



# Modelling (evolving) discontinuities

**Multi-scale fracture and model order reduction** Pierre Kerfriden, Lars Beex, Jack Hale, Olivier Goury, Daniel Alves Paladim, Elisa Schenone, Davide Baroli, Thanh Tung Nguyen

**Advanced discretisation techniques** Danas Sutula, Xuan Peng, Haojie Lian, Peng Yu, Qingyuan Hu, Sundararajan Natarajan, Nguyen-Vinh Phu

**Error estimation** Pierre Kerfriden, Satyendra Tomar, Daniel Alves Paladim, Andrés Gonzalez Estrada

**Biomechanics applications** Alexandre Bilger, Hadrien Courtecuisse, Bui Huu Phuoc

and all the others!

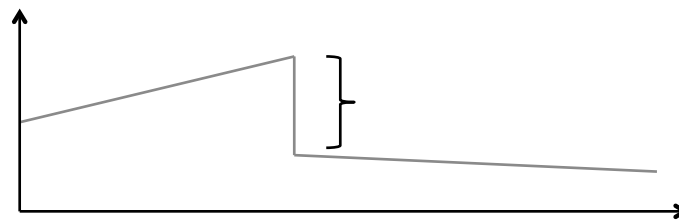
Stéphane P.A. Bordas [stephane.bordas@alum.northwestern.edu](mailto:stephane.bordas@alum.northwestern.edu) <http://legato-team.eu>

CISM Course, Udine, Italy, 2017 June 5-9  
Organised by Gernot Beer & Stéphane Bordas

# Classification of discontinuities

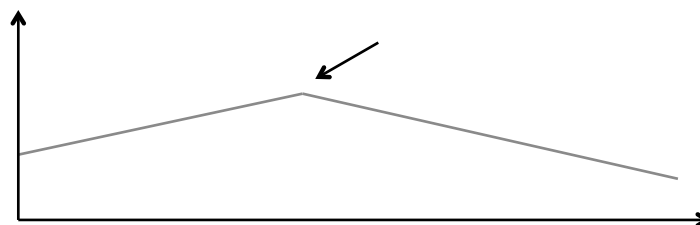
## Strong discontinuities

- The primal field of the solution is discontinuous, e.g. cracks lead to strong discontinuities in the displacement field.



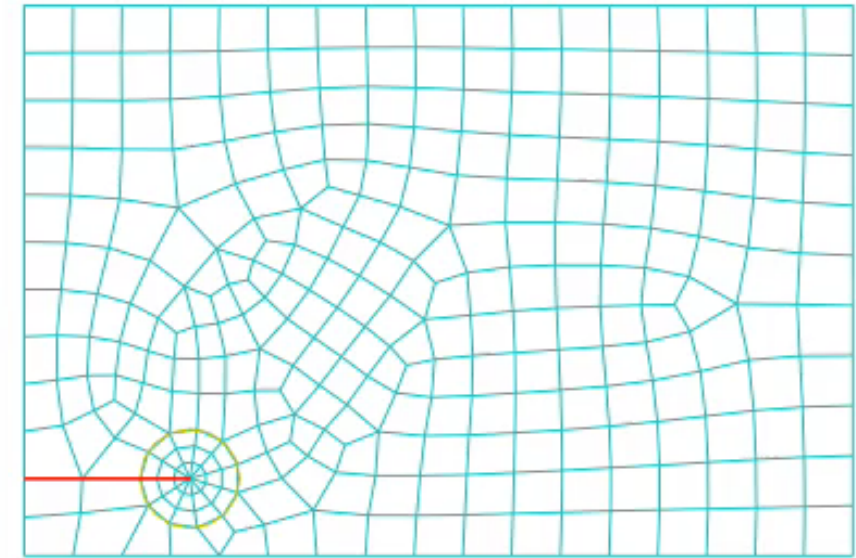
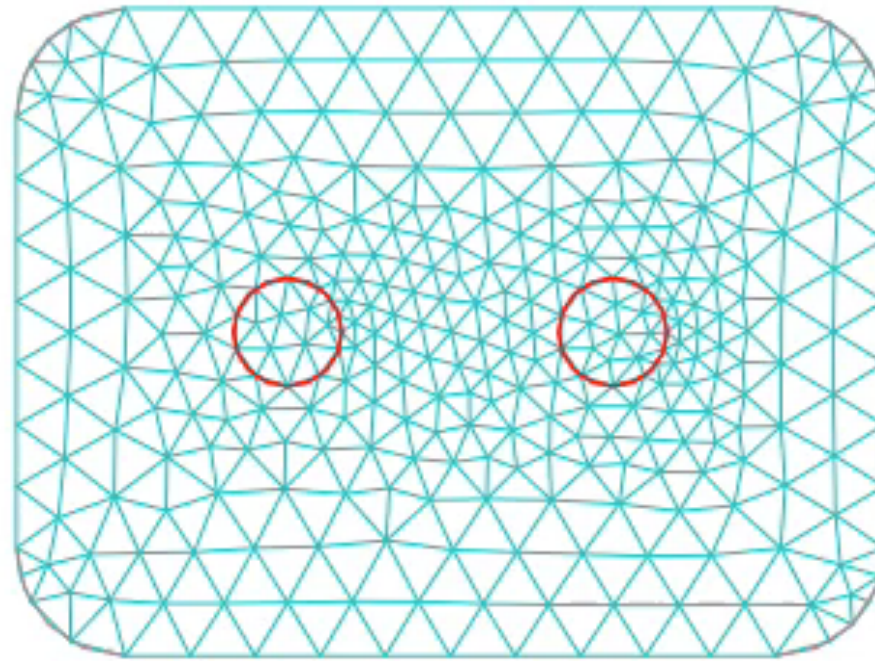
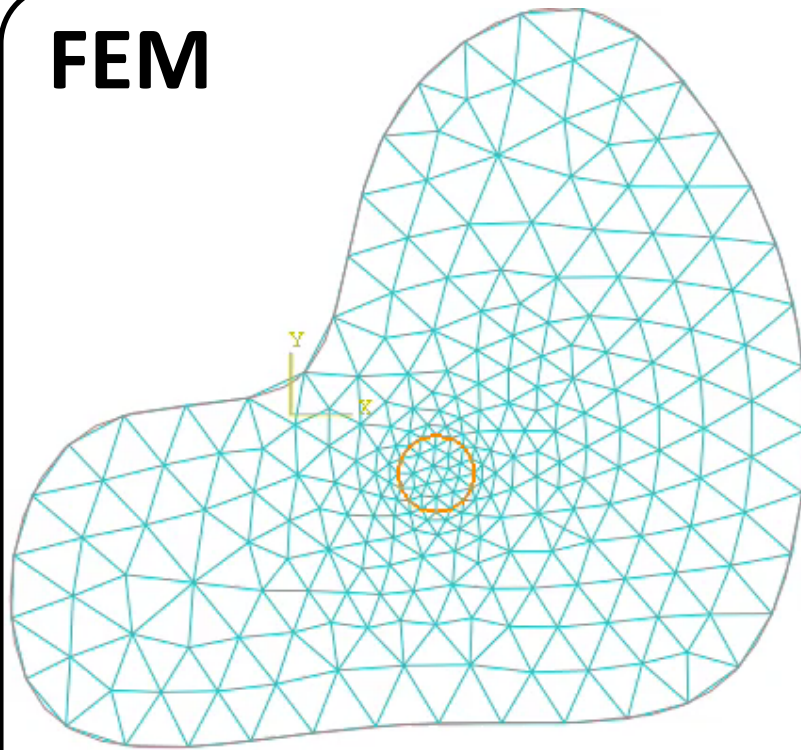
## Weak discontinuities

- The first derivative of the solution is discontinuous, e.g. discontinuities in the strain field through a material interface.

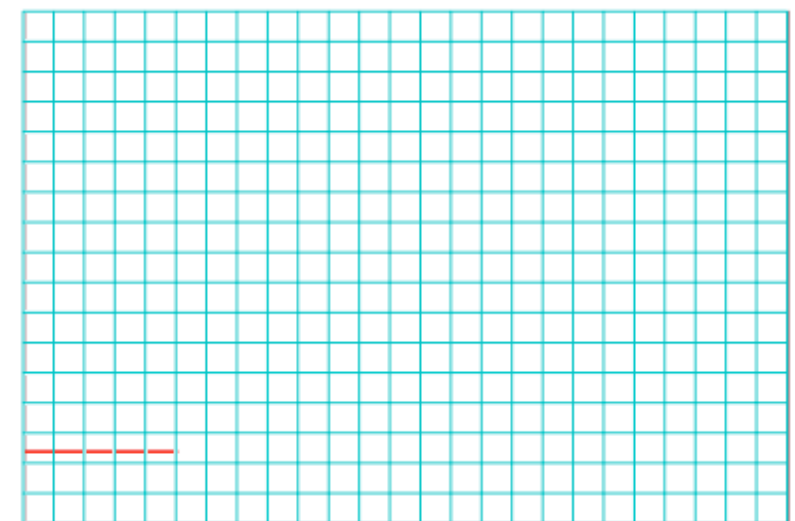
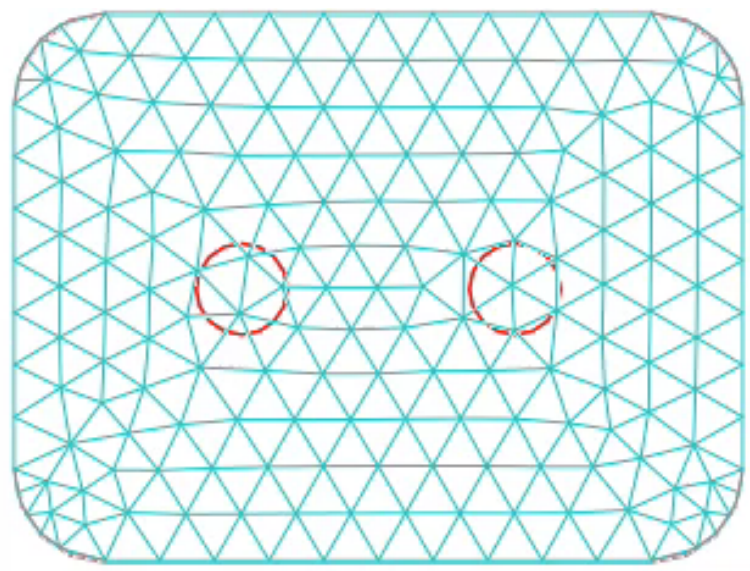
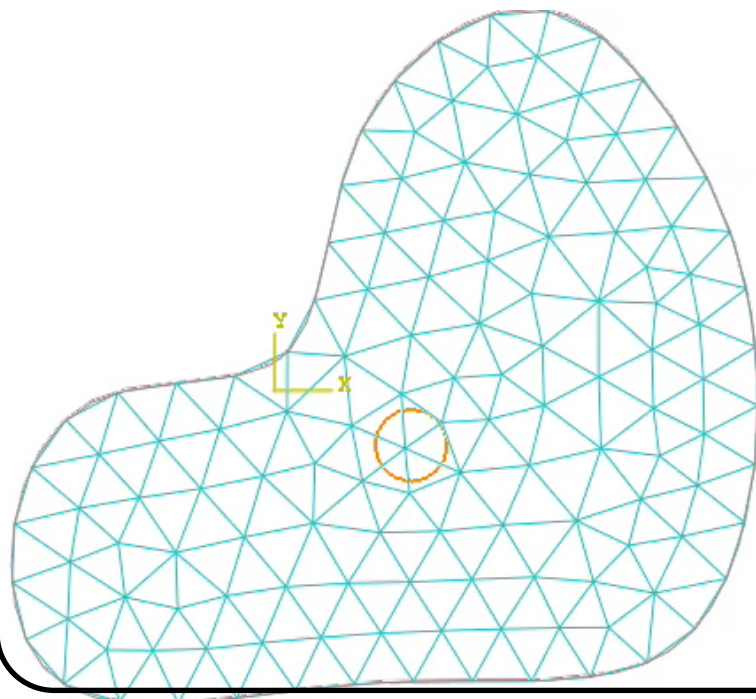




