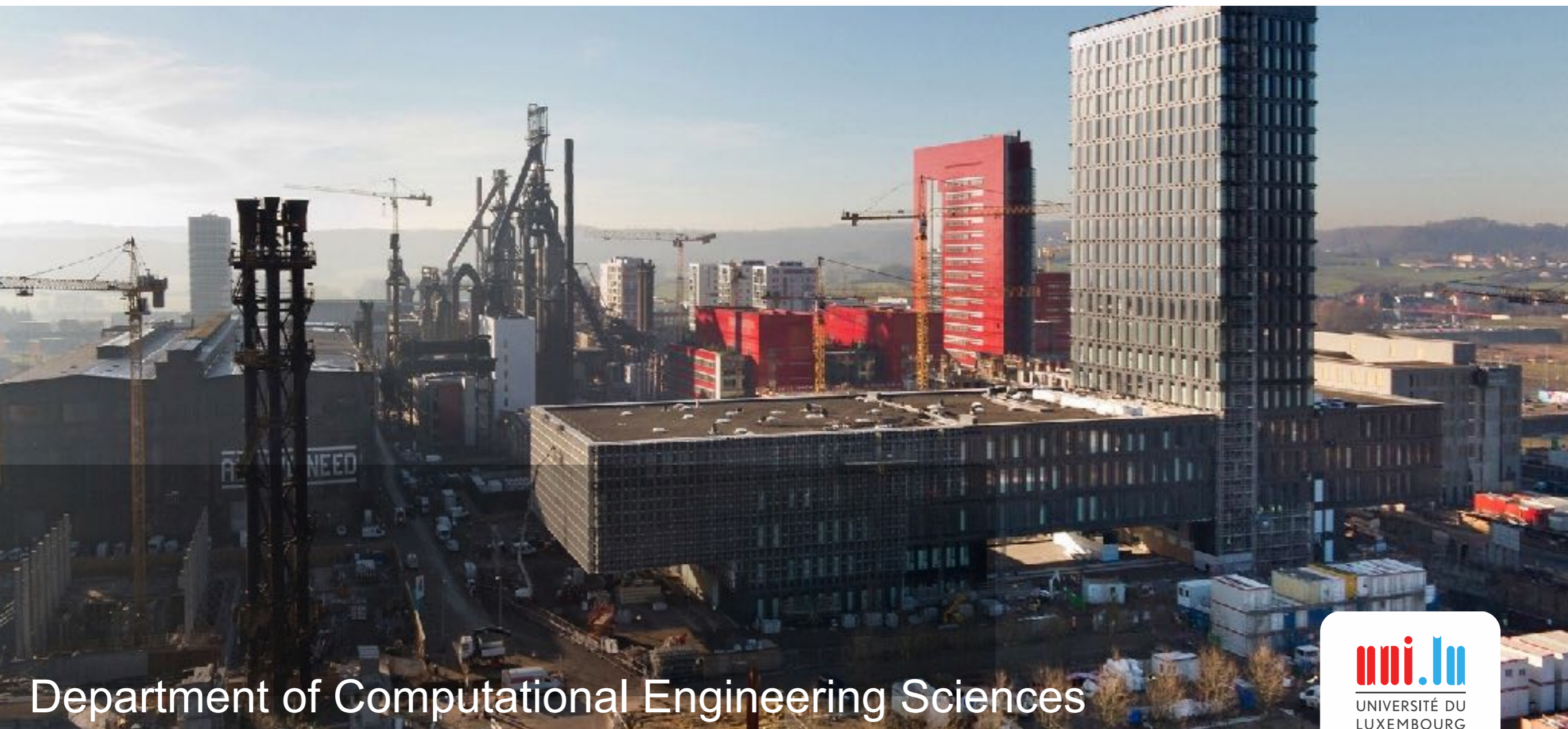


+



Computed in Luxembourg

# Computational Sciences Luxembourg

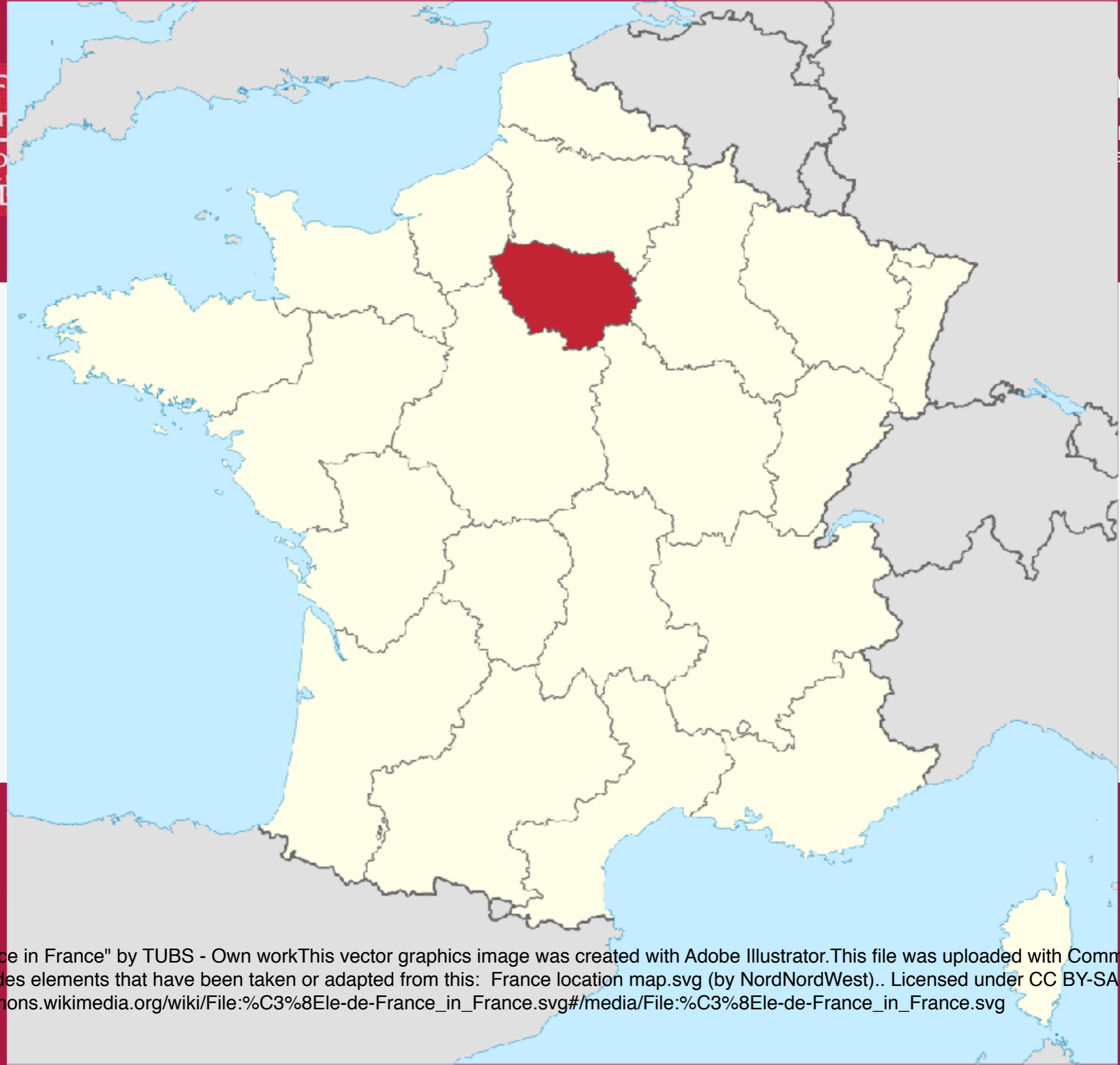


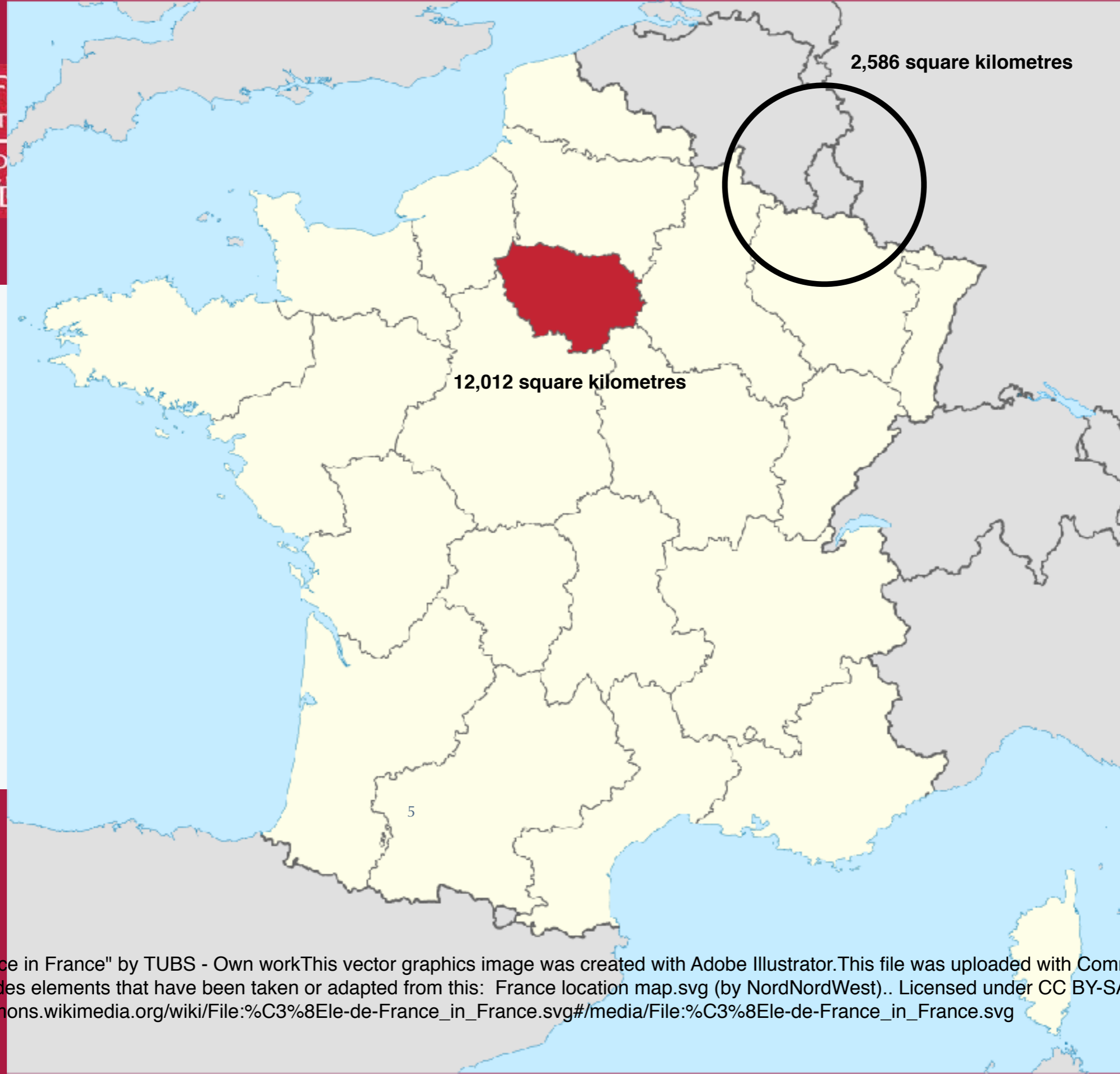
Department of Computational Engineering Sciences











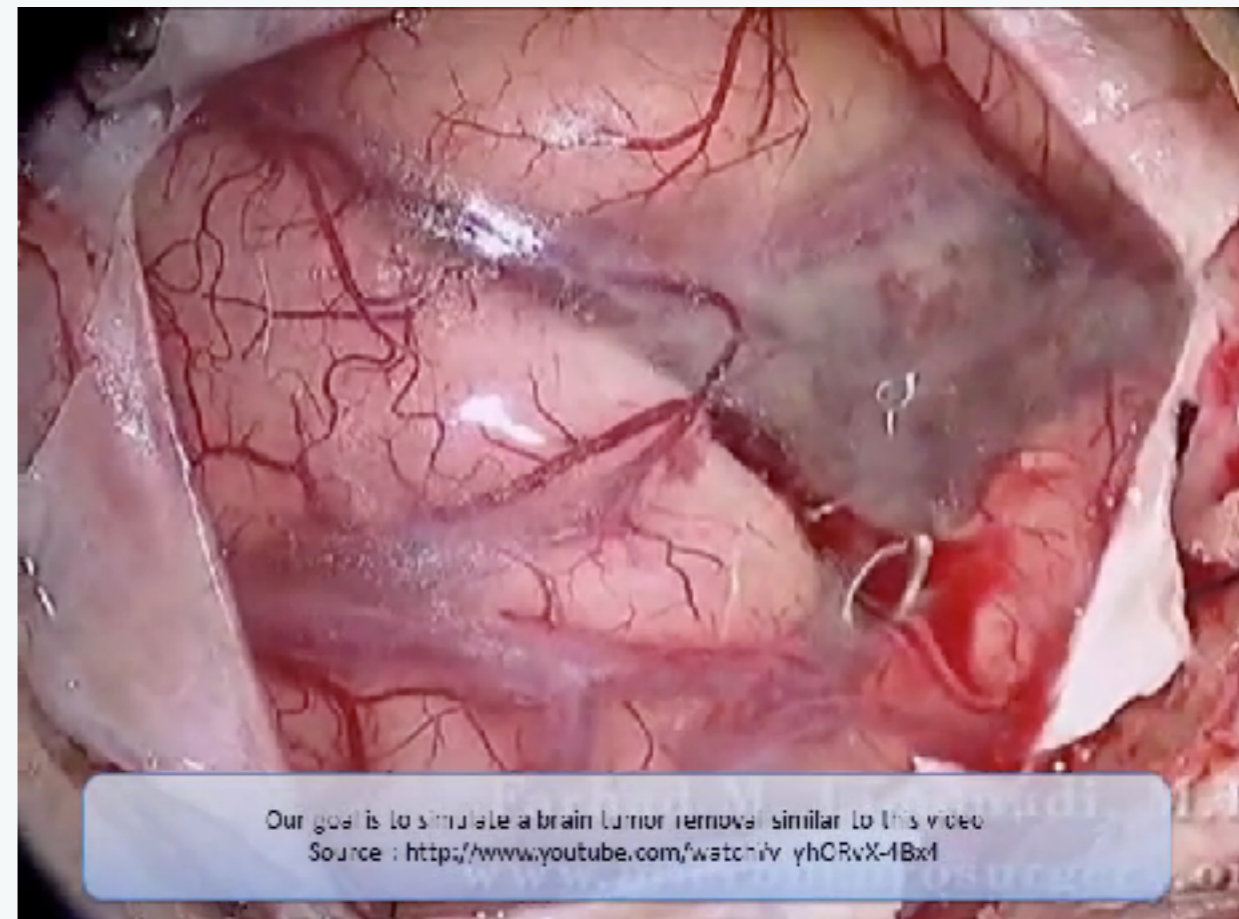
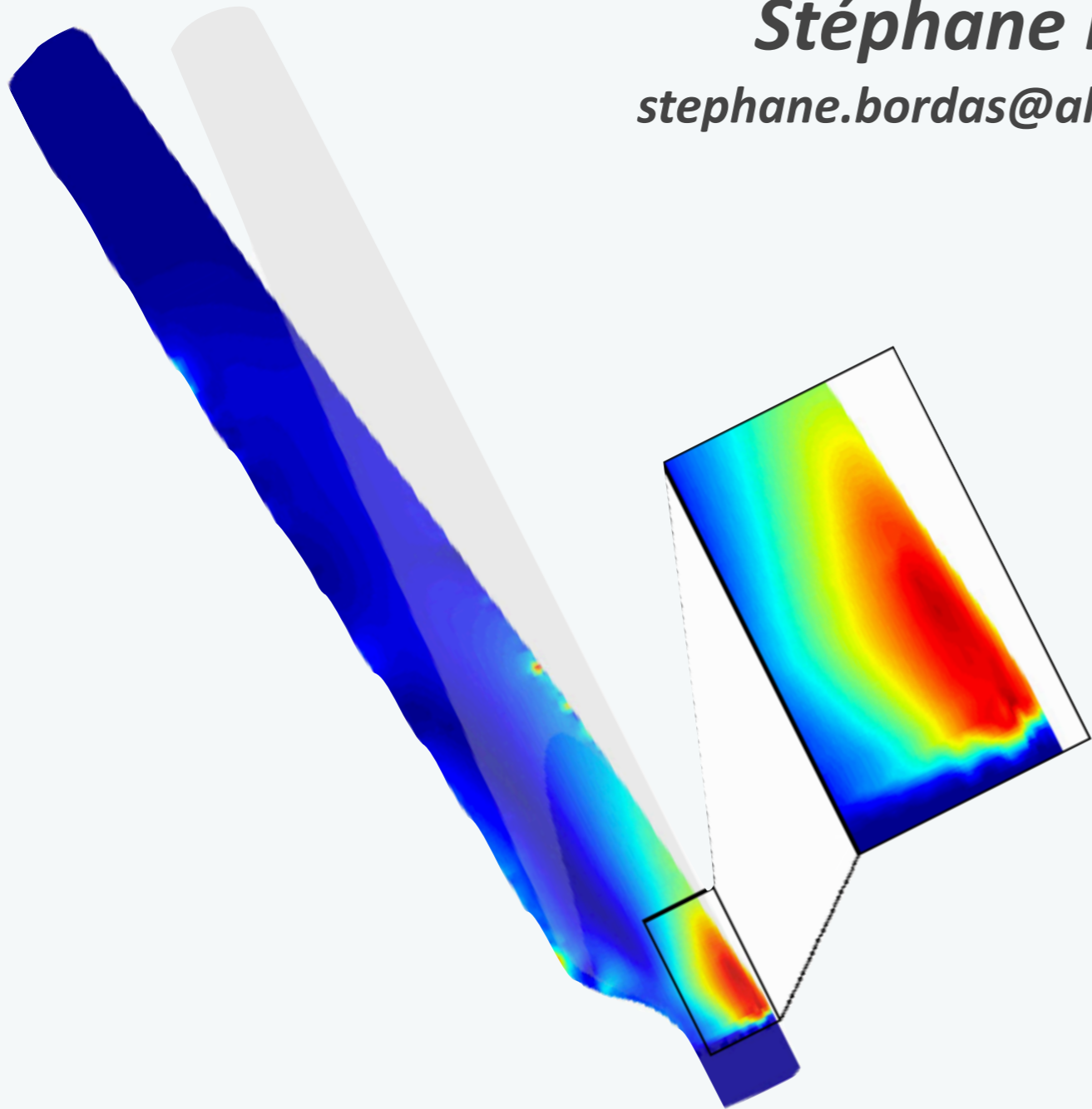


<http://hdl.handle.net/10993/31487>

# Advances in enriched finite element formulations for fracture and cutting: *engineering and surgical simulation applications*

**Stéphane P.A. Bordas**

*stephane.bordas@alum.northwestern.edu*



# Motivation: fracture mechanics

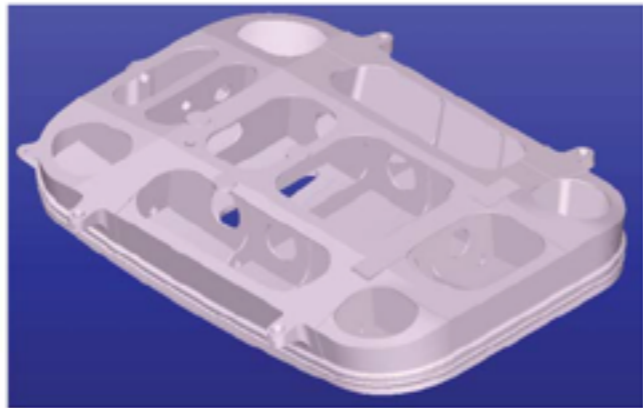


## Fracture of homogeneous materials

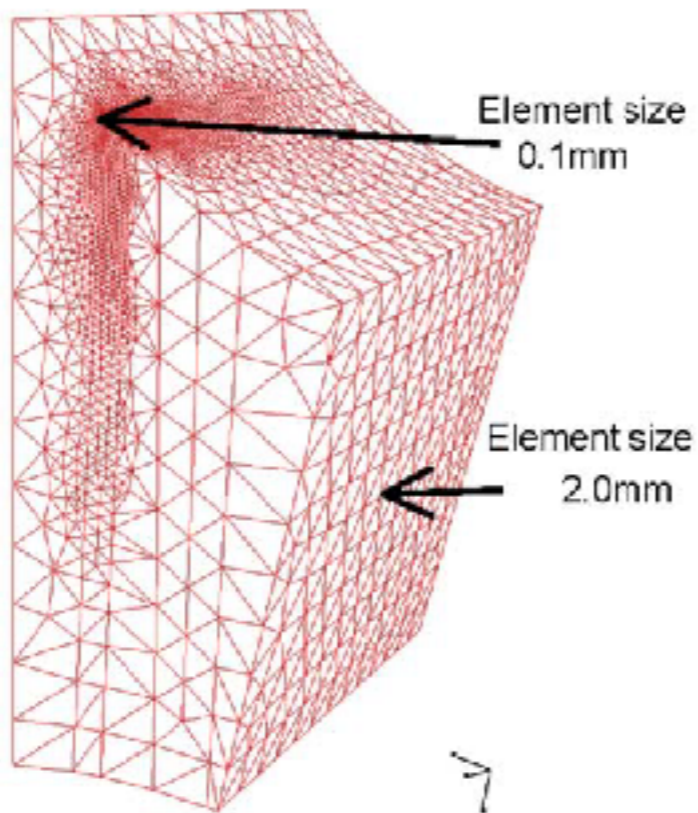
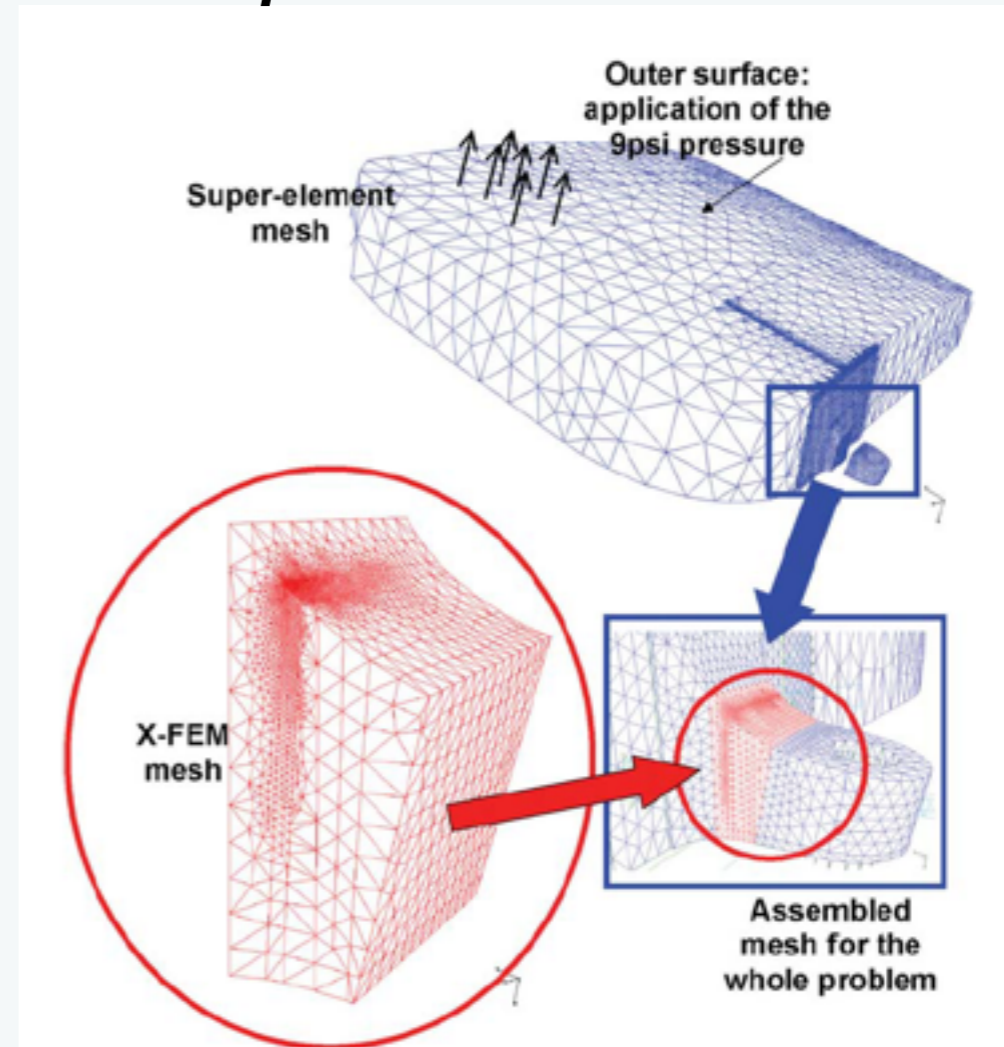
**Question: when should a structure be inspected for flaws?**



(a) Top view



(b) Bottom view



**ad hoc mesh refinement**

SPAB and B. Moran, Engineering Fracture Mechanics, 2006  
 V.P. Nguyen et al. XFEM C++ Library IJNME, 2007  
**Industrial applications of extended finite element methods**  
 See also E. Wyart et al, EFM, IJNME, 2008

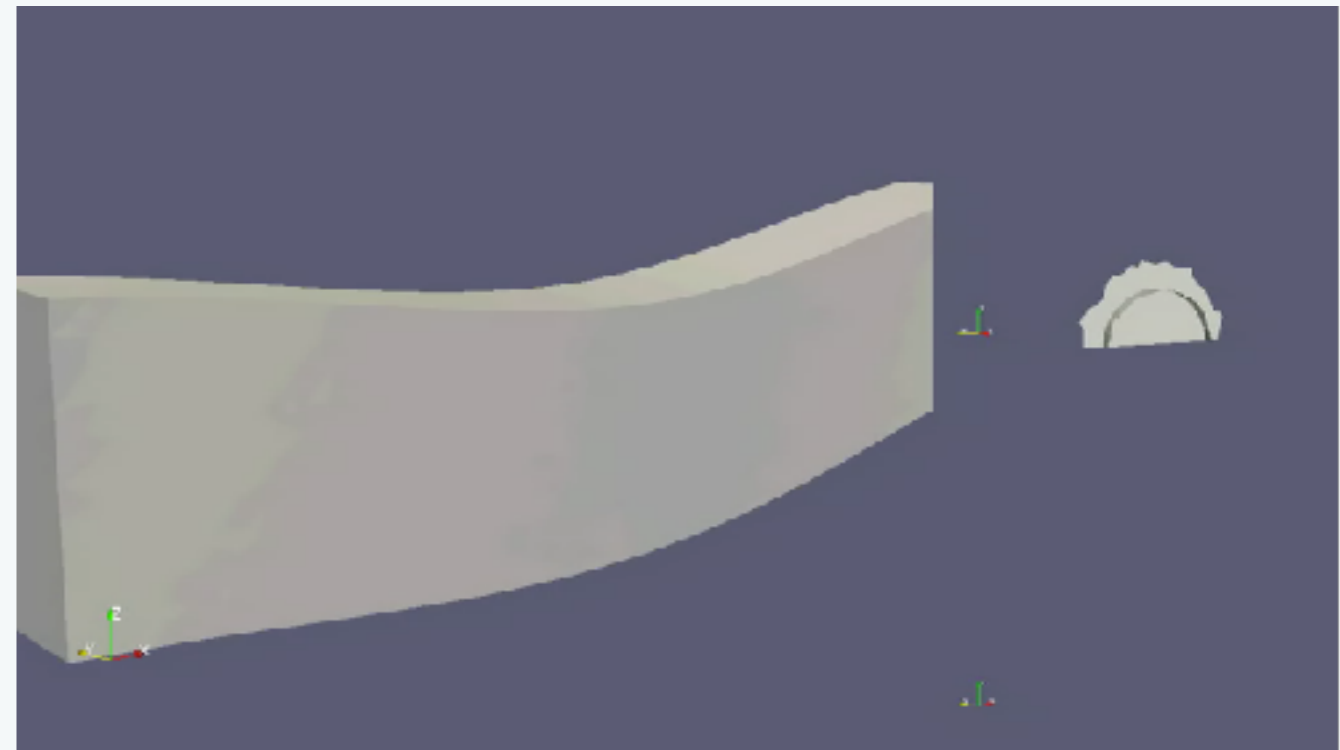
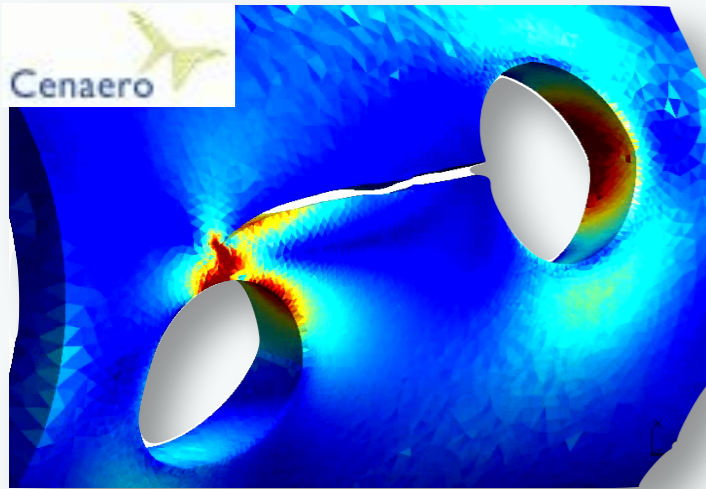
## *Fracture of homogeneous materials*

**Question: How to control accuracy and simplify/avoid meshing?**

- ▶ Partition of Unity - eXtended/Generalized Finite Element Methods
  - Discretisation error governed by the worst approximant
  - Local enrichment of approximations
  - Requires enrichment volumes independent of the mesh
  - Conditioning issues for large enrichment zones or arbitrary enrichment (see stable GFEM, Banerjee, Babuška + Agathos)
  
- ▶ 3D fracture requires **accurate** stress intensity factors (SIFs)
  - Error at each step  $\sim (\text{Error on SIF})^4$
  - Standard enrichment  $\Rightarrow$  oscillations along the front

## *Fracture of homogeneous materials*

**Question: How to control accuracy and simplify/avoid meshing?**



K. Agathos et al. IJNME 2016, CMAME 2016, IJNME 2017, CMAME 2017 with Eleni Chatzi and Giulio Ventura

***How can we use large enrichment radii?***

***How can we control conditioning in large-scale enriched FEM?***

***How can we use higher order terms in the expansion?***



X. Peng et al. IJNME 2016, CMAME 2017  
Enriched Isogeometric Boundary Elements

***How to avoid meshing completely for crack propagation simulations?***

# More details...

*Check out the talk by  
Kostas Agathos (ETHZ)*

16:20 - 17:20	BB	<b>MS4: Enriched finite element methods - Simone</b>
16:20 - 16:40		Generalized Finite Element Method: application to Interface Problems - Uday Banerjee
16:40 - 17:00		Improving conditioning of 3D XFEM for fracture mechanics - Konstantinos Agathos

# More details...

*Check out the talk by  
Kostas Agathos (ETHZ)*

16:20 - 17:20	BB	<b>MS4: Enriched finite element methods - Simone</b>
16:20 - 16:40		Generalized Finite Element Method: application to Interface Problems - Uday Banerjee
16:40 - 17:00		Improving conditioning of 3D XFEM for fracture mechanics - Konstantinos Agathos

Sorry, it was yesterday...

# More details...

*Check out the talk by  
Kostas Agathos (ETHZ)*

16:20 - 17:20	BB	<b>MS4: Enriched finite element methods - Simone</b>
16:20 - 16:40		Generalized Finite Element Method: application to Interface Problems - Uday Banerjee
16:40 - 17:00		Improving conditioning of 3D XFEM for fracture mechanics - Konstantinos Agathos



# Use your favourite time machine...



# Or a more gradual introduction...

Agathos K, Ventura G, Chatzi E, Bordas S. Stable 3D XFEM/vector-level sets for non-planar 3D crack propagation and comparison of enrichment schemes. *International Journal for Numerical Methods in Engineering. Computational Mechanics*, 2017.

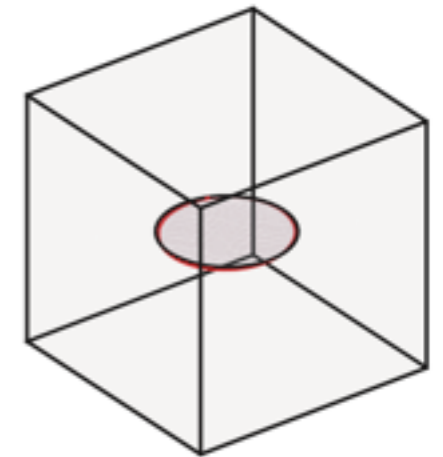
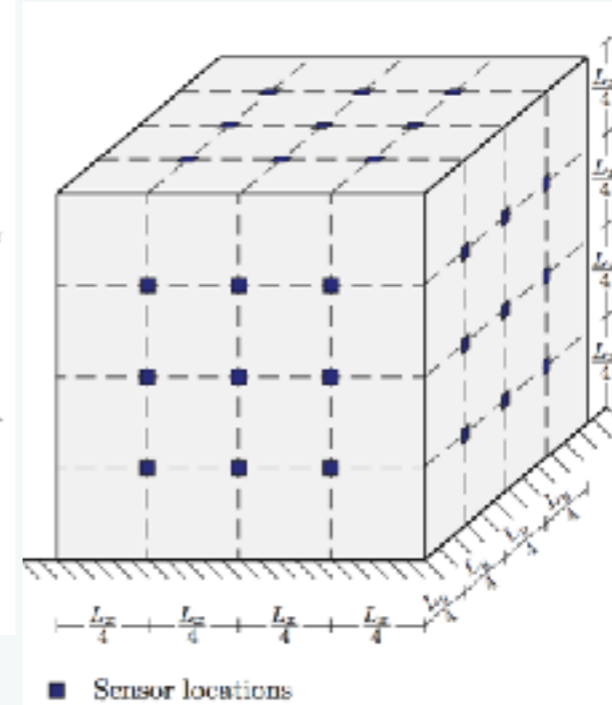
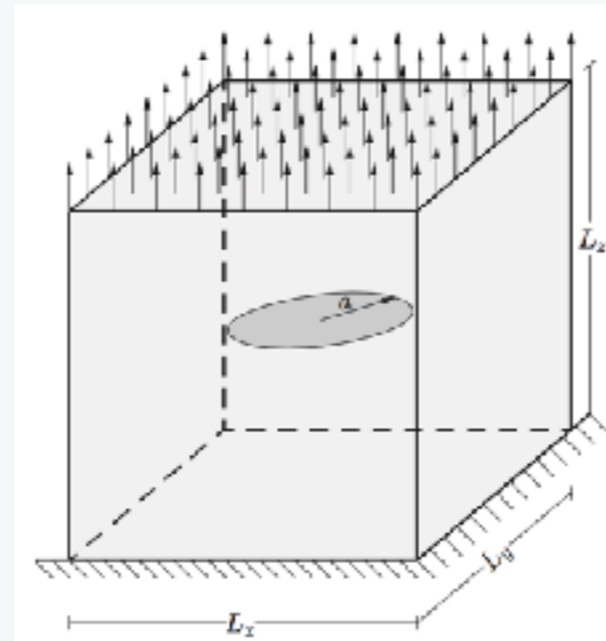
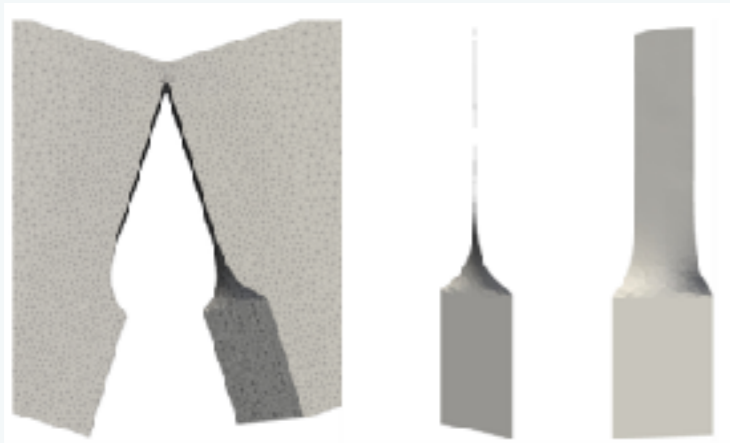
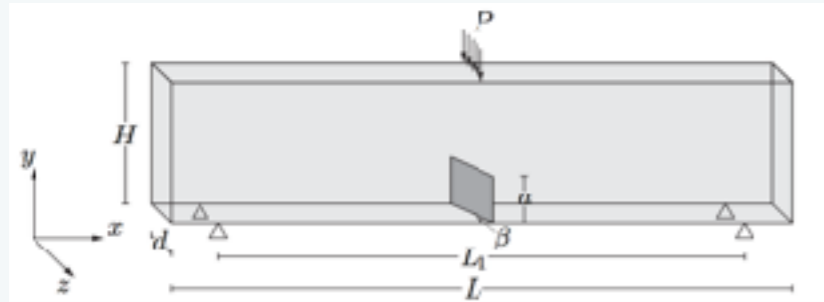
Agathos K, Chatzi E, Bordas S, Talaslidis D. A well-conditioned and optimally convergent XFEM for 3D linear elastic fracture. *International Journal for Numerical Methods in Engineering*. 2016 Mar 2;105(9):643-77.

Agathos, K., E. Chatzi, and SPA Bordas. "Stable 3D extended finite elements with higher order enrichment for accurate non planar fracture." *Computer Methods in Applied Mechanics and Engineering* 306 (2016): 19-46.

<https://orbilu.uni.lu/bitstream/10993/22331/2/paper.pdf>

<http://orbilu.uni.lu/bitstream/10993/22420/1/presentation.pdf>



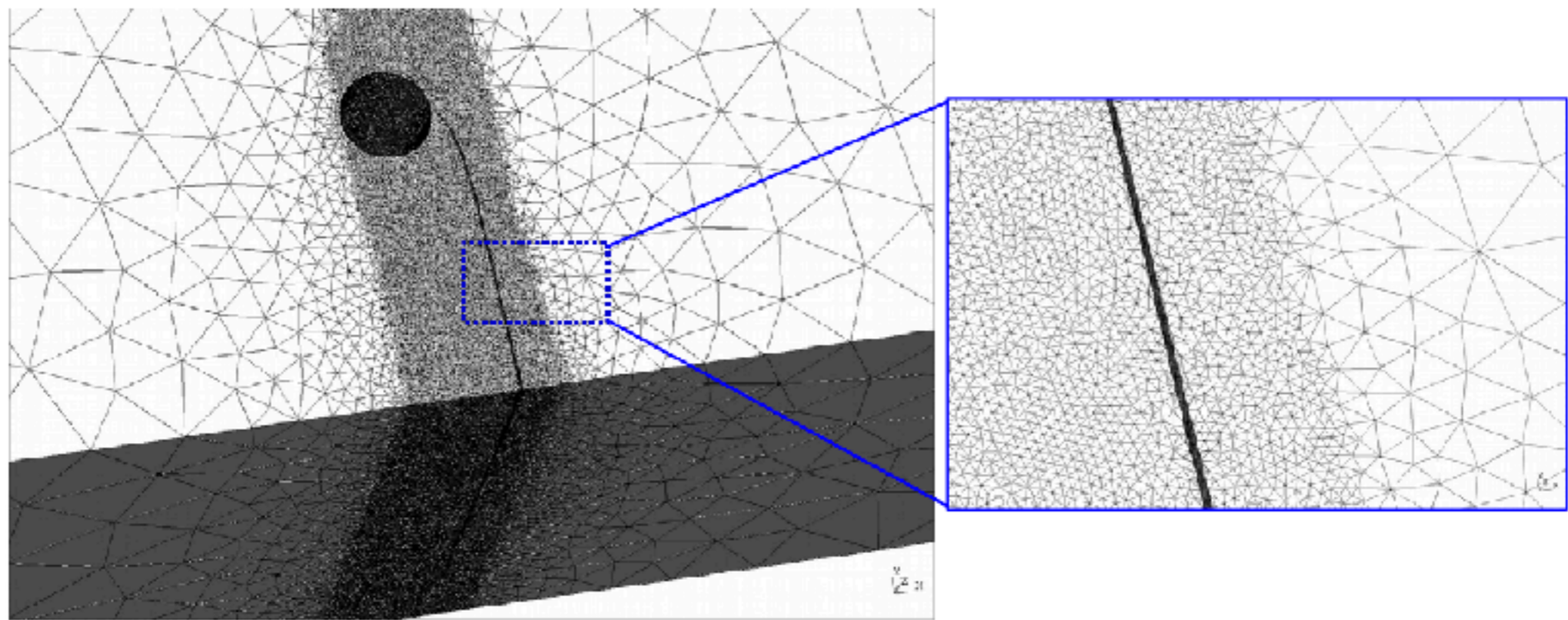


2000 evaluations

- ✓ Introduces a novel form of enrichment.
- ✓ Provides improved conditioning.
- ✓ Enables the use of geometrical enrichment.
- ✓ Enables the use of higher order terms in fracture mechanics
- ✓ Was combined to vector level sets to solve crack propagation problems.
- ✓ Was applied to inverse problems.
- ✓ Provides high accuracy and optimal convergence.

