and secondary obligations, while one of the virtues of non-monotonic logics is that they are intended to cope with conflicting information. One possible reading is that (1) has been formulated as a defeasible rule, and that (2) takes effect in these exceptional circumstances: there is no conflict, because the two rules do not apply to the same circumstances. Another possible reading—which can be regarded as a special case of the first— is that (2) itself expresses an exception to (1): on this reading the problem of inconsistency is resolved by regarding the exceptional rule (2) as *defeating* the general rule (1) in the circumstances in which they both apply. And a natural way of formalising this reading is to adopt or adapt some suitable formalism for non-monotonic reasoning.

Prakken and Sergot argue that a drawback of the use of non-monotonic techniques is that it is difficult to distinguish this case from regular exceptions, like in the rule: if the cottage is by the sea, there may be a fence. Van der Torre and Tan [33] study this problem and distinguish many faces of defeasibility in deontic logic, distinguishing in particular between overshadowing for CTD structures, and cancelling for regular exceptions like the cottage by the sea. What is important for our present discussion is that it even if non-monotonic techniques can deal with the cottage regulations, it is less clear how they deal with the pragmatic oddity. In that sense, the pragmatic oddity is a more challenging example than the traditional CTD examples.

2.3 Requirements

Carmo and Jones [2] formulate a number of requirements that must be met by any adequate treatment of CTDs–we will endorse them all:

(R1) Consistency of the formalisation of the CTD scenario;

(R2) Logical independence;
(R3) Applicability to (at least apparently) timeless CTD examples;
(R4) Uniform representation of norms
(R5) Ability to detach (ideal and actual) obligations
(R6) Ability to avoid the pragmatic oddity
(R7) Ability to represent the fact that a violation has occurred

(R1) and (R2) are self-explanatory. (R3) is motivated by Prakken and Sergot's observation that the primary and CTD rule sometimes pertain to the same point in time. (R4) constitutes a point of disagreement between Carmo/Jones and Prakken/Sergot, and will follow the first ones rather than the second ones. According to Prakken and Sergot, a contrary-to-duty obligation and an according-to-duty obligation must be given a different representation. This is dictated by their own treatment of the pragmatic oddity. Carmo and Jones reject this point of view because it makes the representation dependant on updates: if one wants to introduce new CTDs or remove norms, one has to modify the norms already expressed.

We understand (R7) as an ability to detect violation of a norm. We add the following two requirements:

(R7') No 'drowning' effect
(R8) Ability to allow for a certain amount of agglomeration, like in Horty's example.

(R7') is linked with (R7). This requirement was suggested by M. Sergot (in private, to the first author), with reference to some deontic logics based on non-monotonic logic, like the one developed by Makinson and van der Torre in [25]. Such logics do not face the pragmatic oddity, but this comes at a high price: when a violation occurs, the primary obligation ceases to exist. For instance, in example 2.1, given $\neg k$, $\bigcirc k$ no longer holds. Reflecting on the use of logic programming techniques in legal applications, Herrestad makes the same point:

> "The non-monotonic properties of a logic program using negation-by-failure make a consistent representation [of CTDs] possible. However, the program will have certain counter-intuitive properties. For instance, violated obligations simply vanish. Nothing more can be inferred about them, as the condition for something being obligatory no longer applies. One might argue that in actual life violated obligations do not vanish."[16]

(R8) is motivated by our own treatment of the pragmatic oddity. From Prakken and Sergot, we keep the idea that primary and CTD obligations are of different kind. Our proposal is not to allow them to aggregate using the AND rule, because of this difference in nature.

$$\text{AND}\ \frac{\bigcirc x \qquad \bigcirc y}{\bigcirc (x \wedge y)}$$

However, we agree with Horty [18] that a certain degree of agglomeration should be allowed in order to account for some aspects of normative reasoning in every day life. Consider:

Example 2.4 (Horty).

(1) You ought either to fight in the army or perform alternative service, $\bigcirc(f \vee s)$;
(2) You ought not to fight in the army, $\bigcirc \neg f$.

From these two sentences one should be able to derive the conjoined obligation not to fight in the army and to perform alternative service, $\bigcirc(\neg f \wedge s)$. Obviously, this will not be possible unless one can first derive the conjoined obligation not to fight in the army and to fight in the army or do alternative service, $\bigcirc(\neg f \wedge (f \vee s))$.

3 TWO LOGICS

In this section we introduce two logics, depending on the stance taken on the representation of dilemmas or conflicts between obligations, situations where an agent ought to performs two actions that turn out to be incompatible with one another. We also present sound and complete proof theories for these logics. We show how they handle the pragmatic oddity. The basic idea is to restrict the application of the AND rule.

3.1 Background on input/output (I/O) logic

I/O logic falls within the category of what has been called "norm-based semantics" [14]. The core idea is to explain the laws of deontic logic not by some set of possible worlds among which some are ideal or at least better than others, but with reference to an explicit set of given norms or existing (legal, moral, etc) standards. The meaning of the deontic concepts is given in terms of a set of procedures yielding outputs for inputs. Detachment (or modus-ponens) is the core mechanism of the semantics being used. In I/O logic, a conditional obligation is represented as a pair (a, x) of boolean formulae, where a is the body (antecedent) and x is the head (consequent). A normative system N is a set of such pairs. Our main construct is $x \in O(N, a)$, which intuitively can be read as follows: given input a (state of affairs), x (obligation) is in the output under norms N. We also use the equivalent notation: $(a, x) \in O(N)$. The proof-theory is given in terms of inference rules manipulating pairs of Boolean formulas instead of formulas.

