



Towards a seamless Integration of CAD and Simulation

a posteriori error estimation

Multi-scale fracture and model order reduction Pierre Kerfriden, Lars Beex, Jack Hale, Olivier Goury, Daniel Alves Paladim, Elisa Schenone, Davide Baroli, Thanh Tung Nguyen

Advanced discretisation techniques Danas Sutula, Xuan Peng, Haojie Lian, Peng Yu, Qingyuan Hu, Sundararajan Natarajan, Nguyen-Vinh Phu

Error estimation Pierre Kerfriden, Satyendra Tomar, Daniel Alves Paladim, Andrés Gonzalez Estrada

Biomechanics applications Alexandre Bilger, Hadrien Courtecuisse, Bui Huu Phuoc

and all the others!

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Organised by Gernot Beer & Stéphane Bordas

ERROR ESTIMATION

Reality

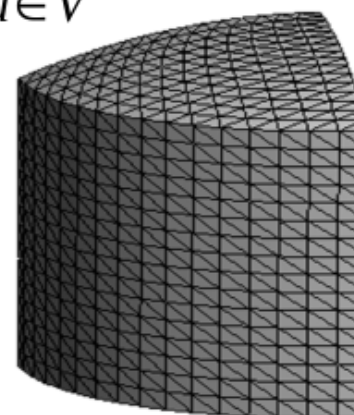


Mathematical model

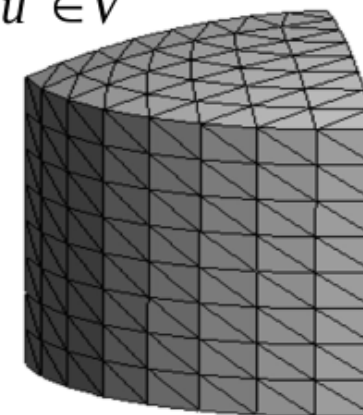
Truth
(Continuum)
 $u \in \tilde{V}$



Refined
(Reference)
 $\hat{u} \in \hat{V}$



Coarse
 $u^h \in V^h$



Model
error

Discretization
error 1

Discretization
error 2

Weak form

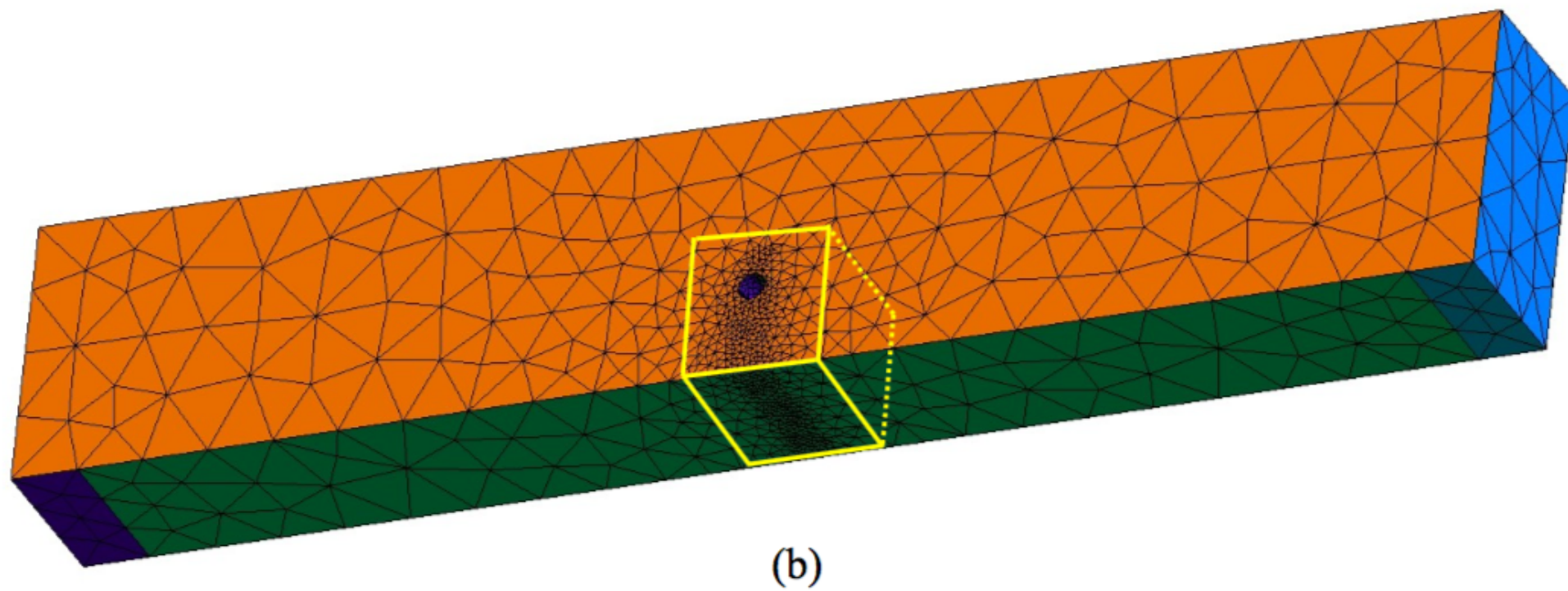
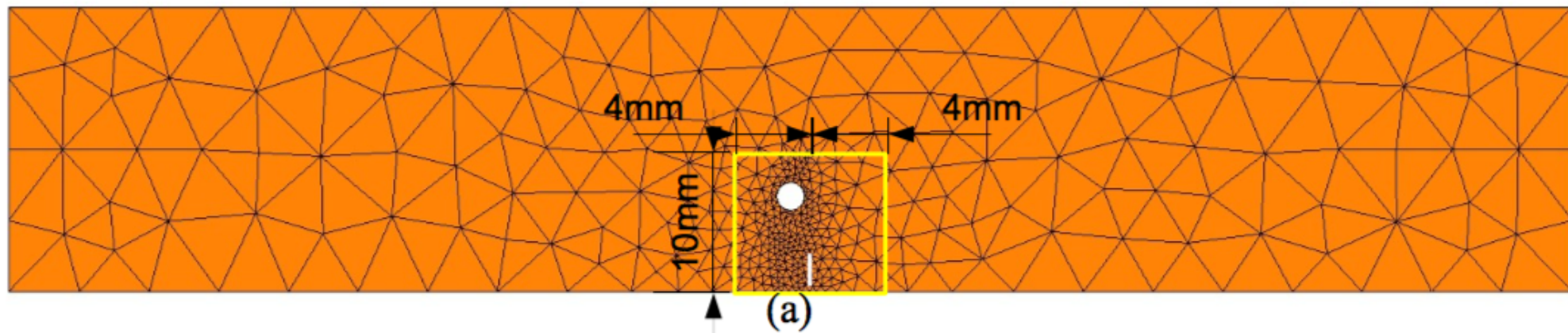
$$\int_{\Omega} (\nabla \tilde{\partial} u \cdot (\underline{D} \nabla u) + \tilde{\partial} u \cdot b \cdot u) d\Omega =: a(u, \tilde{\partial} u) = l(\tilde{\partial} u) := \int_{\Omega} \tilde{\partial} u \cdot f d\Omega + \int_{\Gamma_n} \tilde{\partial} u \cdot g_n d\Gamma_n, \quad \forall \tilde{\partial} u \in \tilde{V}$$

Exact expression for the discretization error (residual form).