**What if you can't add new functions or  
you don't want to increase the  
enrichment radius?**

*(Goal oriented) adaptive computational fracture  
use h-refinement*



**Before: mesh “finely” in the region where the crack is “expected” to propagate**

Y. Jin, O. Pierard, et al. *Comput. Methods Appl. Mech. Engrg.* 318 (2017) 319–348

O.A. González-Estrada et al. *Computers and Structures* 152 (2015) 1–10

O.A. González-Estrada et al. *Comput Mech* (2014) 53:957–976

C. Prange et al. *IJNME* 91.13 (2012): 1459-1474.

M. Duflot, SPAB, *IJNME* 2007, *CNME* 2007, *IJNME* 2008.

J-J. Ródenas Garcia, *IJNME* 2007

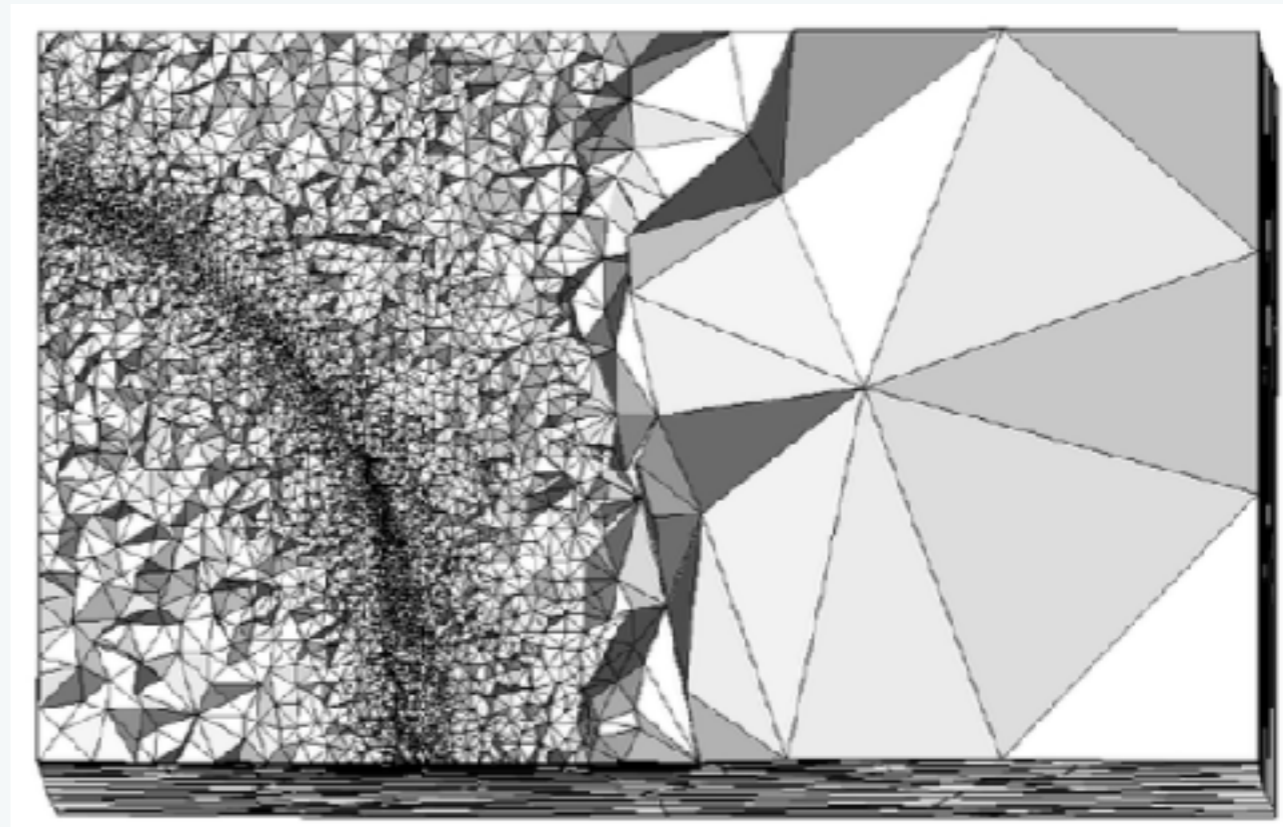
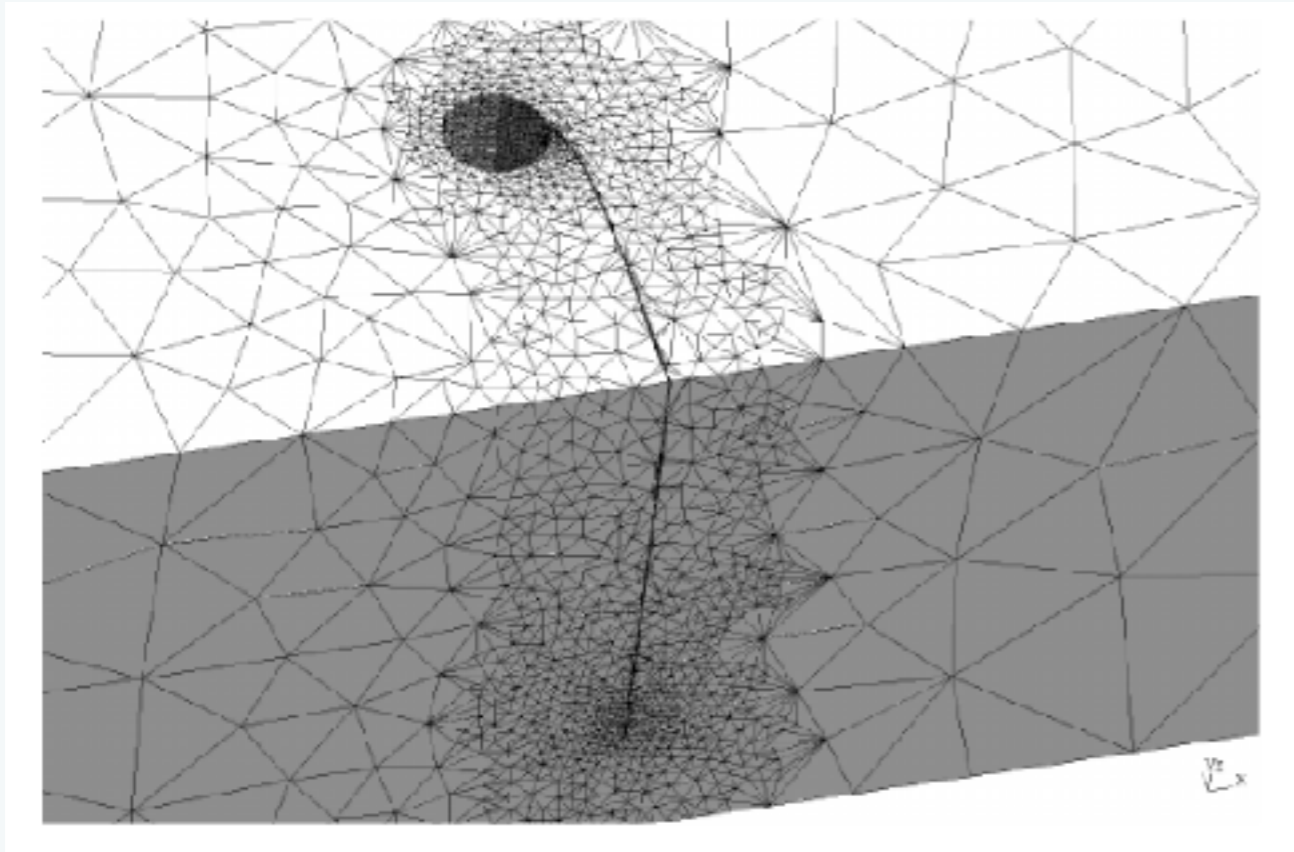
F.B. Barros, et al *IJNME* 60.14 (2004): 2373-2398.

M. Rüter *CMECH* (2013) 1;52(2):361-76.

J. Panetier *IJNME* 81.6 (2010): 671-700.

P. Hild, *CMECH* (2010): 1-28.

## *Fracture of homogeneous materials: error estimation and adaptivity*



**After: determine mesh refinement adaptively using a (goal-oriented) error estimate**

Y. Jin, O. Pierard, et al. Error-controlled adaptive extended finite element method for 3D linear elastic crack propagation Comput. Methods Appl. Mech. Engrg. 318 (2017) 319–348

M. Duflot, SPAB, IJNME 2007, CNME 2007, IJNME 2008.

## More details...

*Check out the talk by Olivier Pierard (Cenaero)  
later this morning MS4 at 11:00*

11:00 - 12:20	BB	<b>MS4: Enriched finite element methods - Aragón</b>
11:00 - 11:20		Error-based mesh adaptation during crack propagation simulation with X-FEM - Olivier Pierard

# More cracks?...

## Extended Finite Element Method (XFEM)

Fracture of “XFEM” using XFEM

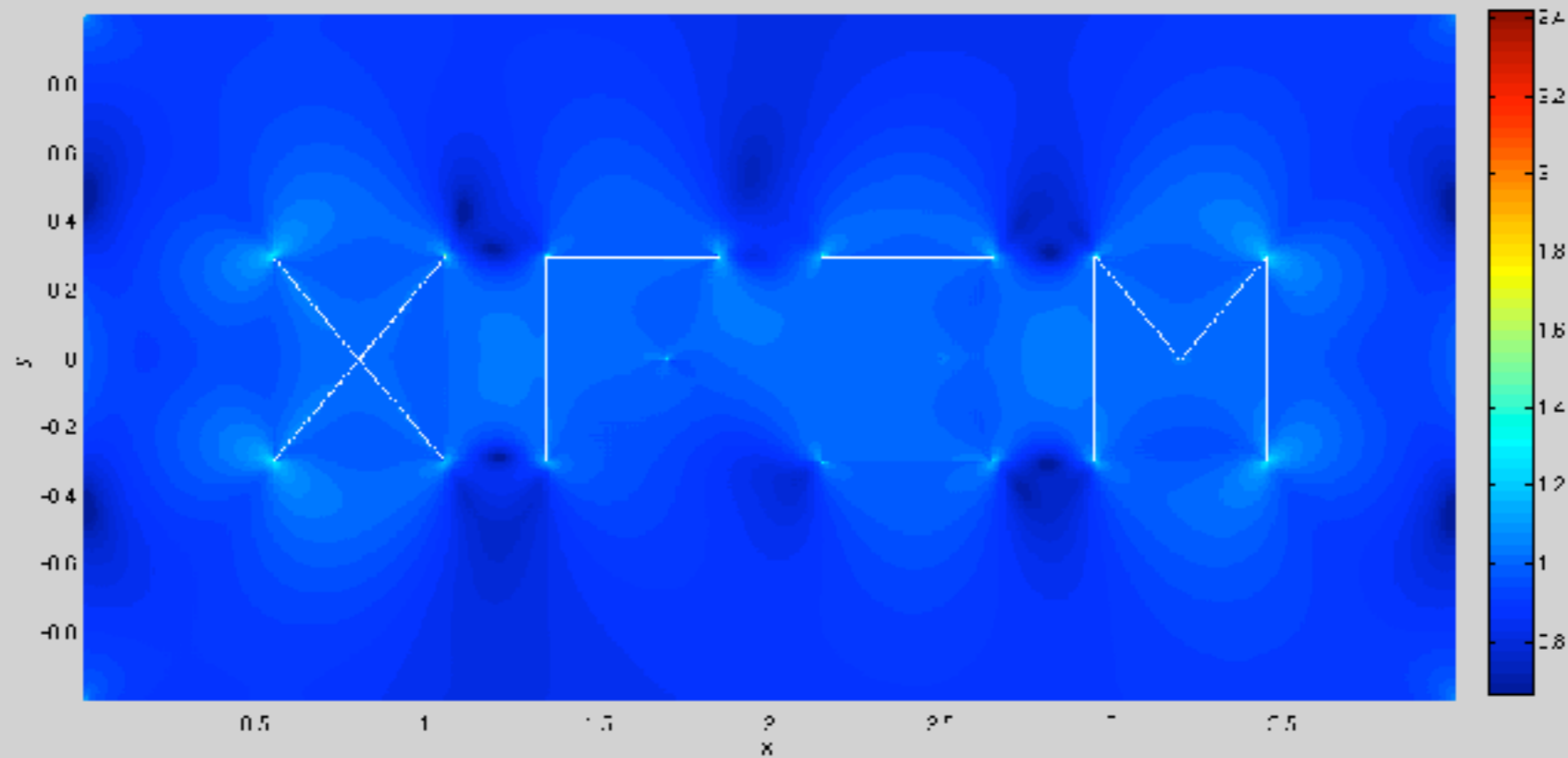
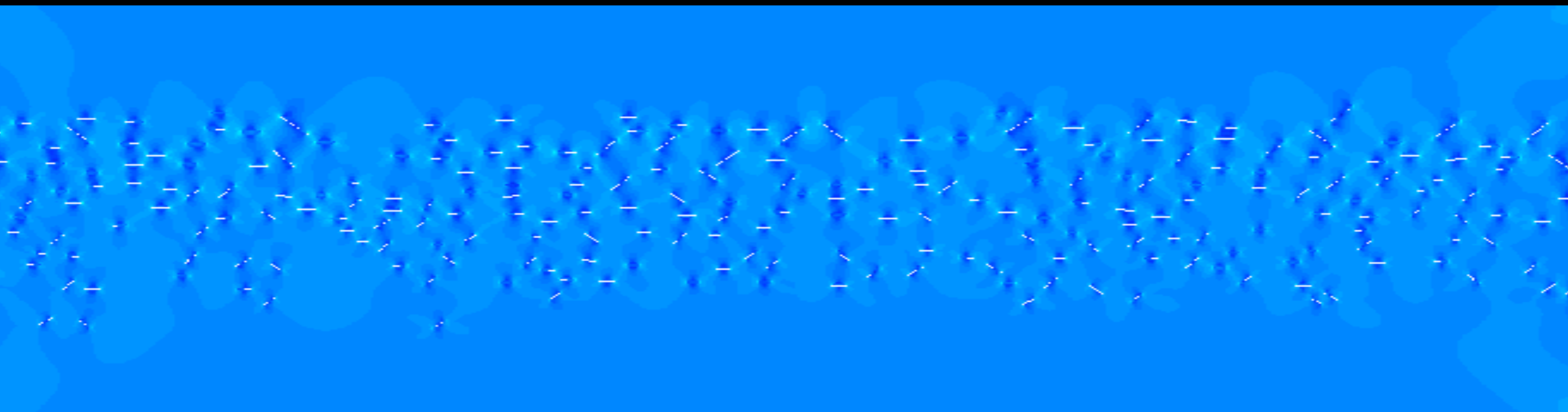


Plate with 300 cracks - vertical extension BCs



Energy-minimal crack growth using XFEM

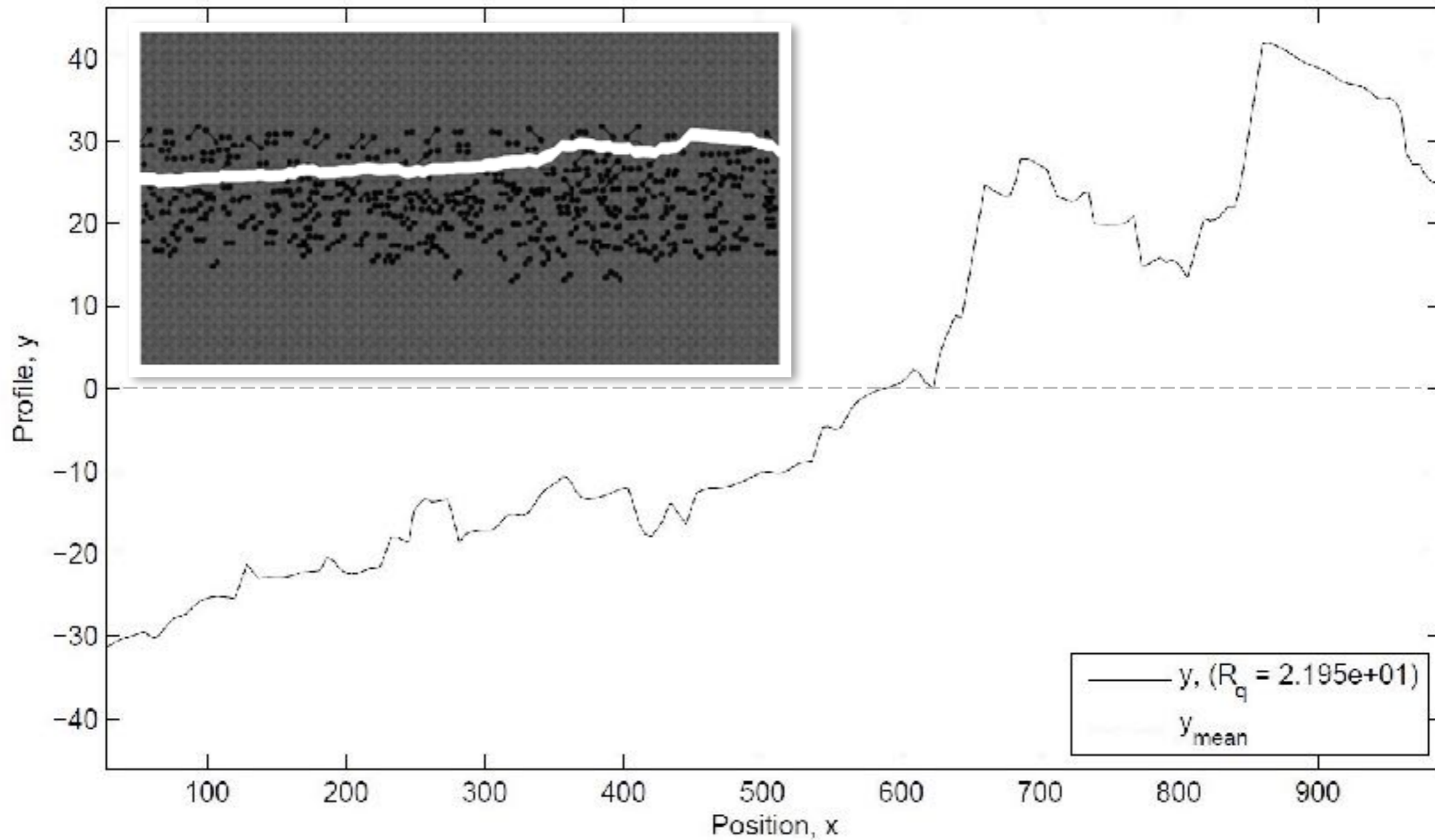


Sutula et al. Preprint of three part EFM paper at  
<http://hdl.handle.net/10993/29414>



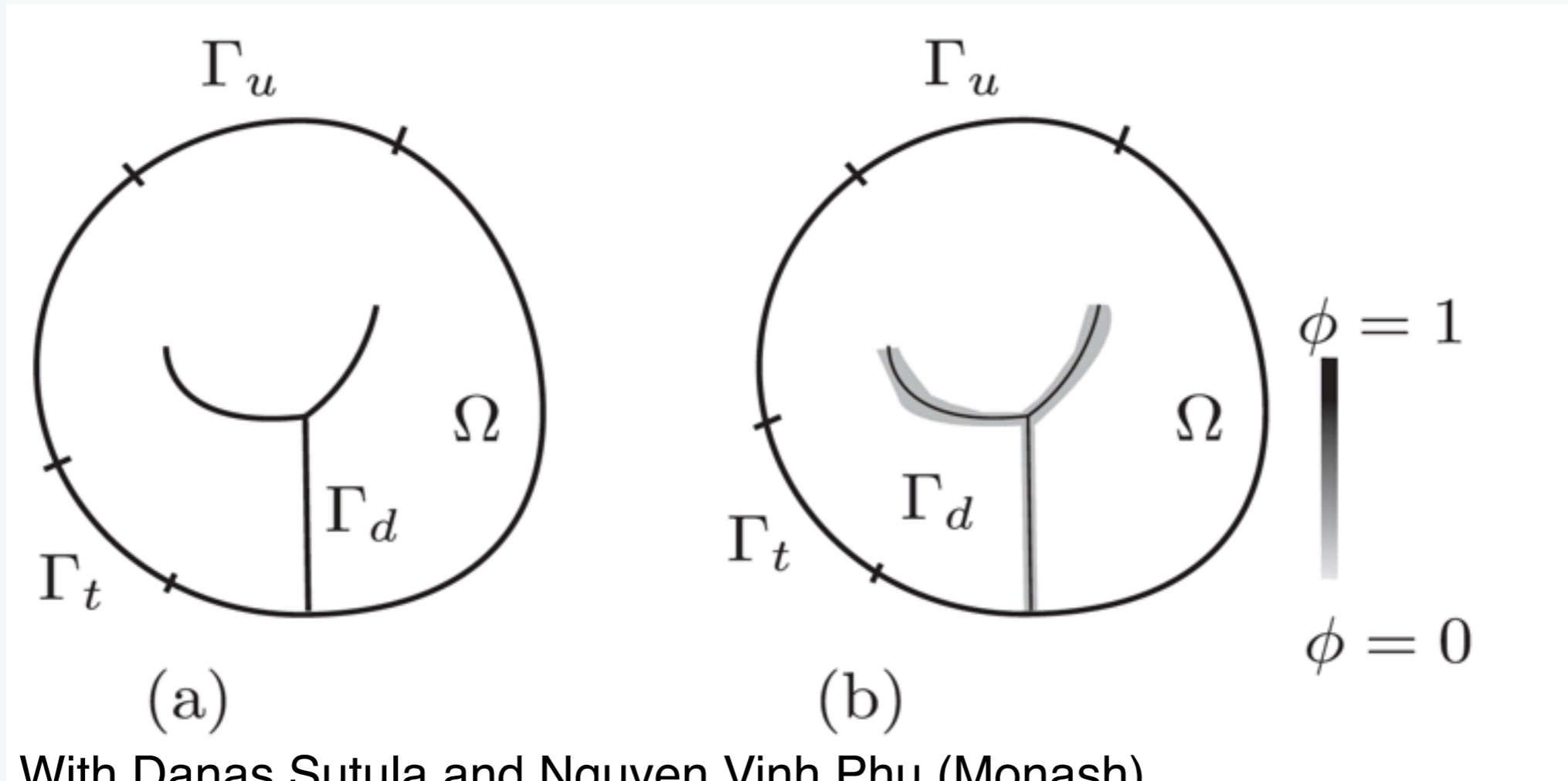
## Vertical extension of a plate with 300 cracks

Post-split roughness

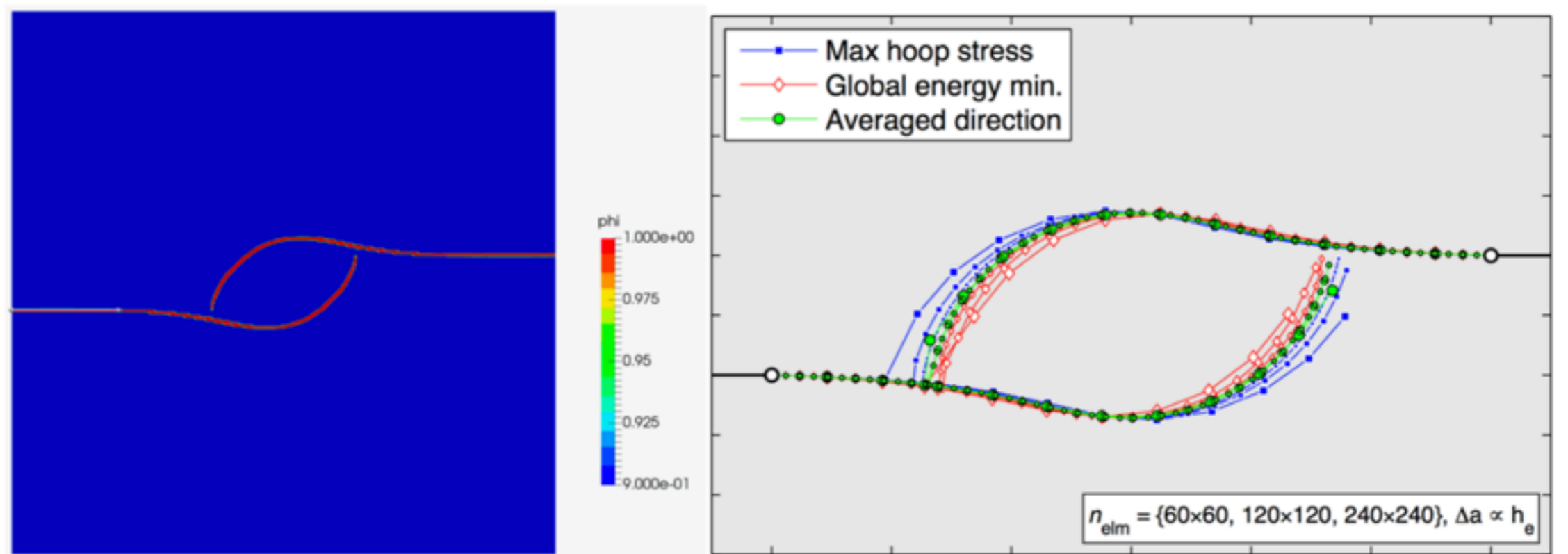


# More cracks?... 3D? ...

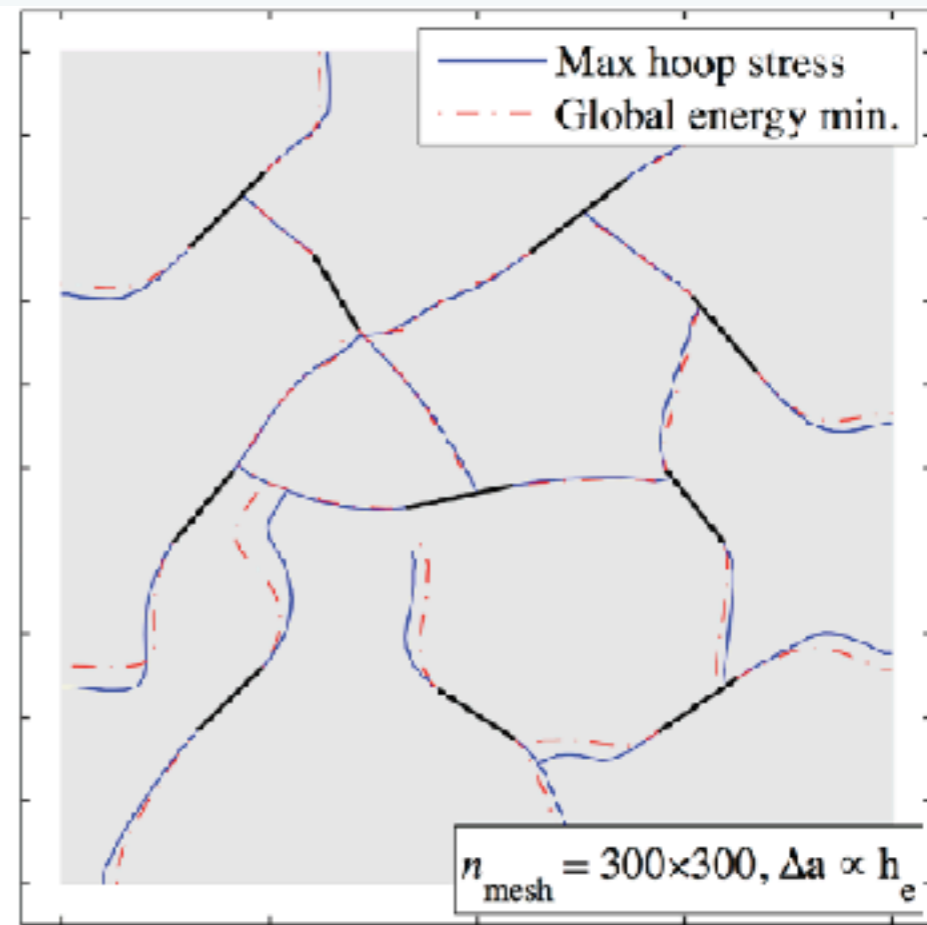
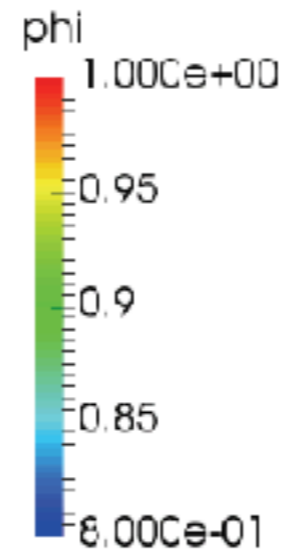
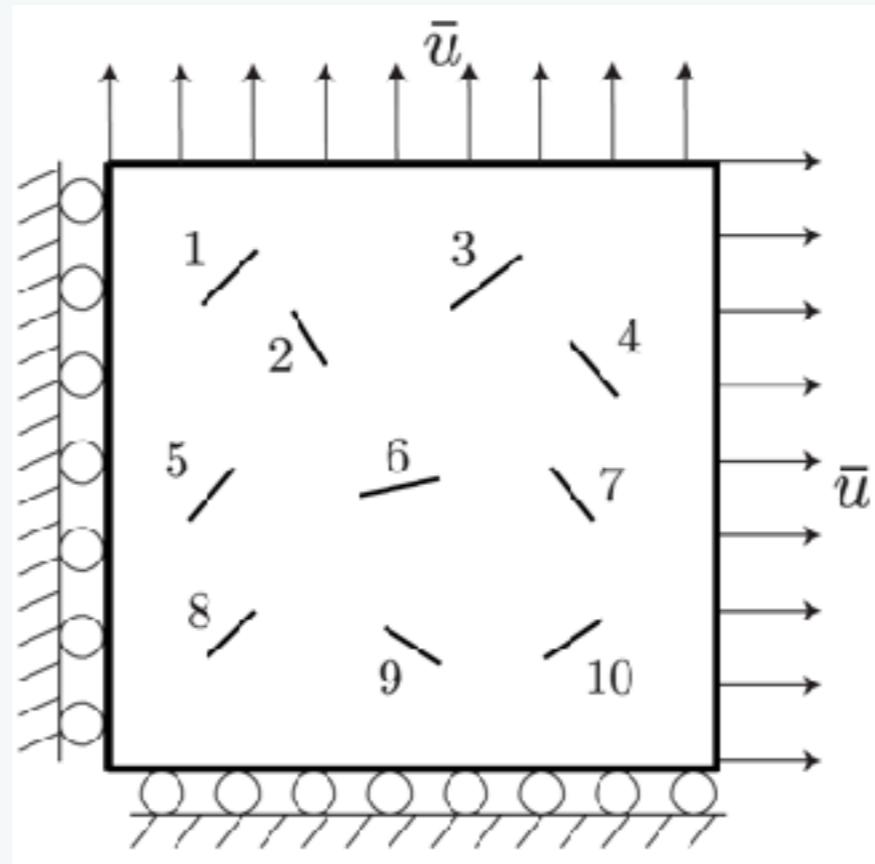
## *Phase field/thick level sets*

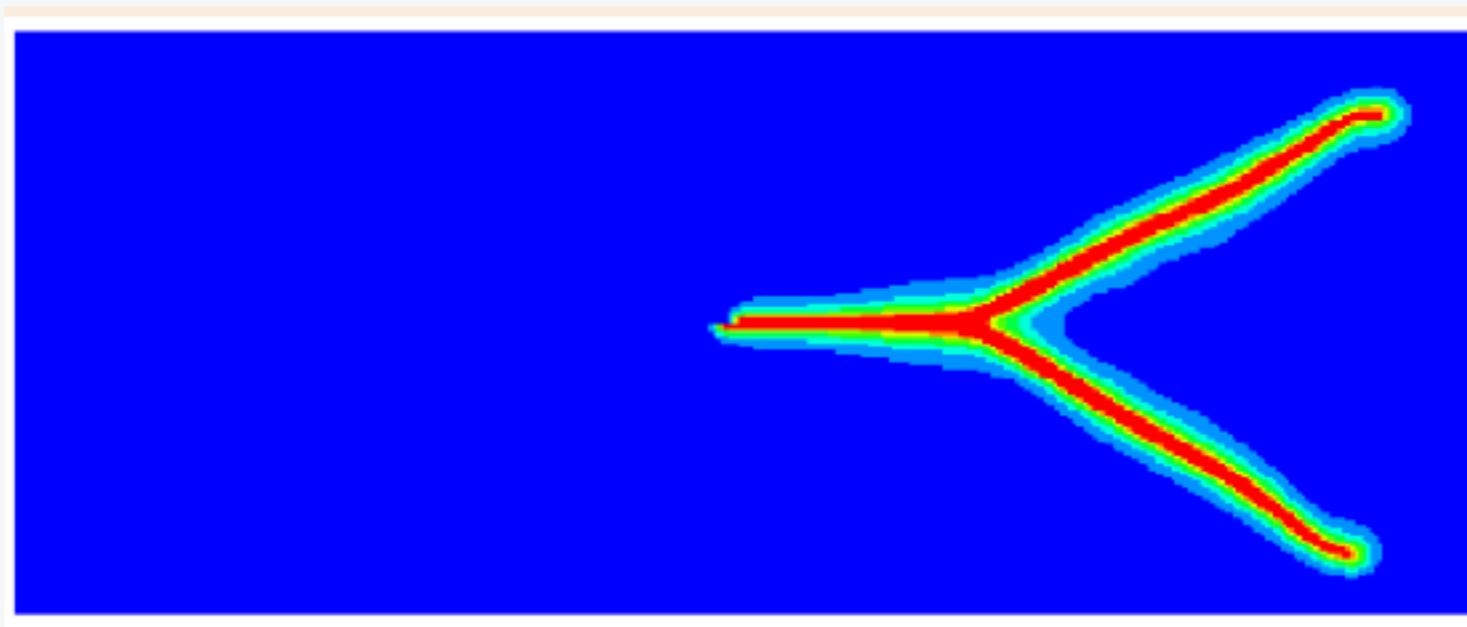
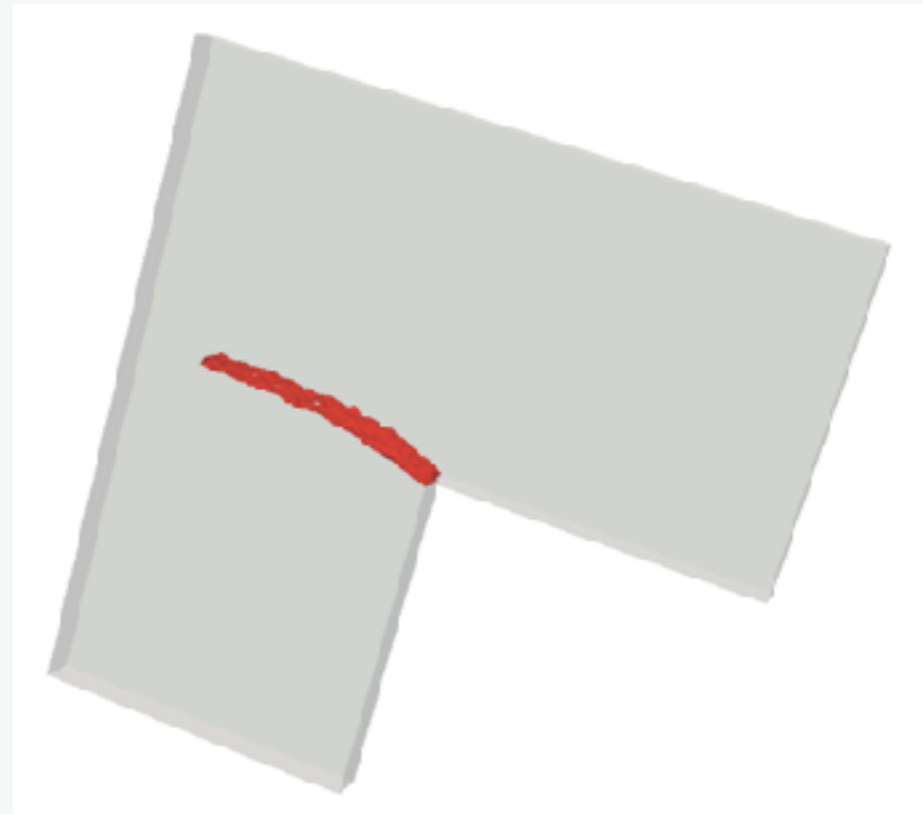
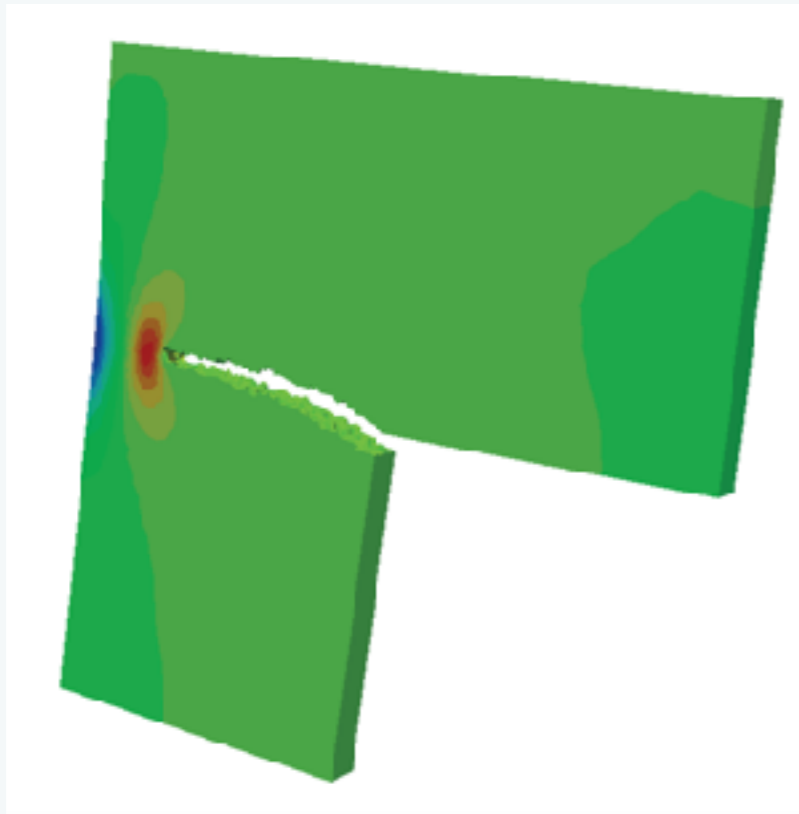


With Danas Sutula and Nguyen Vinh Phu (Monash)  
 9TH Australasian Congress on Applied Mechanics (ACAM9)  
 27 - 29 November 2017  
[phu.nguyen@monash.edu](mailto:phu.nguyen@monash.edu)



With Danas Sutula and Nguyen Vinh Phu (Monash)  
 9TH Australasian Congress on Applied Mechanics (ACAM9)  
 27 - 29 November 2017  
[phu.nguyen@monash.edu](mailto:phu.nguyen@monash.edu)



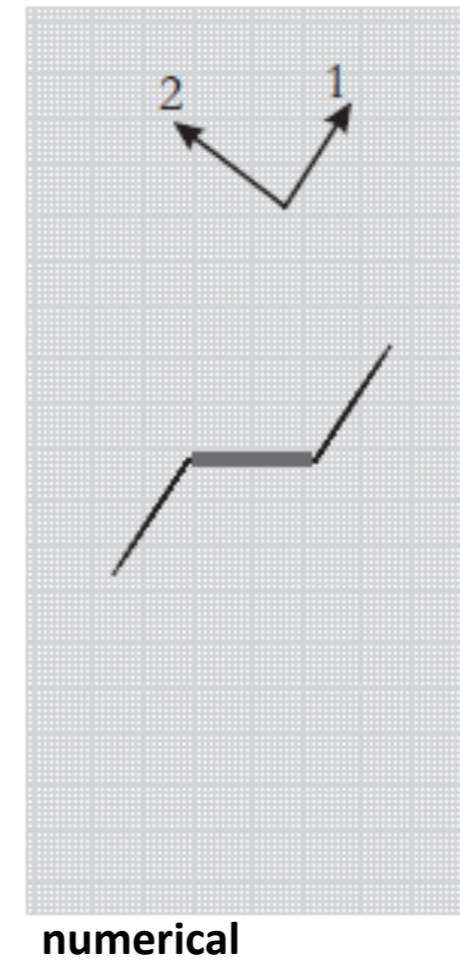
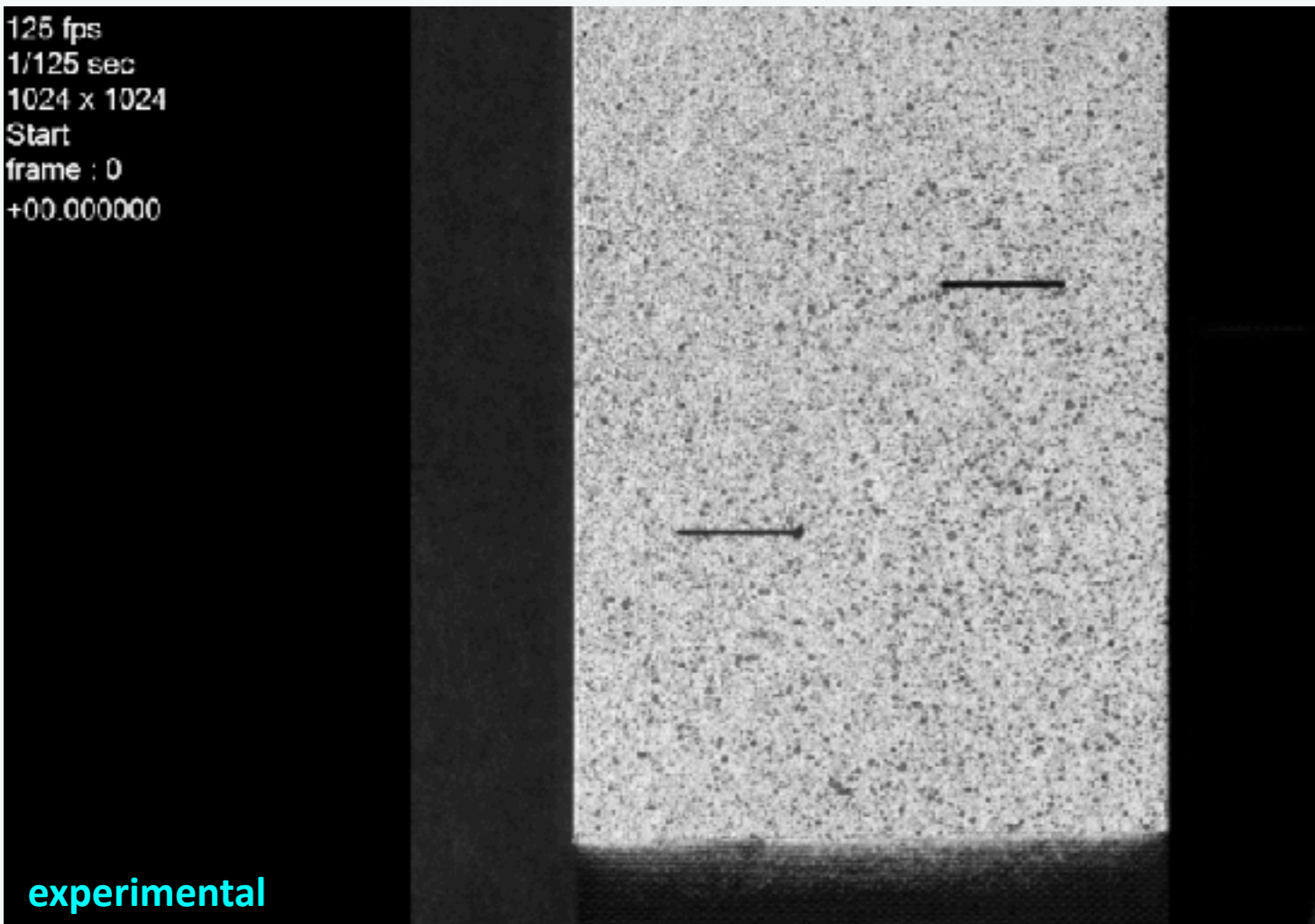


