Geometry Independent Field approximaTion error-driven local adaptivity in elasto-dynamics

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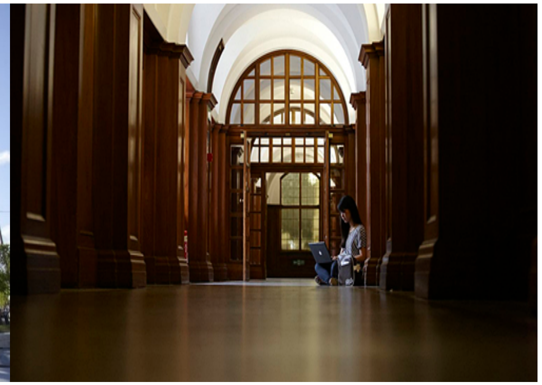

Outline

- Motivation
- Problem statement
- Methodology
- Numerical examples
- Conclusions and Following work

Motivation

- **NURBS-IGA performs better than FEM in dynamics. (High order continuity)**
- **IGA--- I**so**G**eometric **A**nalysis

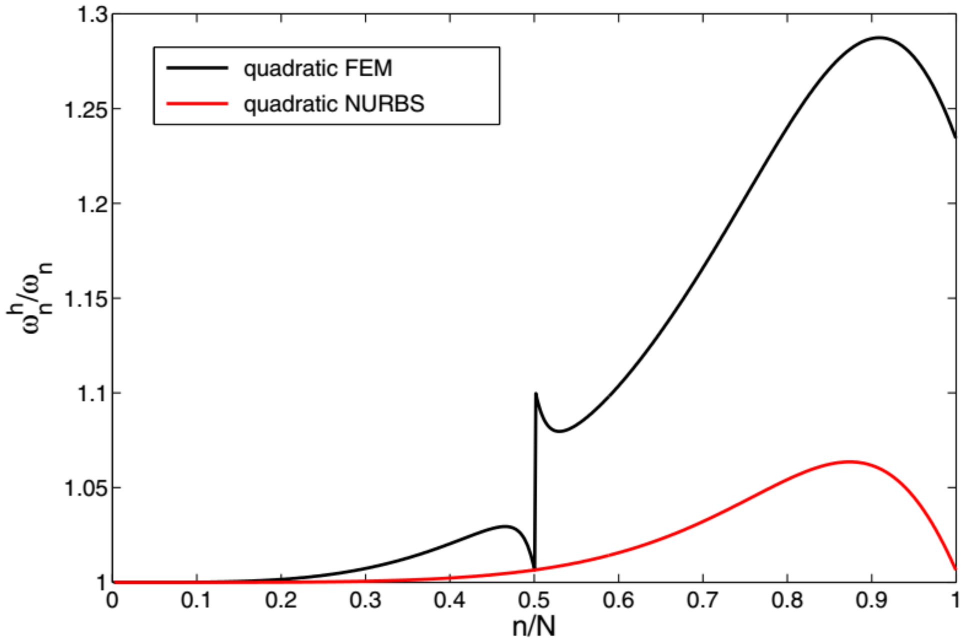

Fig. 8. Fixed–fixed rod. Normalized discrete spectra using quadratic finite elements and NURBS.

Cottrell, J. Austin, et al. "Isogeometric analysis of structural vibrations." *Computer methods in applied mechanics and engineering* 195.41 (2006): 5257-5296.

Motivation

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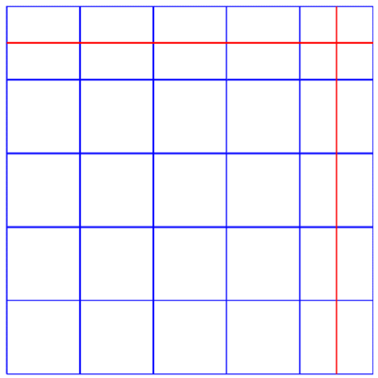
- **NURBS are limited to global refinement. (Tensor product)**

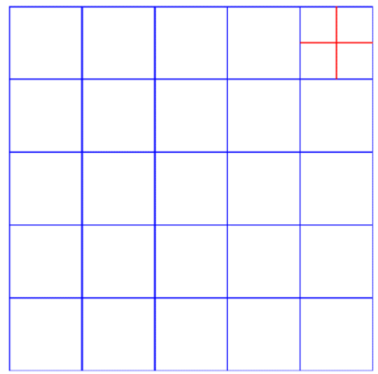
- **PHT are with local refinement. (Hierarchical)**
- **PHT --- P**olynomial splines over **H**ierarchical **T**-meshes

NURBS



Global refinement

PHT



Local refinement

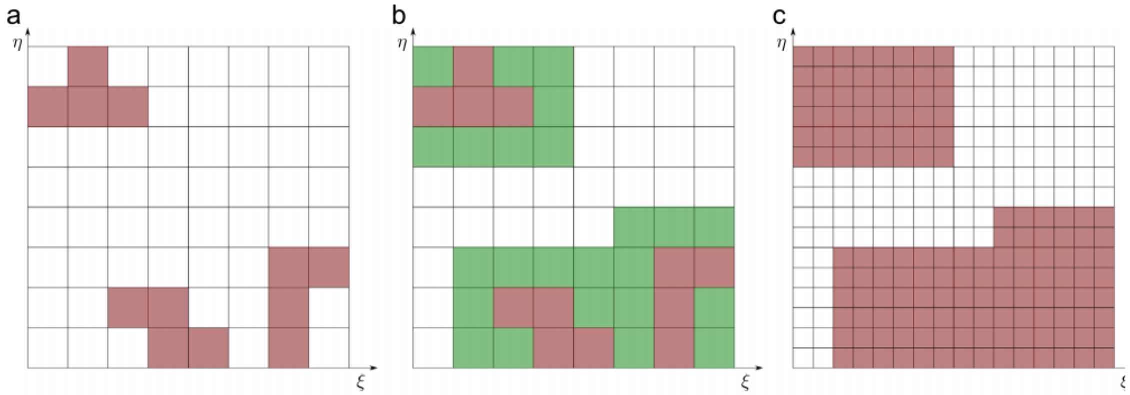
- **But PHT will lose information of geometry for curves because it is not rational**