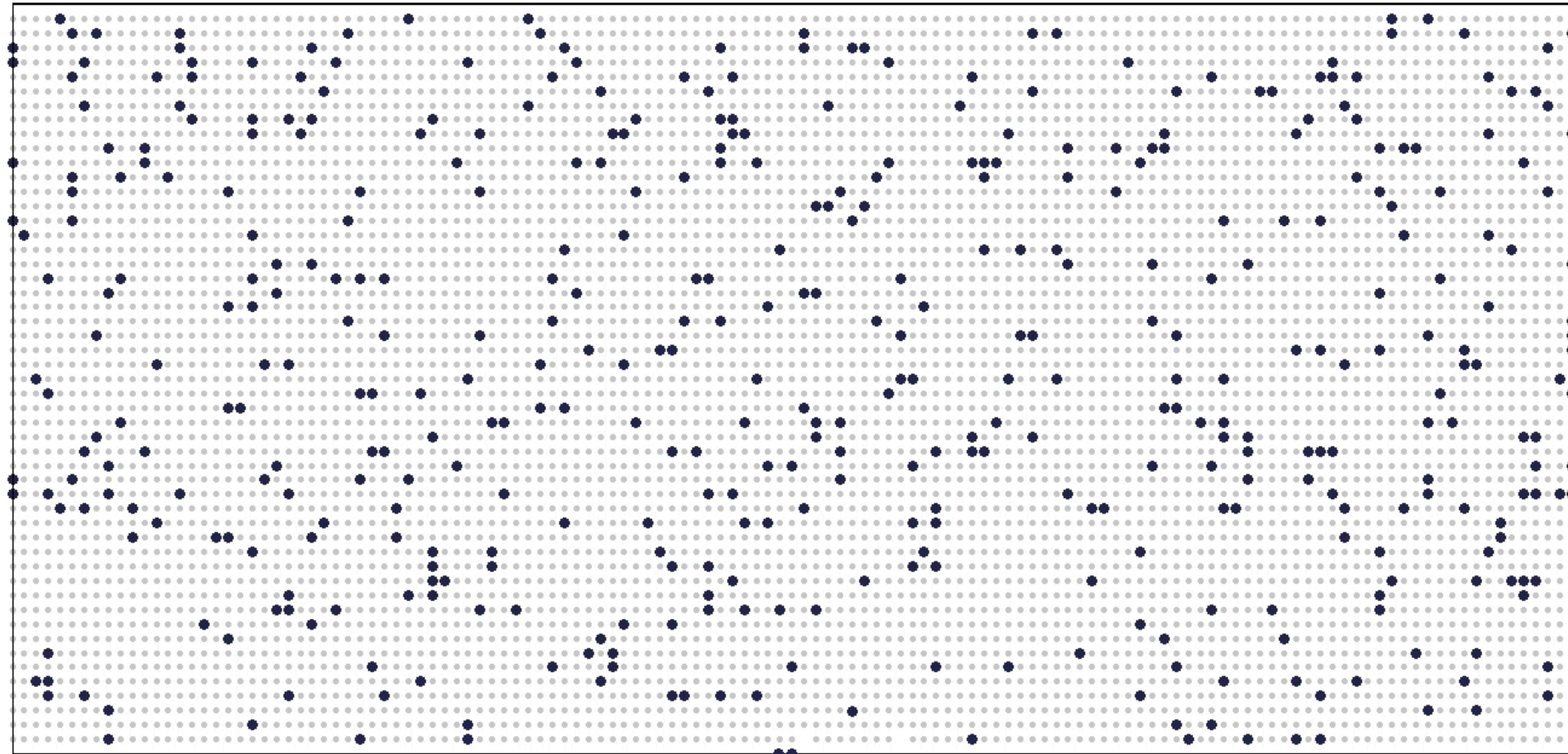
## FEM



## XFEM







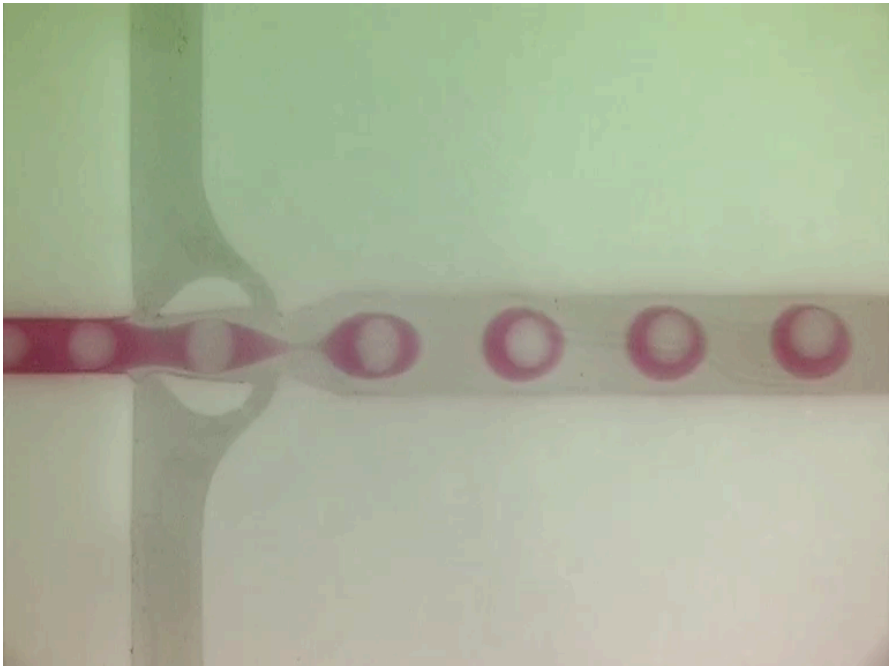
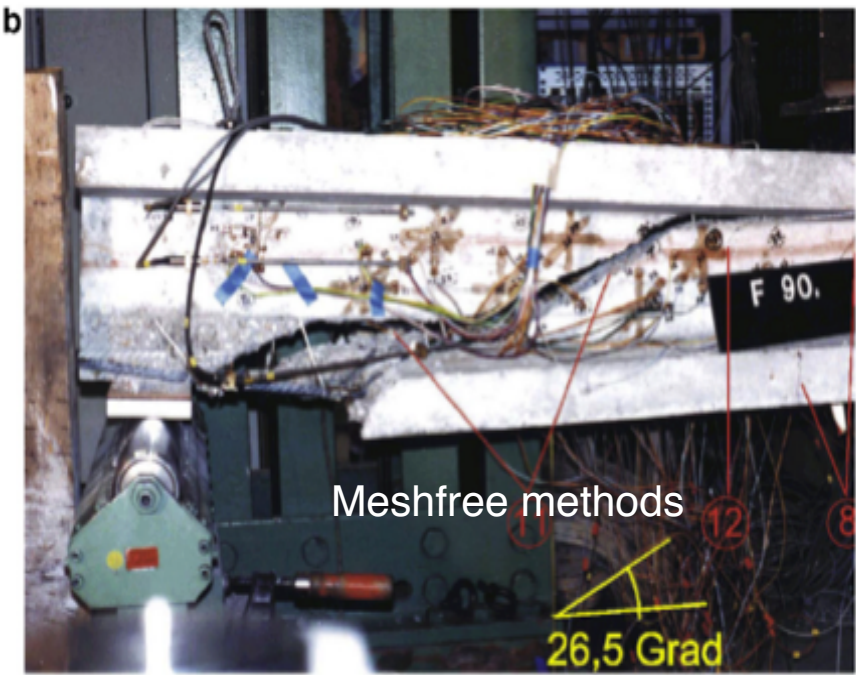
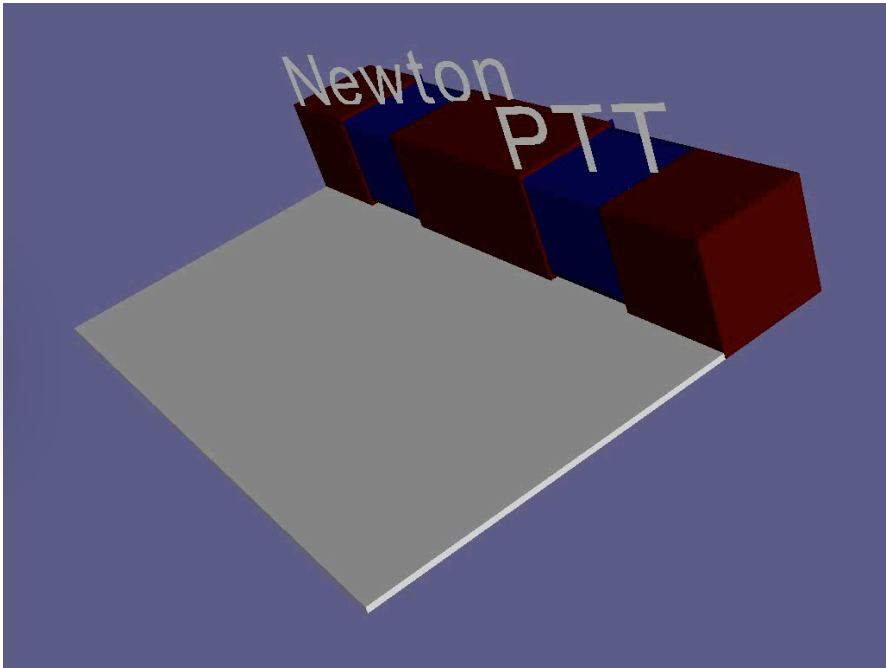
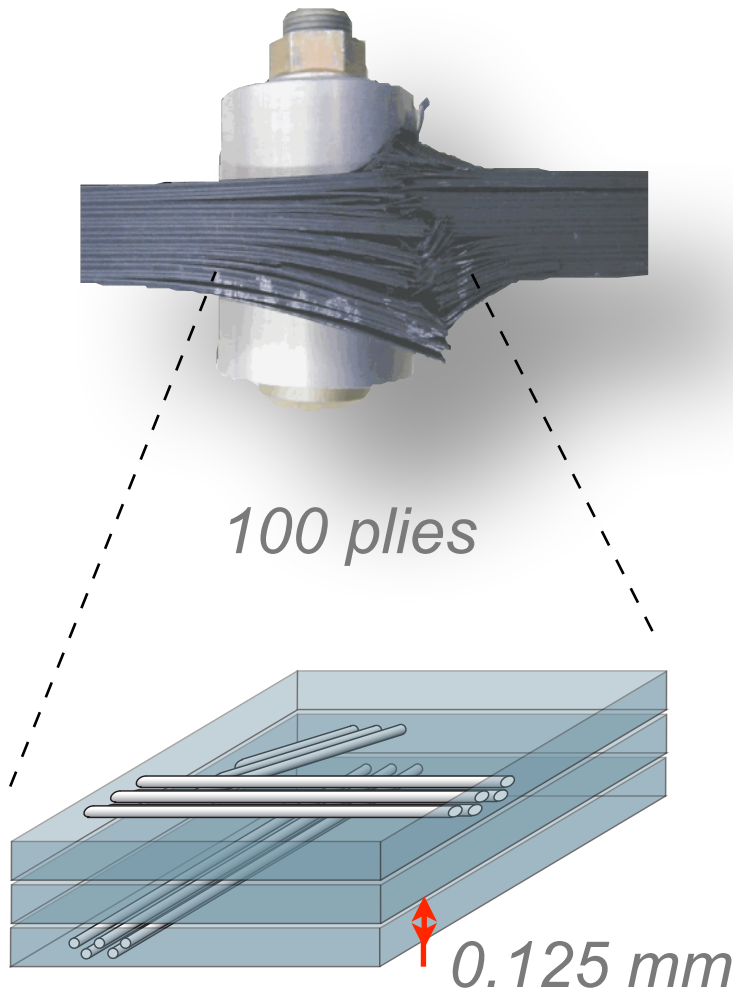
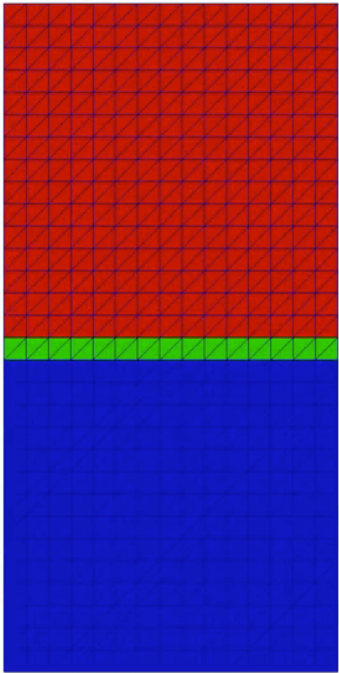
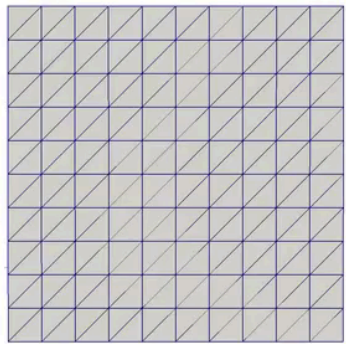
IJNMBE2017 Moh



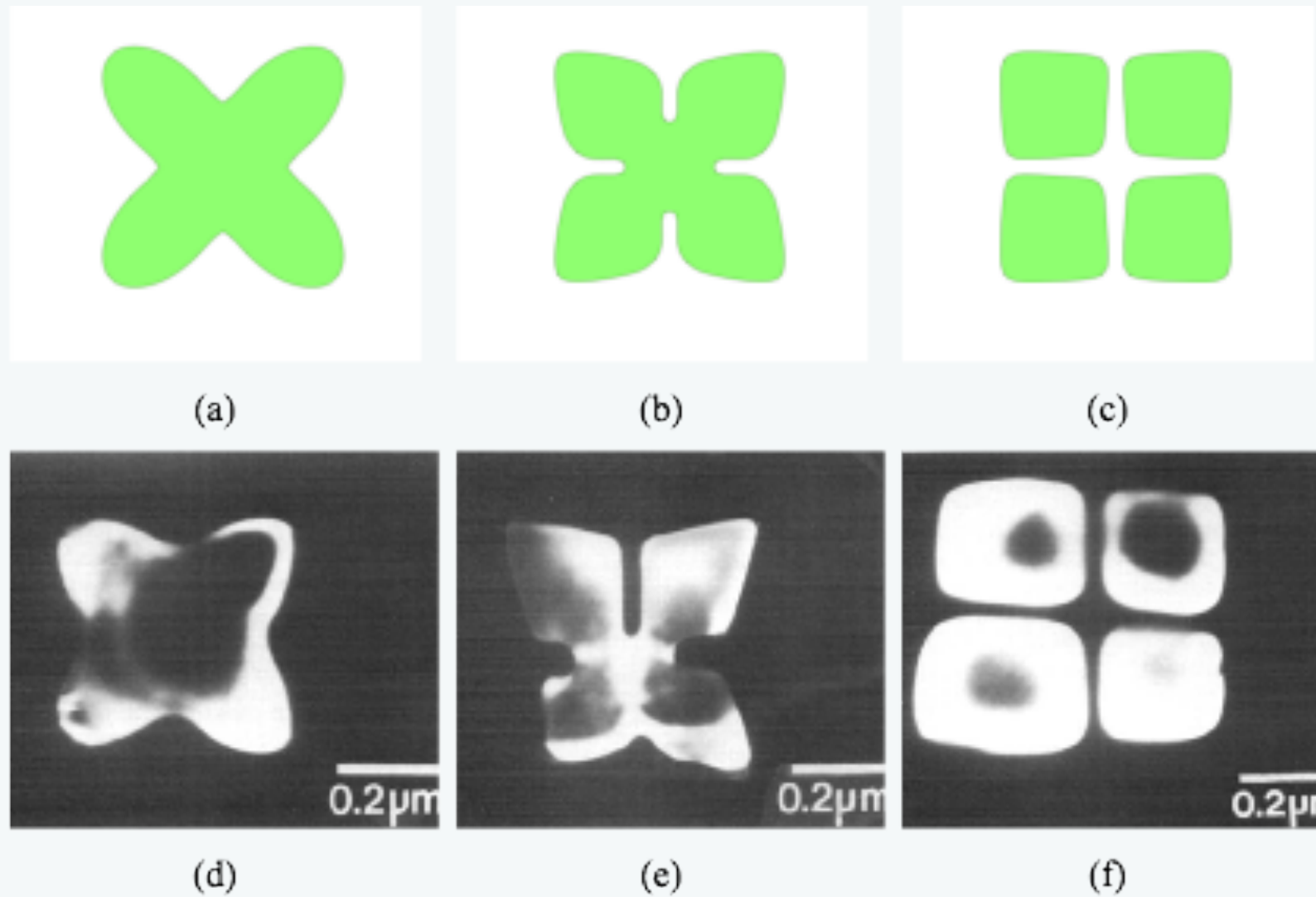
# Interfaces in practical engineering simulations



## Interfaces between phases



# Equilibrium of nano-inhomogeneities



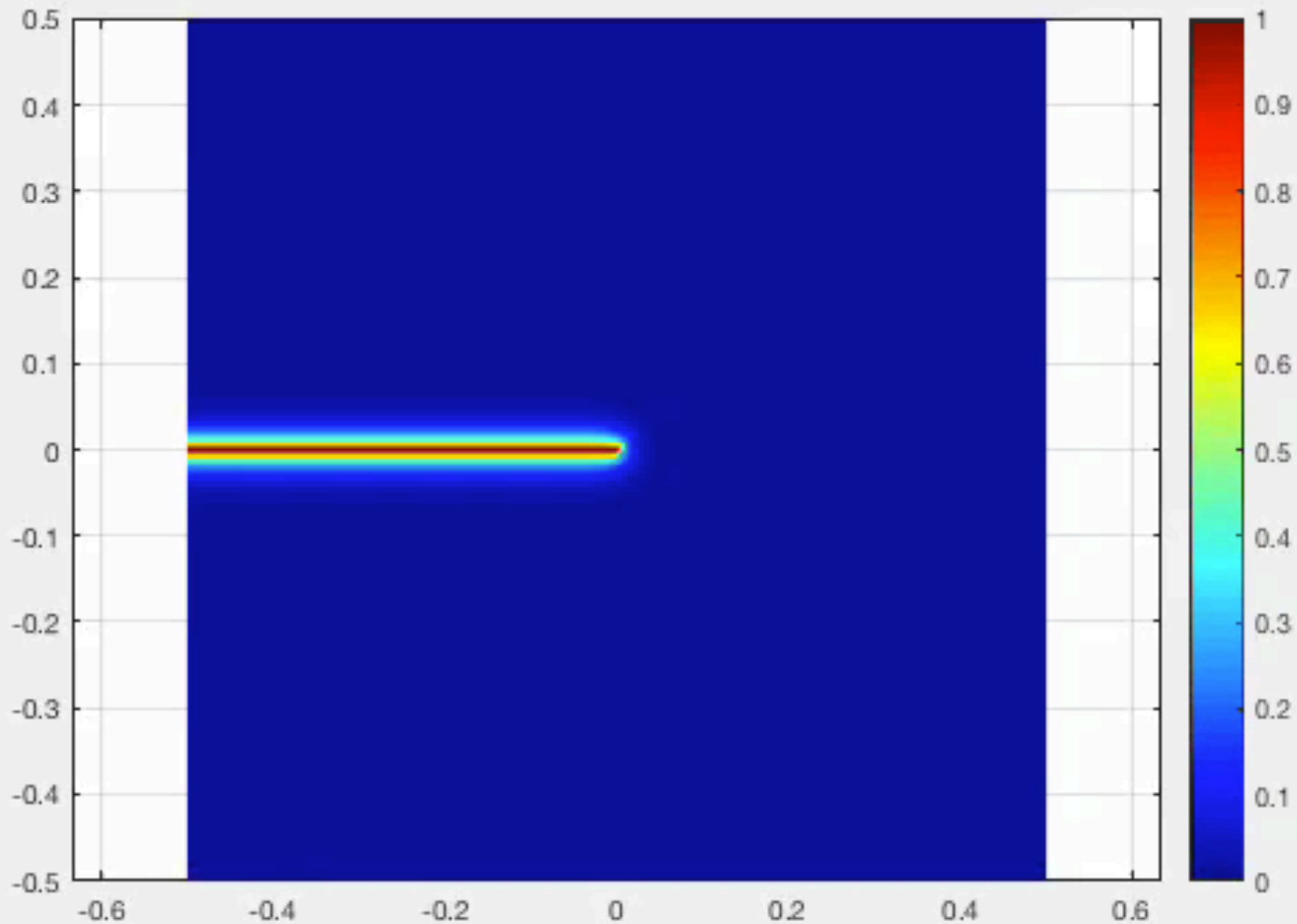
JMPS2015 <http://orbi.lu.uni.lu/bitstream/10993/11024/1/manuscript%20-%20JMPS-D-12-00428.pdf>  
CMECH2013 [http://orbi.lu.uni.lu/bitstream/10993/11022/1/Manuscript\\_XZHAO\\_CMECH\\_revision.pdf](http://orbi.lu.uni.lu/bitstream/10993/11022/1/Manuscript_XZHAO_CMECH_revision.pdf)