With Danas Sutula and Nguyen Vinh Phu (Monash)  
9TH Australasian Congress on Applied Mechanics (ACAM9)  
27 - 29 November 2017  
[phu.nguyen@monash.edu](mailto:phu.nguyen@monash.edu)

## Partial conclusions on fracture of homogeneous materials using enriched FEM

- ◆ Adaptivity for enriched approximations using error estimates
  - ◆ Adapt enrichment radius
  - ◆ Adapt the choice of enrichment
  - ◆ Locally h-adapt the mesh
- ◆ More than a few cracks in 3D may warrant using phase fields models as opposed to discrete cracks
- ◆ Meshfree methods are possible alternatives (See the work of Rabczuk, Belytschko, Zi, SPAB)
- ◆ Next step: heterogeneities

**Question: what main factors govern crack growth in composite laminates?**



L. Cahill et al. Composite Structures, 2014

*Experimental/Numerical approach to determining the driving force for fracture in composites*

## Solder joint durability (microelectronics), Bosch GmbH

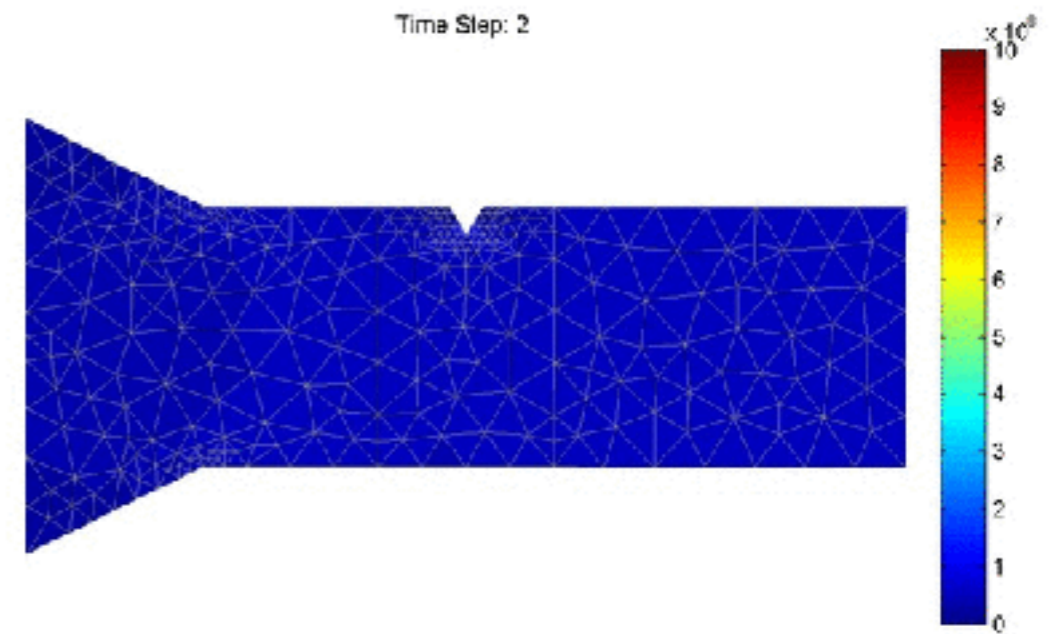
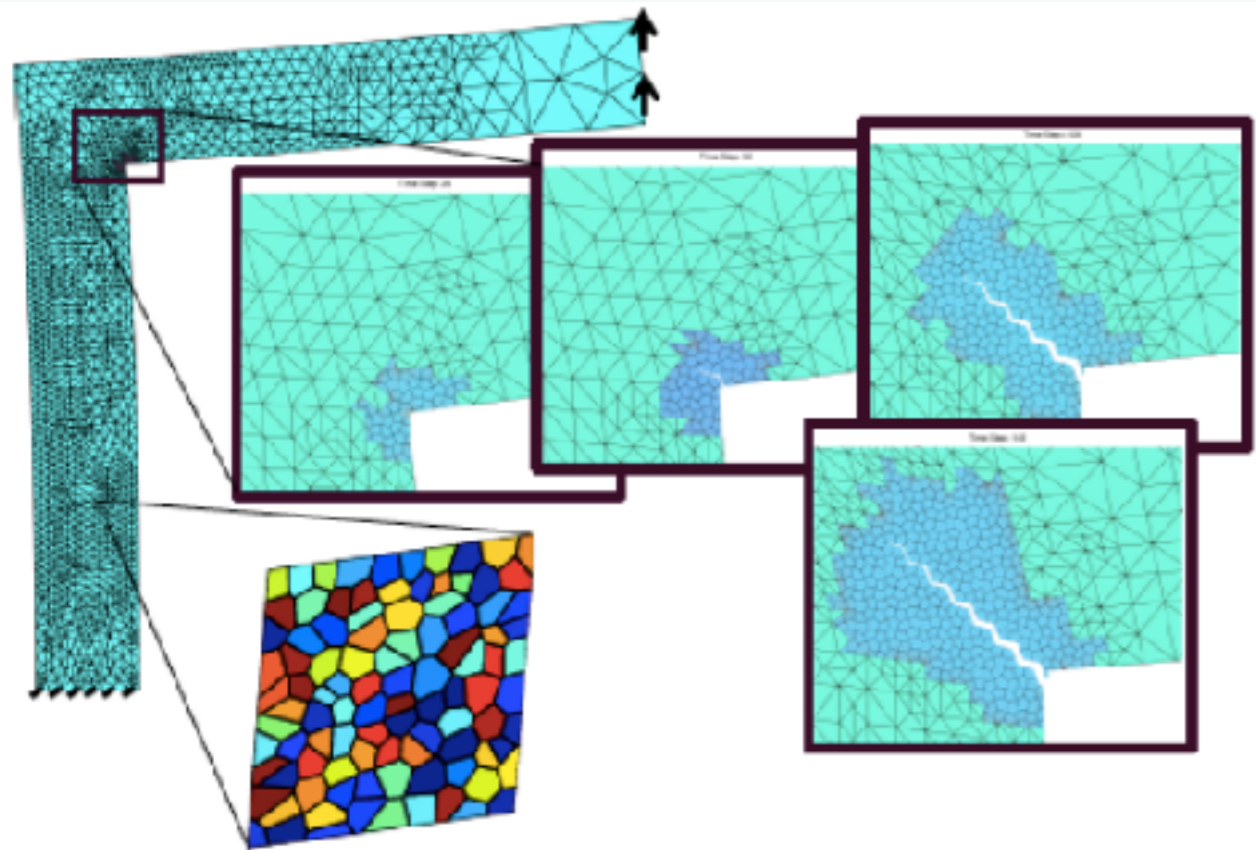


**Question: what is the role of Pb in thermo-mechanical reliability of solder joints?**

- A. Menk and SPAB, IJNME 2011, Comp. Mat. Sci. 2012  
XFEM Preconditioning and application to polycrystalline fracture
- D. A. Paladim et al. Int. J. Numer. Meth. Engng 2017; 110:103–132
- P. Kerfriden et al. Int. J. Numer. Meth. Engng 2014; 97:395–422
- P. Kerfriden et al. Int. J. Numer. Meth. Engng 2012; 89:154–179
- P. Kerfriden et al. Comput. Methods Appl. Mech. Engrg. 200 (2011) 850–866
- K. C. Hoang et al. Num Meth PDEs DOI 10.1002/num.21932



# Fracture over the scales, adaptivity model reduction and selection



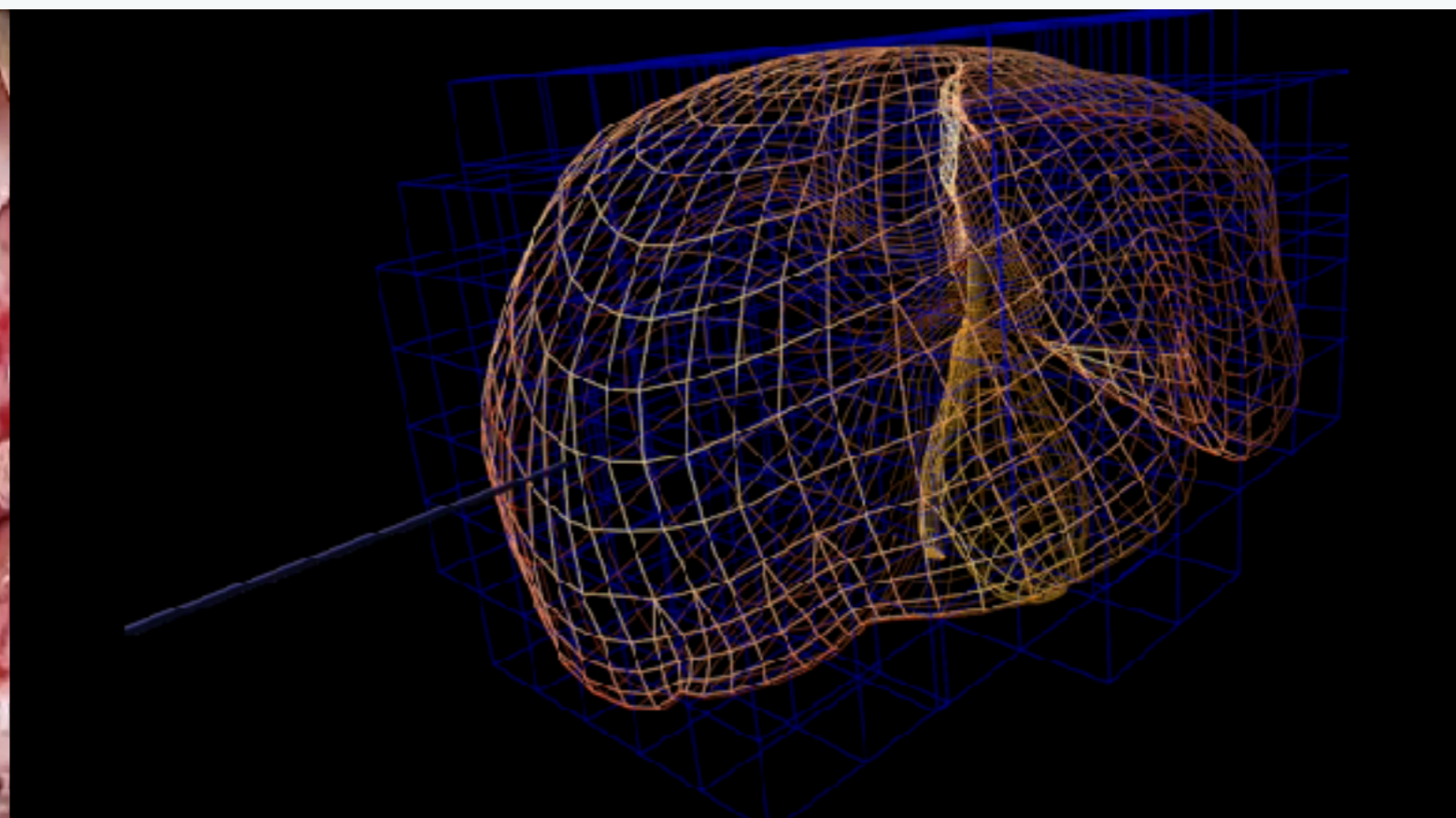
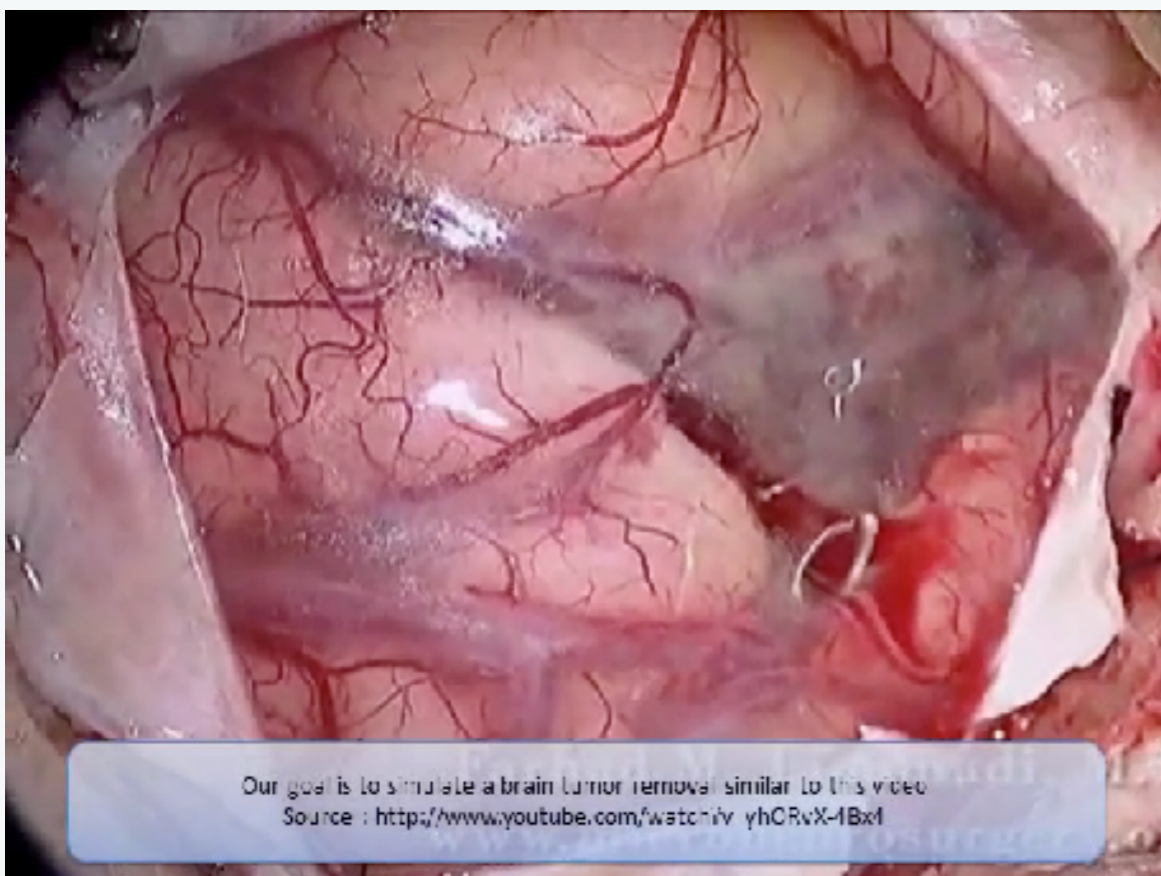
**Question: how can we account for microstructures in a computationally tractable way?**

- O. Goury, P. Kerfriden et al. CMAME, 2016, CMECH (2017) DOI 10.1007/s00466-016-1290-2 - Model reduction for fracture
- C. Hoang et al. Comput. Methods Appl. Mech. Engrg. 298 (2016) 121–158 - Model reduction for elastodynamics
- A. Akbari, P. Kerfriden and SPAB, Philosophical Magazine, (2015) <http://dx.doi.org/10.1080/14786435.2015.1061716>
- P. Kerfriden et al. Comput. Methods Appl. Mech. Engrg. 256 (2013) 169–188 - Model reduction methods for fracture

## Partial conclusions on fracture of heterogeneous materials

- ◆ Simple methods can deal with fracture in unidirectional composites
- ◆ Model + mesh adaptivity for adaptive fracture mechanics simulations: expensive + implementation must be done carefully
- ◆ Model order reduction is ineffective for problems lacking separation of scales
  - ◆ Domain-wise model selection
  - ◆ Adaptive model selection
  - ◆ Machine learning...
- ◆ Next step: biomechanics/real-time

## Cutting and Needle Insertion



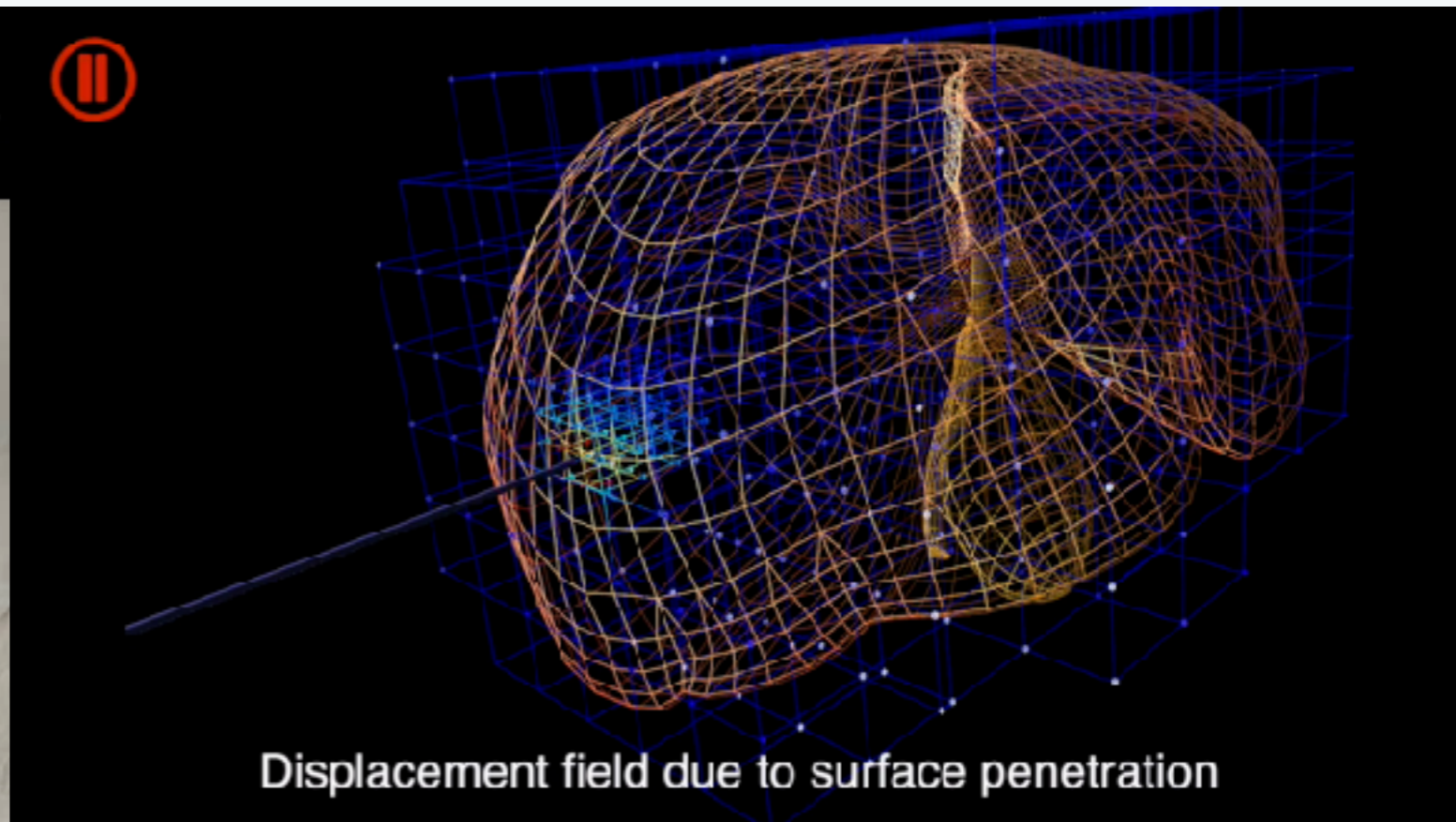
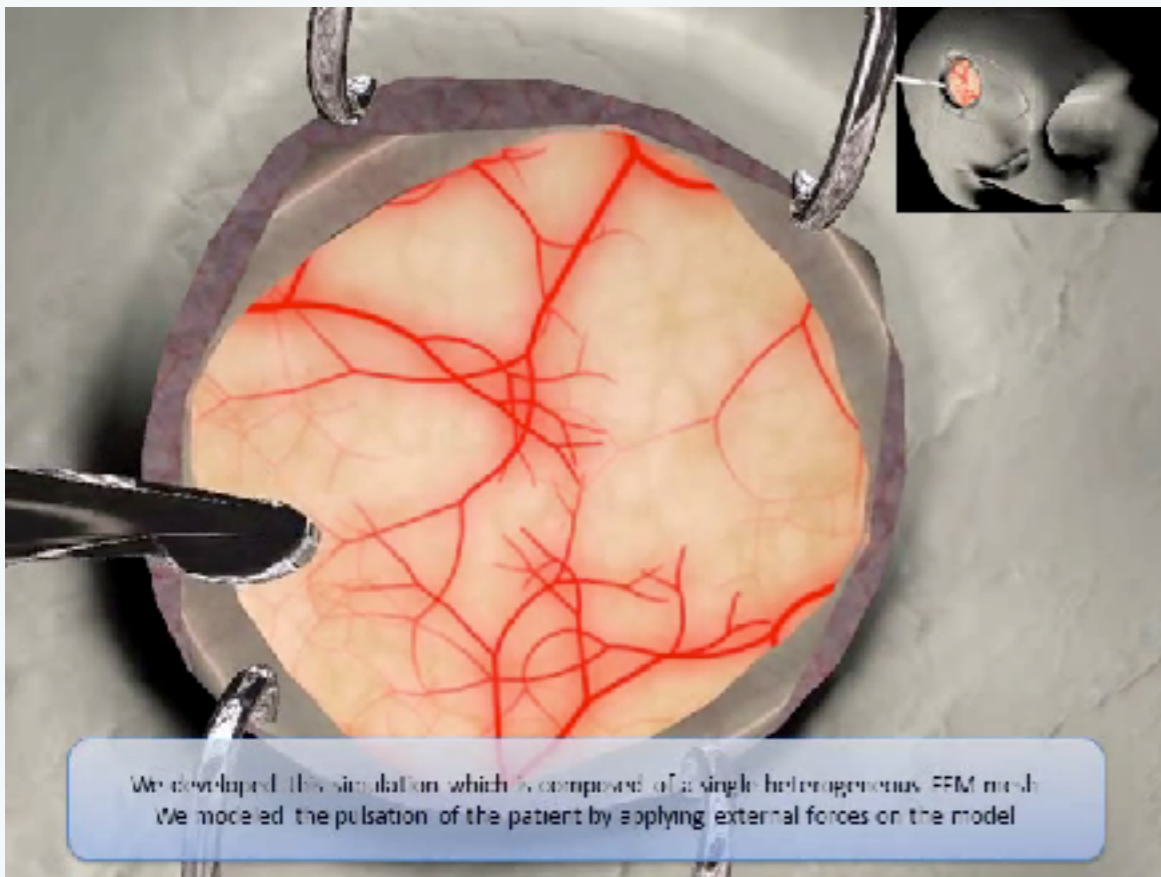
H. Courtecuisse et al. Medical Image Analysis, 2014

P.H. Bui et al. IEEE T. Biomed Eng. 2017 & Frontiers in Surgery, 2017

<http://orbidu.uni.lu/handle/10993/30937>

<http://orbidu.uni.lu/handle/10993/29846>

## Cutting and Needle Insertion



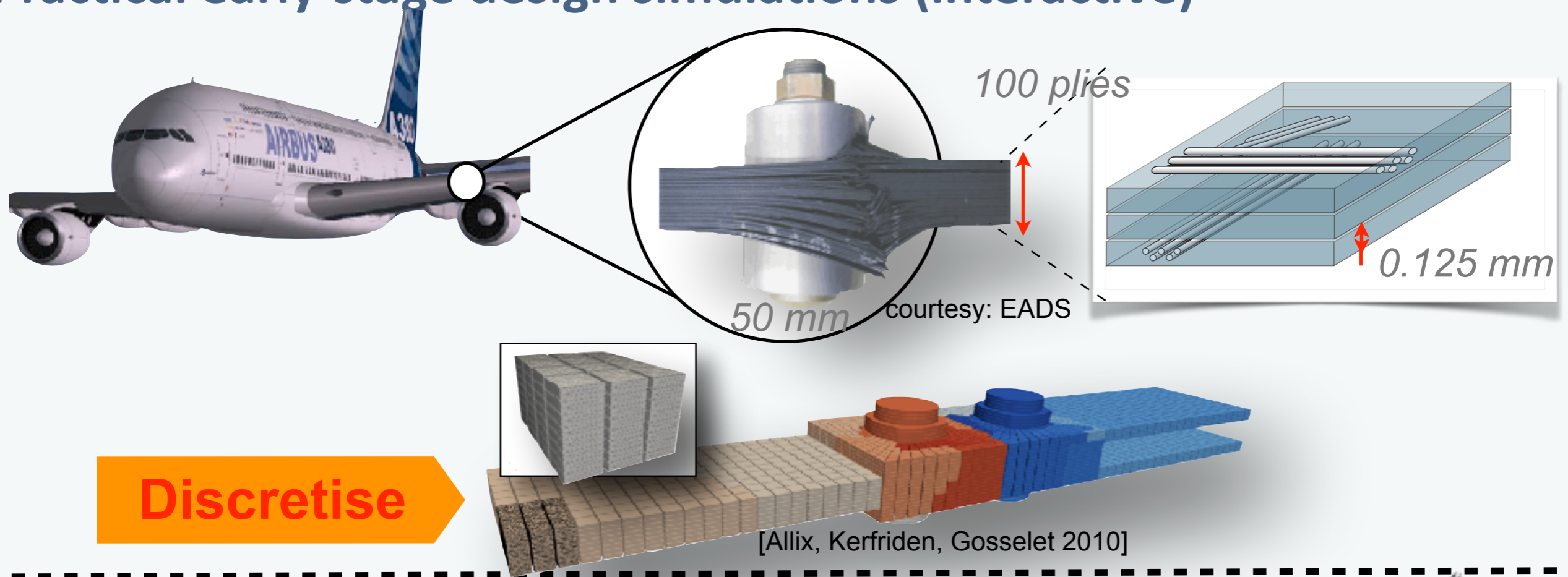
H. Courtecuisse et al. Medical Image Analysis, 2014  
**Question: how can we simulate cutting/fracture in real time using implicit time stepping?**

P.H. Bui et al. IEEE T. Biomed Eng. 2017 & Frontiers in Surgery, 2017  
**Question: how can we adapt the mesh in real time using a posteriori error estimates?**

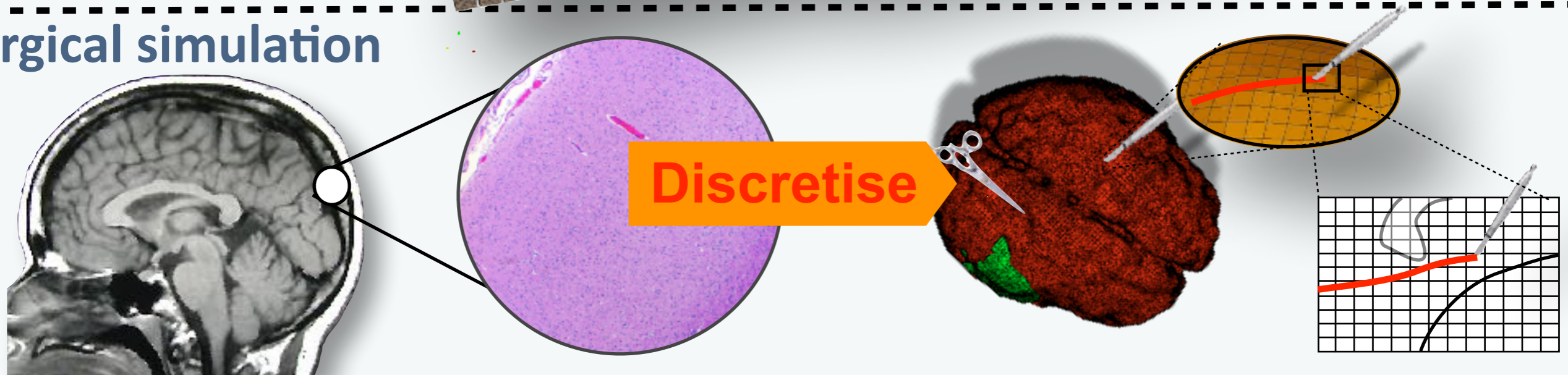
<http://orbilu.uni.lu/handle/10993/30937> <http://orbilu.uni.lu/handle/10993/29846>

# Interfaces in engineering and biomechanics

## Practical early-stage design simulations (interactive)

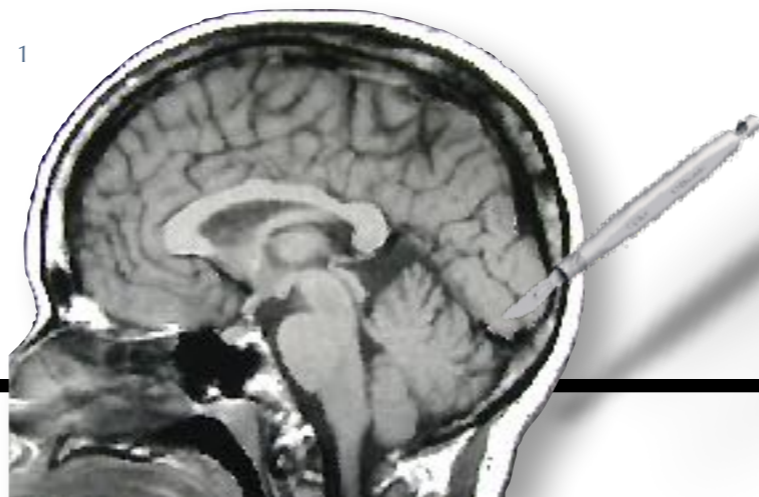


## Surgical simulation



- ▶ Reduce the problem size while controlling the error (in QoI) when solving very large (multiscale) mechanics problems

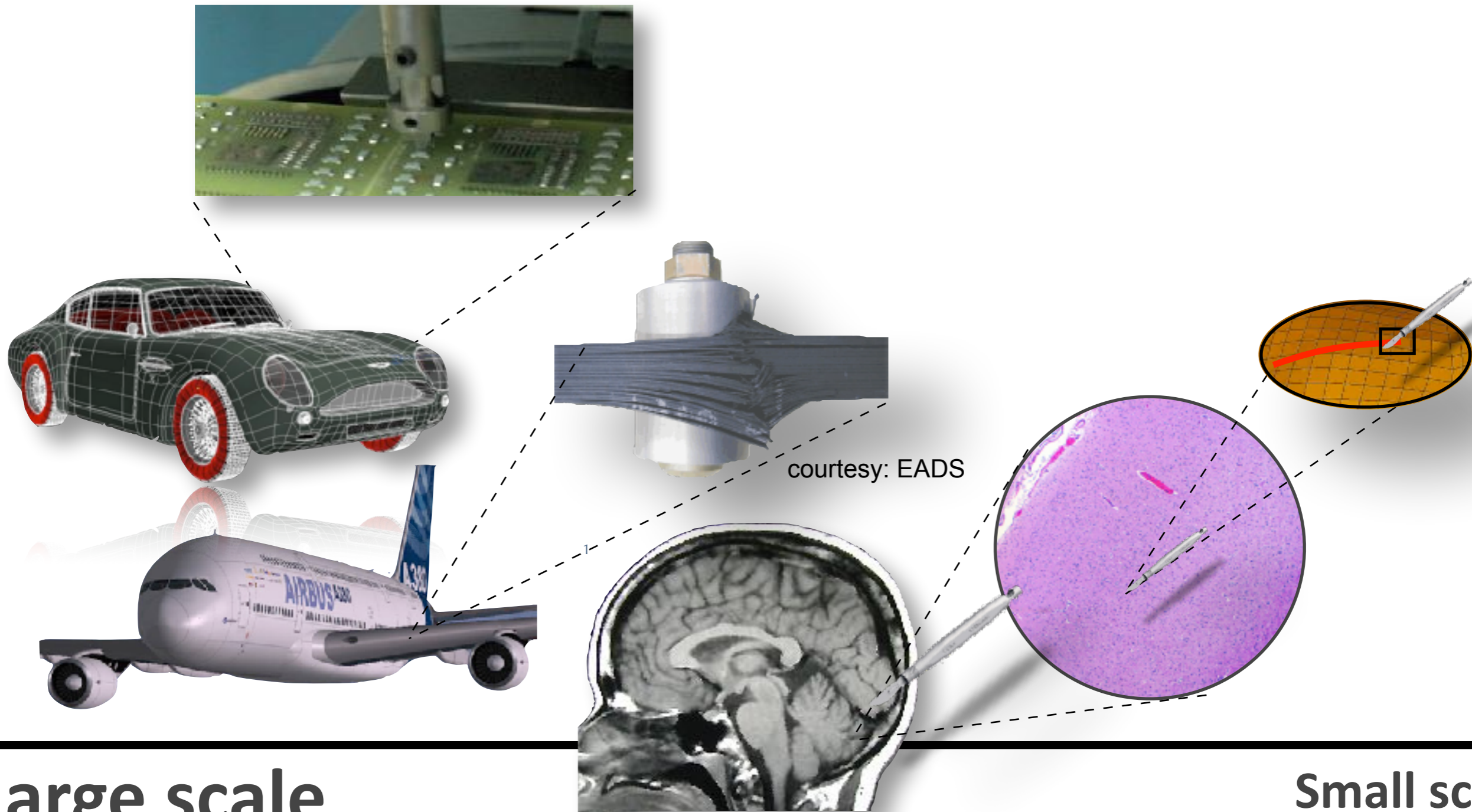
# Discontinuities



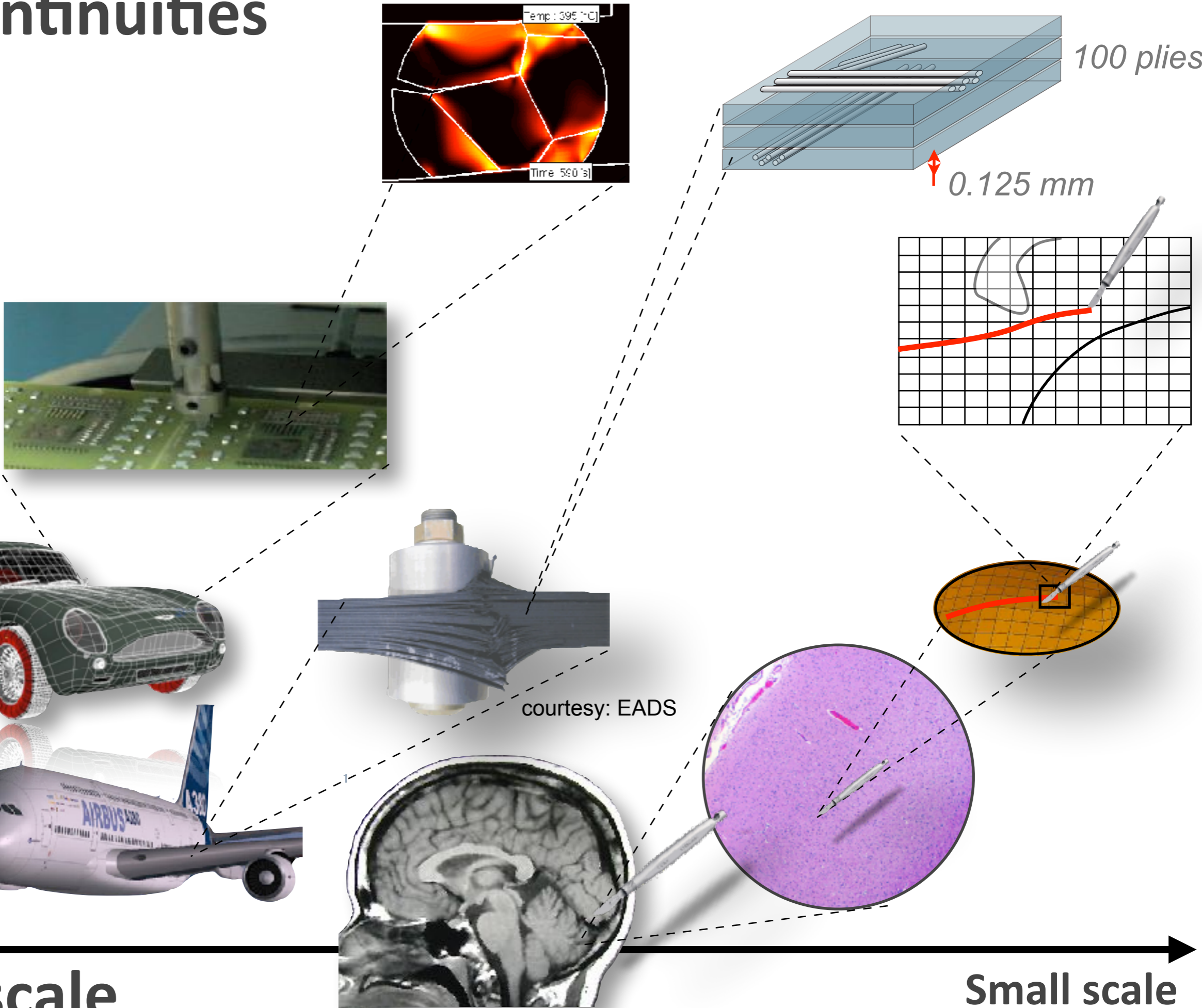
Large scale

Small scale

# Discontinuities



# Discontinuities



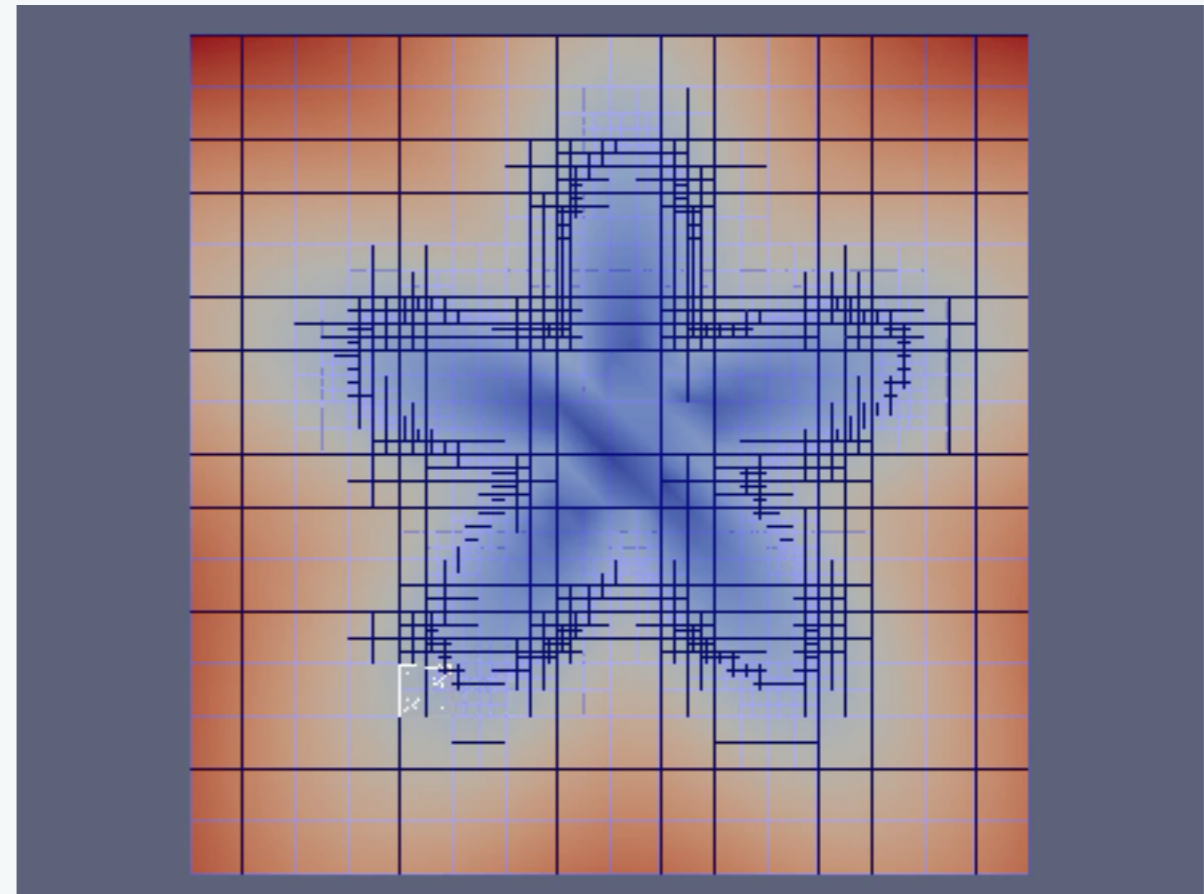
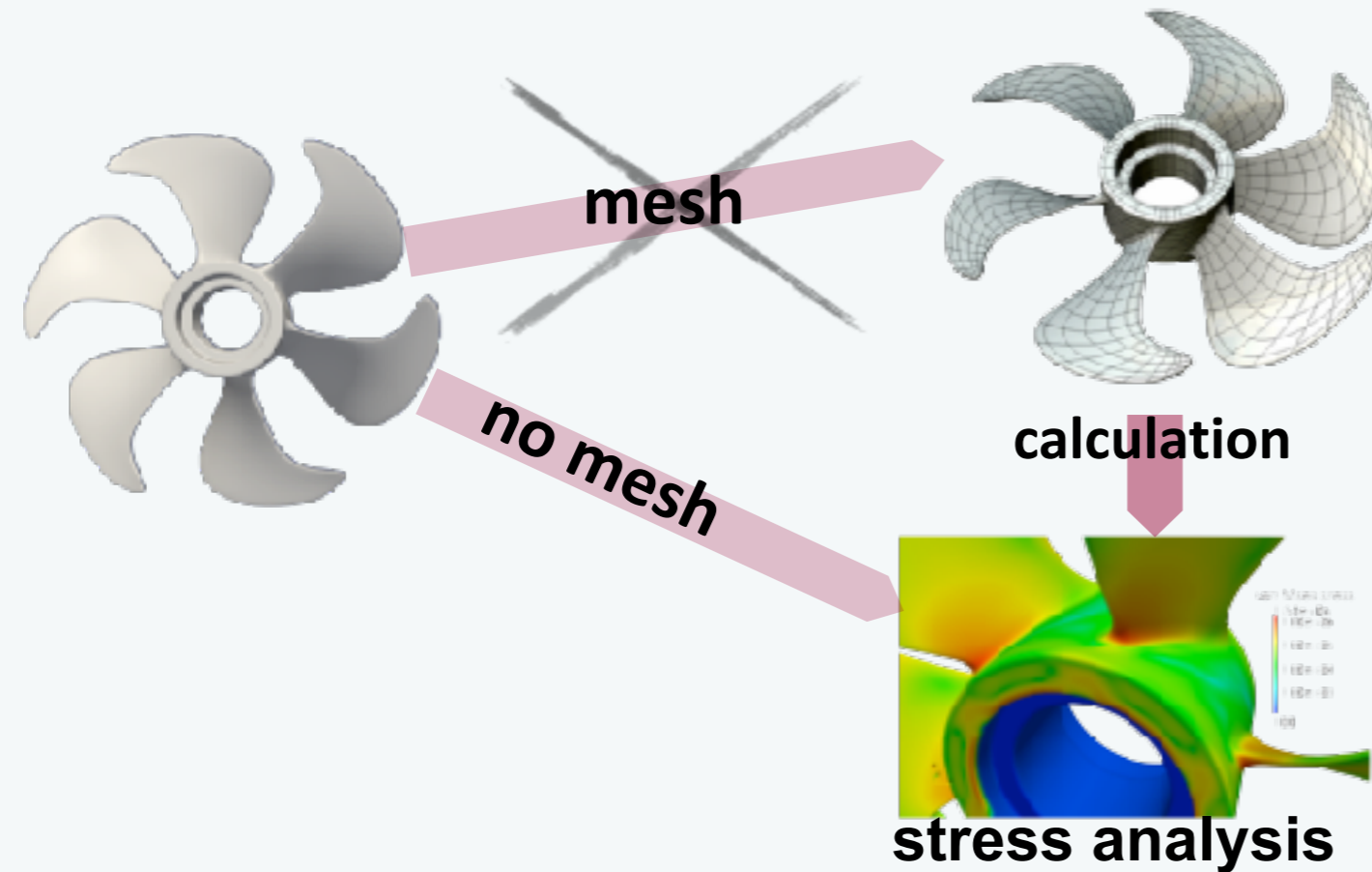