Motivation

- **RHT can describe curves but** **limited to C^1.**
- **RHT --- R**ational splines over **H**ierarchical **T**-meshes

Nguyen-Thanh, N., et al. "An adaptive three-dimensional RHT-splines formulation in linear elasto-statics and elasto-dynamics." *Computational Mechanics* 53.2 (2014): 369-385.

- **Multiple-patch NURBS can achieve local refinement**
- **But over-accuracy and lose continuity at boundary of patch**



Chemin, Alexandre, Thomas Elguedj, and Anthony Gravouil. "Isogeometric local h-refinement strategy based on multigrids." Finite Elements in Analysis and Design 100 (2015): 77-90.

Motivation

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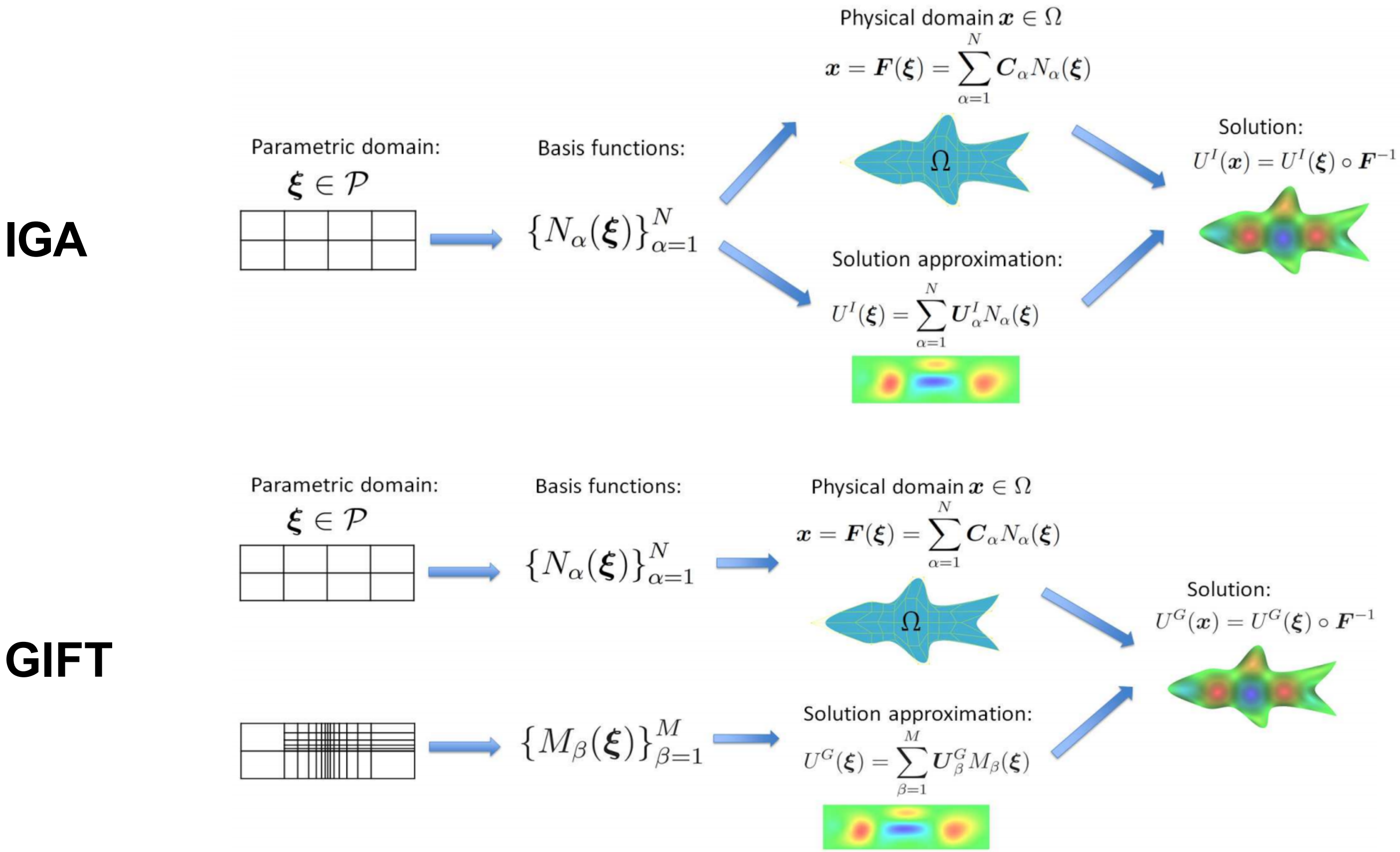
- **GIFT--- G**eometry-**I**ndependent **F**ield approxima**T**ion



