An equation-free multiscale method

a result of extending the quasicontinuum method to irregular structures

Lars Beex — University of Luxembourg

Pierre Kerfriden — Cardiff University

For everybody

For everybody

UNIVERSITÉ DU LUXEMBOURG

1st part:

Explain the ansatz as simply as possible

For everybody

UNIVERSITÉ DU LUXEMBOURG

1st part:

Explain the ansatz as simply as possible

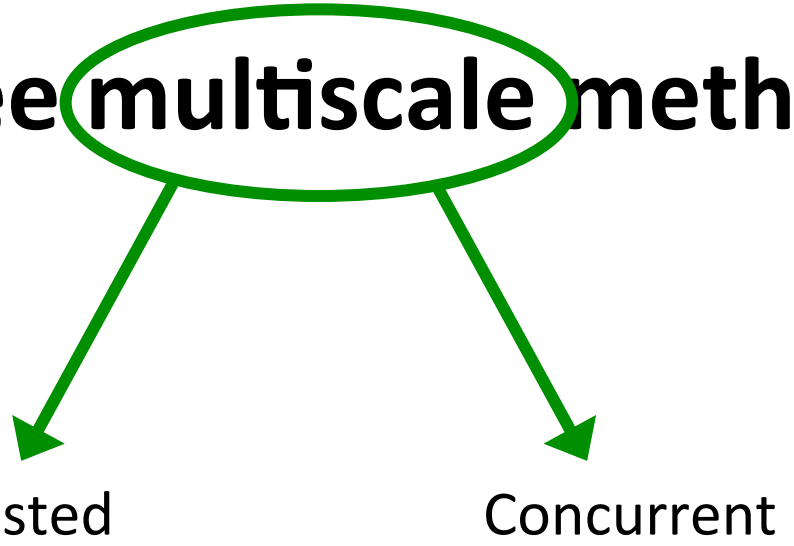
2nd part:

Give an overview of previous works on regular structures

For the people familiar with the FE^2 schemes

UNIVERSITÉ DU LUXEMBOURG

An equation-free multiscale method



UNIVERSITÉ DU LUXEMBOURG

An equation-free multiscale method

No micro-to-macro relations
No macro-to-micro relations
No microstructural BCs

For the people familiar with the FE^2 schemes

UNIVERSITÉ DU LUXEMBOURG

Condition:

Unit cell must be periodic.

For the people familiar with the FE^2 schemes

UNIVERSITÉ DU LUXEMBOURG

Condition:

Unit cell must be periodic.

Advantages compared to other nested multiscale methods:

1. Higher-order macroscale interpolations are as easy to treat as linear ones

2. No scale-separation

For the people familiar with the FE^2 schemes

UNIVERSITÉ DU LUXEMBOURG

Condition:

Unit cell must be periodic.

Advantages compared to other nested multiscale methods:

1. Higher-order macroscale interpolations are as easy to treat as linear ones

2. No scale-separation

Disadvantages compared to other nested multiscale methods:

1. All DOFs in one system, instead of subdivided over the unit cells and the macroscale elements

2. More unit cells required

For the people familiar with reduced-order modelling

The QC method is a ROM approach, BUT

The QC method is a ROM approach, BUT

- No offline snapshots generation

The QC method is a ROM approach, BUT

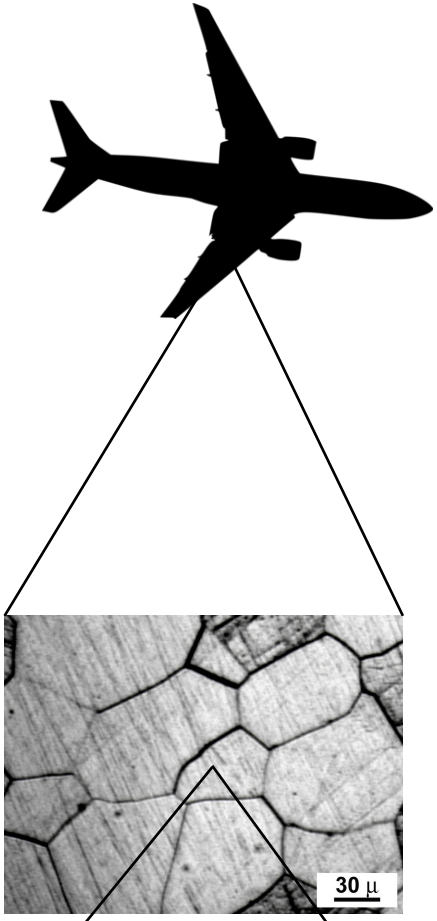
- No offline snapshots generation

- Modes are local and follow partition-of-unity:
    - displacement BCs are easy to apply
    - online adaptivity of modes ‘easy’ to incorporate (ROKOS)