## Partial conclusions

- Zooming into materials/structures reveals discontinuities with complex shapes
- Geometries of domains are complex, even at the continuum level

**Next: a few methods to deal with this complexity**

## *Coupling, or decoupling?*



**Question: When are we better off coupling/decoupling the geometry from the field approximation?**

# Handling (complex) interfaces numerically

## *Coupling geo and field: Isogeometric Analysis*

### Question:

***What is the performance of Isogeometric Analysis in Reducing the Mesh Burden?***

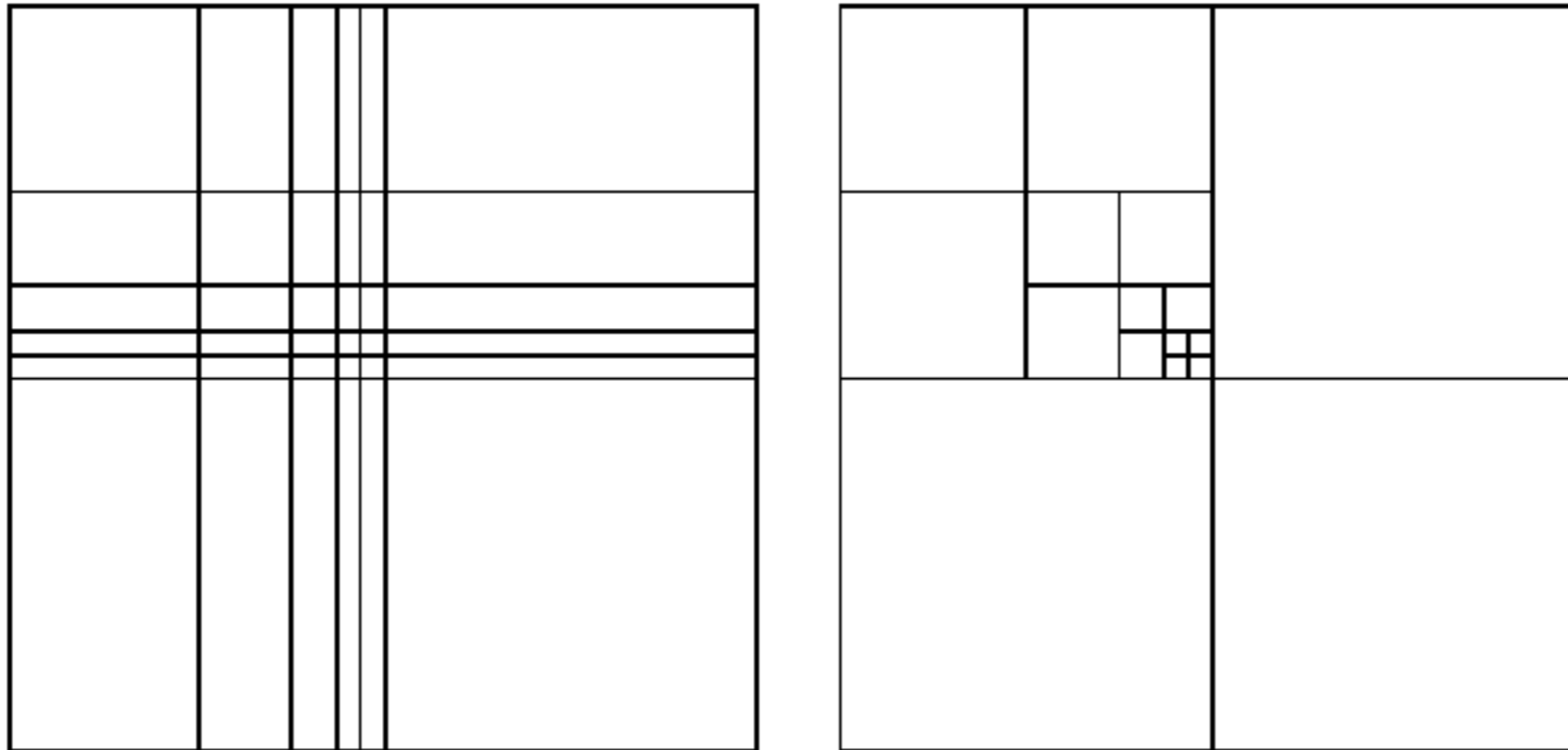
## Isogeometric Finite Element Analysis

- For shell-like domains
- For volumes (needs volume parameterisation)
- Coupling between multiple patches (Nitsche, Mortar...)

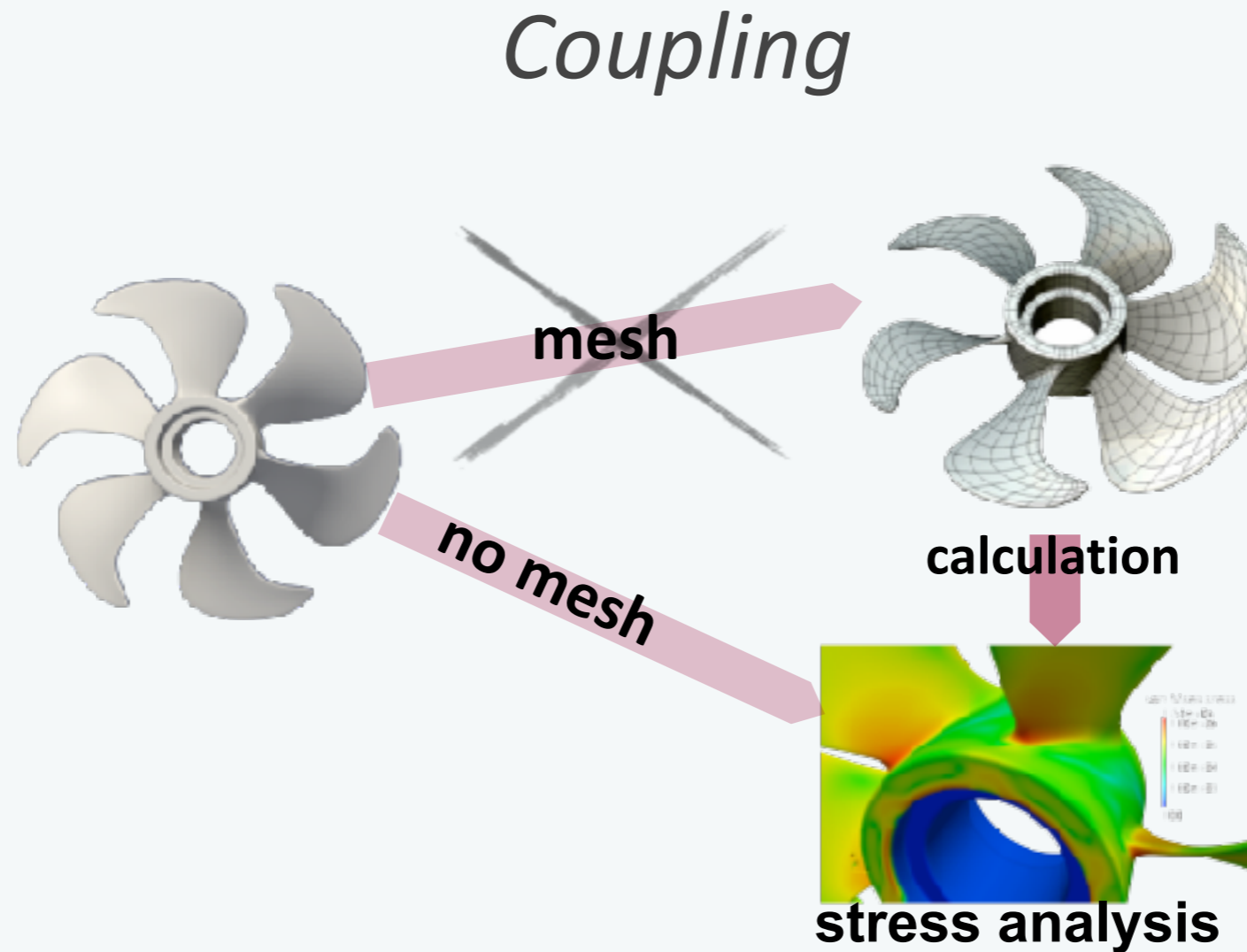
## Adaptivity

- Global refinement - cannot refine field without refining geo...
- Local refinement (not with NURBS)... (PH)T-splines...
- Geometry independent refinement for the field variables?

# Mesh refinement in IGA



Global refinement (tensor-product mesh) vs local refinement (T-mesh)



**Question: How can we fully benefit from the “IGA” concept?**

- Refine the field independently from the geometry
- Suppress the mesh generation and regeneration completely

# Handling (complex) interfaces numerically

## *Coupling geometry and field approximation*

**Question: How can we fully benefit from the “IGA” concept?**

Refine the field independently from the geometry

### Isogeometric Finite Elements

For shell-like domains

For volumes (needs volume parameterisation)

### Geometry Independent Field approximation (GIFT)

Super/Sub-geometric

[REF] Weakening the tight coupling between geometry and simulation in isogeometric analysis: from sub- and super-geometric analysis to Geometry Independent Field approximation (GIFT), IJNME, 2017, submitted [preprint available on arXiv]

**Permalink:** <http://hdl.handle.net/10993/31469>

# Geometry Independent Field approximation (GIFT)

## *Conclusions*

- ☑ Tight link between CAD and analysis
- ☑ The same basis functions, which are used in CAD to represent the geometry, are used in the IGA as shape functions to approximate the unknown solution
- ☑ Geometry is exact at any stage of the solution refinement process
- ☑ Better accuracy per DOF in comparison with standard FEM but higher computational cost (bandwidth...)

# Geometry Independent Field approximation (GIFT) Conclusions

- ✓ Retain the advantages of IGA but decouple the geometry and the field approximation
- ✓ Standard patch tests may not always pass, yet the convergence rates are optimal as long as the geometry is exactly represented by the geometry basis
- ✓ With geometry exactly represented by NURBS, using same degree B-splines or NURBS for the approximation of the solution field yields almost identical results
- ✓ With geometry exactly represented by NURBS, using PHT splines for the approximation of the solution gives additional advantage of local adaptive refinement
- ✓ Any other approximation field can be used for the field variables



## *Coupling*

**Question: How can we fully benefit from the “IGA” concept?**

Suppress the mesh generation and regeneration completely

### Isogeometric Finite Elements

- For shell-like domains
- For volumes (needs volume parameterisation)

#### **Stress analysis and shape optimisation directly from CAD**

- H. Lian et al. (2017). *CMAME*: 317 (2017): 1-41.
- H. Lian et al. (2015). *IJNME*
- H. Lian et al. (2013). *EACM*:166(2):88-99.
- M. Scott et al. (2013) *CMAME* 254: 197-221.
- R. N. Simpson et al. (2013) *CAS* 118: 2-12.
- R. N. Simpson et al. (2012) *CMAME* Feb 1;209:87-100.

### Isogeometric Boundary Element Analysis

- For shell-like domains
- For volumes

#### **Fracture mechanics directly from CAD**

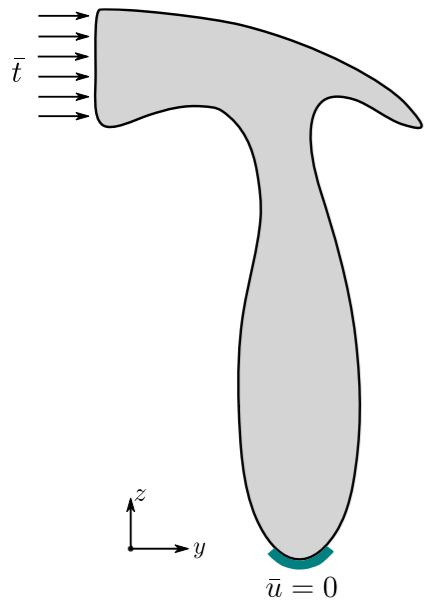
- X. Peng, et al. (2017). *IJF*, 204(1), 55–78.
- X. Peng, et al. (2017). *CMAME*, 316, 151–185.

# Handling (complex) interfaces numerically

*Example applications*

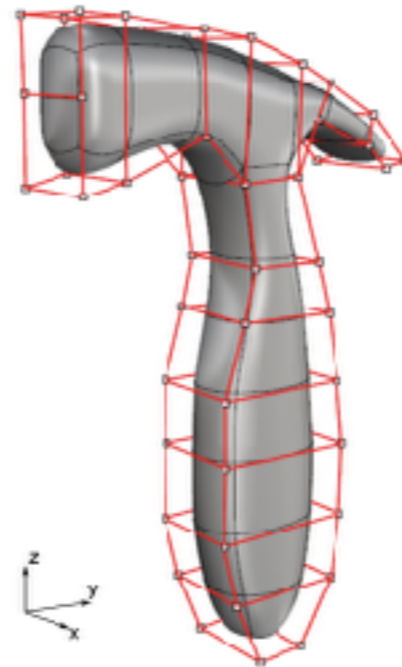
*Isogeometric Boundary Element Analysis  
(IGABEM)*

# Shape optimisation



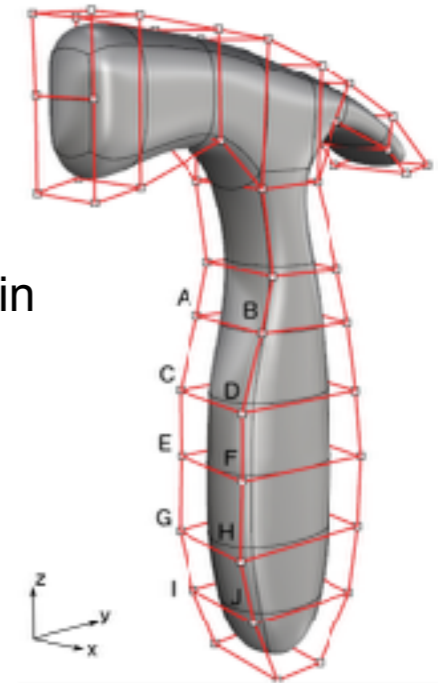
Problem definition

Model construction with CAD



Control points

Design points selection in control points



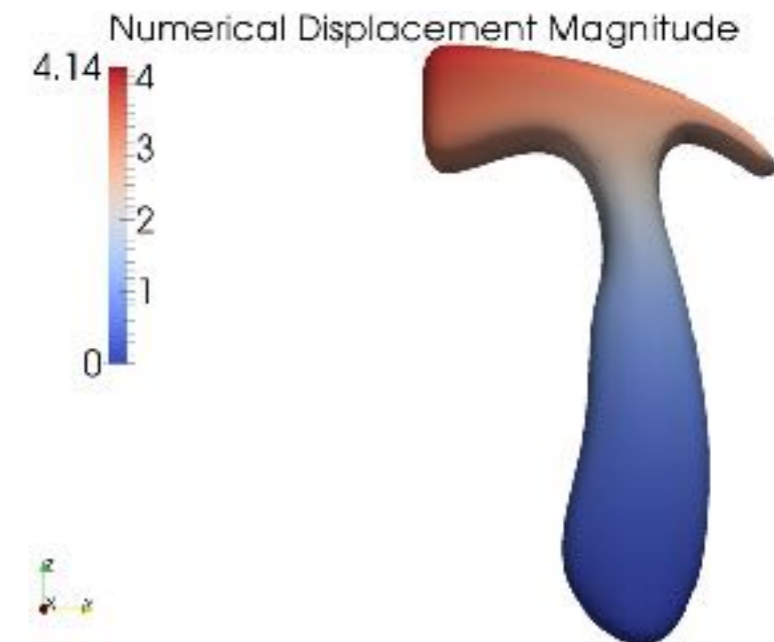
Design points

Objective function:  $\int_S t_i u_i dS$

Volume constraint:  $V - V_0 \leq 1$

Side constraints:

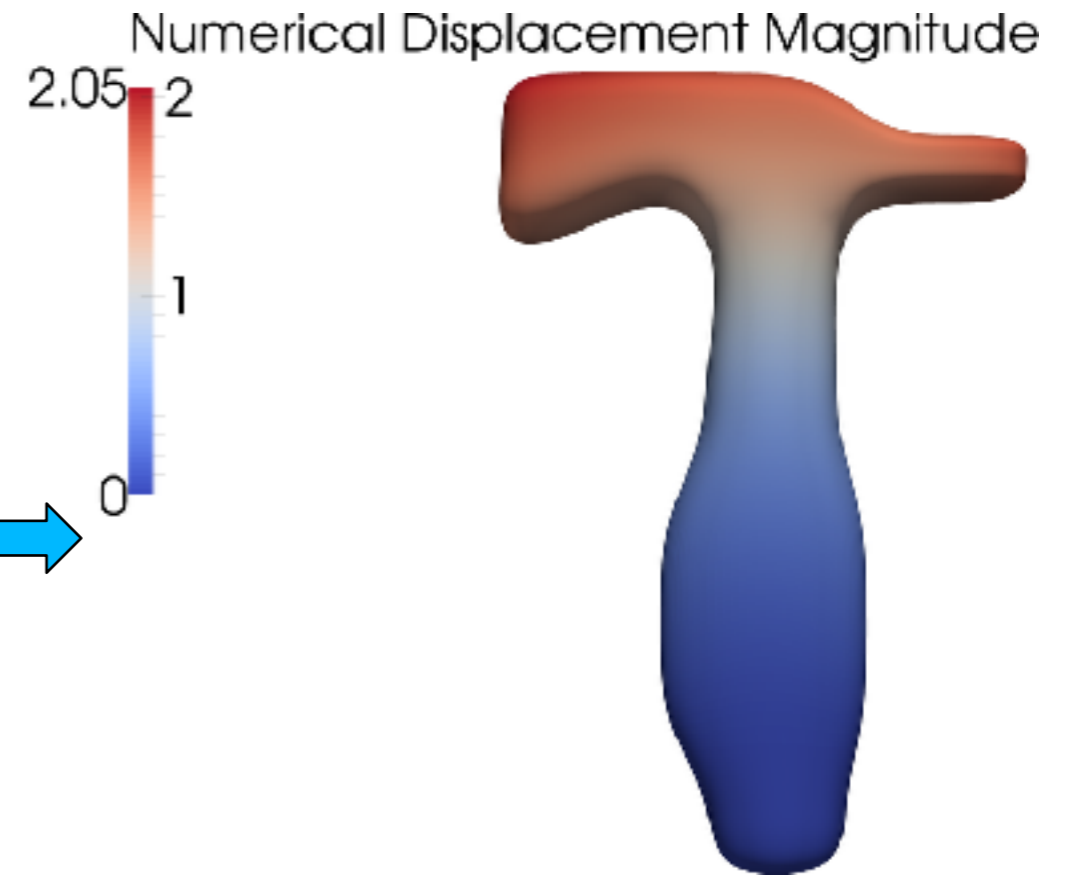
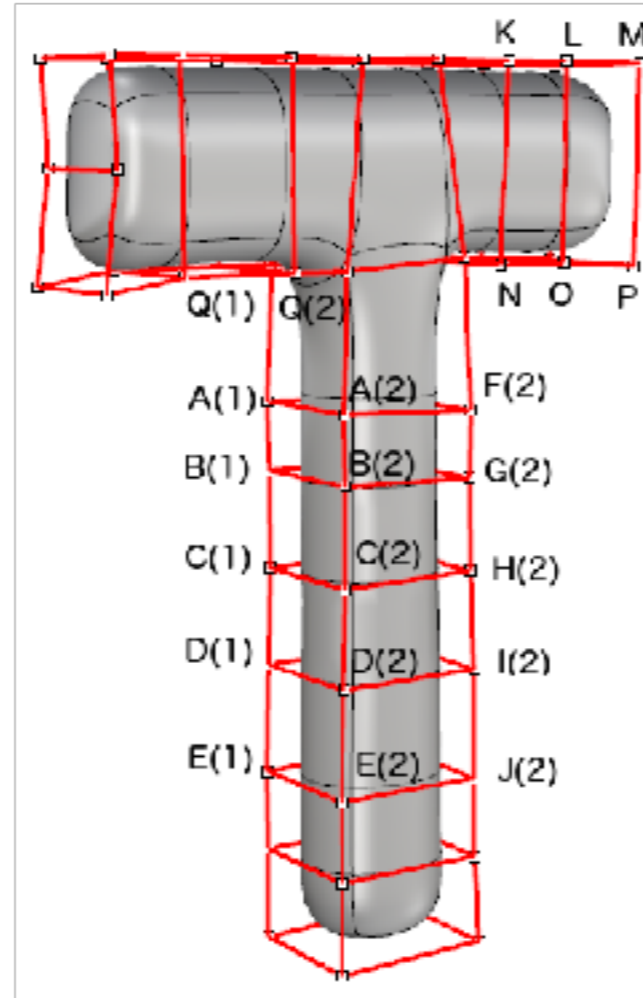
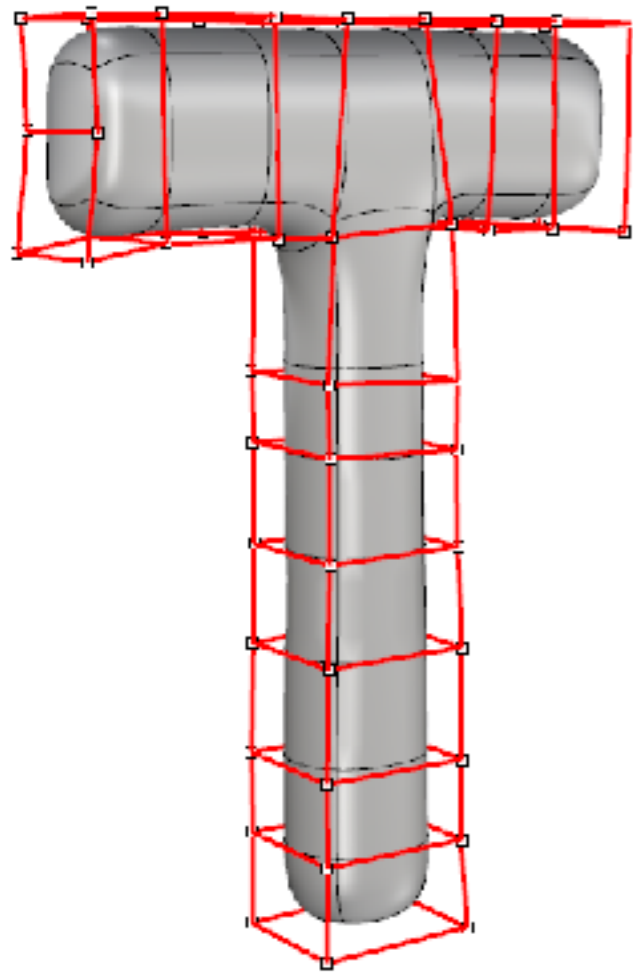
Structural analysis;  
Sensitivity analysis;  
gradient-based optimizer



Optimized solution

Design variable	Lower bound	Upper bound	Initial value
$t_1$	0	4	2.45
$t_2$	0	4	1.25
$t_3$	0	4	1.33
$t_4$	0	4	1.28
$t_5$	0	4	2.30

# Shape optimisation



Construct the geometric model

Choose design points from the control points

Conduct sensitivity analysis to converge to the optimized solution

## Stress analysis and shape optimisation directly from CAD

H. Lian et al. (2017). CMAME: 317 (2017): 1-41.

H. Lian et al. (2015). IJNME

H. Lian et al. (2013). EACM:166(2):88-99.

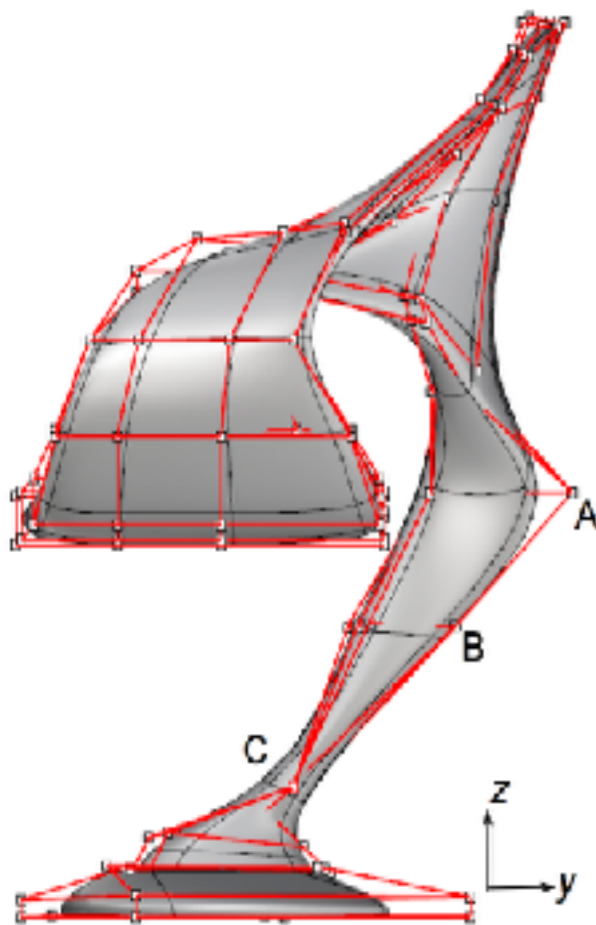
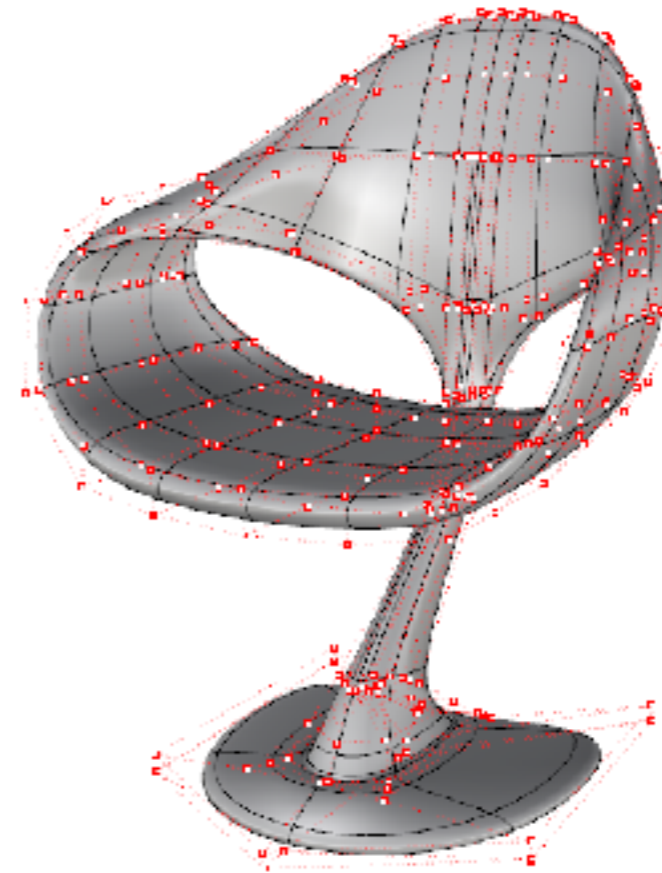
M. Scott et al. (2013) CMAME 254: 197-221.

R. N. Simpson et al. (2013) CAS 118: 2-12.

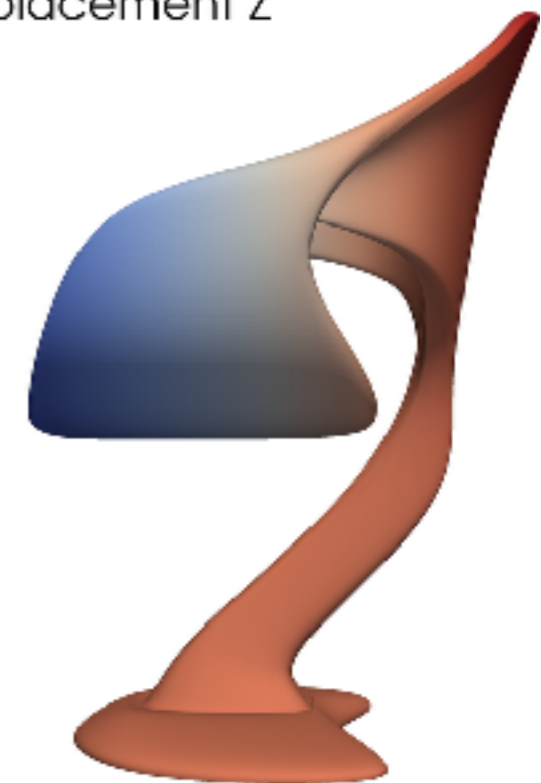
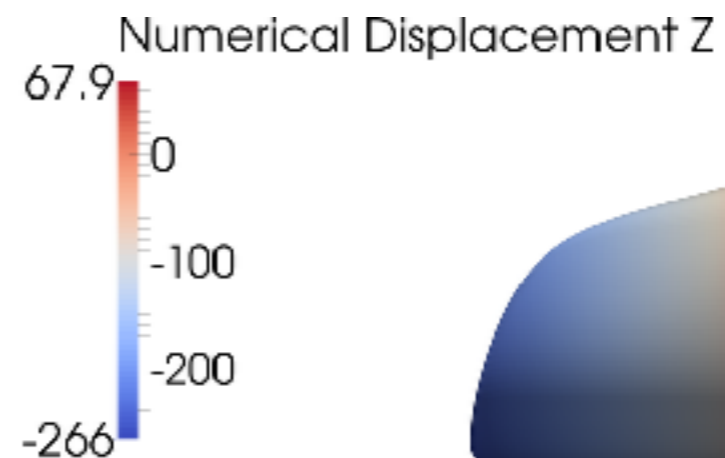
R. N. Simpson et al. (2012) CMAME Feb 1;209:87-100.

# Shape optimisation

Construct the geometric  
model  
(imported from Rhino)

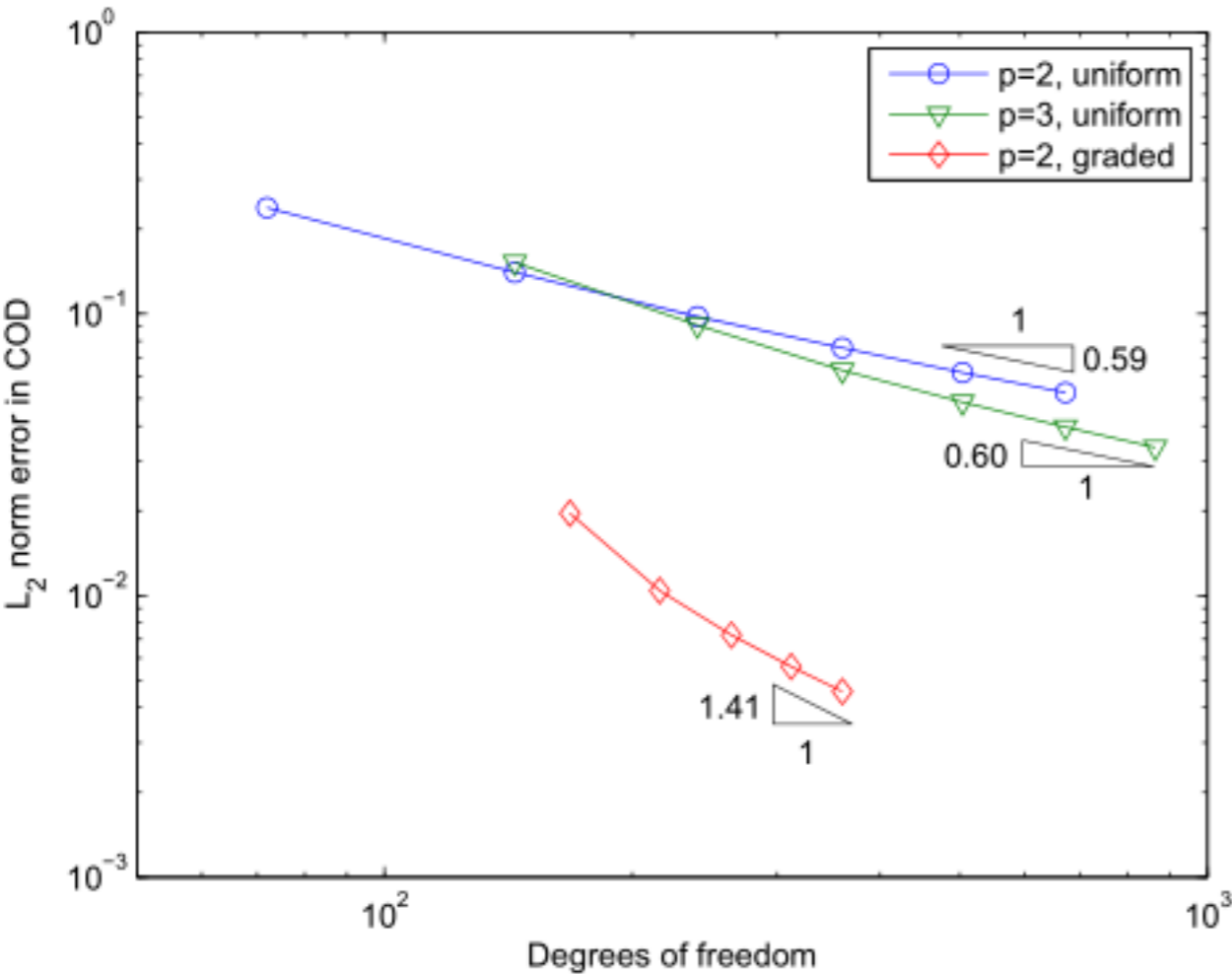


Select design points from  
control points

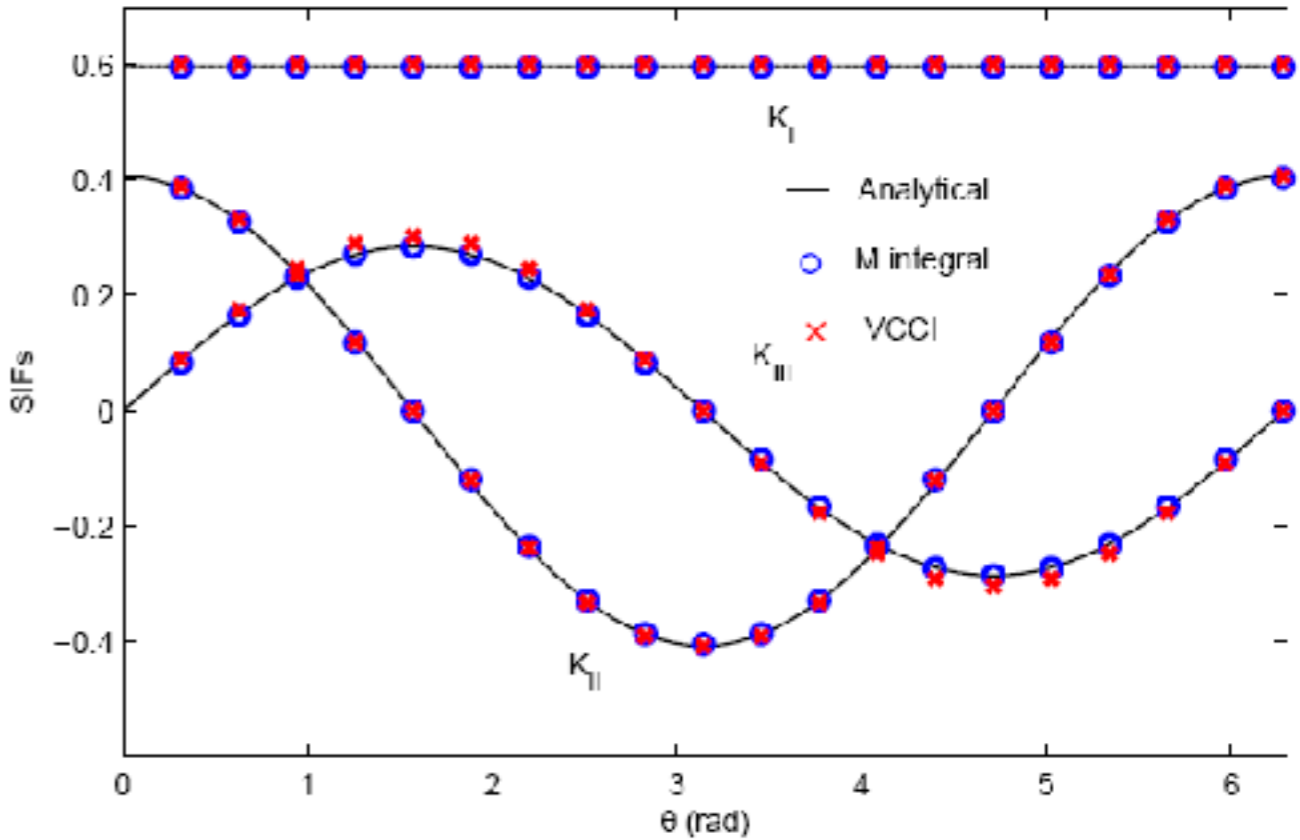


Find optimized solution

# Penny-shaped crack under remote tension



$L_2$  norm error of COD for penny-shaped crack

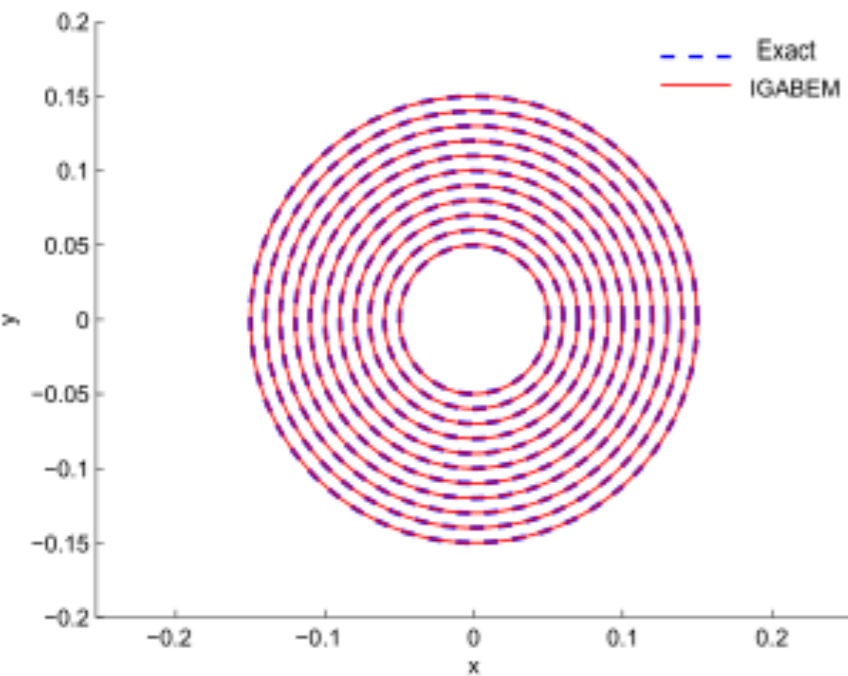


stress intensity factors for penny crack with  $\varphi = \pi/6$

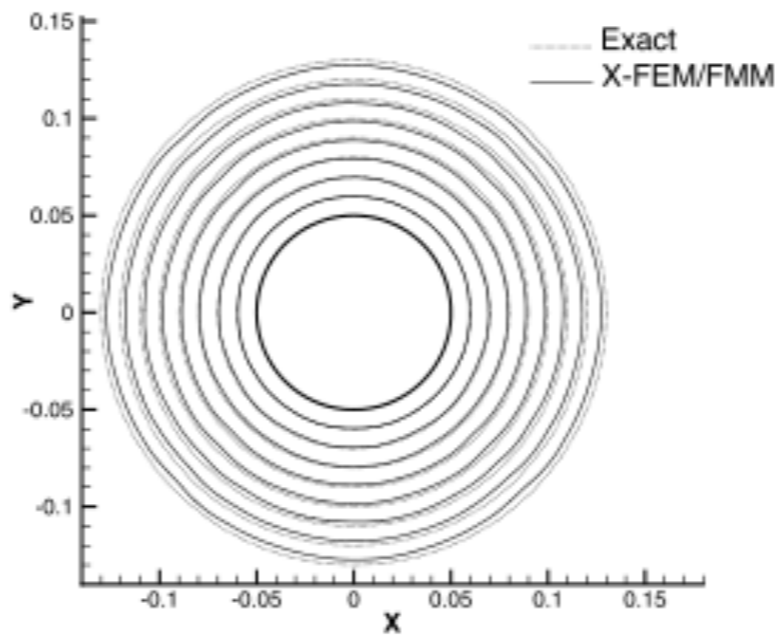
**Fracture mechanics directly from CAD**

