

# Handling discontinuities in isogeometric formulations

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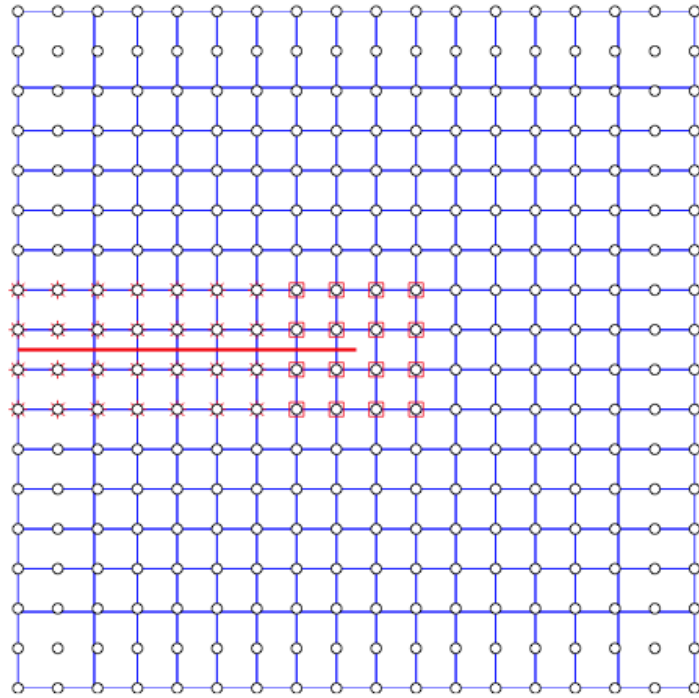
THE UNIVERSITY OF  
WESTERN  
AUSTRALIA



MONASH University

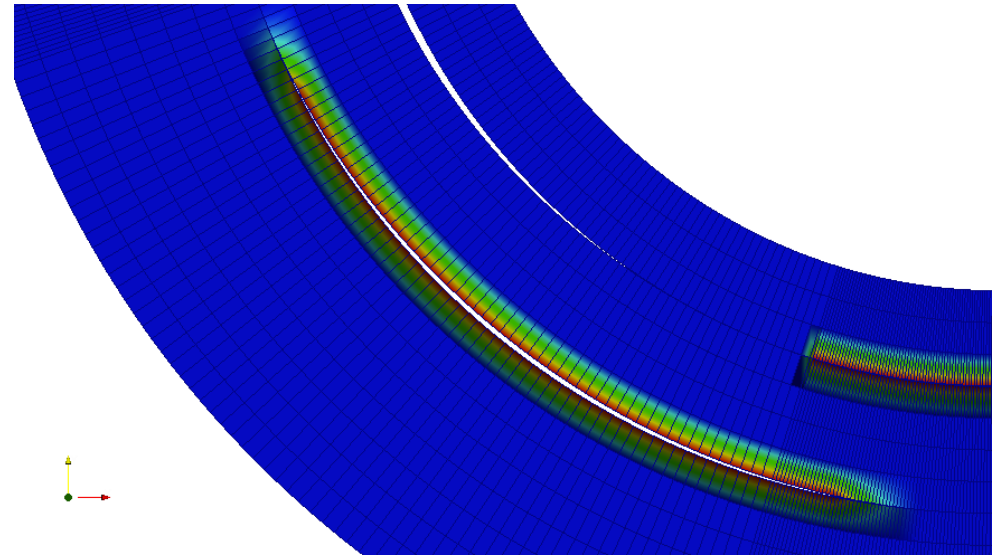


## PUM enriched methods



- IGA: link to CAD and accurate stress fields
- XFEM: no remeshing

## Mesh conforming methods



- IGA: link to CAD and accurate stress fields
- Apps: delamination



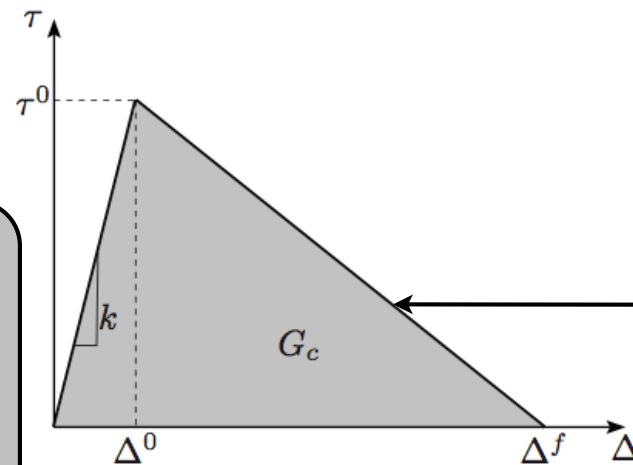
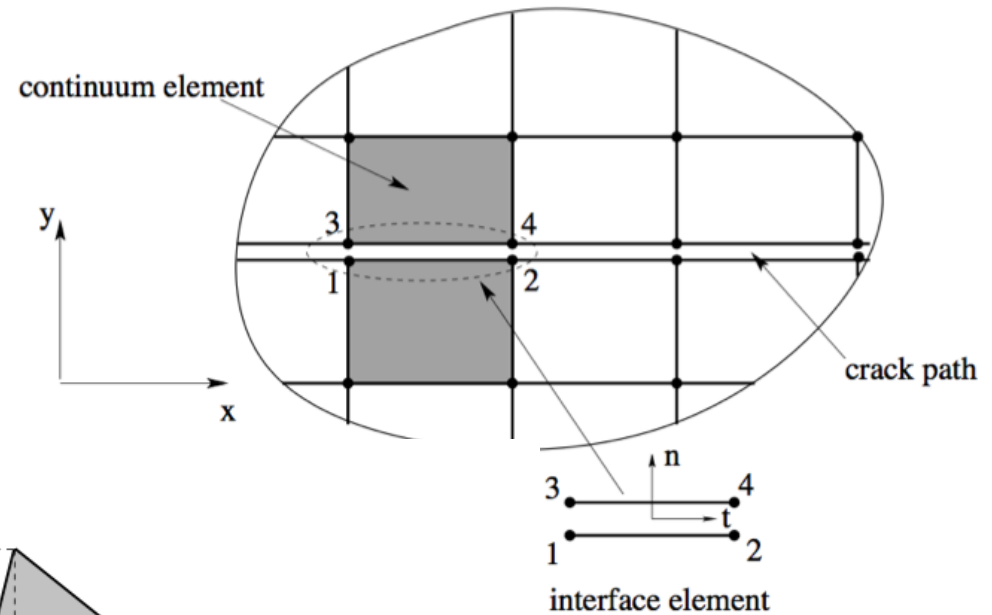
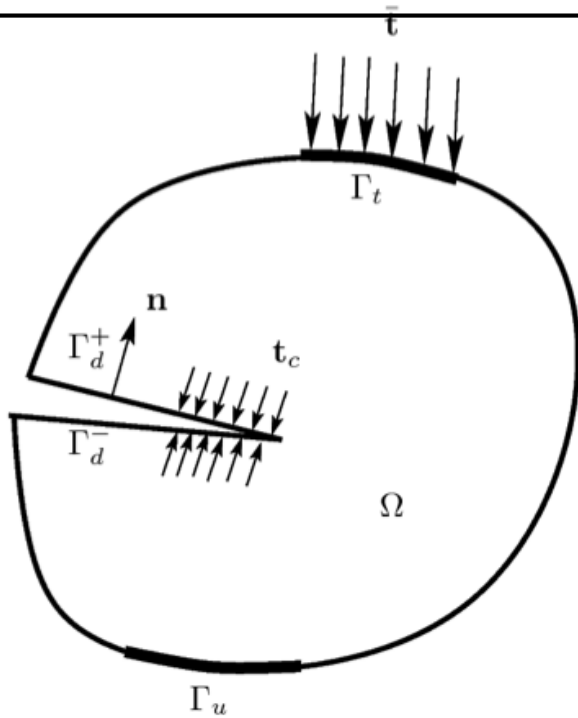
$$\mathbf{u}^h(\mathbf{x}) = \sum_{I \in \mathcal{S}} R_I(\mathbf{x}) \mathbf{u}_I + \sum_{J \in \mathcal{S}^c} R_J(\mathbf{x}) \Phi(\mathbf{x}) \mathbf{a}_J$$

NURBS basis functions

enrichment functions

1. E. De Luycker, D. J. Benson, T. Belytschko, Y. Bazilevs, and M. C. Hsu. X-FEM in isogeometric analysis for linear fracture mechanics. *IJNME*, 87(6):541–565, 2011.
2. S. S. Ghorashi, N. Valizadeh, and S. Mohammadi. Extended isogeometric analysis for simulation of stationary and propagating cracks. *IJNME*, 89(9): 1069–1101, 2012.
3. D. J. Benson, Y. Bazilevs, E. De Luycker, M.-C. Hsu, M. Scott, T. J. R. Hughes, and T. Belytschko. A generalized finite element formulation for arbitrary basis functions: From isogeometric analysis to XFEM. *IJNME*, 83(6):765–785, 2010.
4. A. Tambat and G. Subbarayan. Isogeometric enriched field approximations. *CMAME*, 245–246:1 – 21, 2012.

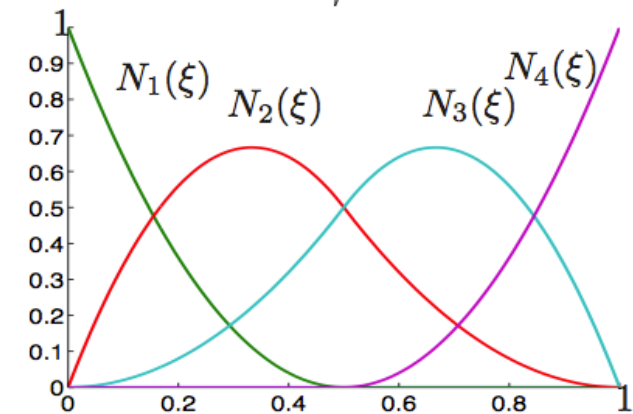
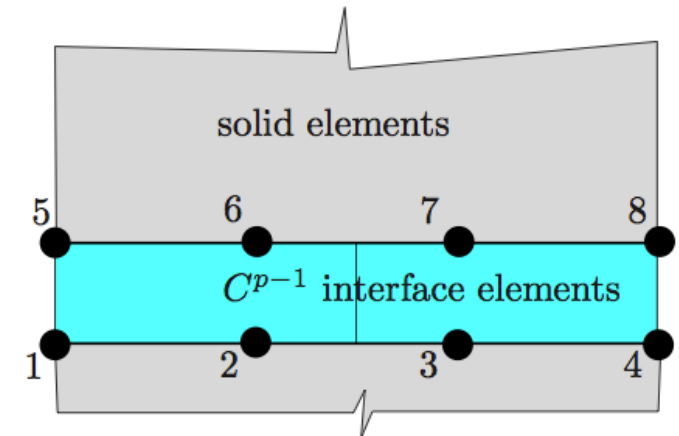
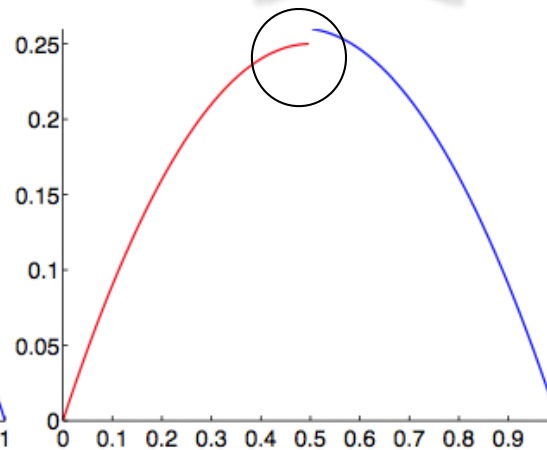
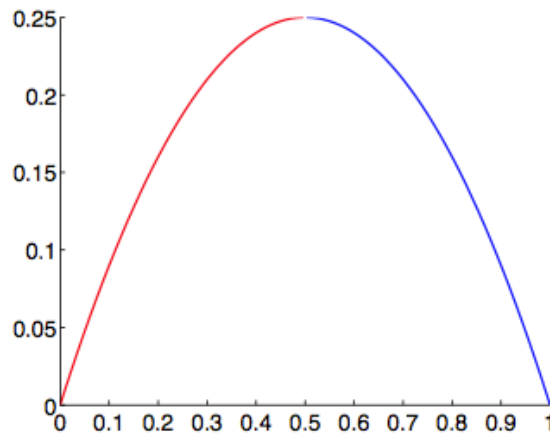
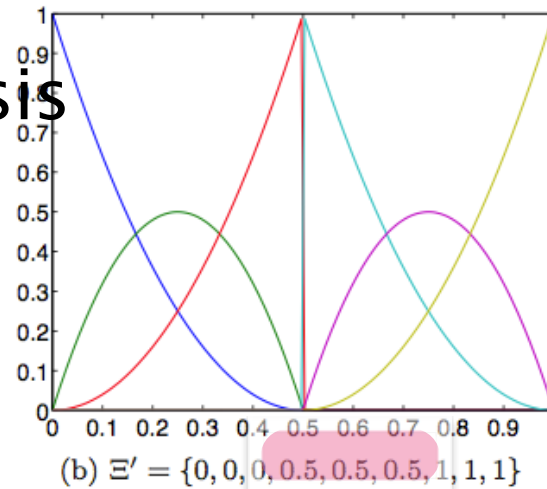
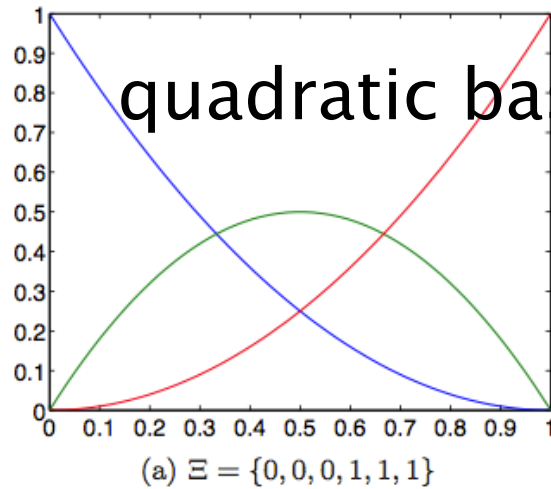
# Delamination analysis with cohesive elements (standard approach)



- No link to CAD
- Long preprocessing
- Refined meshes

$$\int_{\Omega} \delta \mathbf{u} \cdot \mathbf{b} d\Omega + \int_{\Gamma_t} \delta \mathbf{u} \cdot \bar{\mathbf{t}} d\Gamma_t = \int_{\Omega} \delta \boldsymbol{\epsilon} : \boldsymbol{\sigma}(\mathbf{u}) d\Omega + \int_{\Gamma_d} \delta [\![\mathbf{u}]\!] \cdot \mathbf{t}^c([\![\mathbf{u}]\!]) d\Gamma_d$$

# Isogeometric cohesive elements

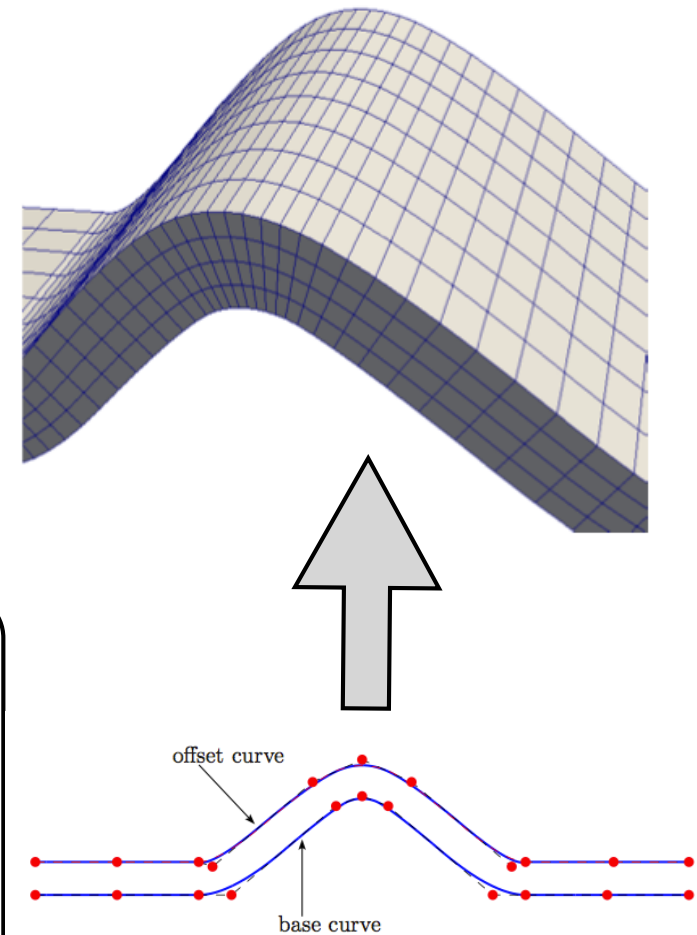


## Knot insertion

1. C. V. Verhoosel, M. A. Scott, R. de Borst, and T. J. R. Hughes. An isogeometric approach to cohesive zone modeling. *IJNME*, 87(15):336–360, 2011.
2. V.P. Nguyen, P. Kerfriden, S. Bordas. Isogeometric cohesive elements for two and three dimensional composite delamination analysis, 2013, Arxiv.