- **GIFT**: NURBS + PHT

exact geometry **local refinement**

Problem statement

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- **Variational equation for vibration problem**

$$\int_{\Omega} \boldsymbol{\varepsilon}^{\boldsymbol{T}}(\boldsymbol{\delta}\mathbf{u})\mathbf{D}\boldsymbol{\varepsilon}(\mathbf{u})d\Omega + \int_{\Omega} \rho\delta\mathbf{u}^T\ddot{\mathbf{u}}d\Omega = 0, \ \forall\delta\mathbf{u} \in \mathscr{V}$$

- **Kirchoff plate theory (thin plate)**

$$\mathbf{u} = \{\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}\}^T = \left\{-z\frac{\partial}{\partial x}, -z\frac{\partial}{\partial y}, 1\right\}^T \boldsymbol{w} = \mathbb{H}w$$

$$\boldsymbol{\varepsilon}(\mathbf{u}) = \left\{\frac{\partial^2}{\partial x^2}, \frac{\partial^2}{\partial y^2}, 2\frac{\partial^2}{\partial x \partial y}\right\}^T \boldsymbol{w} = \mathbb{E}\boldsymbol{w}$$

$\boldsymbol{w}$ is independent variable

$$\int_{\Omega} \mathbb{E}\delta\boldsymbol{w}^T\mathbf{D}\mathbb{E}\boldsymbol{w}d\Omega + \int_{\Omega} \rho\mathbb{H}\delta\boldsymbol{w}^T\mathbb{H}\ddot{\boldsymbol{w}}d\Omega = 0, \ \forall\delta\boldsymbol{w} \in \mathscr{V}$$

Problem statement

- **GIFT form of discrete governing equation**

$$\left.\begin{aligned}\mathbf{K} &= \int_{\mathcal{P}} \mathbb{E}\boldsymbol{M}^T(\boldsymbol{\xi})\mathbf{D}\mathbb{E}\boldsymbol{M}(\boldsymbol{\xi})|\mathbf{J}(\boldsymbol{\xi})|d\boldsymbol{\xi} \\ \mathbf{M} &= \int_{\mathcal{P}} \rho\mathbb{H}\boldsymbol{M}^T(\boldsymbol{\xi})\mathbb{H}\underbrace{\boldsymbol{M}}_{\text{PHT}}(\boldsymbol{\xi})|\mathbf{J}(\boldsymbol{\xi})|d\boldsymbol{\xi}\end{aligned}\right\}$$

- **Mapping**

$$\mathbf{J}(\boldsymbol{\xi}) = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{\xi}} = \sum_{i=1}^{n} \overbrace{\frac{\partial N_i(\boldsymbol{\xi})}{\partial \boldsymbol{\xi}}}^{\text{NURBS}} \boldsymbol{P}_i$$

$$\mathbf{K}\boldsymbol{w} + \mathbf{M}\ddot{\boldsymbol{w}} = 0$$

- **Separation of solution for static state**

$$\boldsymbol{w}(t) = \boldsymbol{\phi}\exp(i\lambda t)$$

- **Eigenvalue problem**

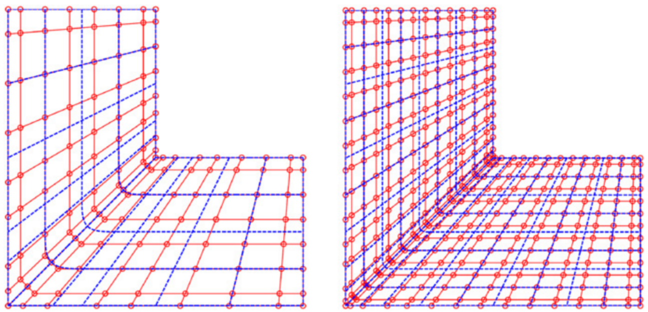
$$(\mathbf{K} - \lambda^2\mathbf{M})\boldsymbol{\phi} = 0$$

$\lambda \longrightarrow$ Natural frequency $\qquad \boldsymbol{\phi} \longrightarrow$ Eigenvector

Methodology

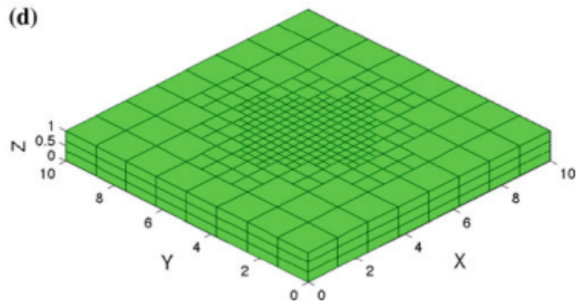
- **Refinement by IGA for vibration**

Shojaee, S., et al. "Free vibration analysis of thin plates by using a NURBS-based isogeometric approach." *Finite Elements in Analysis and Design* 61 (2012): 23-34.



NURBS refinement
(over-accurate and waste CPU time)

Nguyen-Thanh, N., et al. "An adaptive three-dimensional RHT-splines formulation in linear elasto-statics and elasto-dynamics." *Computational Mechanics* 53.2 (2014): 369-385.