X. Peng, et al. (2017). *IJF*, 204(1), 55–78.  
 X. Peng, et al. (2017). *CMAME*, 316, 151–185.

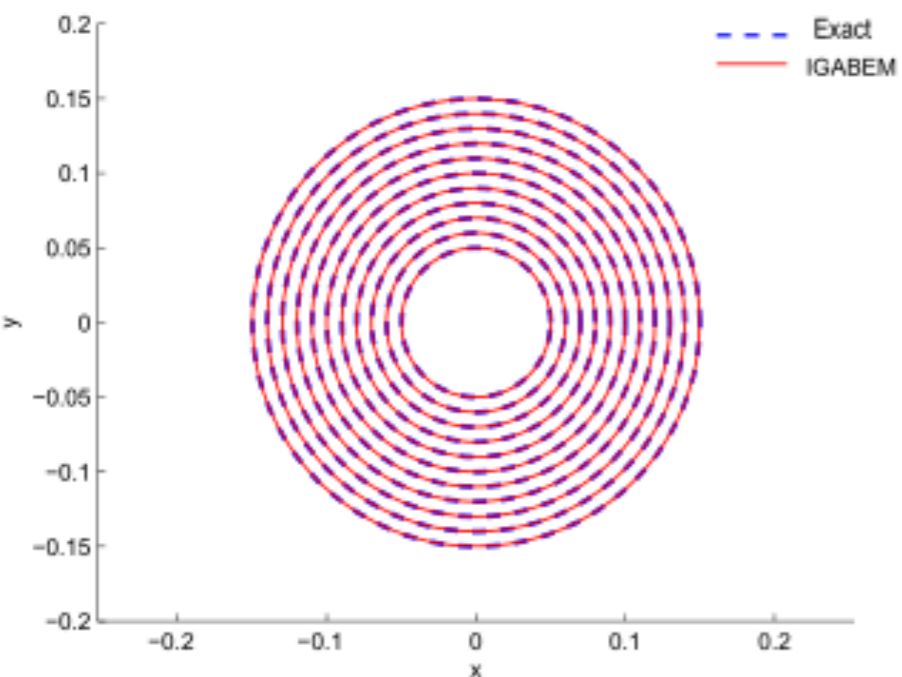
# Numerical example of horizontal penny crack growth (first 10 steps)



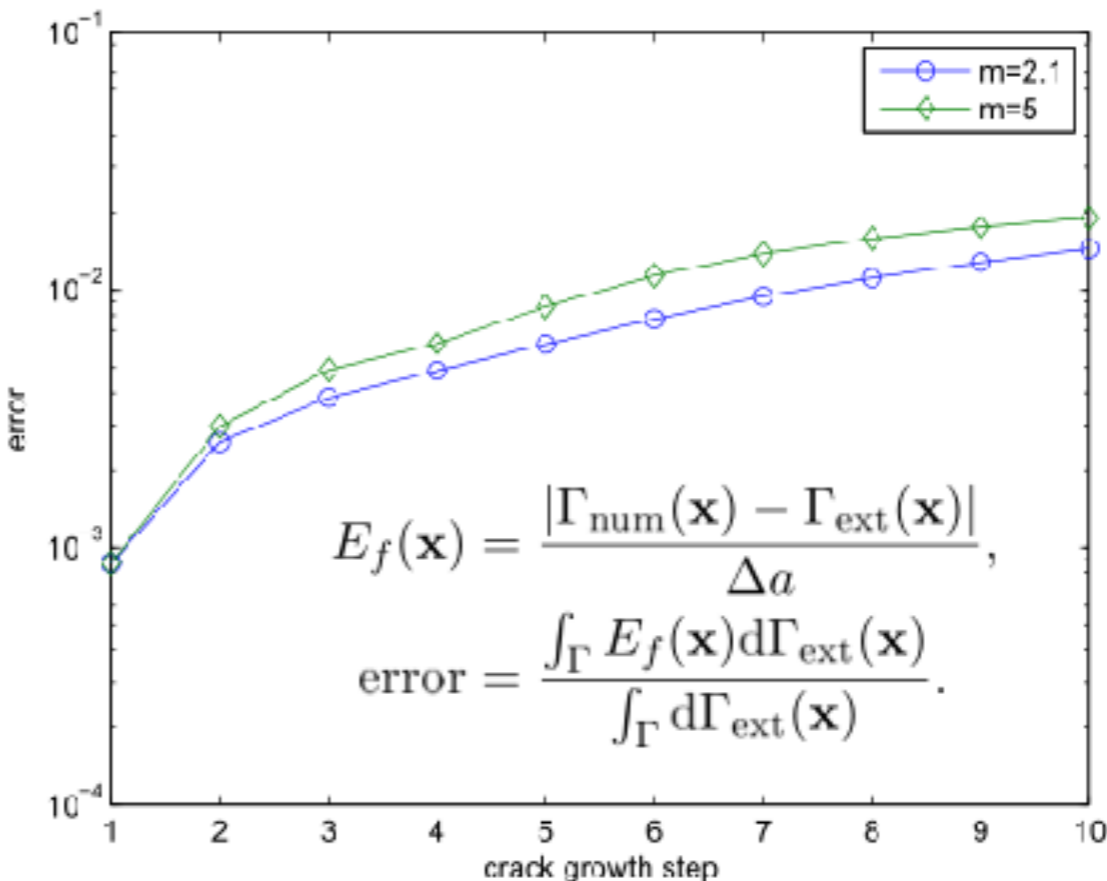
(a) IGABEM,  $m = 2.1$



(b) XFEM/FMM,  $m = 2.1$ , Sukumar *et al* 2003

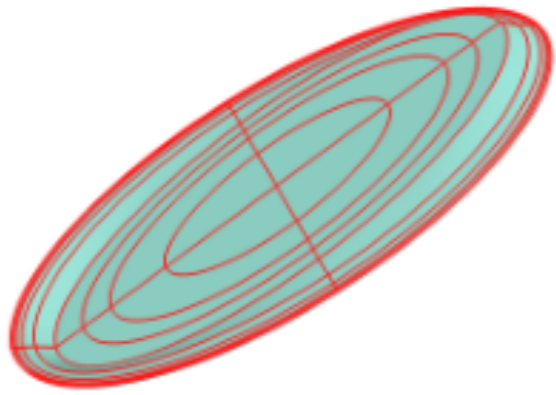


(c) IGABEM,  $m = 5$

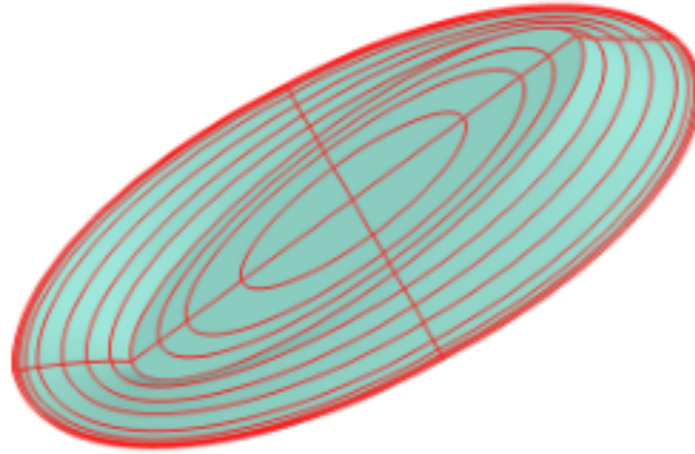


Relative error of the crack front for in each crack growth step by IGABEM

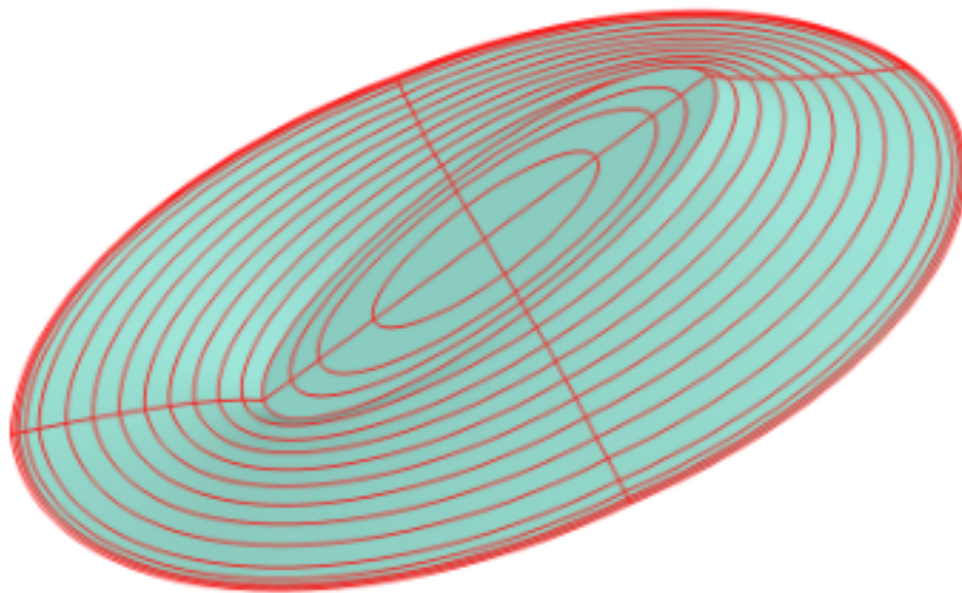
# Numerical example of inclined elliptical crack growth (first 10 steps)



(a) Step 2



(b) Step 5



(c) Step 10

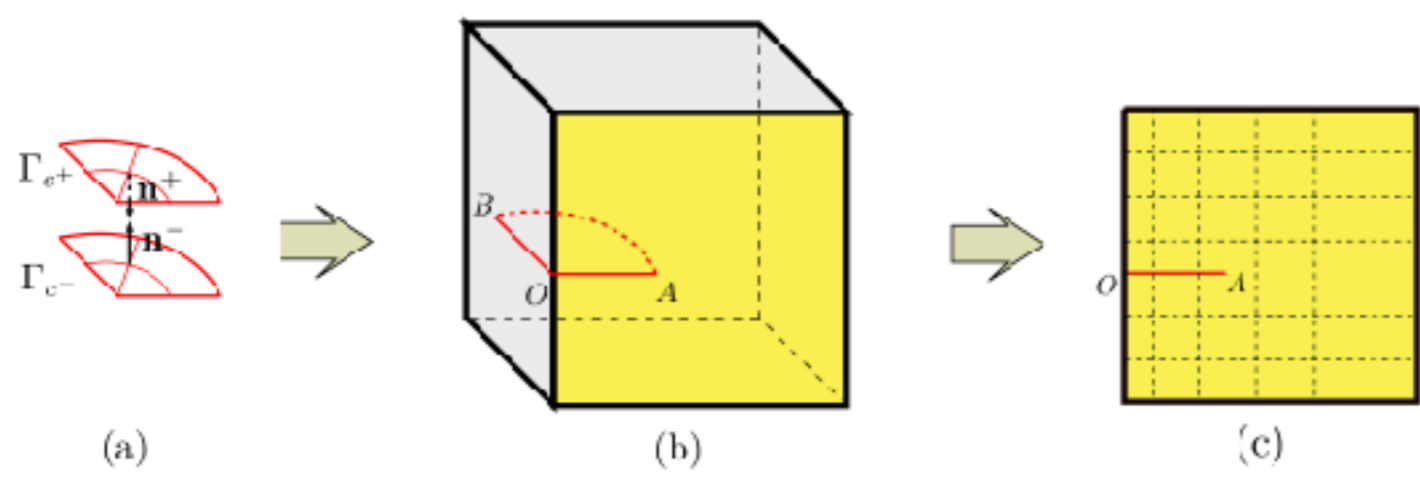
**Fracture mechanics directly from CAD**

X. Peng, et al. (2017). *IJF*, 204(1), 55–78.

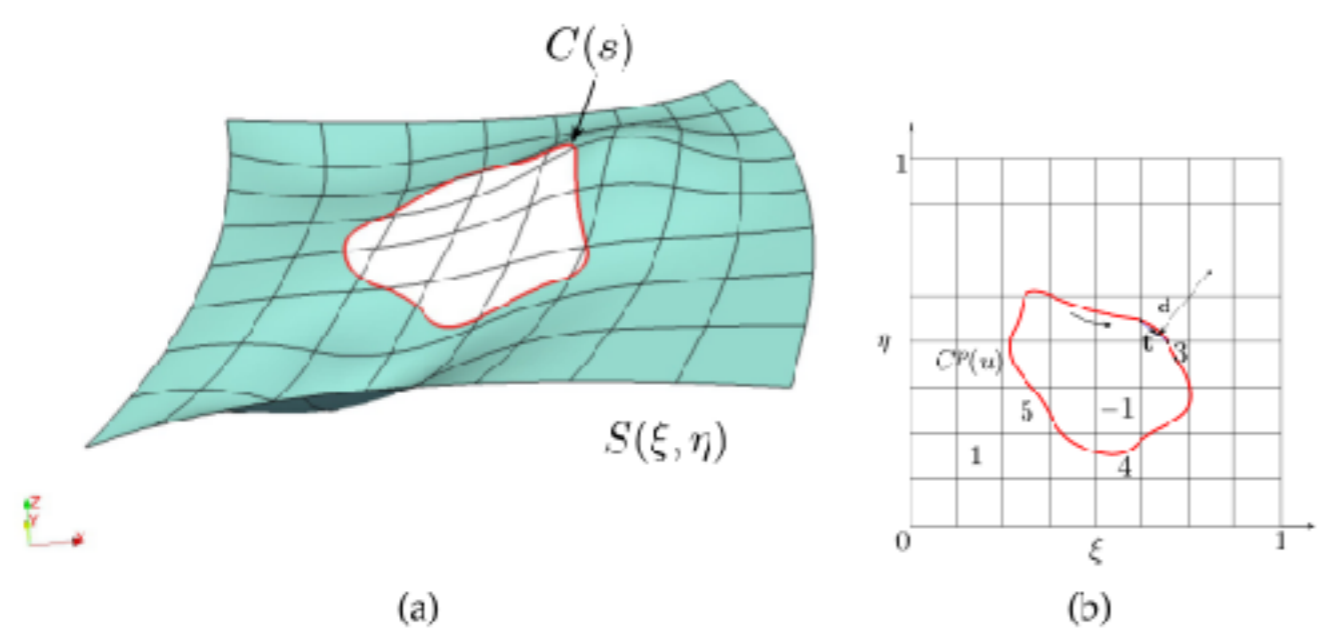
X. Peng, et al. (2017). *CMAME*, 316, 151–185.



# Modeling techniques for surface cracks? Trimmed curves...



Surface discontinuity is introduced

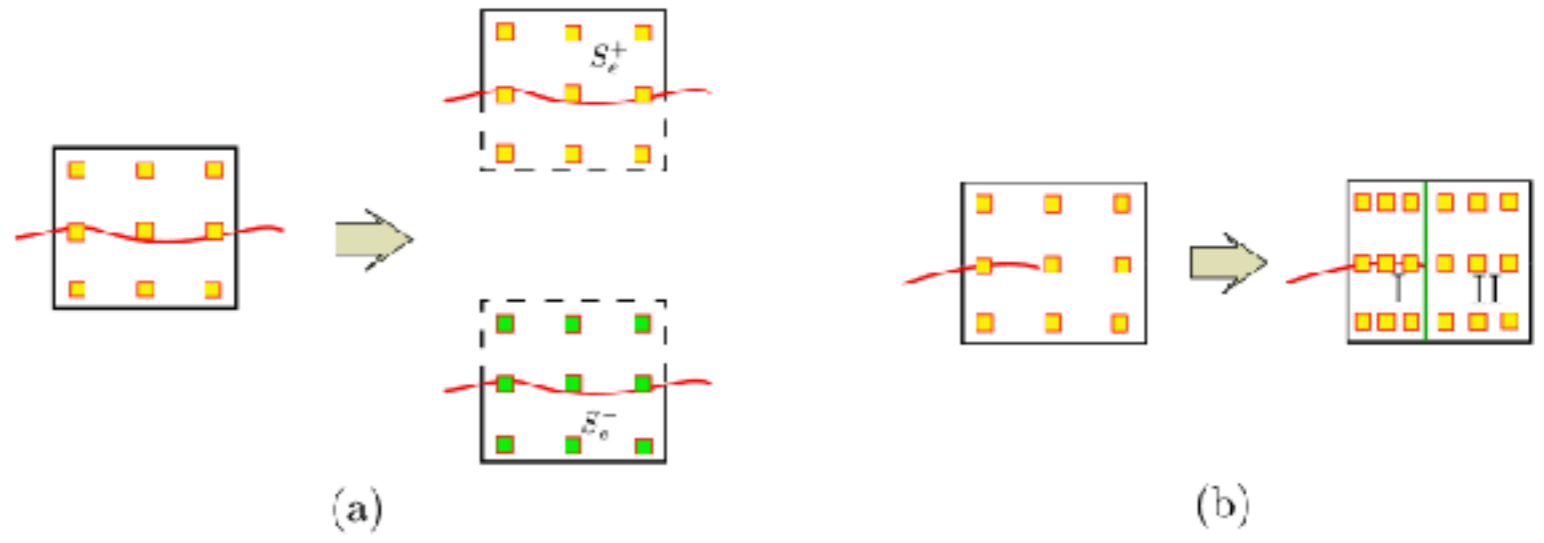


Trimmed NURBS technique

Crack = trimming curve  
Phantom node method

$$\mathbf{u}^+(\mathbf{x}) = \sum_j^{N^e} \mathbf{R}_j(\mathbf{x}) \mathbf{d}_j, \quad \mathbf{x} \in S_e^+$$

$$\mathbf{u}^-(\mathbf{x}) = \sum_k^{N^e} \mathbf{R}_k(\mathbf{x}) \mathbf{d}_k, \quad \mathbf{x} \in S_e^-$$



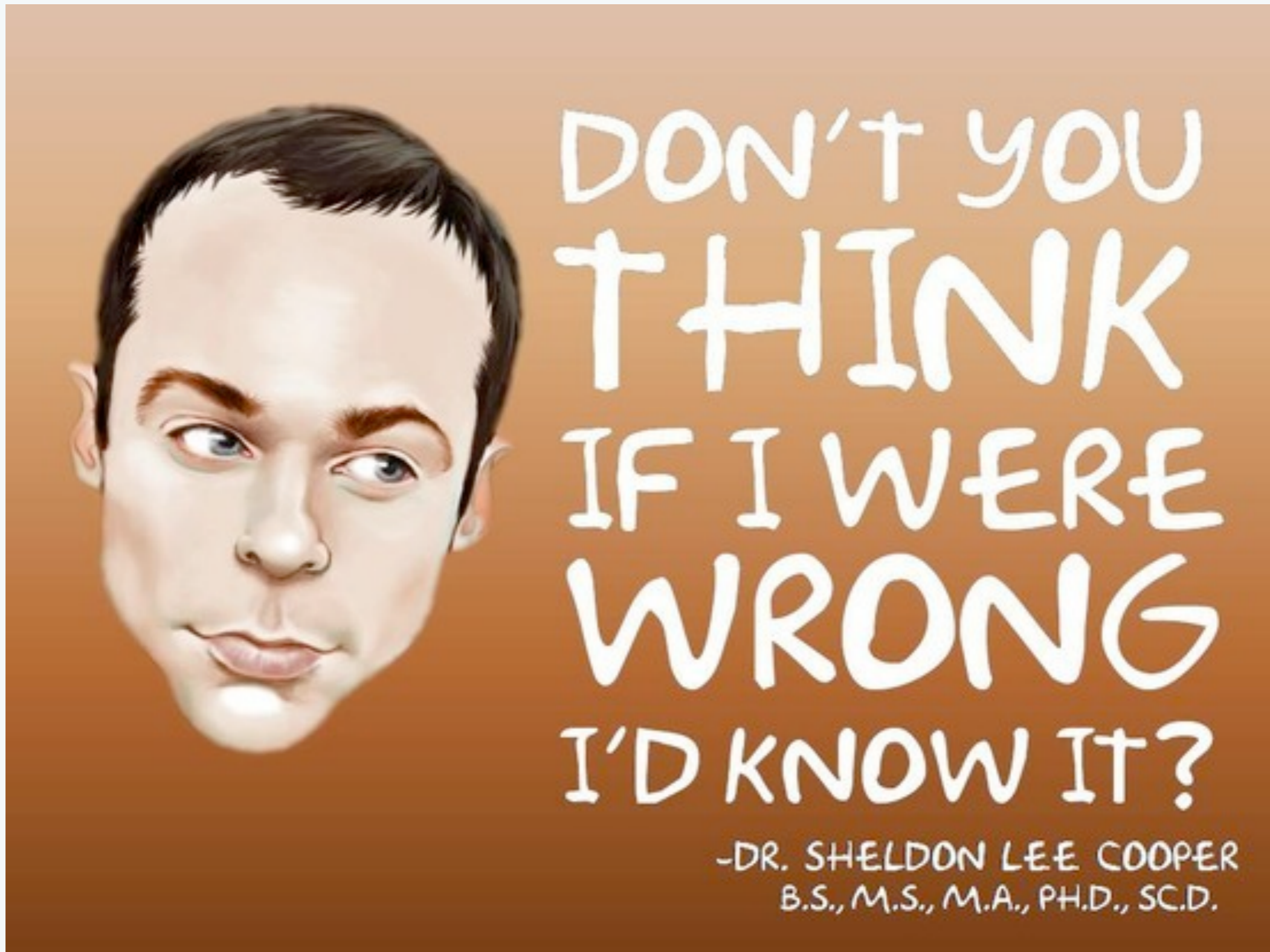
## References

- Peng, X., Atroshchenko, E., Kerfriden, P., & Bordas, S. P. A. (2017). Linear elastic fracture simulation directly from CAD: 2D NURBS-based implementation and role of tip enrichment. *International Journal of Fracture*, 204(1), 55–78.
- Peng, X., Atroshchenko, E., Kerfriden, P., & Bordas, S. P. A. (2017). Isogeometric boundary element methods for three dimensional static fracture and fatigue crack growth. *Computer Methods in Applied Mechanics and Engineering*, 316, 151–185.
- Simpson, R. N., Bordas, S. P. A., Trevelyan, J., & Rabczuk, T. (2012). A two-dimensional Isogeometric Boundary Element Method for elastostatic analysis. *Computer Methods in Applied Mechanics and Engineering*, 209–212(0), 87–100.
- Guiggiani, M., Krishnasamy, G., Rudolphi, T. J., & Rizzo, F. J. (1992). A General Algorithm for the Numerical Solution of Hypersingular Boundary Integral Equations. *Journal of Applied Mechanics*, 59(3), 604–614.
- Rong, J., Wen, L., & Xiao, J. (2014). Efficiency improvement of the polar coordinate transformation for evaluating BEM singular integrals on curved elements. *Engineering Analysis With Boundary Elements*, 38, 83–93.
- Mi, Y., & Aliabadi, M. H. (1992). Dual boundary element method for three-dimensional fracture mechanics analysis. *Engineering Analysis with Boundary Elements*, 10(2), 161–171.
- Becker, A. (1992). *The Boundary Element Methods in Engineering*. McGraw-Hill Book Company.

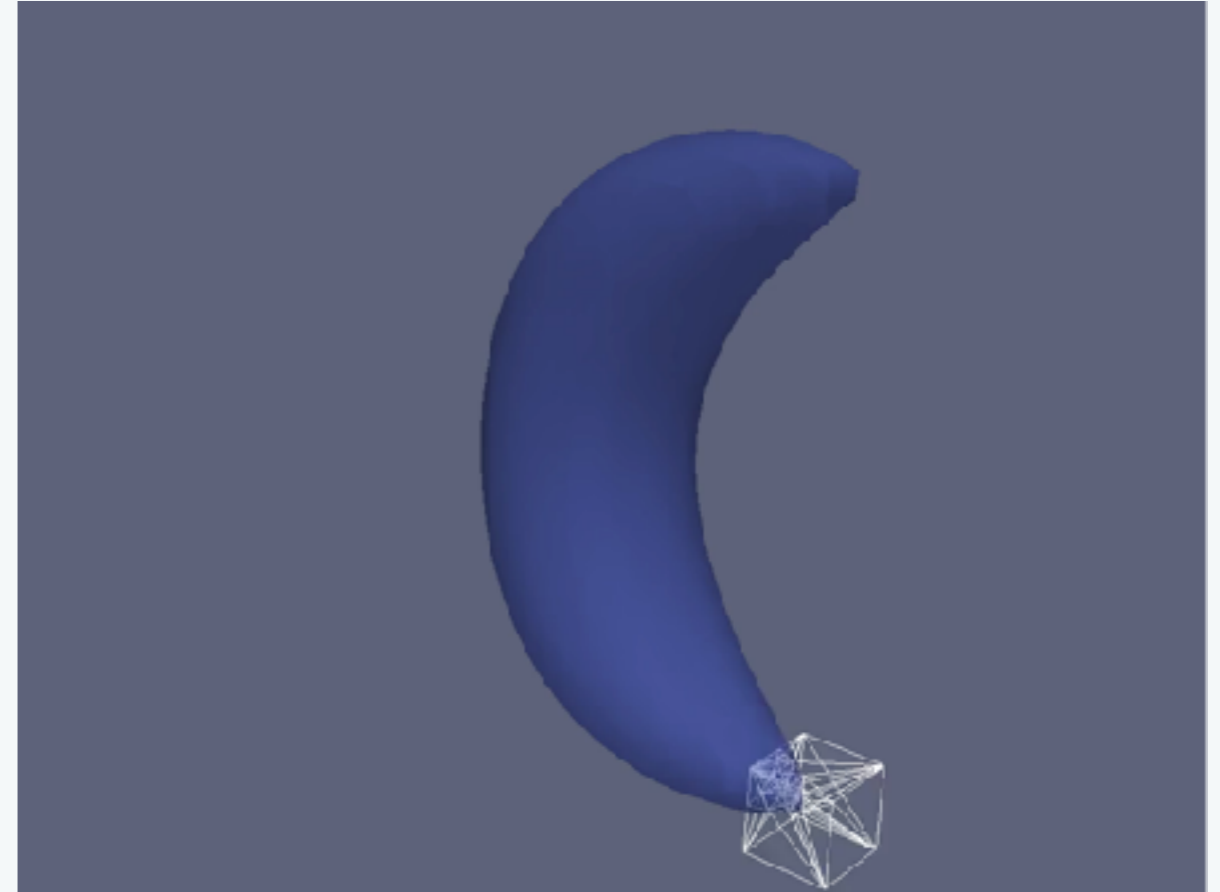
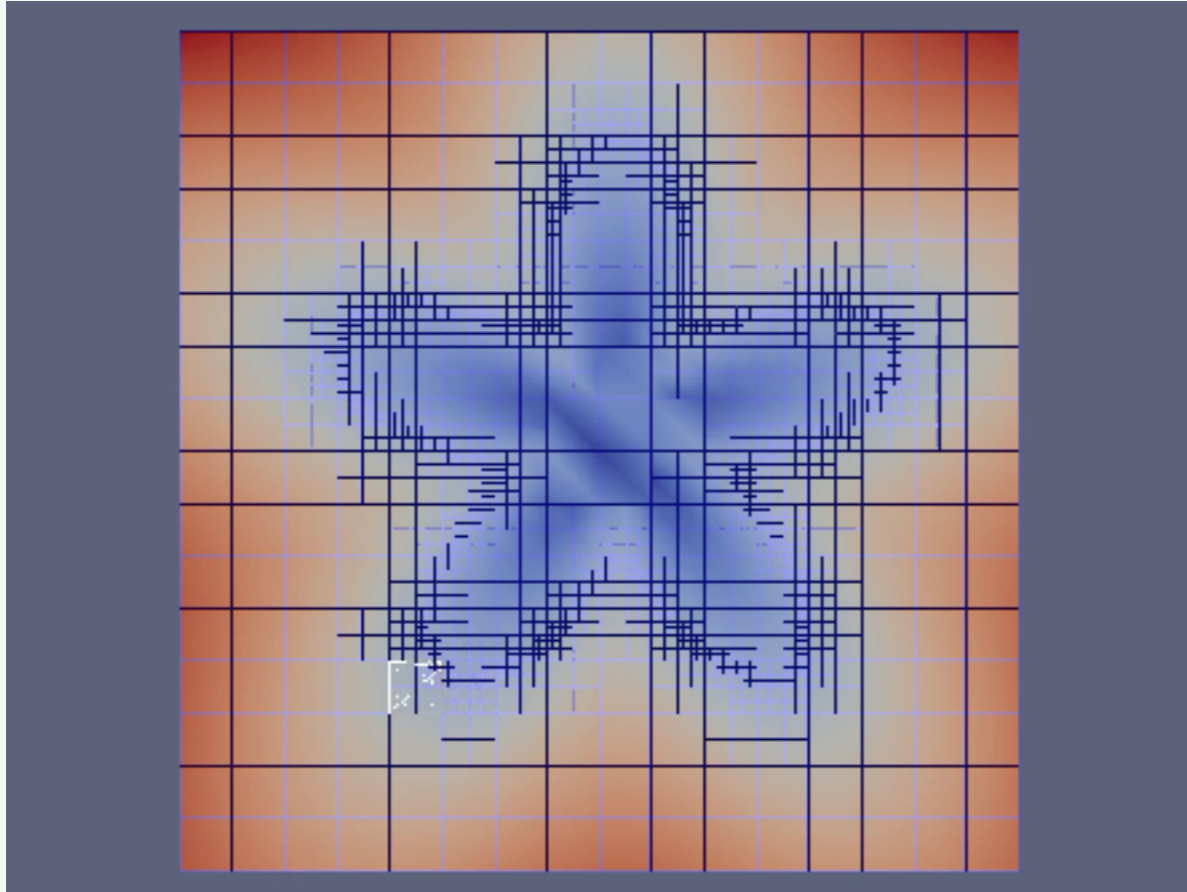
## Partial conclusions on methods coupling geometry and field approximations

- ◆ There are numerous alternatives (subdivision surfaces, IGA, NEFEM, NIGFEM)
- ◆ IGA can offer simulations directly from CAD when used with boundary elements
- ◆ GIFT generalizes this approach by decoupling geometry and field approximations

**Next: methods which decouple geometry and field approximation**



## *Decoupling - Unfitted FEM?*



**Question: for which problems are we better off coupling/decoupling the geometry from the field approximation?**

### **Implicit surfaces**

T. Rüberg (2016) *Advanced Modeling and Simulation in Engineering Sciences* 3 (1), 22

M. Moumnassi (2011) *CMAME* 200(5): 774-796. (CSG and multiple level sets)

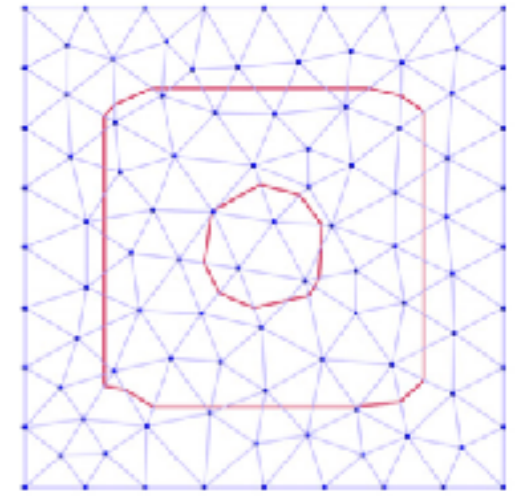
N. Moës (2003) *CMAME* 192.28 (2003): 3163-3177. (Single level set)

T. Belytschko *IJNME* 56.4 (2003): 609-635. (Structured XFEM)

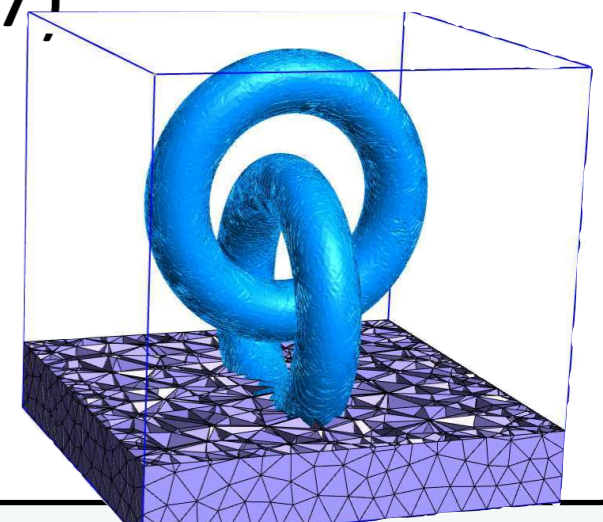
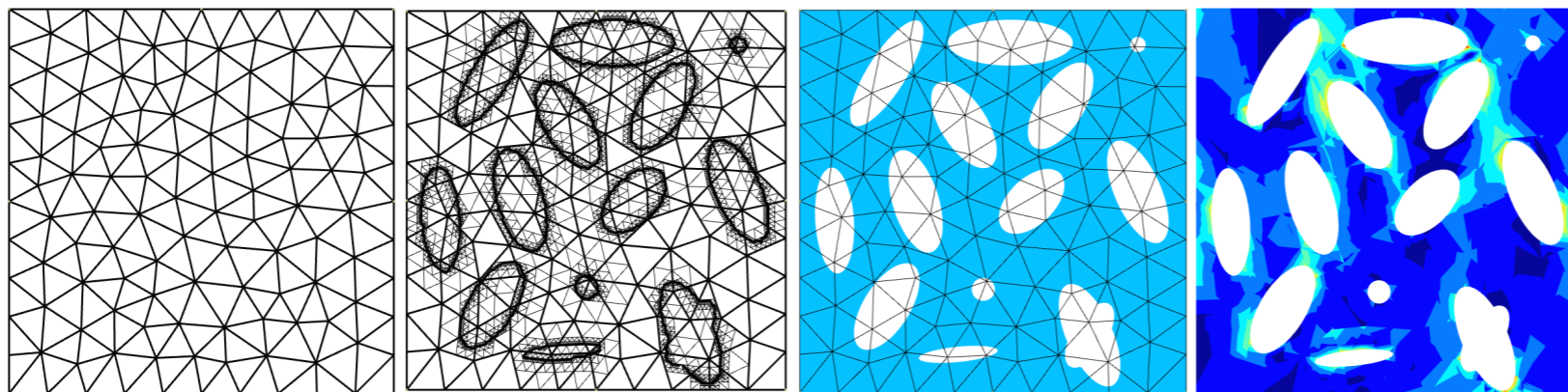
...