For the people familiar with the QC method

UNIVERSITÉ DU LUXEMBOURG

For the people familiar with the QC method



Engineering scale m

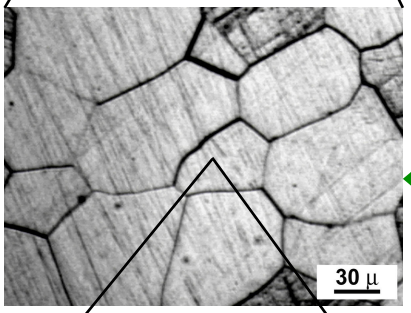
Meso scale mm

Micro scale µm

Nano scale nm

For the people familiar with the QC method

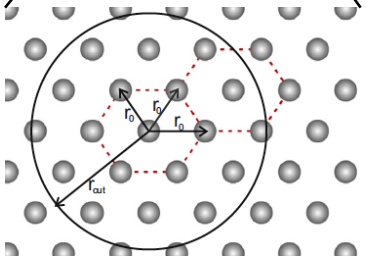
Engineering scale m



Meso scale mm

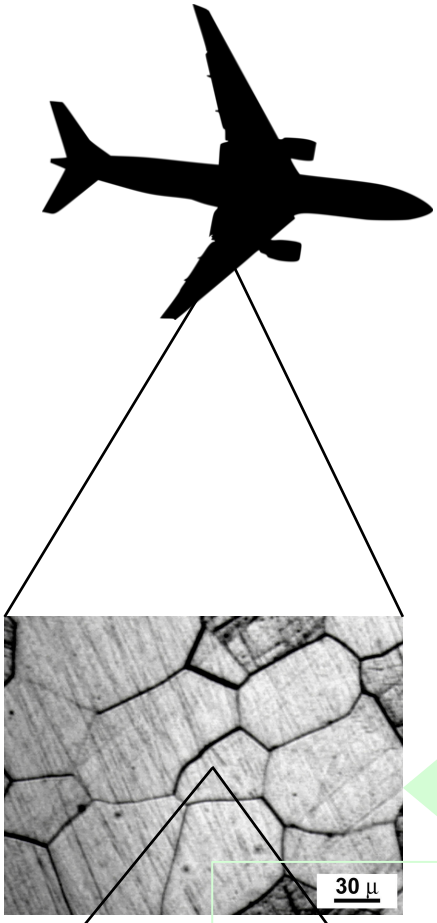
Micro scale µm

QC method

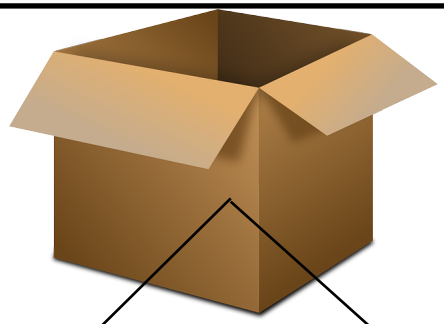


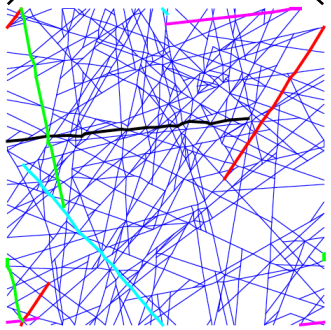
Nano scale nm

For the people familiar with the QC method



Engineering scale m

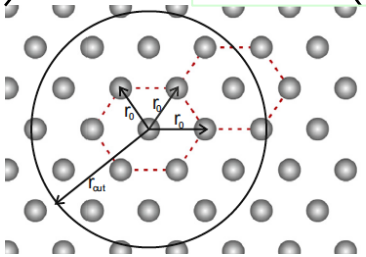