3.2 Semantics

We use the standard notation $(\top, x)$ for the unconditional obligation of x, where $\top$ stands for a tautology like $a \vee \neg a$. $\mathcal{L}$ is the set of all formulae of classical propositional logic. Given an input $A \subseteq \mathcal{L}$, and a normative system N, $N(A)$ denotes the image of N under A, i.e., $N(A) = \{x : (a, x) \in N \text{ for some } a \in A\}$. $Cn(A)$ denotes the set $\{x : A \vdash x\}$, where $\vdash$ is the deducibility relation used in classical propositional logic. The notation $x \dashv\vdash y$ is short for $x \vdash y$ and $y \vdash x$. We use PL as an abbreviation for (classical) propositional logic. $h(M)$ denotes the set of all the heads of the pairs in M, and $b(M)$ denotes the set of all the bodies of the pairs in M. For future reference, we recall some basic facts from PL:

Proposition 3.1.

$$\text{If } A \subseteq B \text{ then } Cn(A) \subseteq Cn(B) \qquad \text{(monotony)}$$
$$\text{If } A \subseteq B \subseteq Cn(A) \text{ then } Cn(B) \subseteq Cn(A) \qquad \text{(cumulative transitivity)}$$
$$Cn(A) = Cn(Cn(A)) \qquad \text{(idempotence)}$$

As usual, a set $A \subseteq \mathcal{L}$ of formulae is said to be consistent if $A \nvdash \bot$, and inconsistent otherwise. ($\bot$ stands for a contradiction like $x \wedge \neg x$.)

In this paper we only consider the simple minded I/O operation from Makinson and van der Torre [25]. The operation is written as O. Compared to their I/O operation, definition 4.4 has three salient features. First, definition 4.4 requires x to be equivalent to the conjunction of heads of rules in some $M \subseteq N$, rather than to be implied by such a conjunction. This has the effect of letting the rule of weakening of the output go (see example 3.3 below). Second, definition 4.4 looks at what is triggered by some $B \subseteq Cn(A)$, instead of looking at what is triggered by A. Third, definition 4.4 uses the consistency proviso ii). The last two features give a "backward-looking" flavour to our account. To determine if x is obligatory, we (so to speak) go back in time, before the violation has occurred, and we check if x was already obligatory at that point in time, in the sense of being equivalent with the conjunction of heads of rules in some $M \subseteq N$.

Definition 3.2. $x \in O(N, A)$ iff there is a finite set of norms $M \subseteq N$ and a set $B \subseteq Cn(A)$ such that $M(B) \neq \emptyset$ and

i) $x \dashv\vdash \wedge M(B)$
ii) For all $(a, x) \in M$, we have $\{a, x\} \cup B$ is consistent

Curly brackets will be omitted for singleton input set A..

In order to give the reader a taste of how the account works, we apply it to a number of examples.

Example 3.3 (Ross' paradox). Let $N = \{(\top, p)$, where p is for posting a letter, $A = \{\top\}$. p is outputted in context $\top$, viz $p \in O(N, \top)$. But $p \vee b$ is not outputted in context $\top$, viz $p \vee b \notin O(N, \top)$. Intuitively, from the obligation to post a letter, one does not derive the obligation to post a letter or burn it.

Example 3.4 (Pragmatic oddity). Let $N = \{(\top, k), (\neg k, a)\}$ and $A = \{\neg k\}$. k is outputted in context $\neg k$, viz $k \in O(N, \neg k)$. Intuitively, once violated, the primary obligation to keep one's promise still holds. Hence the drowning possible is avoided. a is also outputted in context $\neg k$, viz $a \in O(N, \neg k)$. Intuitively, the secondary obligation to apologise is detached. But the joined obligation to keep one's promise and apologise for not keeping it does not hold, viz. $k \wedge a \notin O(N, \neg k)$.

Example 3.5 (Horty). Let $N = \{(\top, f \vee s), (\top, \neg f)\}$ and $A = \{\top\}$. $s \wedge \neg f$ is outputted in context $\top$, viz $s \wedge \neg f \in O(N, \top)$. Intuitively, the joined obligation to perform an alternative military service and not go into the army is detached, as it should be.

In Table 1, we apply the account to some other well-known examples from literature. This will help the reader appreciate what is going on. The first column contains a reference to the paper in which the example was first described. The second and third column show a formalisation of the example in I/O logic, although many of them were first introduced in monadic deontic logic. The last two columns show the output. A "yes" indicates a formula that is outputted. A "no" indicates a formula that is not outputted.

Theorem 3.6 states that O is monotonic with respect to the input set.

Theorem 3.6 (Monotony w.r.t. input). *Given a set A of formulae and a formula a, we have $O(N, a) \subseteq O(N, A)$ whenever $a \in Cn(A)$.*

Proof. Assume $x \in O(N, a)$ and $a \in Cn(A)$. From the first assumption, there is some finite $M \subseteq N$ and some $B \subseteq Cn(a)$ such that $M(B) \neq \emptyset$ and

i) $x \dashv\vdash \wedge M(B)$
ii) For all $(a, x) \in M$, we have $\{a, x\} \cup B$ is consistent

From the second assumption, $\{a\} \subseteq Cn(A)$, and so $Cn(a) \subseteq Cn(A)$, by monotony for $\vdash$ and idempotence. Hence $B \subseteq Cn(A)$, which suffices for $x \in O(N, A)$. □