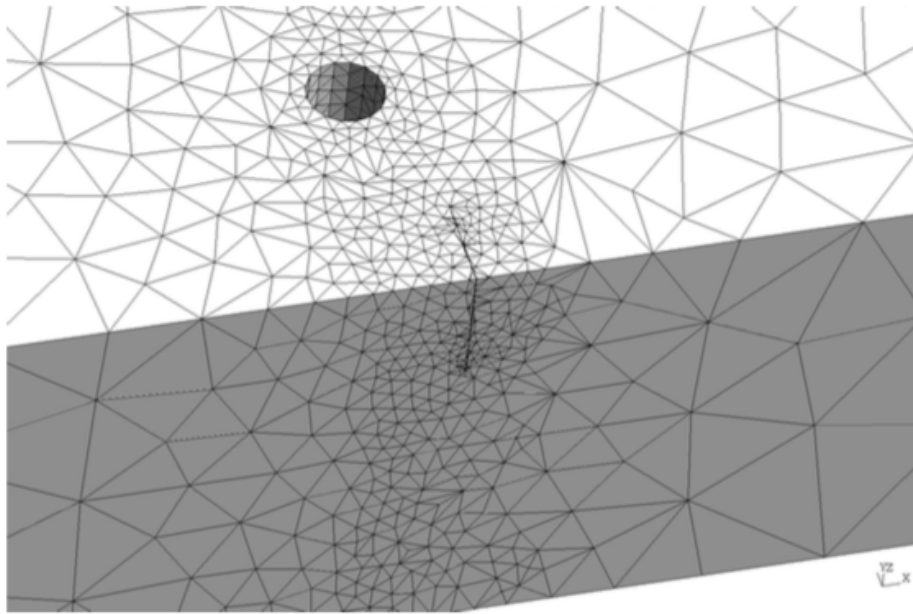
$$a(u, \tilde{\partial} u) - a(u^h, \tilde{\partial} u) = l(\tilde{\partial} u) - a(u^h, \tilde{\partial} u) \quad a(\tilde{e}, \tilde{\partial} u) = R(\tilde{\partial} u) \quad \text{where } \tilde{e} = u - u^h$$

- By Galerkin orthogonality the error in the coarse space is zero
- We need a richer discrete space, to compute any error

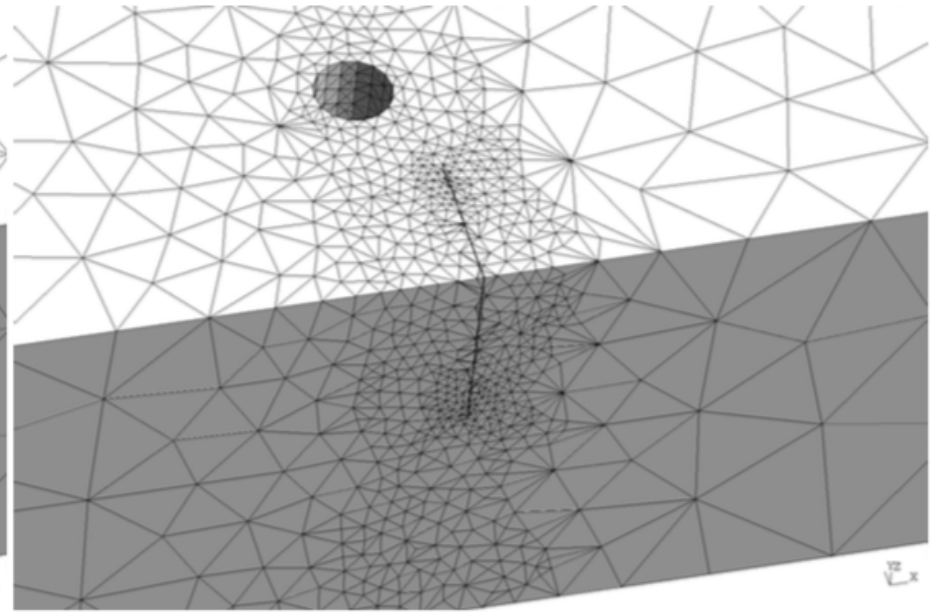
Why error estimation?