RHT prior refinement
(not practical)

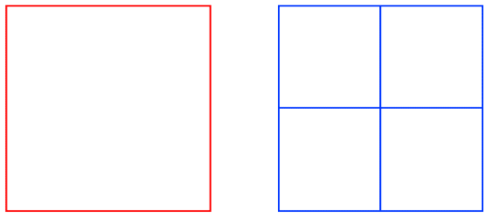
- **Error-driven adaptivity for vibration**

Methodology

- **Error estimation (not limited to vibration)**

Error indicator $\boldsymbol{e} = \tilde{\mathbf{u}} - \mathbf{u}^h$

$\tilde{\mathbf{u}} \longrightarrow$ Solution in coarse mesh $\quad$ $\mathbf{u}^h \longrightarrow$ Solution in reference mesh



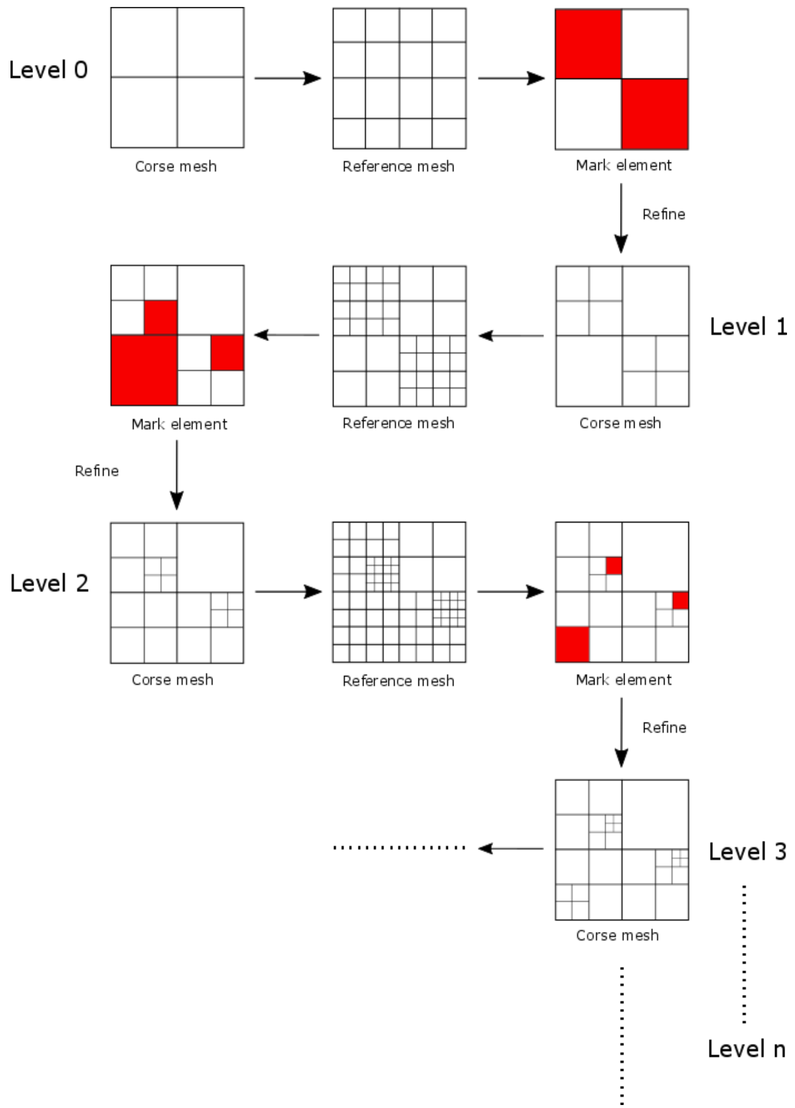
L2 norm of error indicator for element

$$\|\boldsymbol{e}\|_{\Omega_e^i} = \left[\int_{\Omega_e^i} (\tilde{\mathbf{u}} - \mathbf{u}^h)^T (\tilde{\mathbf{u}} - \mathbf{u}^h) d\Omega_e^i\right]^{\frac{1}{2}}, \quad \bigcup_{i=1}^{N} \Omega_e^i = \Omega$$

If $\|\boldsymbol{e}\|_{\Omega_e^i} > \varepsilon_e$ tolerance, refine element i

Methodology

- **Adaptive hierarchical local refinement (Algorithm 1)**



Methodology

- **Error-driven local adaptivity for vibration**

Error indicator for natural frequency

$$e_{\lambda}^{i} = \left| \frac{\tilde{\lambda}^{\tilde{i}} - \lambda^{i}}{\tilde{\lambda}^{\tilde{i}}} \right|$$

$\tilde{\lambda}^{\tilde{i}} \longrightarrow \tilde{i}$ th natural frequency in reference mesh

$\lambda^{i} \longrightarrow i$ th natural frequency in coarse mesh

If $e_{\lambda}^{i} > \varepsilon_{\lambda}$, define error indicator for normalized eigenvector

$$\boldsymbol{e}_{\boldsymbol{\phi}}^{i} = \tilde{\boldsymbol{\phi}}_{N,\tilde{i}} - \boldsymbol{\phi}_{N,i}^{h}$$

- Call **Algorithm 1** $(\tilde{\mathbf{u}} = \tilde{\boldsymbol{\phi}}_{N,\tilde{i}}, \mathbf{u}^{h} = \boldsymbol{\phi}_{N,i}^{h})$ to process local refinement at mode i

?

