Model order reduction

POD and Homogenisation

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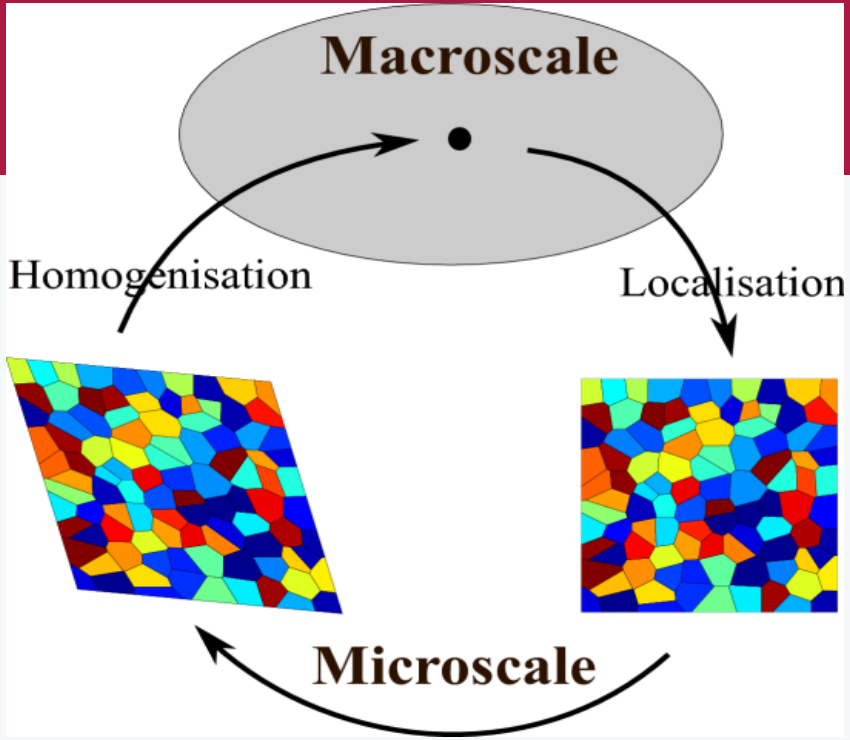
Ways to reduce the models

- Homogenisation (FE^2, etc.) - Hierarchical
- Concurrent and hybrid (bridging domain, ARLEQUIN, etc.)
- Enrichment (PUFEM, XFEM, GFEM)
- Model reduction (algebraic)

Reduction methods based on homogenisation



Definition of an RVE

$$l^c \gg l^f \gg l^g$$

Coupling of macroscopic and microscopic levels

The volume averaging theorem is postulated for:

1) Strain tensor:

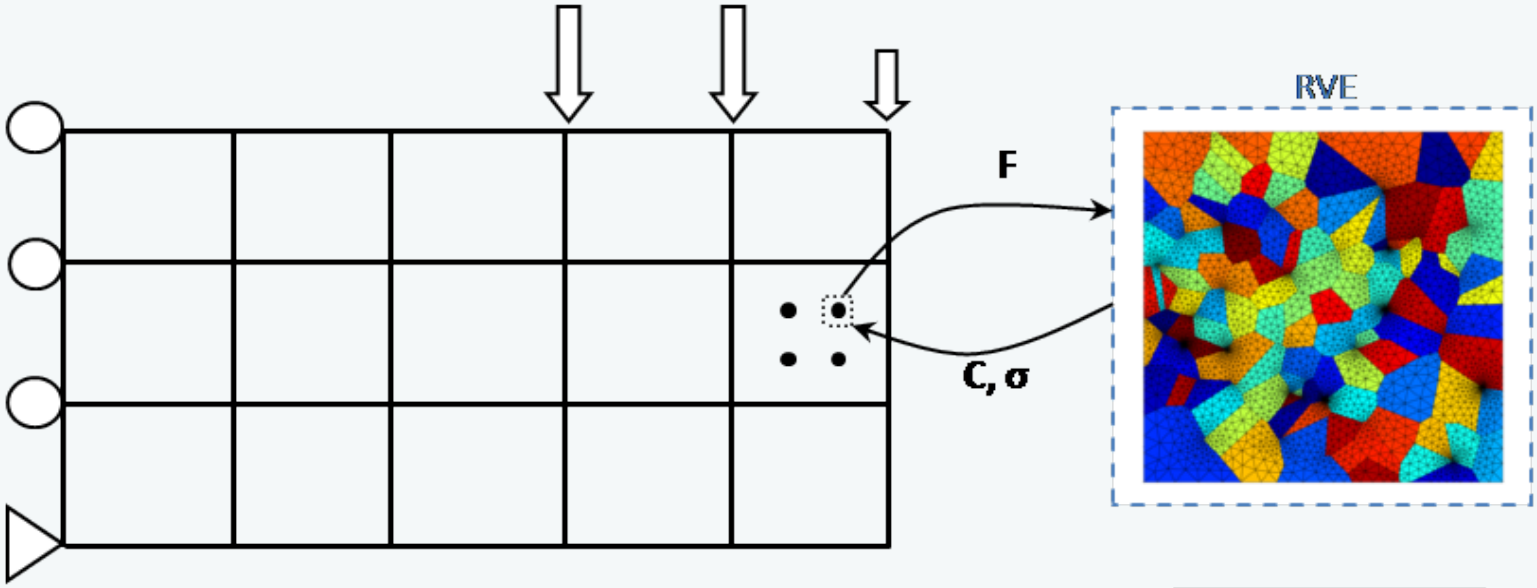
$$\boldsymbol{\epsilon}^c = \frac{1}{|\Omega(\mathbf{x}^c)|} \int_{\partial\Omega(\mathbf{x}^c)} \mathbf{u}^f \otimes_s \mathbf{n}\, d\Gamma$$

2) Virtual work (Hill-Mandel condition):

$$\boldsymbol{\sigma}^c : \delta\boldsymbol{\epsilon}^c = \frac{1}{|\Omega(\mathbf{x}^c)|} \int_{\partial\Omega(\mathbf{x}^c)} \mathbf{t}^f \cdot \delta\mathbf{u}^f \, d\Gamma$$

3) Stress tensor:

$$\boldsymbol{\sigma}^c = \frac{1}{|\Omega(\mathbf{x}^c)|} \int_{\partial\Omega(\mathbf{x}^c)} \mathbf{t}^f \otimes \mathbf{x}^f \, d\Gamma$$



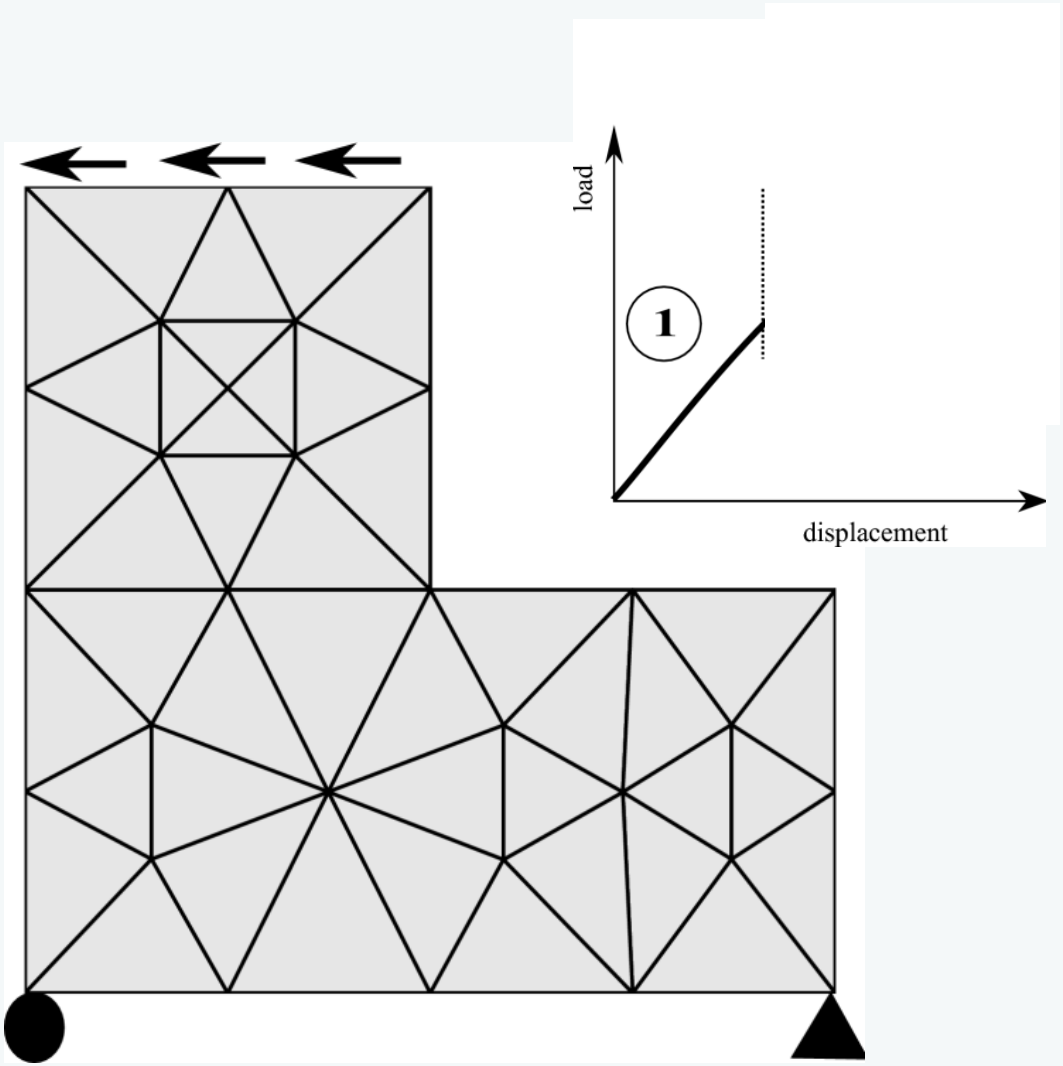
Advantages and abilities:

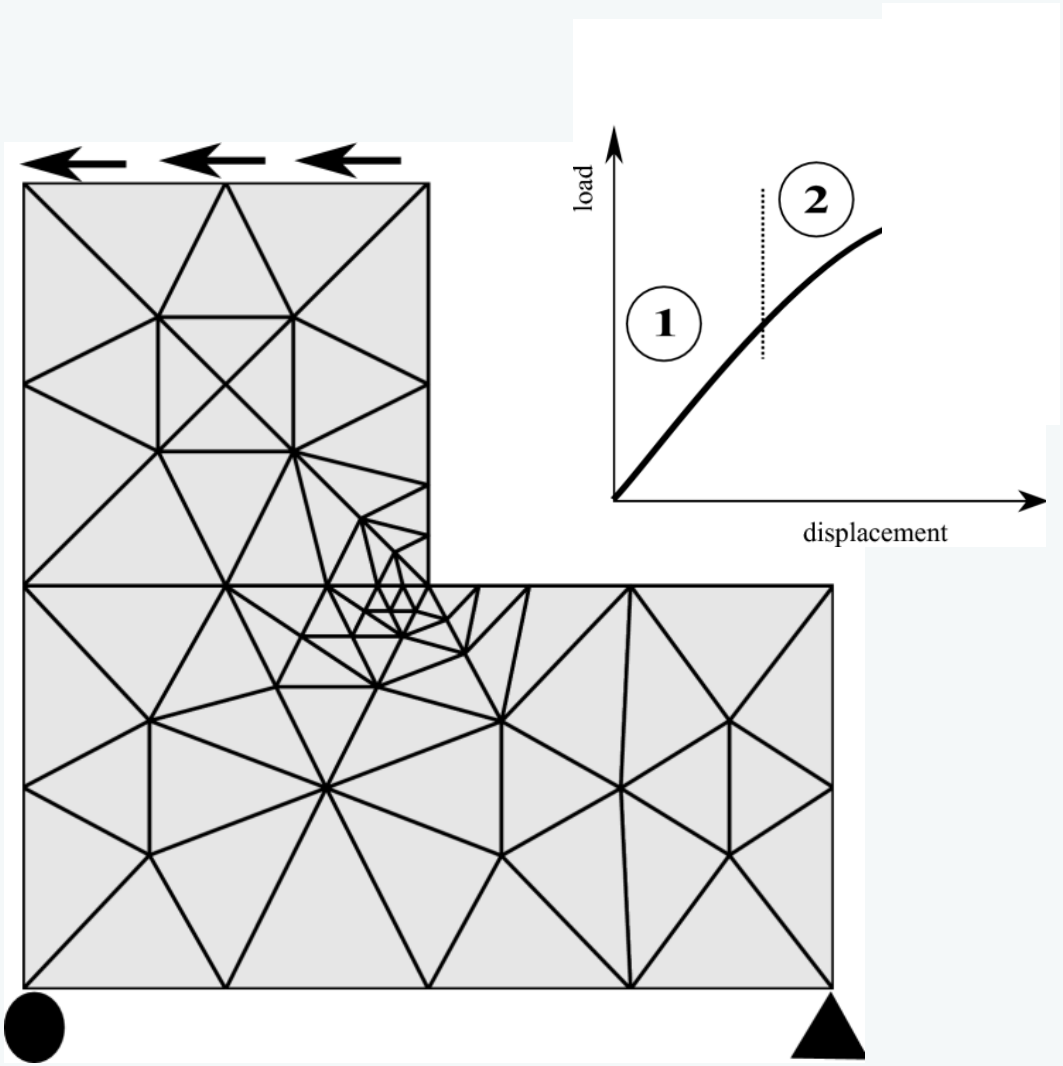
The macroscopic constitutive law is not required
Non-linear material behaviour can be simulated
Microscale behaviour of material is monitored at each load step

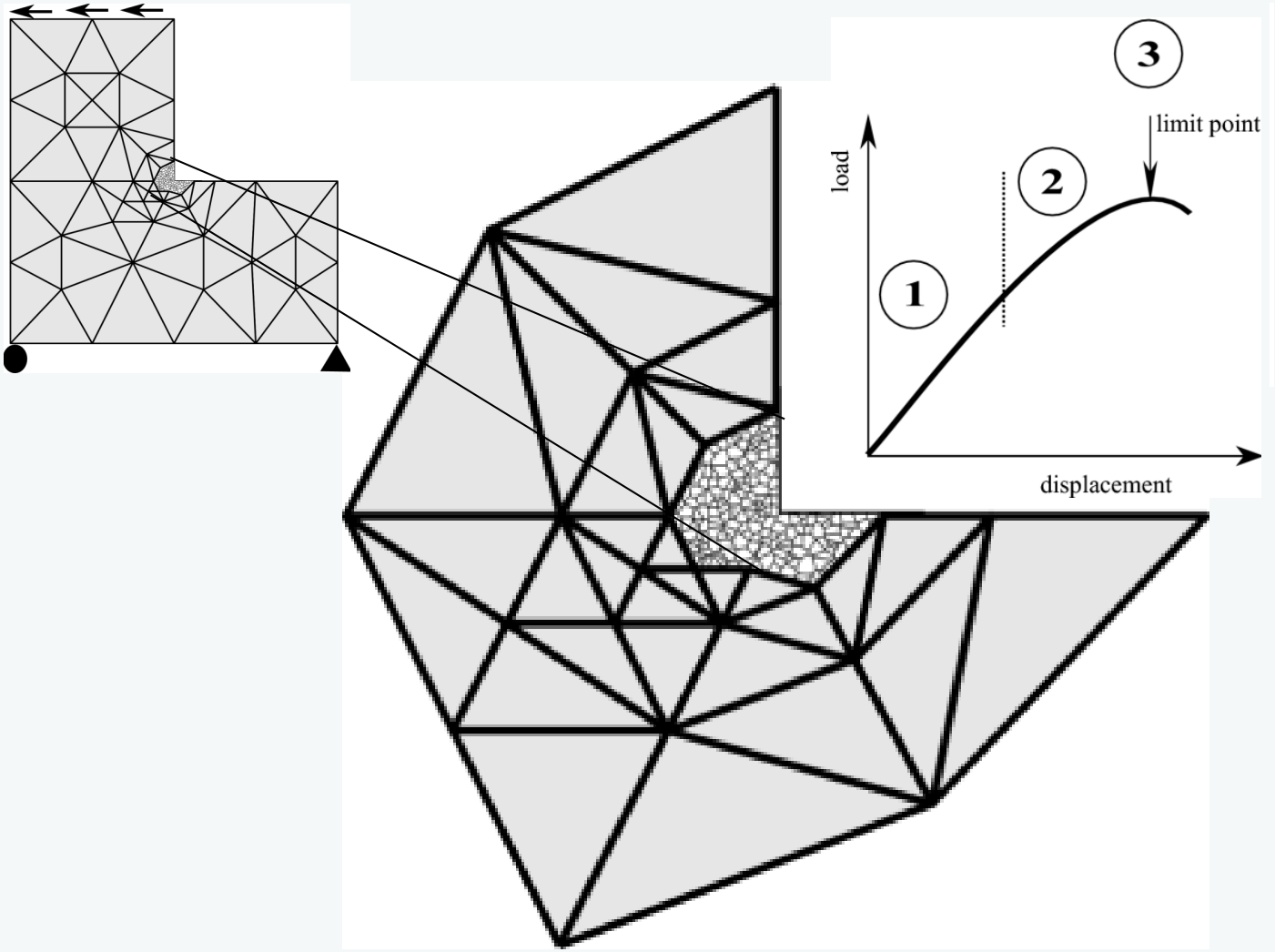
Drawbacks:

In softening regime:

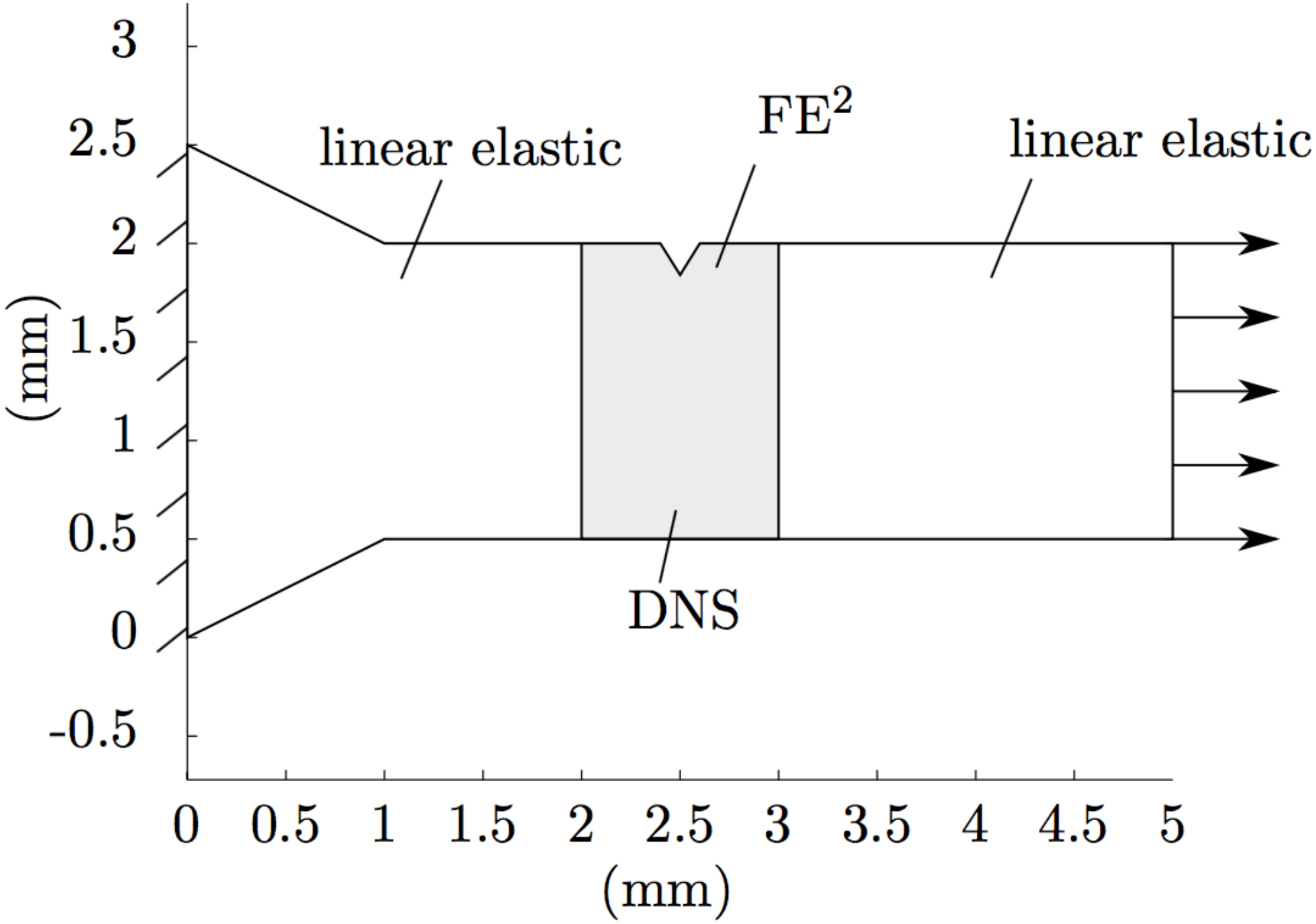
- Lack of scale separation
- Macroscale mesh dependence