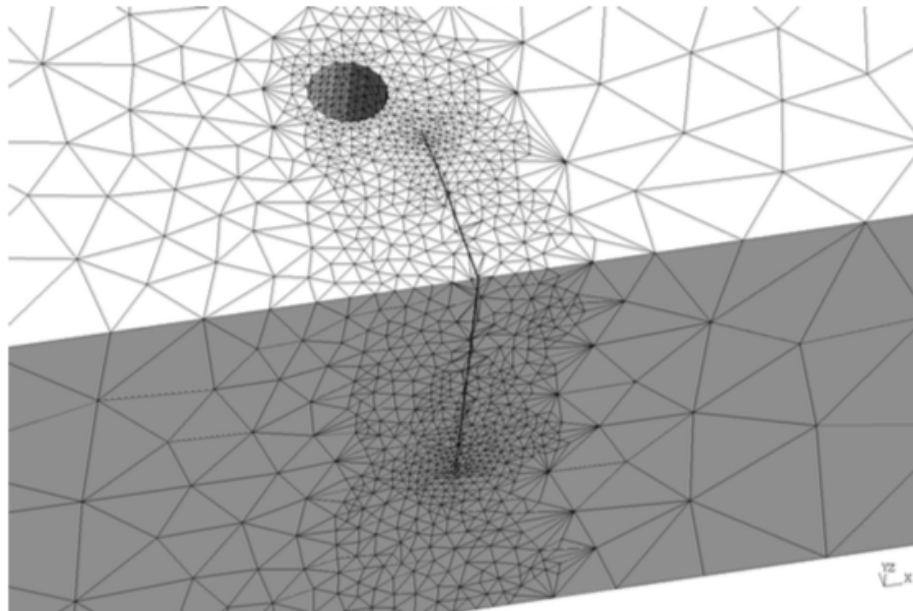
Why error estimation?



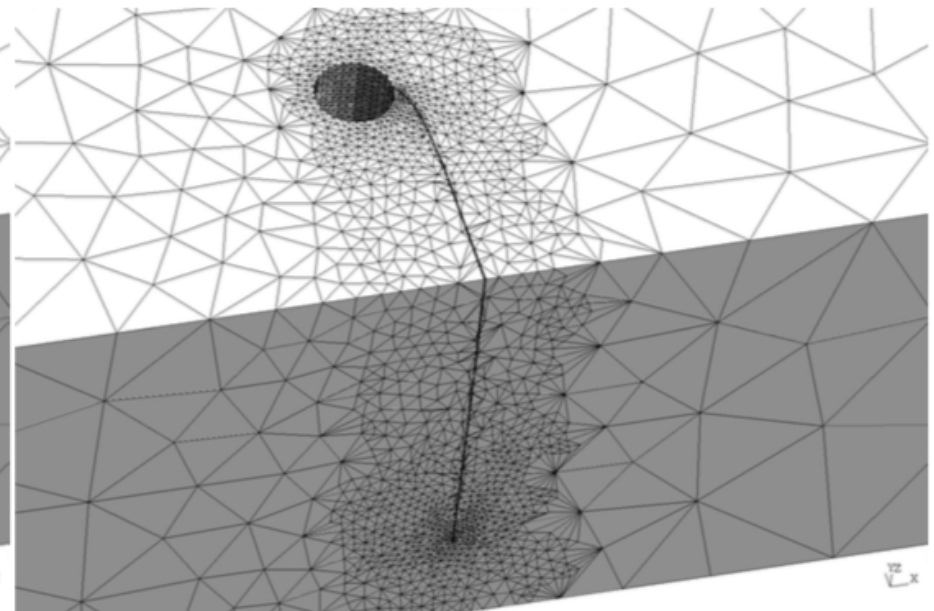
Step 1 (23749)



Step 10 (51864)

