Error-based mesh adaptation during crack propagation simulation with X-FEM

O. Pierard, Y. Jin, S. Bordas

15/06/2017

Olivier Pierard
Senior Research Engineer
Contact: olivier.pierard@cenaero.be

Doc. ref.: CFRAC17_Pierard_Presentation

Context:

- Linear Elastic Fracture Mechanics
- Use of X-FEM
- Propagation of pre-existing crack under fatigue based on SIFs

Even in X-FEM, mesh refinement is needed for:

- Implicit representation of cracks
- Accurate computation of stress fields

Solutions:

- Use fixed mesh with refinement in areas where cracks are expected
- Mesh adaptation
  - Engineering approach based on geometric parameters
  - Scientific approach based on error estimation

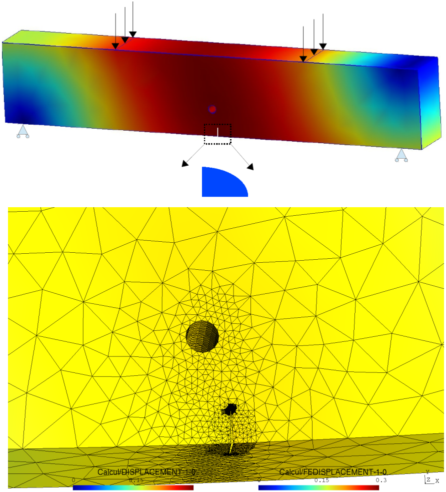
During crack propagation:

- Keep fine mesh along crack front
- Number of DOFs +/- constant

PROD-F-015-02

UNIVERSITÉ DU LUXEMBOURG

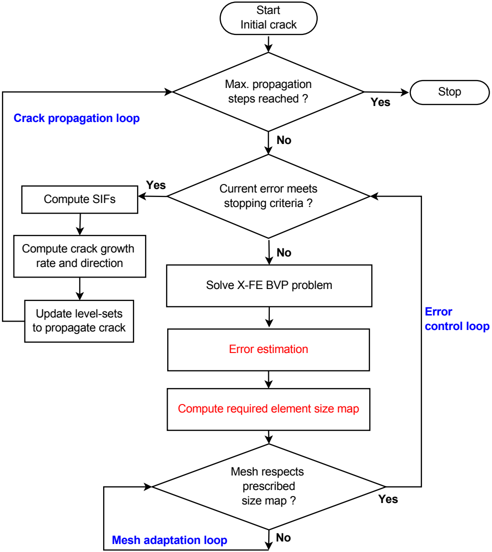
Mesh adaptation during crack propagation
An engineering approach



PROD-F-015-02

UNIVERSITÉ DU LUXEMBOURG

Error control MA - Global strategy



PROD-F-015-02

UNIVERSITÉ DU LUXEMBOURG

1. Solution of the X-FEM problem:

$$\hat{\boldsymbol{u}}(\boldsymbol{x}) = \sum_{I=1}^{n} \phi_I(\boldsymbol{x})\boldsymbol{a}_I + \sum_{J \in \mathcal{J}_h} \phi_J(\boldsymbol{x}) h(\boldsymbol{x}) \boldsymbol{b}_J + \sum_{K \in \mathcal{J}_T} \phi_K(\boldsymbol{x}) \left(\sum_{\ell=1}^{4} F_\ell(\boldsymbol{x}) \boldsymbol{c}_{K\ell} \right)$$

2. eXtended Global Recovery (XGR) - build smoother approximation:

$$\hat{\varepsilon}(\boldsymbol{x}) = \sum_{I=1}^{n} \phi_I(\boldsymbol{x}) d_I + \sum_{J \in \mathcal{J}_h} \phi_J(\boldsymbol{x}) h(\boldsymbol{x}) e_J + \sum_{K \in \mathcal{J}_K} \phi_K(\boldsymbol{x}) \left(\sum_{\ell=1}^{4} G_\ell(\boldsymbol{x}) f_{K\ell} \right)$$