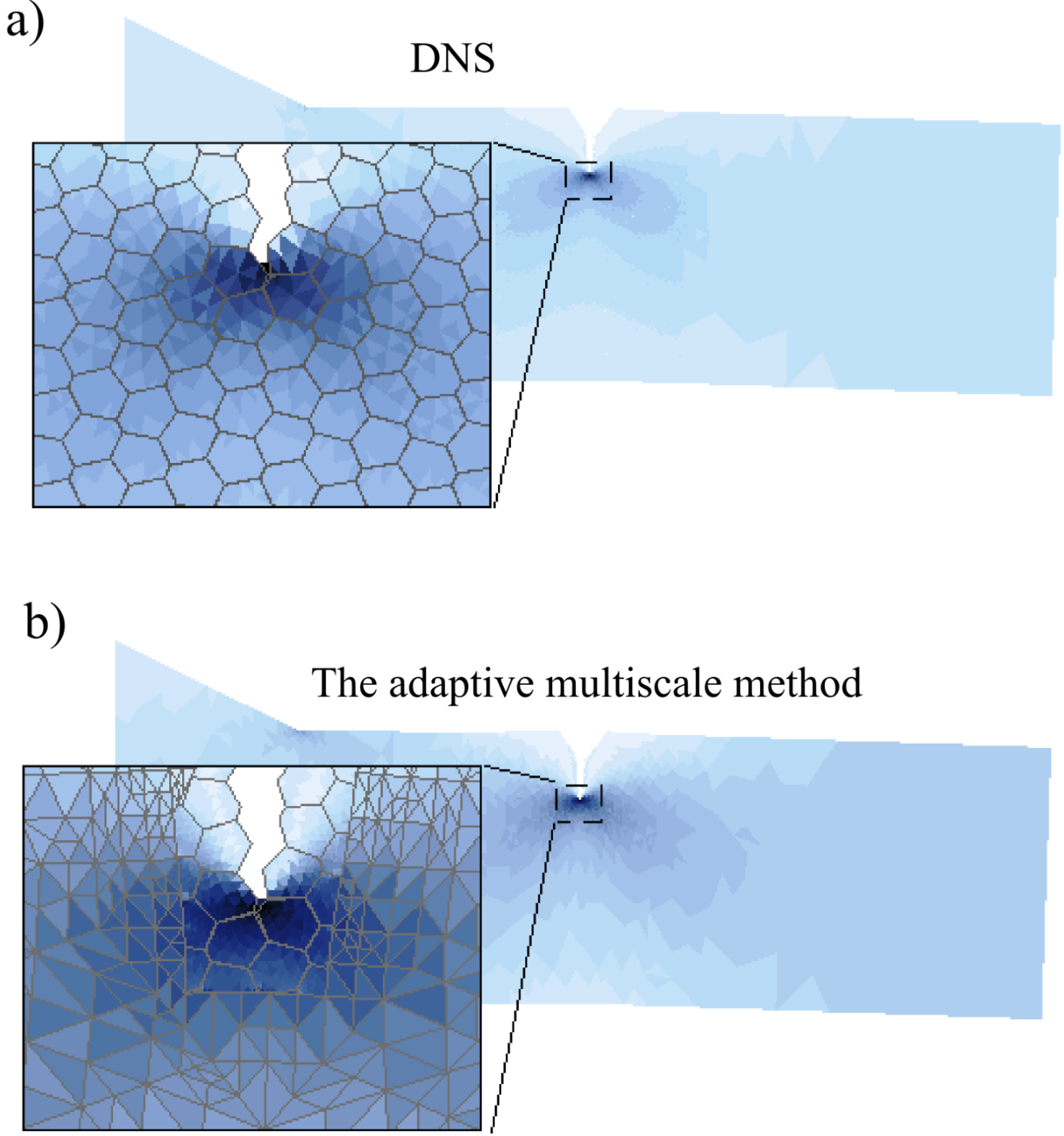

load
1
displacement


load
displacement
1
2

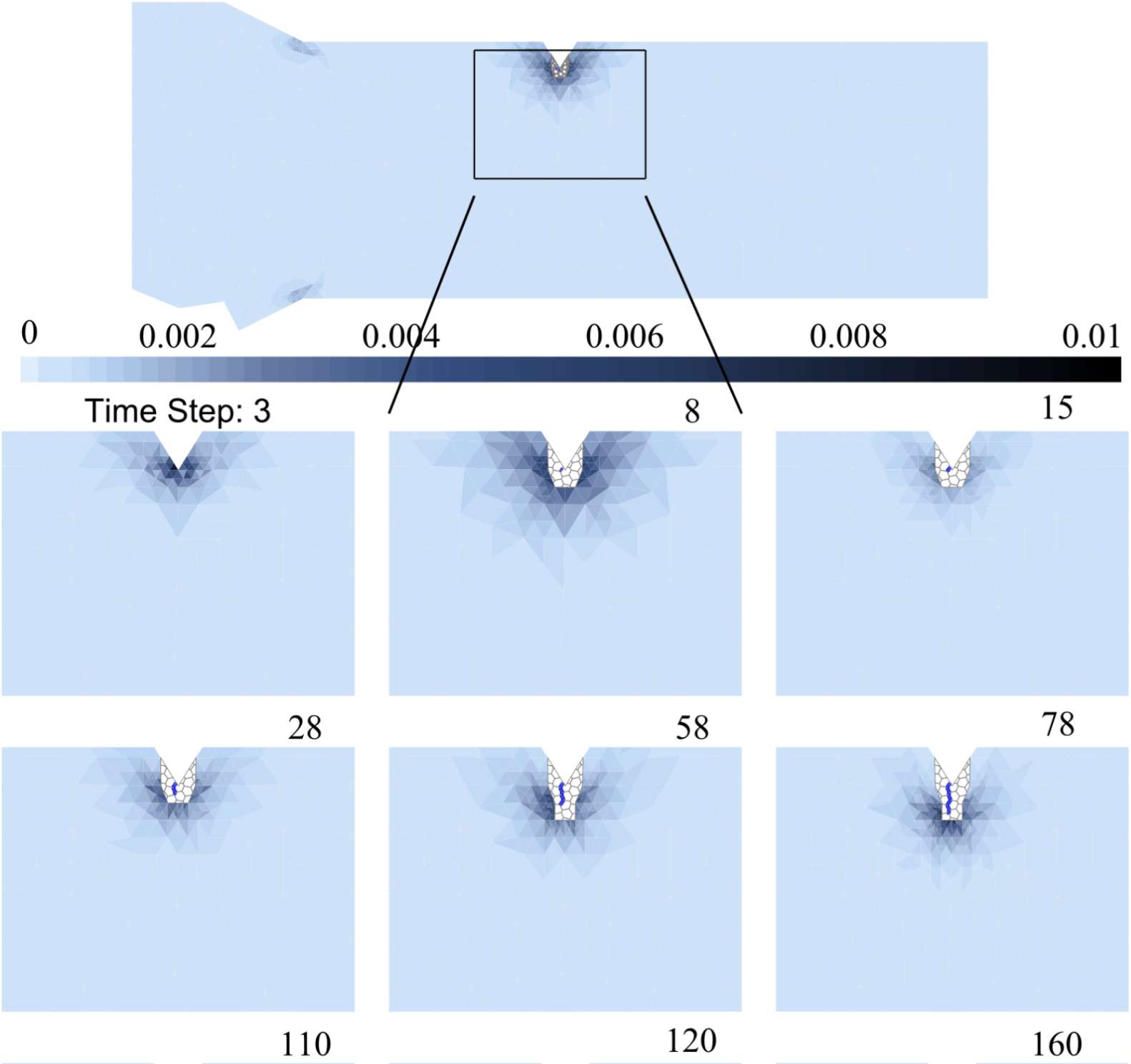


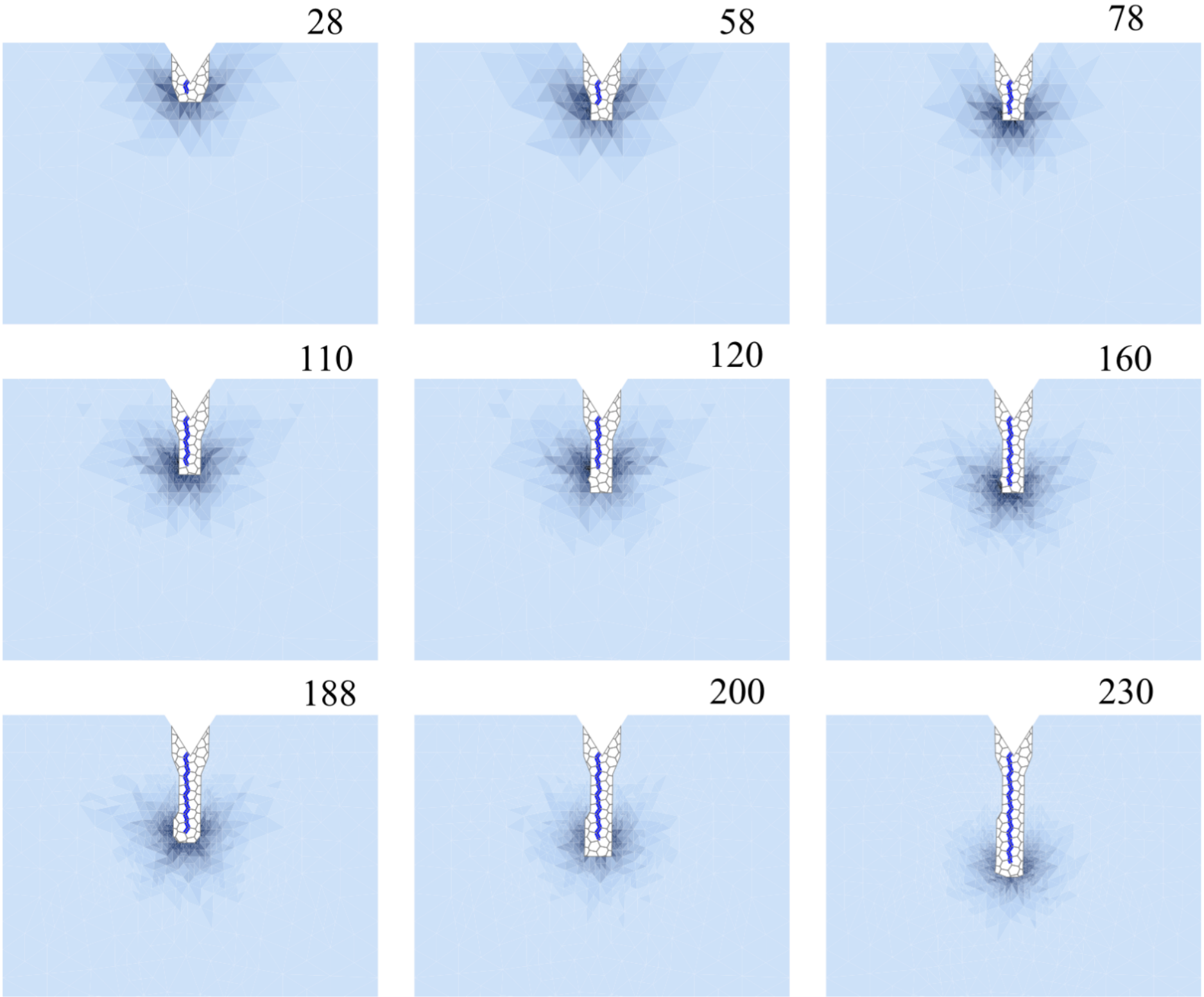
Details in Phil. Magazine, 2015, Akbari, Kerfriden, Bordas

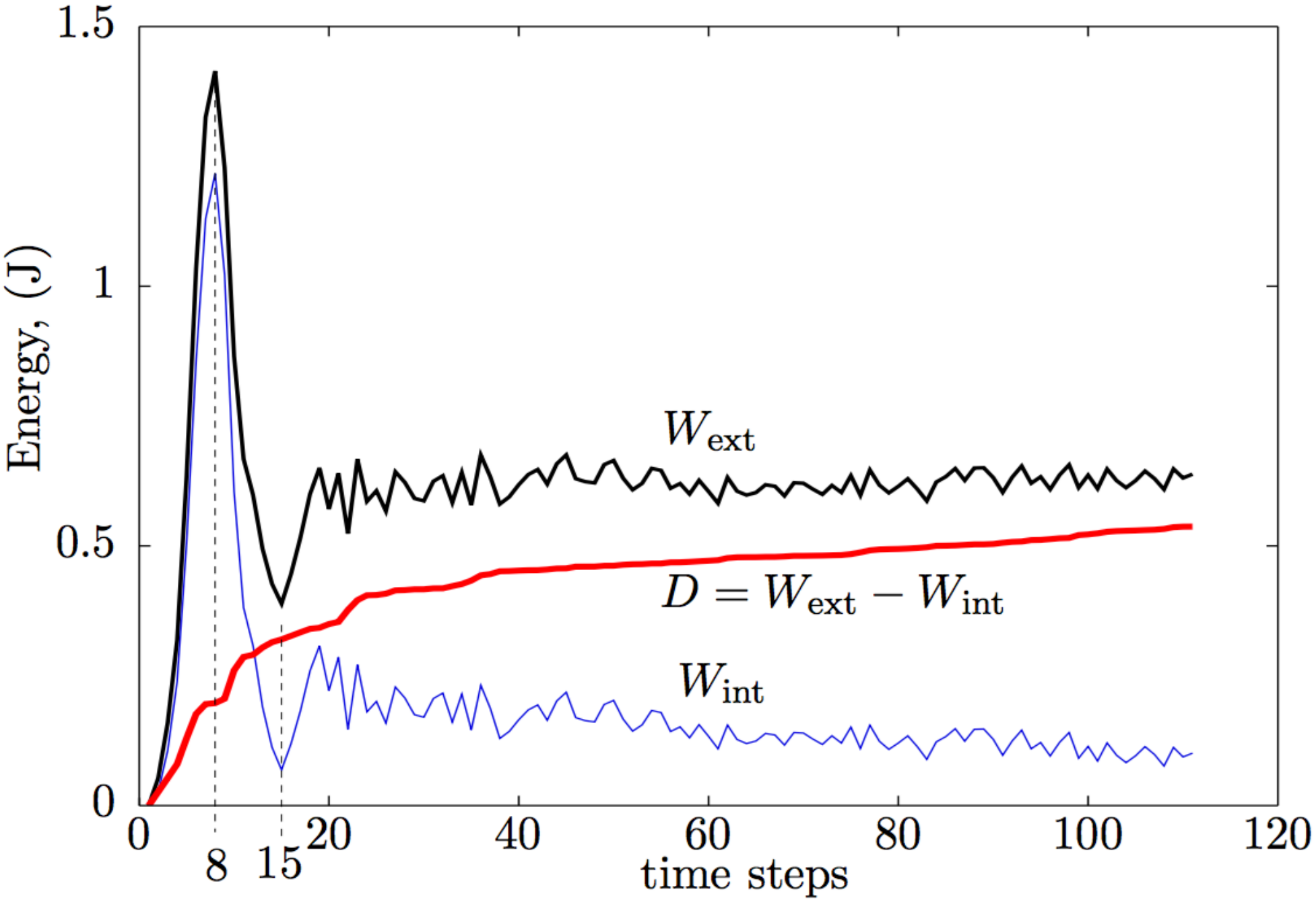

3
2.5
2
1.5
1
0.5
0
-0.5
(mm)
linear elastic
FE^2
linear elastic
DNS
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
(mm)

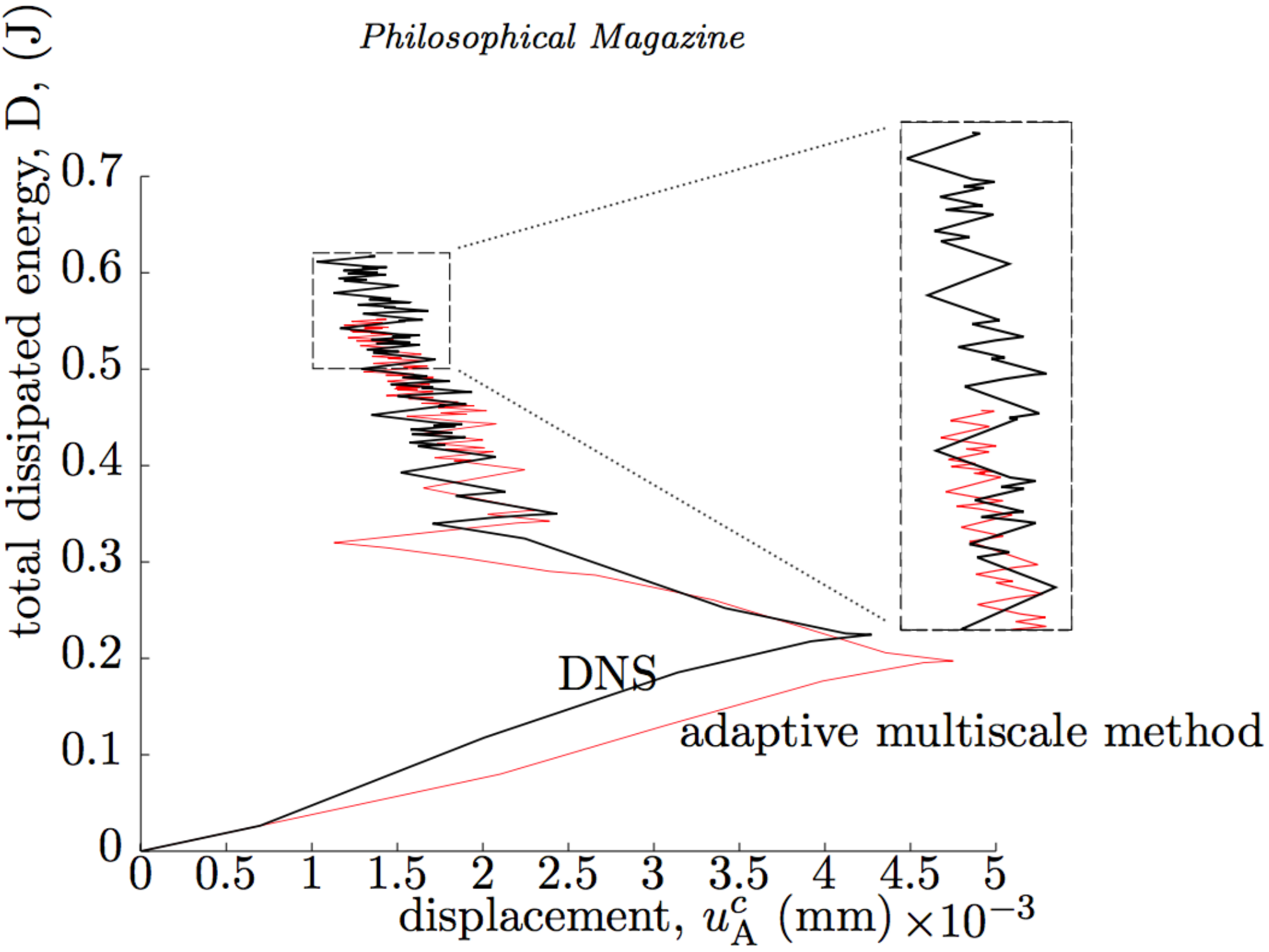

a)
DNS
b)
The adaptive multiscale method

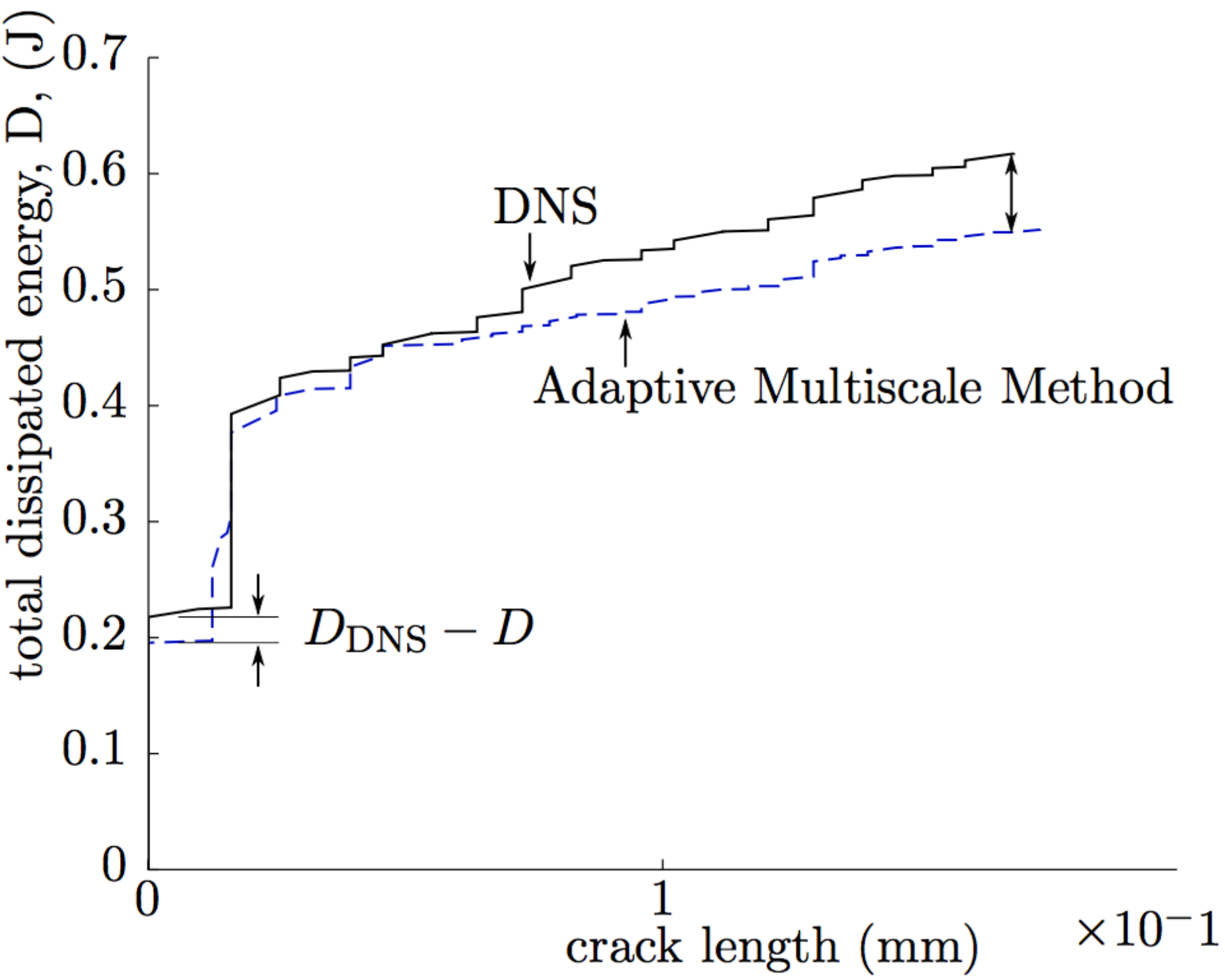
The distribution of strain-gradient sensitivity $L_{\mathcal{V}}||\nabla\nabla\mathbf{u}^c||_e$




28
58
78
110
120
160
188
200
230


1.5
1
0.5
0
Energy, (J)
W_{ext}
$D = W_{\text{ext}} - W_{\text{int}}$
W_{int}
0
8
15
20
40
60
80
100
120
time steps


total dissipated energy, D, (J)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
DNS
adaptive multiscale method
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
displacement, u_{A}^{c} (mm) $\times 10^{-3}$


total dissipated energy, D, (J)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
DNS
Adaptive Multiscale Method
$D_{\mathrm{DNS}} - D$
0
1
crack length (mm)
$\times 10^{-1}$

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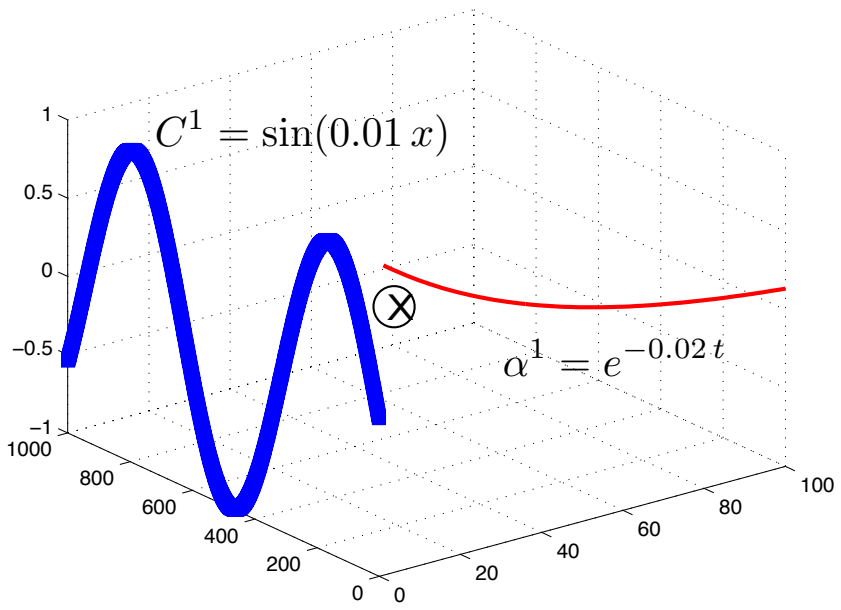
Reduction methods based on algebraic reduction