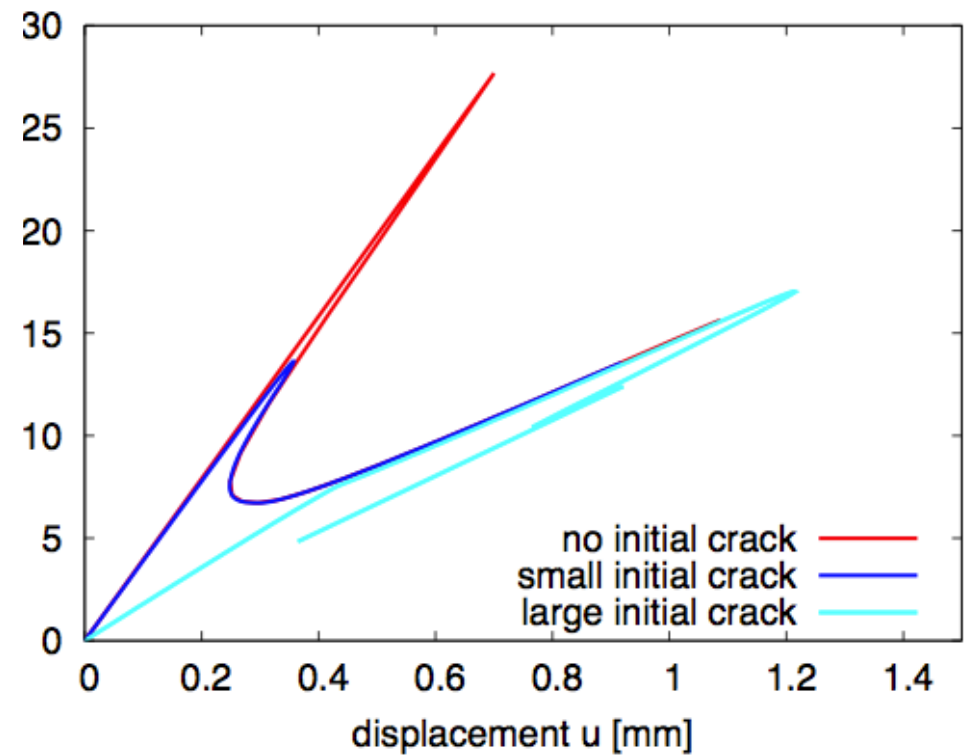
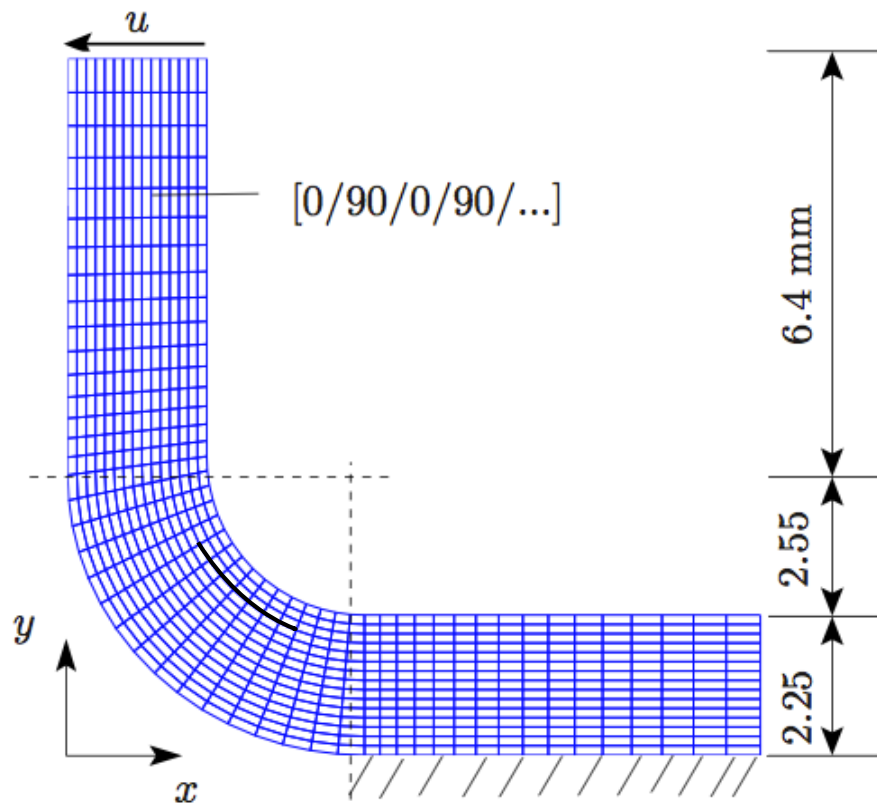
# Isogeometric cohesive elements: advantages

- Direct link to CAD
  - Exact geometry
  - Fast/straightforward generation of interface elements
  - Accurate stress field
  - Computationally cheaper
- 
- 2D Mixed mode bending test (MMB)
  - 2 x 70 quartic-linear B-spline elements
  - Run time on a laptop 4GBi7: 6 s
  - Energy arc-length control

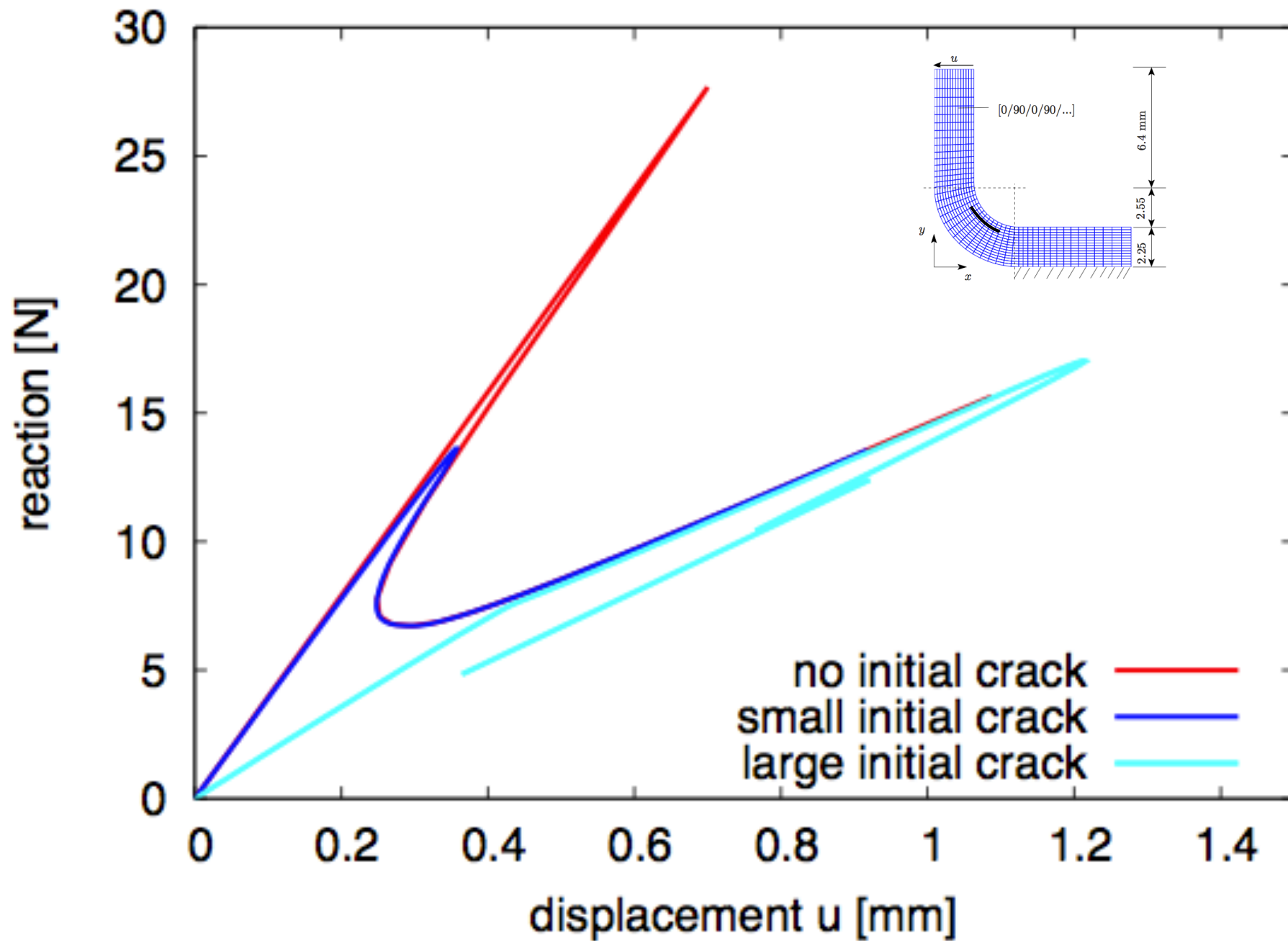


V. P. Nguyen and H. Nguyen-Xuan. High-order B-splines based finite elements for delamination analysis of laminated composites. *Composite Structures*, 102:261–275, 2013.

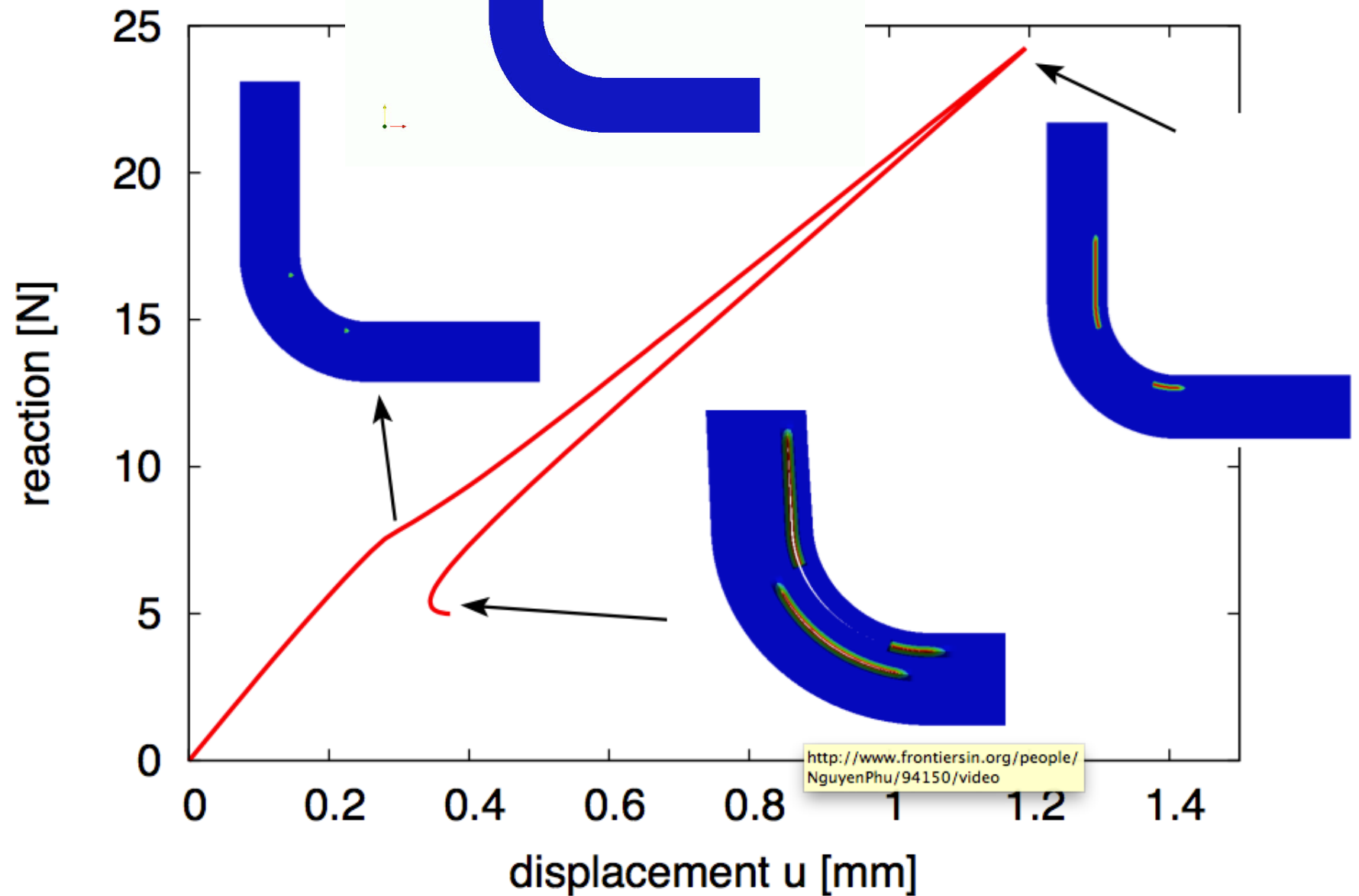
# Isogeometric cohesive elements: 2D example