Table 1: Deontic benchmark examples

	N	A	yes	no
[5]	$(\top, \neg k), (k, k \wedge g)$	k	$\neg k, k \wedge g$	$\bot$
[30]	$(\top, \neg c), (k, c)$	k	$\neg c, c, \bot$	
[18]	$(\top, \neg f'), (a, f')$	a	$\neg f', f', \bot$	
[30]	$(\top, \neg f), (f, f \wedge w), (d, f)$	d	$\neg f, f, \bot$	
[37]	(r, c')	$r \wedge s$	c'	
	$(r, c'), (s, \neg c')$	$r \wedge s$	$c', \neg c'$	$\bot$
[35]	$(\top, p), (\top, \neg p)$	$\top$	$p, \neg p, \bot$	
[36]	$(\top, p)$	$\neg(p \wedge h)$	p	
	$(\top, p), (\top, h)$	$\neg(p \wedge h)$	$p, h, p \wedge h$	$p \wedge \neg h$
[31]	$(\top, \neg d), (d, d \wedge p')$ $(\top, \neg(d \wedge p'))$	d	$\neg d, d \wedge p'$, $\neg(d \wedge p')$	$\bot$

k: kill c: cigarette p: polite
g: gently d: dog h: honest
f: fence r: rain a: asparagus
w : white s: sun c': close
f': finger p': poodle

3.3 Proof theory

Definition 3.7 (Proof system). $(a, x) \in D^\star(N)$ if and only if (a, x) is derivable from N using the rules {SI, EQ, R-AGGR}.

$$\text{SI}\ \frac{(a, x) \qquad b \vdash a}{(b, x)}$$

$$\text{EQ}\ \frac{(a, x) \qquad x \dashv\vdash y}{(a, y)}$$

$$\text{R-AGGR}\ \frac{(a, x) \qquad (a, y)}{(a, x \wedge y)}\ a \wedge x \text{ and } a \wedge y \text{ are consistent}$$

Furthermore, for each leave (b, y) in the derivation, $b \wedge y$ is required to be consistent.

SI stands for "strengthening of the input". EQ stands for "equivalence". R-AGGR stands for "restricted aggregation".

Where A is a set of formulae, $(A, x) \in D^\star(N)$ means that $(a, x) \in D^\star(N)$, for some conjunction a of elements in A. Moreover, $D^\star(N, A)$ is $\{x : (A, x) \in D^\star(N)\}$.

Proposition 3.8. *Given SI, R-AGGR is equivalent to*

$$\textit{R-AGGR}'\ \frac{(a, x) \qquad (b, y)}{(a \wedge b, x \wedge y)}\ a \wedge b \wedge x \textit{ and } a \wedge b \wedge y \textit{ are consistent}$$

Proof.

- R-AGGR $\Rightarrow$ R-AGGR$'$. The last step in the derivation below goes through, because $a \wedge b \wedge x$ and $a \wedge b \wedge y$ are assumed to be consistent.

$$\text{R-AGGR}\ \frac{\text{SI}\ \dfrac{(a, x)}{(a \wedge b, x)} \qquad \text{SI}\ \dfrac{(b, y)}{(a \wedge b, y)}}{(a \wedge b, x \wedge y)}$$

- R-AGGR$'$ $\Rightarrow$ R-AGGR. The first step in the derivation below goes through, because $a \wedge a \wedge x$ and $a \wedge a \wedge y$ are equivalent with $a \wedge x$ and $a \wedge y$, respectively.

$$\text{SI}\ \dfrac{\text{R-AGGR}'\ \dfrac{(a, x) \qquad (a, y)}{(a \wedge a, x \wedge y)}}{(a, x \wedge y)}$$

□

THEOREM 3.9. *O validates the rules of D (for input a).*

PROOF. For SI and EQ, the argument is straightforward, and omitted. We show R-AGGR. Assume that $x \in O(N, a)$ and $y \in O(N, a)$, and that each of $a \wedge x$ and $a \wedge y$ is consistent. From the first hypothesis, there is a finite $M_1 \subseteq N$ and a set $B_1 \subseteq Cn(a)$ such that $M_1(B_1) \neq \emptyset$ and

i) $x \dashv\vdash \wedge M_1(B_1)$
ii) For all $(b, y) \in M_1$, we have $\{b, y\} \cup B_1$ is consistent.

From the second hypothesis, there is a finite $M_2 \subseteq N$ and a set $B_2 \subseteq Cn(a)$ such that $M_2(B_2) \neq \emptyset$ and

i) $x \dashv\vdash \wedge M_2(B_2)$
ii) For all $(b, y) \in M_2$, we have $\{b, y\} \cup B_2$ is consistent.

Define $M_1^- = \{(c, z) \in M_1 : c \in B_1\}$ and $M_2^- = \{(c, z) \in M_2 : c \in B_2\}$. Intuitively, M_1^- is M_1 "stripped of" all the pairs that are not triggered by B_1, and M_2^- is M_2 stripped of all the pairs that are not triggered by B_2. By construction, $M_1^-(B_1) = M_1(B_1)$ and $M_2^-(B_2) = M_2(B_2)$. So $x \dashv\vdash \wedge M_1^-(B_1)$ and $y \dashv\vdash \wedge M_2^-(B_2)$. Put $M_3 = M_1^- \cup M_2^-$ and $B_3 = B_1 \cup B_2$. M_3 is finite. We have $M_3 \subseteq N$ and $B_3 \subseteq Cn(a)$. Furthermore, $M_3(B_3) = M_1^-(B_1) \cup M_2^-(B_2)$, so that

$$x \wedge y \dashv\vdash \wedge M_1^-(B_1) \wedge (\wedge M_2^-(B_2)) \dashv\vdash \wedge M_3(B_3)$$