Illustration of the method of separated representation



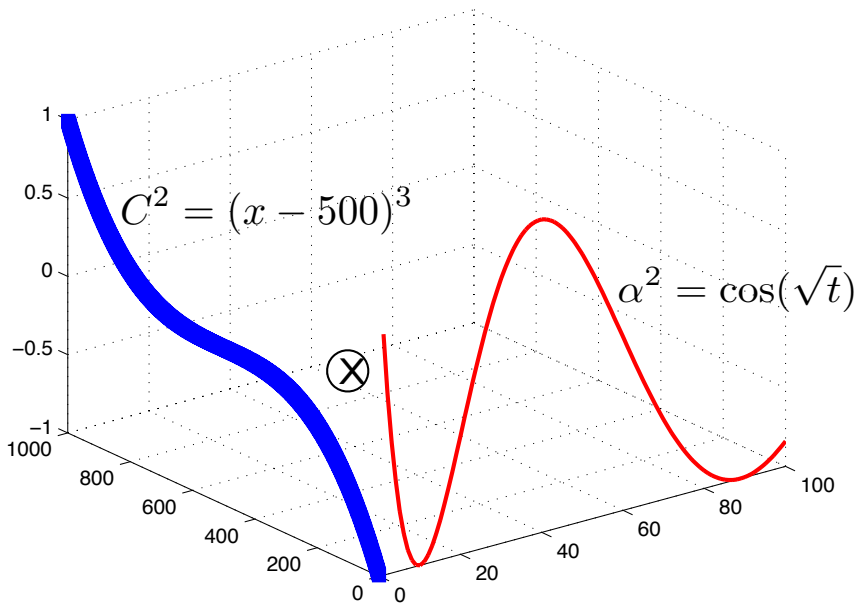
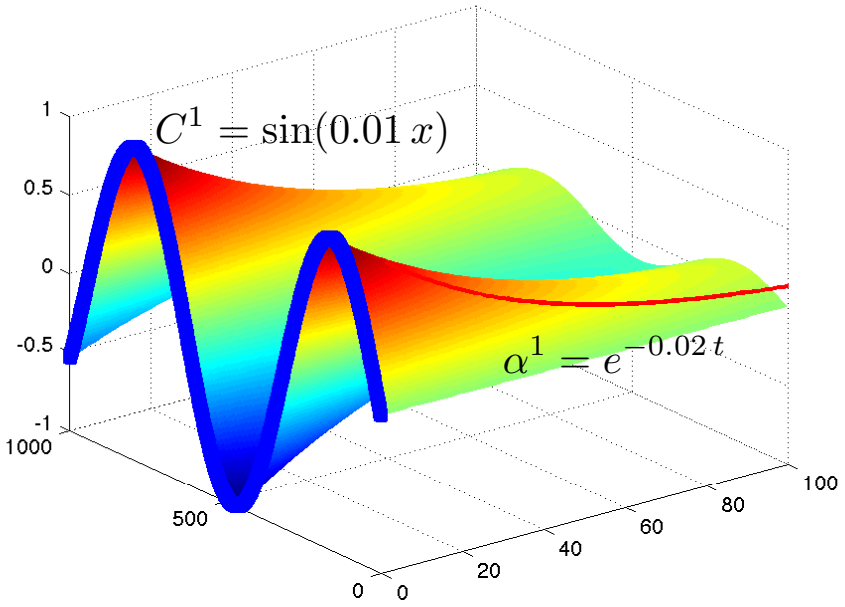


Illustration of the method of separated representation



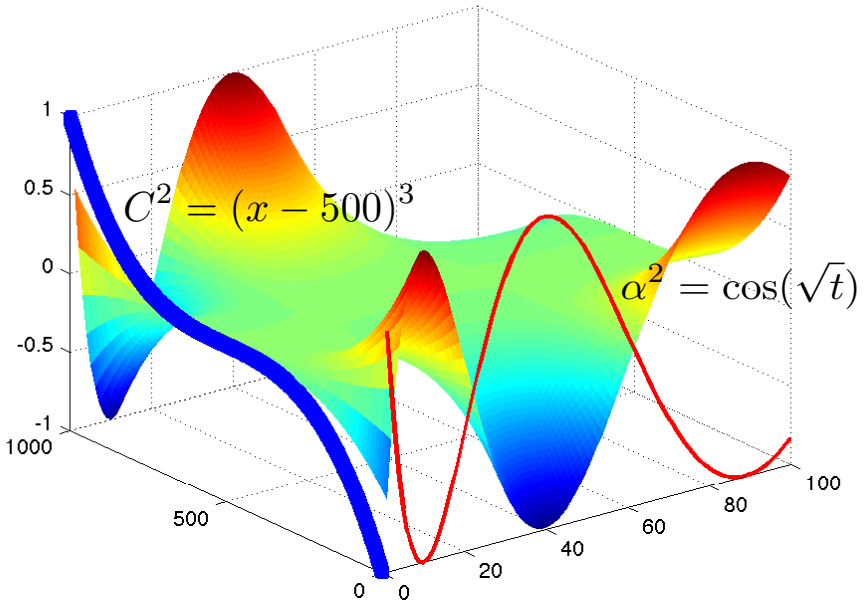
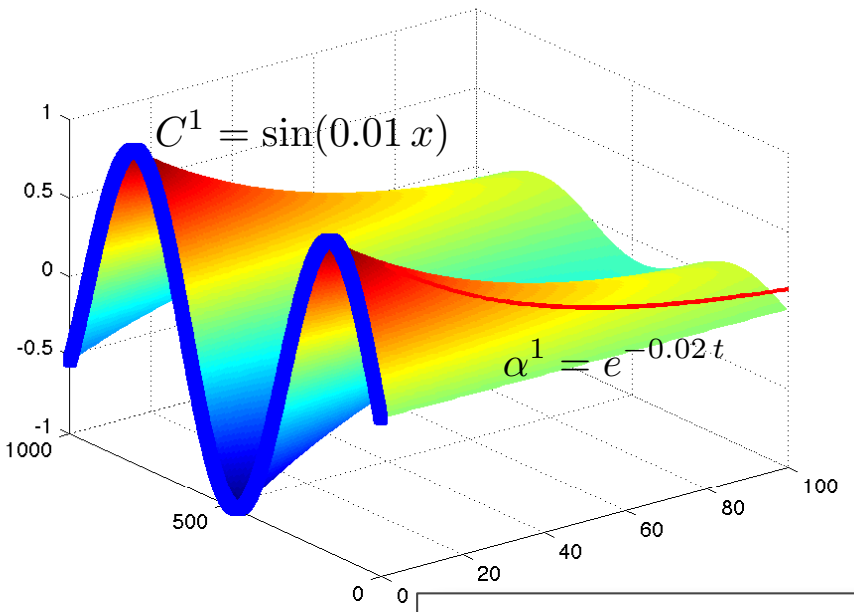


Illustration of the method of separated representation



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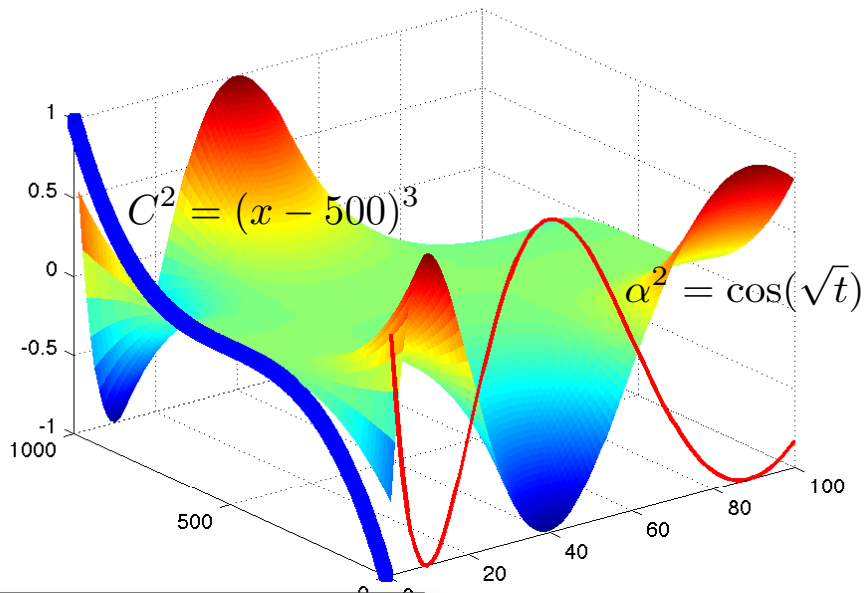

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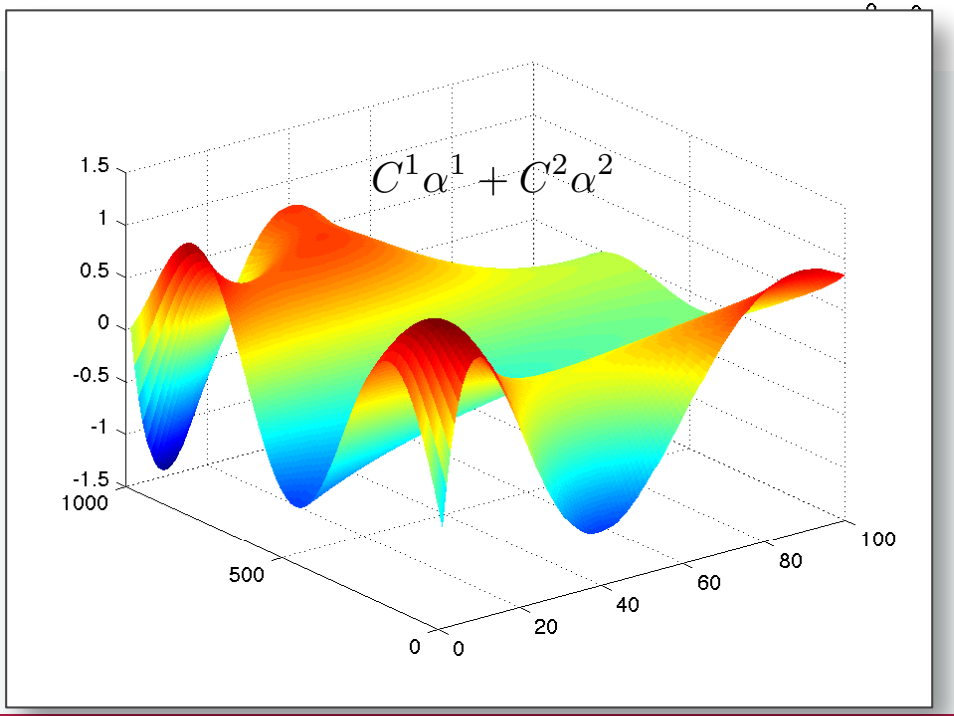
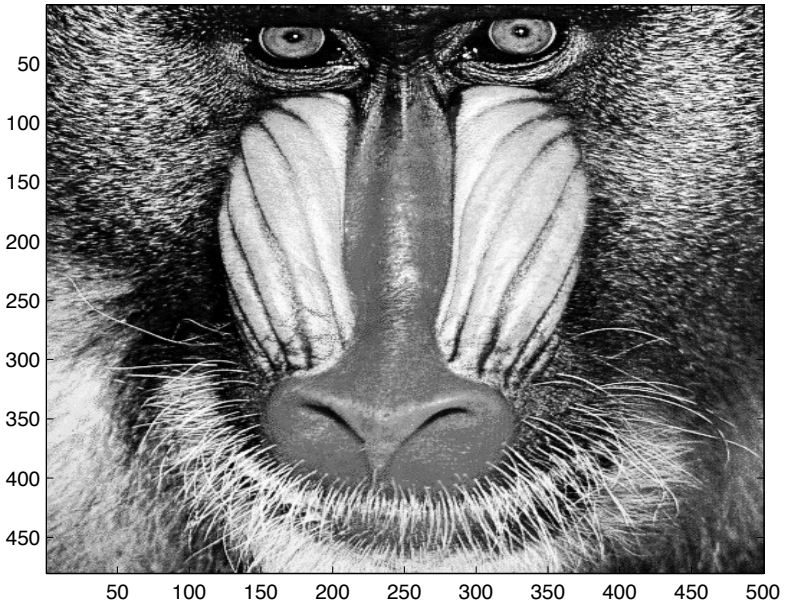
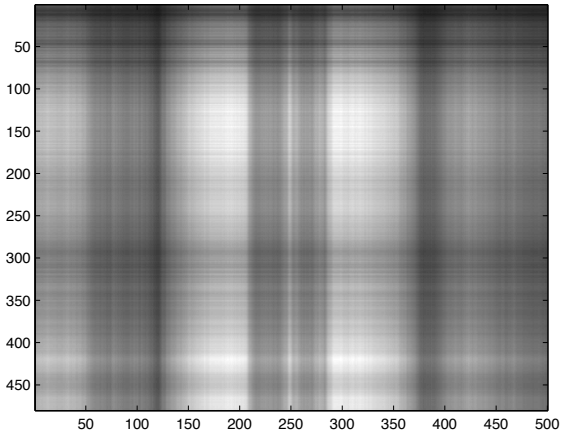


Very rich approximations!

Data compression: get the nose with the POD!



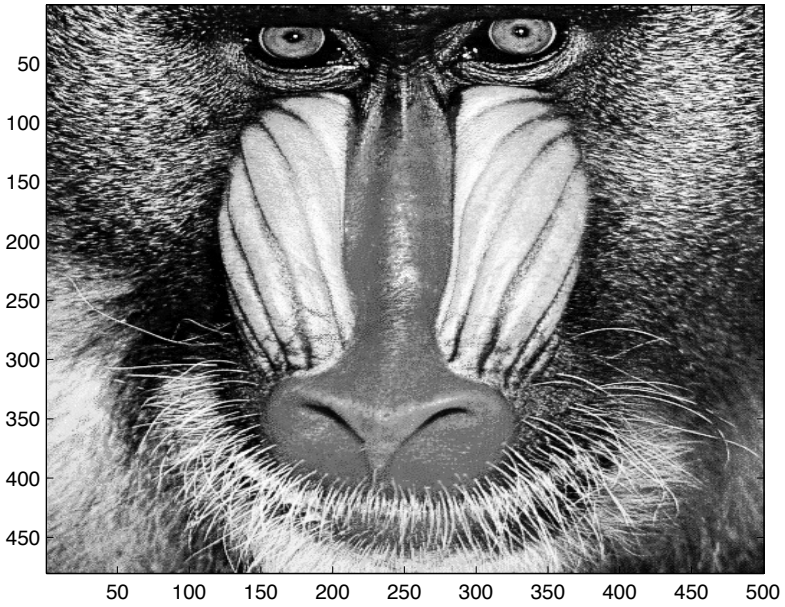


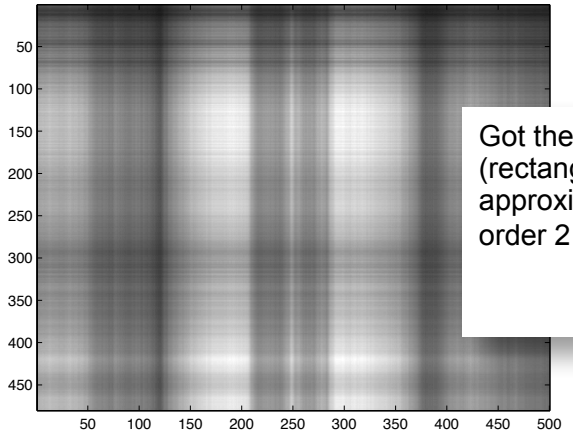
$n_c = 1$

$$\bar{u}(x_i, y_i) = \sum_{i=1}^{n_c} \underline{C}_x^i(x_i)\, \underline{C}_y^i(y_i)$$

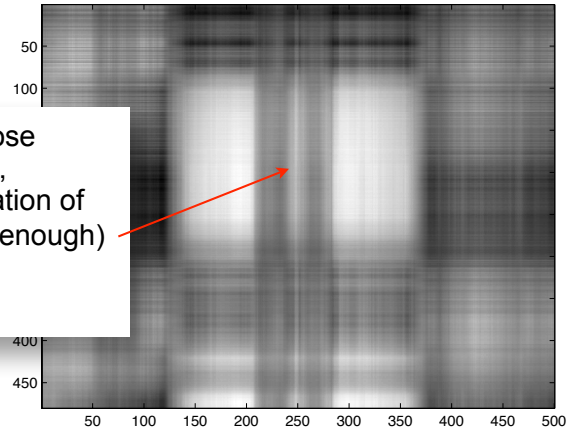
$$(C_x^i, C_y^i)_{i \in [\![1, n_c]\!]} = \operatorname{argmin} \sum_{x_i} \sum_{y_j} (u(x_i, y_j) - \bar{u}(x_i, y_j))^2$$

Data compression: get the nose with the POD!





$n_c = 1$



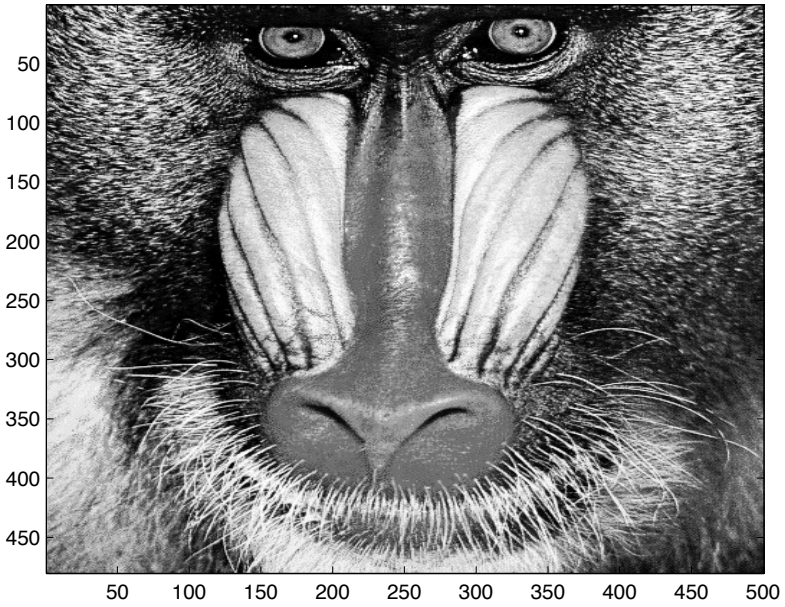
$n_c = 2$

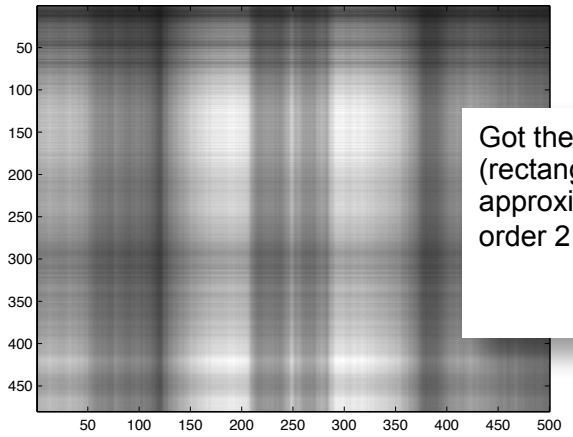
Got the nose (rectangle, approximation of order 2 is enough)

$$\bar{u}(x_i, y_i) = \sum_{i=1}^{n_c} \underline{C}_x^i(x_i)\, \underline{C}_y^i(y_i)$$

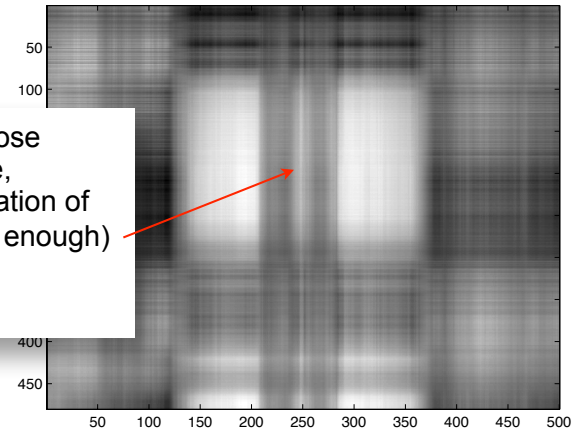
$$(C_x^i, C_y^i)_{i \in [\![1, n_c]\!]} = \operatorname{argmin} \sum_{x_i} \sum_{y_j} (u(x_i, y_j) - \bar{u}(x_i, y_j))^2$$

Data compression: get the nose with the POD!



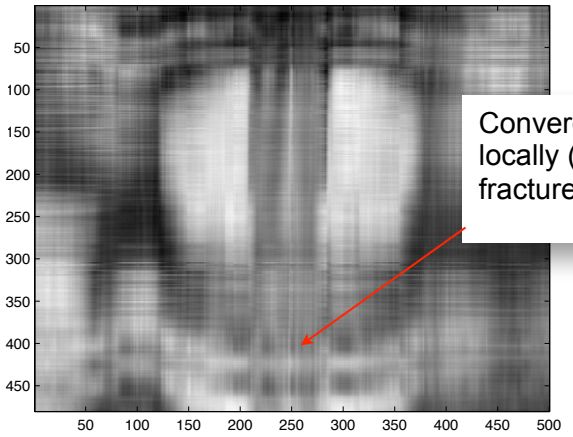


$n_c = 1$

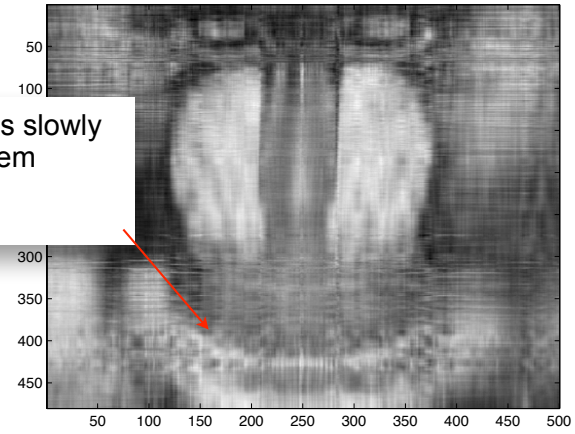


$n_c = 2$

Got the nose (rectangle, approximation of order 2 is enough)

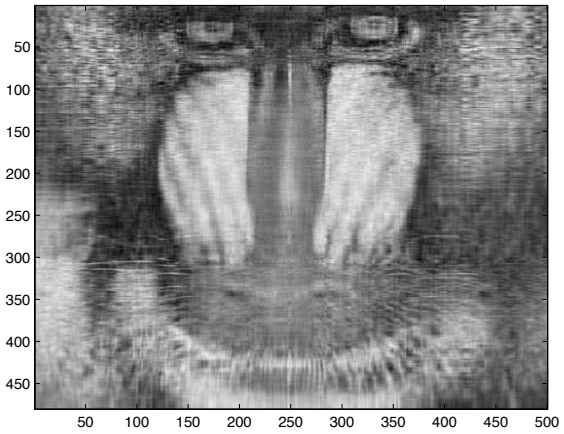


$n_c = 5$



$n_c = 10$

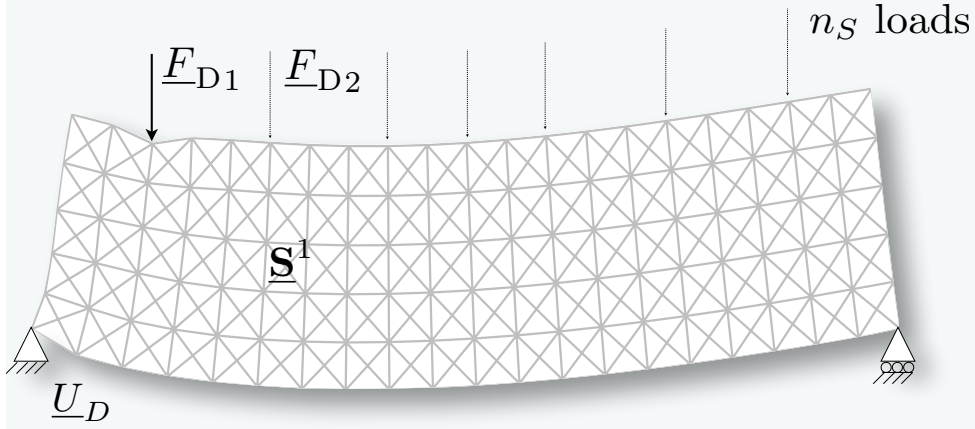
Converges slowly locally (idem fracture)



$n_c = 20$

$$\bar{u}(x_i, y_i) = \sum_{i=1}^{n_c} \underline{C}_x^i(x_i)\, \underline{C}_y^i(y_i)$$

$$(C_x^i, C_y^i)_{i \in [\![1, n_c]\!]} = \operatorname{argmin} \sum_{x_i} \sum_{y_j} (u(x_i, y_j) - \bar{u}(x_i, y_j))^2$$



(1) Solve FINE for n_S parameters (EXPENSIVE!)

$$\underline{\underline{\mathbf{S}}} = \left(\underline{\mathbf{S}}^1 \quad \underline{\mathbf{S}}^2 \quad \dots \quad \underline{\mathbf{S}}^{n_S} \right)$$

(2) Singular value decomposition

$$\underline{\underline{\mathbf{S}}} = \underline{\underline{\mathbf{U}}}\,\underline{\underline{\mathbf{\Sigma}}}\,\underline{\underline{\mathbf{V}}}^T = \sum_{k=1}^{n_S} \Sigma^k \underline{\mathbf{U}}^k \underline{\mathbf{V}}^{kT}$$

n_S solutions, sorted by relevance

where $(\Sigma^k)_{k \in [\![1 \; n_S]\!]}$ in decreasing order

(3) Truncation

Initial set of equations

$$\underline{\mathbf{F}}_{\text{Int}}\left(\underline{\mathbf{U}}\right) + \underline{\mathbf{F}}_{\text{Ext}} = 0$$