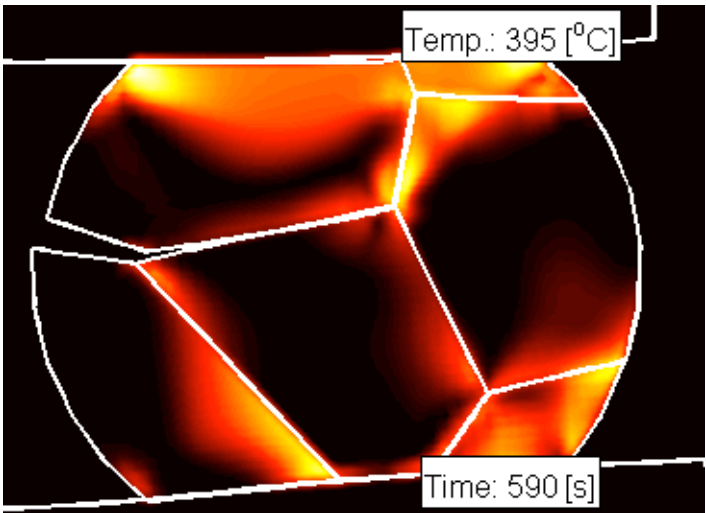
2017 Nguyen-Vinh Phu



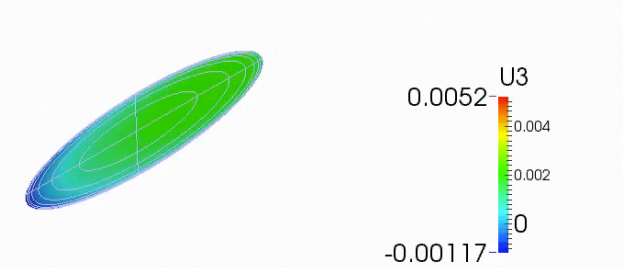
# Cracks and cuts create interfaces



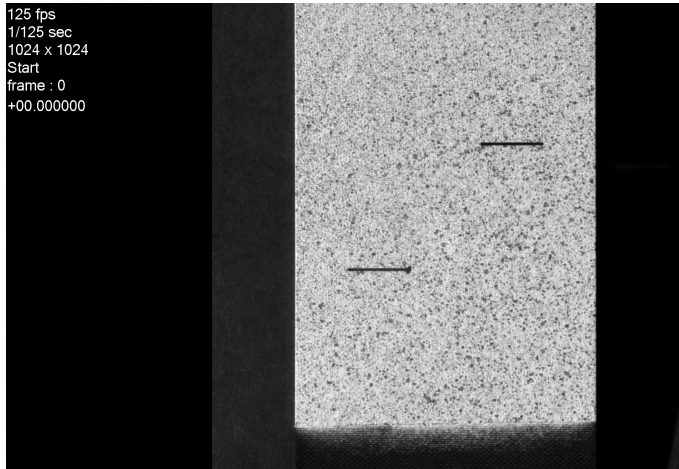
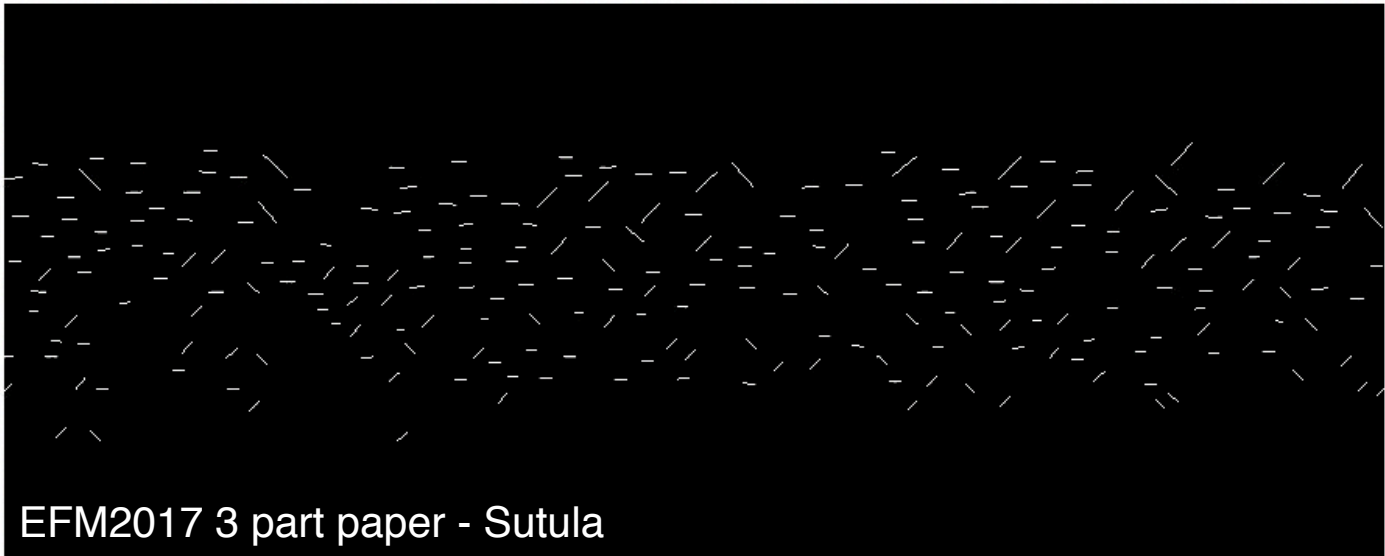
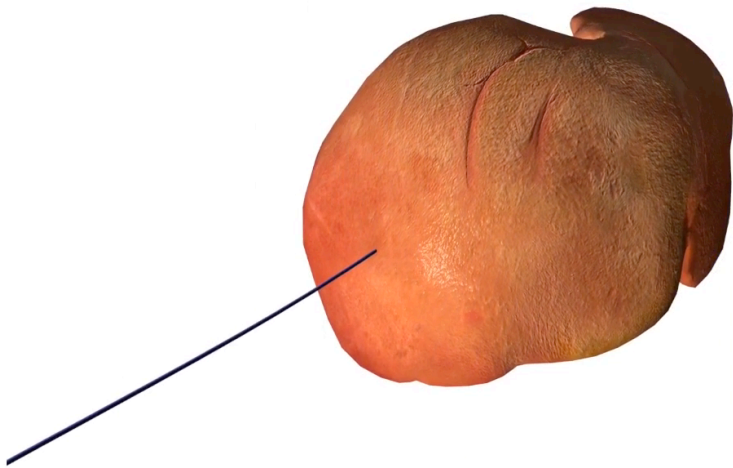
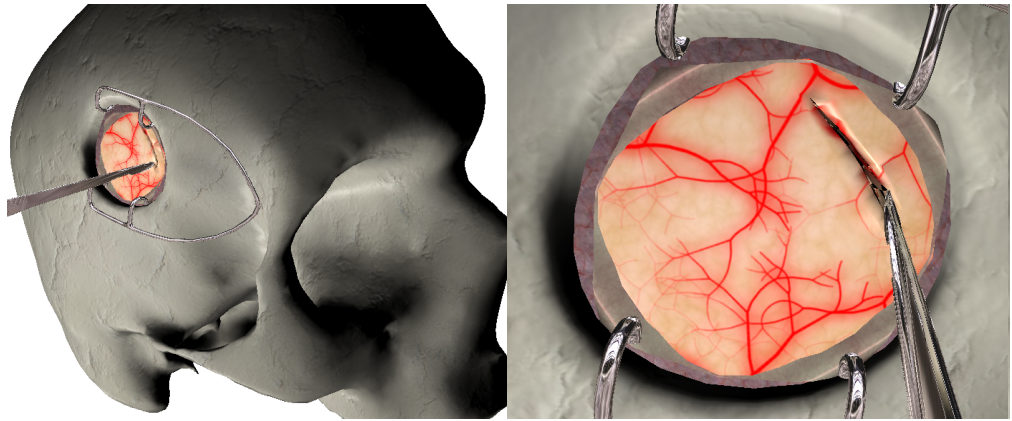
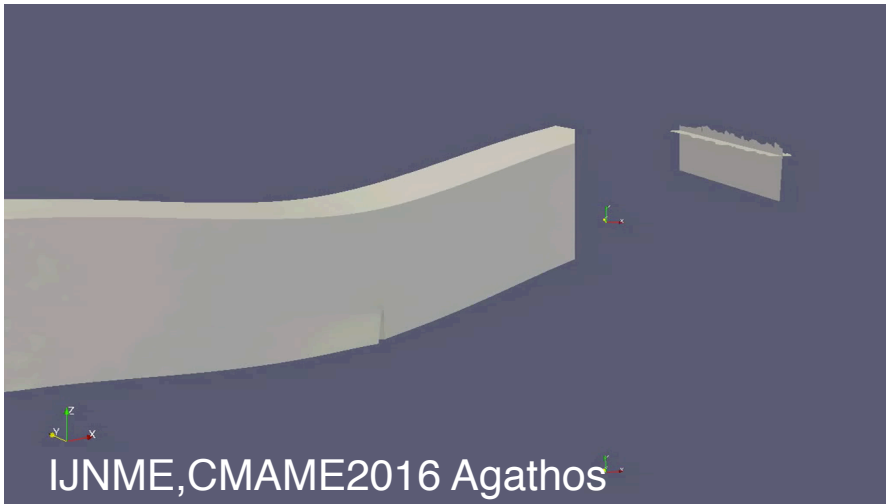
## CRACKS & CUTS



IJNME2011, CMS2012, Menk

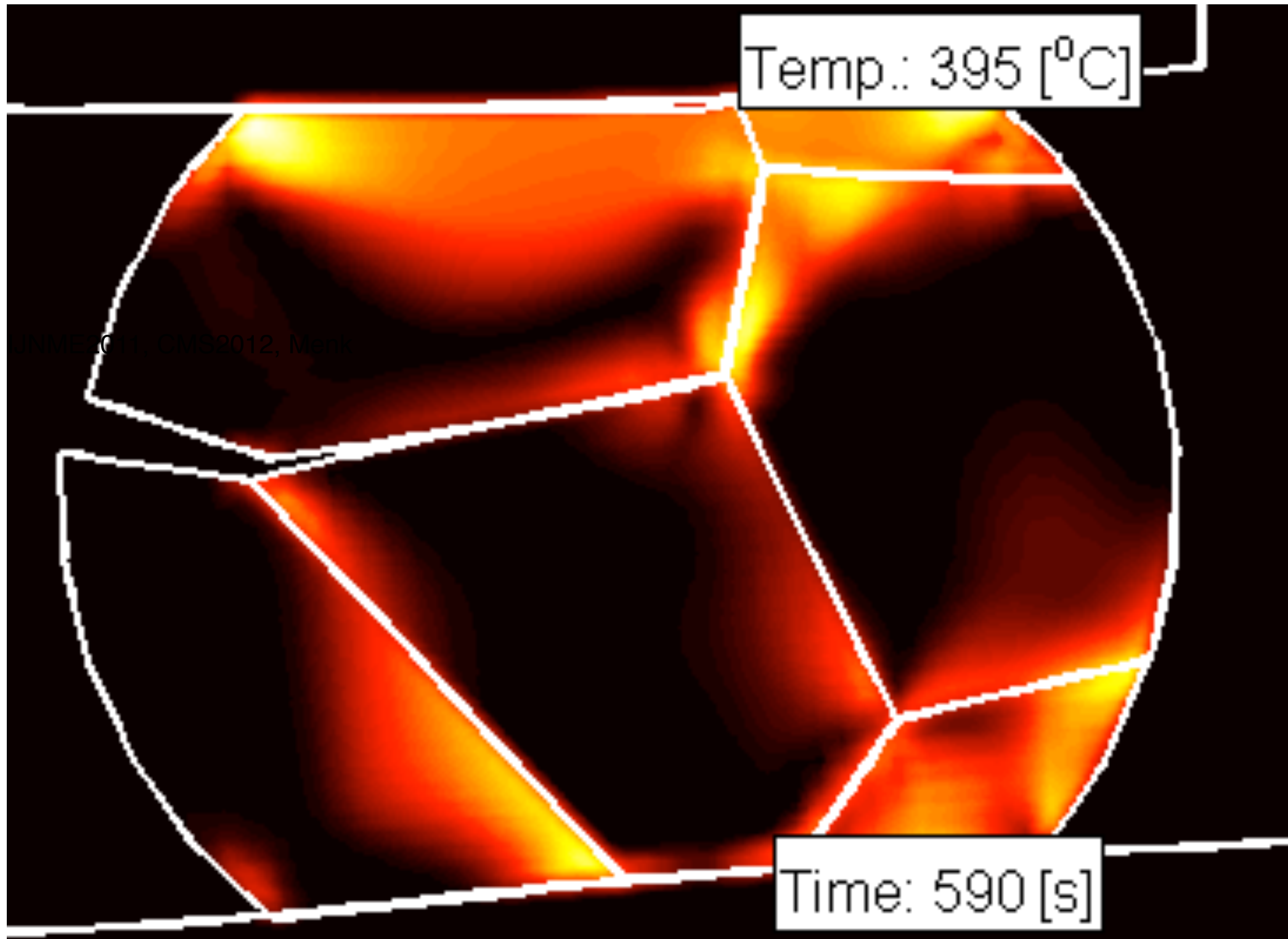


CMAME2016 Peng

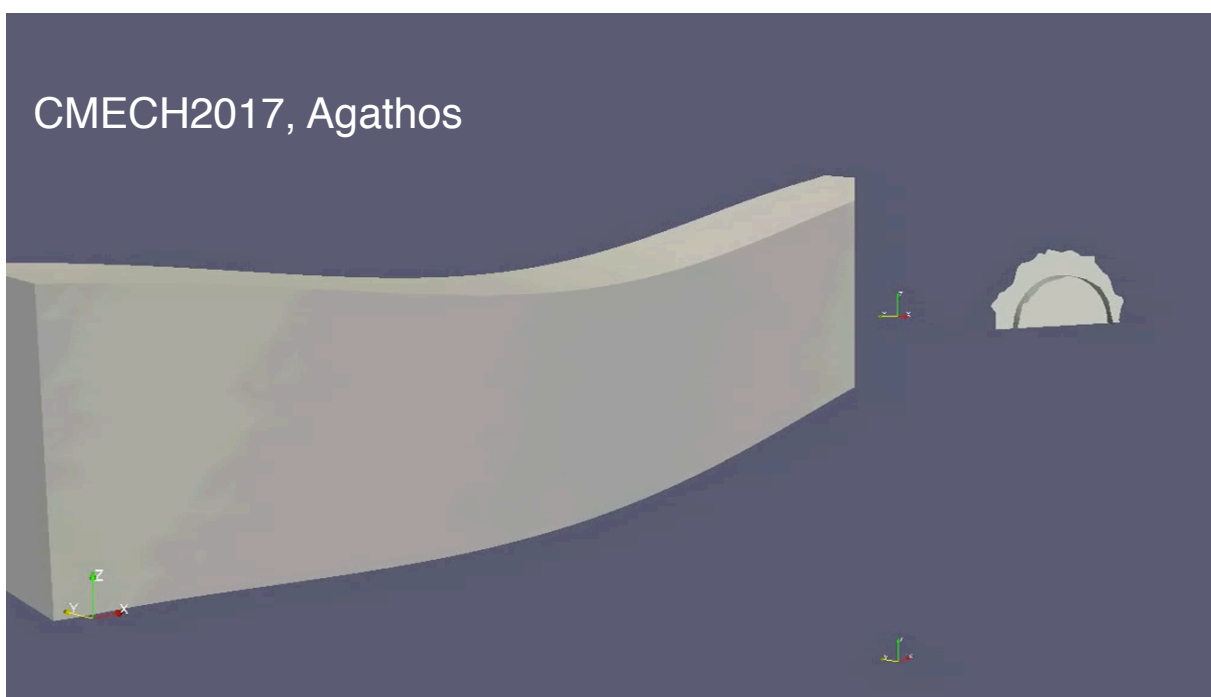
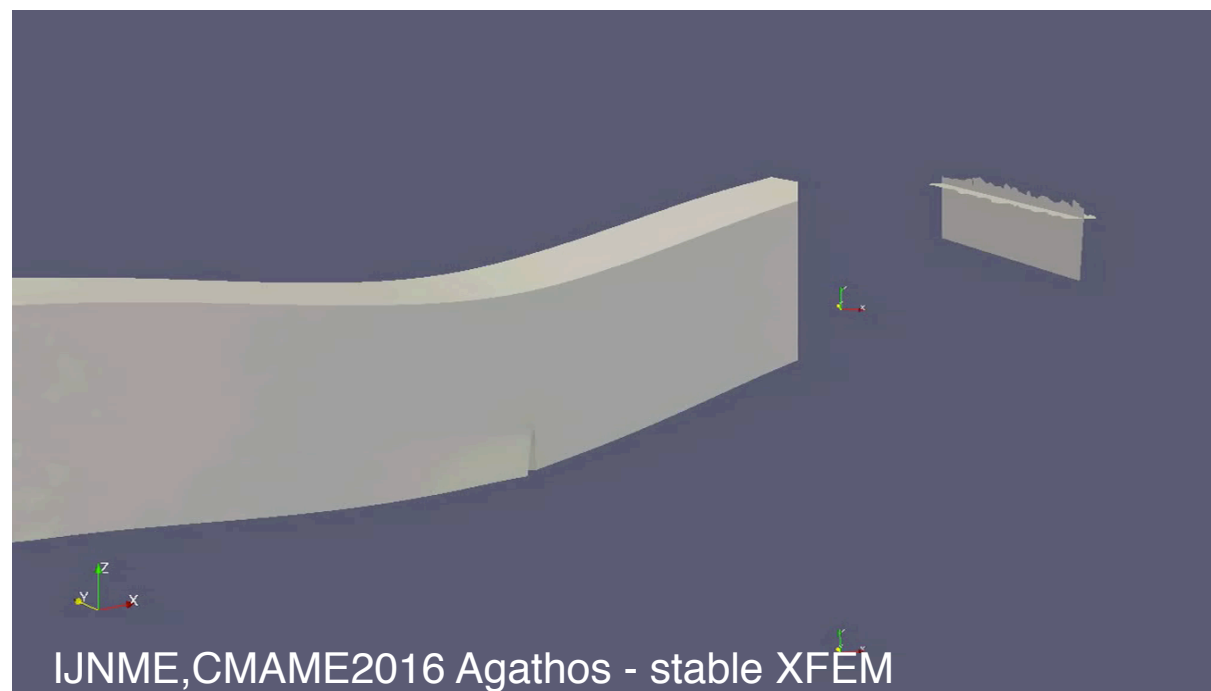




## CRACKS & CUTS

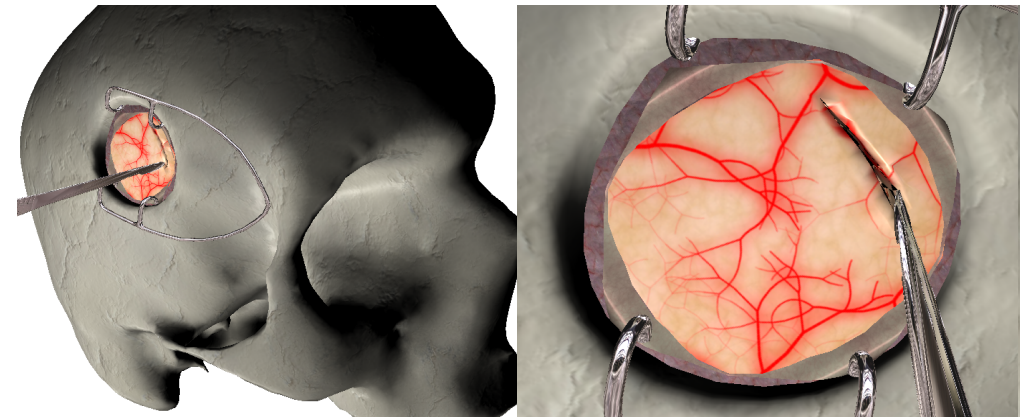


# Cracks and cuts create interfaces

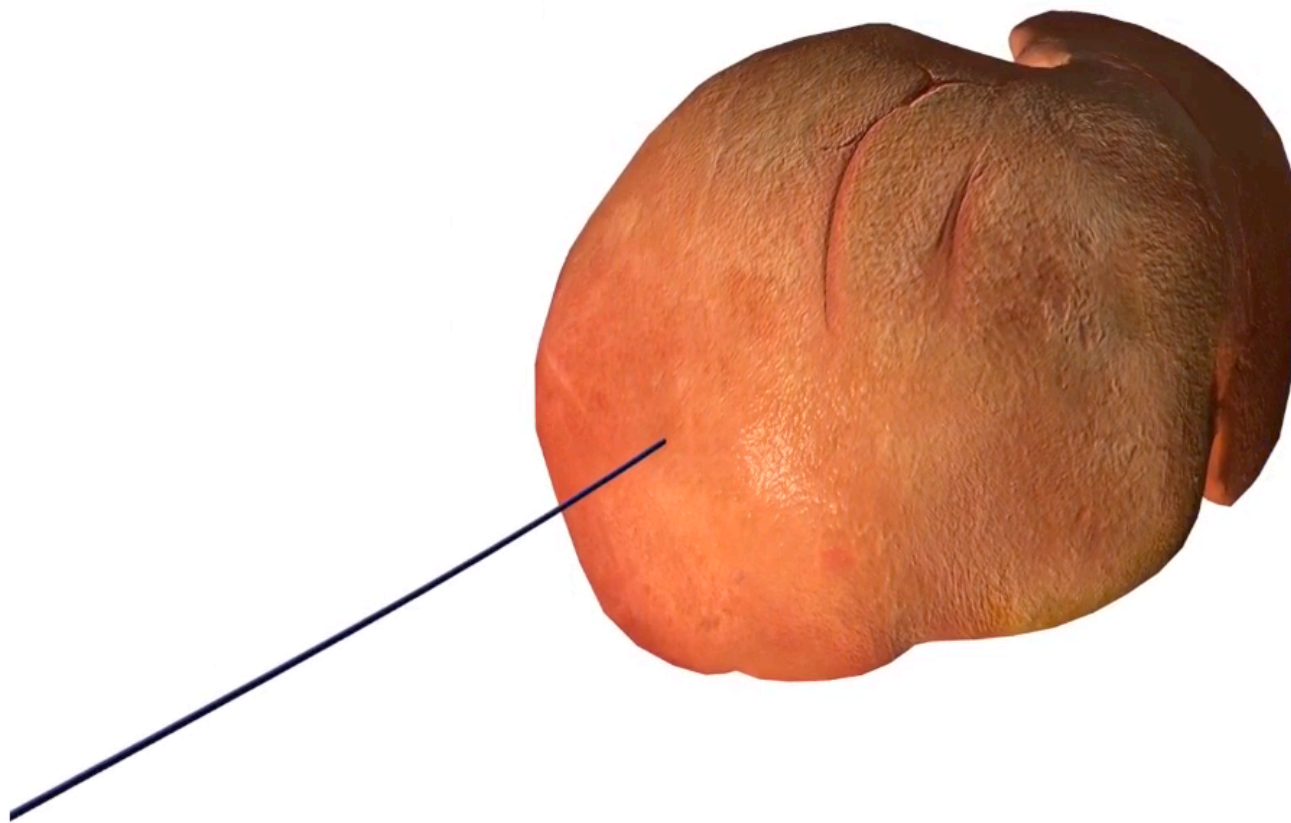




# Cracks and cuts create interfaces



*Real-time simulation of cutting during brain surgery -  
Med. Im. Anal. 2014 Courtecuisse*