- Coefficients determined by minimization of $\int_\Omega \parallel \hat{\varepsilon} - \varepsilon\left(\hat{\boldsymbol{u}}\right) \parallel^2 d\Omega$

3. Estimation of error in energy norm:

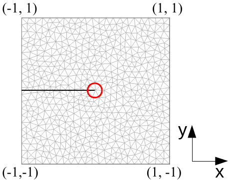
$$\parallel \hat{\varepsilon} - \varepsilon(\hat{\boldsymbol{u}}) \parallel_\Omega = \sqrt{\int_\Omega (\boldsymbol{C} : (\hat{\varepsilon} - \varepsilon(\hat{\boldsymbol{u}}))) : (\hat{\varepsilon} - \varepsilon(\hat{\boldsymbol{u}}))d\backslash}$$

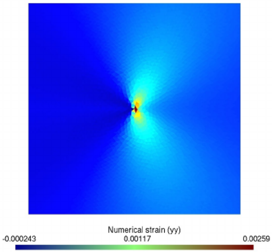
PROD-F-015-02

UNIVERSITÉ DU LUXEMBOURG

XGR - Application to the Westergaard problem

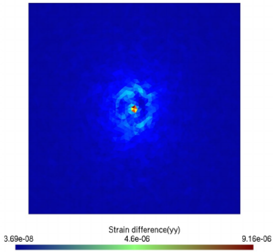
- Single edge crack subjected to pure mode I
- Analytical solution available
- Exact tractions applied on the boundary





$\hat{\varepsilon}_{eq}$

Recovered strain field



$\hat{\varepsilon}_{eq} - \varepsilon_{eq}(\hat{\boldsymbol{u}})$

Estimated loc

(on strai

PROD-F-015-02

UNIVERSITÉ DU LUXEMBOURG

Requested mesh size map computation

Element refinement/coarsening: $h_i^{\text{new}} = \xi_i h_i^{\text{current}}$

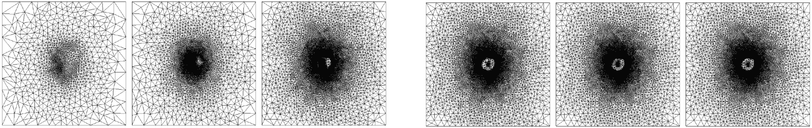
Definitions of ξ_i:

- Equal-distribution criterion: reduce global error + similar local error over all elements
  - Global error: $\xi_i^g = \frac{\max(0.6\|\hat{\boldsymbol{\varepsilon}}-\boldsymbol{\varepsilon}(\hat{\boldsymbol{u}})\|_\Omega, 0.02\|\boldsymbol{\varepsilon}(\hat{\boldsymbol{u}})\|_\Omega)}{\|\hat{\boldsymbol{\varepsilon}}-\boldsymbol{\varepsilon}(\hat{\boldsymbol{u}})\|_\Omega}$
  - Local error:
    - $\xi_i^{\ell e} = \frac{\|\hat{\boldsymbol{\varepsilon}}-\boldsymbol{\varepsilon}(\hat{\boldsymbol{u}})\|_\Omega}{\sqrt{N}} \frac{1}{\|\hat{\boldsymbol{\varepsilon}}-\boldsymbol{\varepsilon}(\hat{\boldsymbol{u}})\|_i}$
    - Volume-weighted: $\xi_i^{\ell v} = \| \hat{\boldsymbol{\varepsilon}} - \boldsymbol{\varepsilon}(\hat{\boldsymbol{u}}) \|_\Omega \sqrt{\frac{\Omega_i}{\Omega}} \frac{1}{\|\hat{\boldsymbol{\varepsilon}}-\boldsymbol{\varepsilon}(\hat{\boldsymbol{u}})\|_i}$
  - $\xi_i = \left(\xi_i^g\right)^{\frac{1}{n}} \left(\xi_i^{\ell e}\right)^{\frac{2}{2n+d}}$ - Global + local [Oñate *et al.*]
  - $\xi_i = \left(\xi_i^g \xi_i^{\ell e}\right)^{\frac{1}{n}}$ - Global + local (simplified) [Zienkiewicz *et al.*]
  - $\xi_i = \left(\xi_i^g \xi_i^{\ell v}\right)^{\frac{1}{n}}$ - Global + local (vol. weighted) [Oñate *et al.*]
- Min-Number criterion: Minimize number of elements for given rel. global error: $\xi_i = \frac{\theta_0^{1/q}}{\theta_i^{1/(q+1)} \left[\sum_i \theta_i^{2/(q+1)}\right]^{1/2q}}$ [Coorevits *et al.*]