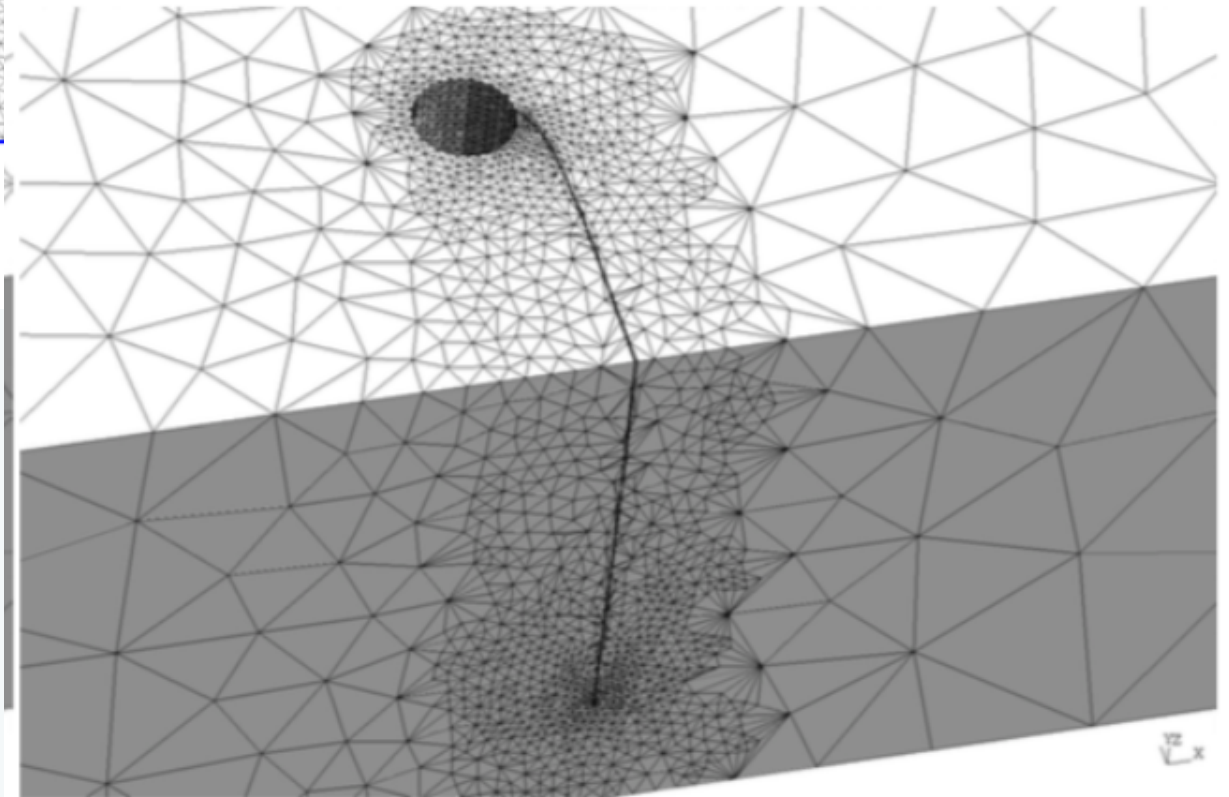
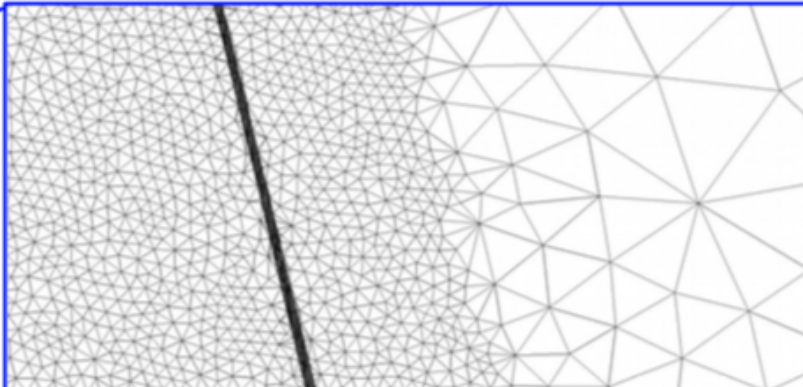
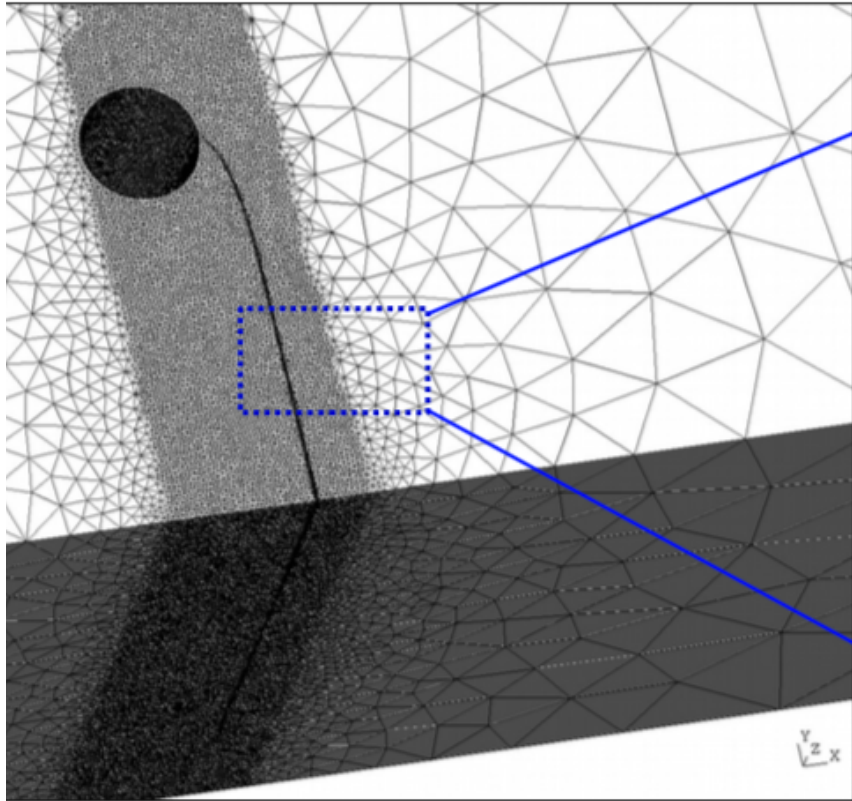