(4) Galerkin orthogonality

$$\underline{\mathbf{C}}^T \underline{\mathbf{F}}_{\text{int}}\left(\underline{\mathbf{C}}\,\underline{\alpha}\right) + \underline{\mathbf{C}}^T \underline{\mathbf{F}}_{\text{ext}} = 0$$

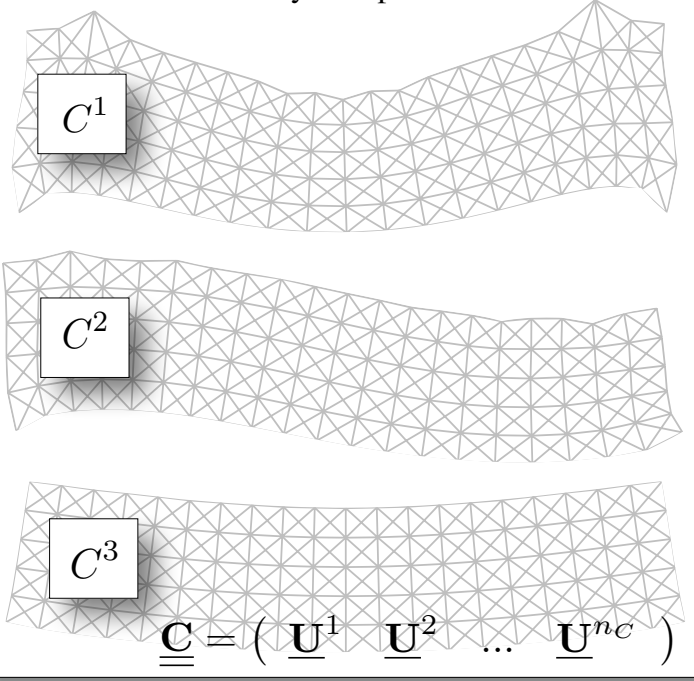
Approximation of the solution in a space of small dimension (n_c)

Family of representative solutions

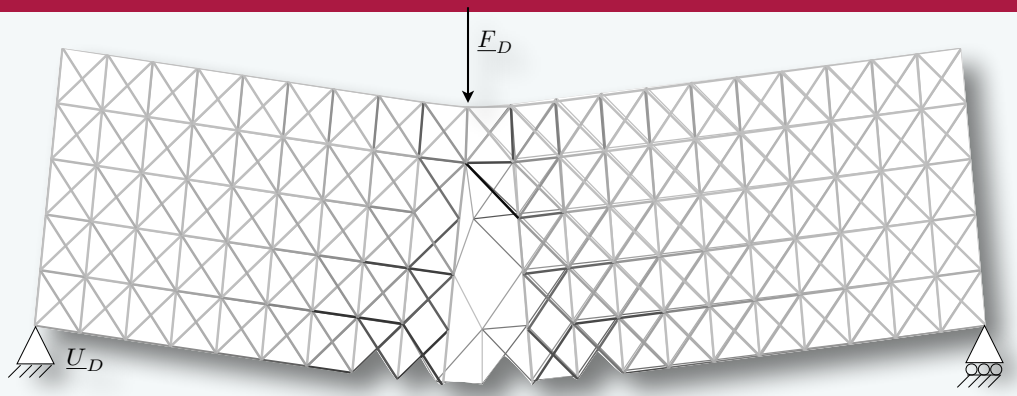
$$\underline{\mathbf{U}} = \underline{\mathbf{C}}\,\underline{\alpha}$$

Solution — Coefficients

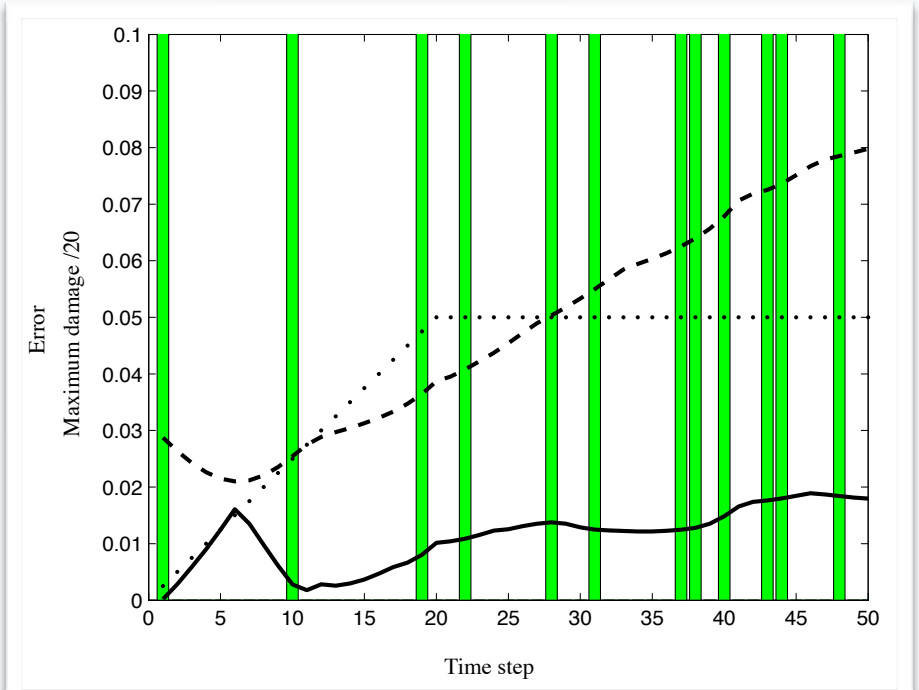
Reduced basis: family of representative solutions

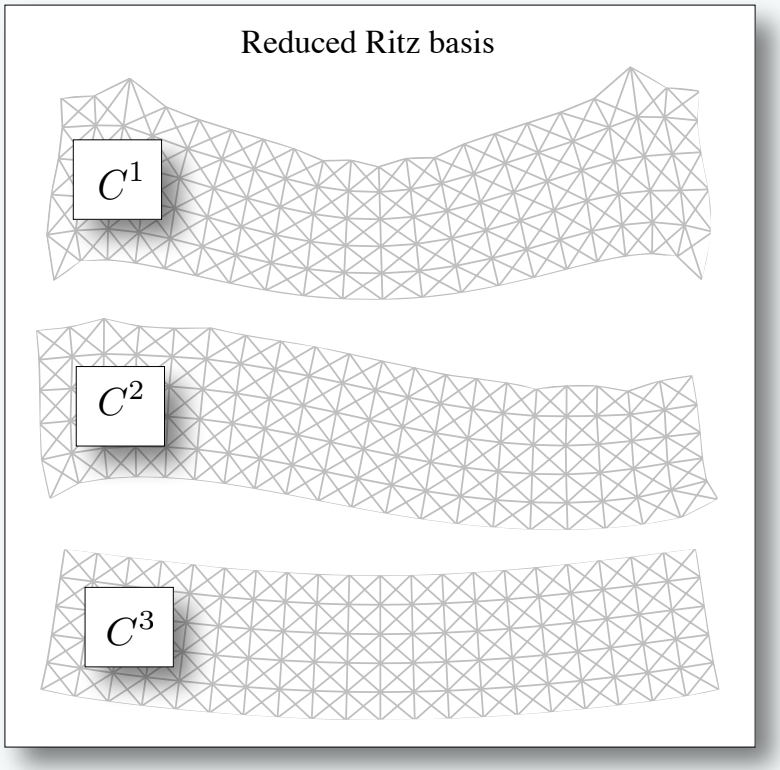


Limitations: case of highly non-linear fracture mechanics pbs



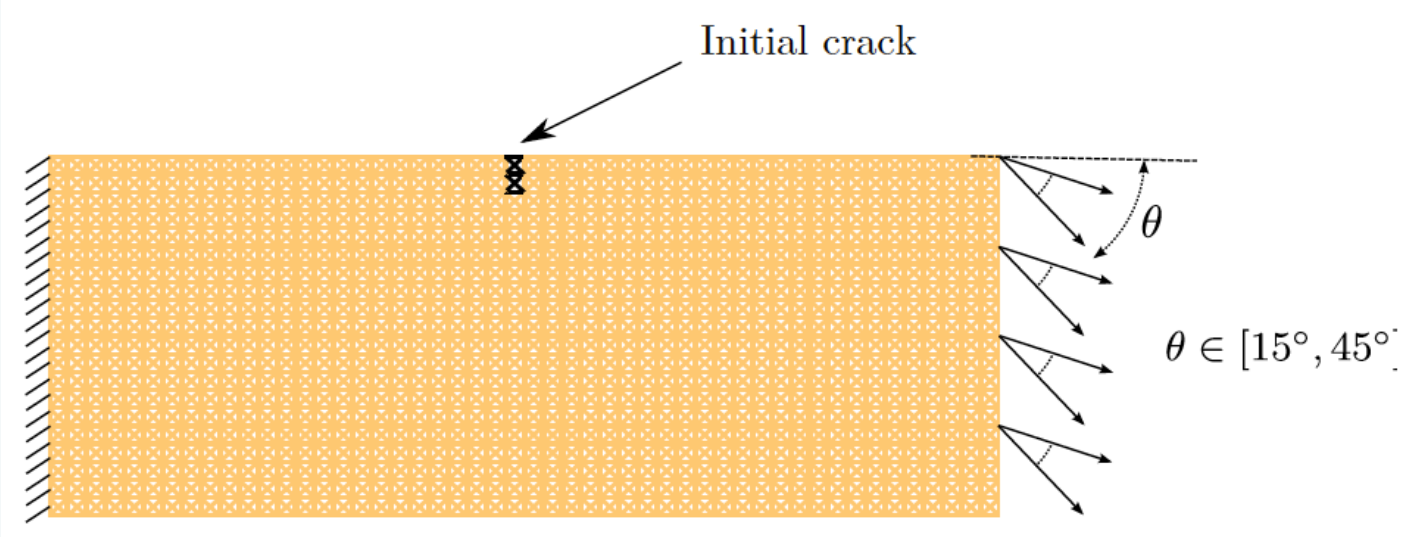
This solution is not in the snapshot !



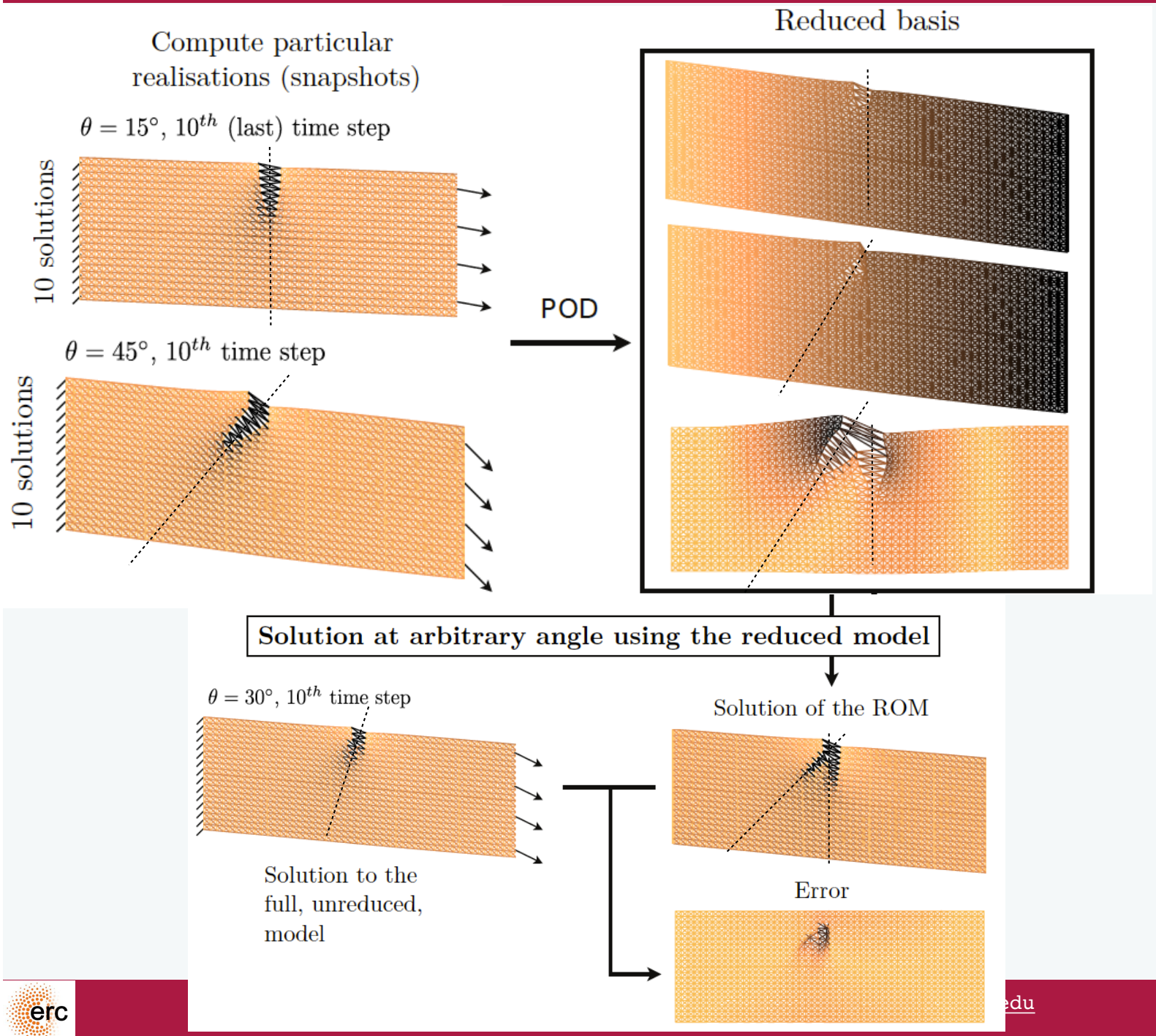


- P. Kerfriden, P. Gosselet, S. Adhikari, and S. Bordas. *Bridging proper orthogonal decomposition methods and augmented Newton-Krylov algorithms: an adaptive model order reduction for highly nonlinear mechanical problems*. Computer Methods in Applied Mechanics and Engineering, 200(5-8):850-866, 2011.

Application to a parametric fracture problem

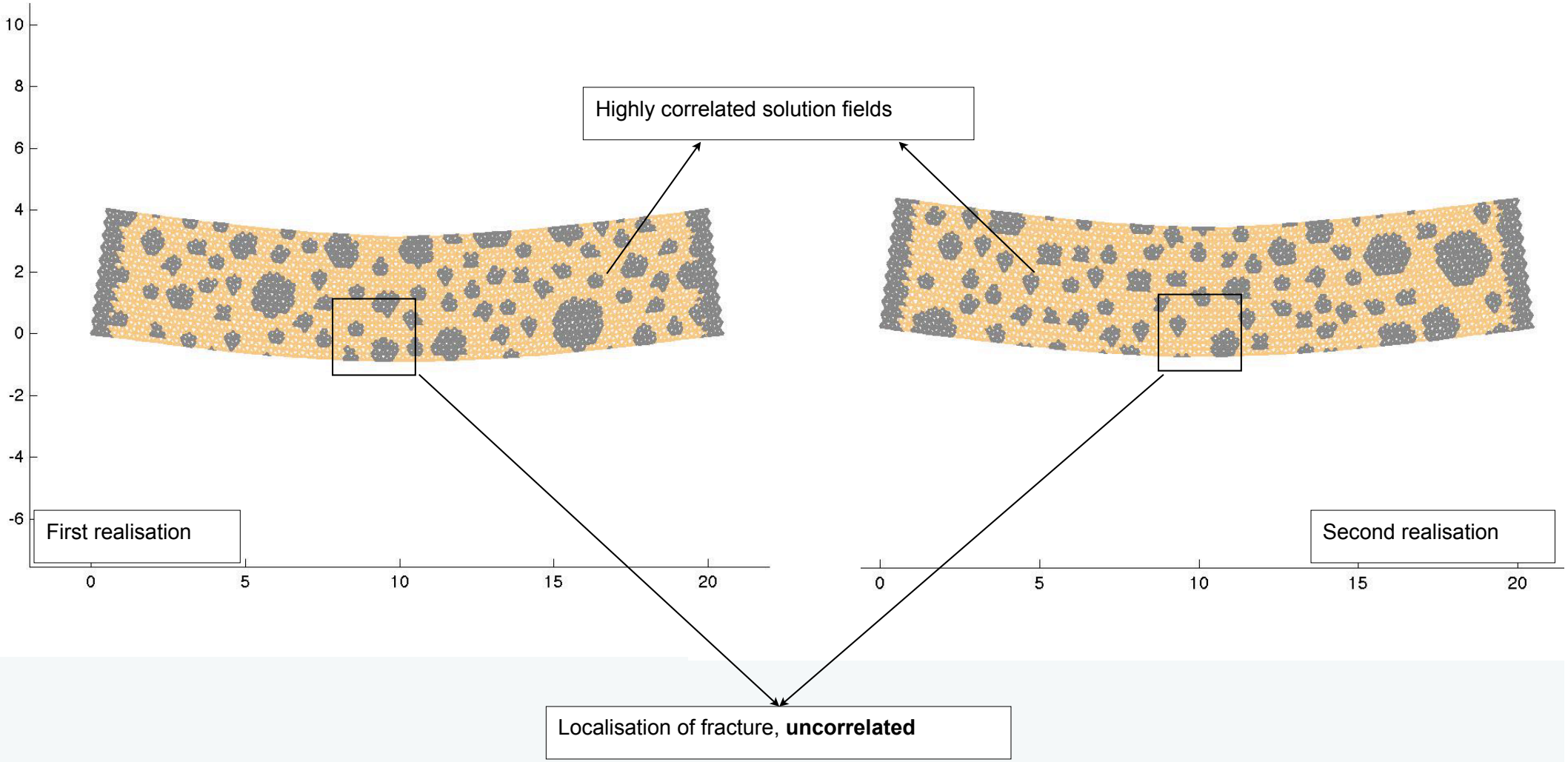


Application to a parametric fracture problem



- The POD solution is not able to reproduce the solution in the cracked area
- Due to lack of correlation introduced by crack growth
- Leads to a local projection error

Parametric / stochastic multiscale fracture mechanics



➡ Direct numerical simulation: efficient preconditioner?

➡ Reduced order modelling?

➡ Adaptive coupling?

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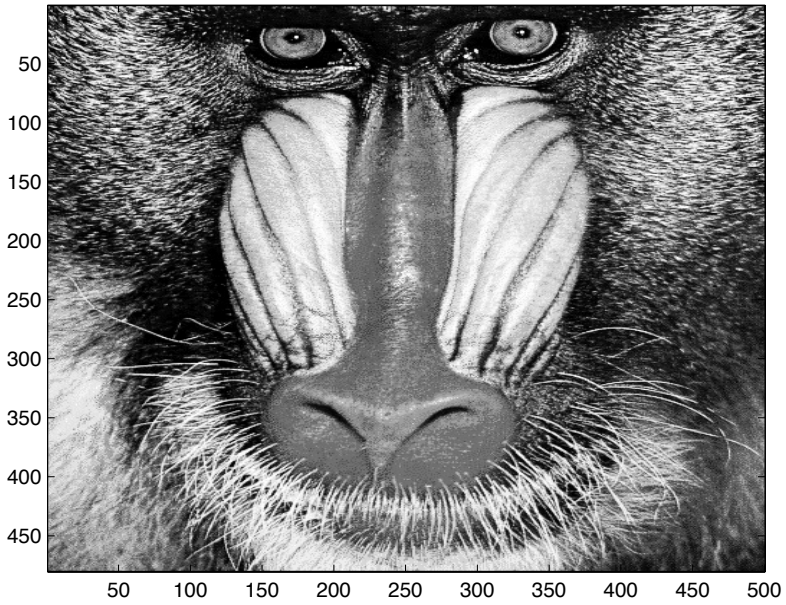
THE RETURN OF THE MONKEY!



What can we do to address this lack of separation of scales/reducibility?

P. Kerfriden, P. Gosselet, S. Adhikari, and S. Bordas. Bridging proper orthogonal decomposition methods and augmented Newton-Krylov algorithms: an adaptive model order reduction for highly nonlinear mechanical problems. *Computer Methods in Applied Mechanics and Engineering*, 200(5-8):850–866, 2011.

P. Kerfriden, J.C. Passieux, and S. Bordas. Local/global model order reduction strategy for the simulation of quasi-brittle fracture. *International Journal for Numerical Methods in Engineering*, 89(2):154–179, 2011.

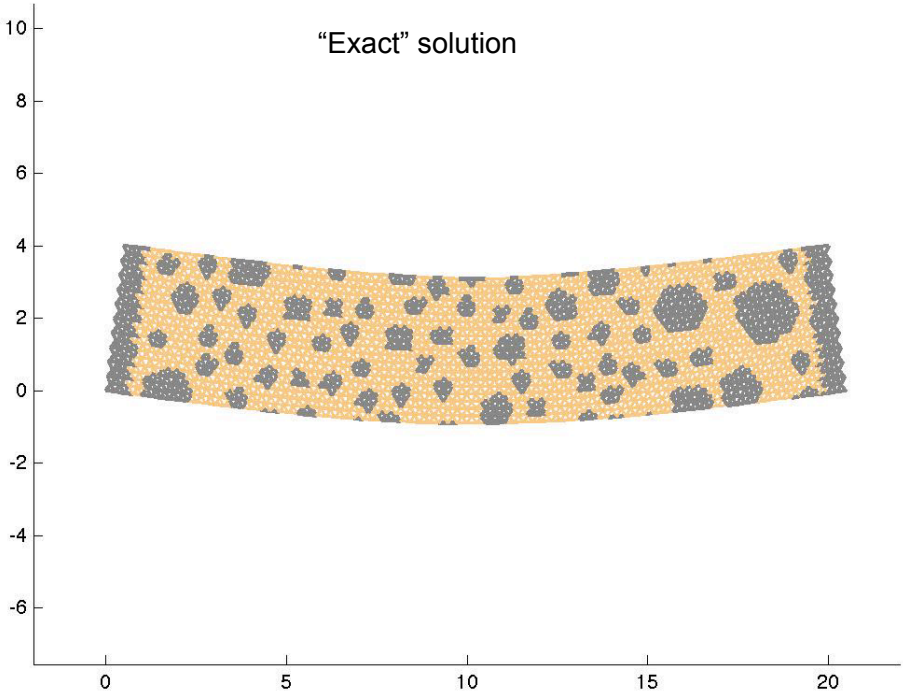
P. Kerfriden, K.M. Schmidt, T. Rabczuk, and Bordas S.P.A. Statistical extraction of process zones and representative subspaces in fracture of random composites. *Accepted for publication in International Journal for Multiscale Computational Engineering, arXiv:1203.2487v2*, 2012.