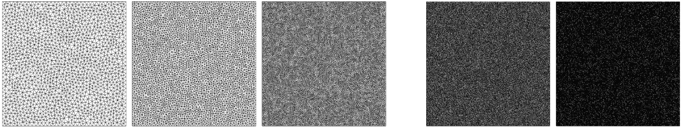
PROD-F-015-02

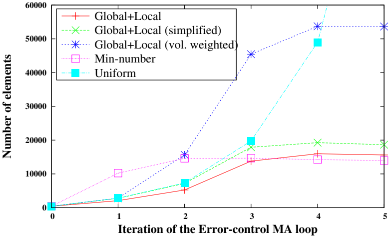
UNIVERSITÉ DU LUXEMBOURG

Global + local (vol. weighted)



Uniform

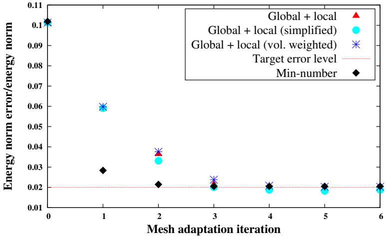


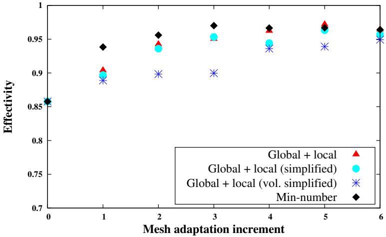


PROD-F-015-02

UNIVERSITÉ DU LUXEMBOURG

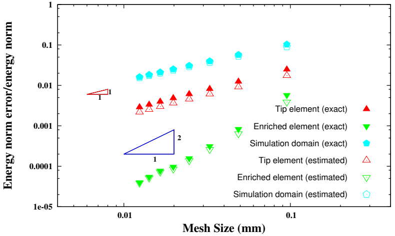
Comparison of the different MA strategies
Evolution of global Errors and effectivities



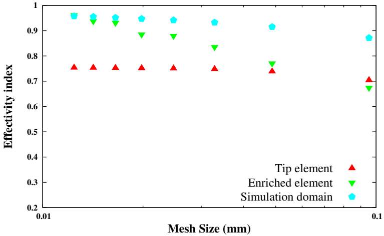


PROD-F-015-02

UNIVERSITÉ DU LUXEMBOURG

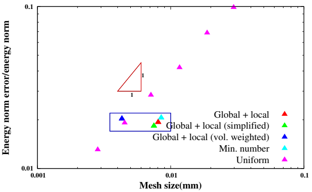


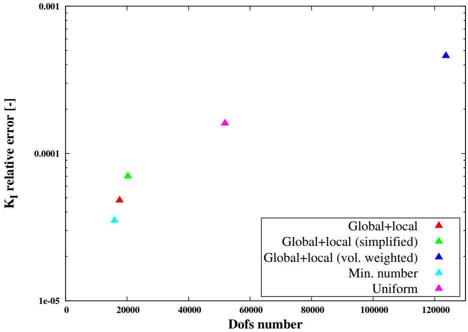
Energy norm error/energy norm
Mesh Size (mm)
1
0.1
0.01
0.001
0.0001
1e-05
0.01
0.1
1
1
2
1
Tip element (exact)
Enriched element (exact)
Simulation domain (exact)
Tip element (estimated)
Enriched element (estimated)
Simulation domain (estimated)