It remains to verify that M_3 and B_3 meet condition ii) in definition 4.4. Let $(b, y) \in M_3$. Note that each of $\{a\} \cup M_1^-(B_1)$ and $\{a\} \cup M_2^-(B_2)$ is consistent, since each of $a \wedge x$ and $a \wedge y$ is consistent. Now, assume, to reach a contradiction, that $\{b, y\} \cup B_3$ is inconsistent, viz. $\{b, y\} \cup B_3 \vdash \bot$. By monotony for $\vdash$,

$$\{a, b, y\} \cup B_3 \vdash \bot \tag{1}$$

But either $(b, y) \in M_1^-$ or $(b, y) \in M_2^-$, so that either $b \in B_1$ or $b \in B_2$, by construction. Hence, $a \vdash b$, since $B_3 = B_1 \cup B_2 \subseteq Cn(a)$. By cut for $\vdash$,

$$\{a, y\} \cup B_3 \vdash \bot \tag{2}$$

Since $B_3 \subseteq Cn(a)$, $a \vdash b'$ for all $b' \in B_3$. By cut again,

$$\{a, y\} \vdash \bot \tag{3}$$

But either $y \in M_1^-(B_1)$ or $y \in M_2^-(B_2)$. So by monotony for $\vdash$, either $\{a\} \cup M_1^-(B_1) \vdash \bot$ or $\{a\} \cup M_2^-(B_2) \vdash \bot$. Contradiction. □

THEOREM 3.10 (SOUNDNESS). $D^\star(N, A) \subseteq O(N, A)$.

PROOF. The proof is on the length of the derivation, using Theorem 3.6 and Theorem 3.9. We only run the verifications for the base case, because it explains the consistency check run on the leaves of a derivation. Given: $(a, x) \in N$; $a \wedge x$ is consistent. To show: $x \in O(N, a)$. M is $\{(a, x)\}$, and B is such that $a \in B \subseteq Cn(a)$. $M(B) = \{x\}$. Because $a \wedge x$ is consistent, $\{a \wedge x\} \cup B$ is consistent. □

THEOREM 3.11 (COMPLETENESS). $O(N, A) \subseteq D^\star(N, A)$

PROOF. Assume $x \in O(N, A)$. Hence there is a finite $M \subseteq N$ and some $B \subseteq Cn(A)$ such that $M(B) \neq \emptyset$ and

i) $x \dashv\vdash \wedge M(B)$
ii) For all $(b, y) \in M$, $\{b, y\} \cup B$ is consistent.

Define $M^- = \{(b, y) \in M : b \in B\}$. We have $x \dashv\vdash \wedge M^-(B)$. Let $(b_1, y_1), ..., (b_n, y_n)$ be an enumeration of all the elements of M^-. By definition of M^-, $\{b_1, ..., b_n\} \subseteq B$. By ii), for all i such that $1 \leq i \leq n$, $y_i \wedge b_1 \wedge .. \wedge b_n$ is consistent. Thus, for all i such that $1 \leq i \leq n$, $y_i \wedge b_i$ is a fortiori consistent. Since $B \subseteq Cn(A)$, $a^\star \vdash b_1 \wedge ... \wedge b_n$, where $a^\star$ is a conjunction of elements in A. A derivation of (A, x) from M^-, and hence from N, is shown below:

$$\text{SI}\ \dfrac{\dfrac{\dfrac{(b_1, y_1) \qquad \ldots\ldots \qquad (b_n, y_n)}{(b_1 \wedge ... \wedge b_n, y_1 \wedge ... \wedge y_n)}\ \text{R-AGGR}'}{(b_1 \wedge ... \wedge b_n, x)}\ \text{EQ}}{(a^\star, x)}$$

This is a derivation of (A, x), as $a^\star$ is a conjunction of elements in A. □

3.4 Normative conflicts

The logic described in the previous section has been tailored for CTDs. Another important issue in deontic logic is the representation of normative conflicts, situations where an agent ought to performs two actions that turn out to be incompatible with one another. These two issues should not be confused. In this section, we introduce a second logic, which is meant to accommodate normative conflicts as well.

For the purpose of this paper, we take the notion of normative conflict in its narrowest sense as suggested by, e.g., Kelsen [22]. We shall assume the instances of normative conflicts are identified through the so-called impossibility-of-joint-compliance test. There is a conflict when it is not possible for a norm subject to comply with two obligations. The notion of normative conflict can also be taken in a broader sense, to cover prohibitions and permissions, or legal powers [17]. But this broader sense goes beyond the expressive power of our formal apparatus, and must be left as a topic for future research.

Jones and Sergot [21] have shown how some of the most subtle and difficult issues currently being investigated in the literature on deontic logic could arise naturally even in apparently mundane examples of law. They have shown it, by taking the example of CTDs. The notion of normative conflict, we believe, provides another good illustration of this. Tenants of so-called legal pluralism [10] would say that a legal order can contain multiple rules of recognition that lead to the order containing multiple, unranked, legal sources. These rules of recognition are inconsistent, and there is the possibility that they will, in turn, identify inconsistent rules addressed to individuals. In addition, pluralist orders lack a legal mechanism able to resolve the inconsistency. Now, the question of how to accommodate the existence of conflicts is not an easy one to answer, as witnessed by the large literature devoted to it in deontic logic. We refer the reader to Goble [6]'s overview chapter for a critical analysis of the different options that have been explored by deontic logicians. In there available formal frameworks

are thoroughly tested against three main requirements, which we will endorse here:

(R9) Conflicting obligations should be consistent:
$$\{\bigcirc x, \bigcirc \neg x\} \nvdash \bot$$
(R10) No deontic explosion:
$$\{\bigcirc x, \bigcirc \neg x\} \nvdash \bigcirc y$$
(R11) Ability to account for the validity of seemingly valid inference patterns, like the one exhibited in Horty's example, example 3.5:
$$\{\bigcirc \neg x, \bigcirc (x \vee y)\} \vdash \bigcirc (\neg x \wedge y)$$

It is not difficult to see that the account described in the previous section meets the last two requirements, but not the first one. This is due to the consistency proviso being used:

$$\text{R-AGGR}\ \frac{(a,x) \qquad (a,\neg x)}{(a,\bot)}$$

We now introduce a second logic that also meets these three requirements all together. The operation is written as $O^\star$. The basic idea is to strengthen the consistency check being used when calculating the output: all the norms in M must be taken into account collectively. On the syntactical side, R-AGGR is replaced with

$$\text{R-AGGR}^\star\ \frac{(a,x) \qquad (a,y)}{(a, x \wedge y)}\ a \wedge x \wedge y \text{ is consistent}$$

We give the formal details below.

Definition 3.12. $x \in O^\star(N, A)$ iff there is a (finite) set of norms $M \subseteq N$ and a set $B \subseteq Cn(A)$ such that $M(B) \neq \emptyset$ and

i) $x \dashv\vdash \wedge M(B)$
ii) $b(M) \cup h(M) \cup B$ is consistent

Proposition 3.13. *$O^\star(N, A) \subseteq O(N, A)$.*

Proof. This follows from the fact that, if the consistency proviso ii) in definition 4.5 is met, then so is the consistency provision ii) in definition 4.4. □

Note that the analogue of theorem 3.6 (monotony w.r.t. input) holds.

The reader may easily verify that in example 3.5 definition 4.5 yields the same result as definition 4.4.

Example 3.14 (Dilemmas, cont'd). Let $N = \{(a, x), (a, \neg x)\}$ and $A = \{a\}$. We have $x \in O^\star(N, a)$. Witness: $M = \{(\top, a)\}$ and $B = \{a\}$. We also have $\neg x \in O^\star(N, a)$. Witness: $M = \{(a, \neg x)\}$ and $B = \{a\}$. But $x \wedge \neg x \notin O^\star(N, a)$. For $x \wedge \neg x$ to be outputted, M must be N and B must be such that $a \in B \subseteq Cn(a)$. But $\{a, x, \neg x\} \cup B$ is inconsistent.

Definition 3.15 (Proof system). $(a, x) \in D^\star(N)$ if and only if (a, x) is derivable from N using the rules {SI, EQ, R-AGGR*}

$$\text{R-AGGR}^\star\ \frac{(a,x) \qquad (a,y)}{(a, x \wedge y)}\ a \wedge x \wedge y \text{ is consistent}$$

For each leave (b, y) of the derivation, $b \wedge y$ is required to be consistent.

The analogue of proposition 3.8 holds:

Proposition 3.16. *Given SI, R-AGGR* is equivalent to*

$$\textit{R-AGGR}^\star_{-}\ \frac{(a,x) \qquad (b,y)}{(a \wedge b, x \wedge y)}\ a \wedge b \wedge x \wedge y \textit{ is consistent}$$

Theorem 3.17. *$O^\star$ validates the rules of $D^\star$ (for input a).*

Proof. For SI and EQ, the argument is straightforward, and omitted. For R-AGGR*, we only need run through the proof for R-AGGR again, and verify that M_3 and B_3 meet condition ii) in definition 4.5. Note that $a \wedge (\wedge M_3(B_3))$ is consistent, because $a \wedge x \wedge y$ is assumed to be consistent. Now, assume, to reach a contraction, that $b(M_3) \cup h(M_3) \cup B_3 \vdash \bot$. By monotony for $\vdash$, $\{a\} \cup b(M_3) \cup h(M_3) \cup B_3 \vdash \bot$. Since $b(M_3) \subseteq B_3 \subseteq Cn(a)$, by cut $\{a\} \cup h(M_3) \vdash \bot$. But $h(M_3) = M_3(B_3)$, and so $a \wedge (\wedge M_3(B_3))$ is inconsistent. Contradiction. □

Theorem 3.18 (Soundness). *$D^\star(N, A) \subseteq O^\star(N, A)$*

Proof. The argument is virtually the same as for theorem 3.10. □

Theorem 3.19 (Completeness). *$O^\star(N, A) \subseteq D^\star(N, A)$*

Proof. We only need run through the proof of theorem , and check that R-AGGR$^\star_{-}$ can be applied where R-AGGR$'$ was applied. From the opening hypothesis, $b(M) \cup h(M) \cup B$ is consistent. It a fortiori follows that $b(M^-) \cup h(M^-)$, and hence $b_1 \wedge .. \wedge b_n \wedge y_1 \wedge \ldots \wedge y_n$, is consistent. □

3.5 Evaluation and formal analysis

In traditional input/output logic, a distinction between unconstrained and constrained output is made. This is useful for the evaluation, so we do the same here. We call the corresponding operator O^- where the minus symbol reflects the absence of the constraint. Note that due to the absence of the constraint, more obligations are derived.

Definition 3.20. $x \in O^-(N, A)$ iff there is a (finite) set of norms $M \subseteq N$ and a set $B \subseteq Cn(A)$ such that $M(B) \neq \emptyset$ and $x \dashv\vdash \wedge M(B)$.

The following result can be derived from results of Parent and van der Torre [29].

Theorem 3.21. *$O^-(N, A)$ is completely characterized by the three rules of strengthening of the input, replacements of logical equivalents in the output, and unrestricted aggregation.*

Table 2 and 3 list some properties, which are motivated and discussed by Parent and van der Torre [28, 29]. We only give a brief explanation here, and refer to these two papers for a more extensive discussion.