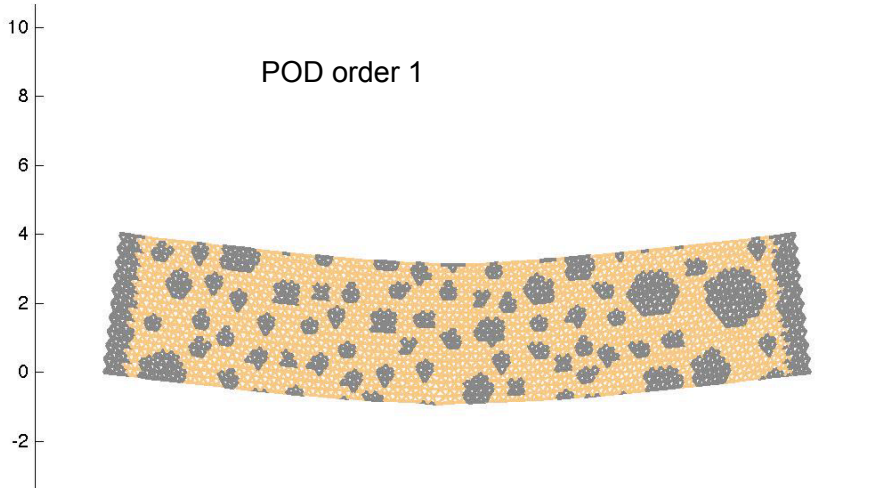
http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3672853/

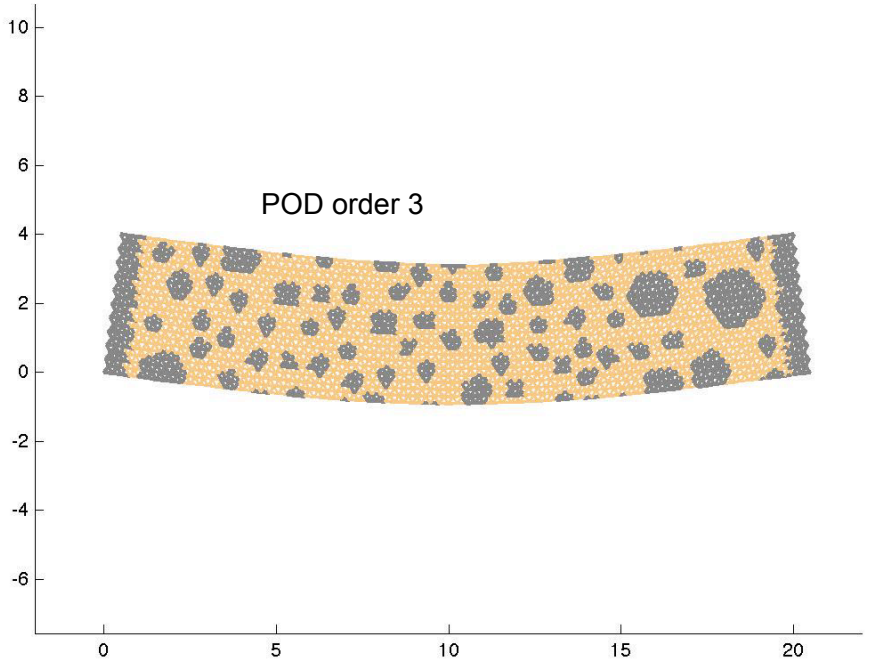
http://orbilu.uni.lu/bitstream/10993/12454/2/presentationUSNCCM

Data compression: fracture

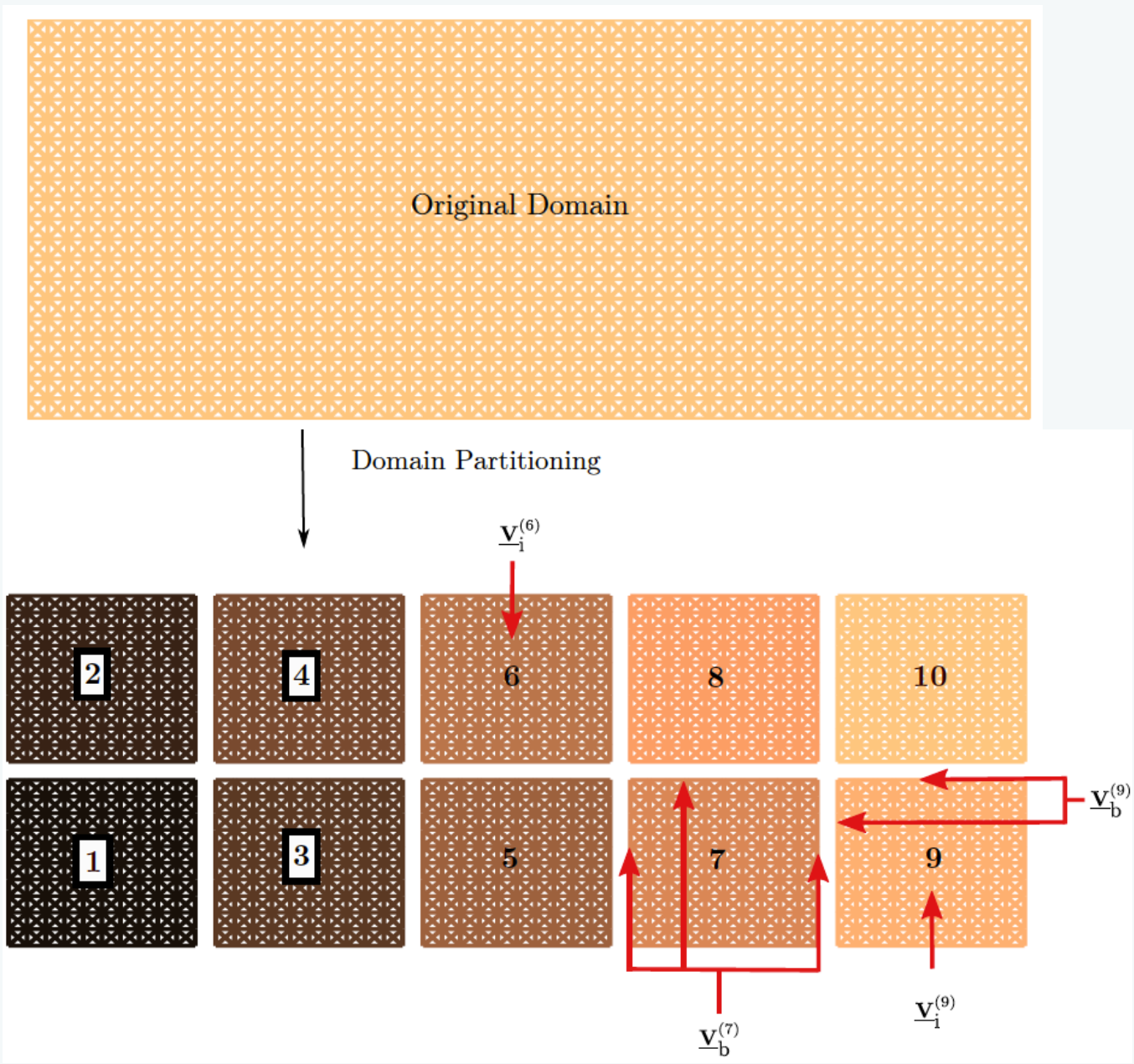
Snapshot POD (snapshot space is spanned by the ensemble of solutions at all time steps)



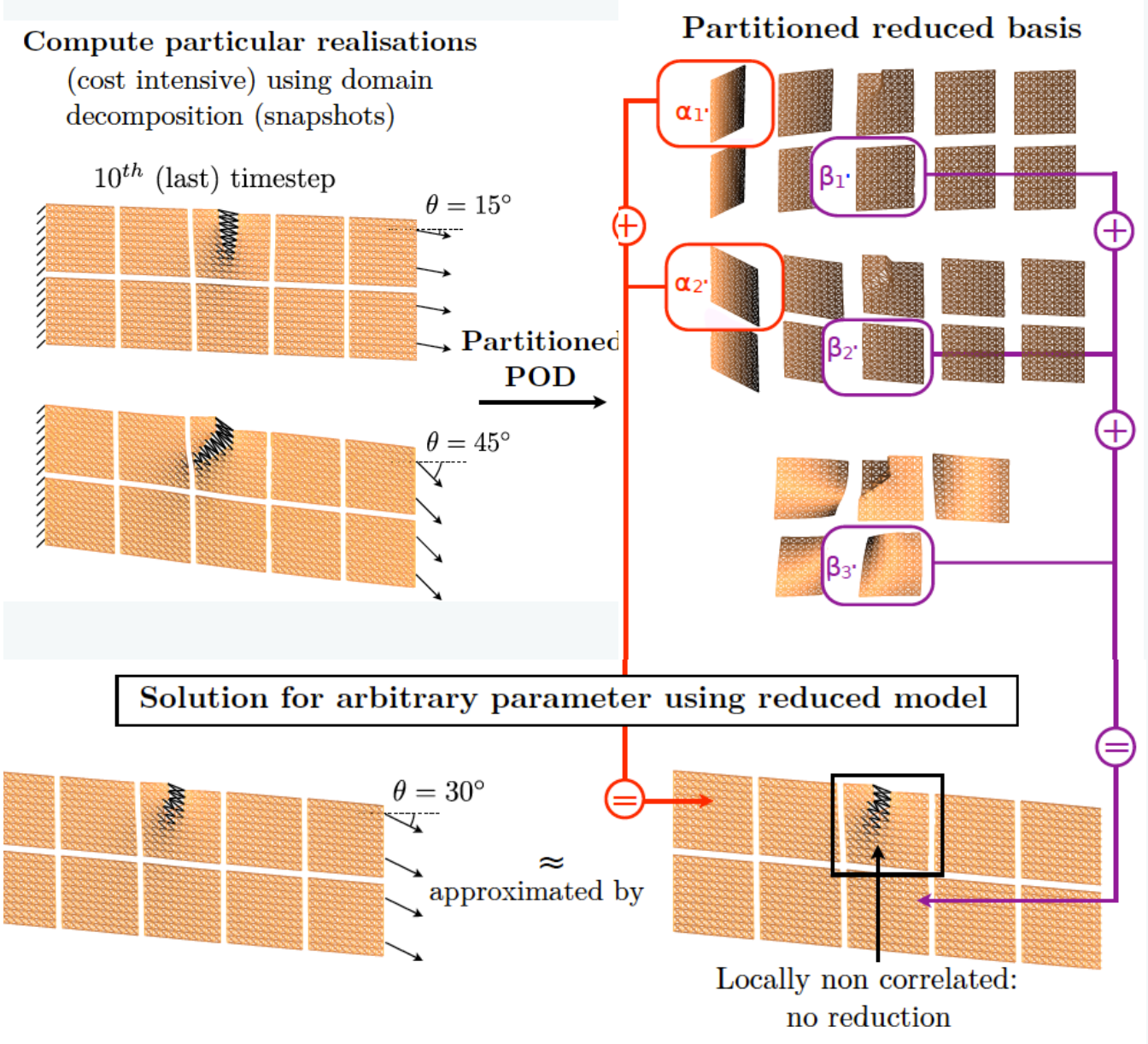




Partitioned POD/DDM

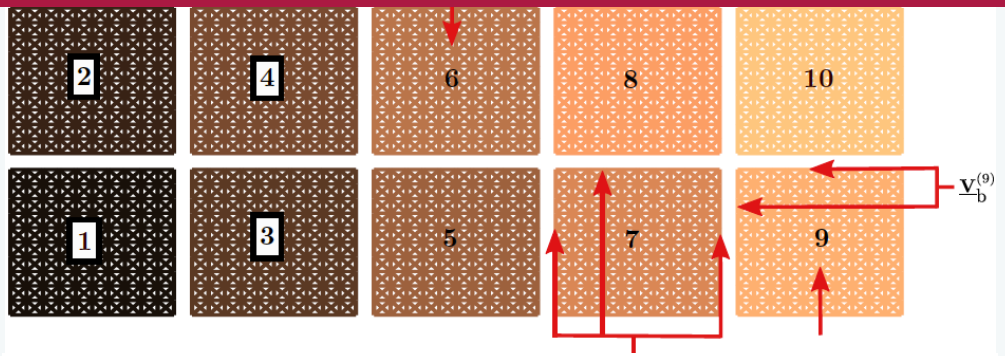


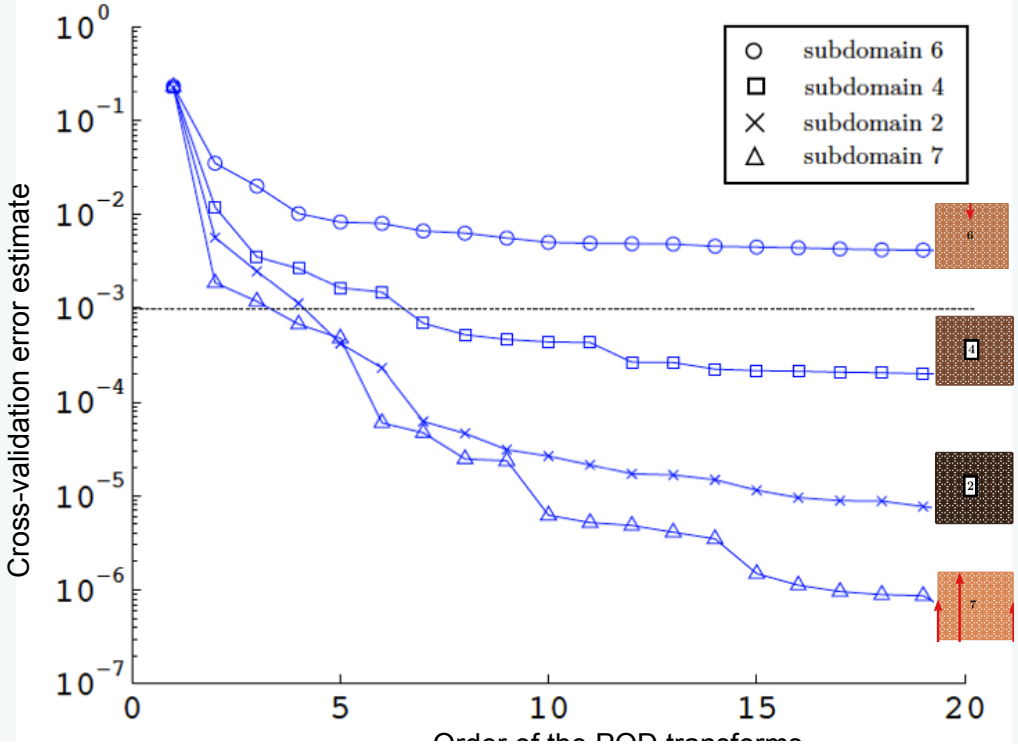
Reduced DDM-POD



- Decompose the structure into subdomains
- Perform a reduction in the highly correlated region
- Couple the reduced to the non-reduced region by a primal Schur complement

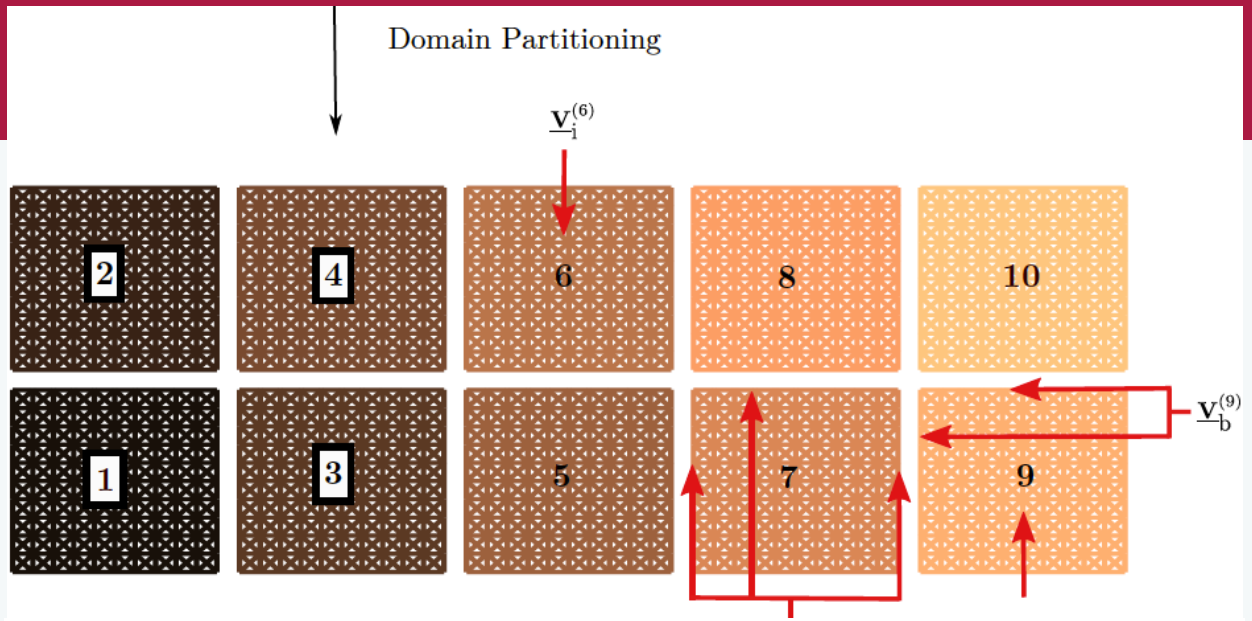
Choice of the reduced subdomains: local error estimation by “leave one out cross-validation” (LOOCV)

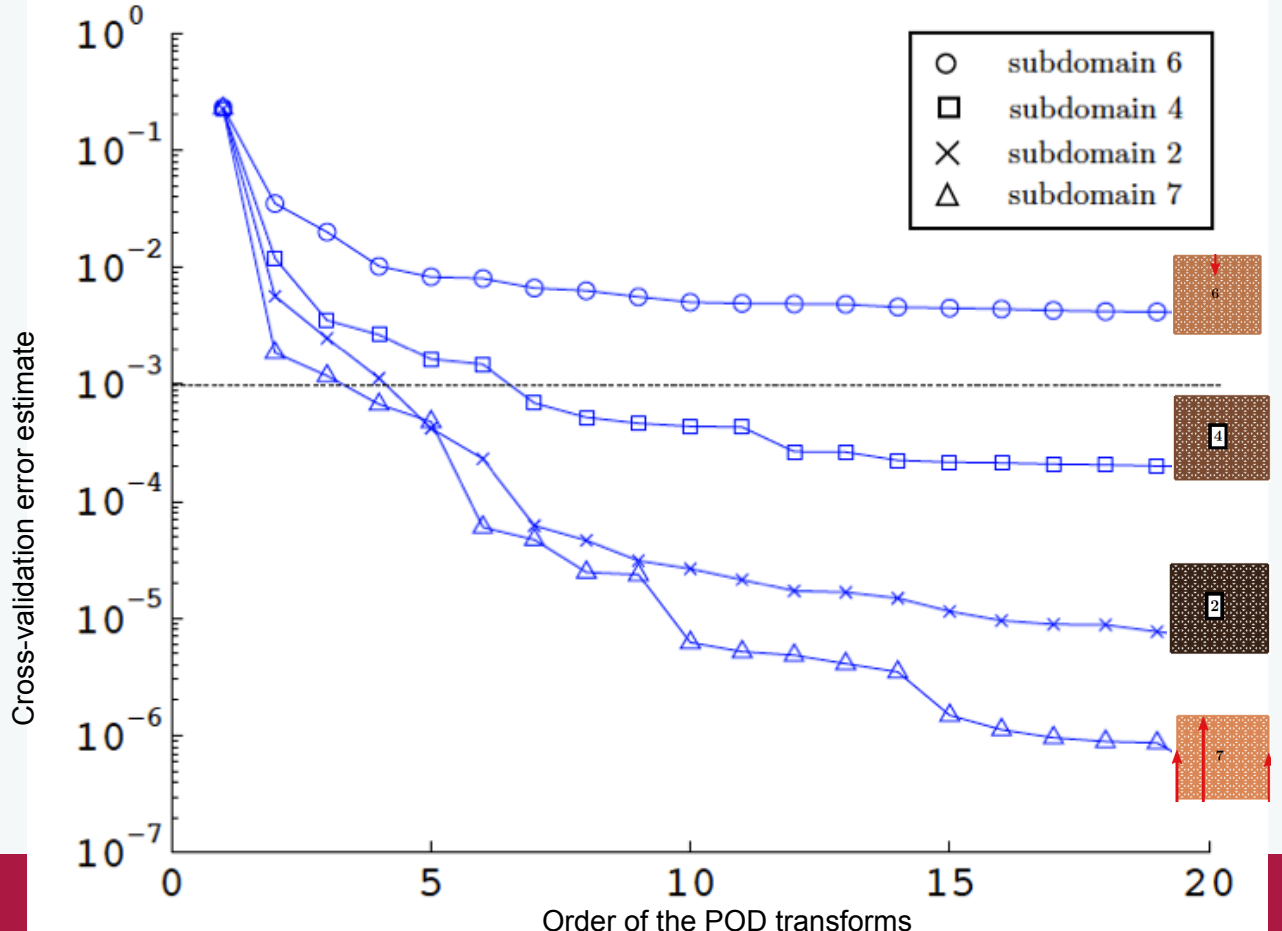




$$\left(\tilde{\nu}_{\mathrm{snap}}^{(e)}\right)^2 = \frac{\sum_{\mu\in\mathcal{P}^{\mathrm{s}}}\sum_{t_n\in\mathcal{T}^{\mathrm{h}}}\left\|\underline{\mathbf{U}}_{\mathrm{i}}(t_n,\mu)-\sum_{j=1}^{n_{\mathrm{c}}^{(e)}}\left(\underline{\tilde{\mathbf{C}}}_{\mathrm{i},j}^{(e),(\mu)\,T}\,\underline{\mathbf{U}}_{\mathrm{i}}(t_n,\mu)\right)\underline{\tilde{\mathbf{C}}}_{\mathrm{i},j}^{(e),(\mu)}\right\|_2^2}{\sum_{t_n\in\mathcal{T}^{\mathrm{h}}}\sum_{\mu\in\mathcal{P}^{\mathrm{s}}}\|\underline{\mathbf{U}}_{\mathrm{i}}(t_n,\mu)\|_2^2}$$

- Reduced subspaces are independent and we assume a snapshot is *a priori* available
  - (1) Dimension of the local space for each subdomain?
  - (2) Is a given subdomain is reducible?
- (1) and (2) will be treated by cross-validation (e.g. W. J. Krzanowski. Cross-validation in principal component analysis. Biometrics, 43(3):575-584, 1987.)
  - **Training set**: snapshot
  - **Validation set**: set of additional finescale solutions
  - Independent training/validation avoids overfitting
  - Cross validation **emulates independence**. Error calculated using the local reduced basis obtained by a snapshot POD transform of all the available snapshot solutions except the one corresponding to the value of the summation variable.
- **NOTE:** If the snapshot is not assumed *a priori* then
  - Assess whether the snapshot contains sufficient information, and generate additional, suitable, data if required
  - Most analysis (mostly by statisticians) assume the snapshot is known *a priori*. Recent review: Hervé Abdi and Lynne J. Williams. Principal component analysis. Wiley Interdisciplinary Reviews: Computational Statistics, 2(4):433{459, 2010.