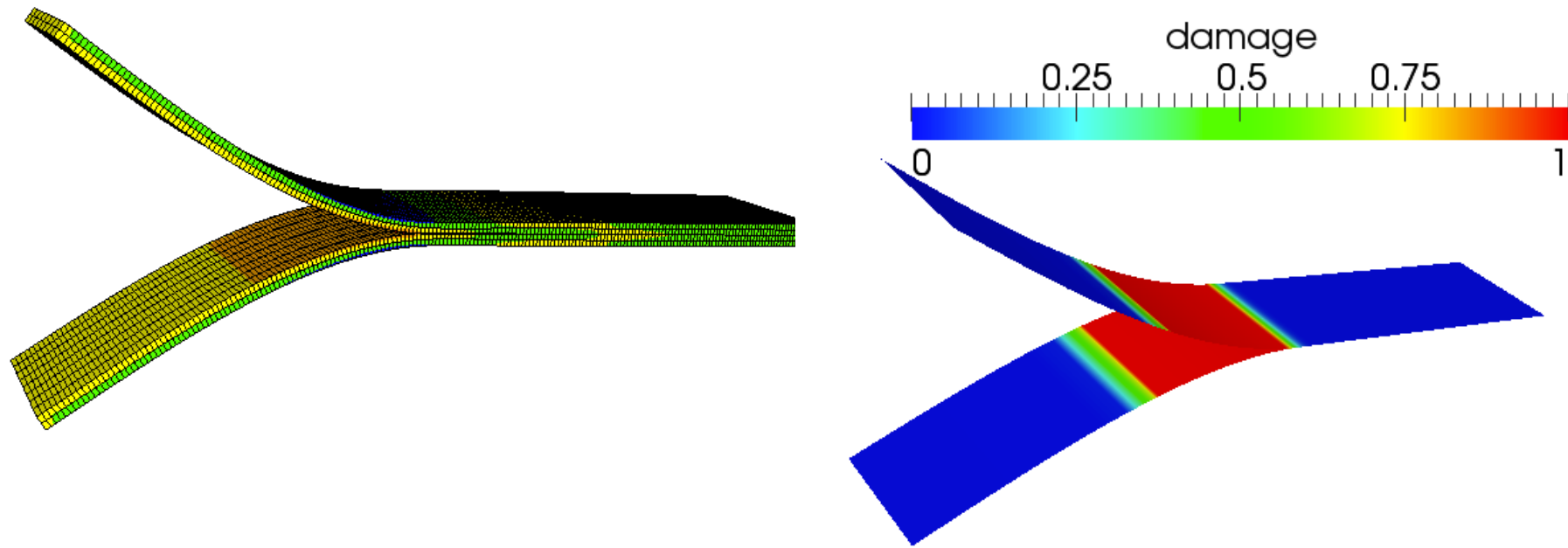
- Exact geometry by NURBS + direct link to CAD
- It is straightforward to vary
  - (1) the number of plies and
  - (2) # of interface elements:
- Suitable for parameter studies/design
- Solver: energy-based arc-length method (Gutierrez, 2007)



# Isogeometric cohesive elements: 2D example

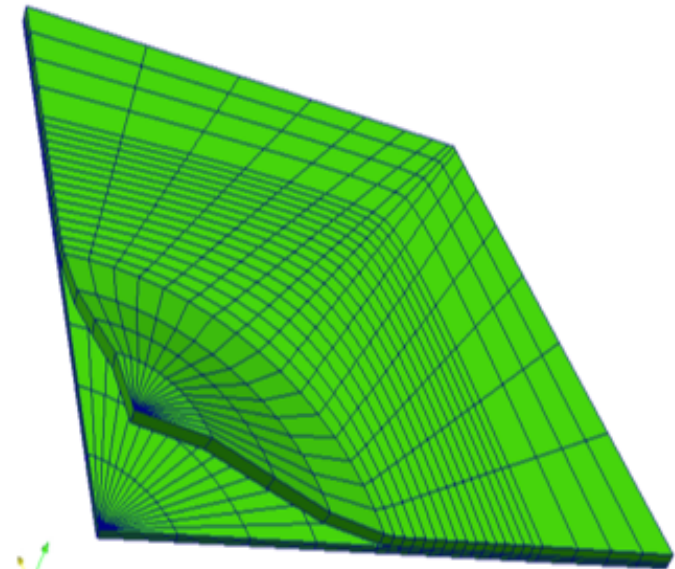
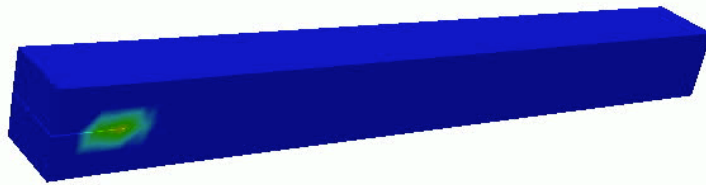


# Isogeometric cohesive elements: 3D example with shells

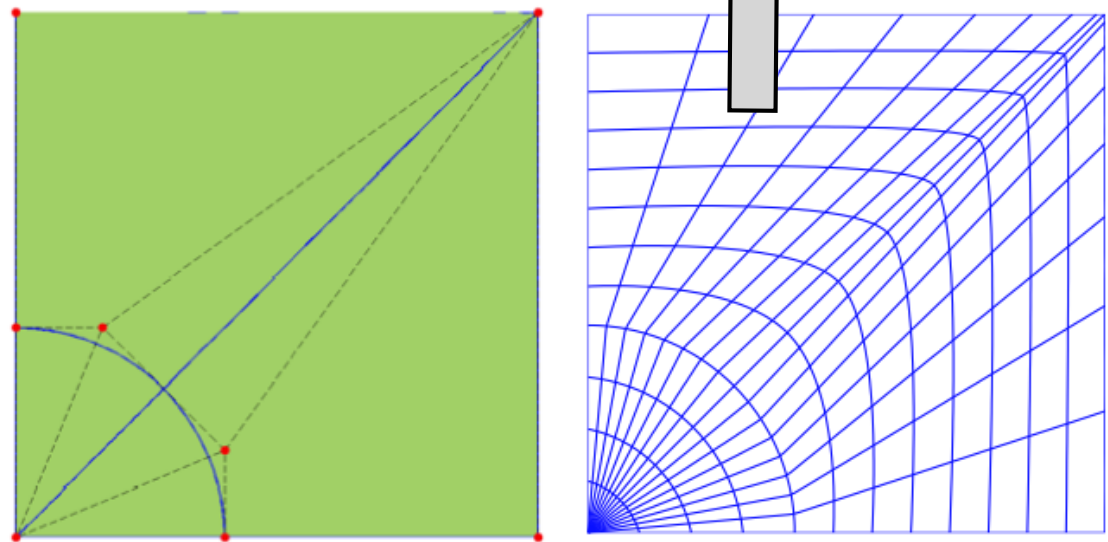


- Rotation free B-splines shell elements (Kiendl et al. CMAME)
- Two shells, one for each lamina
- Bivariate B-splines cohesive interface elements in between

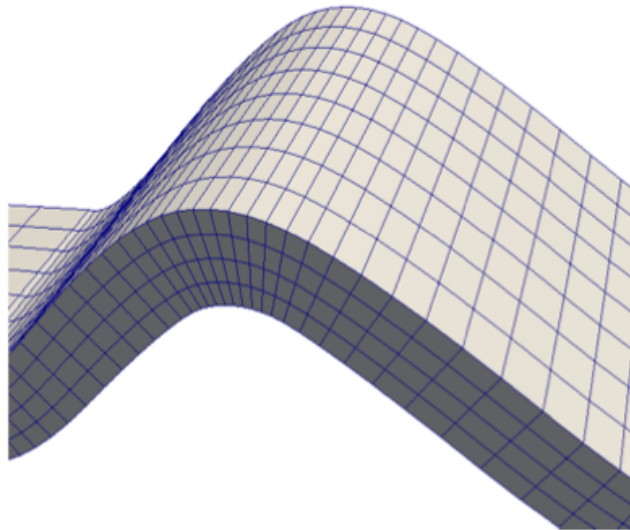
# Isogeometric cohesive elements: 3D examples



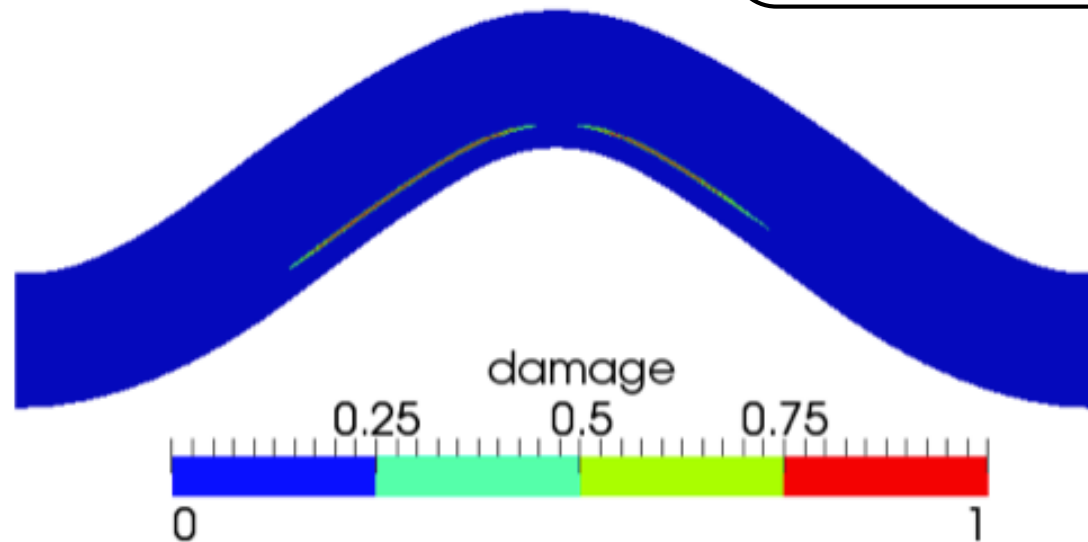
- cohesive elements for 3D meshes the same as 2D
- large deformations



# Isogeometric cohesive elements

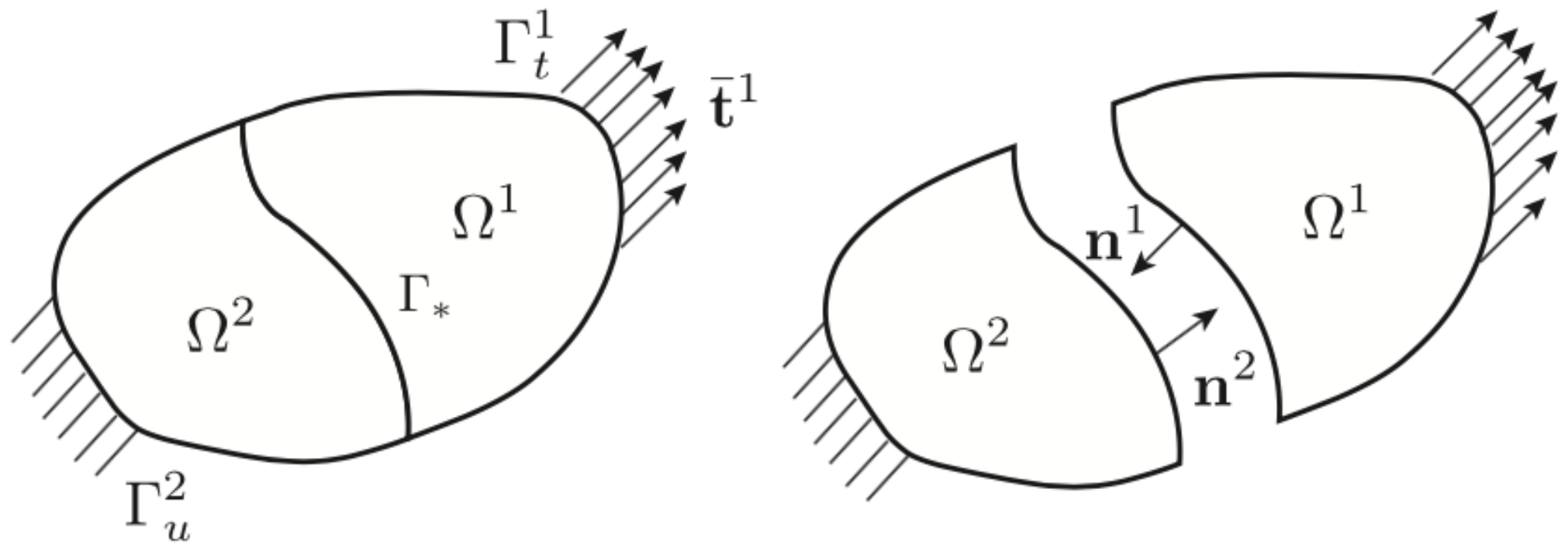


- singly curved thick-wall laminates
- geometry/displacements: NURBS
- trivariate NURBS from NURBS surface(\*)
- cohesive surface interface elements

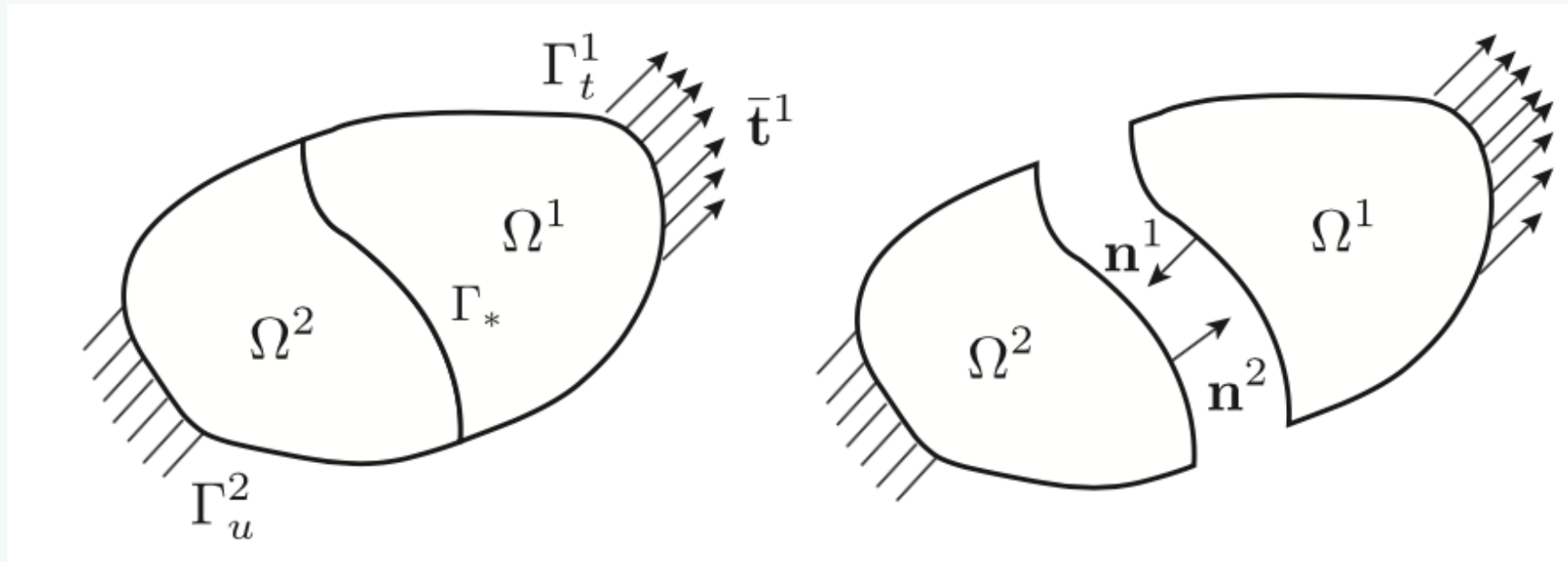


(\*)V. P. Nguyen, P. Kerfriden, S.P.A. Bordas, and T. Rabczuk. An integrated design-analysis framework for three dimensional composite panels. Computer Aided Design, 2013. submitted.