In Table 2, exact factual detachment (EFD) and violation detection (VD) characterise what is special about *deontic* logic, while substitution (SUB), replacements of logical equivalents (RLE), implication (IMP) and paraconsistency (PC) say something about *logic*. We use the notation $x[\sigma]$ to denote a substitution instance of x. Thus, $x[\sigma]$ is obtained from x by replacing uniformly, in x, all occurrences of a propositional letter by the same propositional formula. $A[\sigma]$ and $N[\sigma]$ extend the notion of substitution instance to sets of formulae, and sets of norms in the straightforward way. We write $N \approx M$ whenever M is obtained from N, by replacing each $(b, y) \in N$ with some (c, z) such that b is equivalent with c, and y is equivalent with z. Implication makes use of the so-called

Table 2: Properties [28]

EFD	$(x,y) \in N \Rightarrow y \in O(N,x)$
VD	$(A,y) \in O(N) \Rightarrow (A \cup \{\neg y\}, y) \in O(N)$
SUB	$x \in O(N,A) \Rightarrow x[\sigma] \in O(N[\sigma], A[\sigma])$
RLE	$N \approx M \Rightarrow O(N) \subseteq O(M)$
IMP	$O(N,A) \subseteq Cn(m(N) \cup A)$
PC	$x \in \overline{V}(N,A) \Rightarrow \exists M \subseteq N : x \in O(M,A)$ and $O(M,A) \cup A$ consistent
NM	$O(N) \subseteq O(N \cup M)$
NI	$M \subseteq O(N) \Rightarrow O(N) = O(N \cup M)$

materialisation $m(N)$ of a normative system N, which means that each norm (a,x) is interpreted as a material conditional $a \to x$, i.e. as the propositional sentence $\neg a \vee x$. We distinguish between violated obligations $V(N,A) = \{x \in O(N,A) \mid \neg x \in Cn(A)\}$ and non-violations (or actual obligations) $\overline{V}(N,A) = O(N,A) \setminus V(N,A)$. Moreover, norm monotony (NM) and norm induction (NI) are called "norm change properties", because the normative system N is no longer held constant.

THEOREM 3.22. *The input/output logics O and O^* satisfy the properties in Table 2, except for* SUB *and* NI.

Proof (sketch). EFD *holds due to the consistency constraint on norms requiring for all $(a,x) \in N$ that $a \wedge x$ is consistent in propositional logic,* VD *holds due to monotonicity with respect to input (Th 3.6),* RLE *follows immediately from SI and EQ,* IMP *holds already for the unconstrained operator O^-, and* PC *and* NM *are immediate from the definition of O. A counterexample for* SUB*: $N = \{(a,x)\}$, replace x by $\neg a$. Counterexample* NI*: $(a \wedge \neg x, x) \in O(N)$, but we cannot add $(a \wedge \neg x, x)$ to N.*

The most interesting cases are the properties that do not hold. Lack of substitution may be surprising at first sight, though it is common for non-monotonic logic. Moreover, we can define restricted versions of SUB and NI that do hold. We consider here only NI. We say that M is consistent when each (a,x) in M is consistent, and (a,x) is consistent iff $a \wedge x$ is consistent in propositional logic.

R-NI consistent $M \subseteq O(N) \Rightarrow O(N) = O(N \cup M)$

R-NI is an important property. Together, exact factual detachment, norm monotony and norm induction are equivalent to requiring that $O(N)$ is a closure operator. Though NI does not hold, R-NI is strong enough to prove our completeness results in Section 3. If NI fails completely, then no such completeness result would be possible. We give an example in Section 4.2.

Moreover, the reusability properties in Table 3 relate the system to traditional I/O logic: consequence (CN), inclusion in reusable output (IO), redundancy (R) and strong redundancy (SR). Their formulation appeals to some key notions of so-called constrained input/output logic, developed by Makinson and van der Torre [2001] in order to reason about norm violation. Following Parent and van der Torre [28, 29], we only consider the input/output constraint. One of the distinguishing properties of traditional constrained input/output logic is that it leads to a set of extensions, where each extension is a set of formulas.

$$out(N,A) = Cn(N(Cn(A)))$$
$$conf(N,A) = \{N' \subseteq N \mid out(N',A) \cup A \text{ consistent}\}$$
$$maxf(N,A) = \{N' \in conf(N,A) \mid N' \subseteq \text{-maximal}\}$$
$$outf(N,A) = \{O(N.A) \mid N' \in maxf(N,A)\}$$

Table 3: Properties [28, 29]

CN	$out(N,A) = Cn(O(N,A))$
IO	$O(N) \subseteq O^-(N) \subseteq out(N)$
R	$outf(N,A) = outf(O(N),A)$
SR	$outf(N \cup M, A) = outf(O(N) \cup M, A)$

THEOREM 3.23. *The input/output logics O and O^* satisfy the properties in Table 3.*

Proof (sketch). Note that N can be assumed to be consistent. CN *and* IO *can be shown by inspecting the semantic conditions, and* R *and* SR *can be shown by structural induction on $O(N)$.*

We end the evaluation with a final observation. It makes use again of $\overline{V}(N,A)$ representing the actual obligations in circumstances A. Though aggregation is restricted, it says that actual obligations are closed under aggregation. Thus, it is only for obligations of distinct contexts, such as primary and secondary obligations, that aggregation is restricted.