Domain Partitioning
$\underline{\mathbf{v}}_{\mathrm{i}}^{(6)}$
2
4
6
8
10
1
3
5
7
9
$\underline{\mathbf{v}}_{\mathrm{b}}^{(9)}$

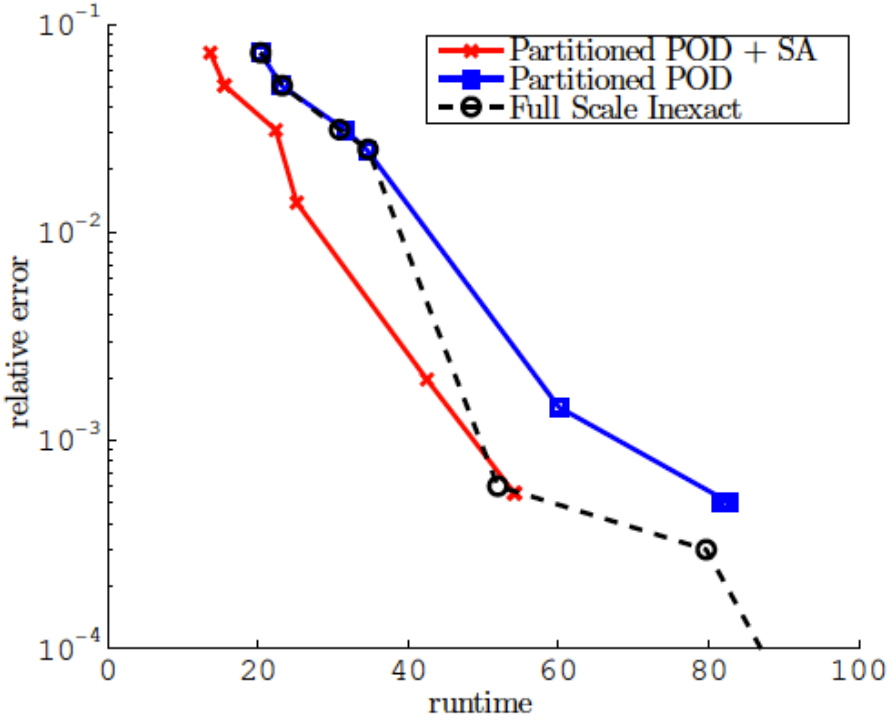

subdomain 6
subdomain 4
subdomain 2
subdomain 7
Cross-validation error estimate
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
5
10
15
20
Order of the POD transforms

Performance: load angle 40 | 27 - 121 nodes

- Relative error

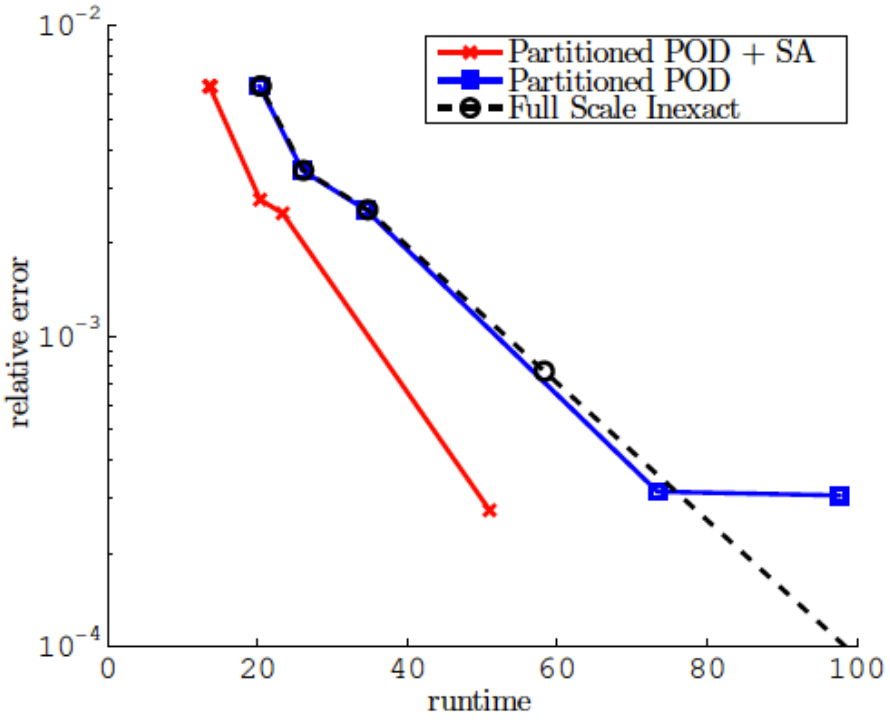
$$\nu^{\mathrm{app},(\mu)}\left(\underline{\mathbf{U}}^{\mathrm{app}}\right)^2 = \frac{\sum_{t_n \in \mathcal{T}^h} \|\underline{\mathbf{U}}^{\mathrm{app}}(t_n,\mu) - \underline{\mathbf{U}}^{\mathrm{ex}}(t_n,\mu)\|_2^2}{\sum_{t_n \in \mathcal{T}^h} \|\underline{\mathbf{U}}^{\mathrm{ex}}(t_n,\mu)\|_2^2}$$

40^o



(a) Relative error for the different models using 121 nodes per subdomain

27^o



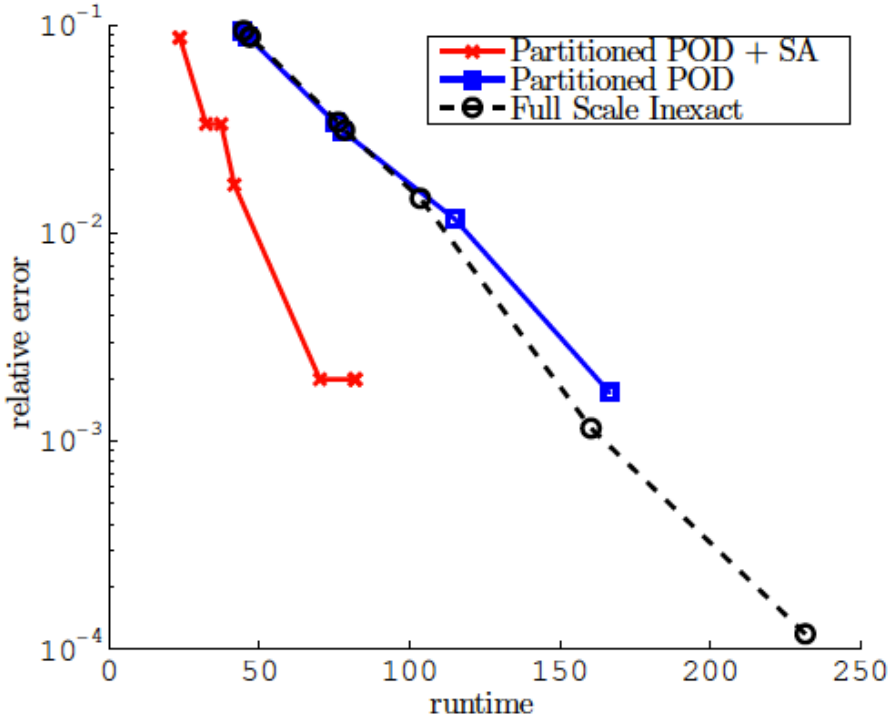
(a) Relative error for the different models using 121 no per subdomain

Performance: load angle 40 | 27 - 256 nodes

- Relative error

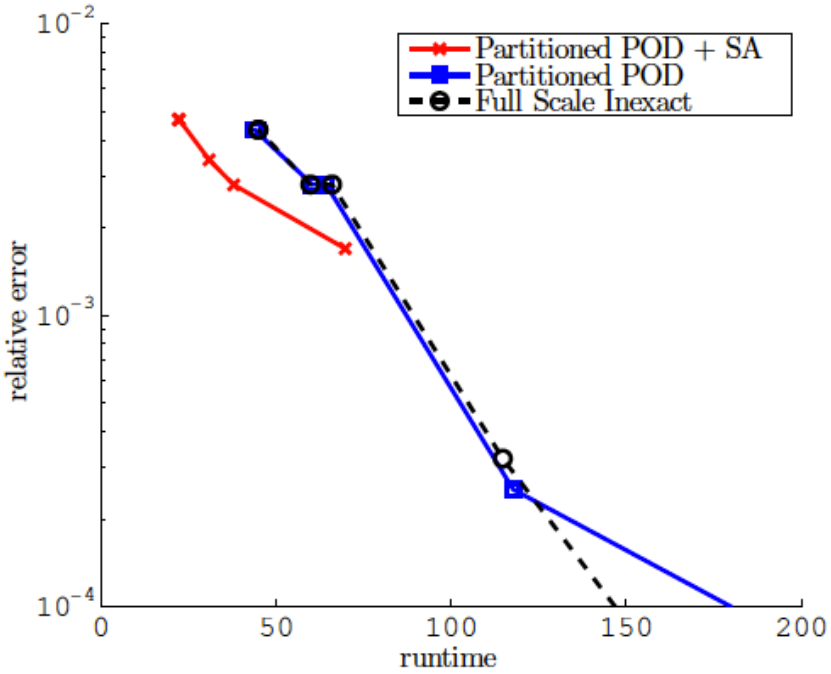
$$\nu^{\mathrm{app},(\mu)}\left(\underline{\mathbf{U}}^{\mathrm{app}}\right)^2 = \frac{\sum_{t_n \in \mathcal{T}^h} \left\| \underline{\mathbf{U}}^{\mathrm{app}}(t_n,\mu) - \underline{\mathbf{U}}^{\mathrm{ex}}(t_n,\mu) \right\|_2^2}{\sum_{t_n \in \mathcal{T}^h} \left\| \underline{\mathbf{U}}^{\mathrm{ex}}(t_n,\mu) \right\|_2^2}$$

40^o



(b) Relative error for the different models using 256 nodes per subdomain

27^o



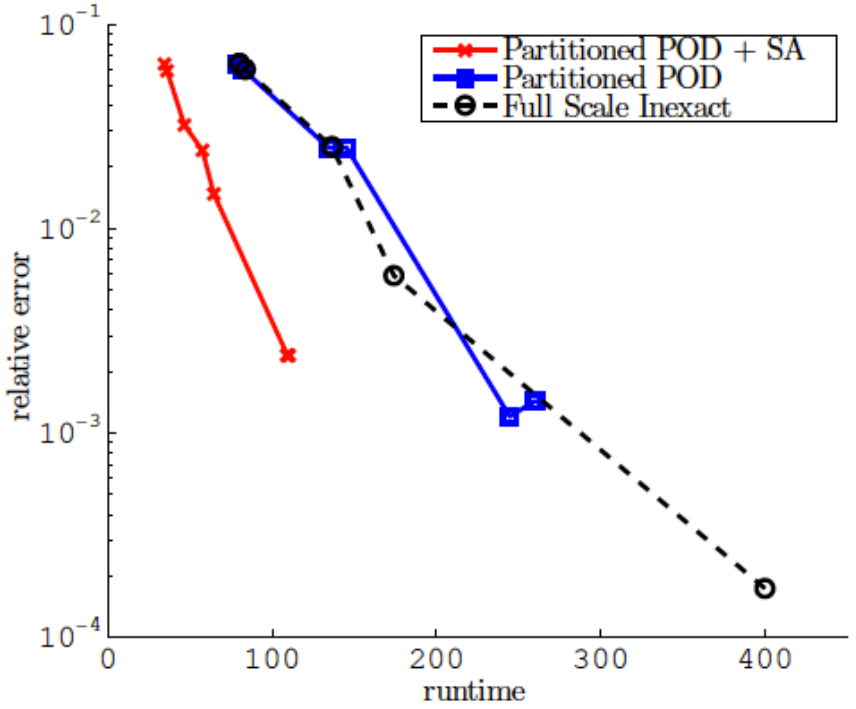
(b) Relative error for the different models using 256 nodes per subdomain

Performance: load angle 40 | 27 - 441 nodes

- Relative error

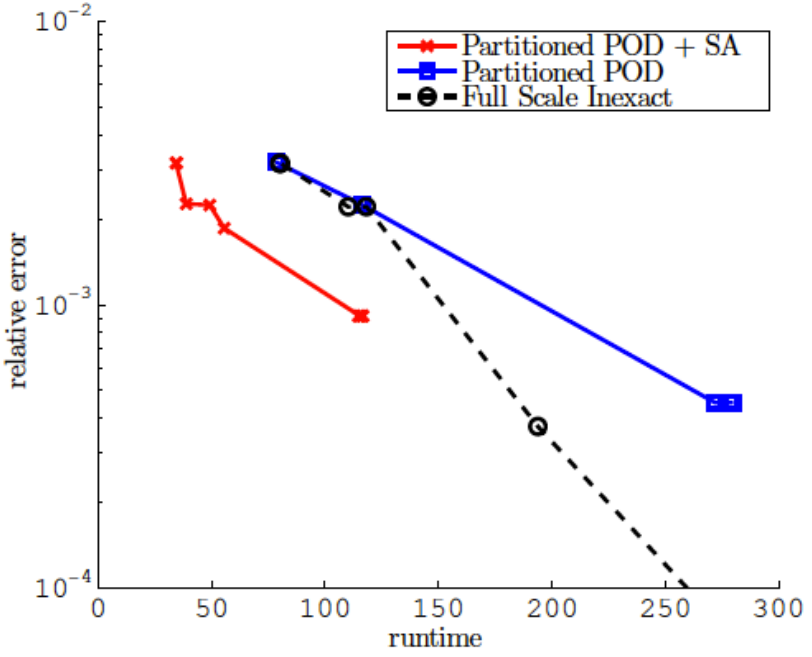
$$\nu^{\mathrm{app},(\mu)}\left(\underline{\mathbf{U}}^{\mathrm{app}}\right)^2 = \frac{\sum_{t_n \in \mathcal{T}^h} \|\underline{\mathbf{U}}^{\mathrm{app}}(t_n,\mu) - \underline{\mathbf{U}}^{\mathrm{ex}}(t_n,\mu)\|_2^2}{\sum_{t_n \in \mathcal{T}^h} \|\underline{\mathbf{U}}^{\mathrm{ex}}(t_n,\mu)\|_2^2}$$

40^o



(c) Relative error for the different models using 441 nodes per subdomain

27^o

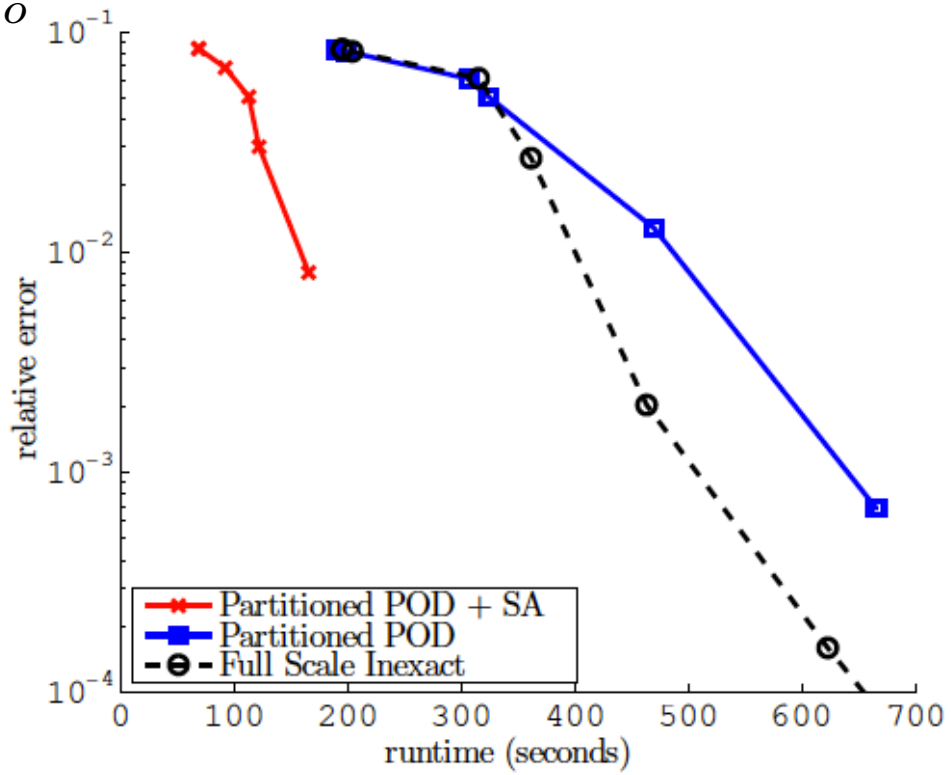


(c) Relative error for the different models using 441 nodes per subdomain

- Relative error

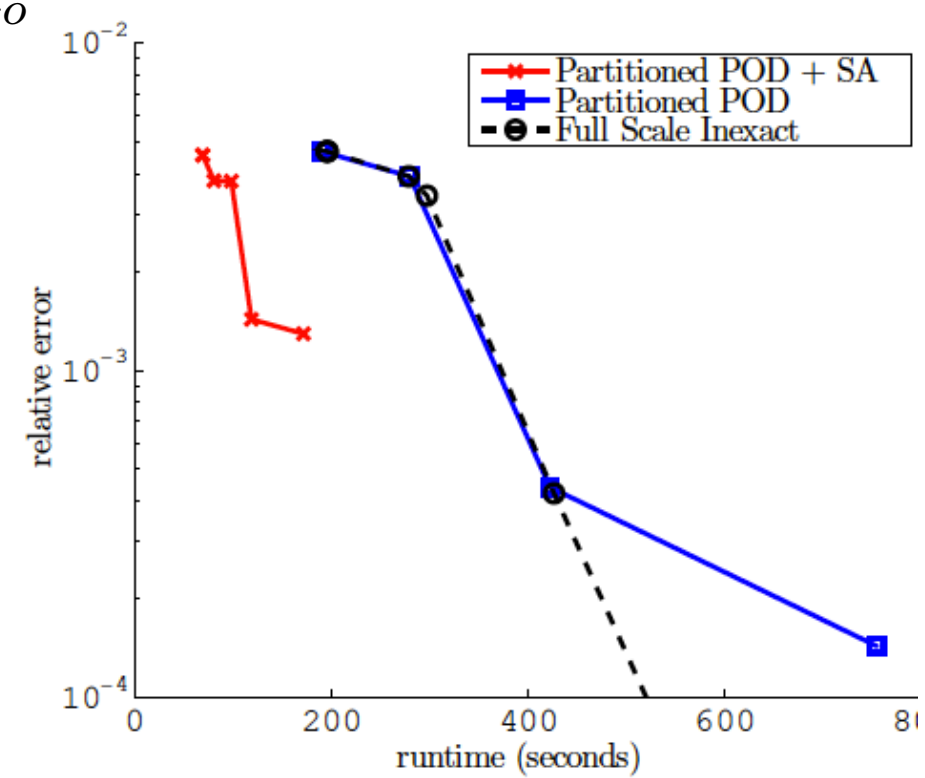
$$\nu^{\mathrm{app},(\mu)}\left(\underline{\mathbf{U}}^{\mathrm{app}}\right)^2 = \frac{\sum_{t_n \in \mathcal{T}^h} \|\underline{\mathbf{U}}^{\mathrm{app}}(t_n,\mu) - \underline{\mathbf{U}}^{\mathrm{ex}}(t_n,\mu)\|_2^2}{\sum_{t_n \in \mathcal{T}^h} \|\underline{\mathbf{U}}^{\mathrm{ex}}(t_n,\mu)\|_2^2}$$

40^o



(d) Relative error for the different models using 961 nodes per subdomain

27^o



(d) Relative error for the different models using 961 per subdomain

CARDIFF UNIVERSITY
PRIFYSGOL CAERDYDD

Applications to surgical simulation

with INRIA, France; Karol Miller, UWA.

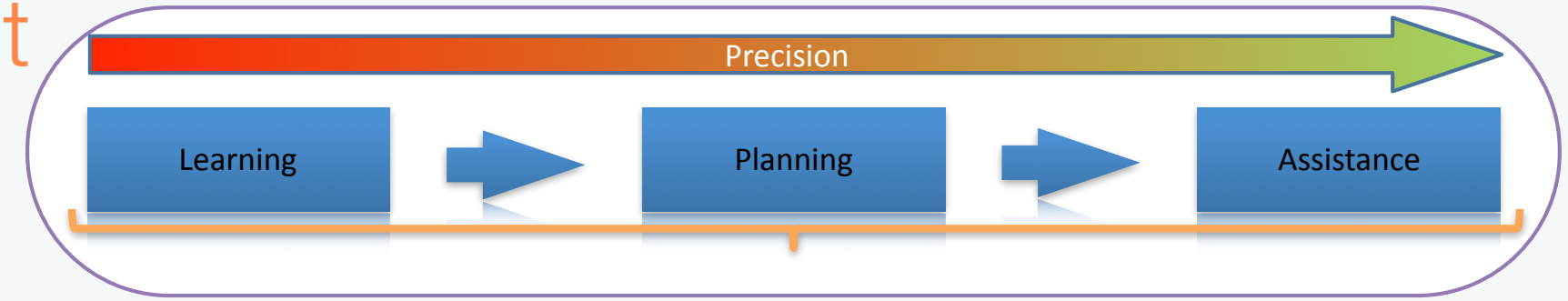
erc

RealTcut

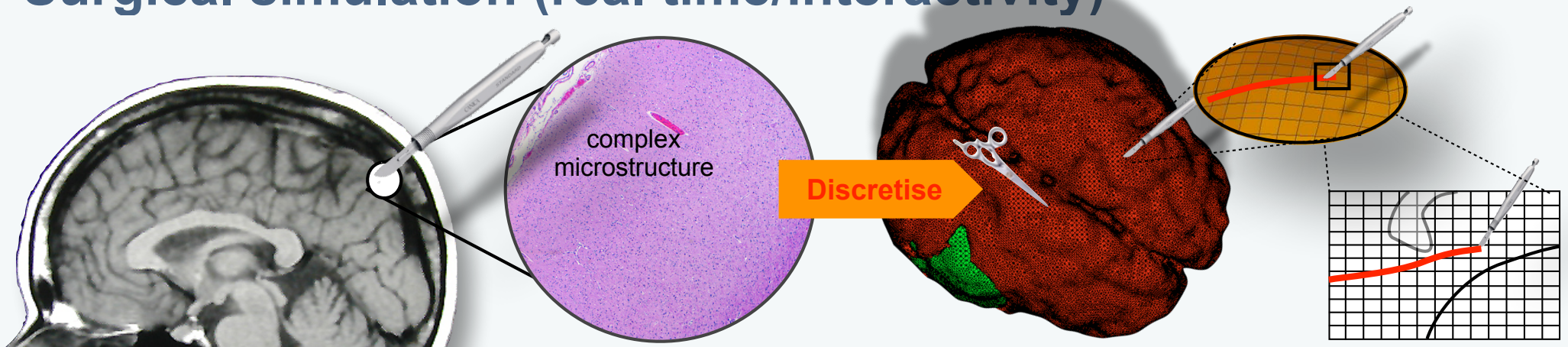
Interactive multiscale cutting simulations

IMAM
Institute of Mechanics & Advanced Materials

RealTcut

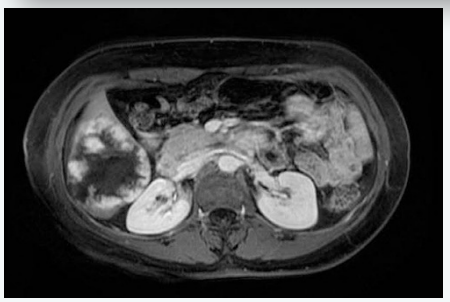
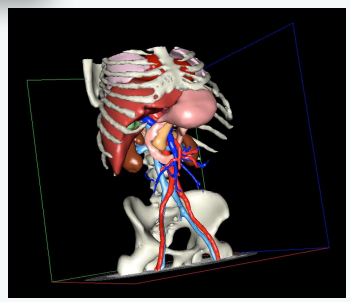
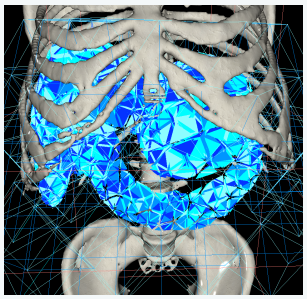
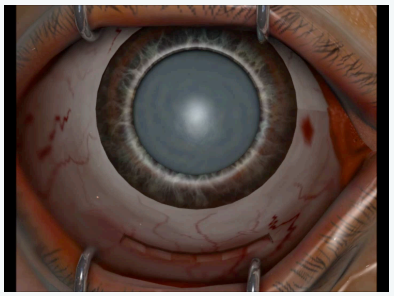
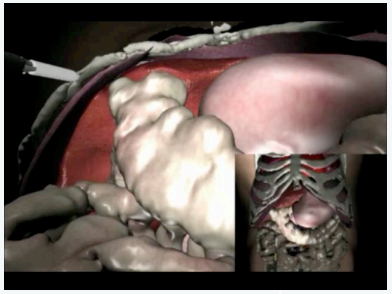


Surgical simulation (real time/interactivity)



- Reduce the problem size while controlling error in solving very large multiscale mechanics problems