# Non-matching interface elements for delamination and contact



# Non-matching interface elements for delamination and contact



$$-\nabla \cdot \boldsymbol{\sigma}^m = \mathbf{b}^m \quad \text{on } \Omega^m$$

$$\mathbf{u}^m = \bar{\mathbf{u}}^m \quad \text{on } \Gamma_u^m$$

$$\boldsymbol{\sigma}^m \cdot \mathbf{n}^m = \bar{\mathbf{t}}^m \quad \text{on } \Gamma_t^m$$

$$\mathbf{u}^1 = \mathbf{u}^2 \quad \text{on } \Gamma_*$$

$$\boldsymbol{\sigma}^1 \cdot \mathbf{n}^1 = -\boldsymbol{\sigma}^2 \cdot \mathbf{n}^2 \quad \text{on } \Gamma_*$$

$$-\nabla \boldsymbol{\sigma}^m = \mathbf{b}^m \quad \text{on } \Omega^m$$

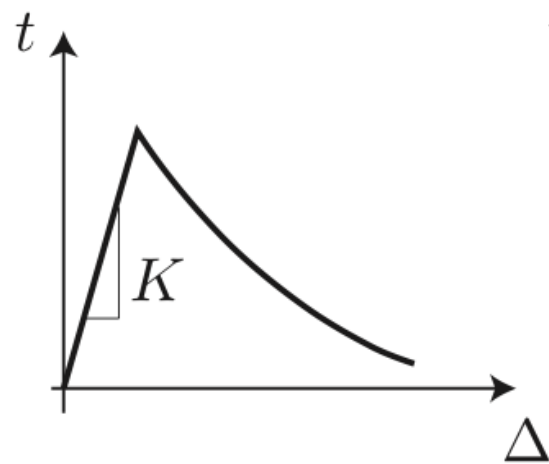
$$\mathbf{u}^m = \bar{\mathbf{u}}^m \quad \text{on } \Gamma_u^m$$

$$\boldsymbol{\sigma}^m \cdot \mathbf{n}^m = \bar{\mathbf{t}}^m \quad \text{on } \Gamma_t^m$$

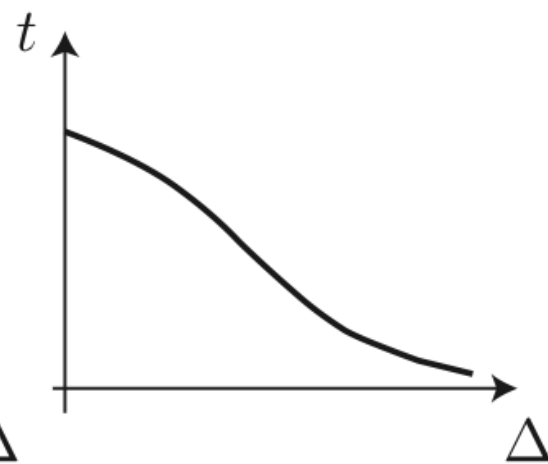
$$-\boldsymbol{\sigma}^1 \cdot \mathbf{n}^1 = \boldsymbol{\sigma}^2 \cdot \mathbf{n}^2 = \mathbf{t} \quad \text{on } \Gamma_*$$

$$\mathbf{t} = \mathbf{t}(\llbracket \mathbf{u} \rrbracket, \zeta) \quad \text{on } \Gamma_*$$

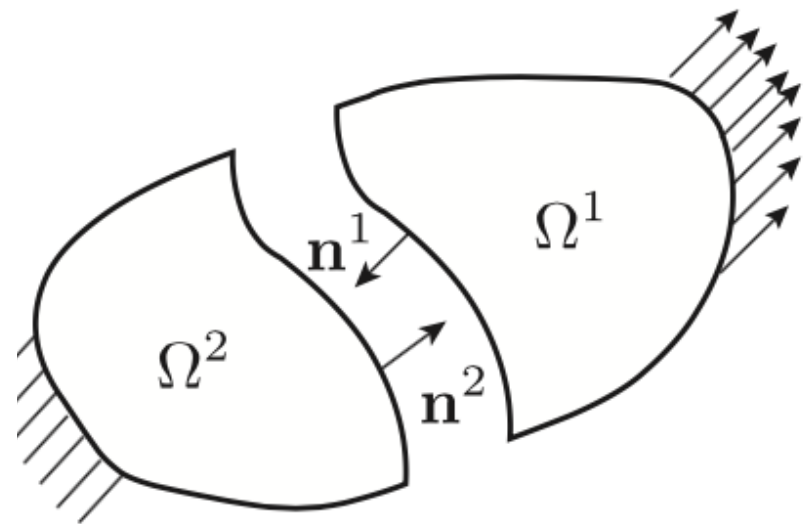
# Non-matching interface elements for delamination and contact



(a) intrinsic TSL



(b) extrinsic TSL



$$-\nabla \cdot \boldsymbol{\sigma}^m = \mathbf{b}^m \quad \text{on } \Omega^m$$

$$\mathbf{u}^m = \bar{\mathbf{u}}^m \quad \text{on } \Gamma_u^m$$

$$\boldsymbol{\sigma}^m \cdot \mathbf{n}^m = \bar{\mathbf{t}}^m \quad \text{on } \Gamma_t^m$$

$$\mathbf{u}^1 = \mathbf{u}^2 \quad \text{on } \Gamma_*$$

$$\boldsymbol{\sigma}^1 \cdot \mathbf{n}^1 = -\boldsymbol{\sigma}^2 \cdot \mathbf{n}^2 \quad \text{on } \Gamma_*$$

$$-\nabla \boldsymbol{\sigma}^m = \mathbf{b}^m \quad \text{on } \Omega^m$$

$$\mathbf{u}^m = \bar{\mathbf{u}}^m \quad \text{on } \Gamma_u^m$$

$$\boldsymbol{\sigma}^m \cdot \mathbf{n}^m = \bar{\mathbf{t}}^m \quad \text{on } \Gamma_t^m$$

$$-\boldsymbol{\sigma}^1 \cdot \mathbf{n}^1 = \boldsymbol{\sigma}^2 \cdot \mathbf{n}^2 = \mathbf{t} \quad \text{on } \Gamma_*$$

$$\mathbf{t} = \mathbf{t}(\llbracket \mathbf{u} \rrbracket, \zeta) \quad \text{on } \Gamma_*$$

## Weak form

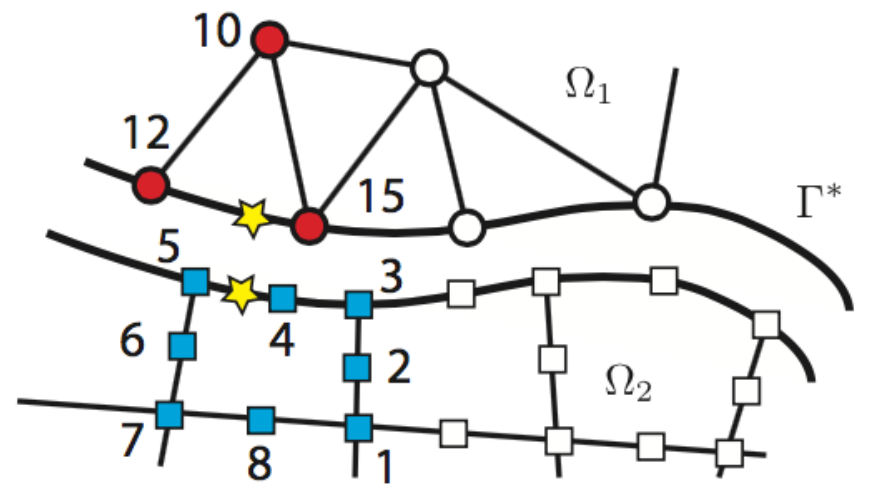
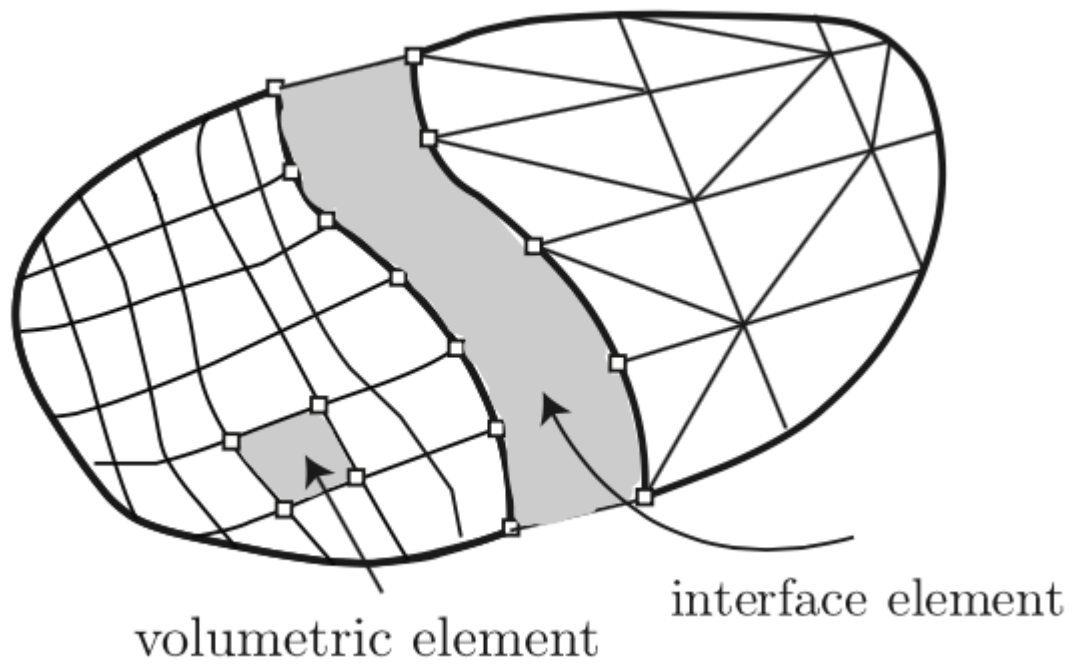
$$\mathbf{S}^m = \{\mathbf{u}^m(\mathbf{x}) | \mathbf{u}^m(\mathbf{x}) \in \mathbf{H}^1(\Omega^m), \mathbf{u}^m = \bar{\mathbf{u}}^m \text{ on } \Gamma_u^m\}$$

$$\mathbf{V}^m = \{\mathbf{w}^m(\mathbf{x}) | \mathbf{w}^m(\mathbf{x}) \in \mathbf{H}^1(\Omega^m), \mathbf{w}^m = \mathbf{0} \text{ on } \Gamma_u^m\}$$

Find  $(\mathbf{u}^1, \mathbf{u}^2) \in \mathbf{S}^1 \times \mathbf{S}^2$  such that

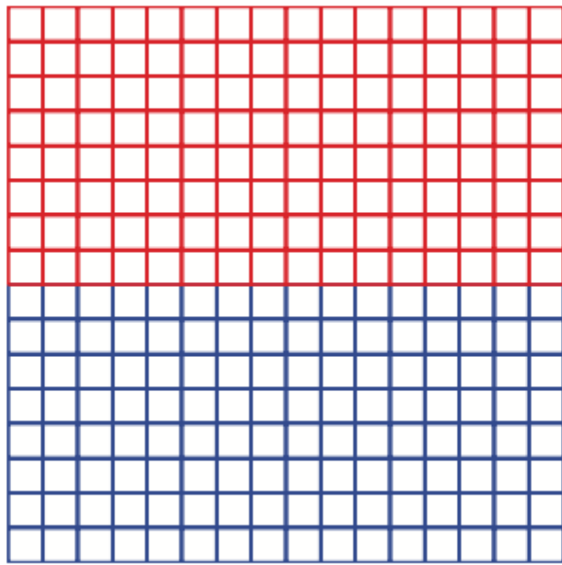
$$\begin{aligned} & \sum_{m=1}^2 \int_{\Omega^m} (\boldsymbol{\epsilon}(\mathbf{w}^m))^T \boldsymbol{\sigma}^m d\Omega + (1-\beta) \left[ - \int_{\Gamma_*} \llbracket \mathbf{w} \rrbracket^T \mathbf{n} \{ \boldsymbol{\sigma} \} d\Gamma - \int_{\Gamma_*} \{ \boldsymbol{\sigma}(\mathbf{w}) \}^T \mathbf{n}^T \llbracket \mathbf{u} \rrbracket d\Gamma + \int_{\Gamma_*} \alpha \llbracket \mathbf{w} \rrbracket^T \llbracket \mathbf{u} \rrbracket d\Gamma \right] \\ & + \beta \int_{\Gamma_*} \llbracket \mathbf{w} \rrbracket^T \mathbf{t}(\llbracket \mathbf{u} \rrbracket) d\Gamma = \sum_{m=1}^2 \int_{\Gamma_t^m} (\mathbf{w}^m)^T \bar{\mathbf{t}}^m d\Gamma + \sum_{m=1}^2 \int_{\Omega^m} (\mathbf{w}^m)^T \mathbf{b}^m d\Omega \quad \text{for all } (\mathbf{w}^1, \mathbf{w}^2) \in \mathbf{V}^1 \times \mathbf{V}^2 \end{aligned}$$

$$\llbracket \mathbf{u} \rrbracket = \mathbf{u}^1 - \mathbf{u}^2, \quad \{ \boldsymbol{\sigma} \} = \gamma \boldsymbol{\sigma}^1 + (1 - \gamma) \boldsymbol{\sigma}^2$$

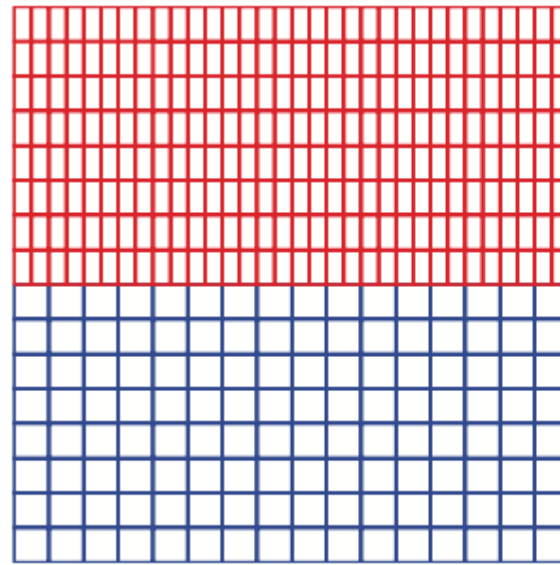


The interface elements are of zero thickness.