Step 20 (125031)



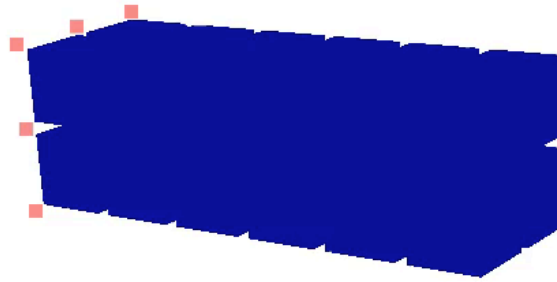
Step 32 (296055)

Why error estimation?



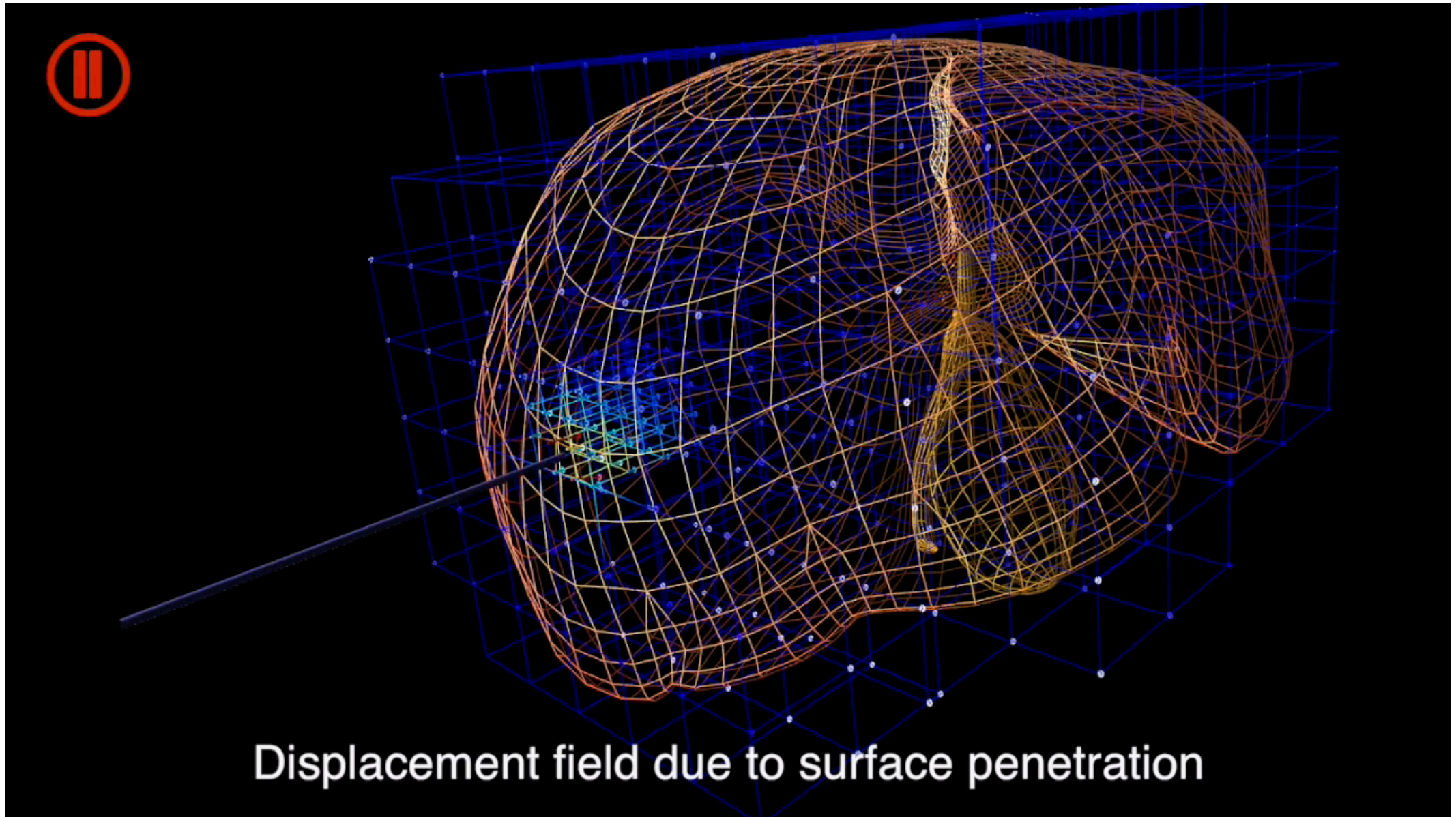
Without error estimation: the whole “potential path” of the crack is meshed in advance leading to large increases in the computational time millions of elements as opposed to a few hundred of thousands