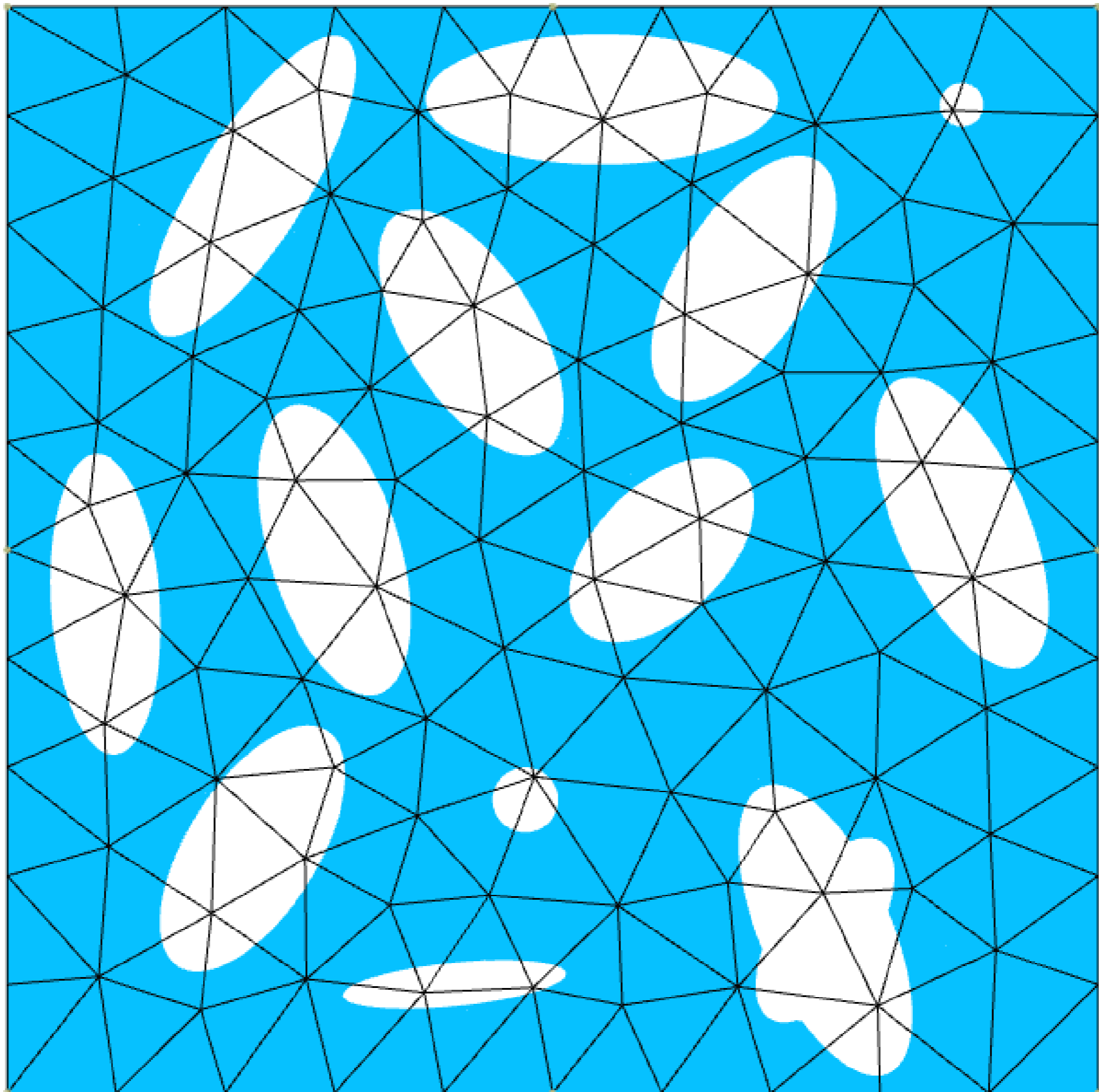
# Separate field and boundary discretisation

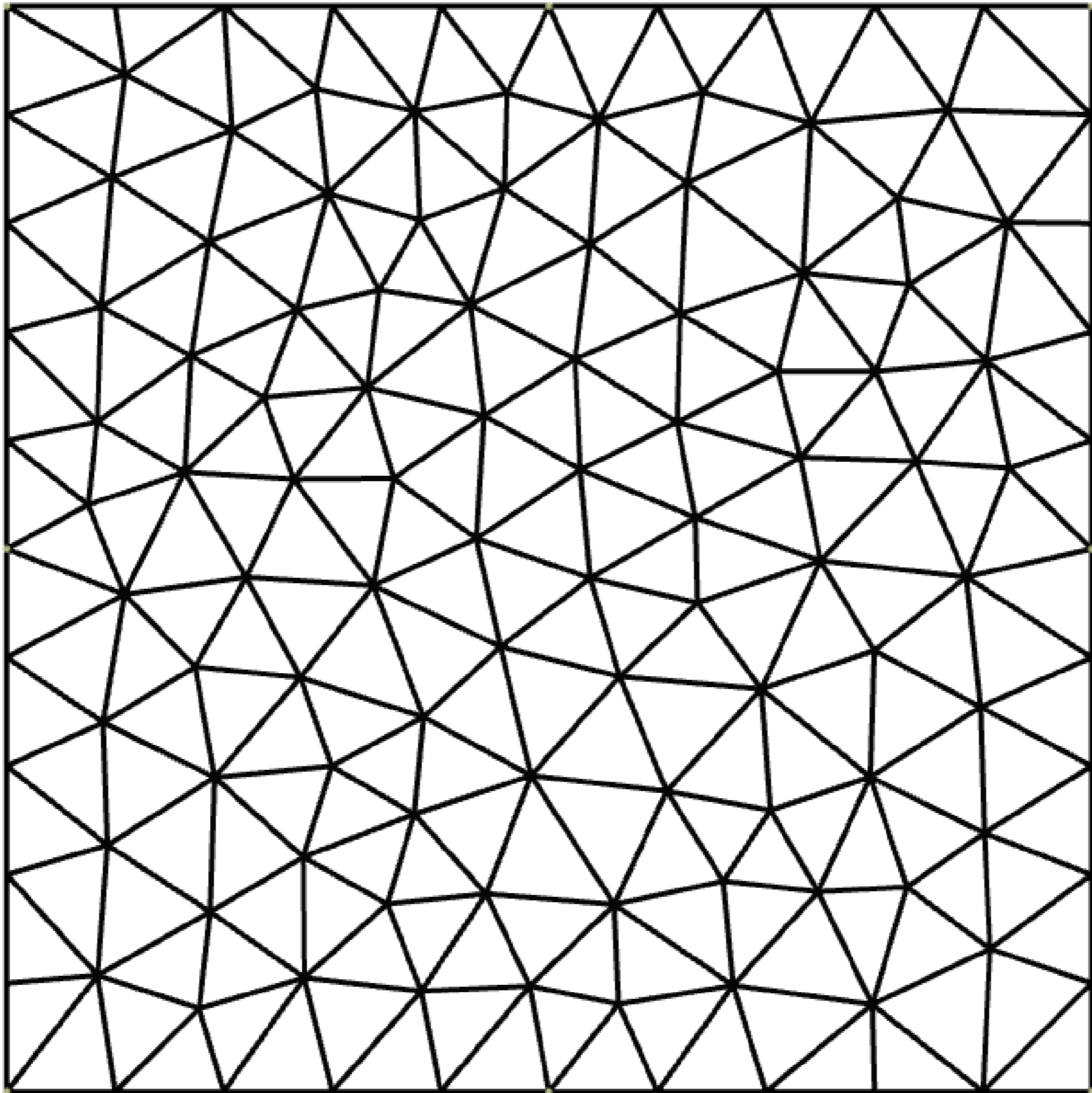
- Immersed boundary method (Mittal, *et al.* 2005)
- Fictitious domain (Glowinski, *et al.* 1994)
- Embedded boundary method (Johansen, *et al.* 1998)
- Virtual boundary method (Saiki, *et al.* 1996)
- Cartesian grid method (Ye, *et al.* 1999, Nadal, 2013)



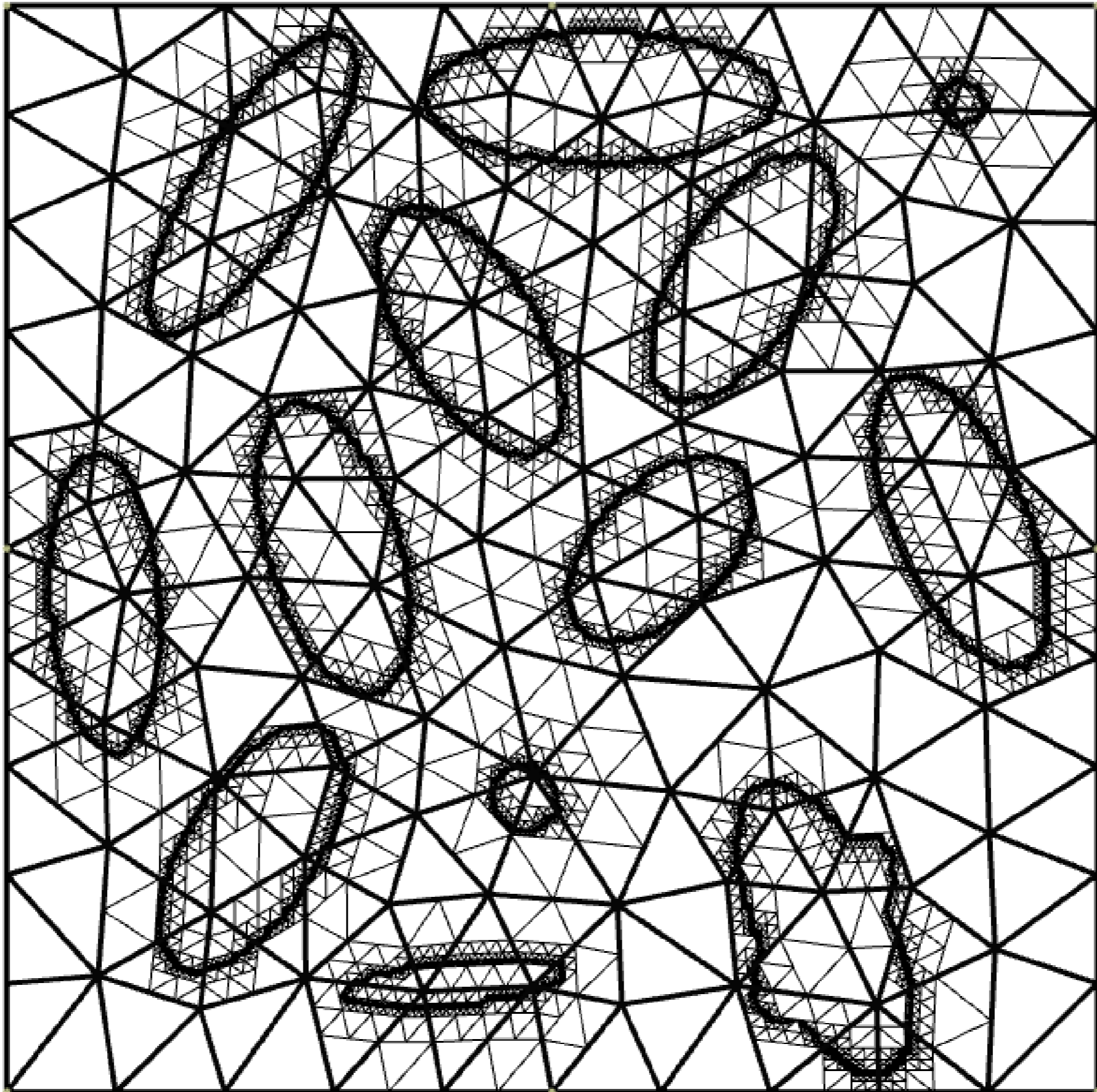
- ✓ Easy adaptive refinement + error estimation (Nadal, 2013)
- ✓ Flexibility of choosing basis functions
- Accuracy for complicated geometries? BCs on implicit surfaces?
- ➔ An accurate and implicitly-defined geometry from arbitrary parametric surfaces including corners and sharp edges (Moumnassi 2011; Ródenas Garcia 2016; Fries 2017)

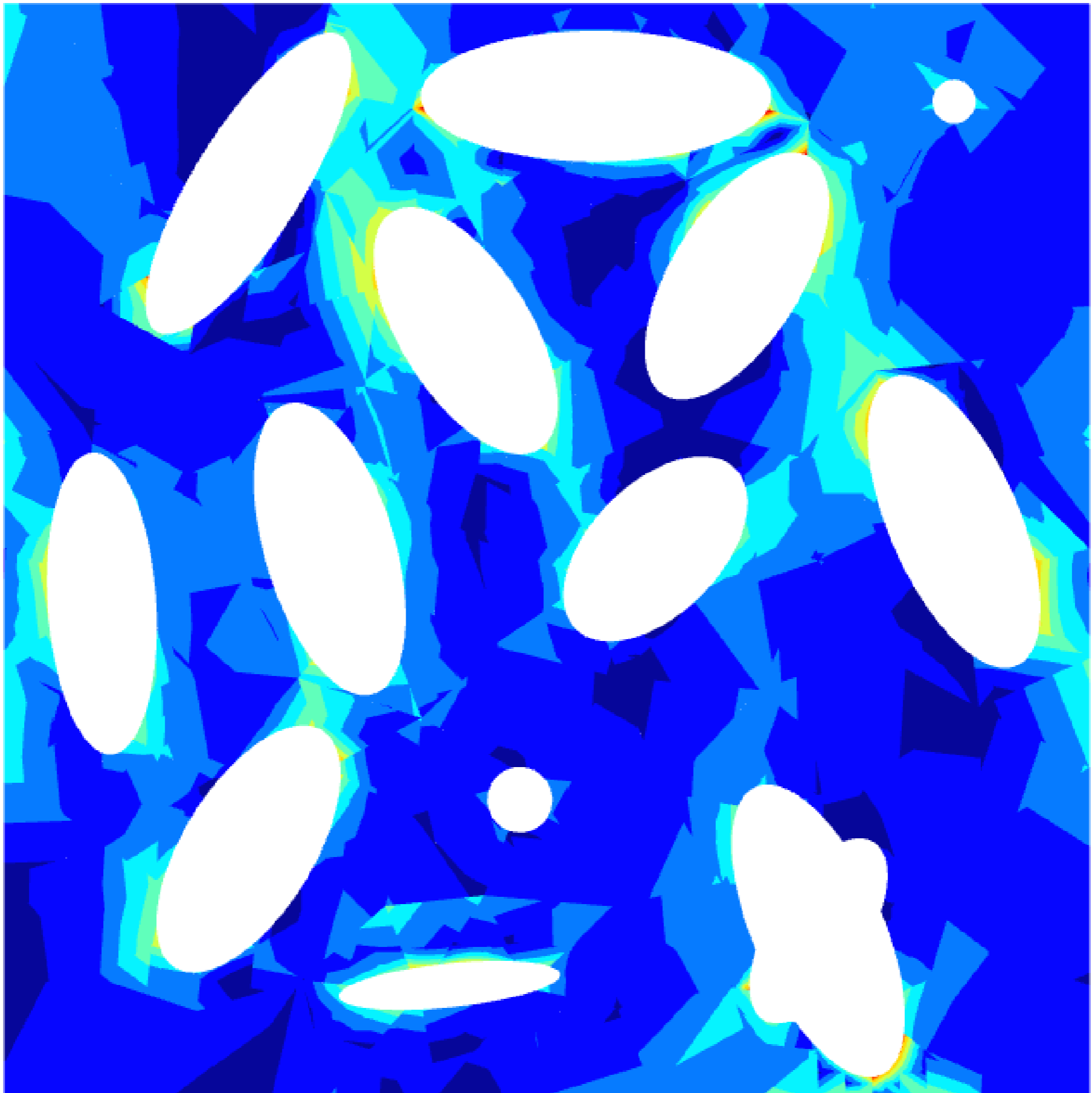


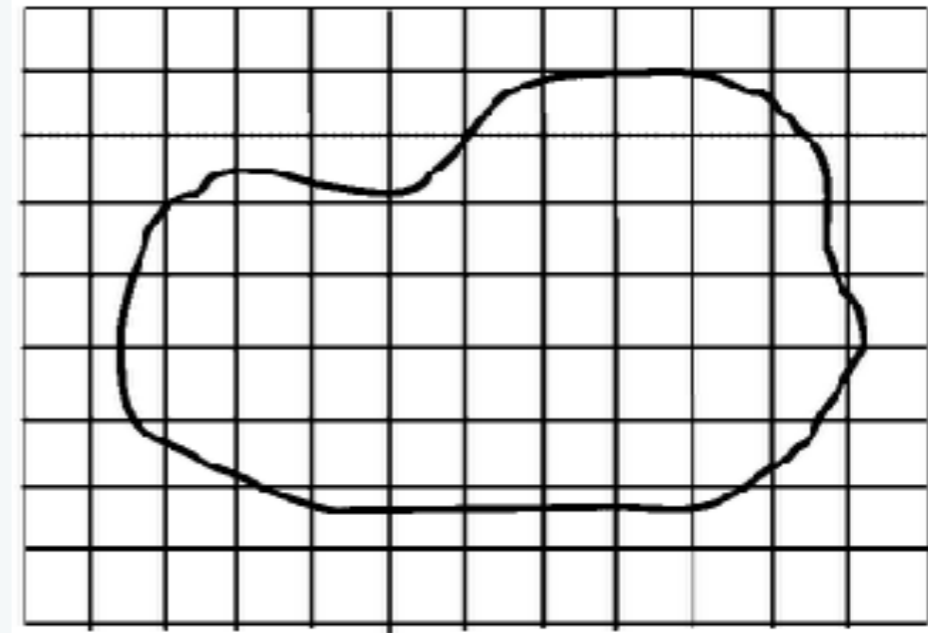
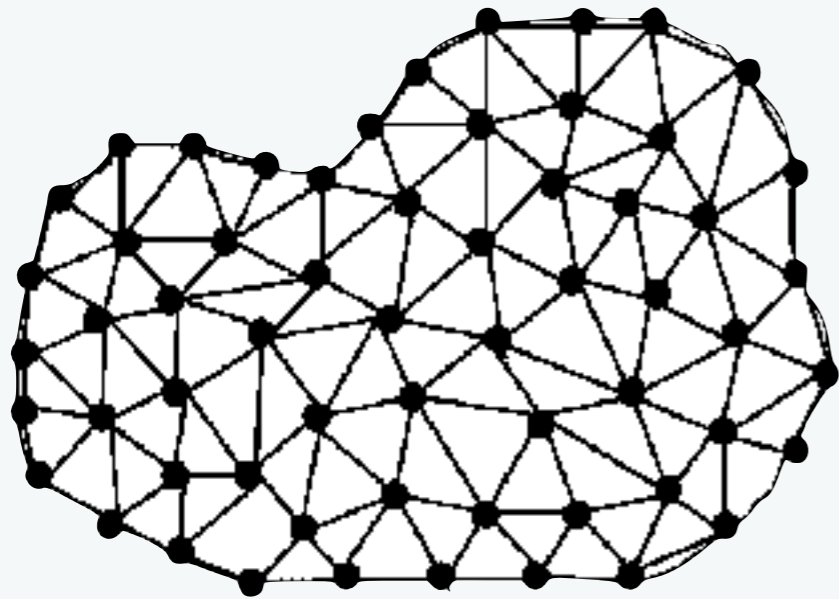








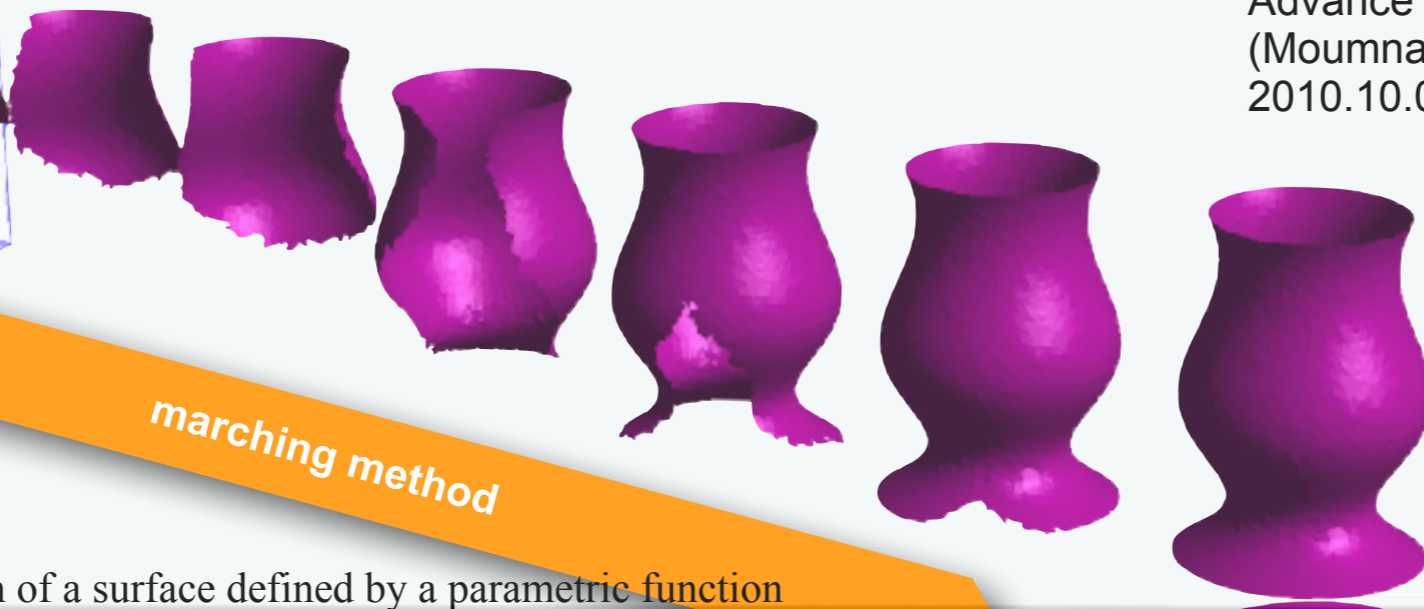




**Question: How can we generate level set functions from CAD descriptions (including corners/vertices)?**



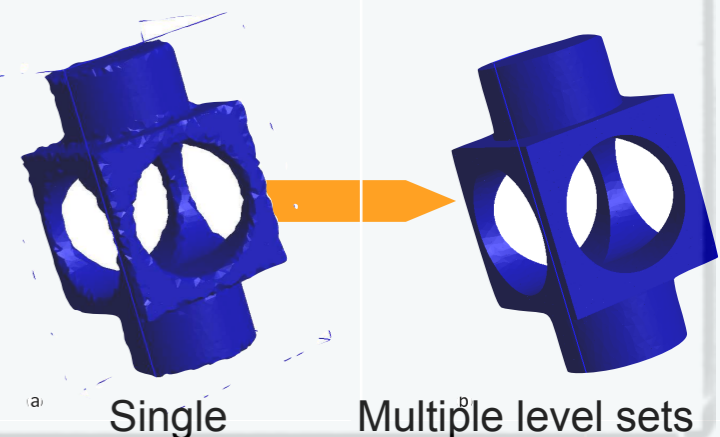
seed point(s) - requires **one single** global search

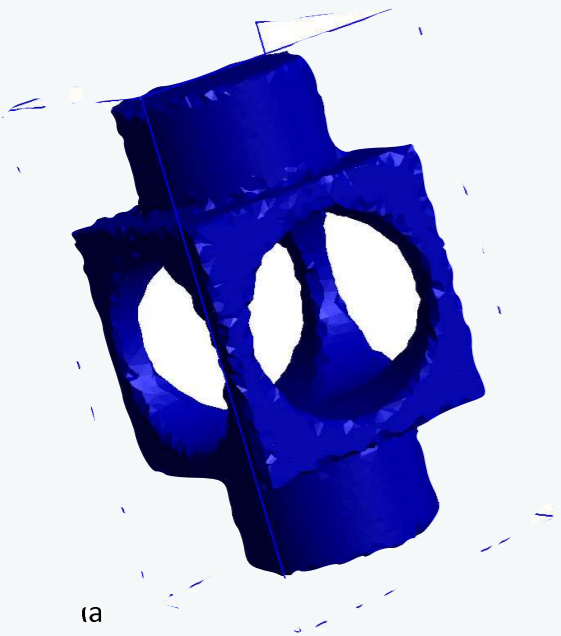
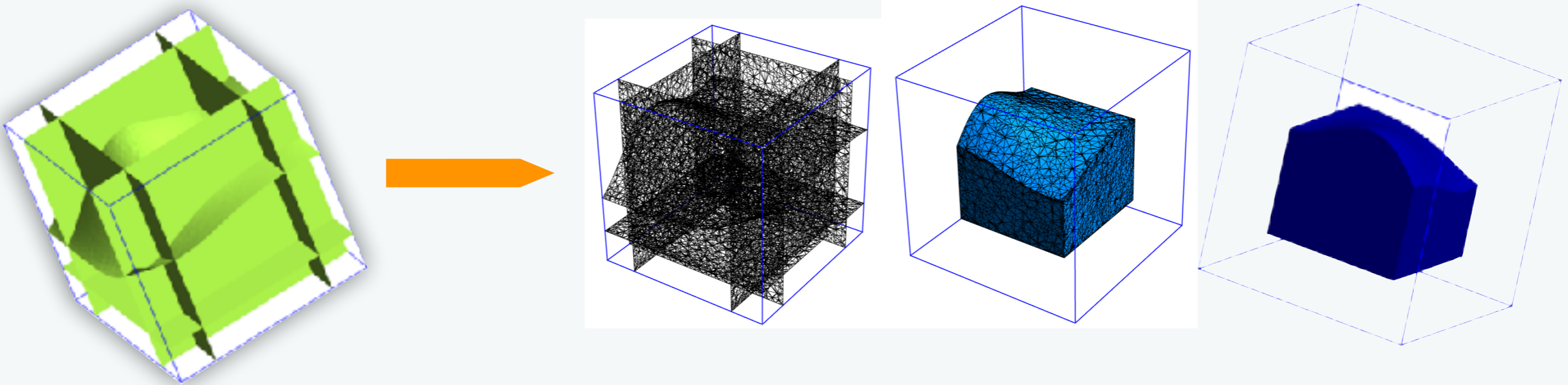


marching method

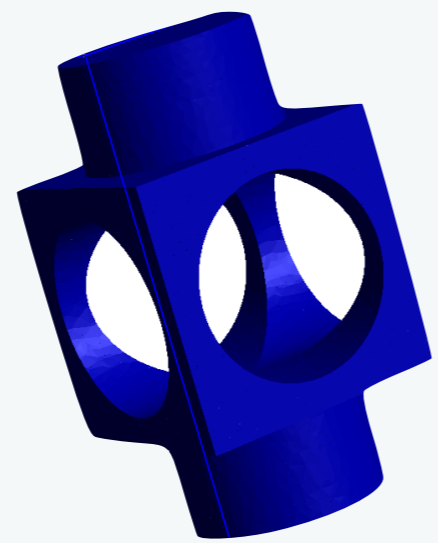
Level Set representation of a surface defined by a parametric function

Advance by CRP Henri Tudor in 2011 (Moumnassi et al, CMAME DOI: 10.1016/j.cma.2010.10.002)

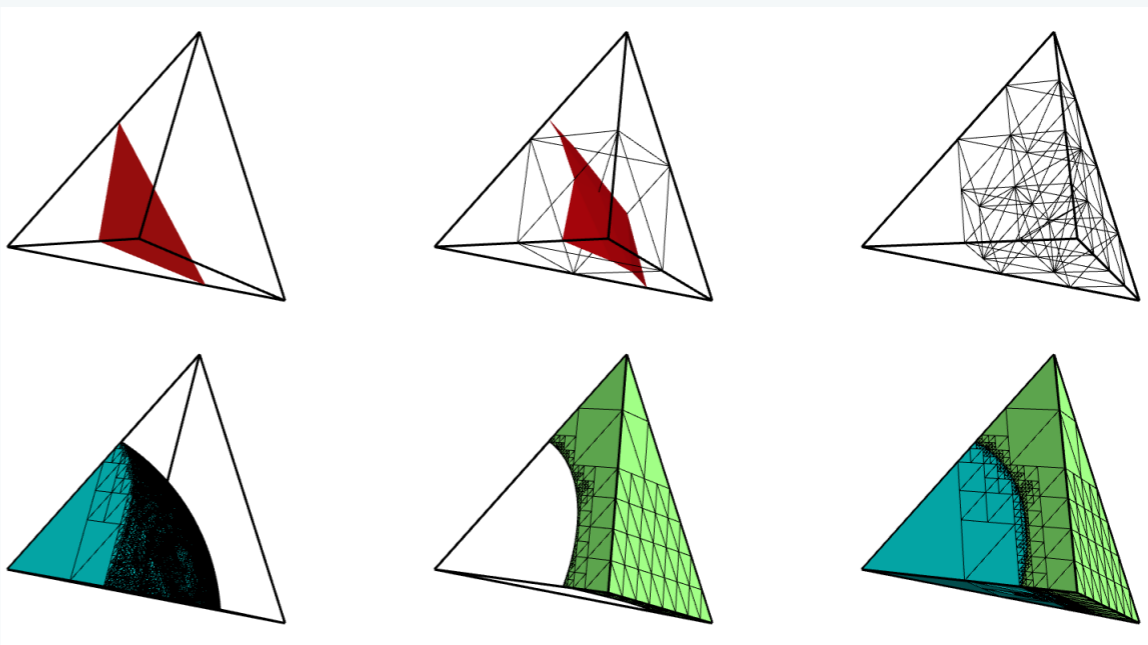




Single level set

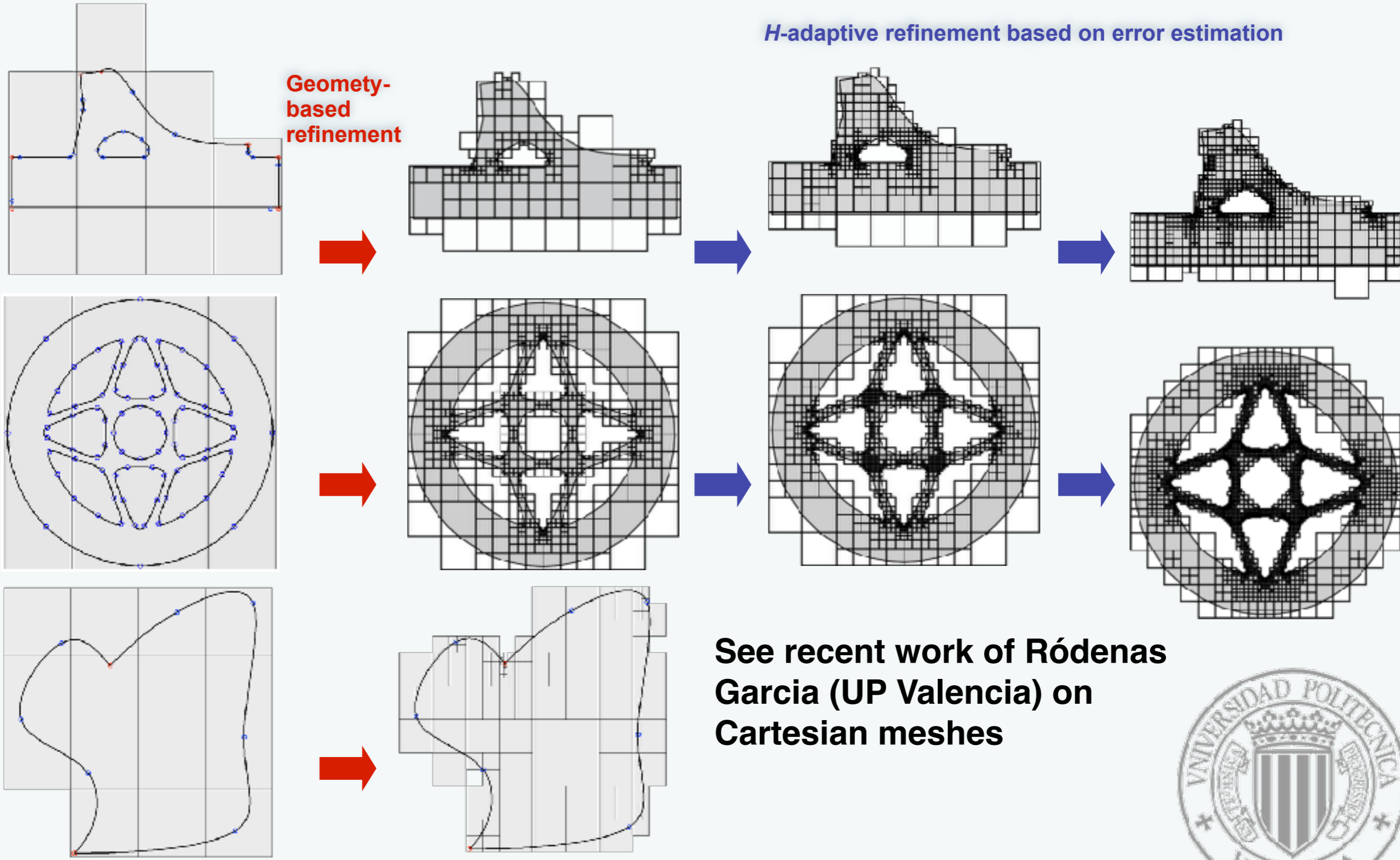


Multi level sets



# Examples

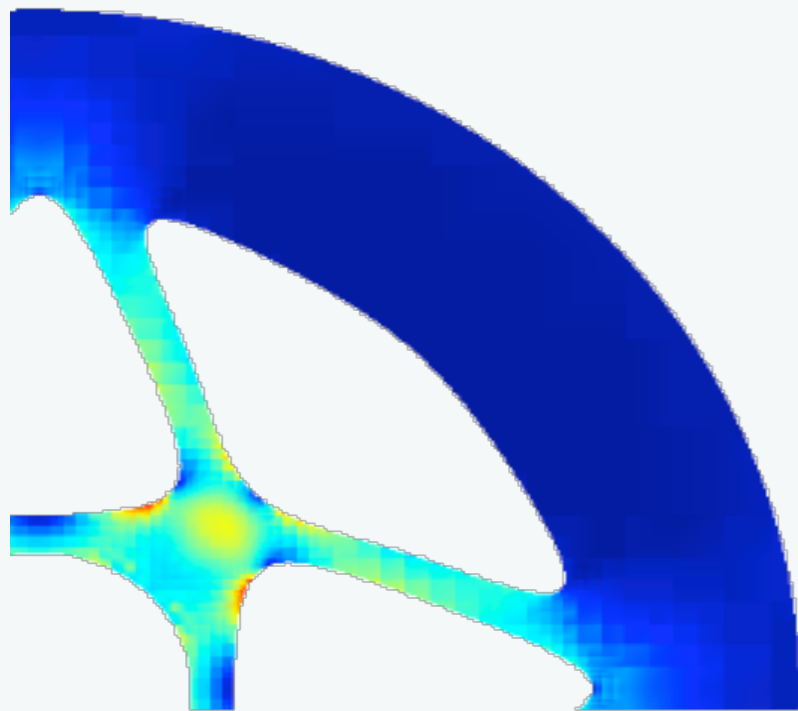
*H*-adaptive refinement based on error estimation



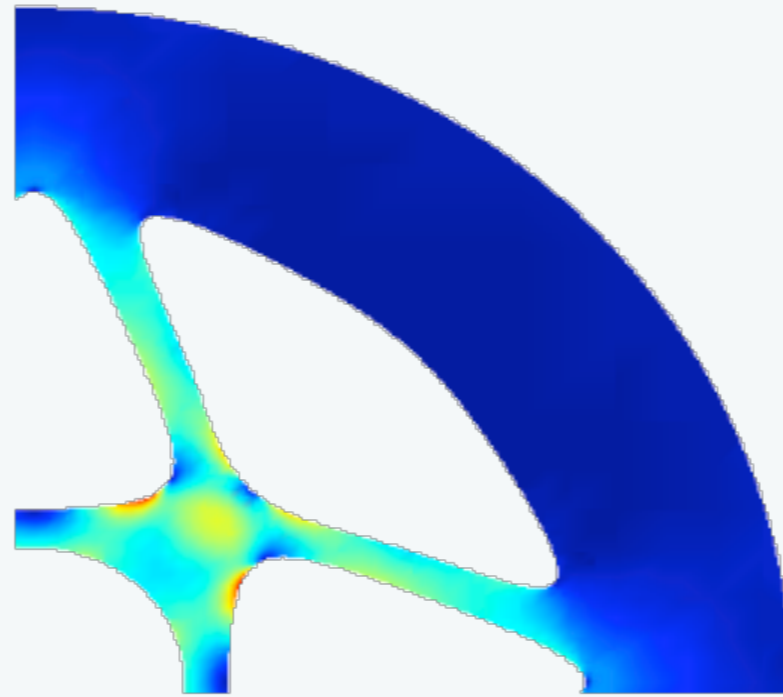
See recent work of Ródenas Garcia (UP Valencia) on Cartesian meshes



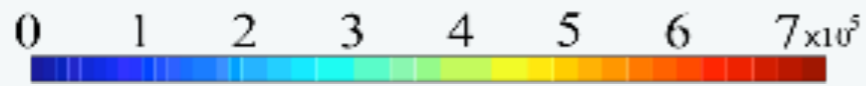
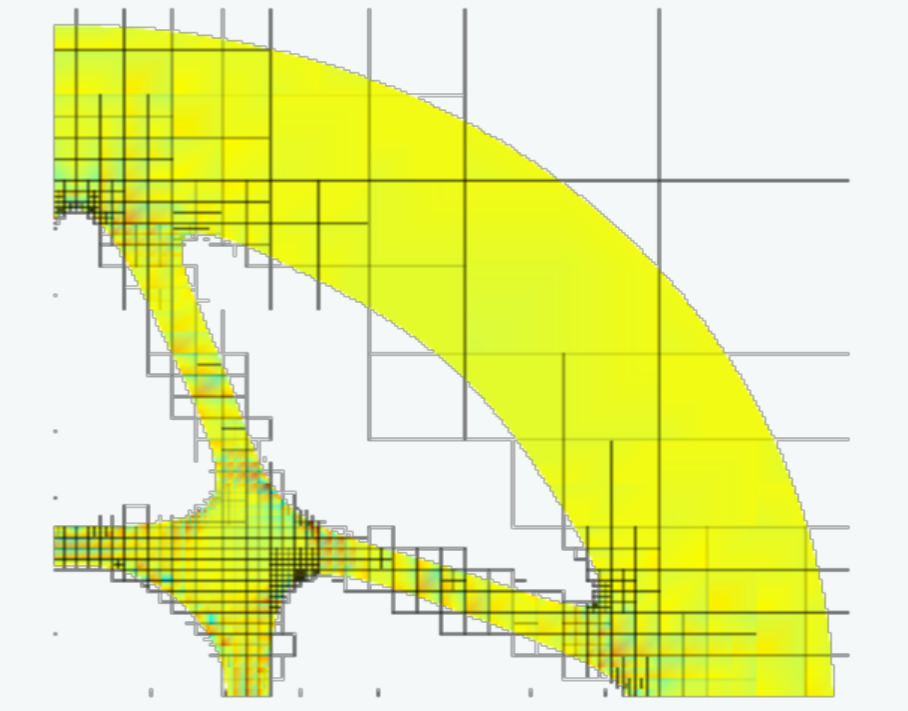
FEM