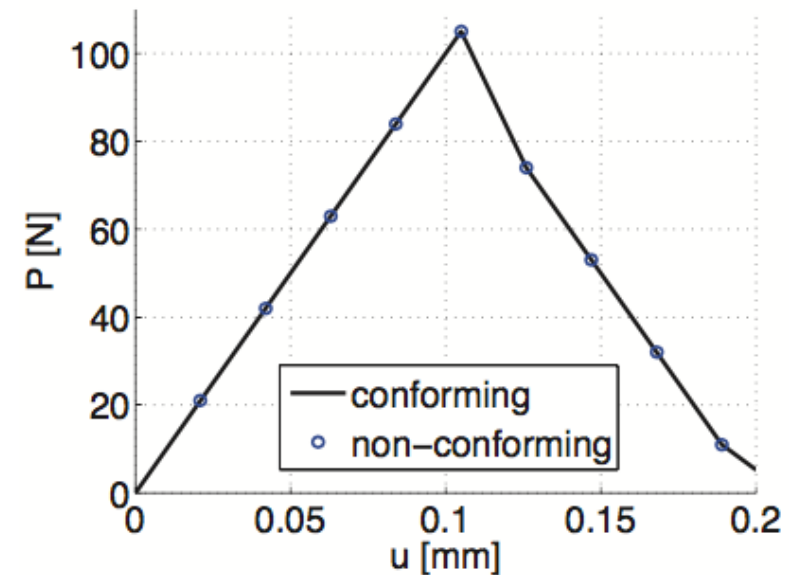
# 2D uniaxial tension test



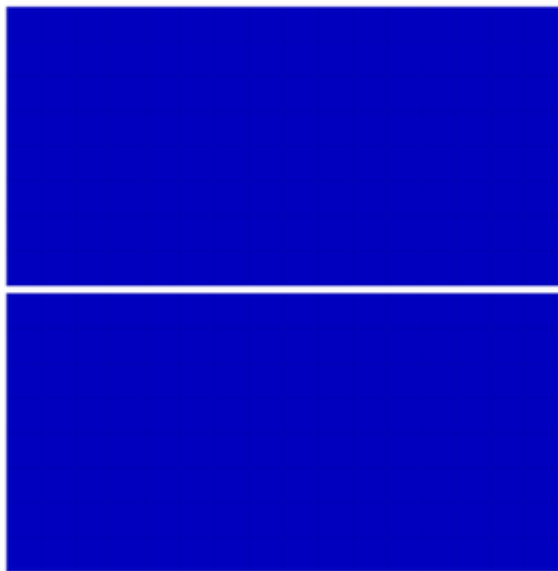
(a) conforming mesh



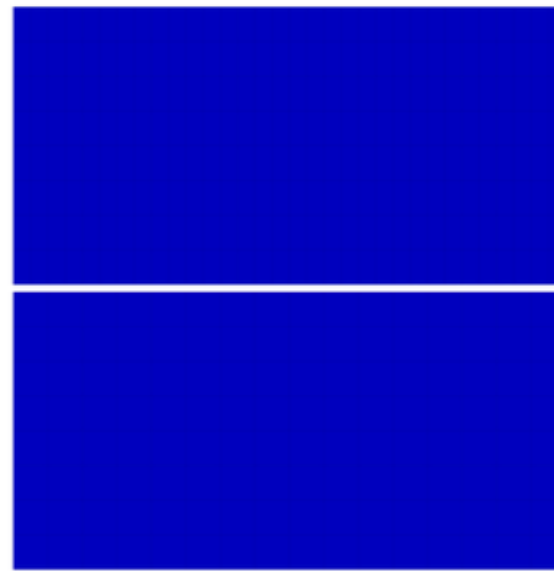
(b) nonconforming mesh



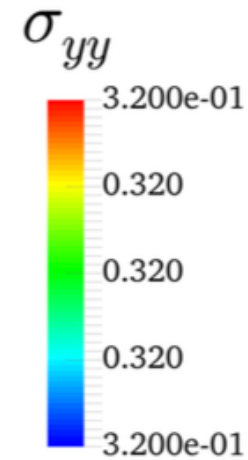
(c) load-displacement



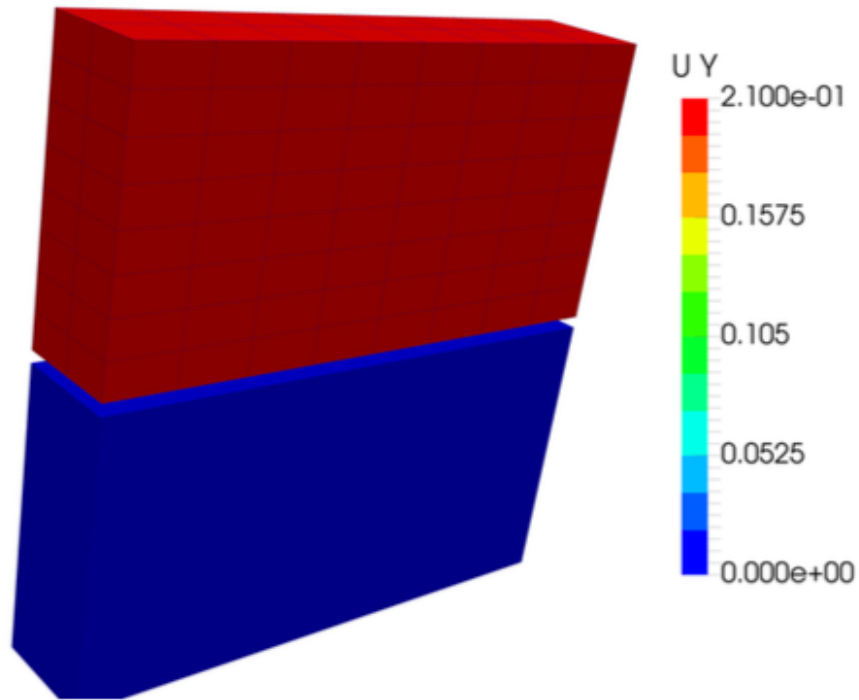
(a) matching mesh



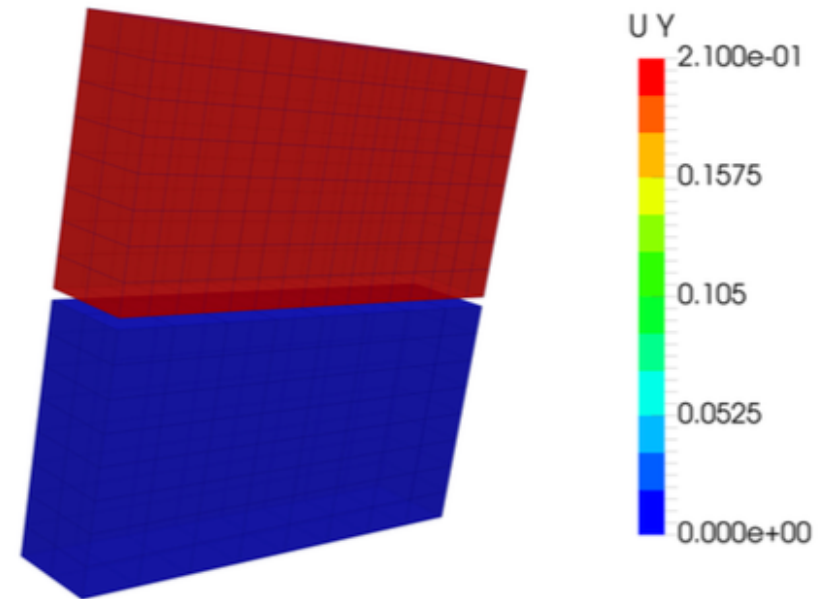
(b) non-matching mesh



# 3D uniaxial tension



(a) matching mesh



(b) non-matching mesh

## 2D peeling test

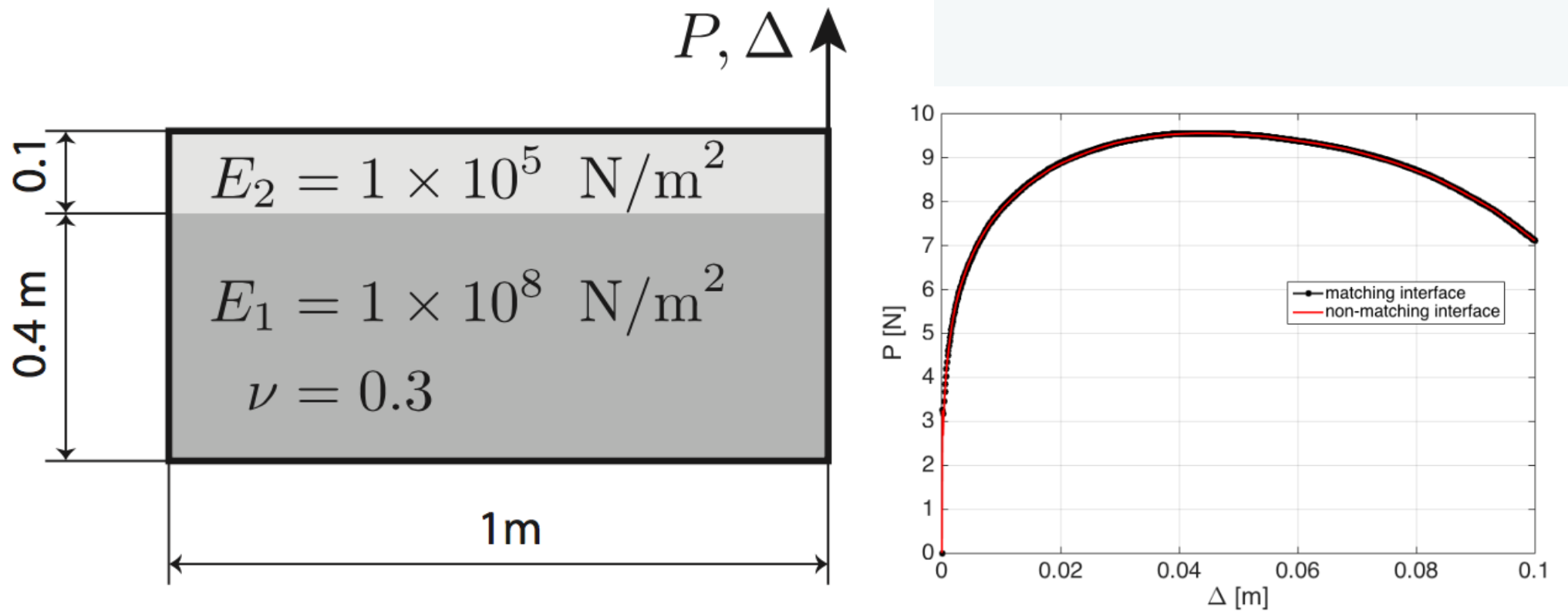
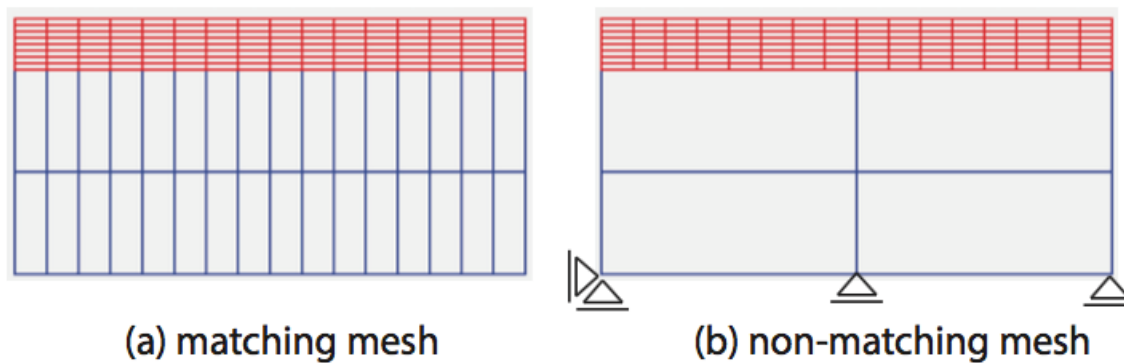
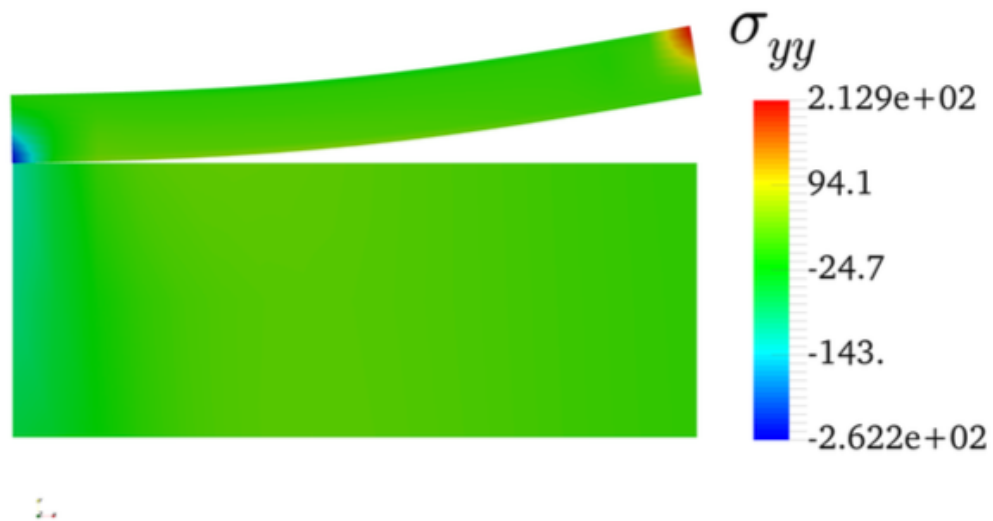


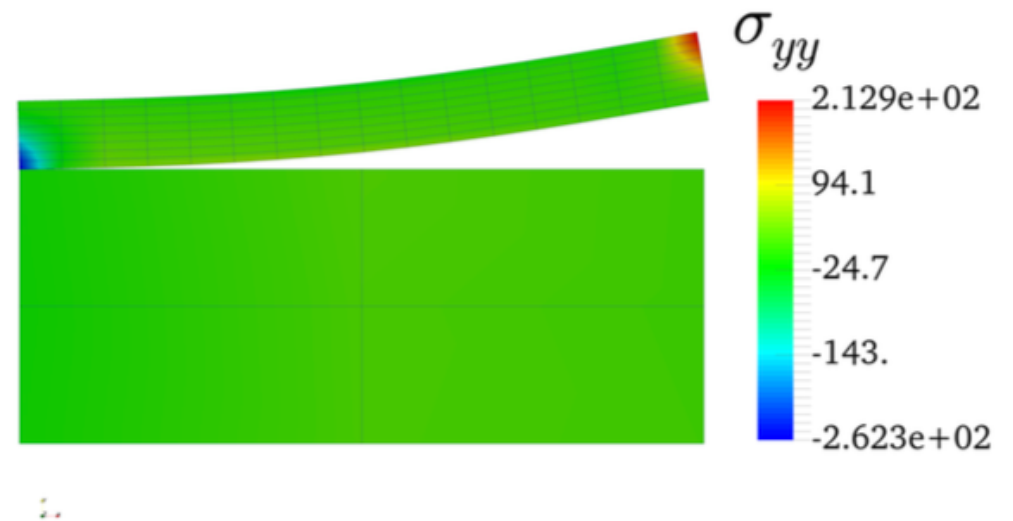
Figure 12: Peeling test: problem configuration.