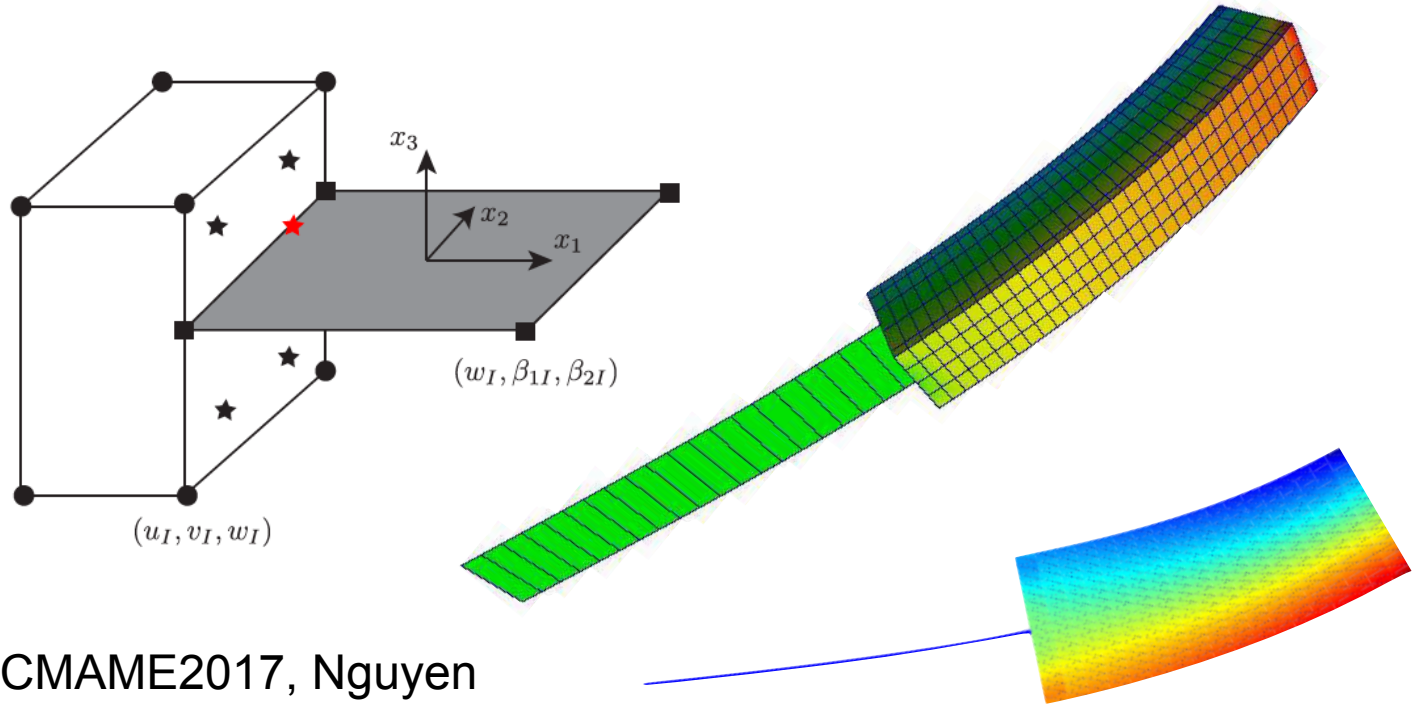
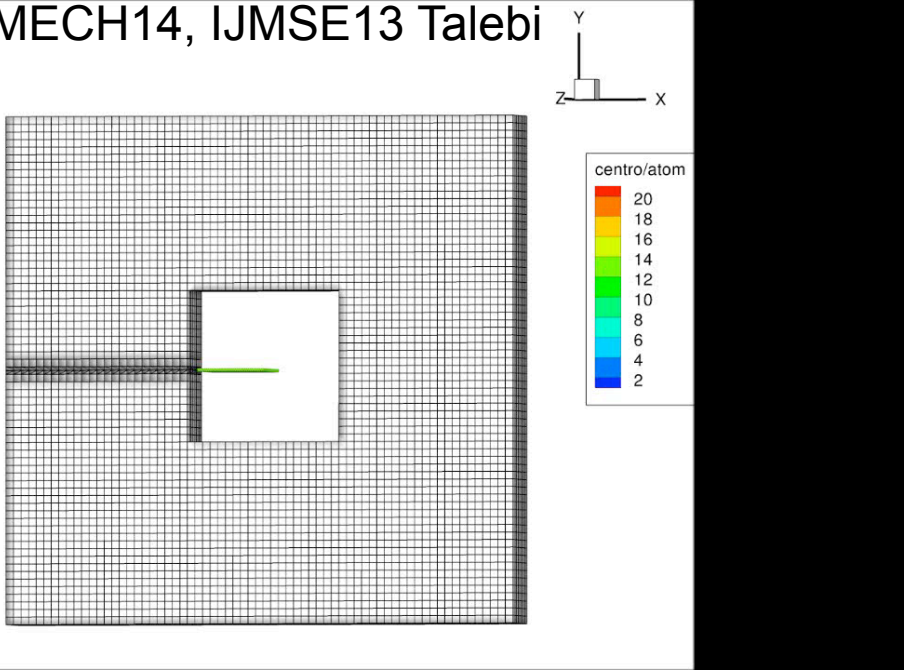
**Needle tissue interaction with breathing motion**

*IEEE J. Biomed. Engng. 2017 Bui*

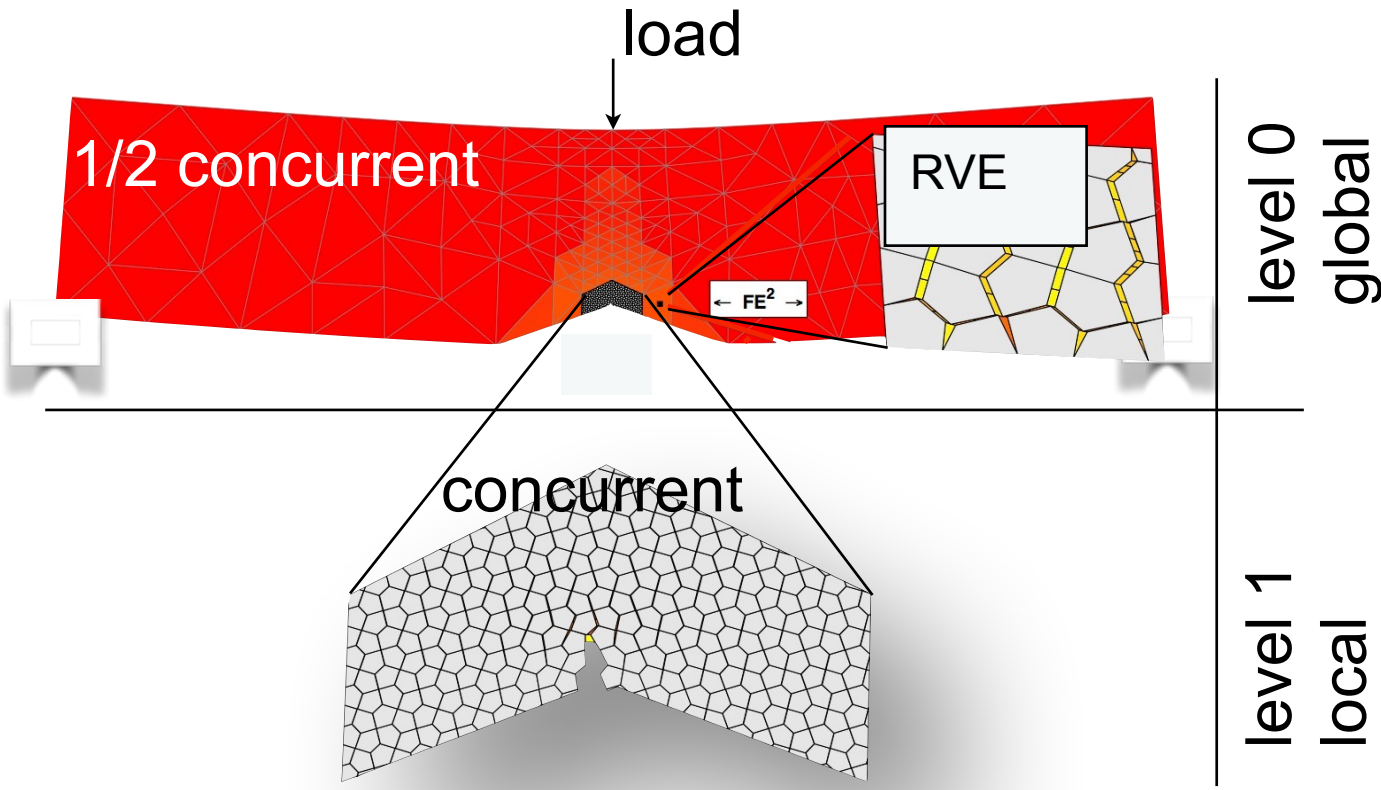
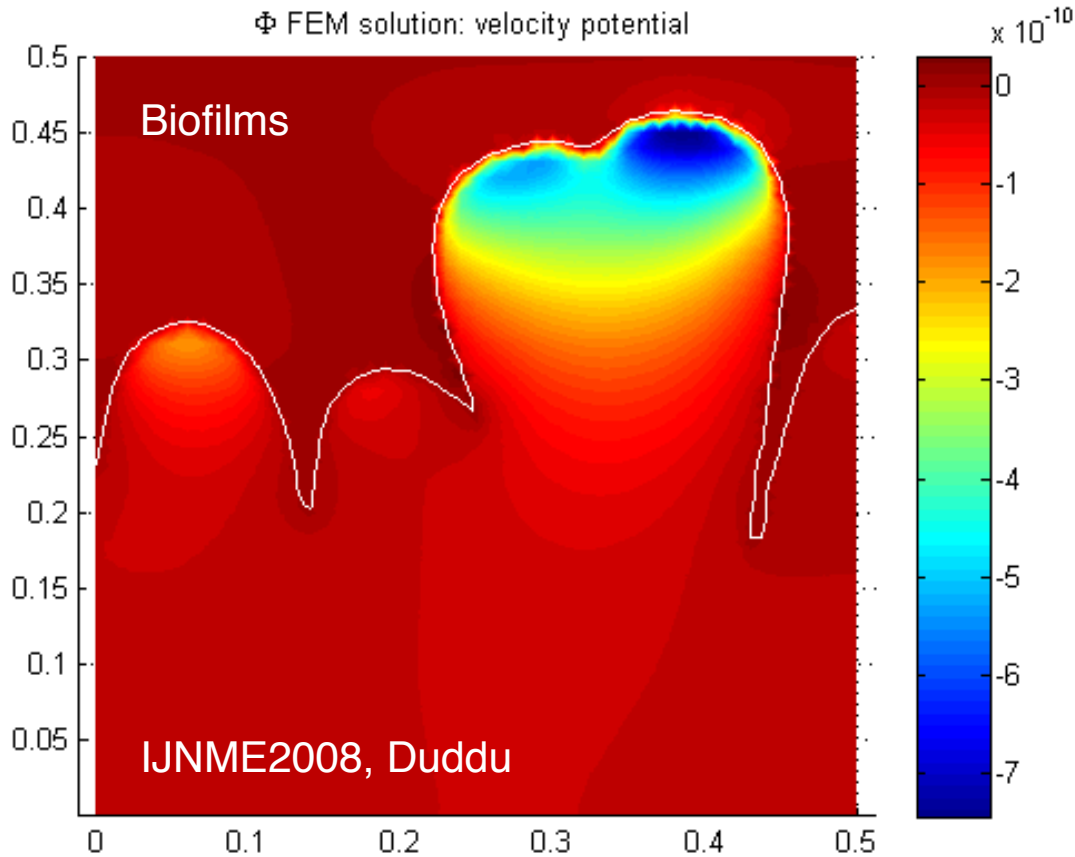


MODELS

CMECH14, IJMSE13 Talebi



CMAME2017, Nguyen



PhilMag15, Akbari  
CMAME13,CMECH16, Goury  
NMPDES13,CMAME15, Chi

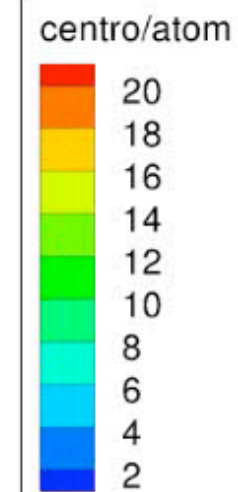
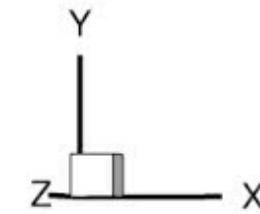
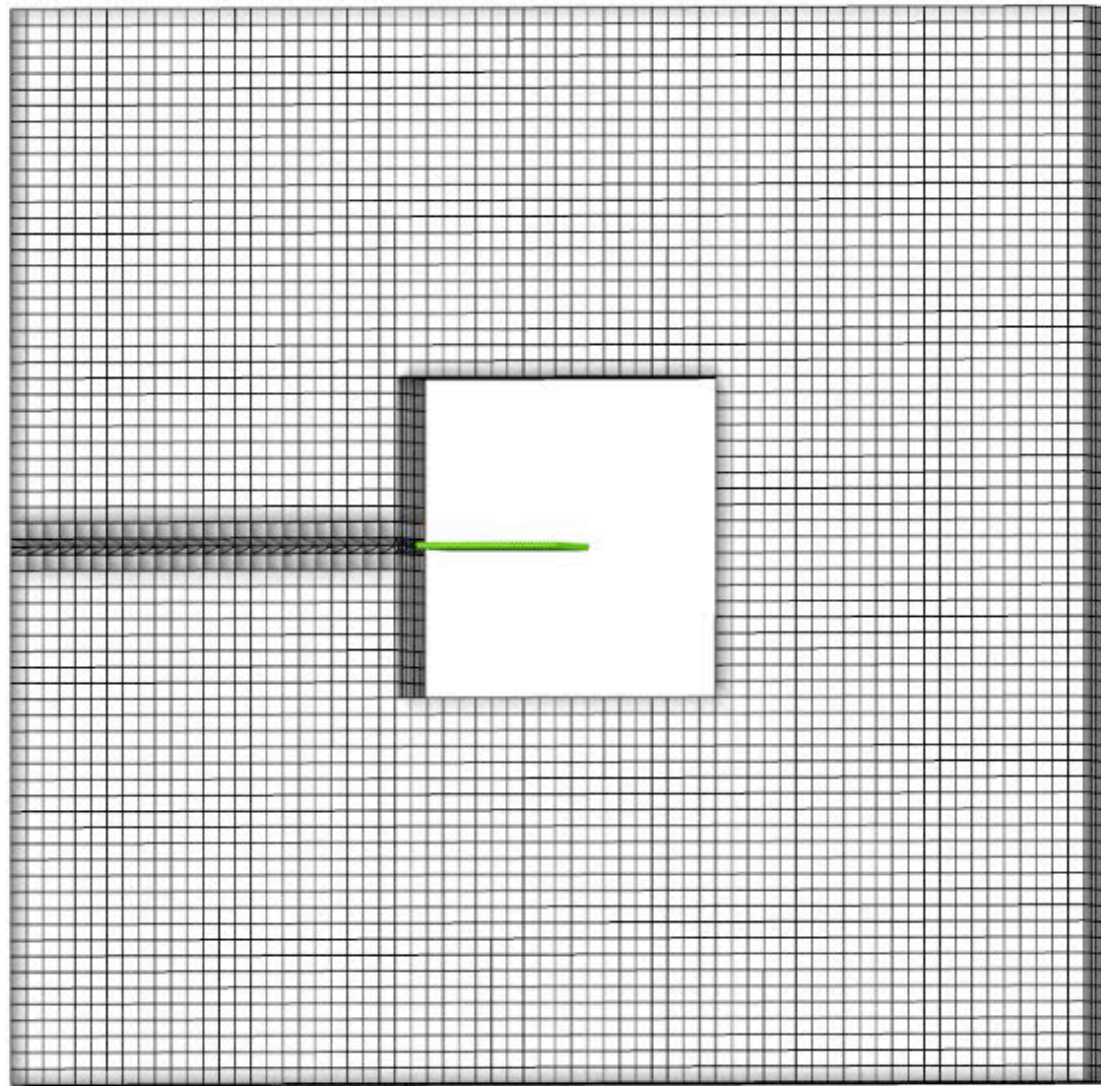