SPR-C

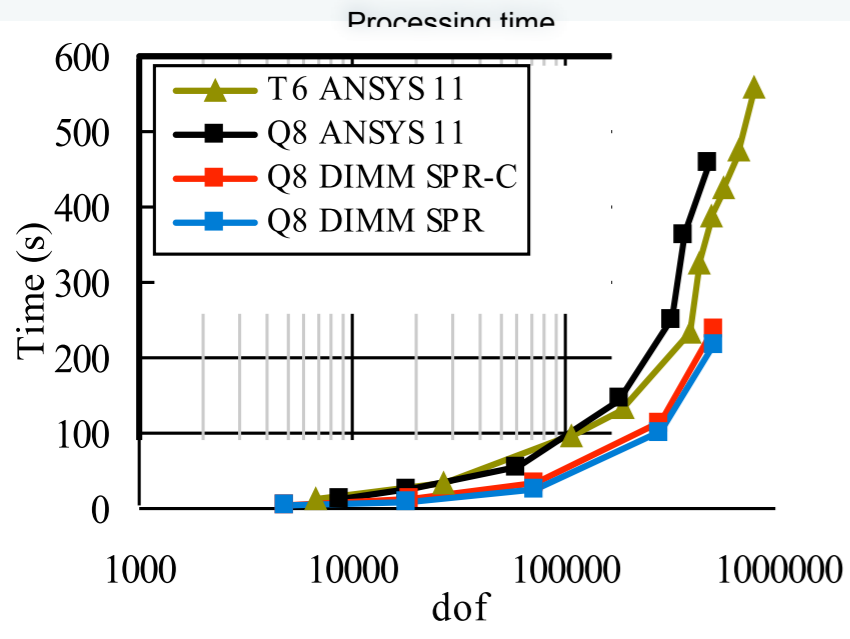


SPR-C-FEM



Quad8 uniform refinement

45

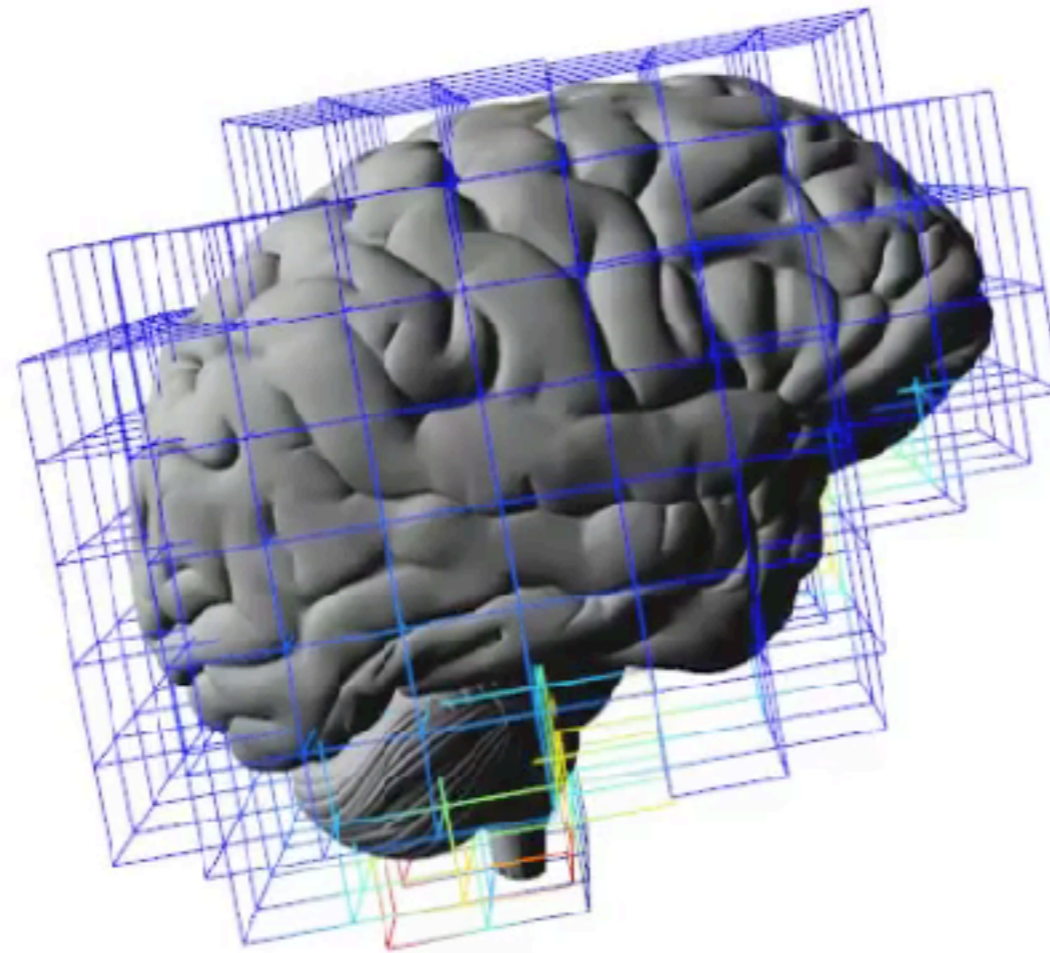


See recent work of Ródenas Garcia (UP Valencia) on Cartesian meshes



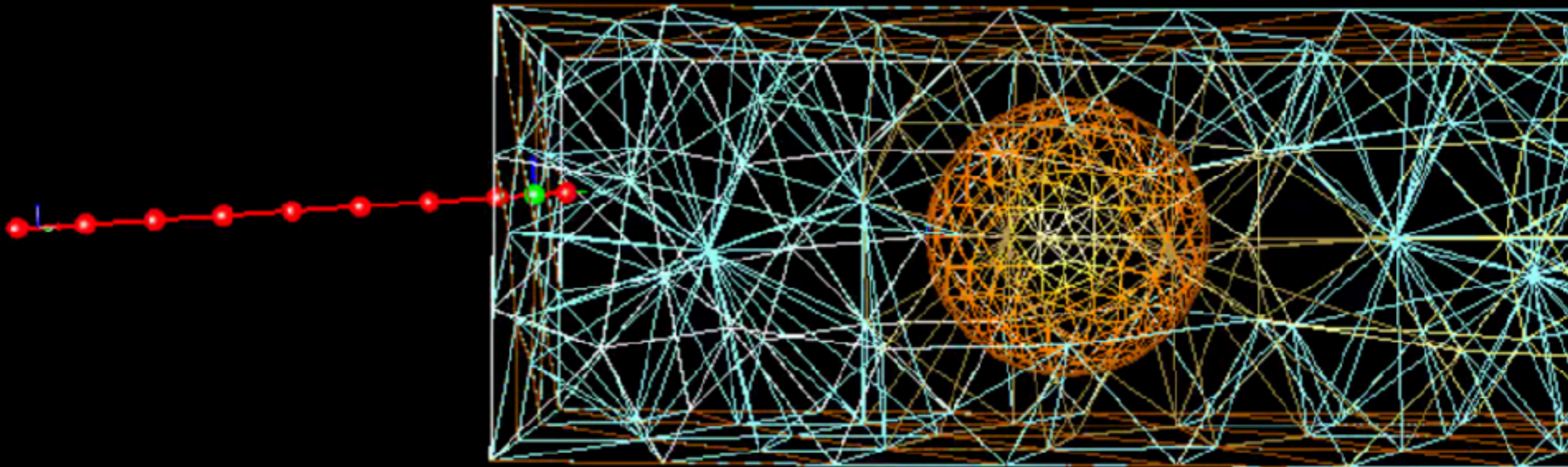
## *With implicit boundaries*

Brain shift occurs  
prior to cannula insertion



# Real-time needle insertion simulation

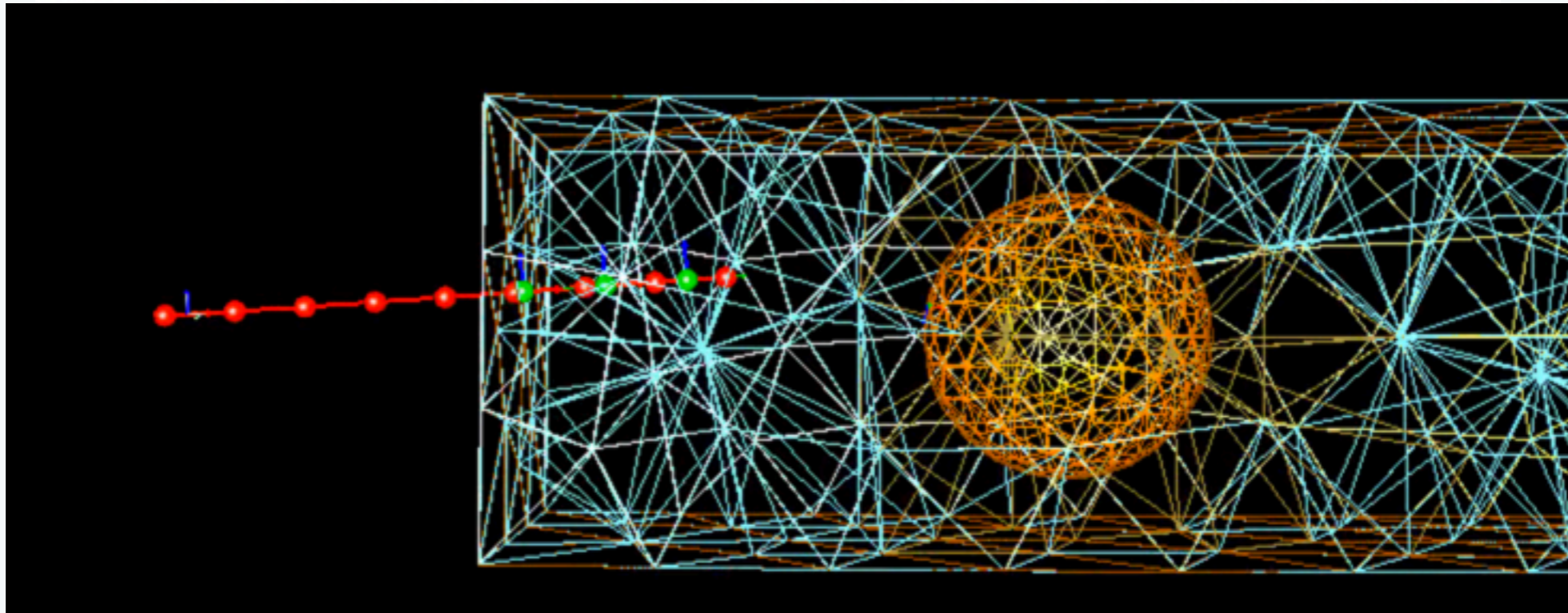
*With implicit boundaries*





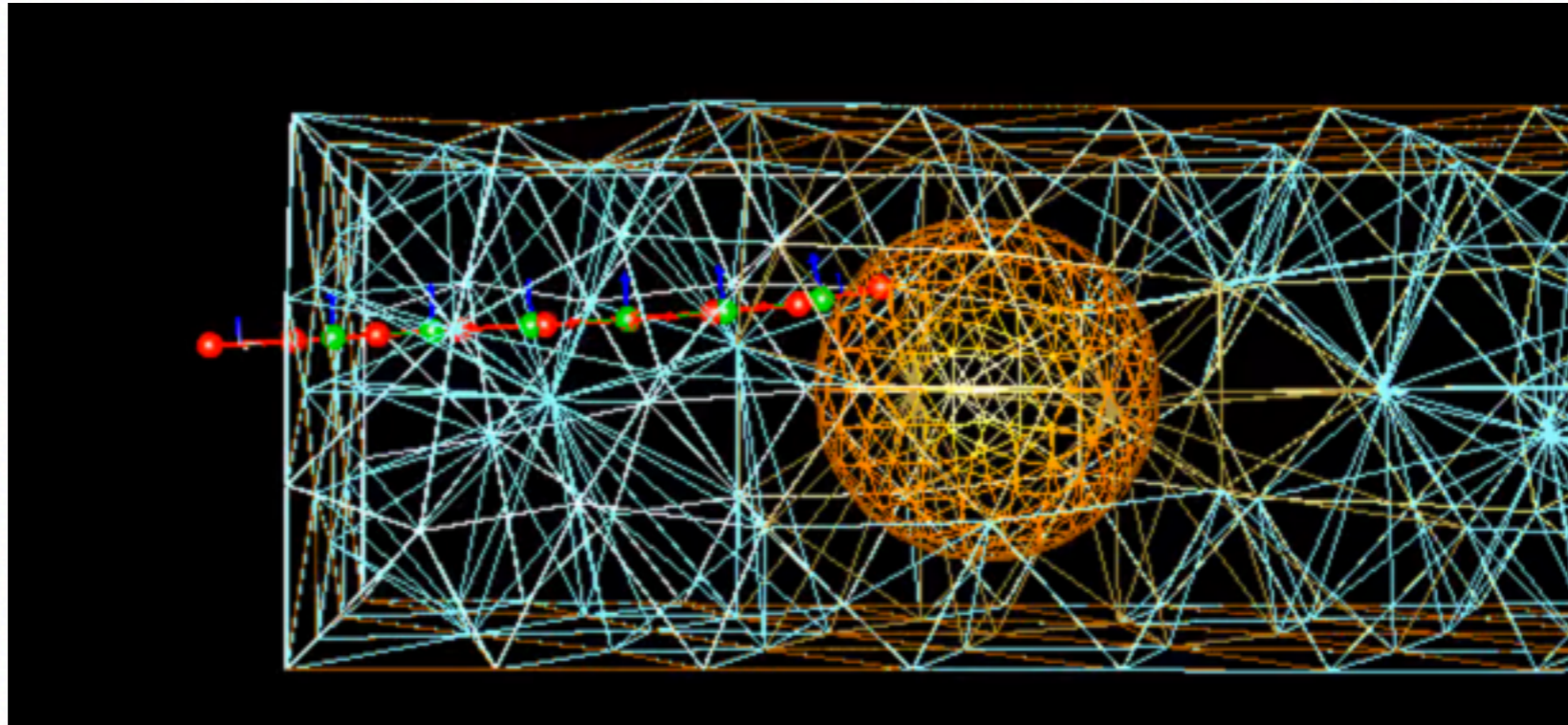
# Real-time needle insertion simulation

*With implicit boundaries*

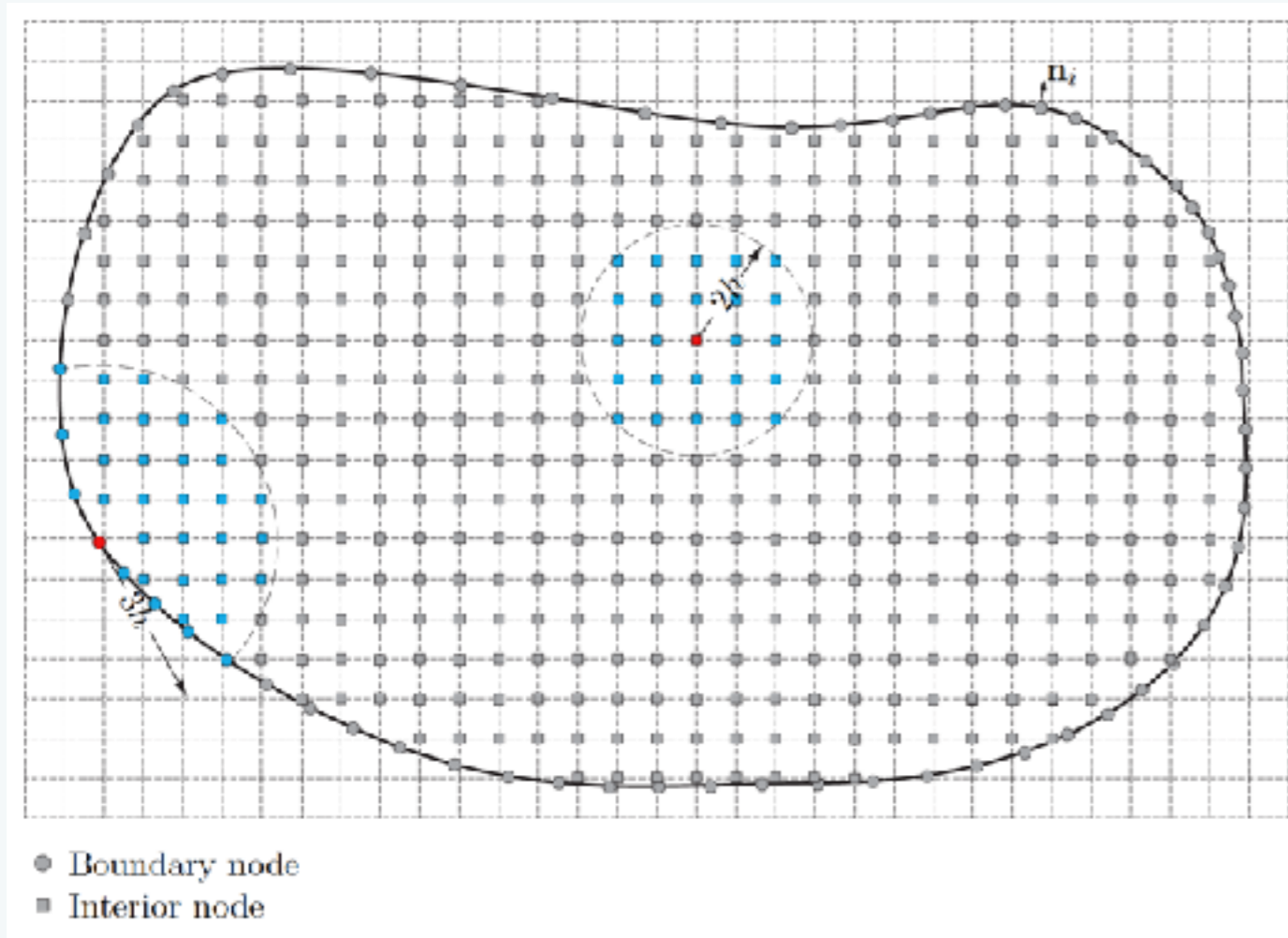


# Real-time needle insertion simulation

*With implicit boundaries*



# Discretisation Correction of Particle Strength Exchange Collocation

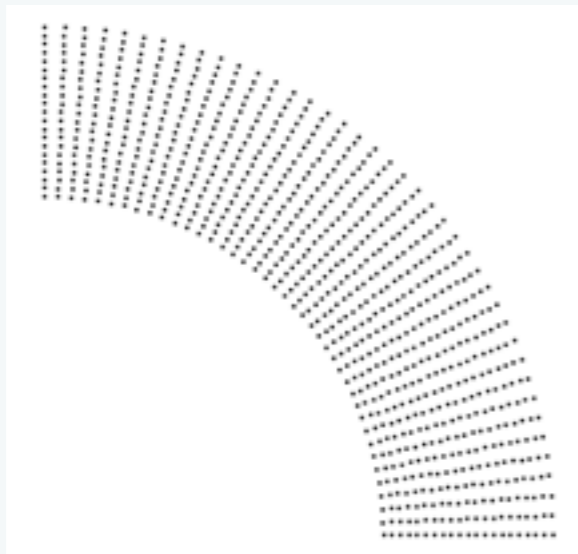
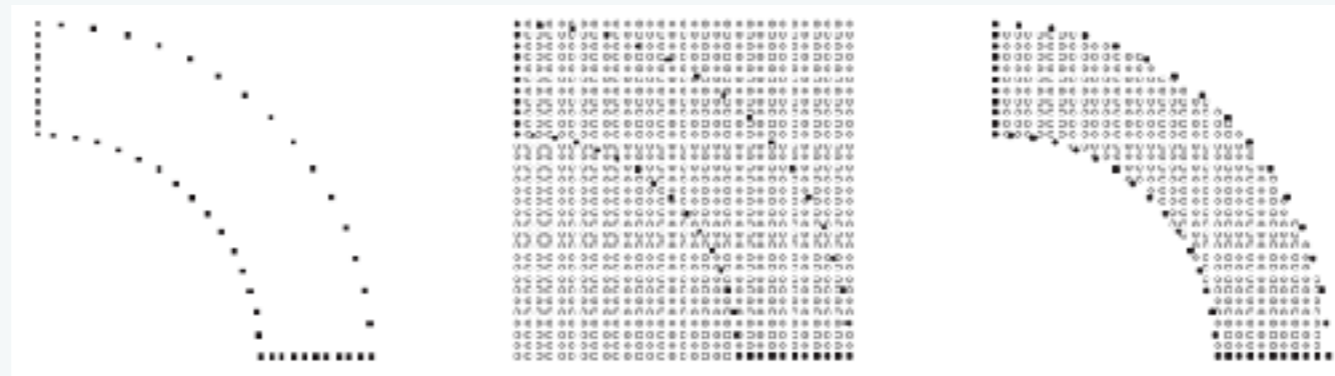


Key idea: use “generalized” finite differences

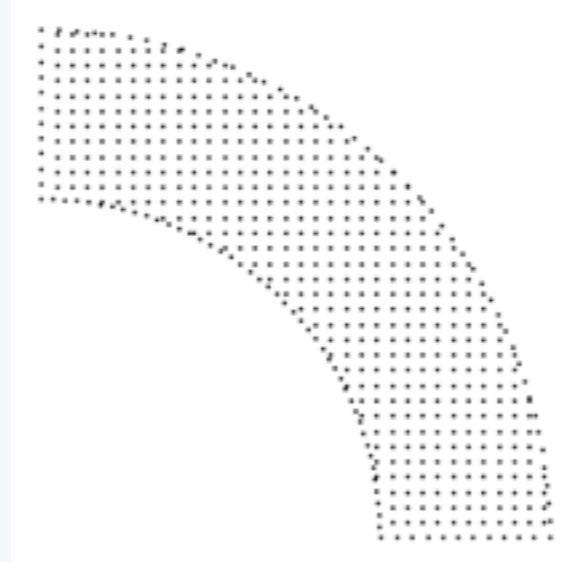
Key difficulty: stability and complex boundaries

Birte Schrader, Sylvain Reboux, and Ivo F Sbalzarini. Discretization correction of general integral PSE operators for particle methods. *Journal of Computational Physics*, 229(11):4159–4182, 2010.

K. Agathos et al. Stable immersed collocation method for elasto-static analysis directly from CAD [preprint available on [orbi.uni.lu](http://orbi.uni.lu)]



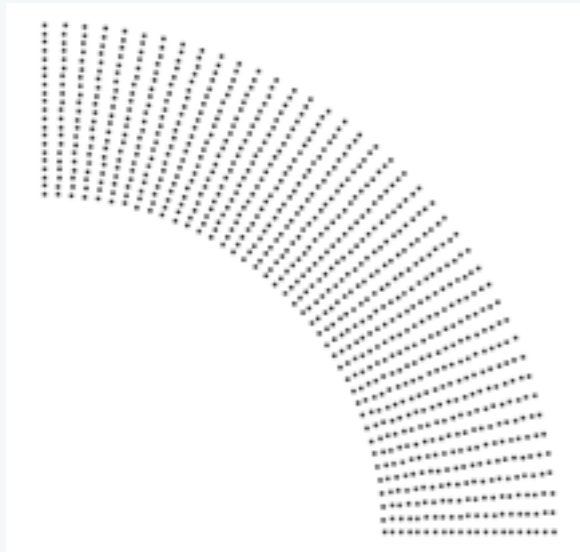
Symmetric



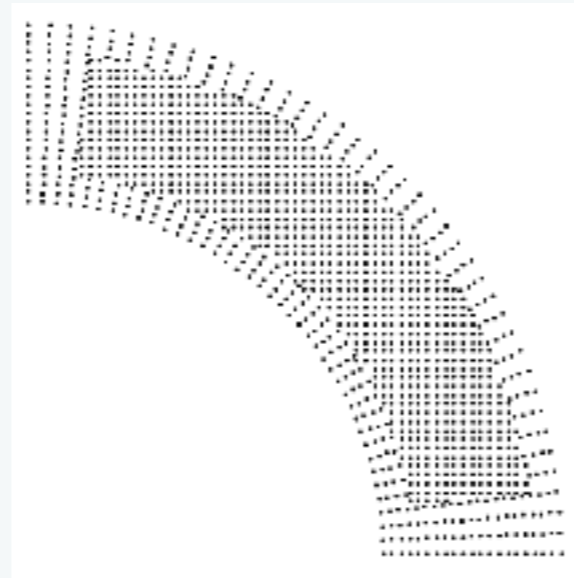
Cartesian



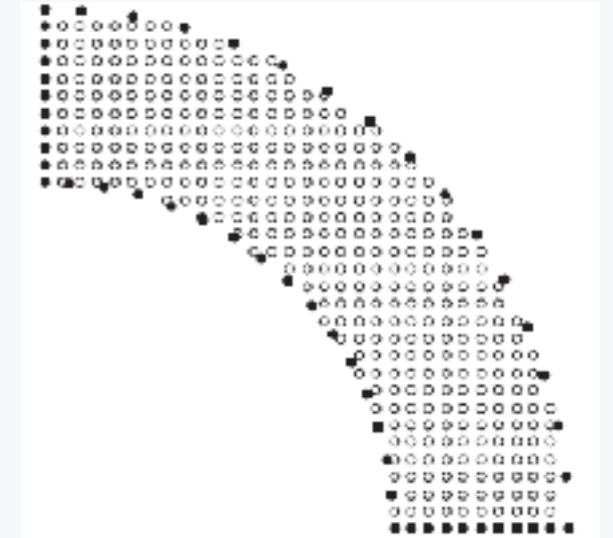
Hybrid



Symmetric



Hybrid

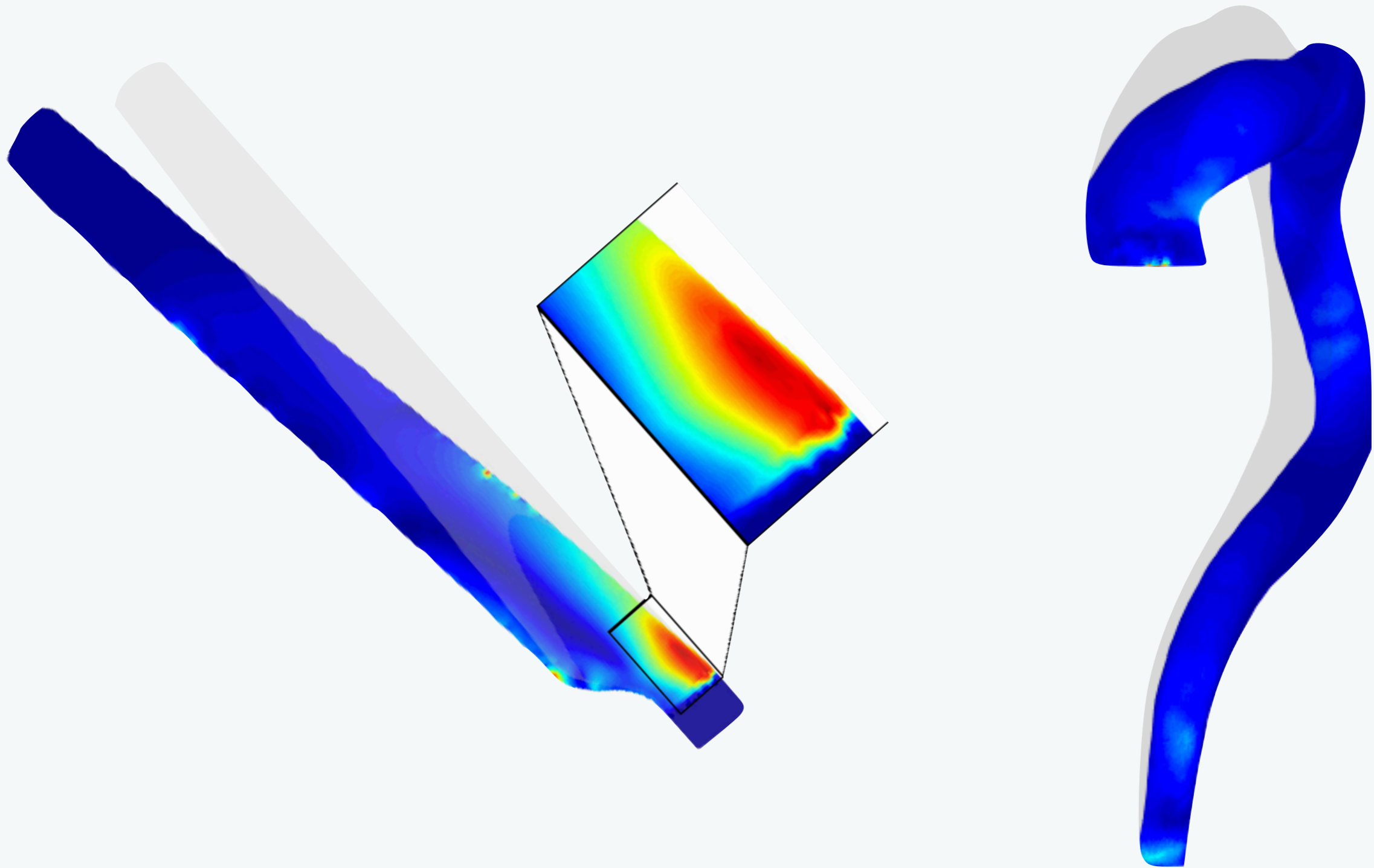


Cartesian

The symmetric node distribution is the most accurate whilst the Cartesian distribution is the worst, the Cartesian-symmetric distribution is intermediate.

Convergence rates of the collocation approach is similar to that of the P1 FEM we compared to whilst the error level is slightly higher. This corroborates results of the isogeometric point collocation method.

## Wind turbine blade & aorta



## Partial conclusions on methods decoupling geometry and field approximations

- ◆ There are numerous alternatives (immersed, CutFEM, structured XFEM, collocation...)
- ◆ Discussions on higher order boundaries (see XDMS2017 book of abstracts!)
- ◆ Using CAD geometries within a structured mesh/grid is a versatile approach

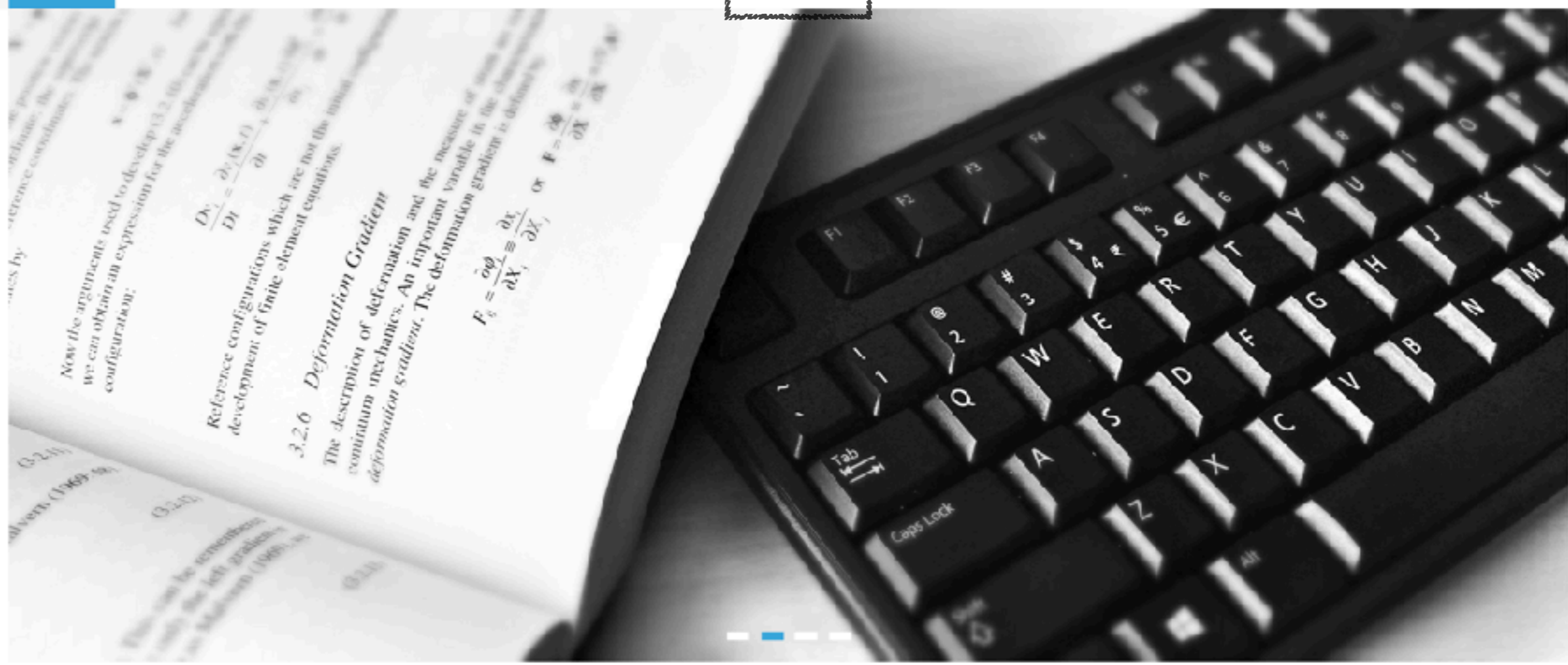
**Next: beyond discretisations...**

# Legato Team

Computational Mechanics



- Home
- Mission & Projects
- News
- People
- Publications
- Resources
- Media
- Contacts





# Legato Team

Computational Mechanics

[Home](#)

[Mission & Projects](#)

[News](#)

[People](#)

[Publications](#)

[Resources](#)

[Media](#)

[Contacts](#)

## Our software

- [Open Source Codes on Sourceforge](#)
- [Open Source Codes on Bitbucket](#)



Share this:



The work of Stéphane Bordas was supported in part by the European Research Council under the European Union's S Grant Agreement n. 279578

## Personal Data

**Username:** cmechanicosos  
**Joined:** 2012-04-13 14:29:25

## Projects



### ElemFreGalerkin

A tutorial Galerkin meshfree code

*Last Updated: 2017-01-29*



### OpenXfem++

OpenXfem++ is an XFEM (eXtended Finite Element Method) written in C++.

*Last Updated: 2017-01-28*



### XFEM

XFEM implementation in MATLAB

*Last Updated: 2017-02-08*



### ciGen

ciGen is a short C++ code to generate cohesive interface elements.

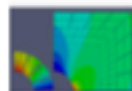
*Last Updated: 2017-01-25*



### igabem

Isogeometric boundary element analysis with matlab

*Last Updated: 2017-03-02*



### igafem

Open source 3D Matlab Isogeometric Analysis Code

*Last Updated: 2017-02-05*



### igafemgui

*Last Updated: 2017-05-10*

★ 5.0 Stars (2)  
↓ 33 Downloads (This Week)  
📅 Last Update: 2015-08-07

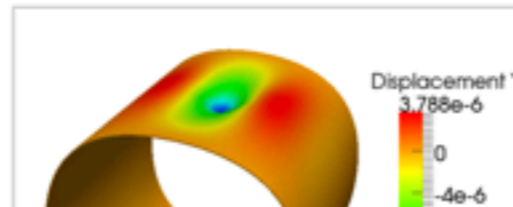
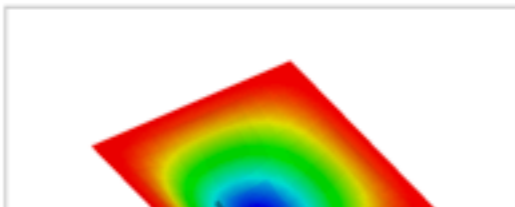
sf **Download**  
lgafem-v2.zip

[Tweet](#) [G+1](#) [0](#) [Like 4](#)

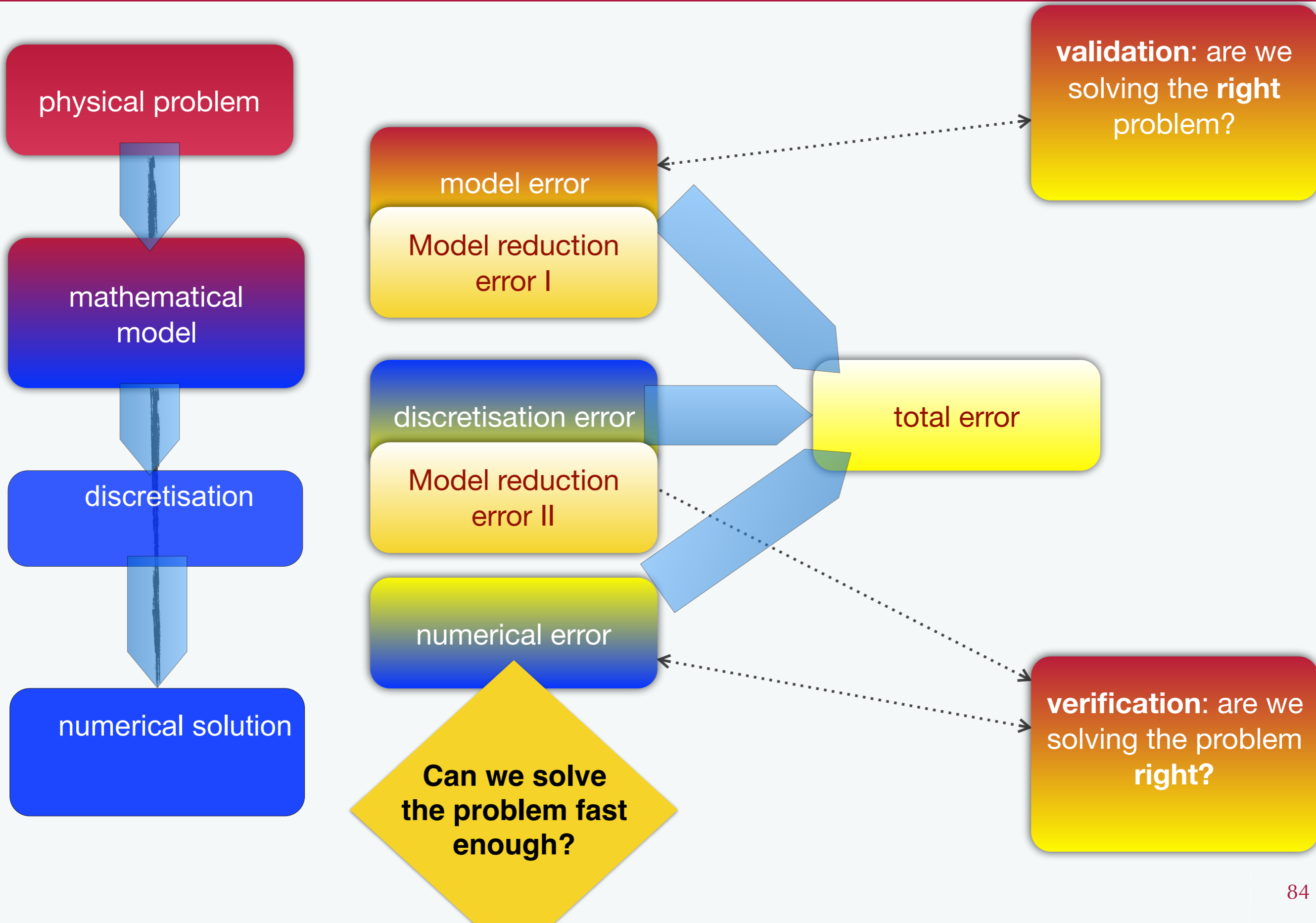


[Browse All Files](#)

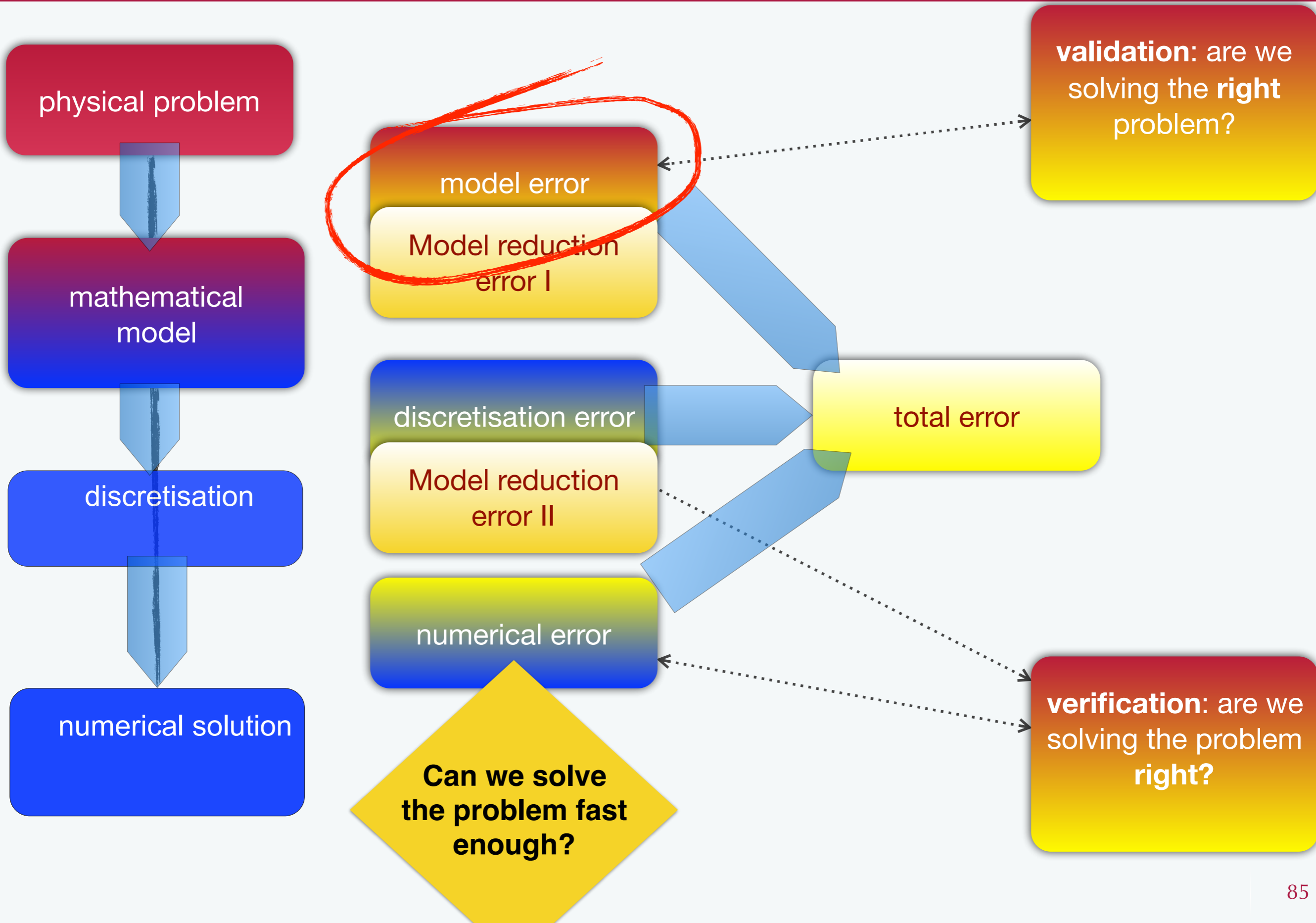
freeends

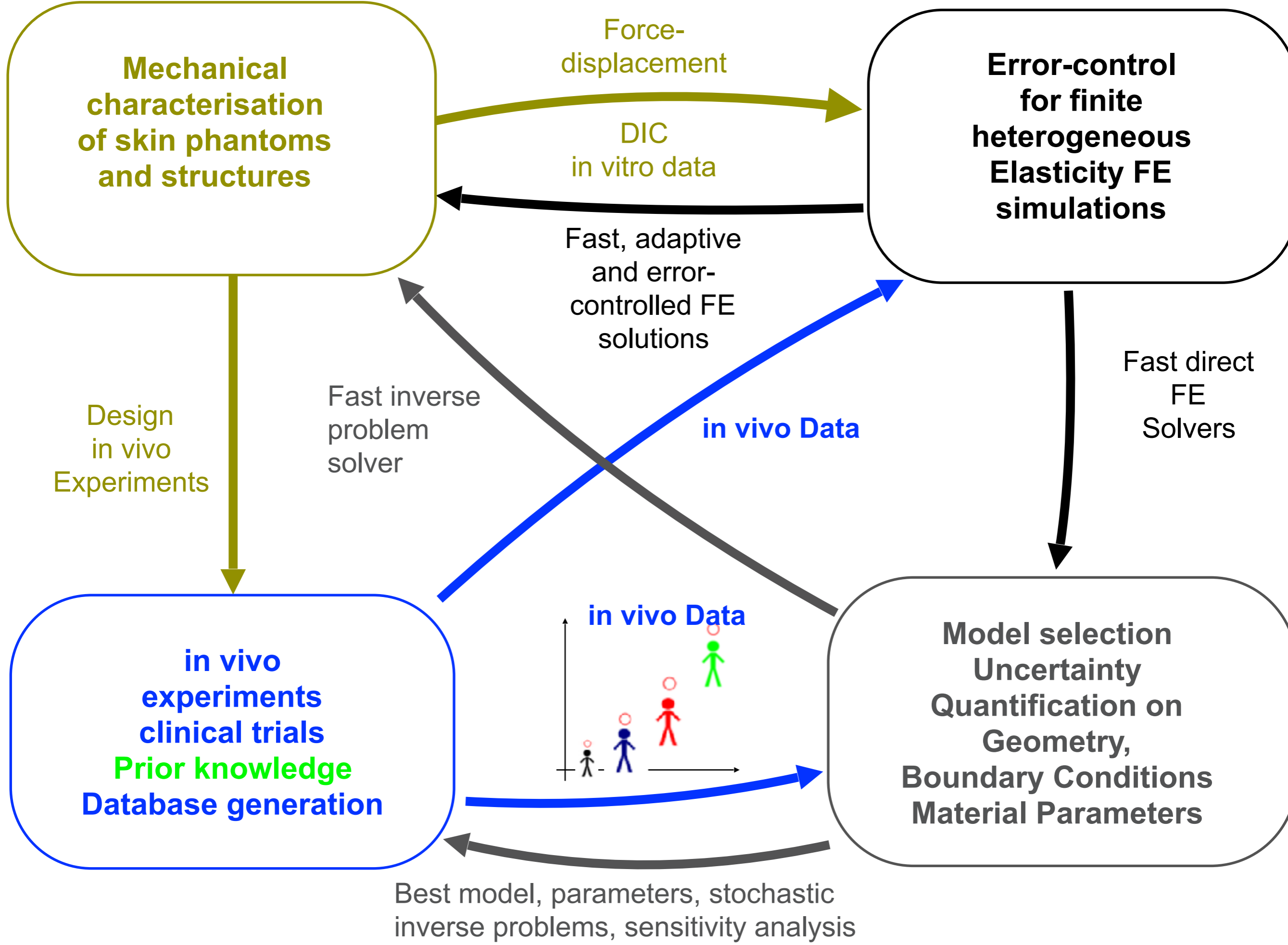


# Modelling and simulation



# Modelling and simulation





# Ingredient #1 - Error control

Error-control  
for finite  
heterogeneous  
Elasticity FE  
simulations

## Ingredient #2 - Uncertainty quantification

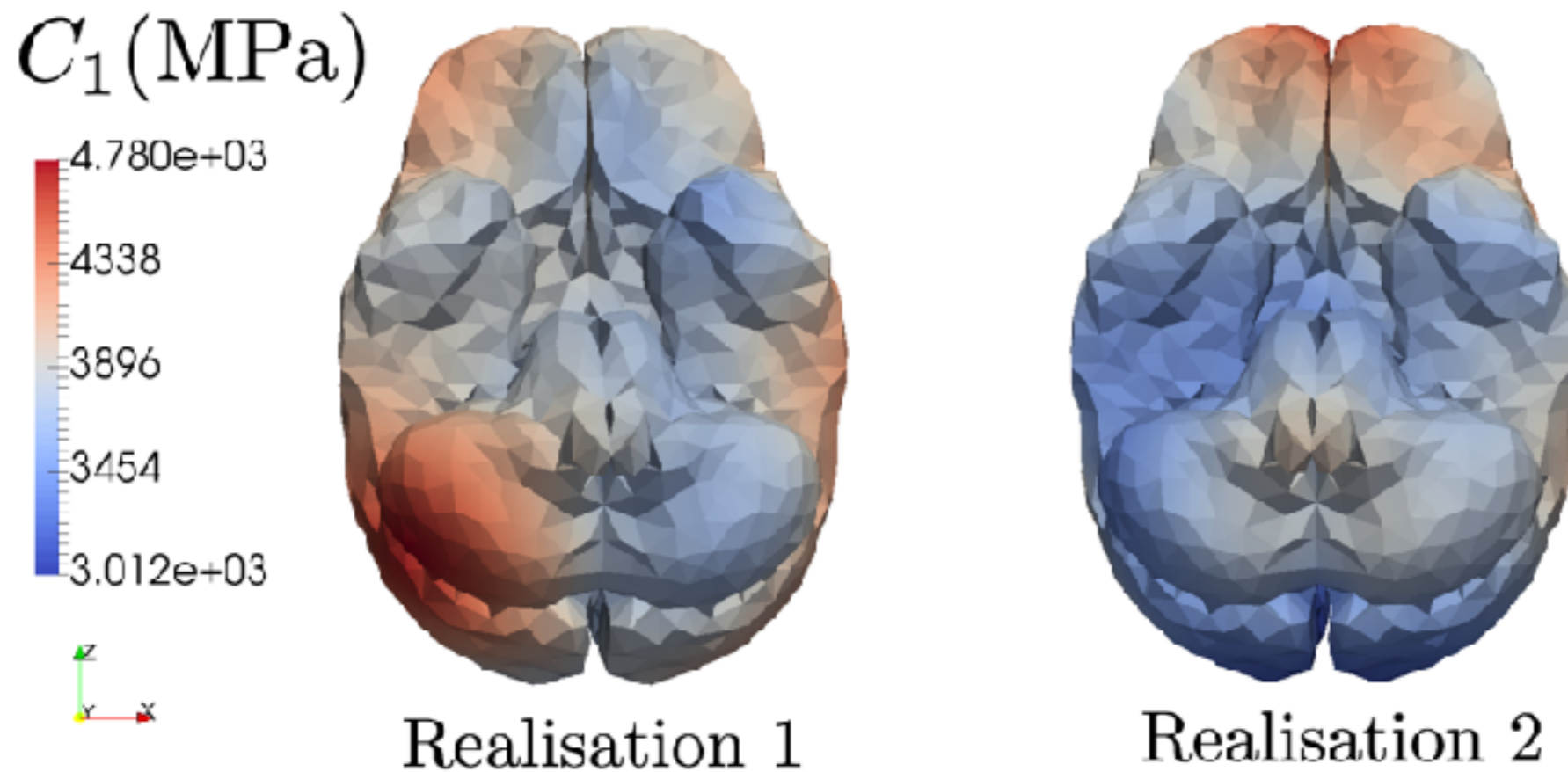
Model selection  
Uncertainty  
Quantification on  
Geometry,  
Boundary Conditions  
Material Parameters



# Random Fields

- ▶ Different methods: Karhunen–Loève expansion [Adler 2007], Fast Fourier transform [Nowak 2004].