Know mode i $\Rightarrow$ Locate mode i

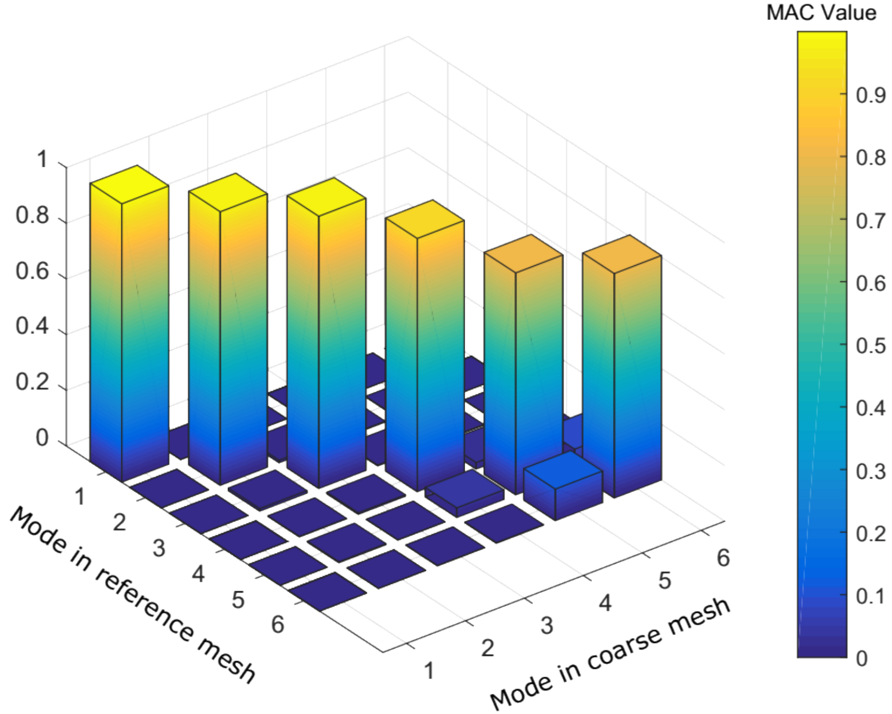
Methodology

- **Modal Assurance Criterion (MAC)**

$$\mathcal{M}(\tilde{i},i)|_{\tilde{\phi}_{N,\tilde{i}},\phi^h_{N,i}} = \frac{(\tilde{\phi}^T_{N,\tilde{i}} \cdot \phi^h_{N,i})^2}{\|\tilde{\phi}_{N,\tilde{i}}\|^2 \|\phi^h_{N,i}\|^2}$$

Consistent correspondence check between mode shapes

- **MAC matrix values**

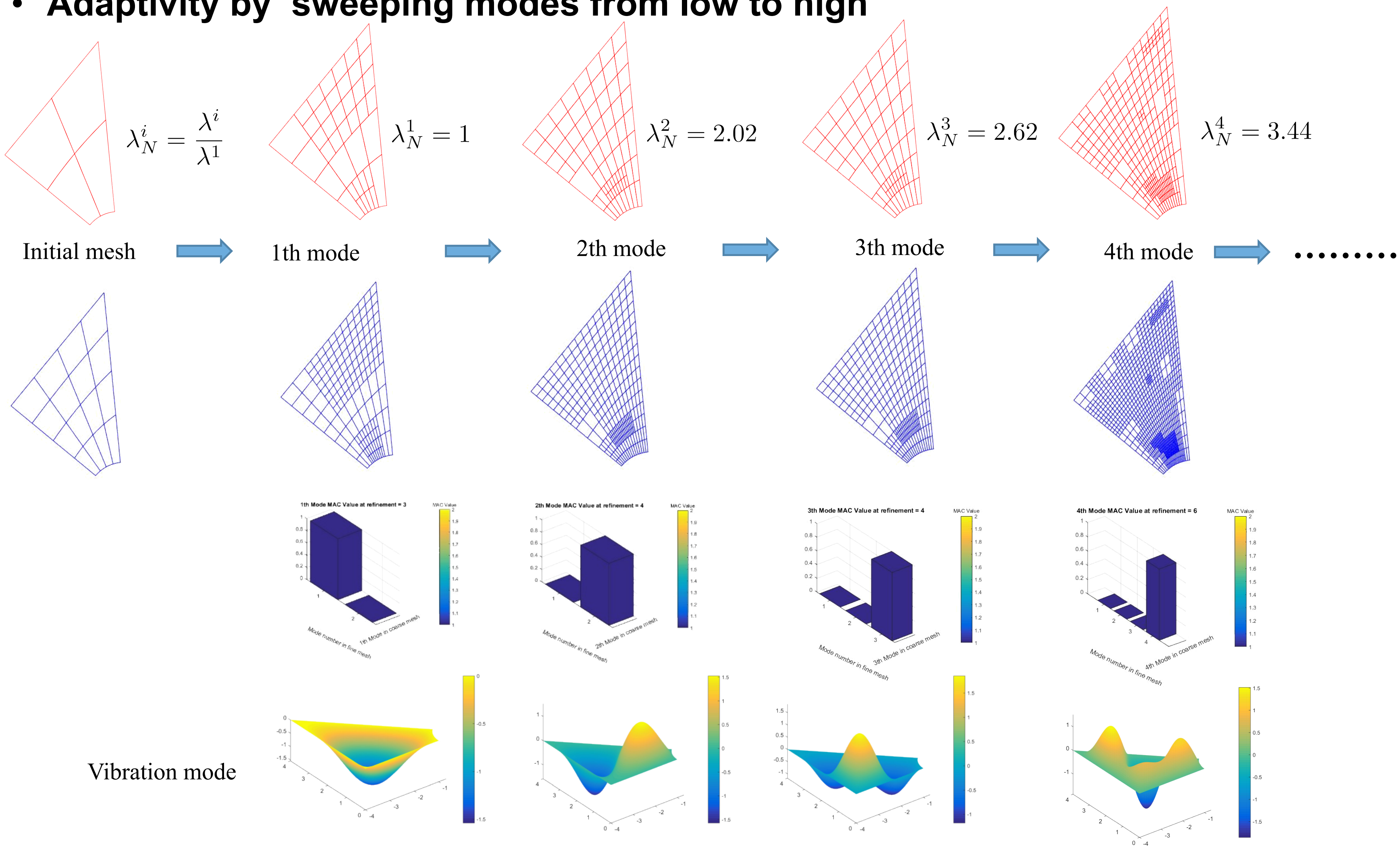


Values of MAC matrix $\mathcal{M}$ (row is for $\tilde{i}$, column is for i)