## MODELS

CMECH14, IJMSE13 Talebi

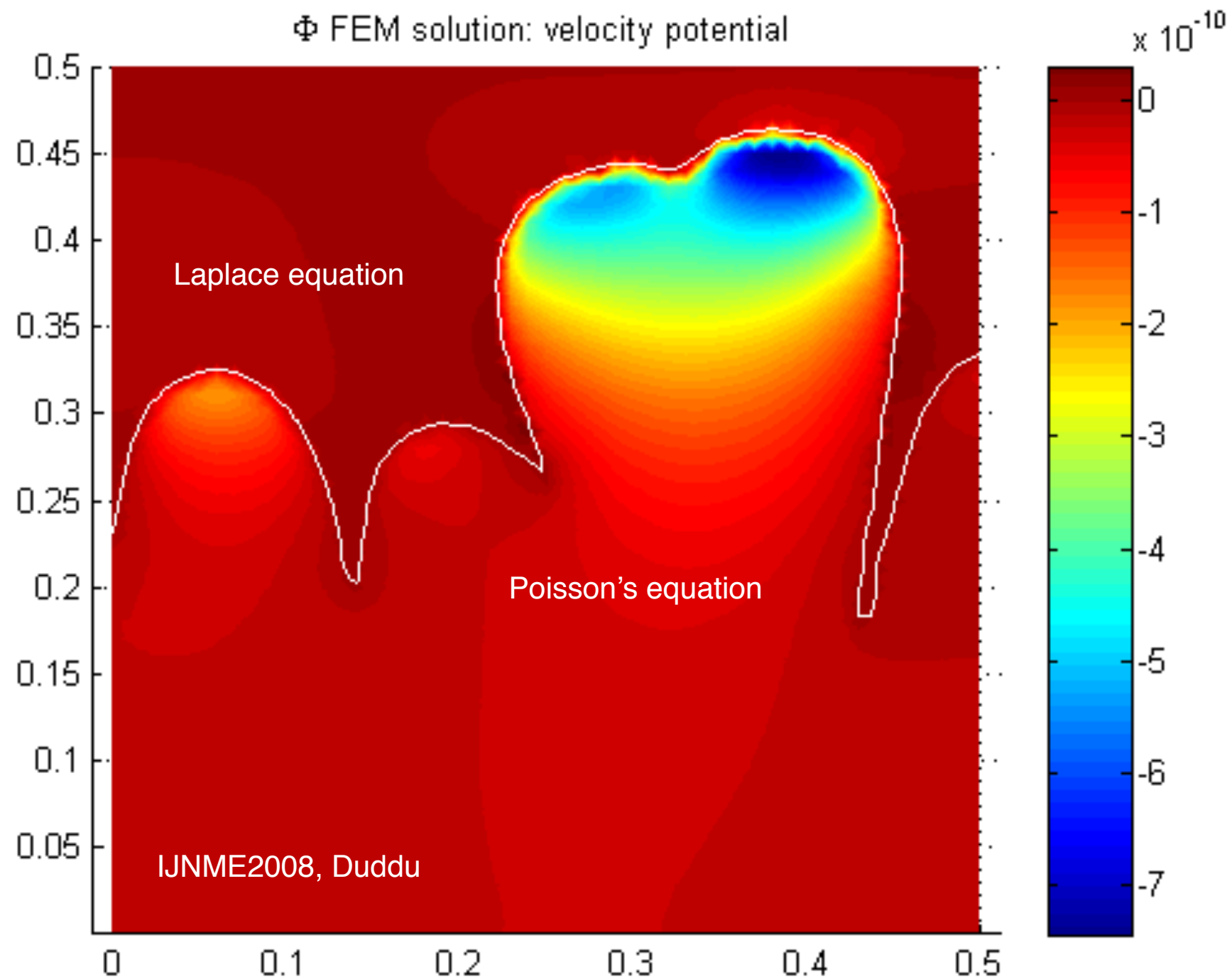
Biofilms

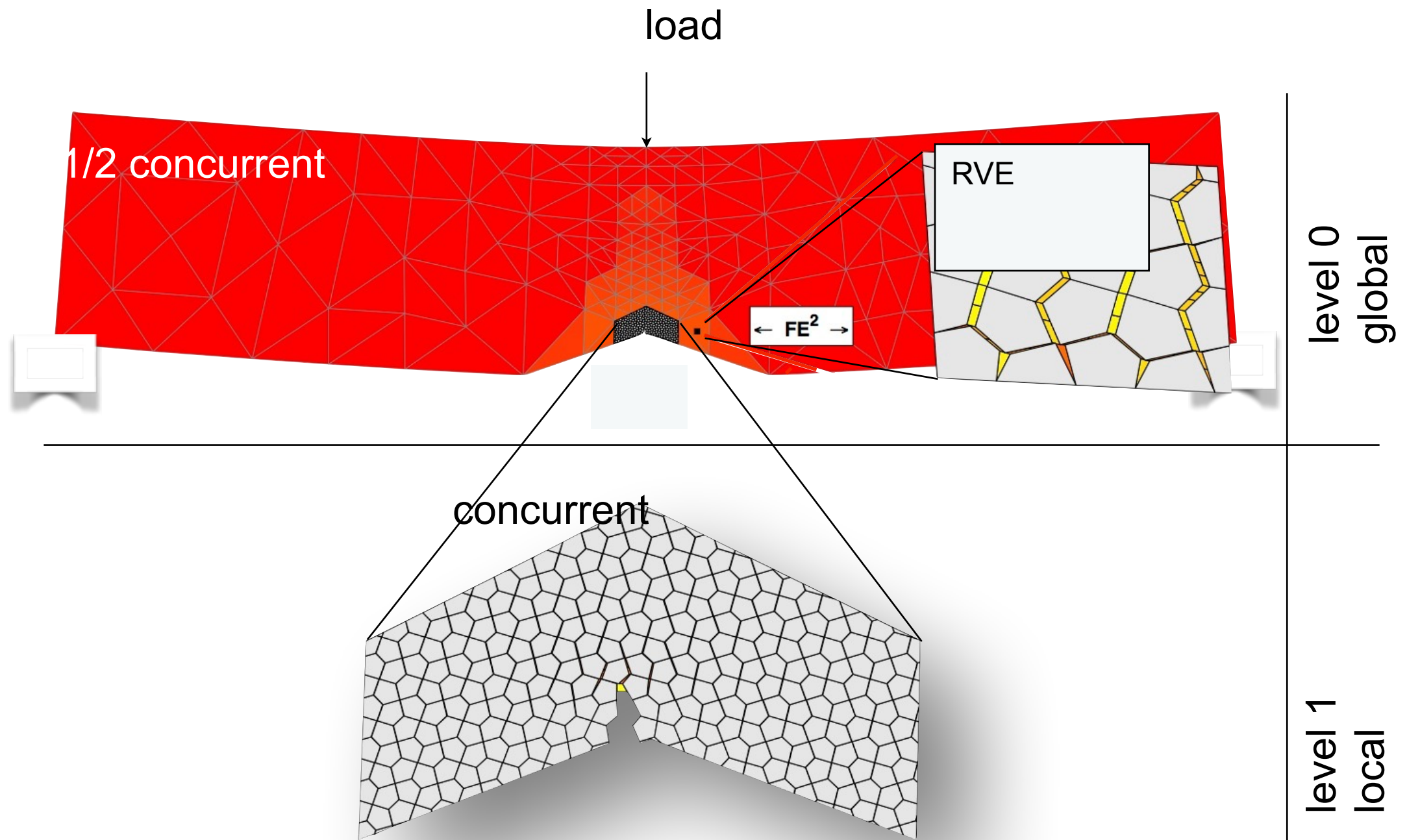






## MODELS

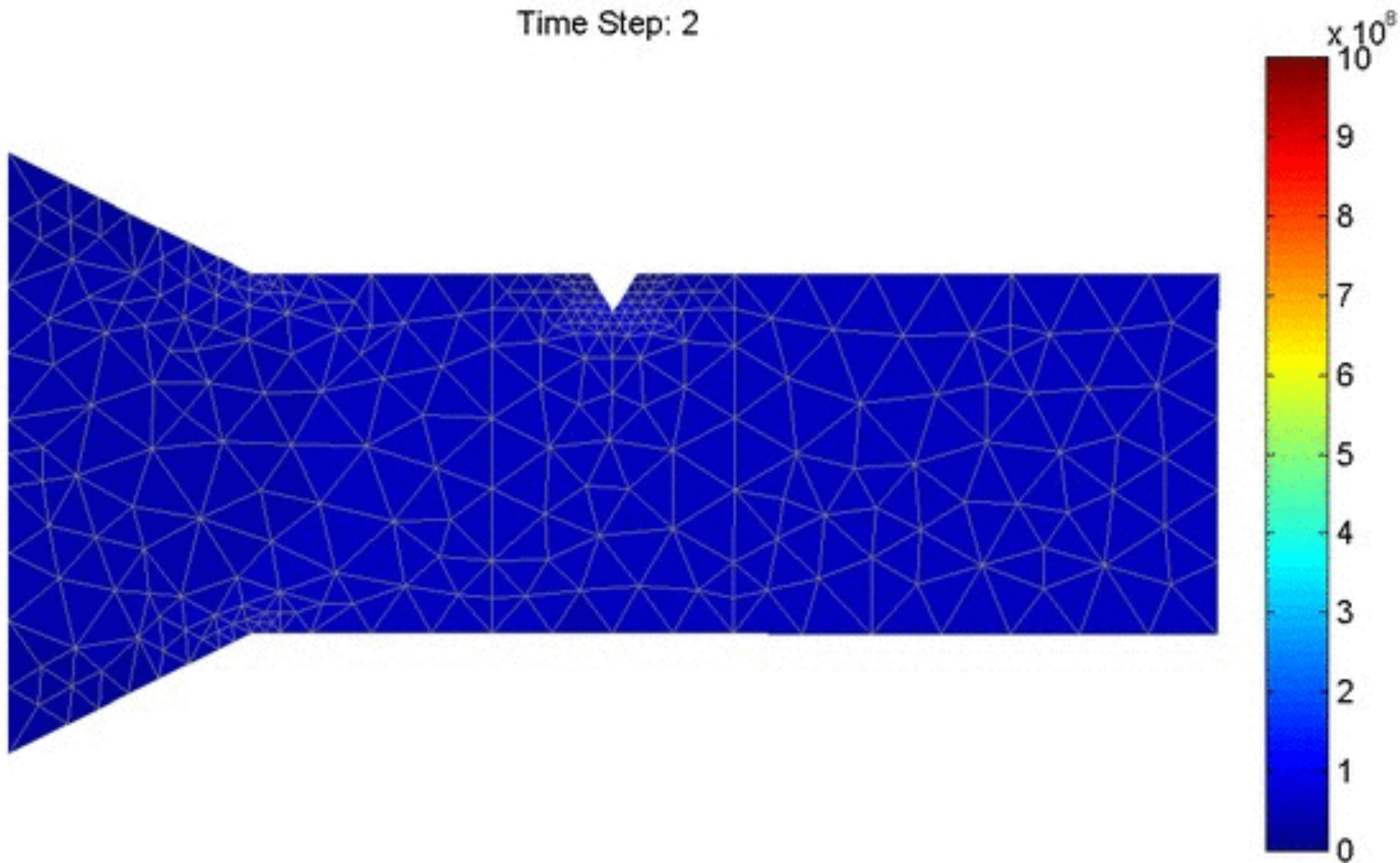




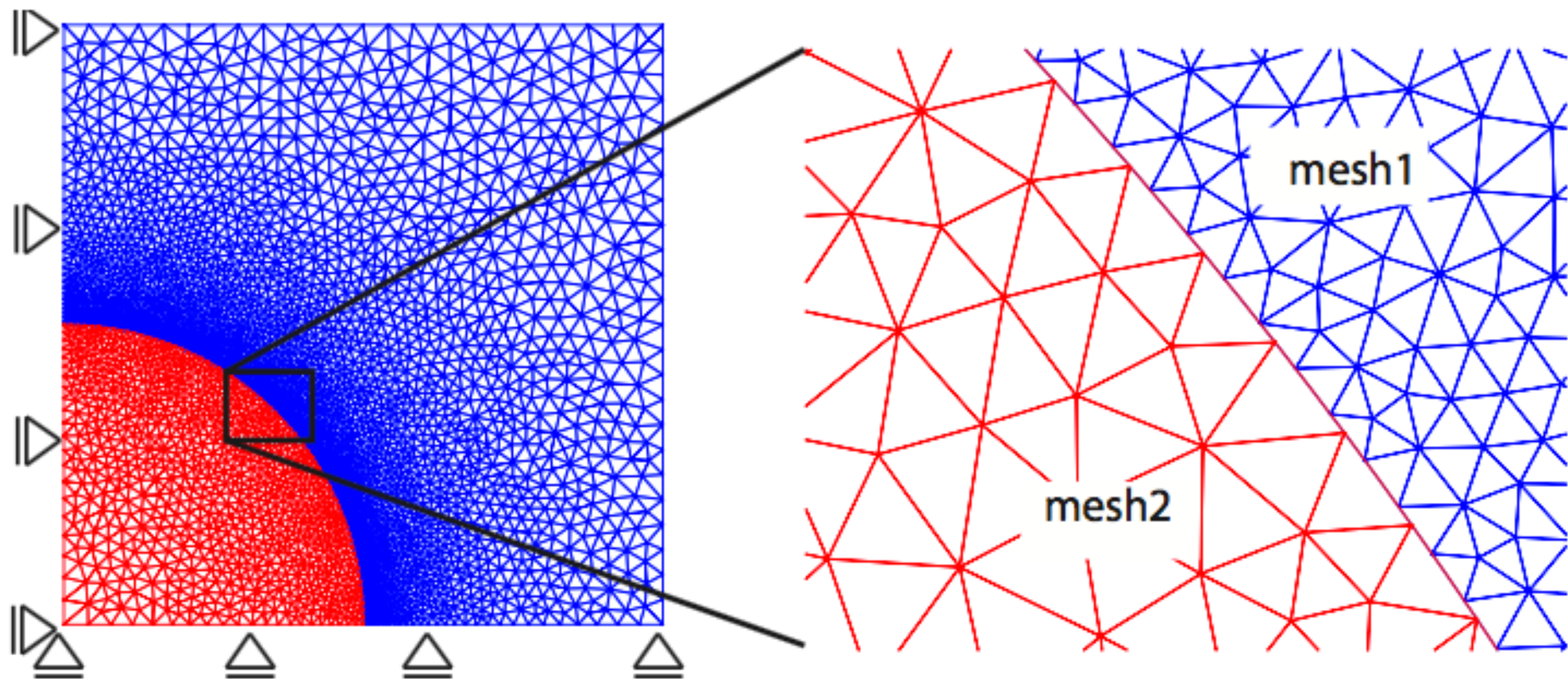
PhilMag15, Akbari  
CMAME13,CMECH16, Goury  
NMPDES13,CMAME15, Chi



Time Step: 2







CMECH2014, CAD2014, CMECH2016, MatCompSim2016, CMAME2017, Nguyen-Vinh Phu

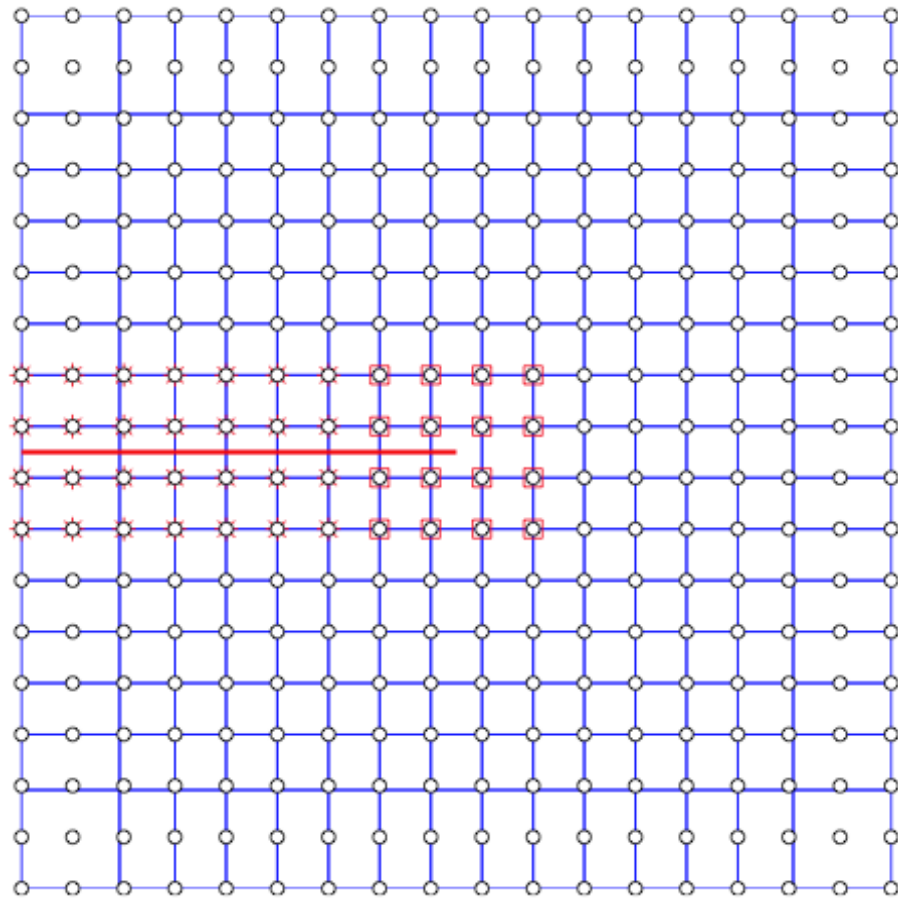
<http://publications.uni.lu/bitstream/10993/13726/1/phu-meshless.pdf>