THEOREM 3.24. *If $p,q \in \overline{V}(N,A)$, then $p \wedge q \in \overline{V}(N,A)$.*

We finally note that Jones and Porn [20] introduced the distinction between actual and ideal obligations, and their analysis has played a major role in the history of deontic logic, including the work of Prakken and Sergot and the work of Carmo and Jones on the pragmatic oddity. A more detailed comparison is left to further research.

4 EXTENSIONS

In the previous section we introduced the weakest logics of normative systems. As done in the traditional input/output logic framework, the minimal system can be extended to handle for example reasoning by cases and deontic detachment. In this section we consider such extensions.

4.1 Adding consequential closure

It seems most straightforward to extend the semantics with consequential closure, as follows.

Definition 4.1. $x \in O_1(N,A)$ iff there is a (finite) set of norms $M \subseteq N$ and a set $B \subseteq Cn(A)$ such that $M(B) \neq \emptyset$ and

i) $x \in Cn(M(B))$
ii) For all $(a,x) \in M$, we have $\{a,x\} \cup B$ is consistent

Definition 4.2. $x \in O_1^{\star}(N,A)$ iff there is a (finite) set of norms $M \subseteq N$ and a set $B \subseteq Cn(A)$ such that $M(B) \neq \emptyset$ and

i) $x \in Cn(M(B))$
ii) $b(M) \cup h(M) \cup B$ is consistent

THEOREM 4.3. *These two input/output operations satisfy all properties in Table 1 and 2, except for* SUB *and* NI. *Moreover, they do not satisfy* R-NI.

Proof. The proof of the properties is analogous to the proof of the properties for the operators without consequential closure. The main difference is R-NI. *We give a counterexample. Let* $N = \{(\top, p \wedge q), (\neg q, r)\}$. *We have* $(\top, p) \in O_1(N)$ *and* $(\neg q, p \wedge r) \notin O_1(N)$. *Moreover, we have* $(\neg q, p \wedge r) \in O_1(N \cup \{(\top, p)\})$.

This has far reaching consequences. At first sight it may seem that for the proof system of this I/O operation we just have to add the following rule of weakening of the output (WO) to the above set of proof rules of O.

$$\frac{(a, x \wedge y)}{(a, x)} WO$$

However, if we would add WO, we would derive unsound consequences. For example, if we write D_1 for D plus WO, then we would have $(\neg q, p \wedge r) \in D_1(N)$ in the example in the proof. In general we have that $O_1(N)$ and $O_1^*(N)$ are not consequence operators, and thus we cannot axiomatize it in the way we axiomitized O and O^*.

4.2 Reasoning by cases

Reasoning by cases can easily be implemented by adding the disjunction rule:

$$\frac{(a, x), (b, x)}{(a \vee b, x)}$$

The challenge is to find a suitable semantics. We conjecture it means generating cases in the semantics in the following way, where a complete set B is a maxi-consistent set, that is, it is consistent and each strict extension is inconsistent.

Definition 4.4. $x \in O_2(N, A)$ iff for all complete B containing A, we have $x \in O_1(N, B)$.

Definition 4.5. $x \in O_2^\star(N, A)$ iff for all complete B containing A, we have $x \in O_1^\star(N, B)$.

4.3 Deontic detachment

Deontic detachment is more difficult to implement in the semantics, but it can easily be realized in the proof theory. As argued by Parent and van der Torre [29], the following aggregative version of deontic detachment could be adopted.

$$ACT \frac{(a, x), (a \wedge x, y)}{(a, x \wedge y)} \text{ if } a \wedge x \wedge y \text{ are consistent}$$

The challenge is to define a semantics for it. We note here that this approach does not suffer from Stolpe's so-called irrelevance obligation problem, which derives from $(\top, a)$ and (b, c) the obligation for c in the context $\neg a$. For example, the consistency constraint blocks the following derivation.

$$\dfrac{\text{SI}\,\dfrac{(\top, a)}{(\neg a, a)} \qquad (b, c)}{(\neg a, a \wedge c)}\ \text{ACT}$$

5 MODAL INPUT/OUTPUT LOGIC

A natural next step is to change the base logic from PL to the modal logic KD. This has the advantage that we can represent not only regulative norms (with $O\alpha$ in head), but also permissive norms (with $P\alpha$ in head), and we can distinguish regulative norms from constitutive ones (with factual sentence α in head). Moreover, this move would give us the required expressive power to talk and reason about conditional norms having a deontic formula as antecedent, as in Governatori's paradox [7]. it contains a conditional norm whose body and head are permissions: "the collection of medical information is permitted provided that the collection of personal information is permitted." The above is a topic for future research, as there are also various challenges. In this section, we highlight the challenge that, in order to handle the pragmatic oddity, one would need to run the consistency check in modal logic KT, obtained by replacing the D axiom $\bigcirc x \rightarrow Px$ with the axiom $\bigcirc x \rightarrow x$. This would lead to the following definition:

Definition 5.1. $x \in O(N, A)$ iff there is a finite set of norms $M \subseteq N$ and a set $B \subseteq Cn_{KD}(A)$ such that $M(B) \neq \emptyset$ and:

i) $x \dashv\vdash_{KD} \wedge M(B)$
ii) For all $(a, x) \in M$, we have $\{a, x\} \cup B$ is consistent in KT

The following example illustrates the modal input/output logic.