Mode	1	2	3	4	5	6
1	0.9995	0.0001	0.0003	0.0000	0.0000	0.0000
2	0.0001	0.9825	0.0087	0.0030	0.0018	0.0000
3	0.0003	0.0089	0.9777	0.0029	0.0024	0.0001
4	0.0000	0.0016	0.0052	0.9093	0.0233	0.0085
5	0.0000	0.0034	0.0008	0.0343	0.7970	0.0830
6	0.0000	0.0001	0.0013	0.0020	0.1133	0.8062

Methodology

- **Adaptivity by sweeping modes from low to high**



(a) 1th mode, $\lambda_N^1 = 1$ (b) 2th mode, $\lambda_N^2 = 2.02$ (c) 3th mode, $\lambda_N^3 = 2.62$ (d) 4th mode, $\lambda_N^4 = 3.44$

Numerical examples

- **Heterogeneous platehole**

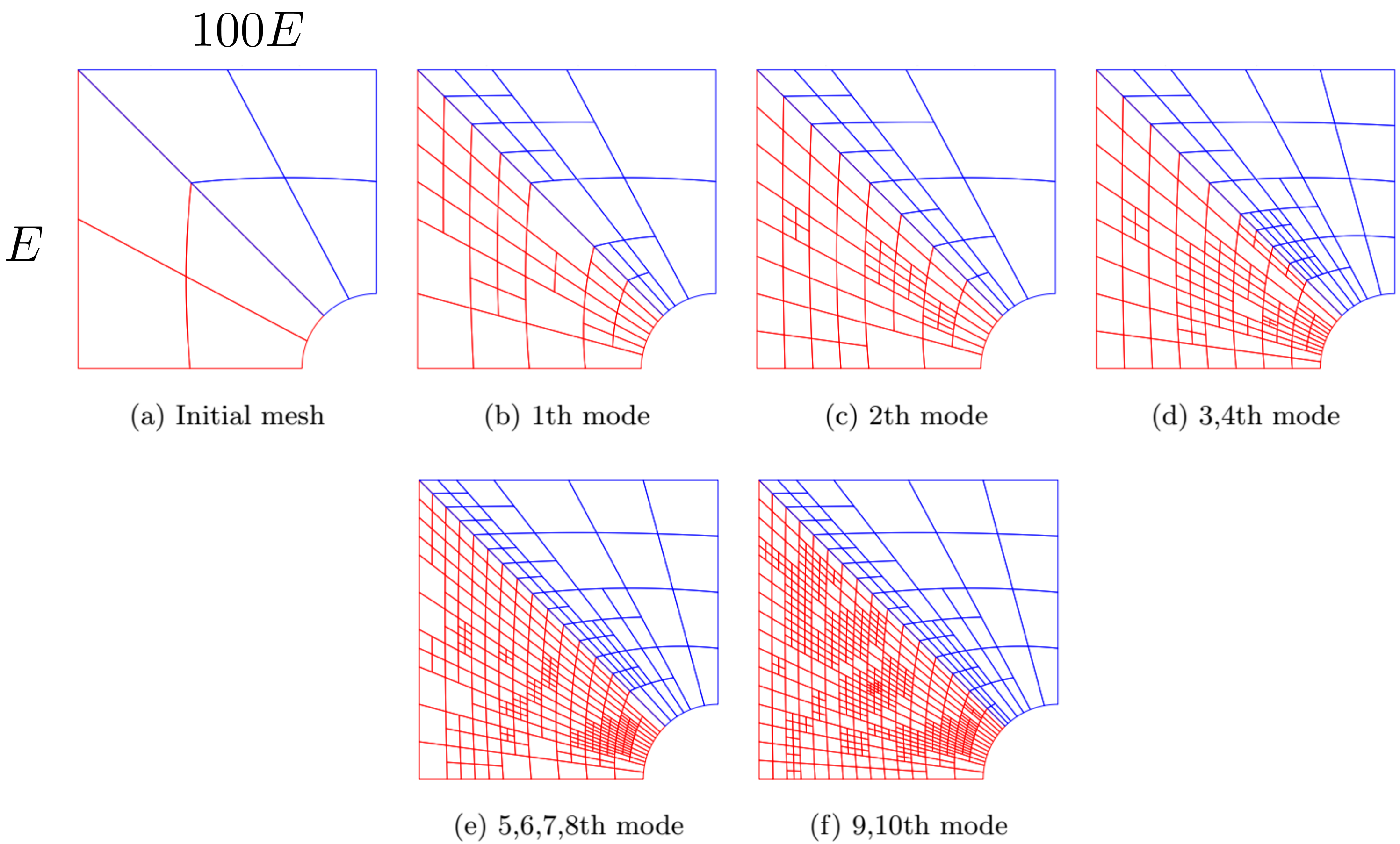
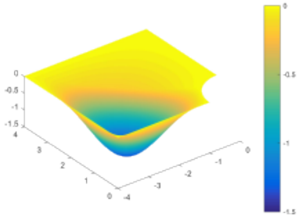
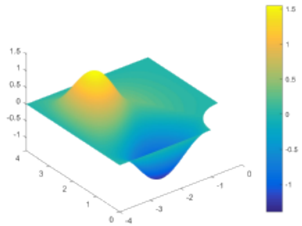


Figure 8: Adaptive refinement process of 1-10th mode for vibration of Kirchoff composite platehole ($E_2 = 100E_1$) in coarse mesh. Geometry-NURBS, Solution Field-PHT, p =3, q =3.