<https://orbilu.uni.lu/bitstream/10993/15234/1/bordasphu.pdf>



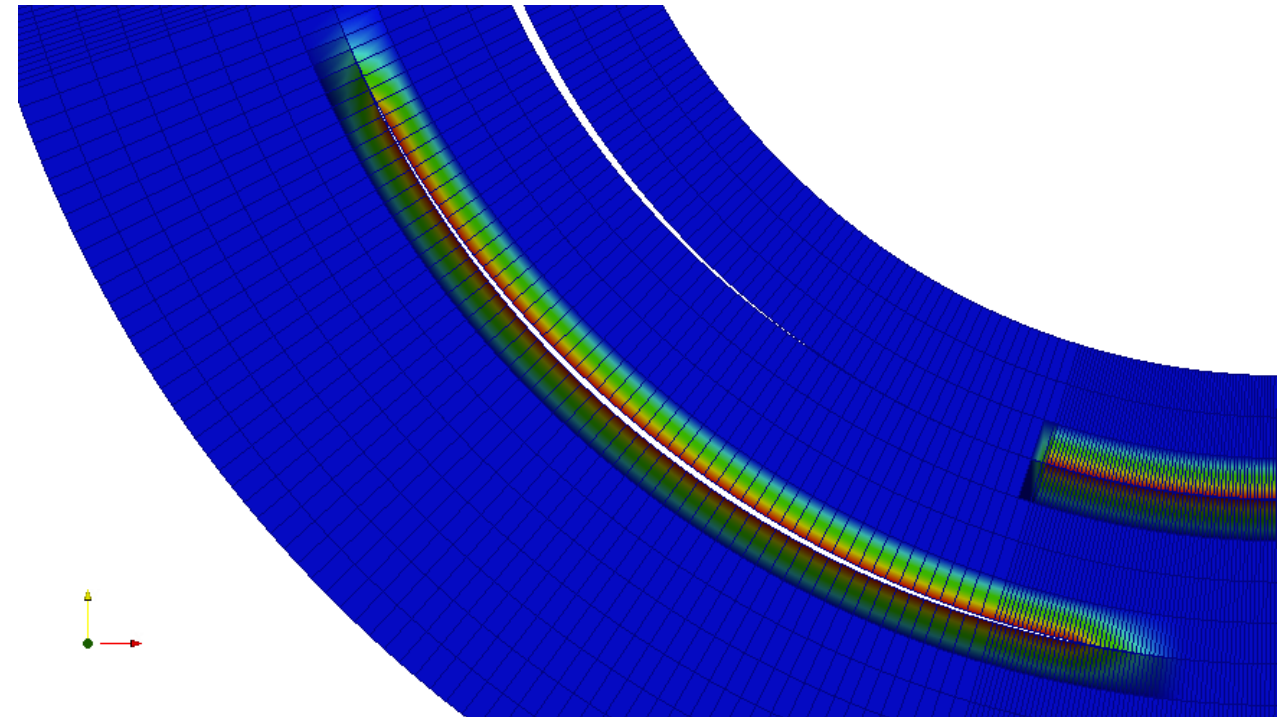


## PUM enriched methods



- IGA: link to CAD and accurate stress fields
- XFEM: no remeshing

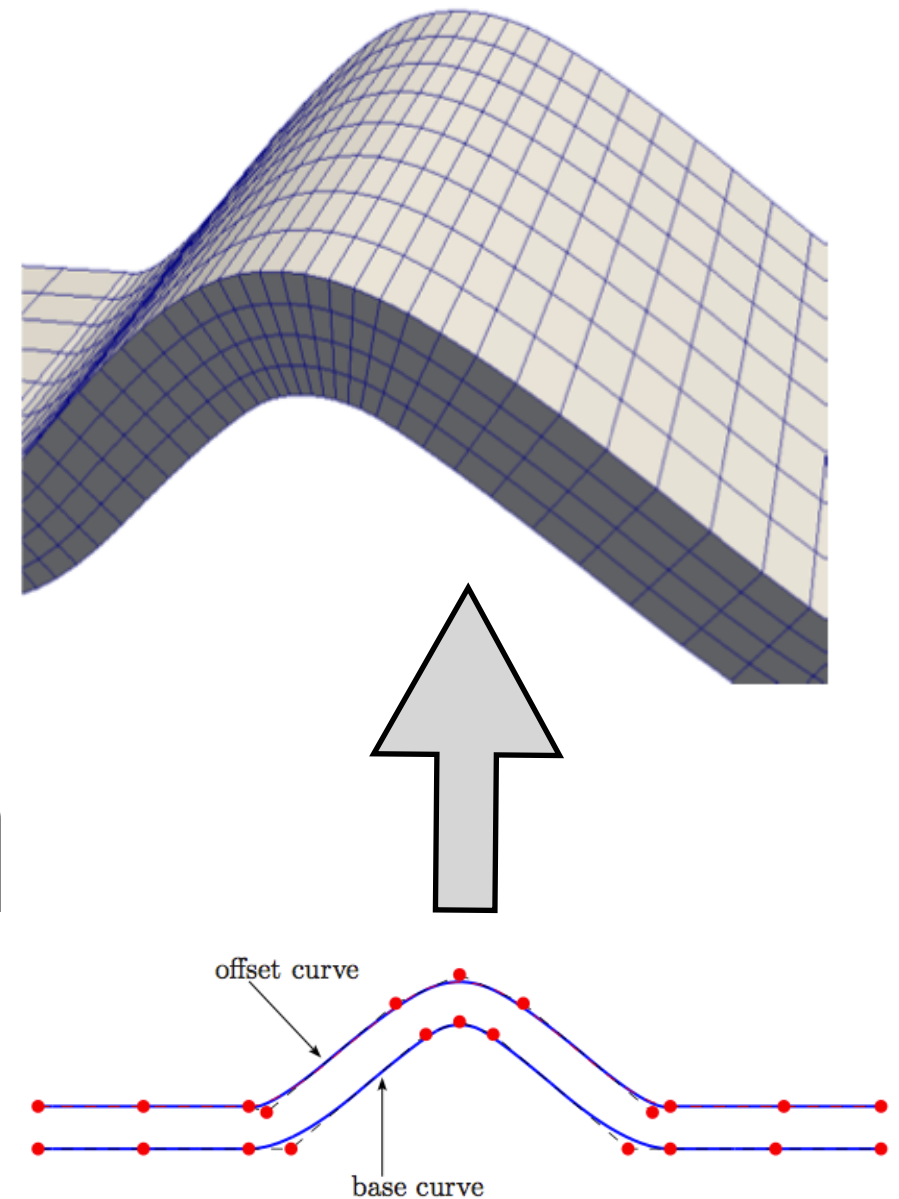
## Mesh conforming methods



- IGA: link to CAD and accurate stress fields
- Apps: delamination

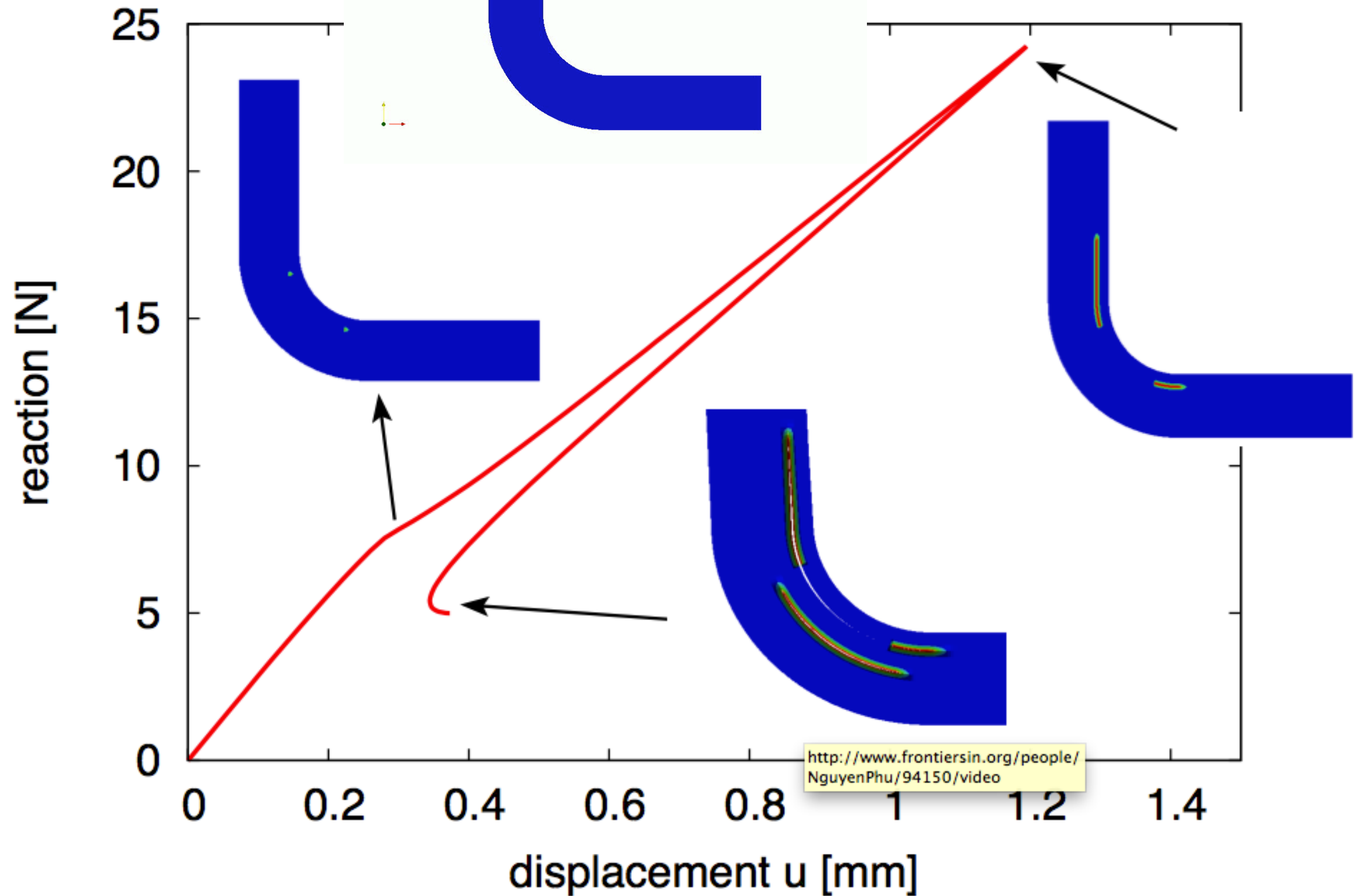
# Isogeometric cohesive elements: advantages

- Direct link to CAD
  - Exact geometry
  - Fast/straightforward generation of interface elements
  - Accurate stress field
  - Computationally cheaper
- 
- 2D Mixed mode bending test (MMB)
  - 2 x 70 quartic-linear B-spline elements
  - Run time on a laptop 4GBi7: 6 s
  - Energy arc-length control

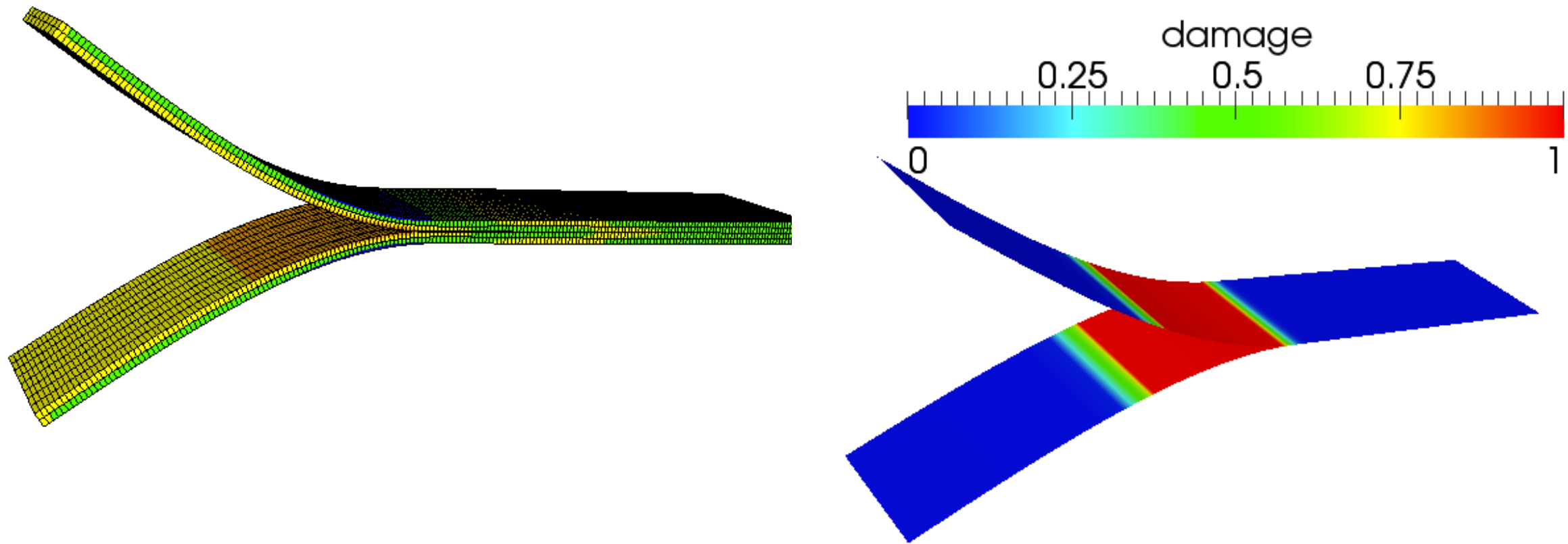


V. P. Nguyen and H. Nguyen-Xuan. High-order B-splines based finite elements for delamination analysis of laminated composites. *Composite Structures*, 102:261–275, 2013.

# Isogeometric cohesive elements: 2D example



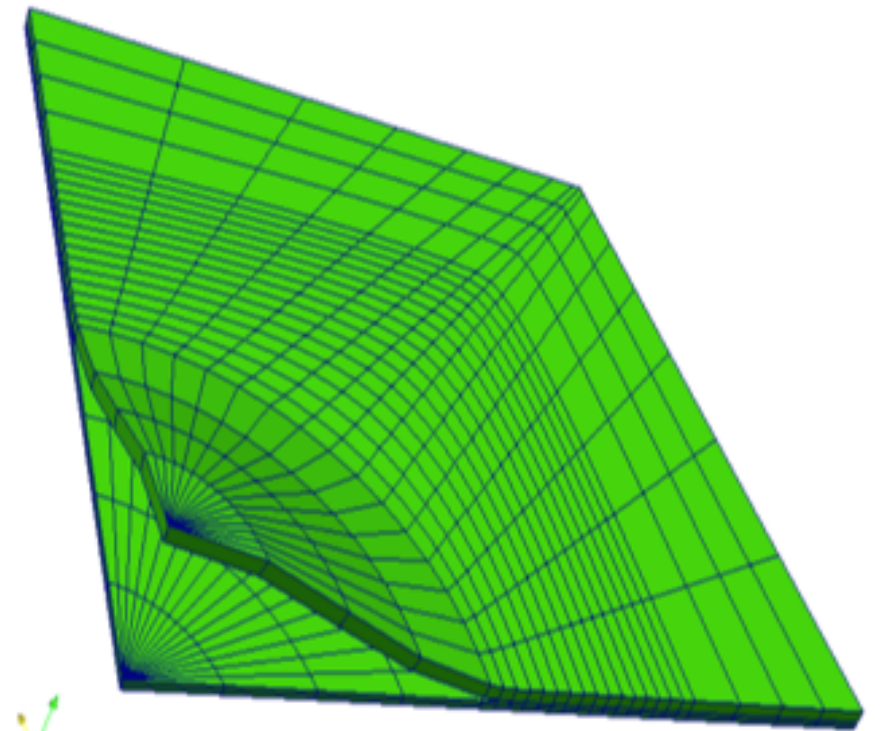
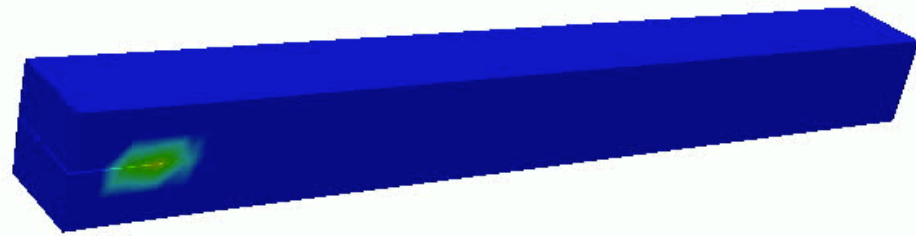
# Isogeometric cohesive elements: 3D example with shells



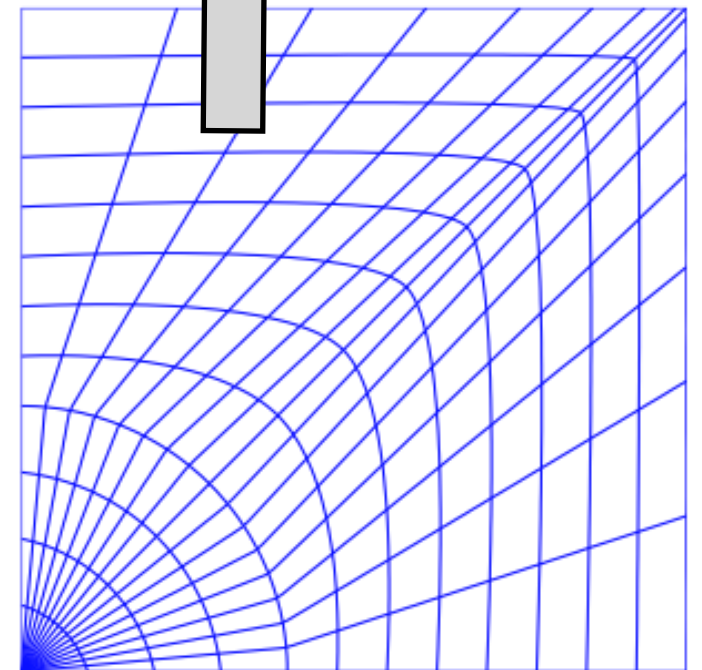
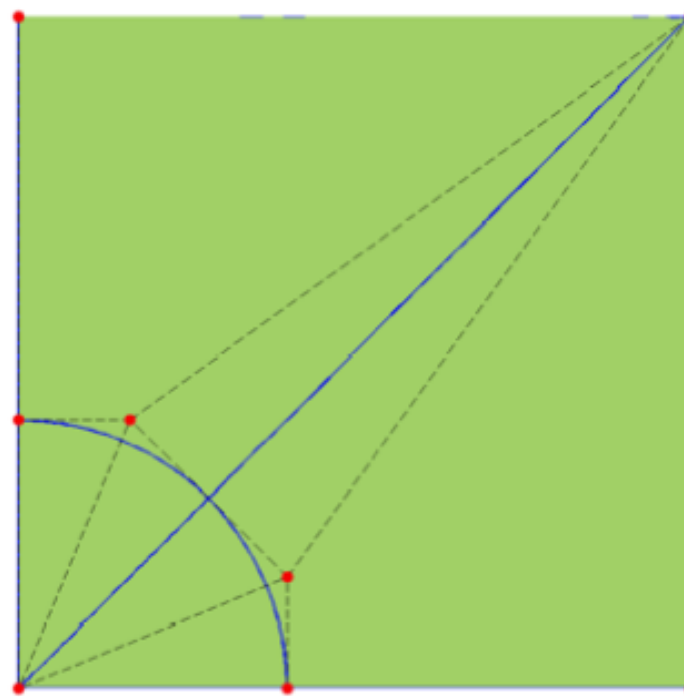
- Rotation free B-splines shell elements (Kiendl et al. CMAME)
- Two shells, one for each lamina
- Bivariate B-splines cohesive interface elements in between