## 2D peeling test

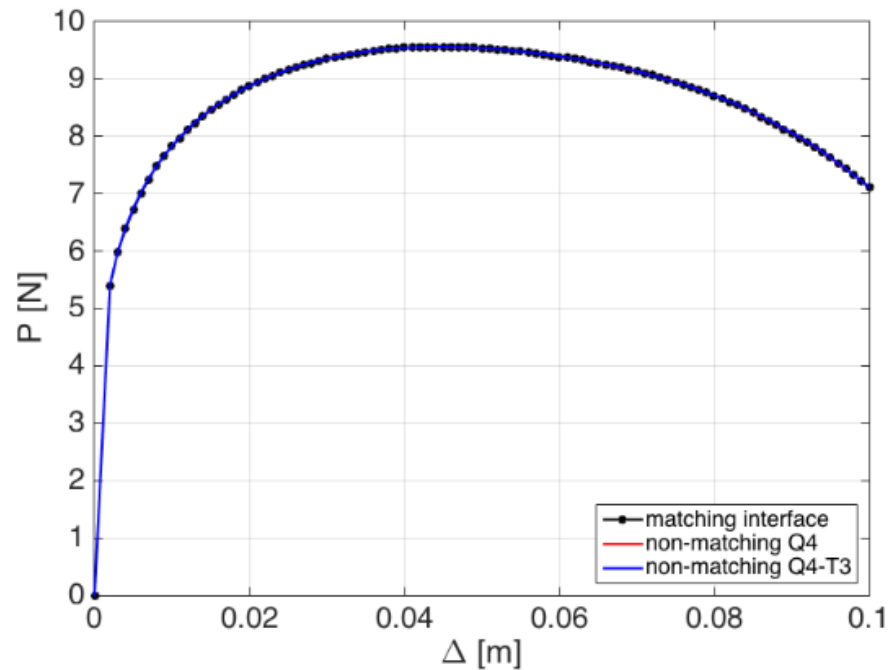


(a) matching mesh

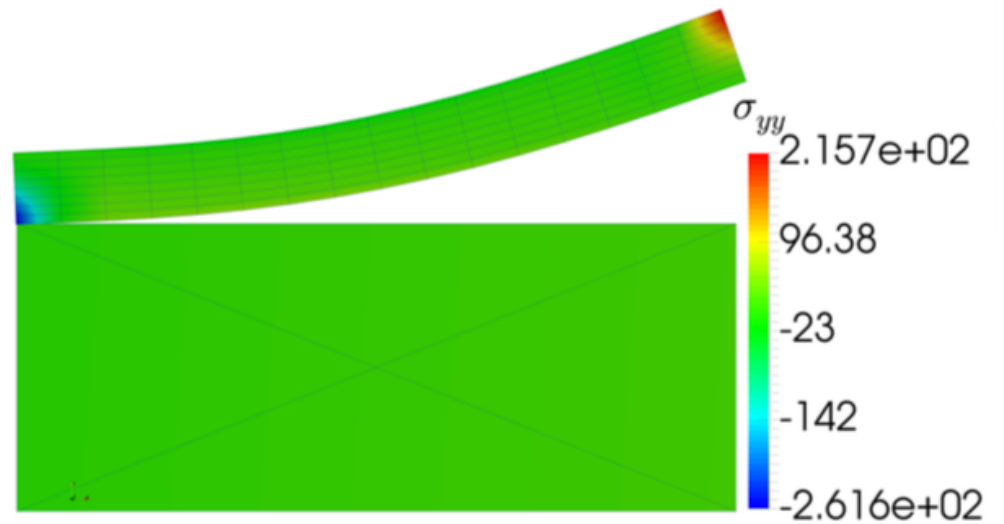


(b) non-matching mesh

## 2D peeling test



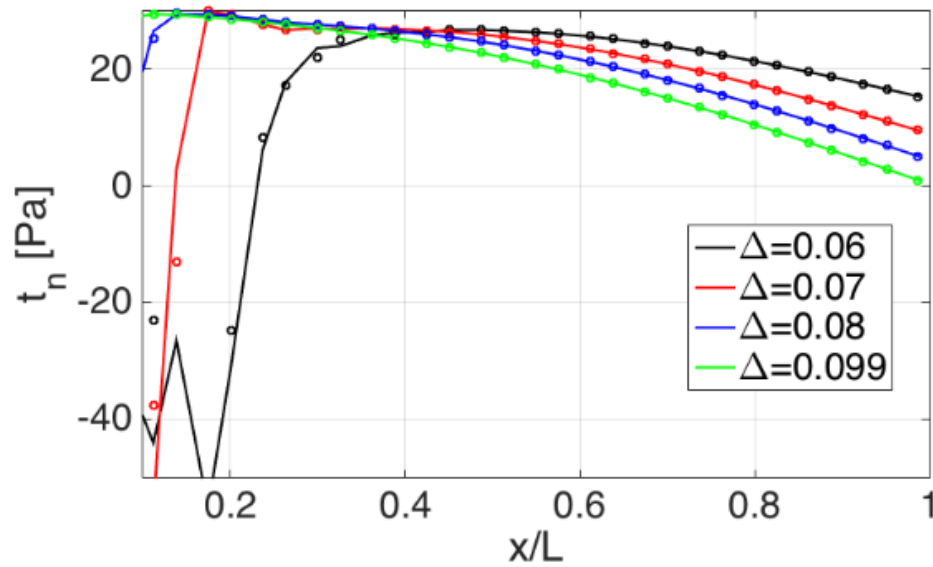
(a) load-displacement



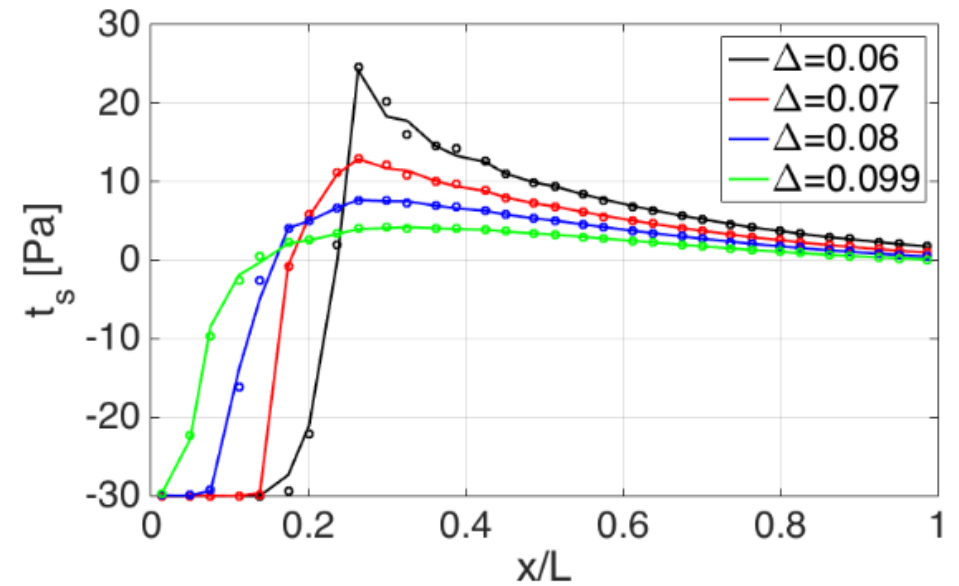
(b) stress contour

Figure 17: Peeling test: substrate discretised by three-node triangular elements whereas layer is meshed by Q4 elements. Note that there is a slight difference with the  $P - \Delta$  curves in Fig. 14 as displacement increments that are ten times larger were used.

## 2D peeling test F(D) curves



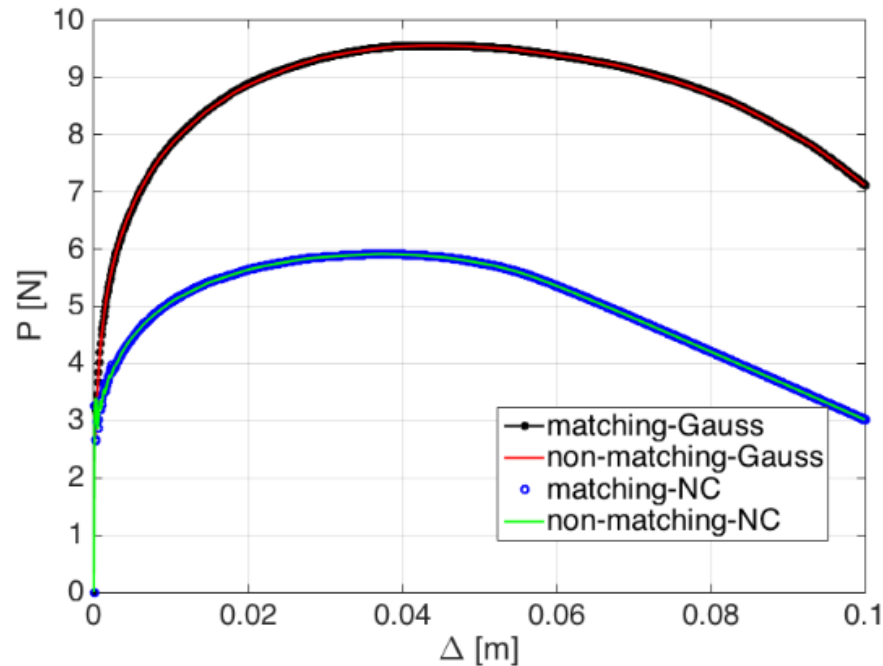
(a) normal cohesive traction



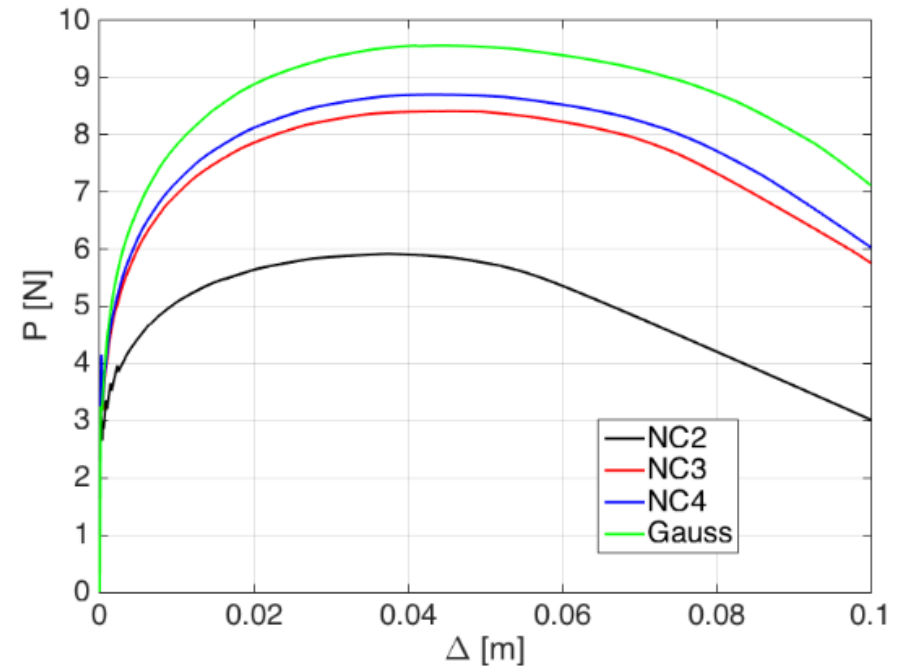
(b) tangential cohesive traction

Figure 18: Peeling test: local response of the proposed interface element (solid lines) vs. standard interface element (circles) for different imposed displacements  $\Delta$ .

## 2D peeling test - role of integration



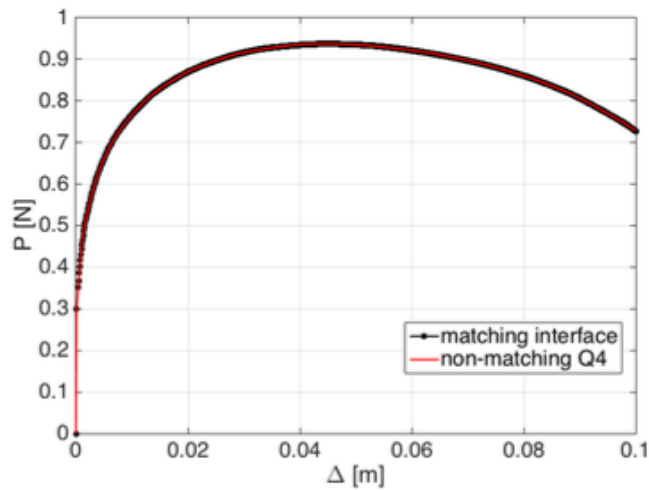
(a)



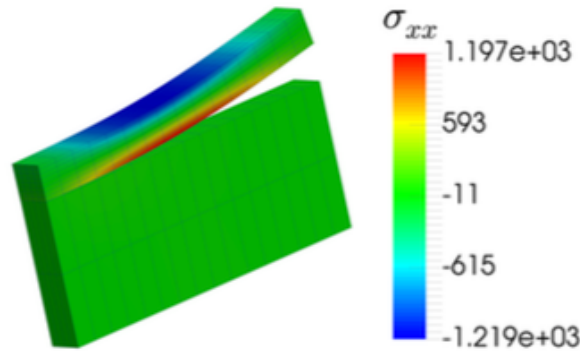
(b)

Figure 19: Peeling test:  $P - \Delta$  curves obtained with matching and non-matching FE meshes with Gauss and Newton-Cotes (NC) quadrature rules. Increasing the number of NC integration points shift the  $P - \Delta$  curves to the Gauss-based curve (right).