Meso scale mm

Micro scale µm

QC method

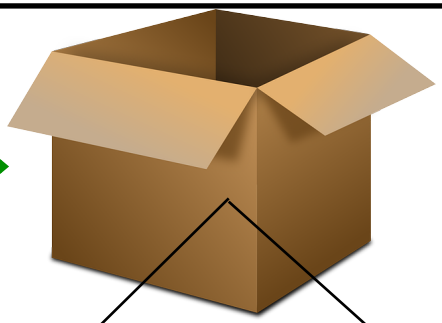


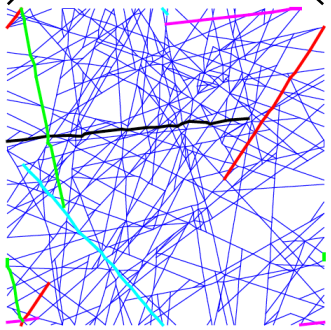
Nano scale nm

For the people familiar with the QC method

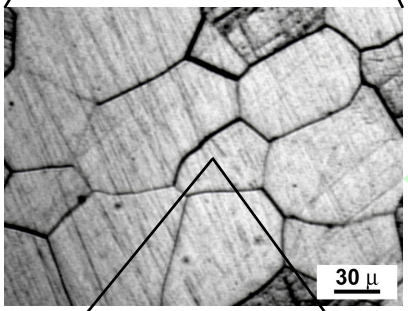
Engineering scale m

QC method



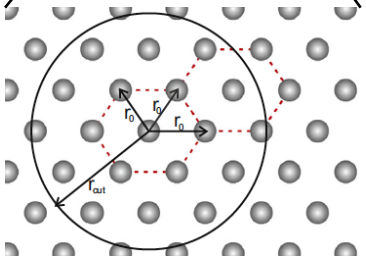


Meso scale mm



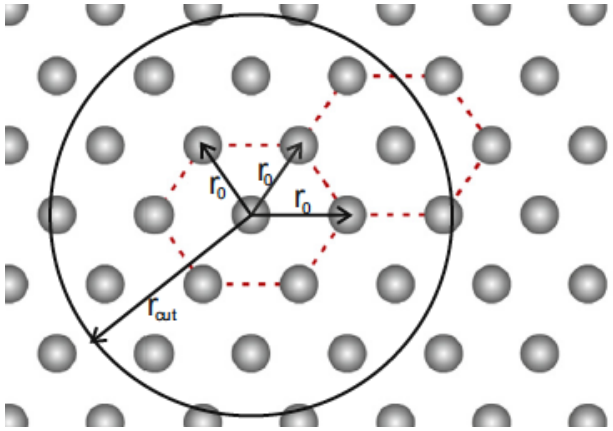
Micro scale µm

QC method

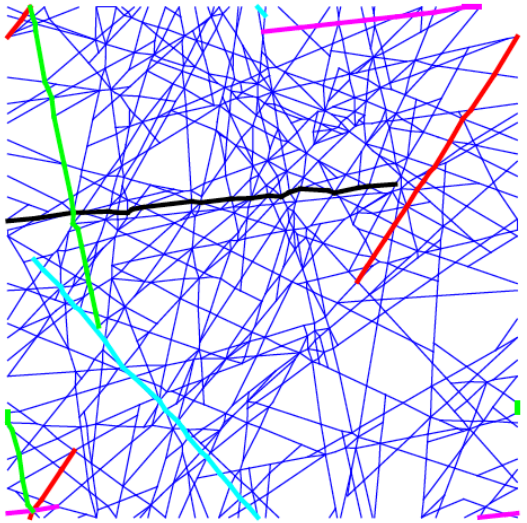


Nano scale nm

For the people familiar with the QC method



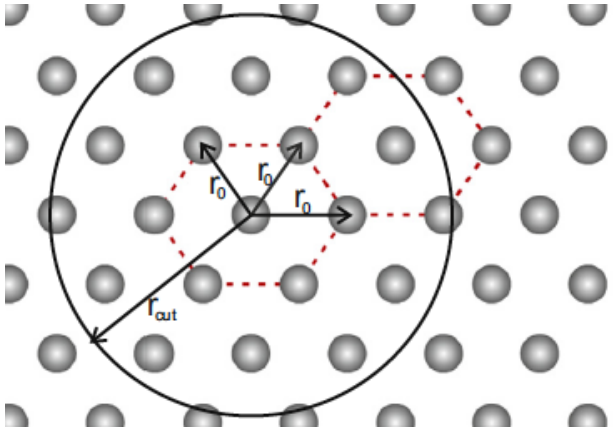
Structural differences



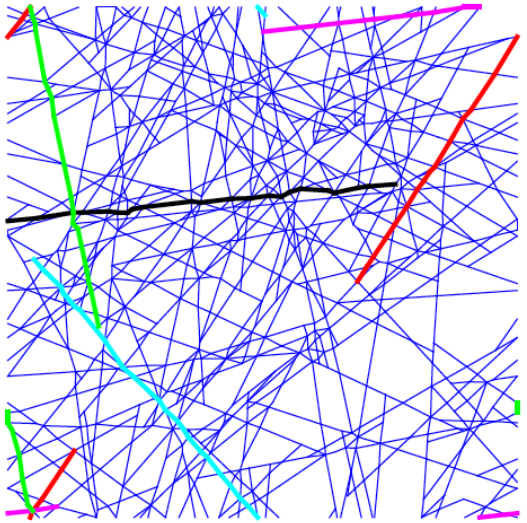
Next-to-nearest neighbours etc.