# Isogeometric cohesive elements: 3D examples



- cohesive elements for 3D meshes the same as 2D
- large deformations



ITN  
INSIST

# Streamlining the CAD-analysis transition

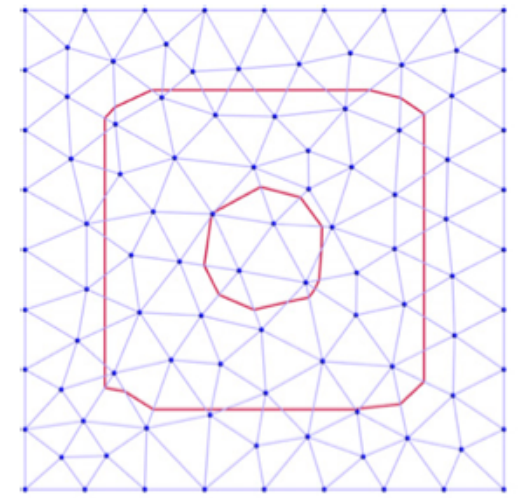
## *Coupling, or decoupling?*

ITN  
INSIST

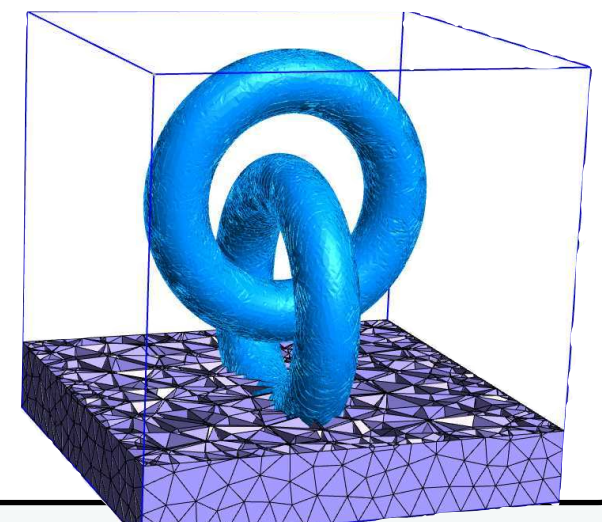
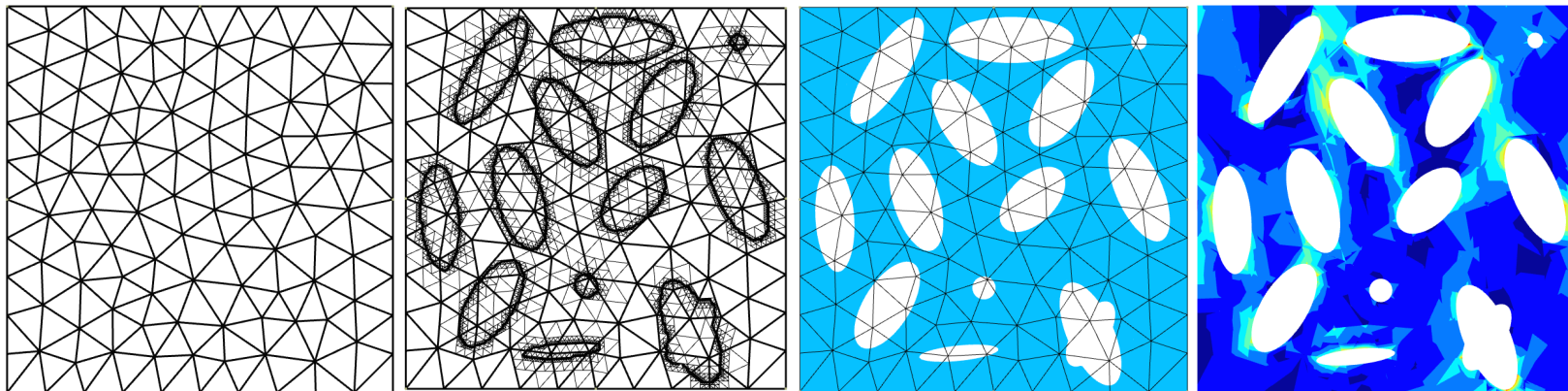
*Decoupling CAD and Analysis.*

# Separate field and boundary discretisation

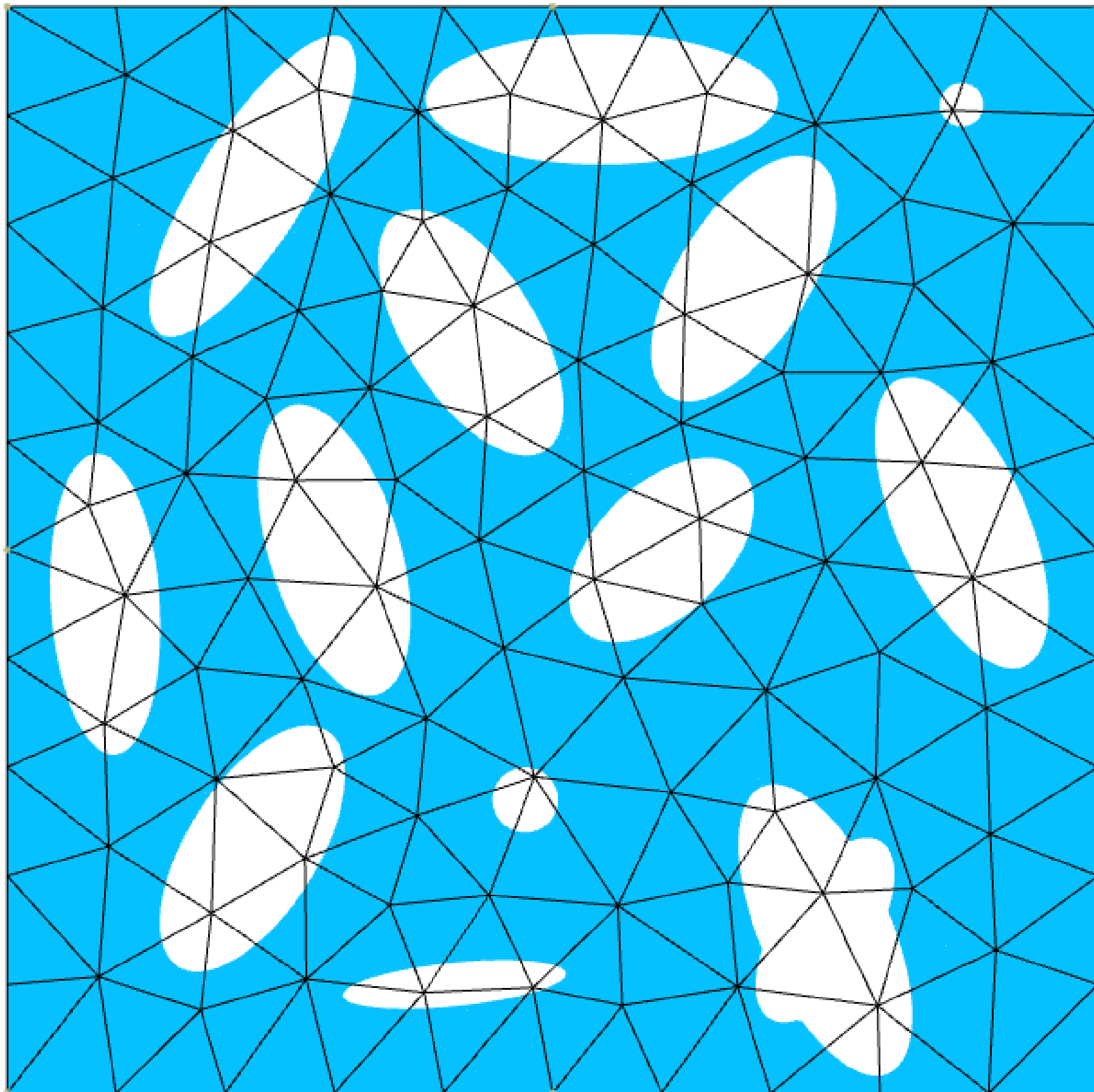
- Immersed boundary method (Mittal, *et al.* 2005)
- Fictitious domain (Glowinski, *et al.* 1994)
- Embedded boundary method (Johansen, *et al.* 1998)
- Virtual boundary method (Saiki, *et al.* 1996)
- Cartesian grid method (Ye, *et al.* 1999, Nadal, 2013)

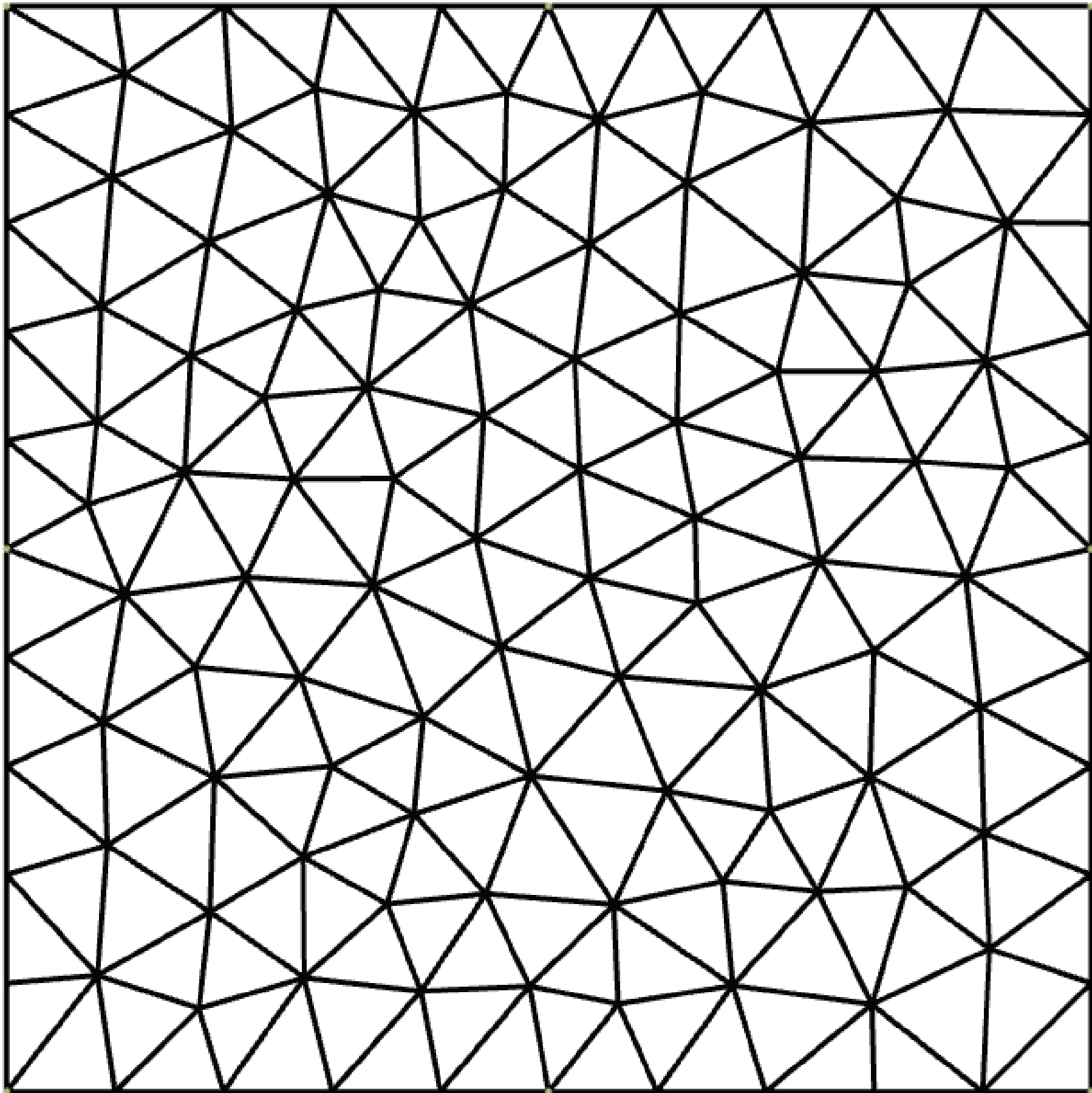


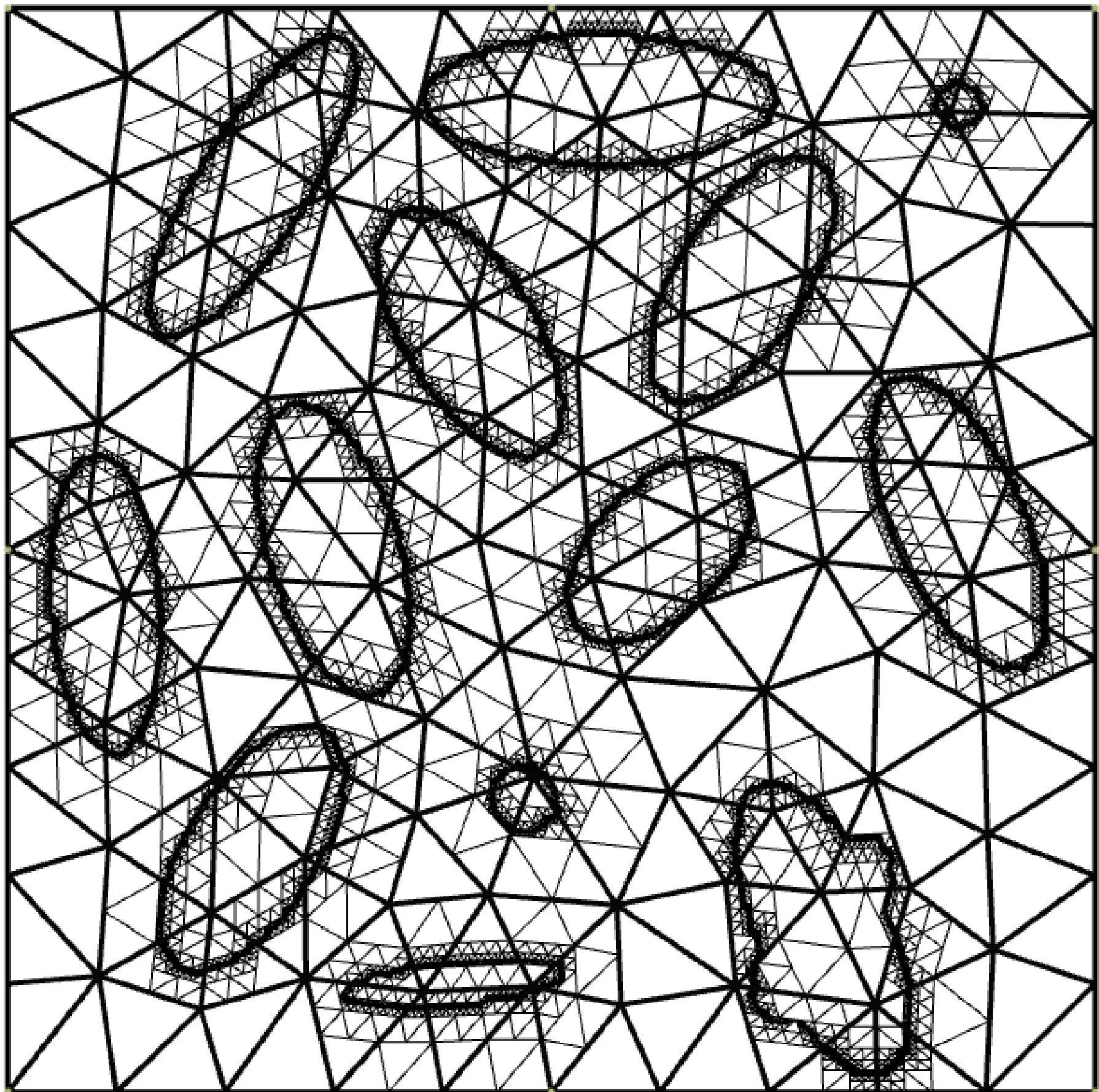
- ✓ Easy adaptive refinement + error estimation (Nadal, 2013)
- ✓ Flexibility of choosing basis functions
- Accuracy for complicated geometries? BCs on implicit surfaces?
- ➔ An accurate and implicitly-defined geometry from arbitrary parametric surfaces including corners and sharp edges (CMAME2011, Moumnassi)