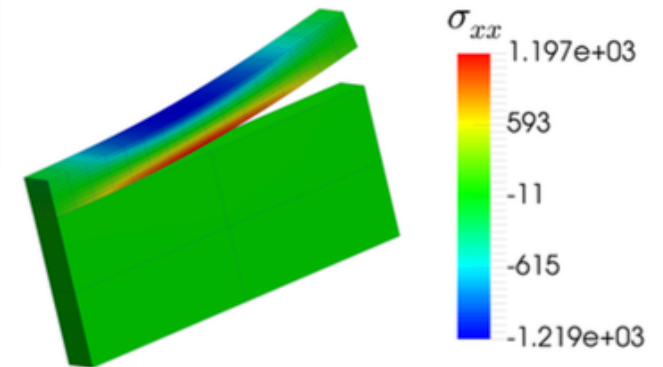
# 3D peeling test



(a) load-displacement



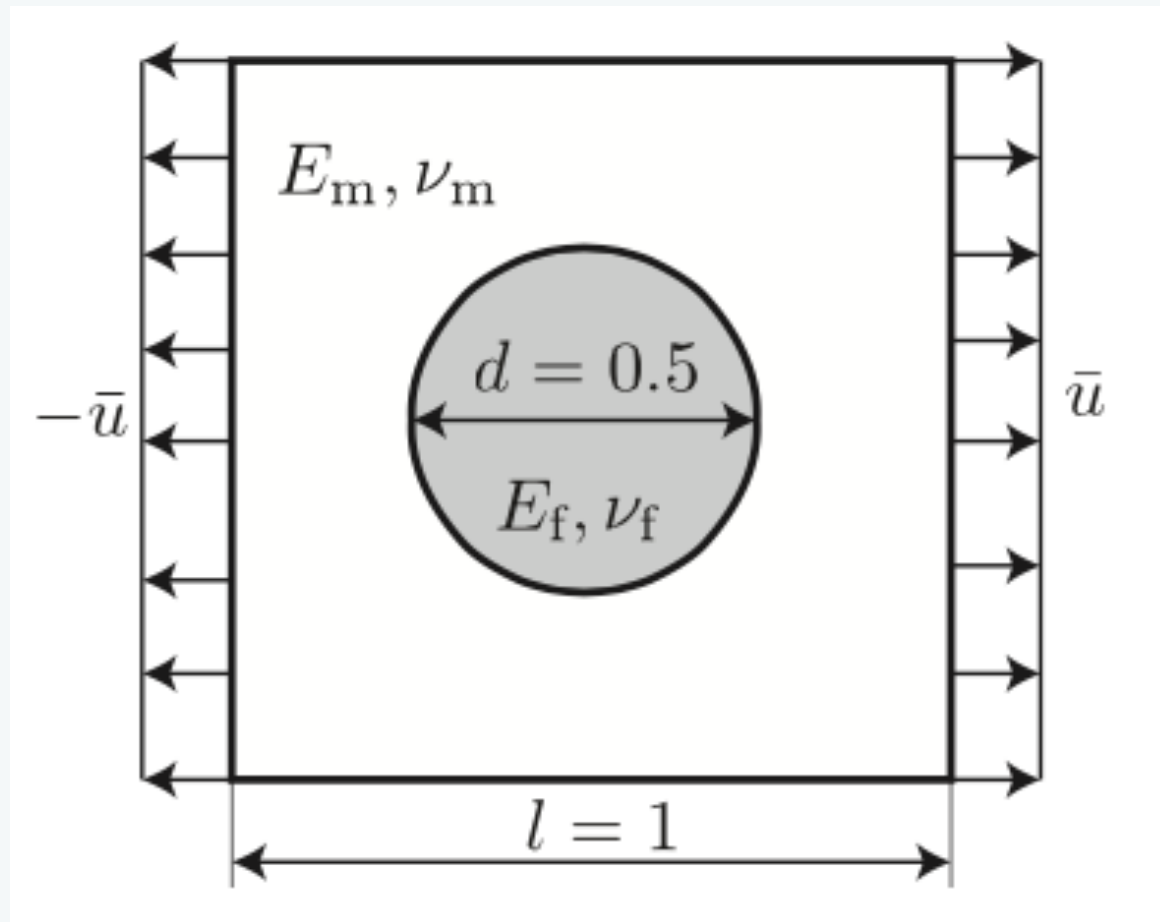
(b) matching mesh



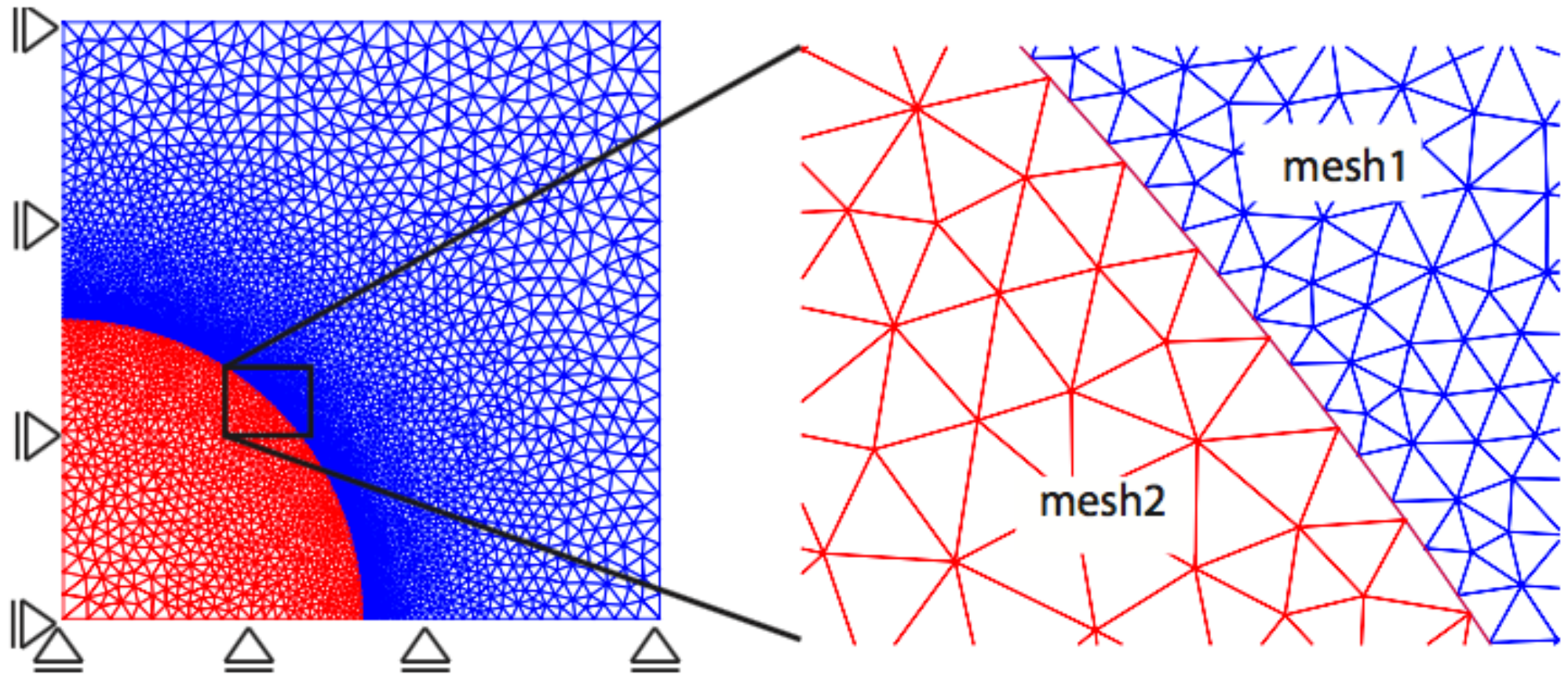
(c) non-matching

Figure 20: Three dimensional peeling test:  $P - \Delta$  curves and stress distribution.

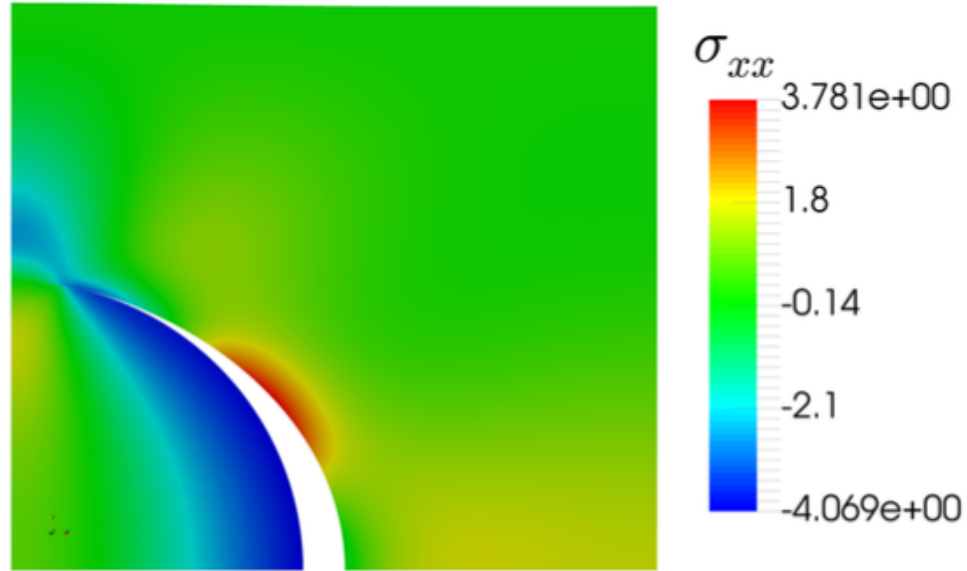
# Fibre-reinforced composite - debonding