Nearest neighbours only

For the people familiar with the QC method



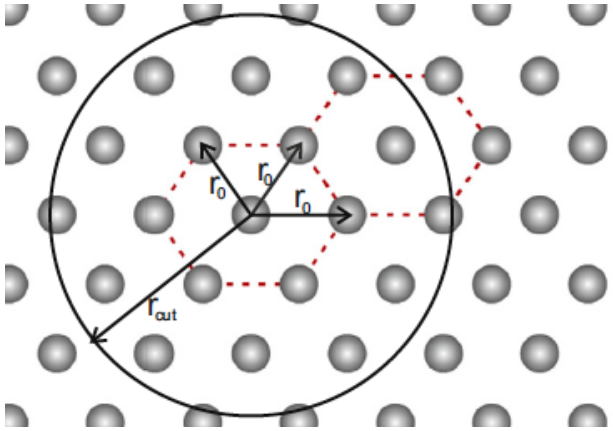
Structural differences



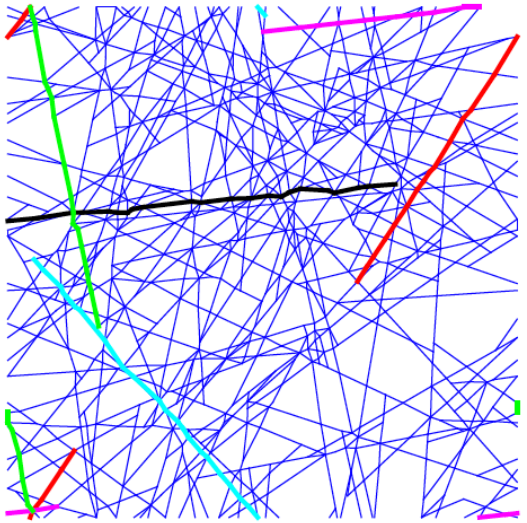
Next-to-nearest neighbours etc.
Non-local interactions

Nearest neighbours only
Local interactions

For the people familiar with the QC method



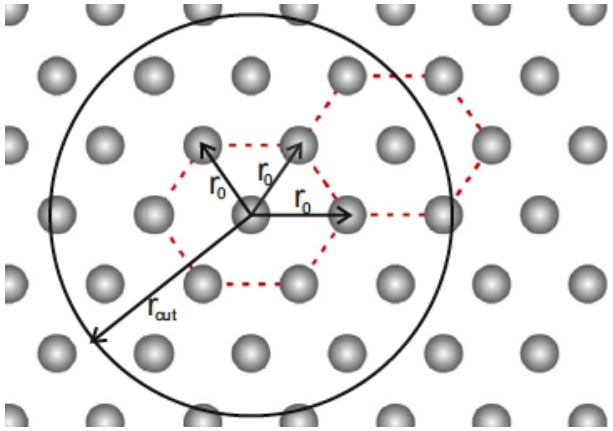
Structural differences



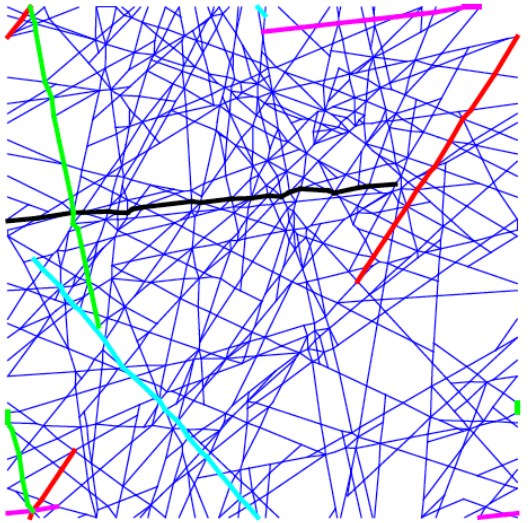
Next-to-nearest neighbours etc.
Non-local interactions
Elastic interactions