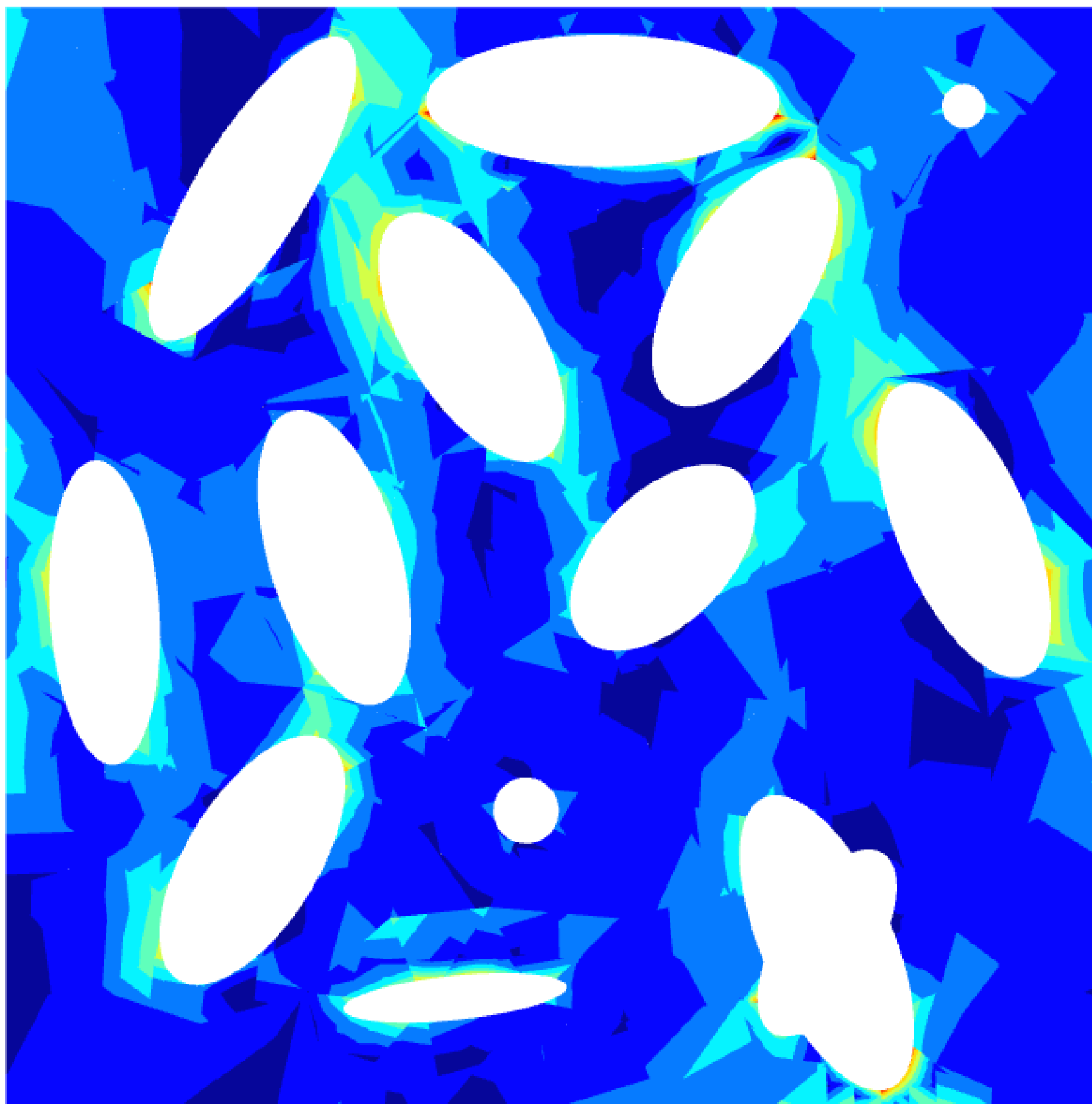




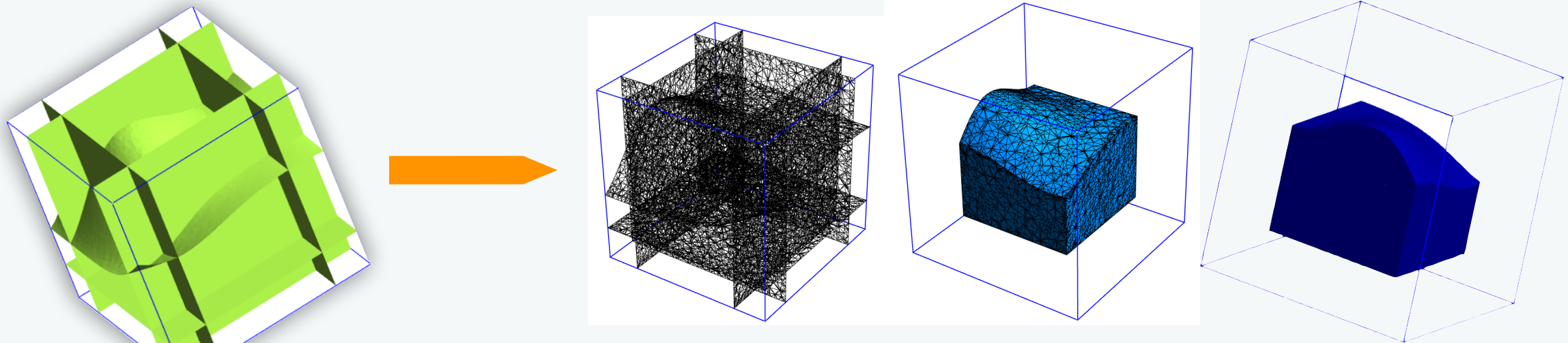




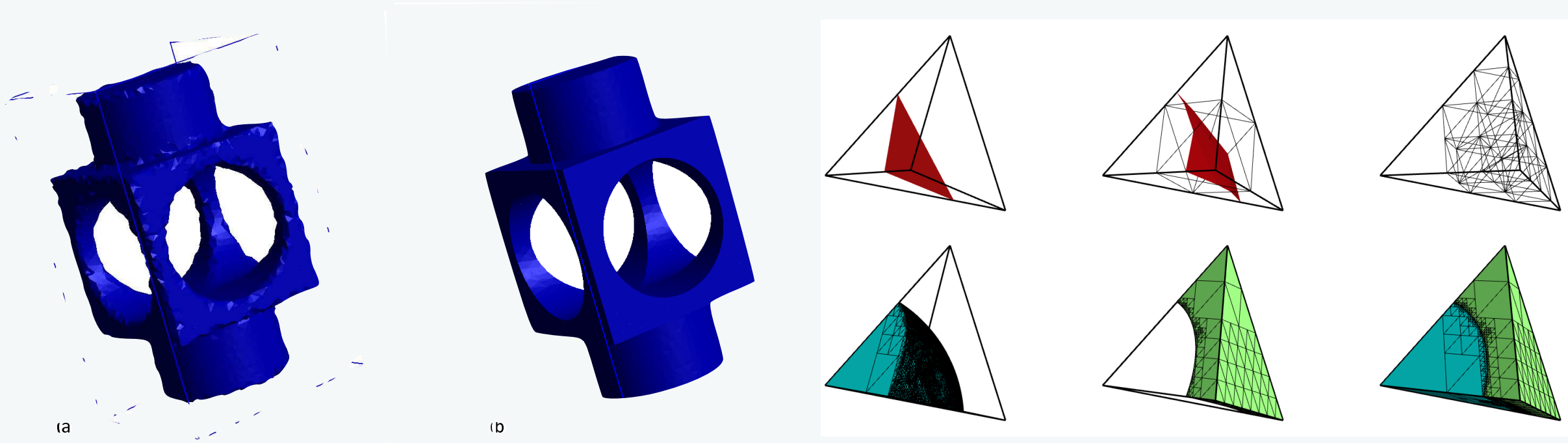




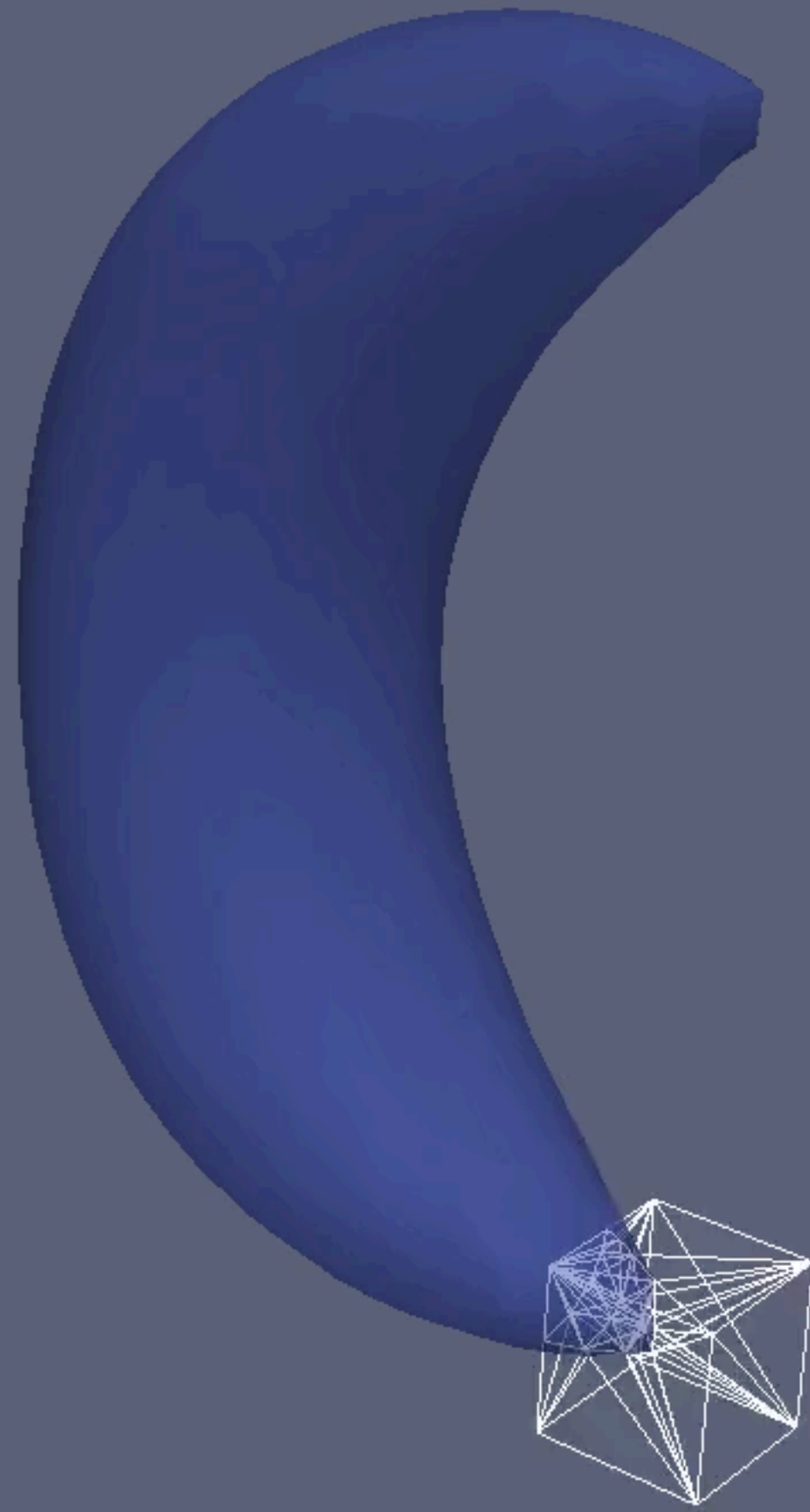
- multiple level sets



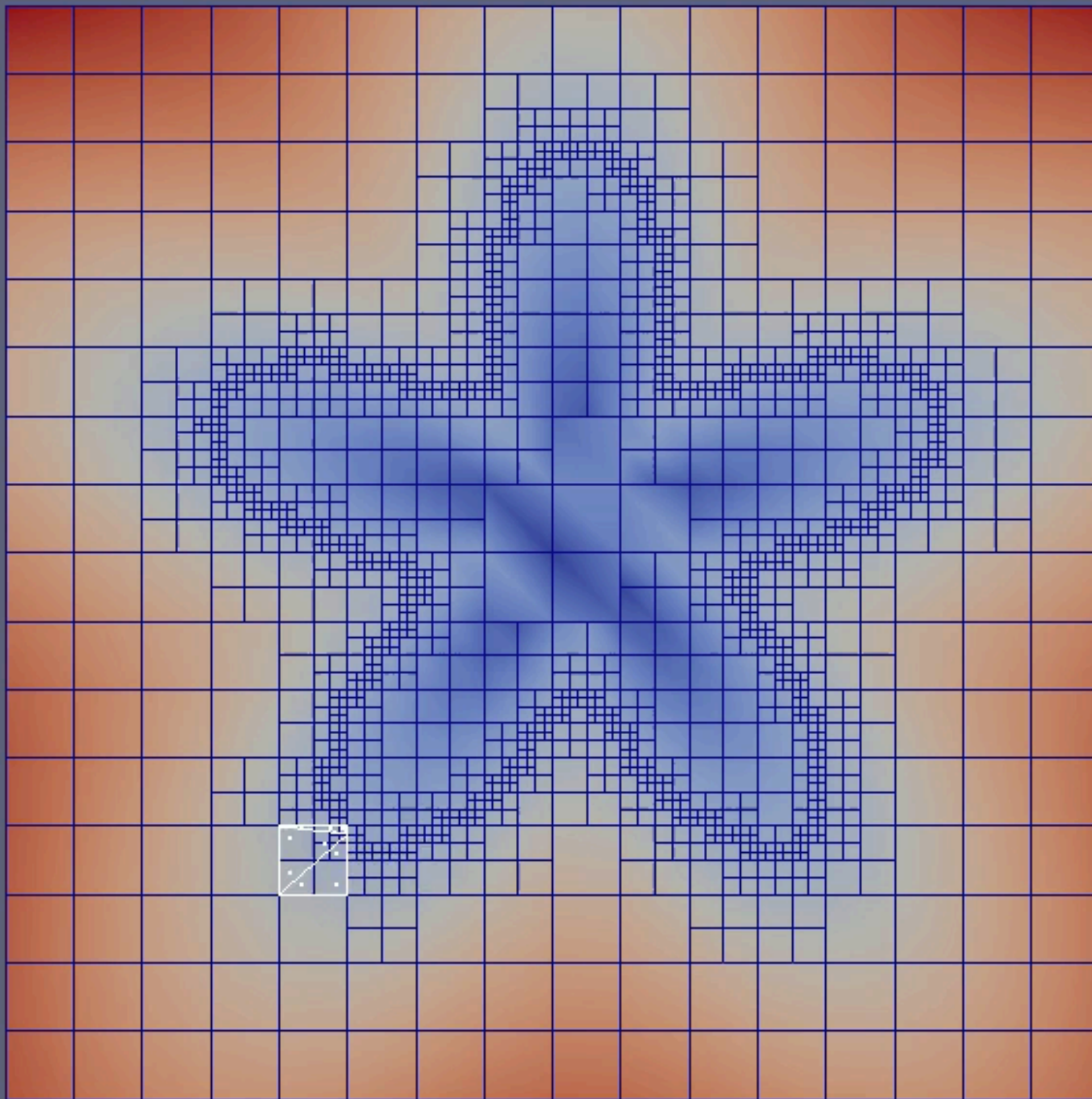
- single (left) versus multiple (right)



<http://legato-team.eu/project/mesh-burden/quadtree-and-octree-implicit-boundary-methods/>





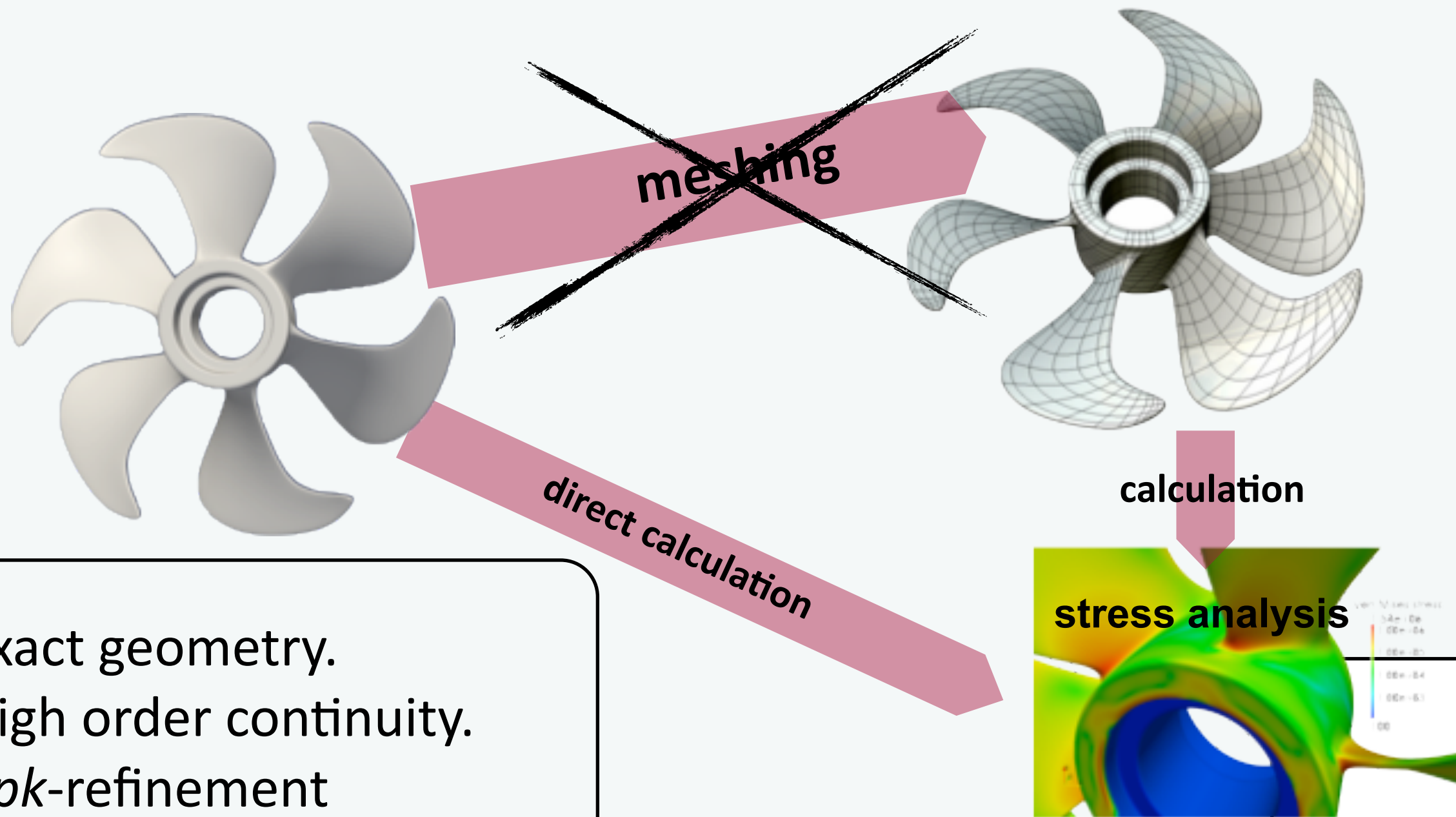


ITN  
INSIST

*Coupling CAD and Analysis.*



Approximate the unknown fields with the same basis functions ( NURBS, T-splines ... ) as that used to generate the CAD model



- Exact geometry.
- High order continuity.
- *hpk*-refinement