Effectivity index
Mesh Size (mm)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.01
0.1
Tip element
Enriched element
Simulation domain

PROD-F-015-02



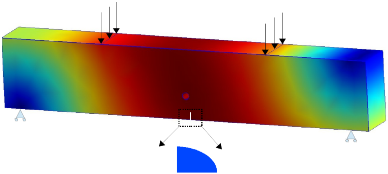


Similar energy error can give signifively different errors on SIFs

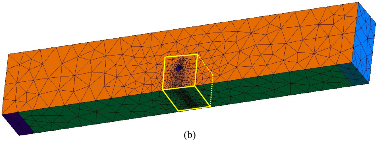
PROD-F-015-02

Application to 3D crack propagation

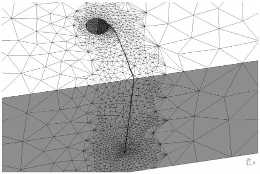
4-points bending test



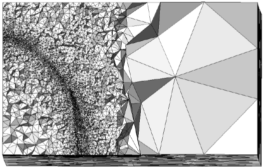
Initial mesh



Surface mesh (step 32)



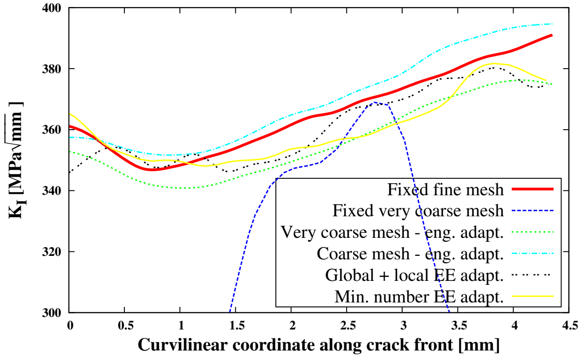
Mesh cross-section around crack front (step 32)



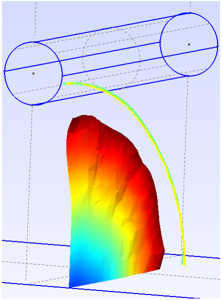
PROD-F-015-02

UNIVERSITÉ DU LUXEMBOURG

Sifs along crack cront (step 0)



Crack fronts (Step 30)



Crack growth rate: $\frac{da}{dn} = C\Delta K_{eq}^{4.75} \rightarrow$ Error on K_i strongly ampl

PROD-F-015-02

UNIVERSITÉ DU LUXEMBOURG

Rough comparison of the computational cost

	Fixed fine	Very coarse Eng.	Global + local	Min-number
Initial mesh [dofs]	883.000	11.400	3.300	3.300
Nbr. propa. steps	35	35	35	35
Nbr it Error est.	0	0	3	3
XGR [dofs]	N/A	N/A	1.200.000	420.000
X-FEM [dofs]	883.000	18.000	400.000	140.000
Time per step [s]	1000-2400	70-80	8.000-20.000	1.000-5.200

PROD-F-015-02

UNIVERSITÉ DU LUXEMBOURG

Conclusions:

- Global + local & Min-number stategies give more reliable predicitions
- Sub-optimal convergence rate in crack tip element
- Similar energy error can give signifively different errors on SIFs
- Our engineering approach is (surprisingly?) an extremely efficient solution

Perspectives:

- Error-based mesh adaptation on initial configuration only
- Use recovery technique limited to a volume around the crack front to reduce XGR computation time
- Goal-oriented mesh adaptation is a must

PROD-F-015-02

UNIVERSITÉ DU LUXEMBOURG