## Randoms fields

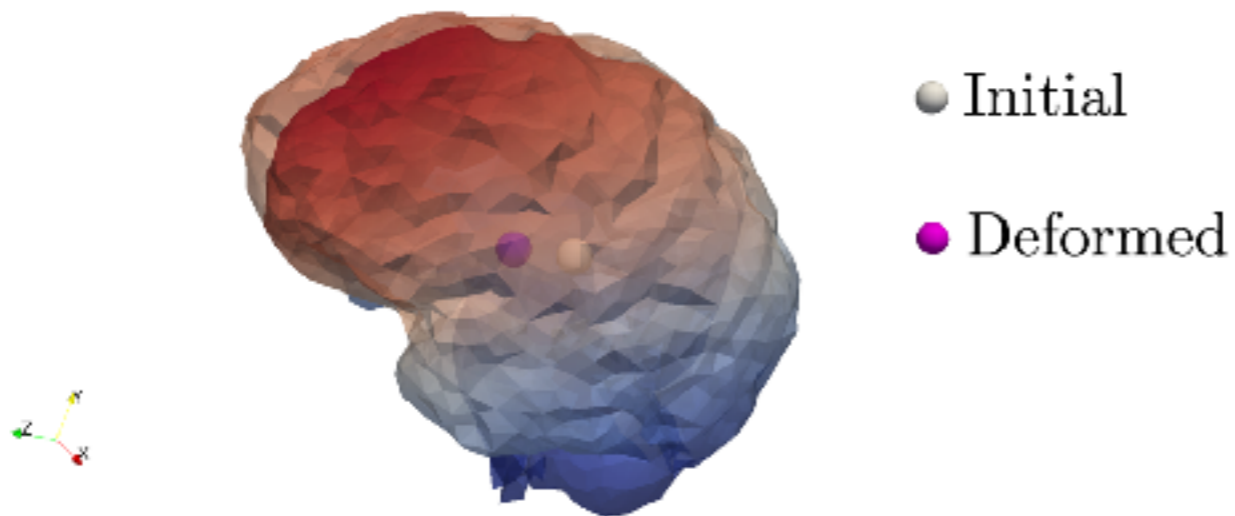
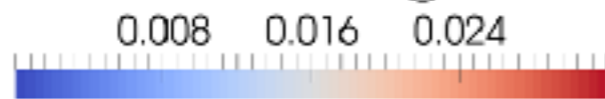


Two realisations of RF, with a log-normal distribution, for the parameter  $C_1$  (in MPa).

# Stochastic FE analysis of brain deformation

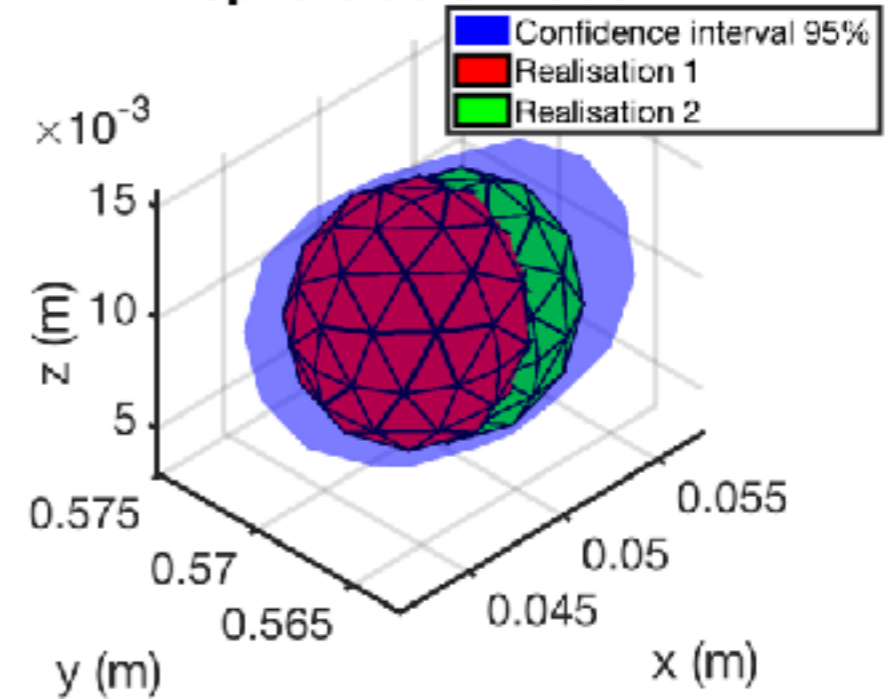
## Numerical results (8 RV, Holzapfel model)

Displacement magnitude (m)



Brain deformation with random parameters  
1 MC realisation.

**Sphere deformation**



Confidence interval 95%  
MC simulations.

# Numerical results: convergence

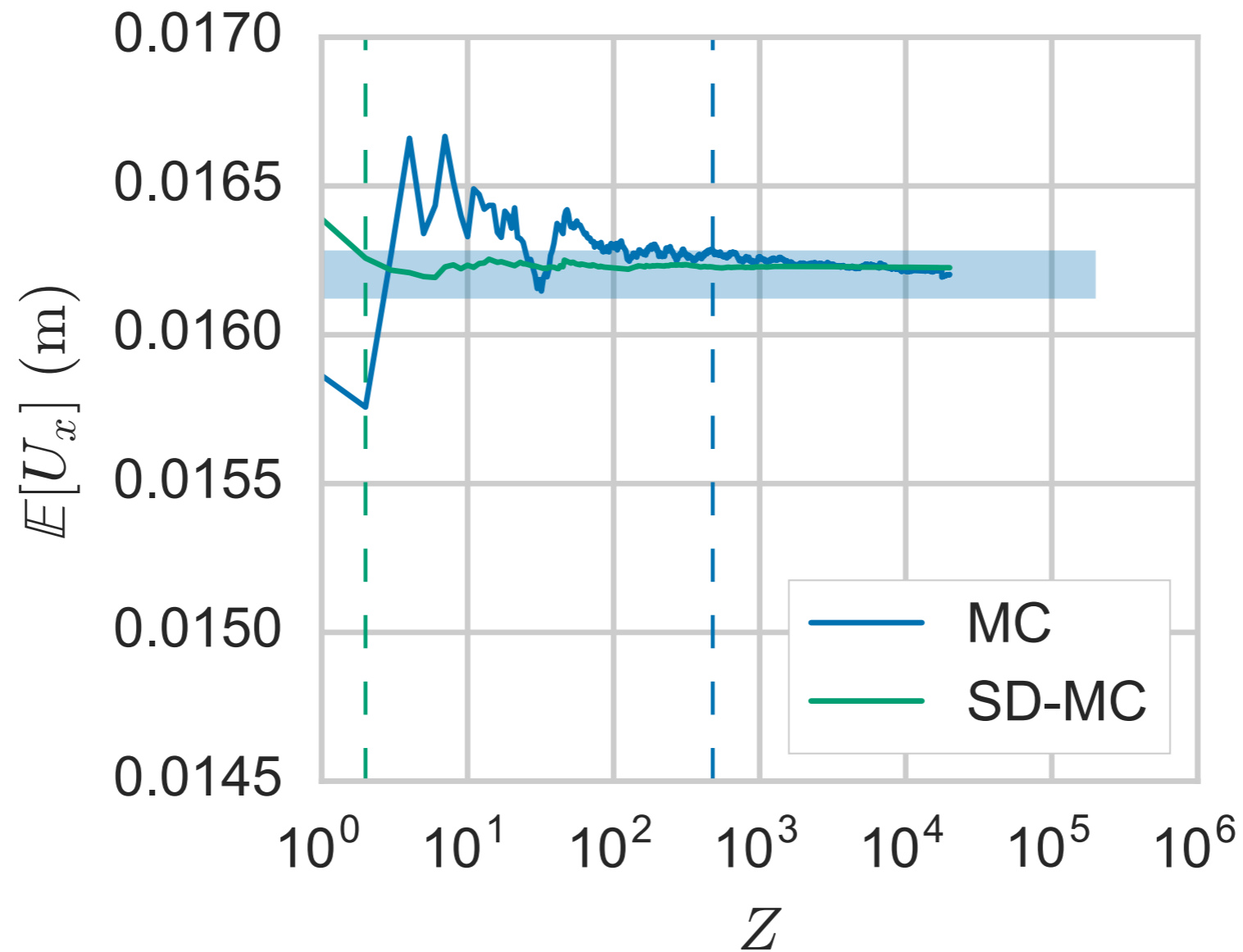
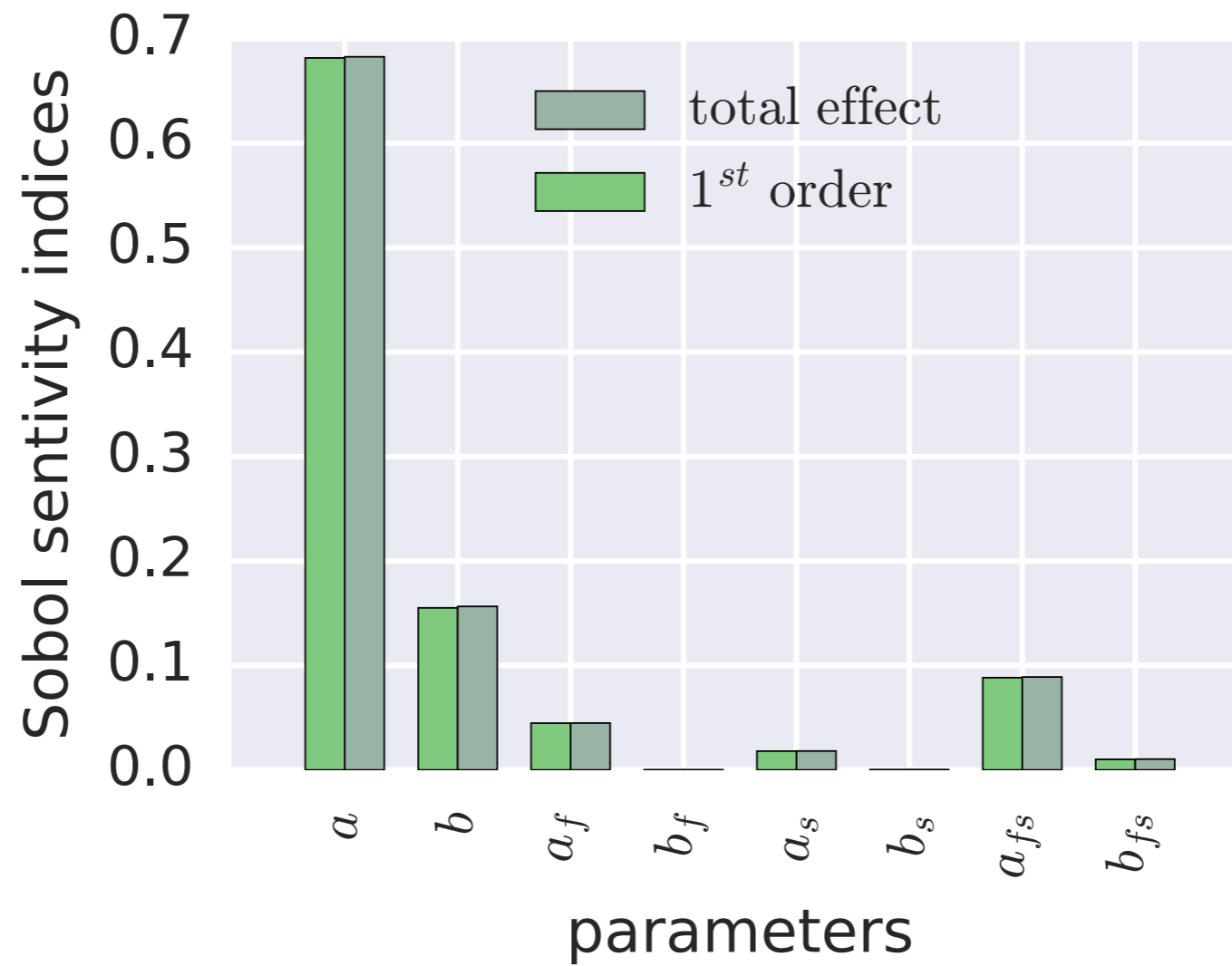


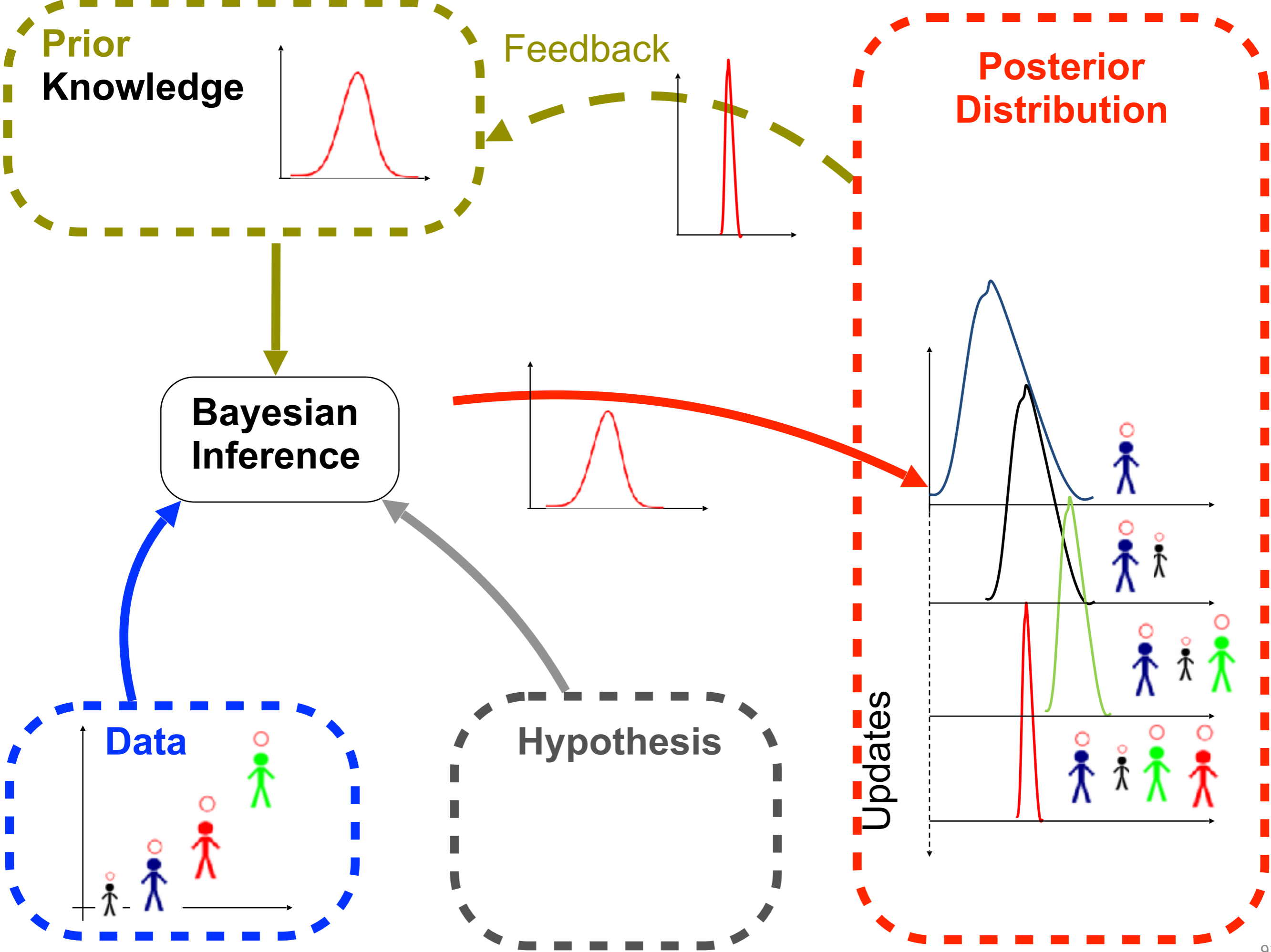
Fig. Center of the sphere: expected value of the displacement in the x direction as a function of  $Z$ .

# Global sensitivity analysis

► Sobol sensitivity indices [Sobol 2015, Saltelli 2002]



Quantity of interest: displacement magnitude of the target.



# MODEL PROBLEM

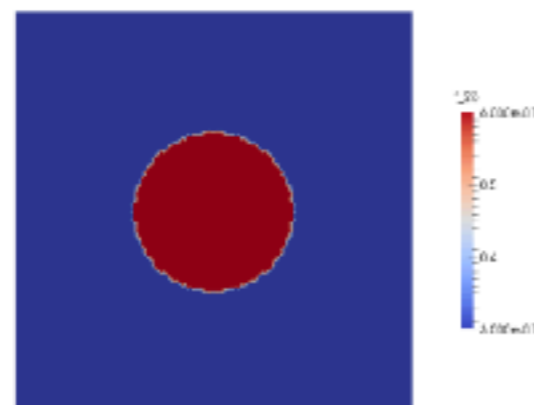
Given displacement observations on the surface of a block of soft tissue, possibly containing a stiff tumor, what can we infer about the material parameters of the tissue inside? How sure are we about what we infer?

## FIGURE 1



Left: Three virtual experimental results from applying three different loads to the same non-homogeneous block of soft tissue. We are only given the observations on the exterior surface, and they are corrupted by random white noise.

## FIGURE 2



Left: The true material parameter field used to generate the experimental data in Figure 1. A stiff circular tumour is surrounded by softer healthy tissue.

# Elastography under uncertainty

## METHODOLOGY

- ▶ We use the Bayesian framework for statistical inference (Stuart, 2010).
- ▶ Allows for rigorous statistical quantification of uncertainty arising from:
  - ▶ Partial observations.
  - ▶ Noisy instruments.
  - ▶ Model inadequacy.
- ▶ Soft tissue modelled by a fully non-linear hyperelastic PDE.
- ▶ Flexible Gaussian noise and prior modelling.
- ▶ We use derivatives of the finite element model to find the most likely material parameters and approximate the covariance structure.

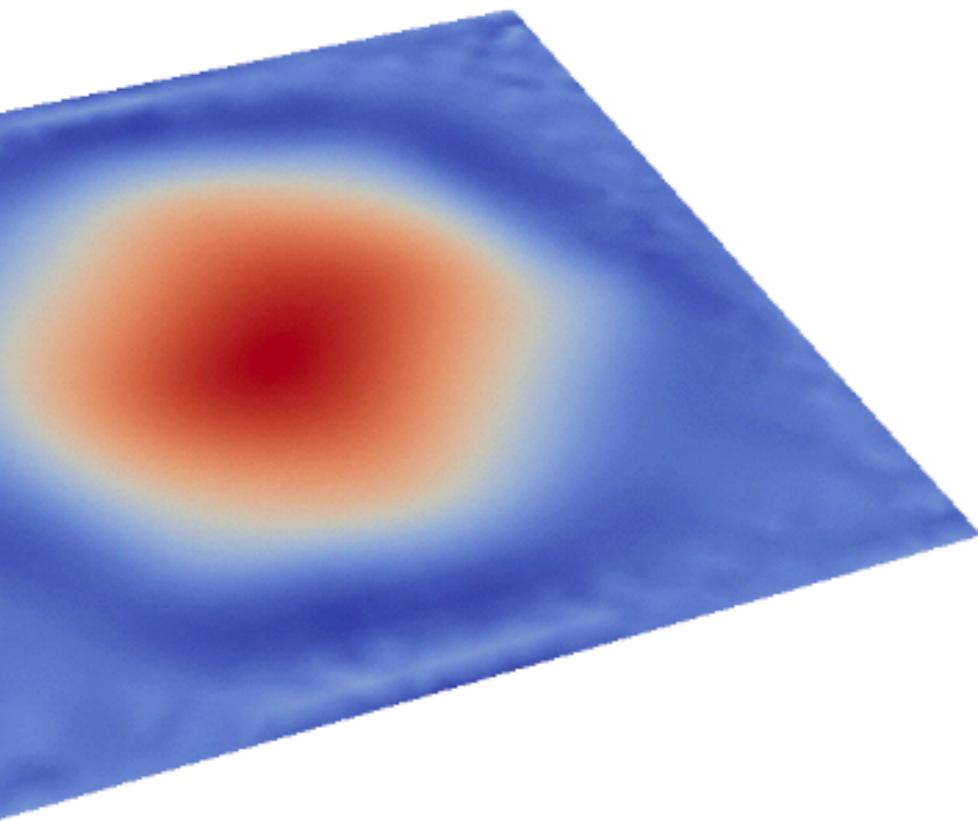
# Elastography under uncertainty

## COMPUTATIONAL TECHNIQUES

- ▶ Automatic construction of forward and adjoint models with dolfin-adjoint (Farrell et al., 2013). *Easy to change physical model.*
- ▶ Efficient algebraic multigrid preconditioning of forward and adjoint models. *Forward runs dominate overall cost, reduce as much as possible.*
- ▶ Gauss-Newton Conjugate-Gradient method to find maximum a posteriori point. *Scales well on mesh refinement.*
- ▶ Matrix-free Krylov-Schur algorithm for principal component analysis of prior pre-and-post-conditioned Hessian of likelihood. *Fixed cost for given observations/model.*
- ▶ Optimal low-rank update from prior to posterior covariance (Spantini et al., 2014). *Reduces Hessian actions/forward model runs.*



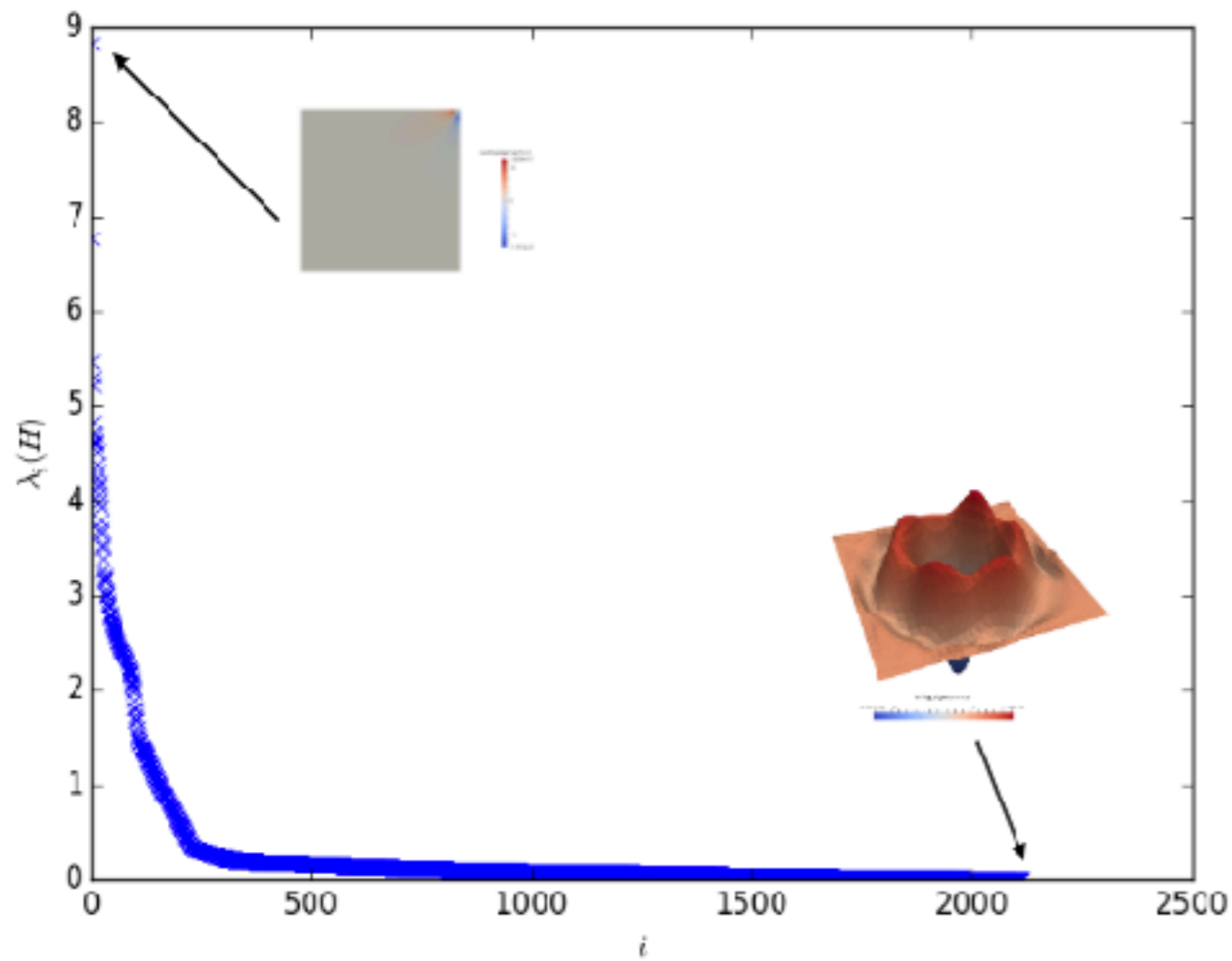
# Elastography under uncertainty



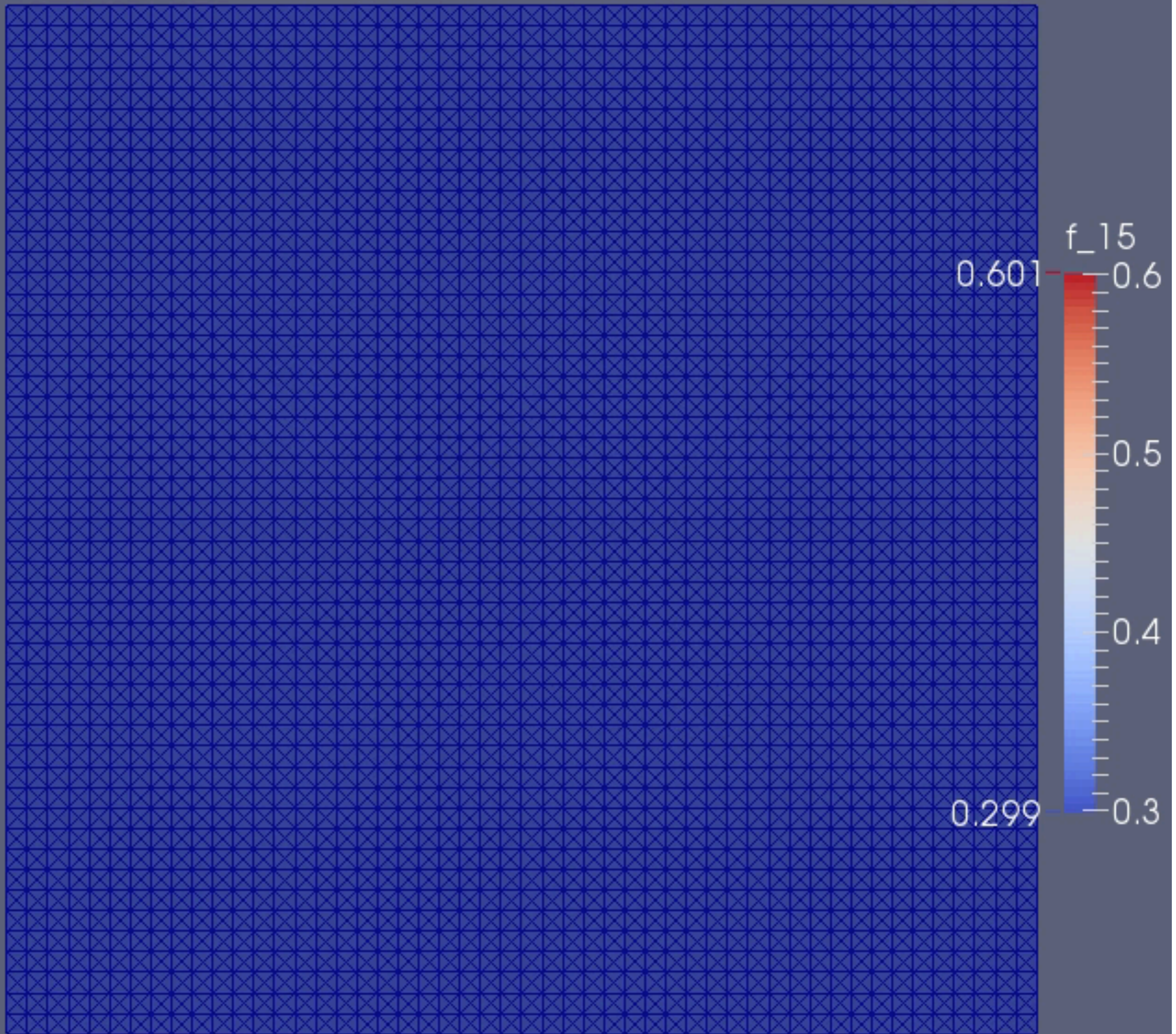
Left: Recovered MAP point, cf. Figure 1.  
We *can* detect the stiff inclusion inside the object just from the noisy surface observations.

# Elastography under uncertainty

FIGURE 5



Left: Low-rank structure of spectrum of posterior covariance. Data is only informative on low-rank subspace of original parameter space. Top left eigenvector points towards direction in parameter space most-constrained by the observations, bottom right towards least-constrained.



Thank you for your attention!

You can download these slides here

<http://hdl.handle.net/10993/31487>

or

[http://orbilu.uni.lu/bitstream/10993/31487/1/  
XDMS\\_2017\\_Bordas.pdf](http://orbilu.uni.lu/bitstream/10993/31487/1/XDMS_2017_Bordas.pdf)

or/and email me

[stephane.bordas@alum.northwestern.edu](mailto:stephane.bordas@alum.northwestern.edu)

Innovative Training Network RAINBOW  
funded by Horizon 2020

15 PhD studentships available !



Patient-Specific Data



Expert Knowledge



Guidance

Design of Implants & Prosthetics

Diagnosis

Surgical Training

Prognosis

Medical Devices

Planning

Monitoring

<p>Cardiovascular Devices</p> <p>u </p>	<p>Scoliosis</p> <p></p>	<p>Eye surgery</p> <p></p>	<p>Neurology</p> <p></p>	<p>Spine Braces</p> <p>u </p>
<p>Hip growth</p> <p></p>	<p>Prostate Cancer</p> <p></p>	<p>Intraoperative radiotherapy</p> <p>u </p>	<p>Surgical guidance</p> <p></p>	<p>Soft organ diagnosis</p> <p></p>
<p>Dental prostheses</p> <p></p>	<p>Breast Cancer</p> <p></p>	<p>Surgical navigation</p> <p>u </p>	<p>Surgical planning</p> <p></p>	<p><b>Apps</b></p>



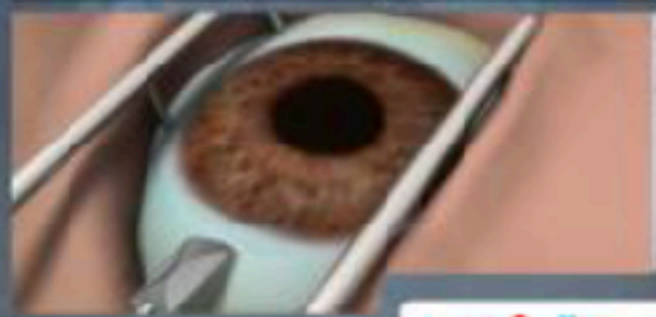
Patient-Specific Data



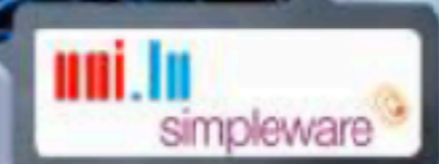
Expert Knowledge



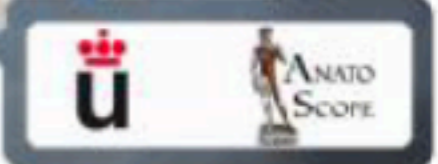
Eye surgery



Neurology



Spine Braces



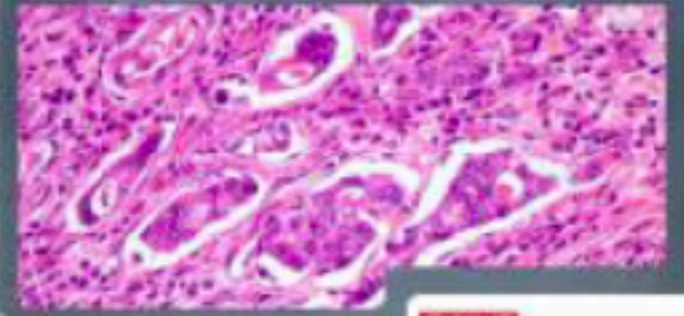
Intraoperative radiotherapy



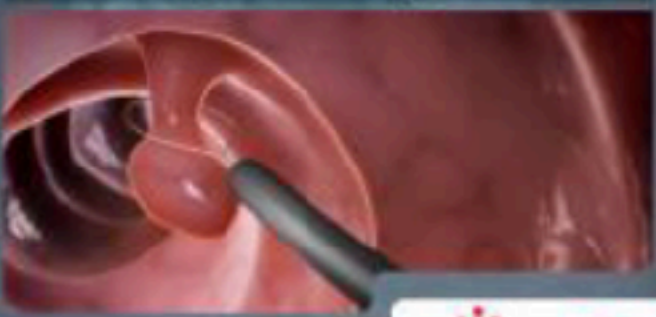
Surgical guidance



Soft organ diagnosis



Surgical navigation



Surgical planning



# Apps



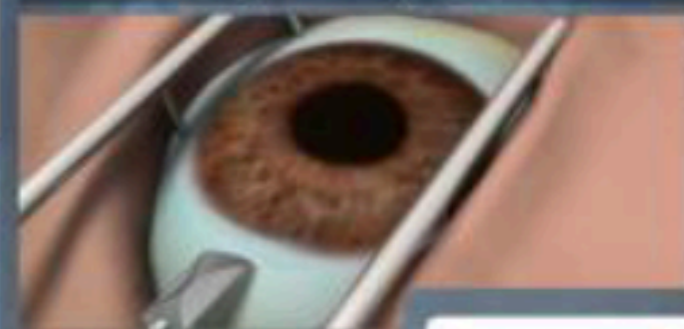
Cardiovascular Devices



Scoliosis



Eye surgery



Hip growth



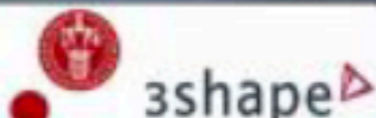
Prostate Cancer



Intraoperative radiotherapy



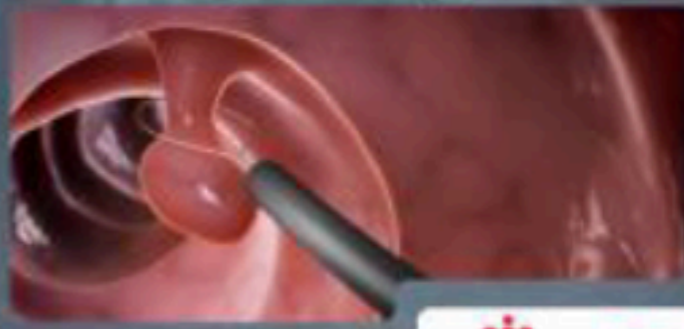
Dental prostheses



Breast Cancer



Surgical navigation



### **Multi-scale fracture and model order reduction**

Pierre Kerfriden, Lars Beex, Jack Hale, Olivier Goury, Daniel Alves Paladim, Elisa Schenone, Davide Baroli, Thanh Tung Nguyen, Hoang Khac Chi, Timon Rabczuk

### **Advanced discretisation techniques**

Elena Atroshchenko, Danas Sutula, Xuan Peng, Haojie Lian, Peng Yu, Qingyuan Hu, Sundararajan Natarajan, Nguyen-Vinh Phu

### **Error estimation**

Pierre Kerfriden, Satyendra Tomar, Daniel Alves Paladim, Andrés Gonzalez Estrada

### **Biomechanics applications**

Alexandre Bilger, Hadrien Courtecuisse, Bui Huu Phuoc

## *Fracture of homogeneous materials*

[https://publications.uni.lu/bitstream/10993/10039/1/2013stressAnlaysiaWithoutMeshingIGABEM3D\\_ICE.pdf](https://publications.uni.lu/bitstream/10993/10039/1/2013stressAnlaysiaWithoutMeshingIGABEM3D_ICE.pdf)

[https://orbilu.uni.lu/bitstream/10993/14029/2/superGA\\_upd26\\_05\\_2014.pdf](https://orbilu.uni.lu/bitstream/10993/14029/2/superGA_upd26_05_2014.pdf)

<https://arxiv.org/pdf/1210.8216>

<https://arxiv.org/pdf/1610.01694>

[https://orbilu.uni.lu/bitstream/10993/12915/1/2011implicitDomainFEM\\_CMAME.pdf](https://orbilu.uni.lu/bitstream/10993/12915/1/2011implicitDomainFEM_CMAME.pdf)

[http://orbilu.uni.lu/bitstream/10993/17993/2/mi130149\\_proof.pdf](http://orbilu.uni.lu/bitstream/10993/17993/2/mi130149_proof.pdf)

[http://orbilu.uni.lu/bitstream/10993/17383/1/meshburdenreduction2010v6%20\(2b\)Enviado.pdf](http://orbilu.uni.lu/bitstream/10993/17383/1/meshburdenreduction2010v6%20(2b)Enviado.pdf)

# Recent open access publications

**What makes Data Science different?**

<http://hdl.handle.net/10993/30235>

**Energy-minimal crack growth**

<http://hdl.handle.net/10993/29414>

**Uncertainty quantification for soft tissue biomechanics**

<http://orbilu.uni.lu/handle/10993/28618>

<http://orbilu.uni.lu/handle/10993/30946>

**Real-time simulation and error control**

<http://orbilu.uni.lu/handle/10993/29846>

<http://orbilu.uni.lu/handle/10993/30937>

**Bayesian parameter identification in mechanics**

<http://orbilu.uni.lu/bitstream/10993/29561/3/template.pdf>

<http://orbilu.uni.lu/bitstream/10993/28631/1/1606.02422v4.pdf>

<http://orbilu.uni.lu/handle/10993/28631>