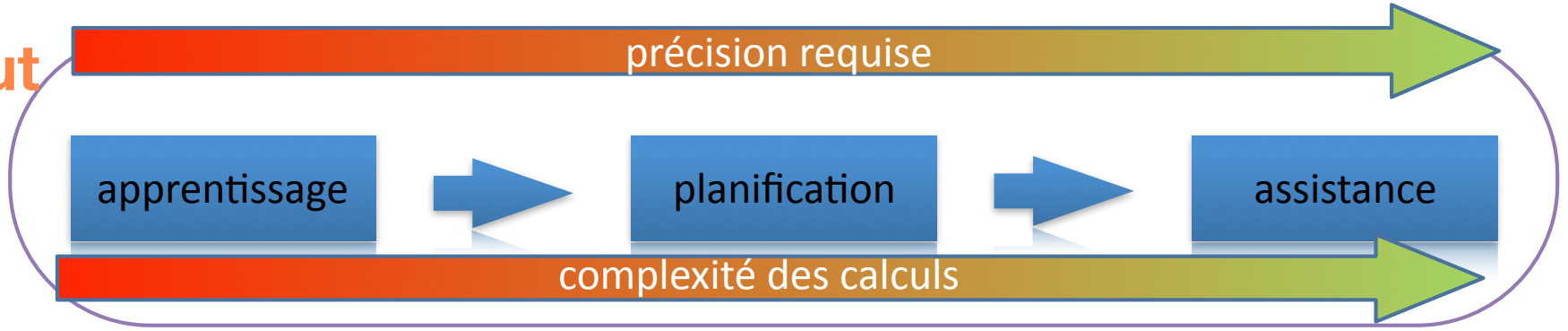
Courtecuisse et al. PBMB 2011

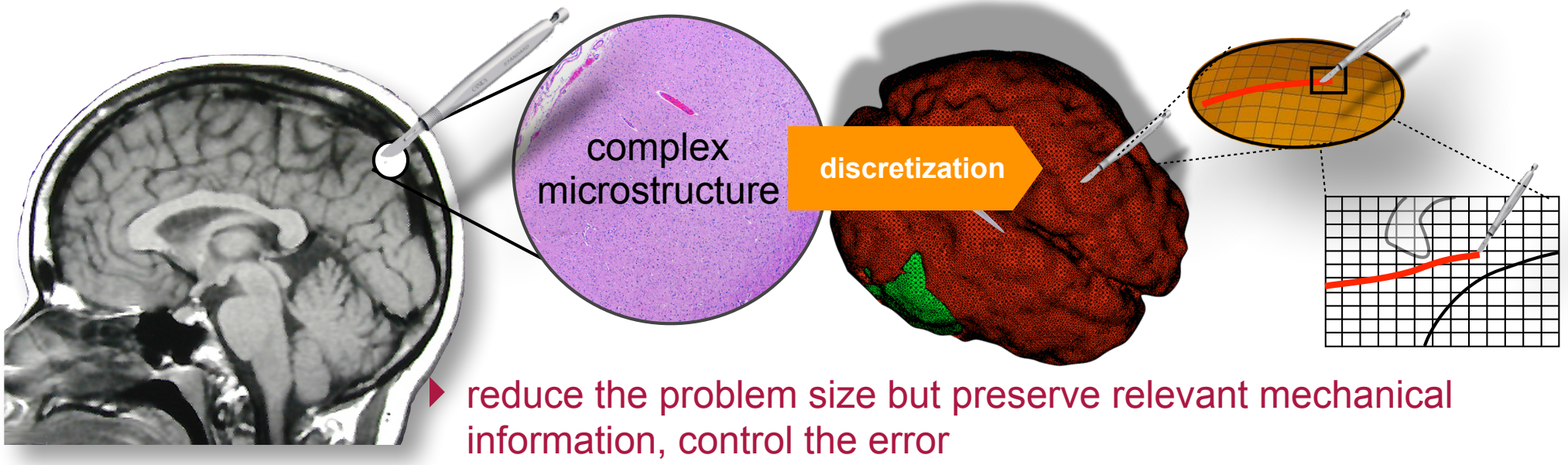
Interactive simulation of cutting in soft tissue

RealTcut

erc



Real-time/interactivity for non-linear problems involving topological changes



- reduce the problem size but preserve relevant mechanical information, control the error

Approach

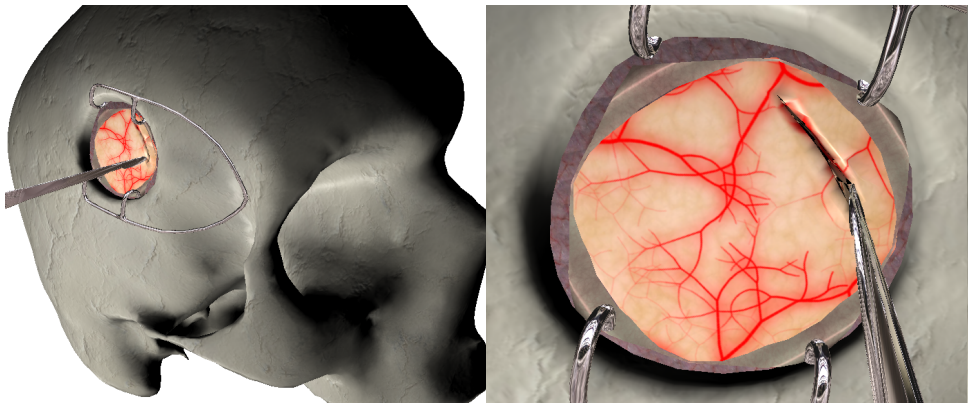
Model reduction

Concrete objective: compute the response of organs during surgical procedures (including cuts) in real time (50-500 solutions per second)

Two schools of thought

- constant time
  - ➡ accuracy often controlled visually only
- model reduction or “learning”
  - ➡ scarce development for biomedical problems
  - ➡ no results available for cutting

First implicit, interactive method for cutting with contact

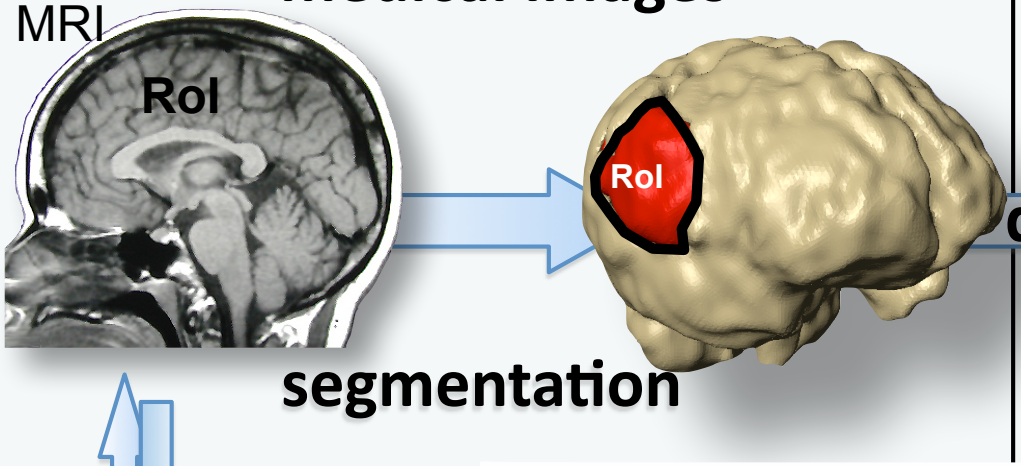


[Courtecuisse et al., MICCAI, 2013]
Collaboration INRIA

Proposed approach: maximize accuracy for given computational time. Error control

Four main difficulties

Complex geometries from medical images

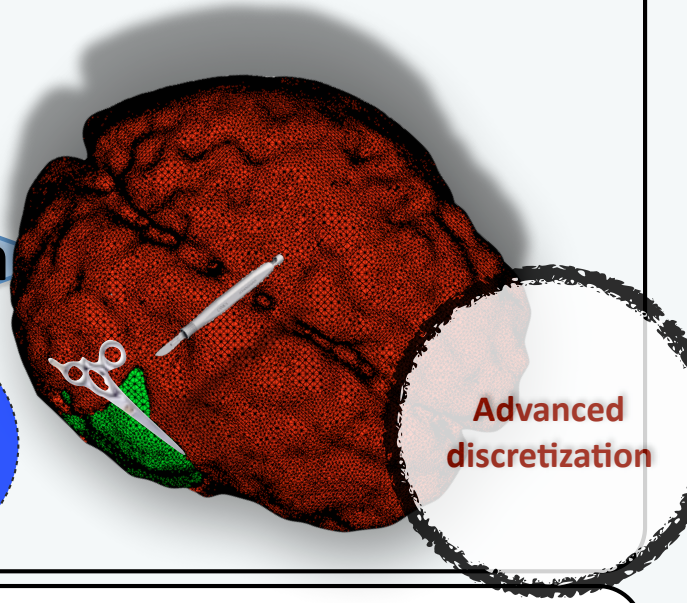


segmentation

Region of interest (RoI)

discretization

Topological changes & contact



Model reduction

adaptivité

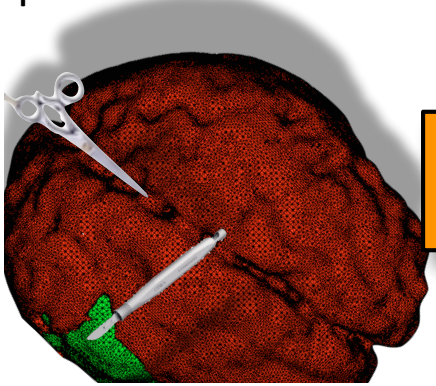
Error control

- interactivity
- space-time discretization?
- optimize use of compute resources

Verification & Validation

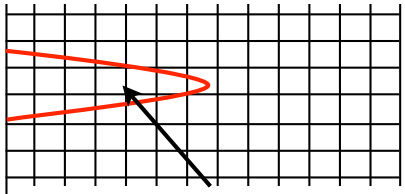
calculs **offline**

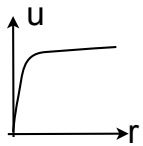
génération solutions particulières



~10^6 snapshots

calcul champs asymptotiques





action de l'instrument

tri
pré-opératoire

~10^3 snapshots

"mapping" spécifique patient

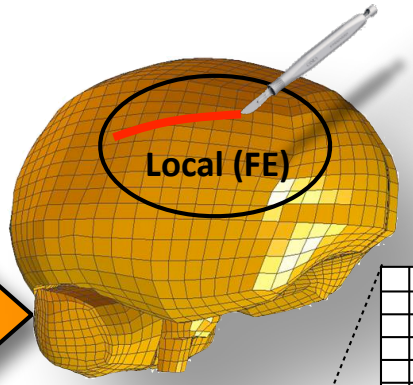
POD

O(10) fonctions

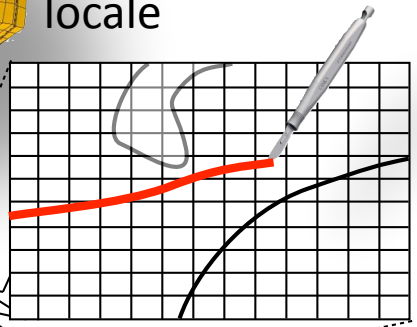
espace réduit de petite dimension

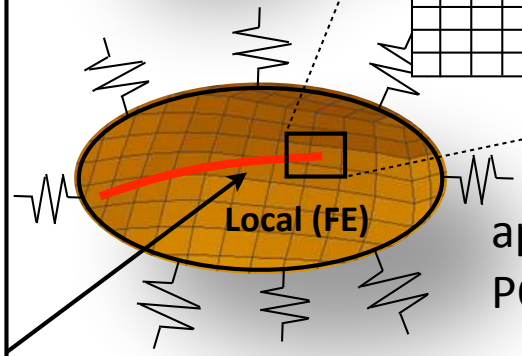
enrichissement "pointe de coupe"

calculs **online**: interactivité



représentation locale





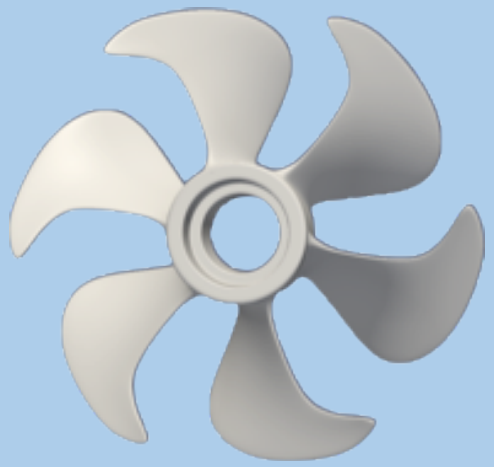
approximation POD globale

A semi-implicit method for real-time deformation, topological changes, and contact of soft tissues

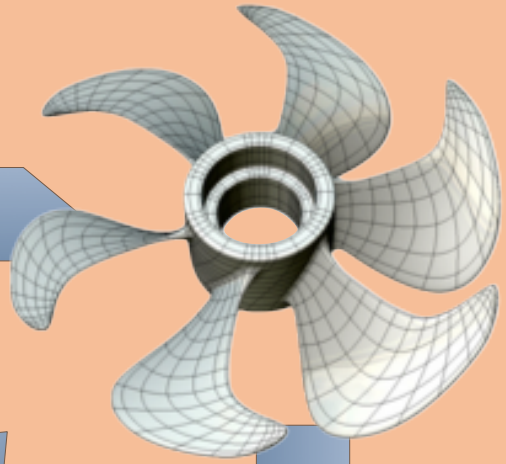
Paper ID : 269


There's a fine line between wrong and visionary.
Unfortunately, you have to be a visionary to see it.
Sheldon Cooper,
The Big Bang Theory: The Pirate Solution

GEOMETRICAL MODEL



DISCRETISATION



Verification

A POSTERIORI ERROR CONTROL

MATERIAL MODELS

Phenomenological

Elasticity/Plasticity

Crack growth law (Paris…)

Fracture energy

Maximum tensile strength

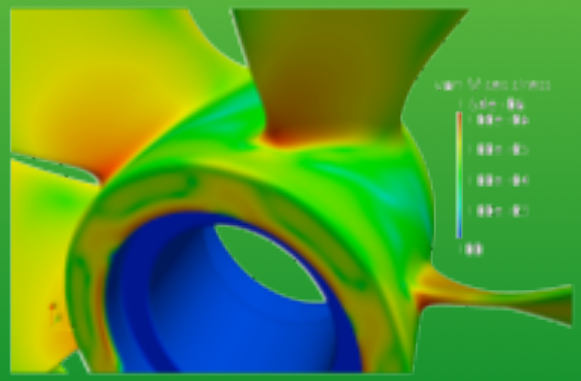
Multi-scale

Debonding, Fibre pull-out

Fibre breakage, interface fracture,

grains, dislocations, MD, quantum…

NUMERICAL SOLUTION



Validation & parameter identification

EXPERIMENTS

CONVENTIONAL APPROACH

Worst load combination

Scales of interest

Crack growth rate

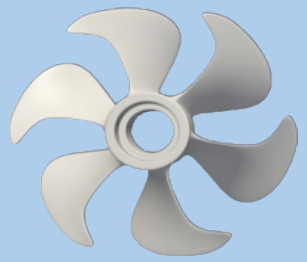
INFORMATION

Inspection interval

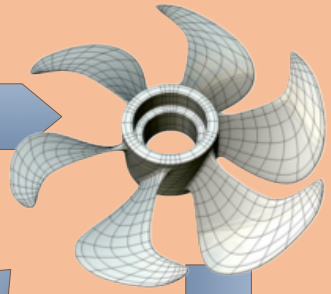
Mission?

DIGITAL TWIN OF THE SYSTEM

GEOMETRICAL MODEL



DISCRETISATION

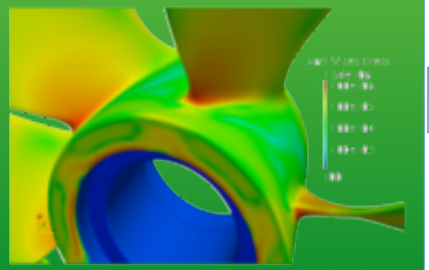


A POSTERIORI ERROR

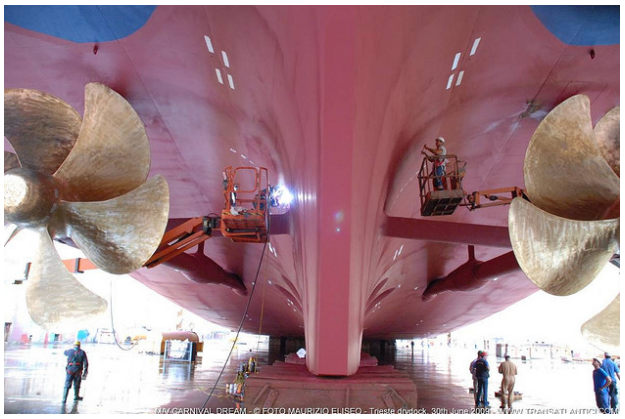
LEARN MATERIAL MODELS

which scales?
what models?
what parameters?
what scale transition?
what data is missing?