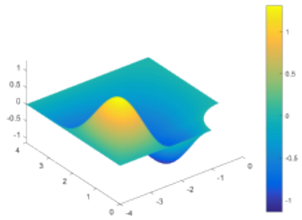
Numerical examples



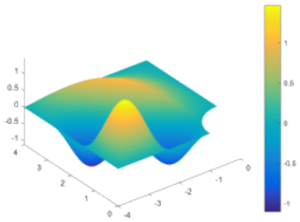
(a) 1th mode, $\lambda_N^1 = 1$



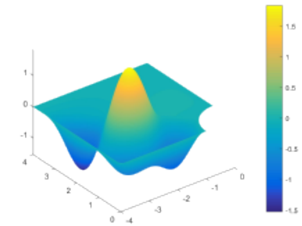
(b) 2th mode, $\lambda_N^2 = 2.03$



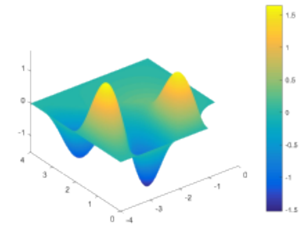
(c) 3th mode, $\lambda_N^3 = 2.30$



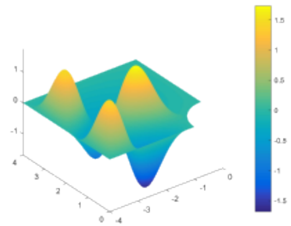
(d) 4th mode, $\lambda_N^4 = 3.12$



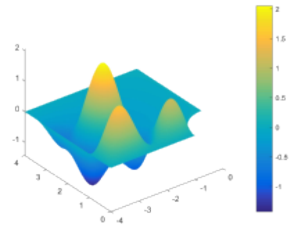
(e) 5th mode, $\lambda_N^5 = 3.52$



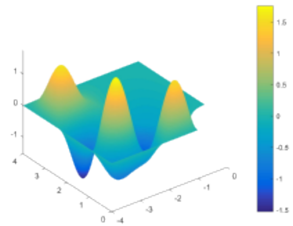
(f) 6th mode, $\lambda_N^6 = 4.20$



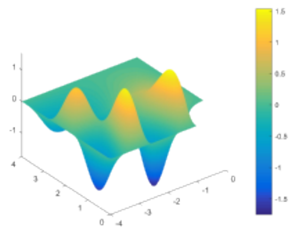
(g) 7th mode, $\lambda_N^7 = 5.32$



(h) 8th mode, $\lambda_N^8 = 5.45$



(i) 9th mode, $\lambda_N^9 = 6.09$

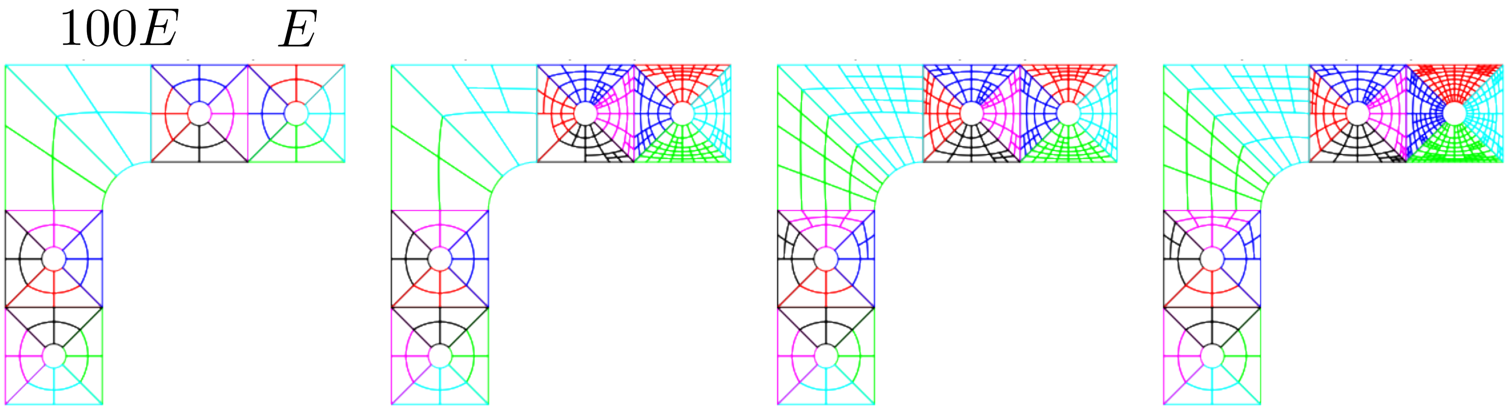


(j) 10th mode, $\lambda_N^{10} = 6.93$

Figure 9: Vibration at 1-10th mode of Kirchoff composite platehole ($E_2 = 100E_1$).