Example 5.2 (Pragmatic oddity, ct'd). Consider the modal norms $N = \{(\top, \bigcirc k), (\neg k, \bigcirc a)\}$ and $A = \{\neg k\}$. Put $M = N$ and $B = \{\top, \neg k\}$. We have $\bigcirc(k \wedge a) \dashv\vdash_{KD} \wedge M(B)$. Suppose the consistency check ii) is run in KD. $\{\top, \bigcirc k, \neg k\}$ is consistent in KD, so that $\bigcirc(k \wedge a) \in O(N, A)$. Suppose the consistency check ii) is run in KT. $\{\top, \bigcirc k, \neg k\}$ is not consistent in KT, and so $\bigcirc(k \wedge a) \notin O(N, A)$.

6 FURTHER RESEARCH

We believe that there are still new questions and challenges regarding contrary to duty reasoning. We mention three of these challenges in this section.

6.1 Not so odd

We first give an example from the well studied library regulations [21], where the pragmatic oddity does *not* sound odd.

Example 6.1.

(1) Bring back the books within two weeks, $\bigcirc b$.
(2) If you haven't brought back the books within two weeks, pay a penalty, $\neg b \rightarrow \bigcirc p$
(3) You haven't brought back the books within two weeks, $\neg b$.

The derivation of the obligation to bring back the goods and pay a penalty $\bigcirc(b \wedge p)$ does not sound odd at all.

This is a particular property of the library regulations, and it also holds in many examples of commerce—but not always. For example, suppose you need to deliver a wedding cake, then once the obligation is violated, there is no longer the need to deliver it. Thus, the challenge is to formally distinguish the library regulation example from the other examples of pragmatic oddities discussed in this paper. We believe that we should make time and in particular deadlines explicit to formally distinguish these cases.

6.2 Sanctions

We may define sanctions as obligations conditional on a violation: $\neg\alpha \wedge \bigcirc\alpha \rightarrow \bigcirc\beta$. For example, in the library example just above, the second line could be formalised as a sanction:

(2) If you haven't brought back the books within two weeks but you should have, pay a penalty, $\neg b \wedge \bigcirc b \rightarrow \bigcirc p$.

Also, in the pragmatic oddity, the apology could be represented as a sanction, i.e. $\neg k \wedge \bigcirc k \rightarrow \bigcirc a$.

This class of contrary to duty examples can be studied in modal input/output logic.

6.3 Fluents

Fluent are properties of the world which can change over time, and are sometimes distinguished from events and actions. A typical example discussed in the deontic logic literature is the cooler of a fence in the cottage regulations. The challenge with fluents in the context of contrary to duty reasoning arises in the context of a violation. For example, suppose we have the rules that there should be no fence, but if there is a fence it should be white. Moreover, assume there is a black fence. We can derive two conflicting instructions:

(1) Remove the fence
(2) Paint the fence white

This becomes even more challenging for the rarely discussed multi level obligations, where besides a primary and a secondary obligation there are also tertiary and higher level obligations. Consider again an example by Prakken and Sergot:

(1) You should not go on the road with your bike
(2) If you're on the road, you should be staying on the extreme right side of the road
(3) If you're not on the extreme right side of the road, you should be staying on the extreme left side of the road.

If, for one reason or the other, you are in the middle of the road, (1) tells you should not be on the road, (2) tells you should keep your right, and (3) that you should keep your left. We are convinced that no parent will give his or her child this contradictory advice. The imperatives are extremely confusing, and the child will not be able to decide what to do based on these commands.

7 SUMMARY

The pragmatic oddity has been presented by Prakken and Sergot as a problem for the possible worlds semantics of deontic logic: what does it mean to say that the ideal worlds satisfy both that you keep your promise (k), and that you apologise (a) for not keeping it? This may suggest that the pragmatic oddity does not occur in norm-based semantics, as we do not have ideal worlds. However, as we argue in this paper, in norm-based semantics we can define the pragmatic oddity as the derivation of the obligation $\bigcirc(k \wedge a)$.

In addition, we argue that both $\bigcirc(k)$ and $\bigcirc(a)$ should be detached, the first to represent that $\neg k$ is a violation, the second to represent the cue for action to apologise. The problem of the pragmatic oddity is thus turned into a problem of aggregation. This reflects that the aggregation rule should not combine primary and secondary obligations. They refer to different contexts, sometimes called the ideal and actual obligations.

We present two logics, one in the tradition of standard deontic logic in which dilemmas are inconsistent, as represented by the so-called deontic D axiom $\neg(\bigcirc p \wedge \bigcirc\neg p)$, and one in which such dilemmas can be represented in a consistent way. We prove completeness, give an evaluation by a list of formal properties, and we sketch how to extend the logics with other reasoning patterns such as reasoning by cases and deontic detachment.

A particularly interesting case is the addition of consequential closure. The input/output logics of Makinson and van der Torre satisfy this rule, to stay as close as possible to classical logic. Carmo and Jones reject it in their analysis of contrary-to-duty reasoning, following Chellas. Following Stolpe [32] also several input/output logics without consequential closure have been defined. As we show in this paper, constrained output with consequential closure does not satisfy norm induction, and thus $O(N)$ is not a closure operator. This explains why Makinson and van der Torre only provide a proof system for their unconstrained logics, but not for the constrained ones: it is impossible. We believe this provides another argument for the adoption of deontic logics without consequential closure.

We have listed several topics of further research, such as input/output logics which have a modal logic as their base logic, and new challenges regarding contrary-to-duty reasoning. Moreover, an important question is whether the ways of handling the pragmatic oddity in possible worlds semantics and in norm-based semantics are related. Here the analogy of violated obligation and actual obligation with ideal and actual obligations may be useful.

Finally, we have shown that the pragmatic oddity as an aggregation problem comes down to separating the primary and secondary obligations. Governatori and Rotolo [9] have introduced a logic where this is explicit in the syntax. How such an approach could be incorporated in the norm-based semantics is another topic for further research.

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