Step 32 (296055)





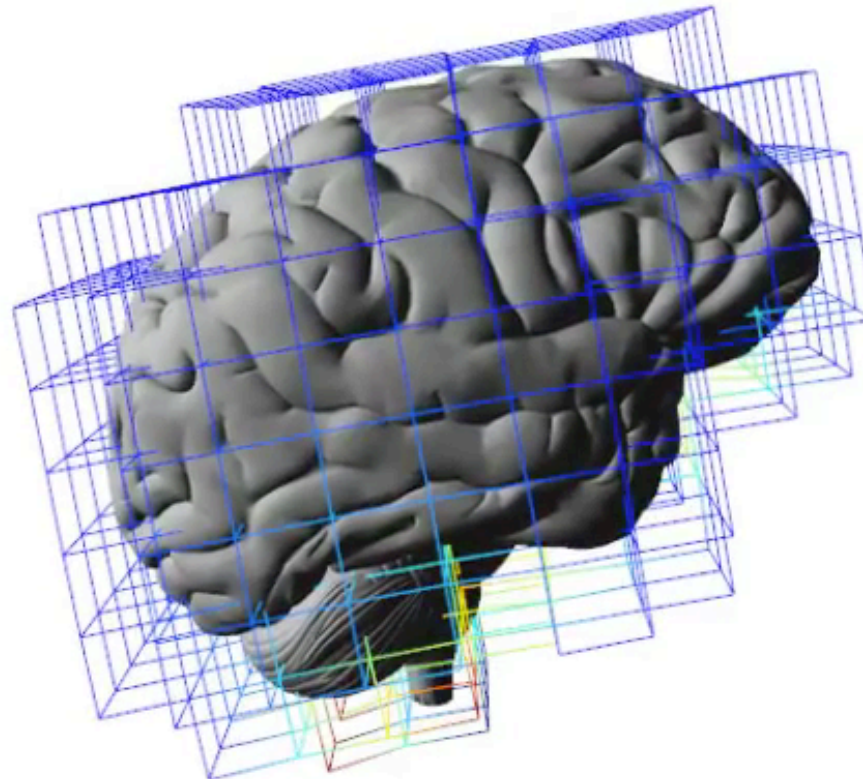
Real-time error estimation



Needle insertion in the liver

Real-time error estimation

Brain shift occurs
prior to cannula insertion



Needle insertion in the brain for deep brain stimulation