NUMERICAL SOLUTION



REAL SYSTEM



DATA

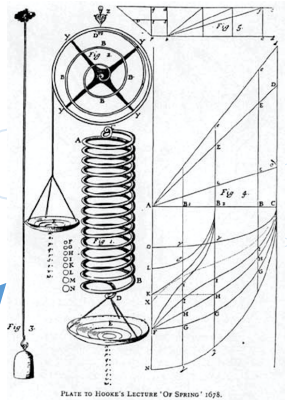
Strain

Environment Conditions

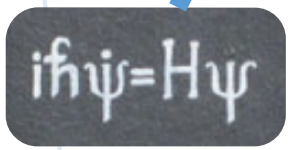
Structural Health

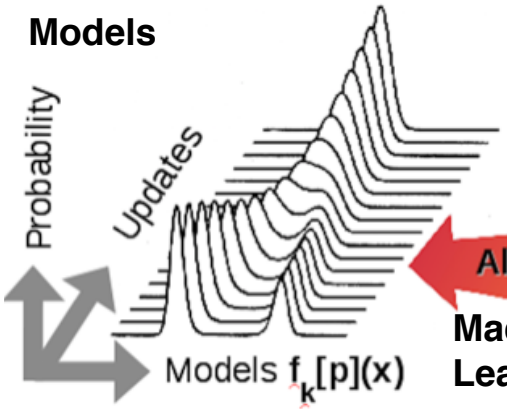
Cracks

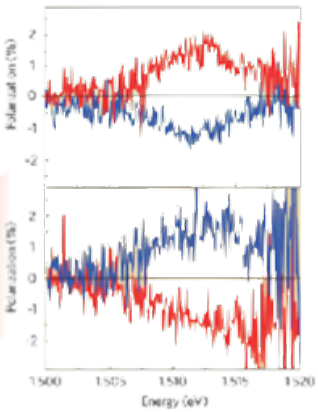
VISION



Scales++







data

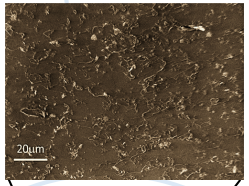
a-priori knowledge

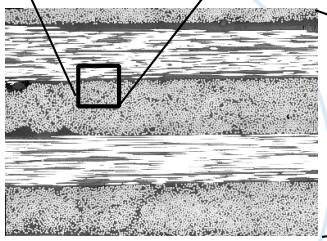
automatic model selection

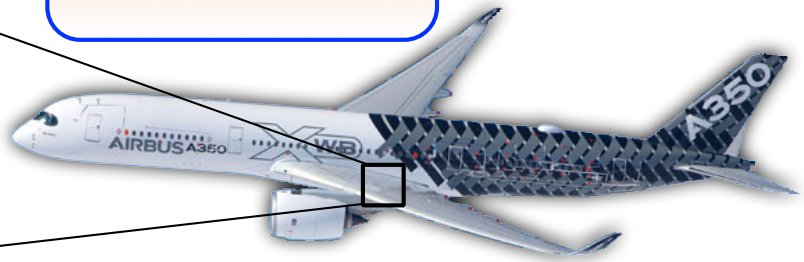
General approach
No predefined model

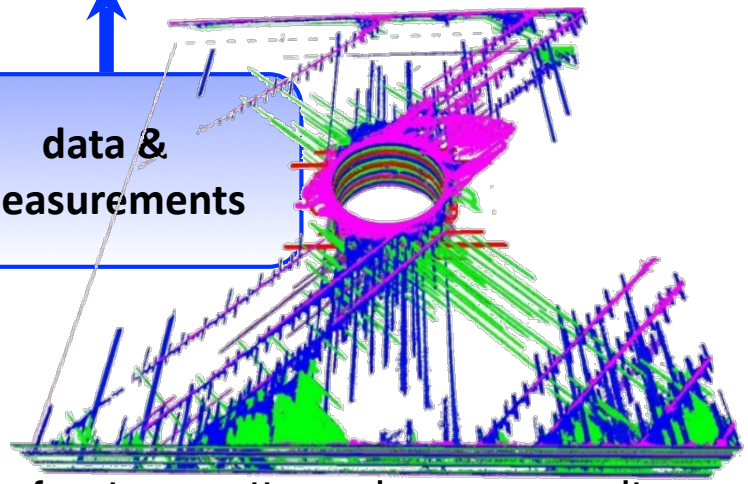
upscaling techniques

data & measurements





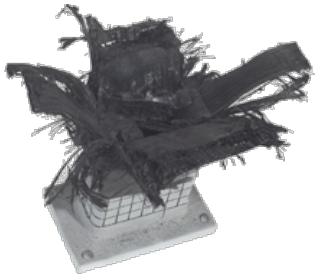


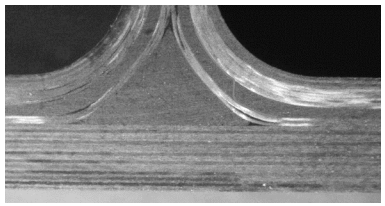


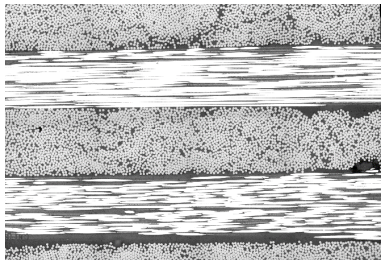
fracture patterns in a composite pan

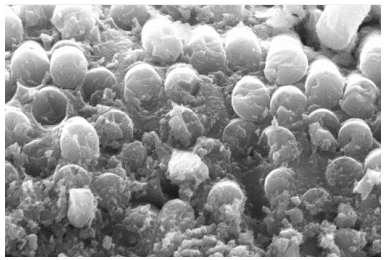
Digital Twins…

Characterisation

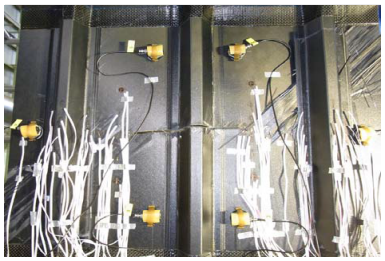


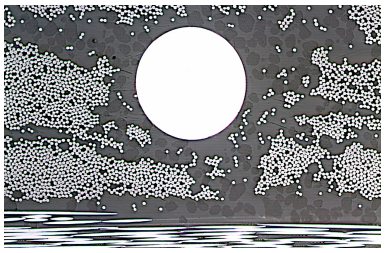


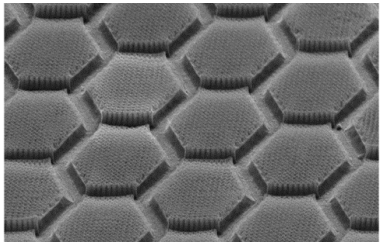




Monitoring







Multiscale models are **unreliable**

Quantitative **predictions ?**

Fracture/lack of scale separation

Learn better models

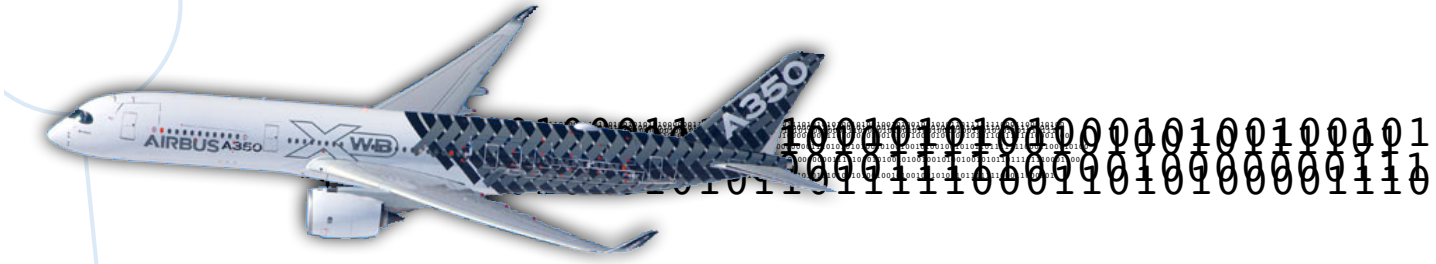
Measure

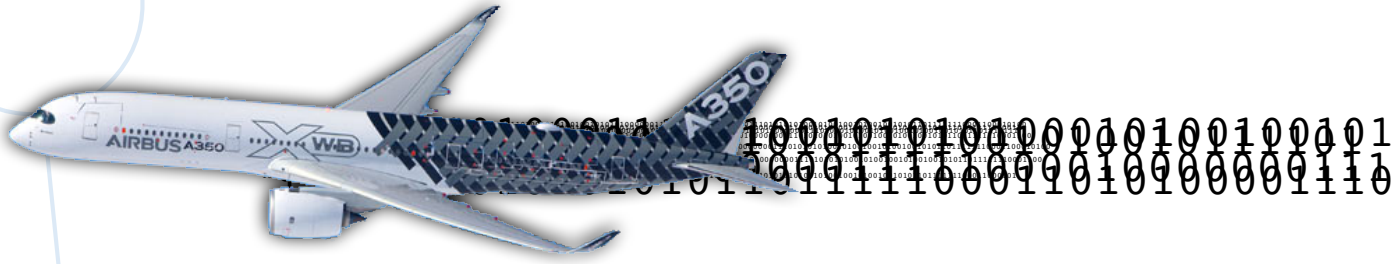
Analyse & **Learn** from **data**

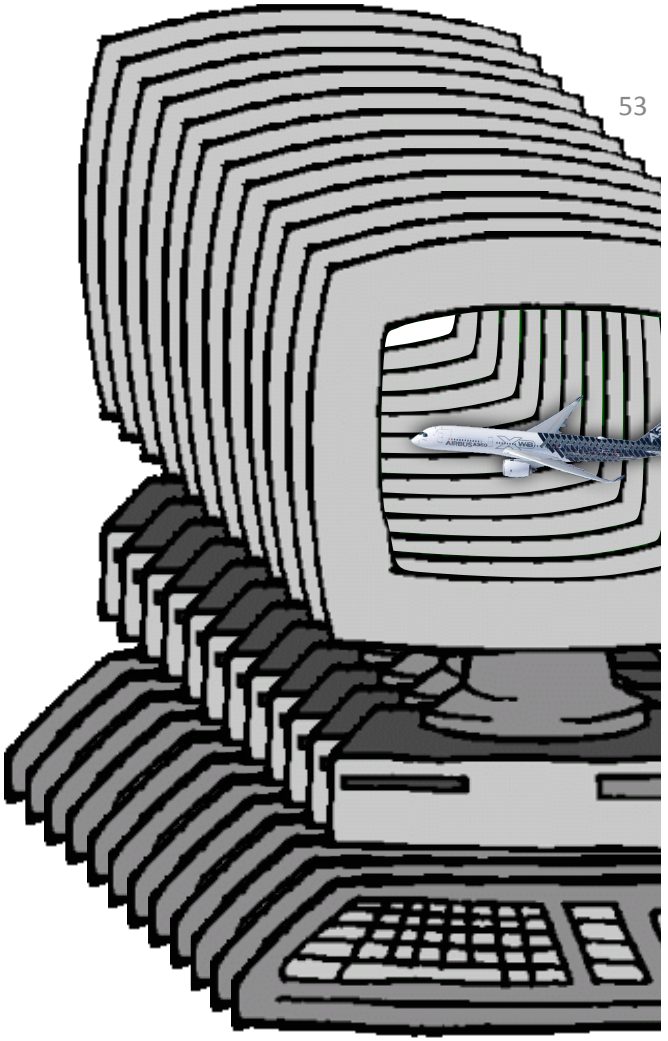
Improved model

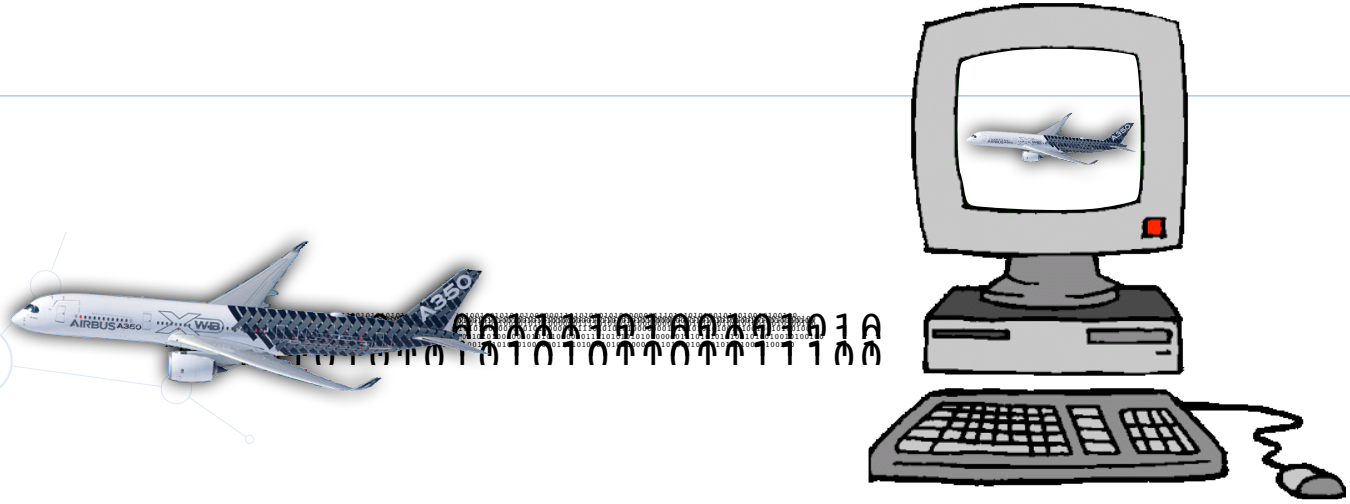
Validate

Identify missing data


AIRBUS A350
XWB
A350


AIRBUS A350
XWB
A350





- Experience every event that its flying twin experiences
- Will revolutionise certification, fleet management and support (mirrors life of the “as-built” state)
- Will decrease weight
  - no reliance on statistical distribution of material properties
  - no reliance on heuristic design methods
  - less reliance on physical testing (environment?)
  - no assumed similitude between testing and operational conditions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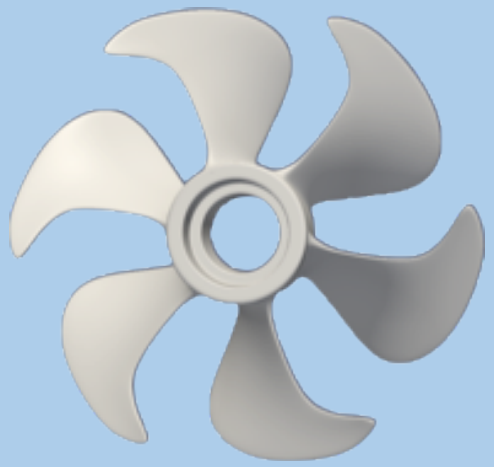
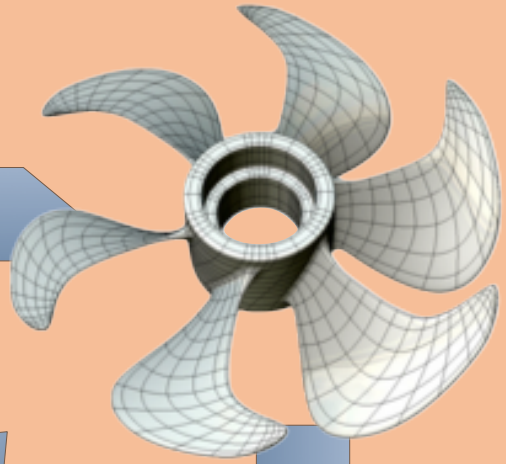
GEOMETRICAL MODEL



DISCRETISATION



Verification

A POSTERIORI ERROR CONTROL

MATERIAL MODELS

Phenomenological

Elasticity/Plasticity

Crack growth law (Paris…)

Fracture energy

Maximum tensile strength

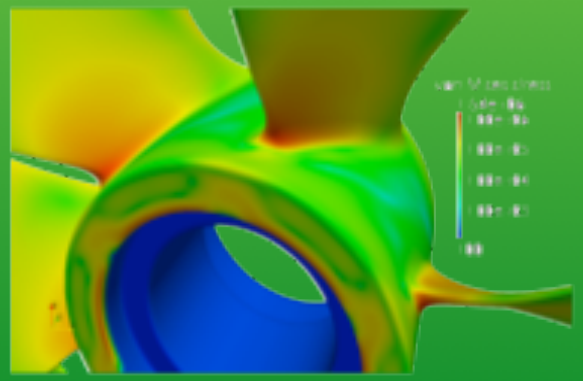
Multi-scale

Debonding,Fibre pull-out

Fibre breakage, interface fracture,

grains, dislocations, MD, quantum…

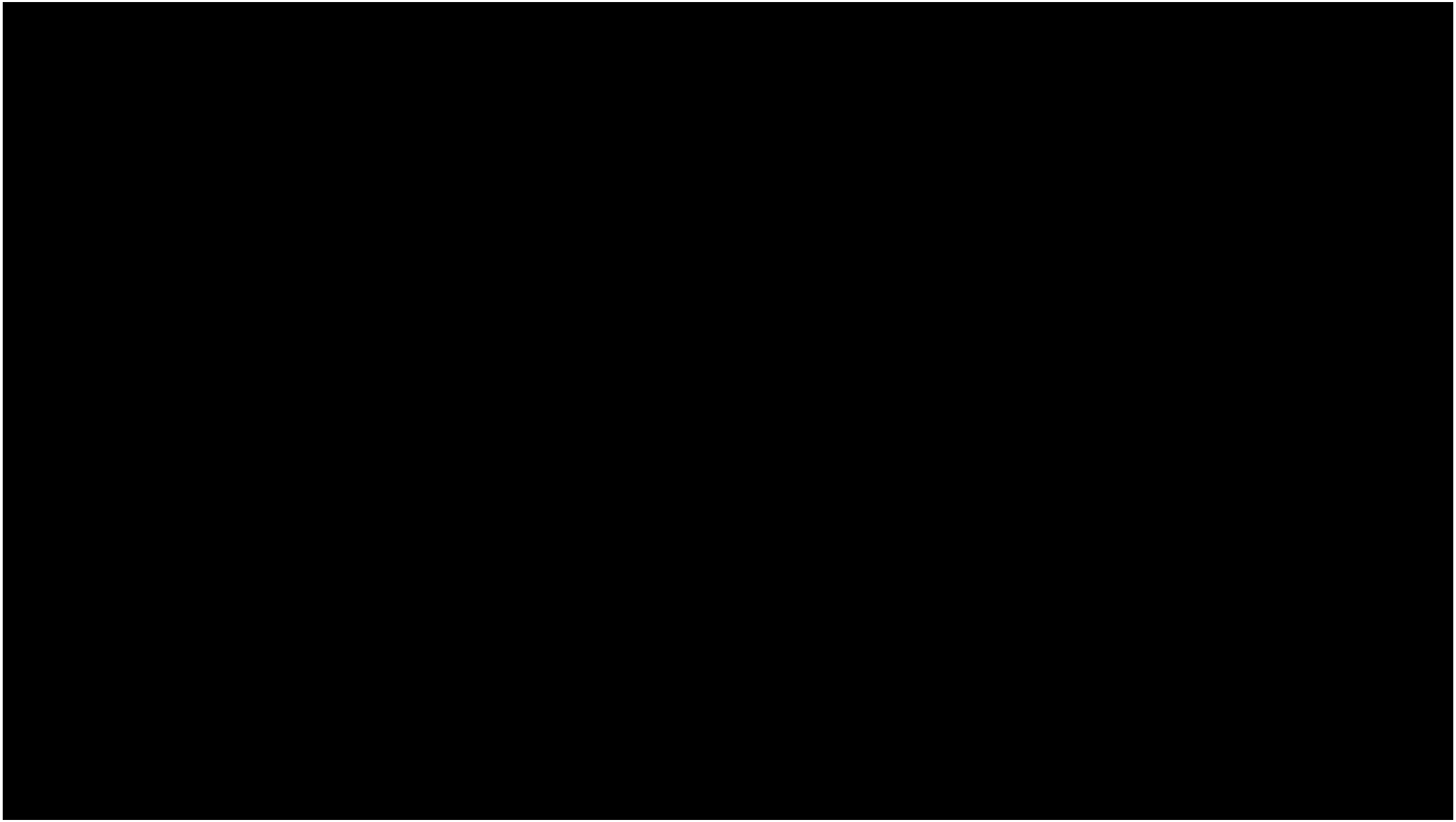
NUMERICAL SOLUTION

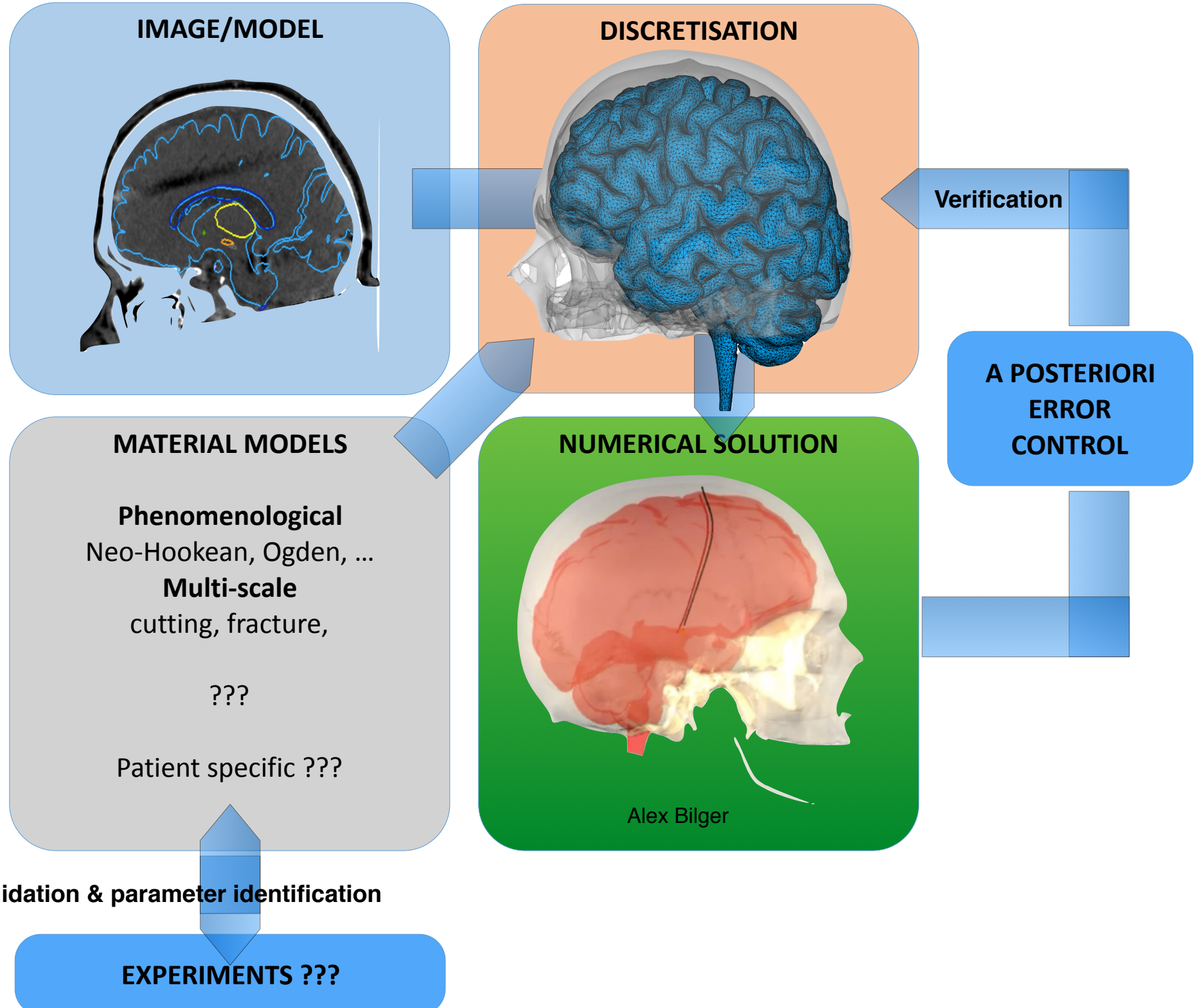


Validation & parameter identification

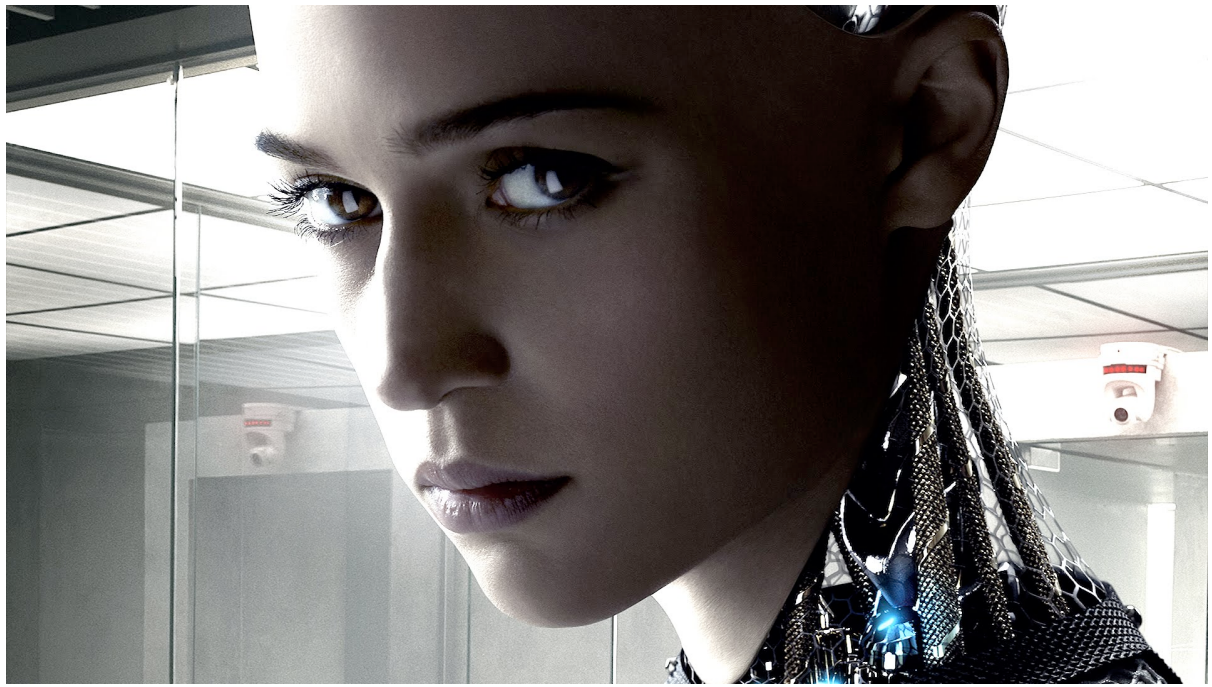
EXPERIMENTS

CONVENTIONAL APPROACH




IMAGE/MODEL
DISCRETISATION
Verification
A POSTERIORI ERROR CONTROL
MATERIAL MODELS
Phenomenological
Neo-Hookean, Ogden, …
Multi-scale
cutting, fracture,
???
Patient specific ???
NUMERICAL SOLUTION
Alex Bilger
Validation & parameter identification
EXPERIMENTS ???

DIGITAL TWIN OF THE PATIENT



Treatment simulation

Scales of interest

Disease evolution

INFORMATION

"Inspection"interval

Fitness

REAL PATIENT

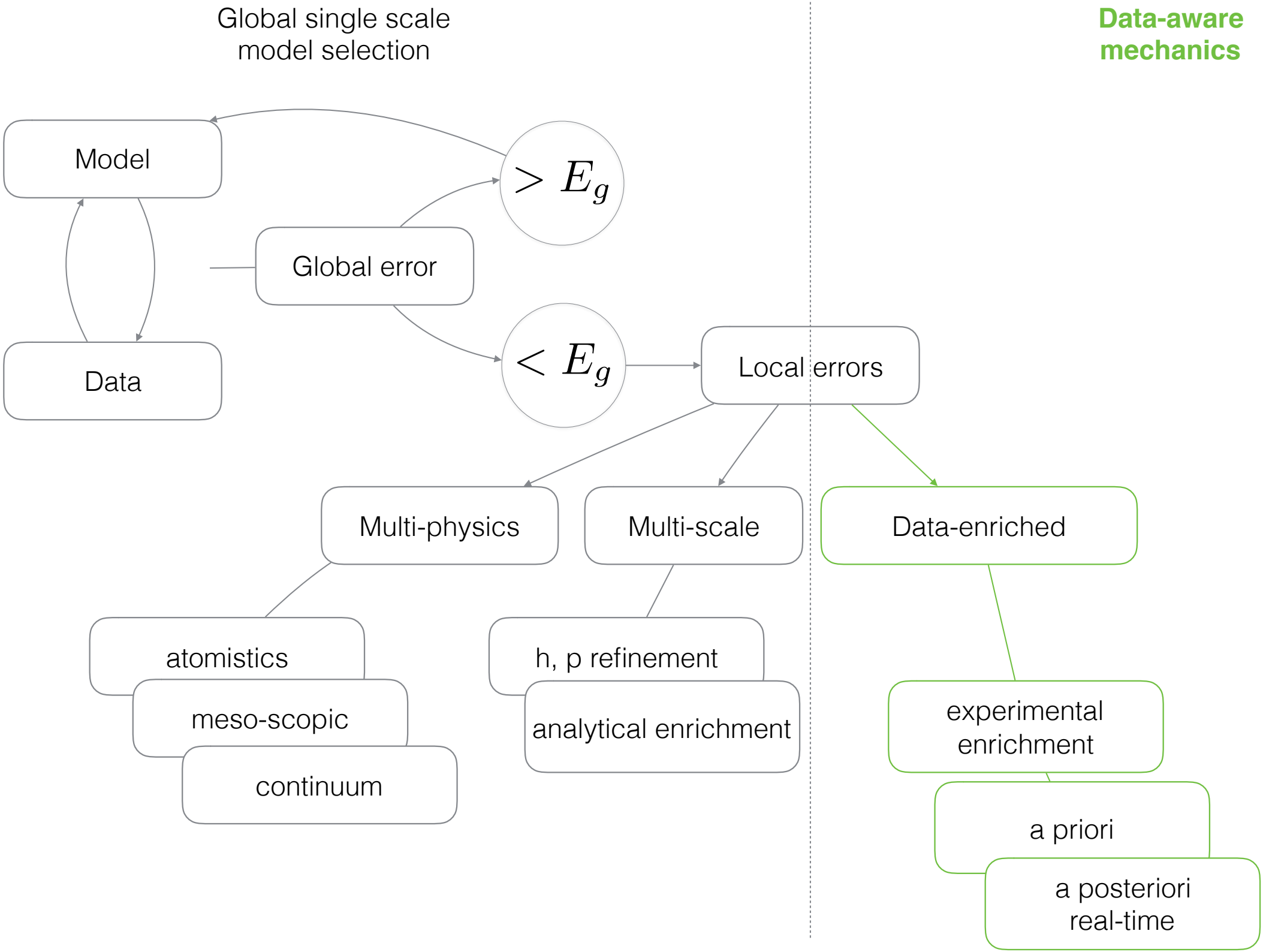
DATA

Organ state

Environment Conditions

Health

Disease


Global single scale
model selection
Data-aware
mechanics
Model
Data
Global error
$> E_g$
$< E_g$
Local errors
Multi-physics
Multi-scale
Data-enriched
atomistics
meso-scopic
continuum
h, p refinement
analytical enrichment
experimental
enrichment
a priori
a posteriori
real-time

Papers on fracture

http://orbilu.uni.lu/handle/10993/26421

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Papers on fracture

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Other related documents

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Plenary talk at XDMS2017
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What makes Data Science different?
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Energy-minimal crack growth
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Uncertainty quantification for soft tissue biomechanics
http://orbilu.uni.lu/handle/10993/28618
http://orbilu.uni.lu/handle/10993/30946
Needle insertion real-time simulation and error control
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