Numerical examples

- **Heterogeneous Lshaped-bracket non-symmetric boundary condition**



(a) Initial mesh (b) 6-10th mode (c) 11th mode (d) 12-16th mode

Figure 12: Adaptive refinement process of 6-16th mode for vibration of Kirchoff composite LShaped-bracket (right-up 4 patches are soft) in coarse mesh. Geometry-NURBS, Solution Field-PHT, p =3, q =3.

Numerical examples

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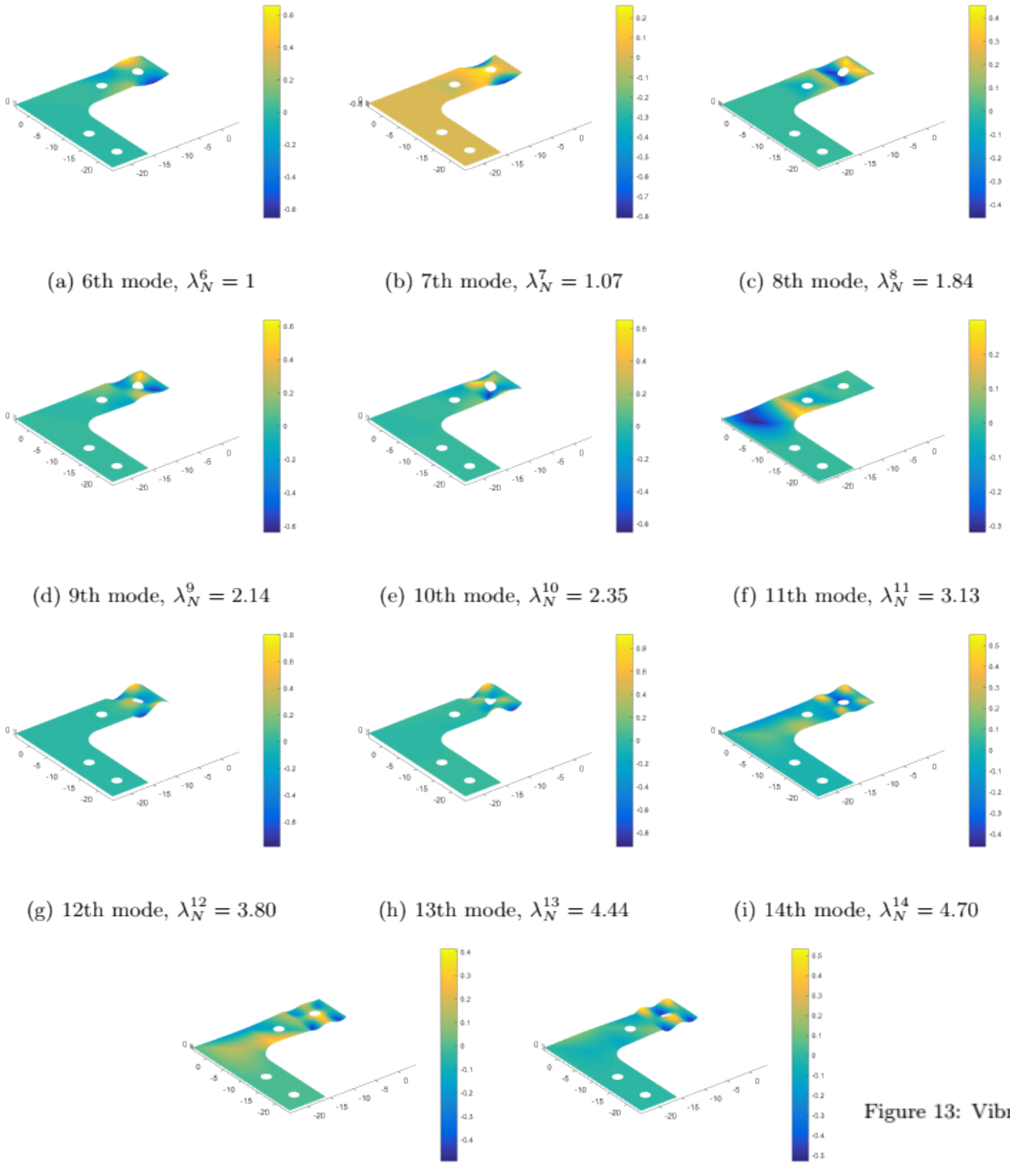

(a) 6th mode, $\lambda_N^6 = 1$ (b) 7th mode, $\lambda_N^7 = 1.07$ (c) 8th mode, $\lambda_N^8 = 1.84$

(d) 9th mode, $\lambda_N^9 = 2.14$ (e) 10th mode, $\lambda_N^{10} = 2.35$ (f) 11th mode, $\lambda_N^{11} = 3.13$

(g) 12th mode, $\lambda_N^{12} = 3.80$ (h) 13th mode, $\lambda_N^{13} = 4.44$ (i) 14th mode, $\lambda_N^{14} = 4.70$

(j) 15th mode, $\lambda_N^{15} = 4.72$ (k) 16th mode, $\lambda_N^{16} = 5.23$

Figure 13: Vibration at 6-16th mode of Kirchoff composite LShaped-bracket (right-up 4 patches are soft).

Conclusions and Following work

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- **Contributions**
  I. **GIFT for dynamics**
  II. **Error-driven adaptive hierarchical local refinement**
  III. **Modal analysis**

- **Following work**
  I. **3D vibration**
  II. **Space-time adaptivity**

Thank you!