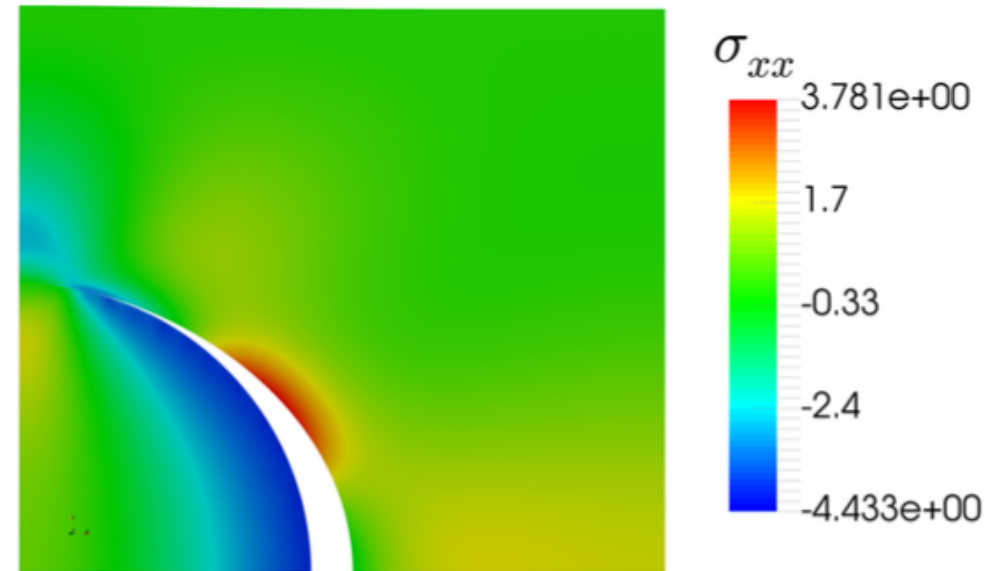
# Non-matching interfaces



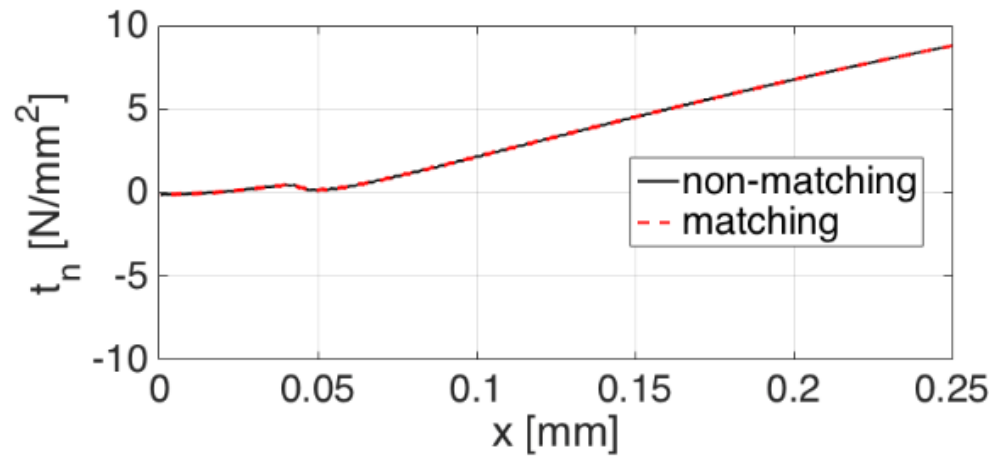
# Fibre-debonding



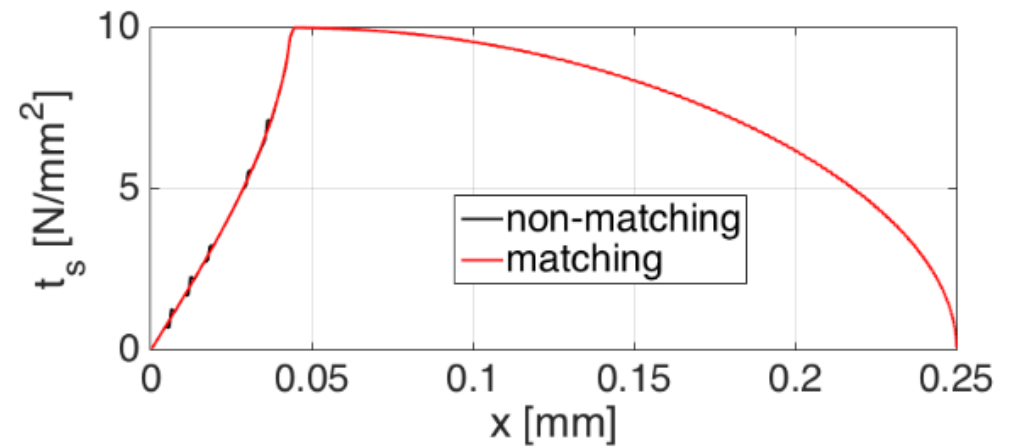
(a) matching mesh



(b) non-matching mesh



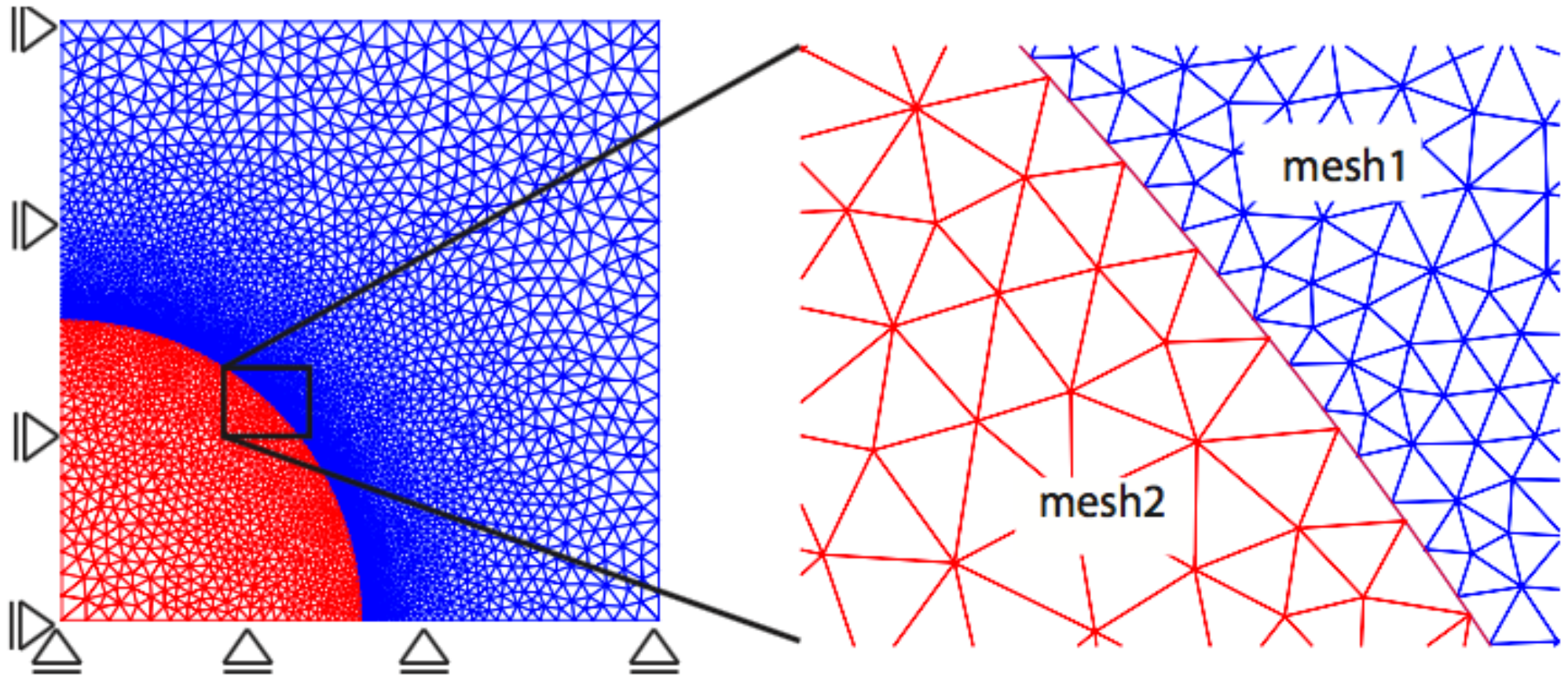
(a) normal stress



(b) tangential stress

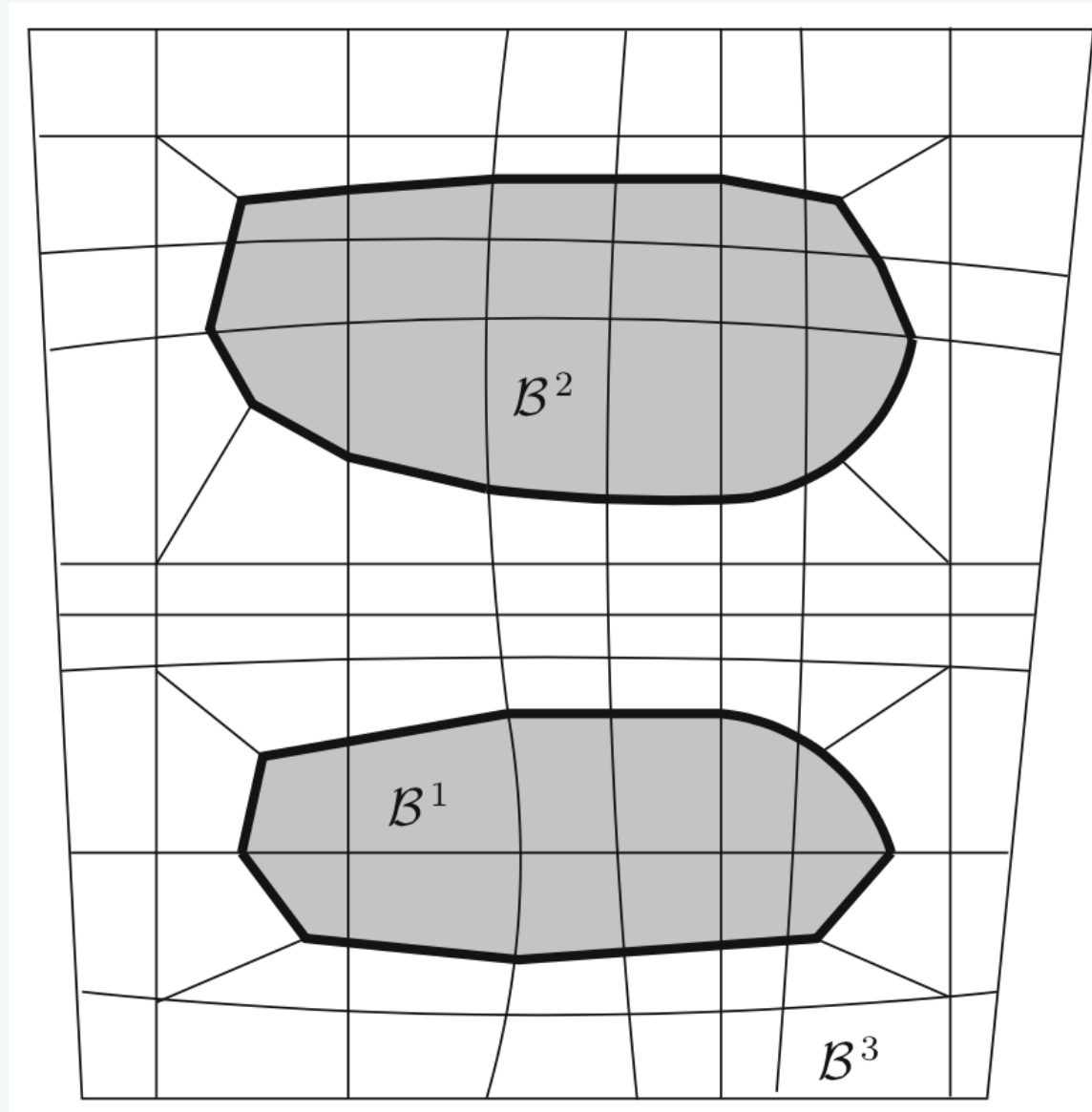
# Conclusions

- ▶ Incompatible/non-matching elements
- ▶ Small strain interfacial fracture
  - No need for conforming meshes along the interface
  - non-matching interface
  - no high-dummy stiffness
  - fewer elements (up to twice as fast)
  - Newton-Cotes integration leads to premature failure

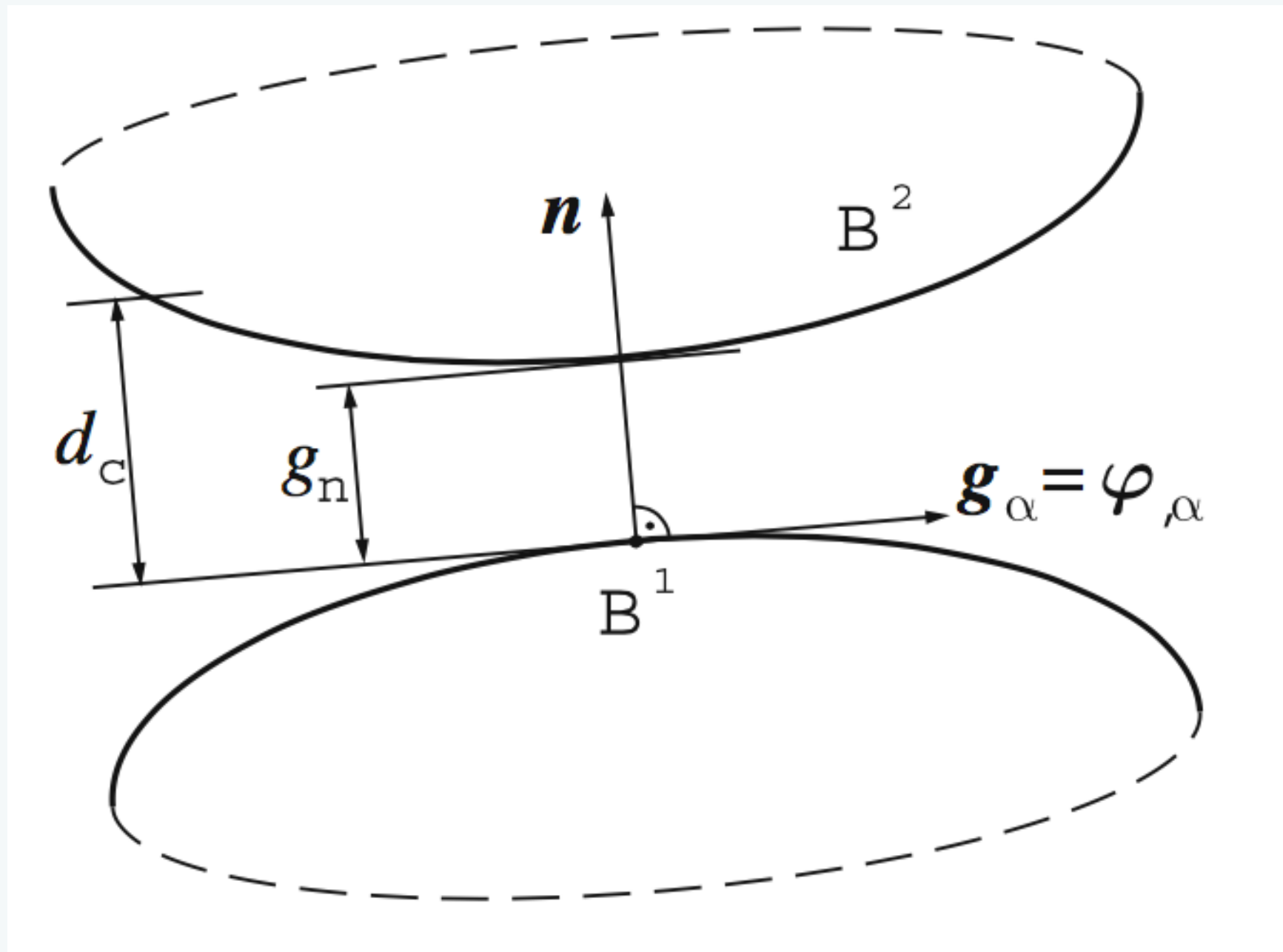


# Third medium contact formulation, Wriggers

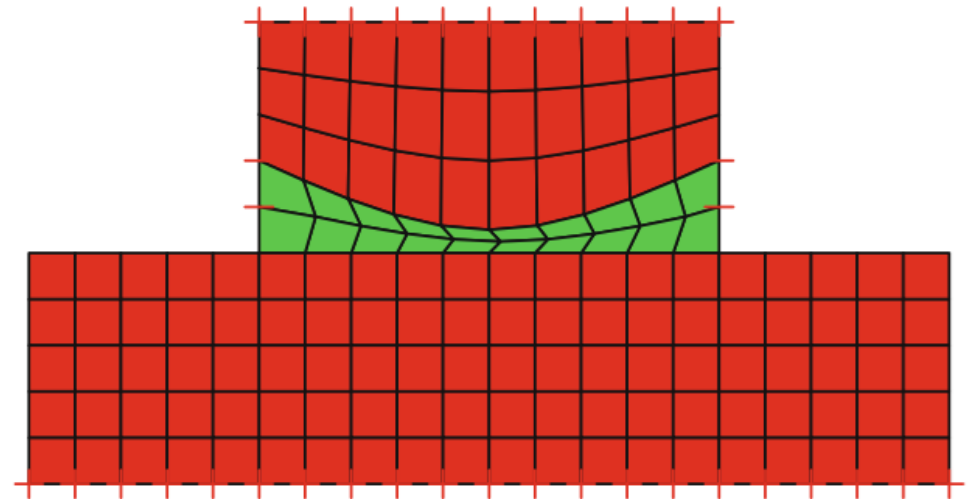
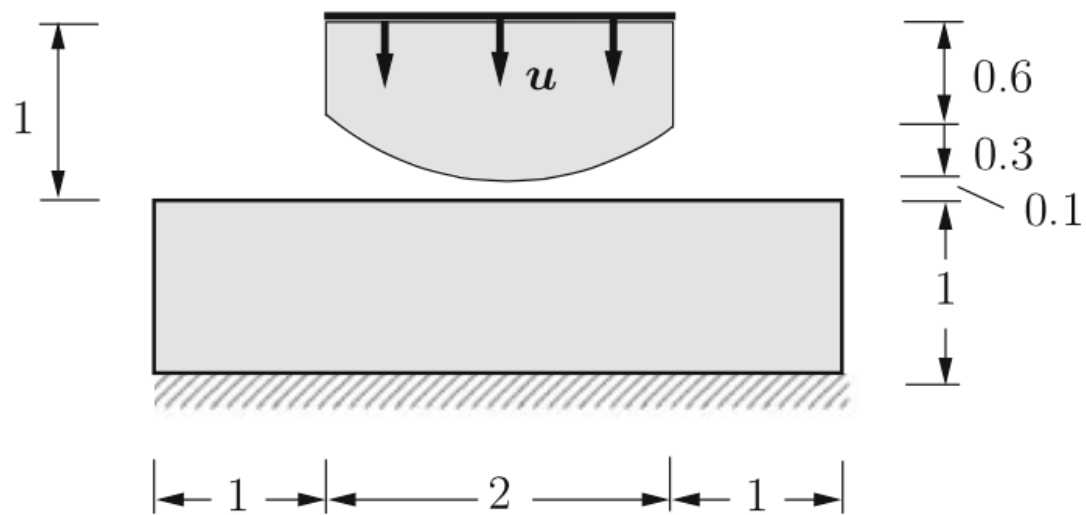
A finite element method for contact using a third medium P. Wriggers · J. Schröder · A. Schwarz  
Comput Mech (2013) 52:837–847



# Gap function



# Contacts as interfaces

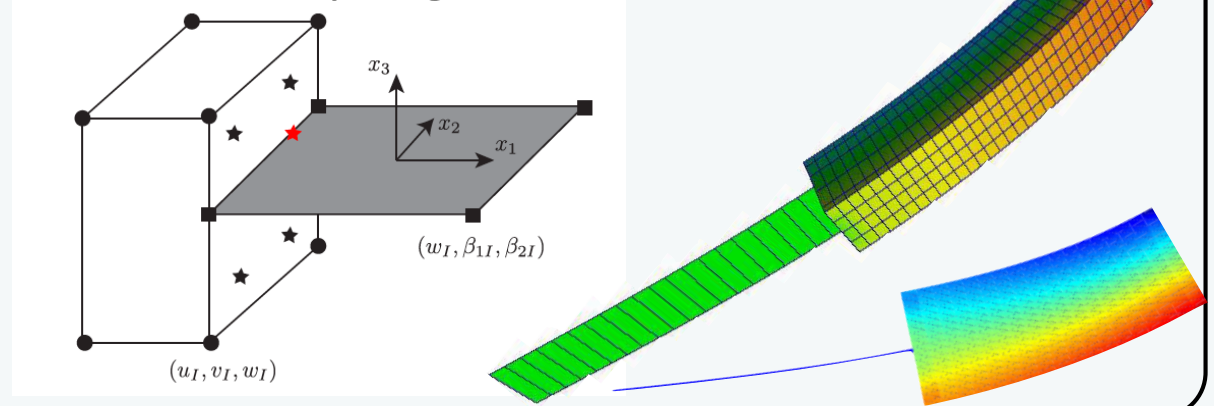


# Future work: model selection (continuum, plate, beam, shell?)

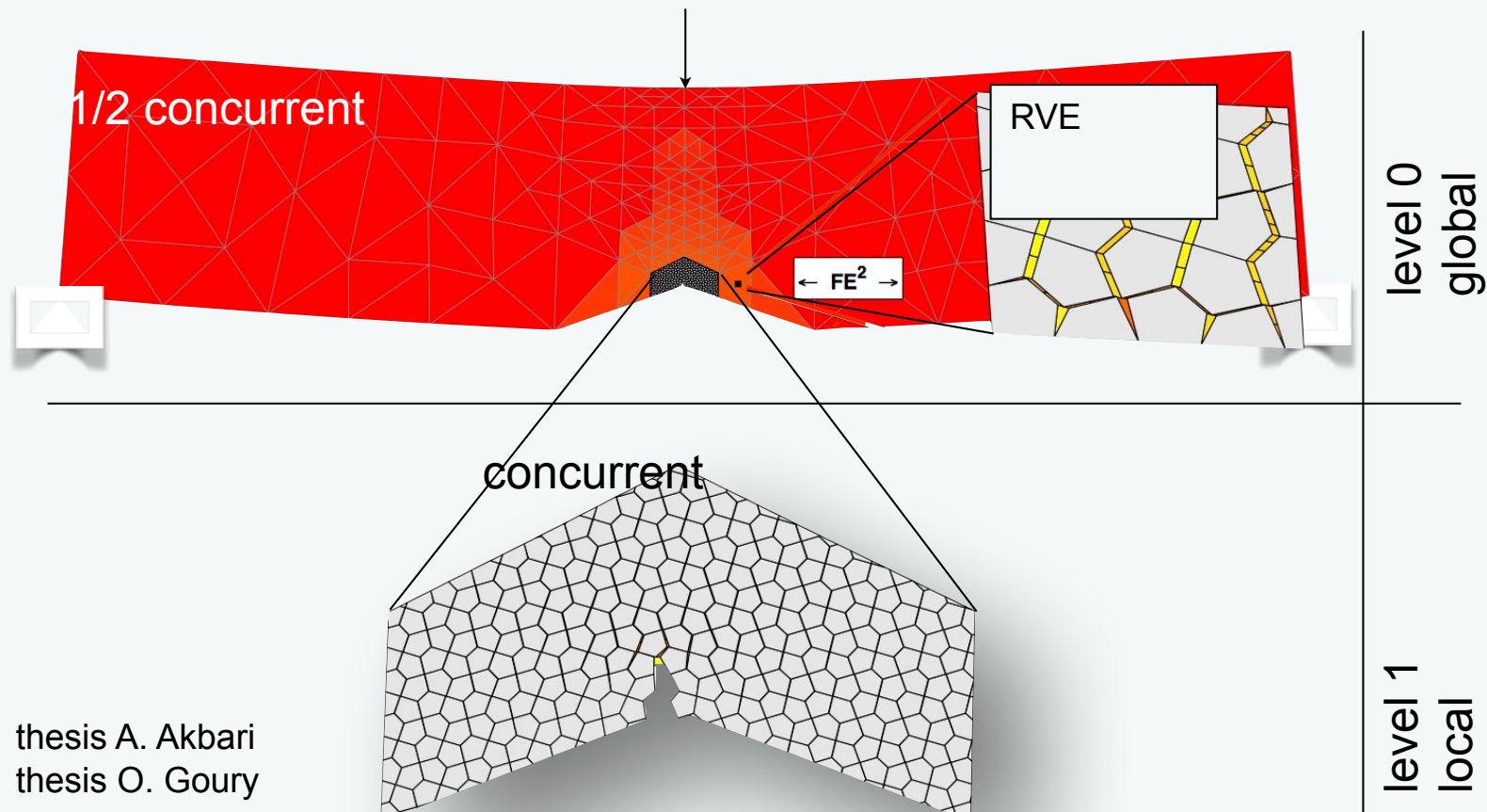
## Model selection

- Model with shells
- Identify “hot spots” - dual
- Couple with continuum
- Coarse-grain

## • Nitsche coupling - NURBS-NURBS



load



## *Fracture of homogeneous materials*

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