Nearest neighbours only
Local interactions
Dissipative interactions

For the people familiar with the QC method



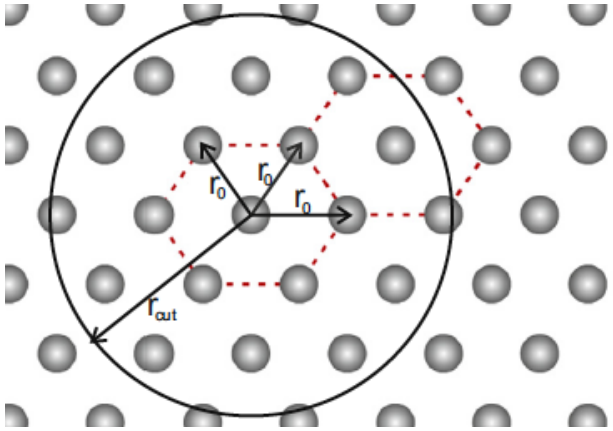
Structural differences



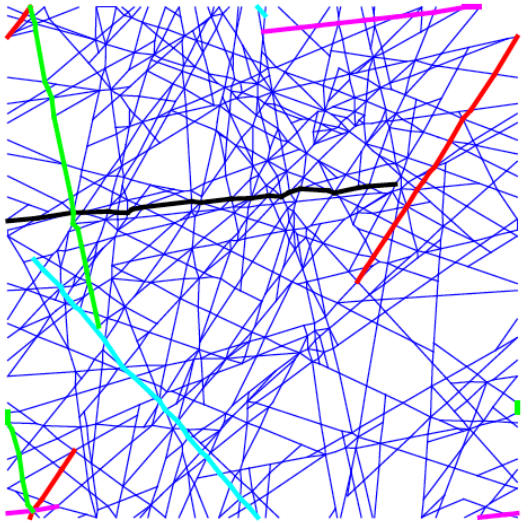
Next-to-nearest neighbours etc.
Non-local interactions
Elastic interactions
Interactions are springs

Nearest neighbours only
Local interactions
Dissipative interactions
Interactions may be beams

For the people familiar with the QC method



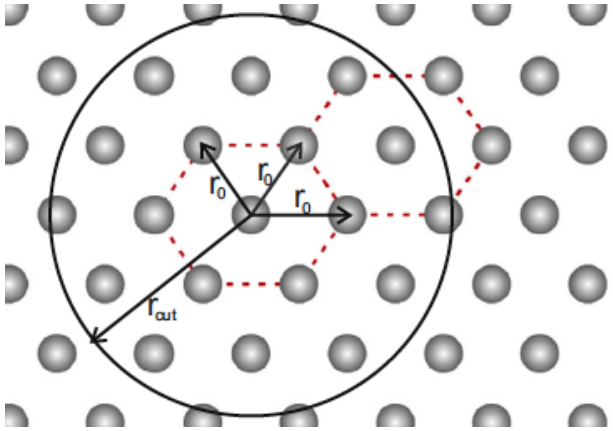
Structural differences



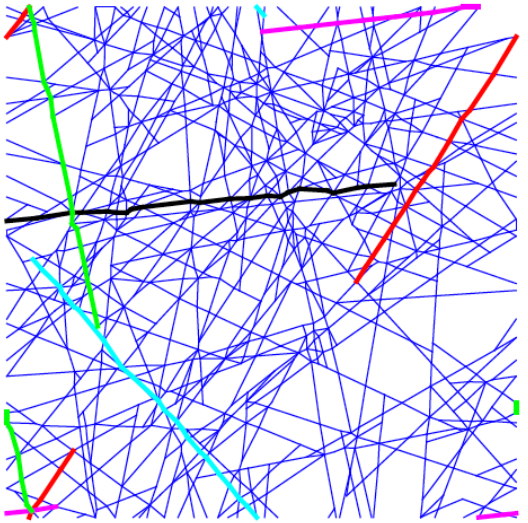
Next-to-nearest neighbours etc.
Non-local interactions
Elastic interactions
Interactions are springs
Connectivity may change

Nearest neighbours only
Local interactions
Dissipative interactions
Interactions may be beams
Connectivity is constant

For the people familiar with the QC method



Structural differences



Next-to-nearest neighbours etc.
Non-local interactions
Elastic interactions
Interactions are springs
Connectivity may change
LATTICES

Nearest neighbours only
Local interactions
Dissipative interactions
Interactions may be beams
Connectivity is constant
IRREGULAR NETWORKS

Agenda

UNIVERSITÉ DU LUXEMBOURG

- The QC method in a nutshell

Agenda

UNIVERSITÉ DU LUXEMBOURG

- The QC method in a nutshell

- From regular lattice to irregular networks
    - string of 1D LE springs
    - string of LE EB beams
    - planar unit cells of beams

Agenda

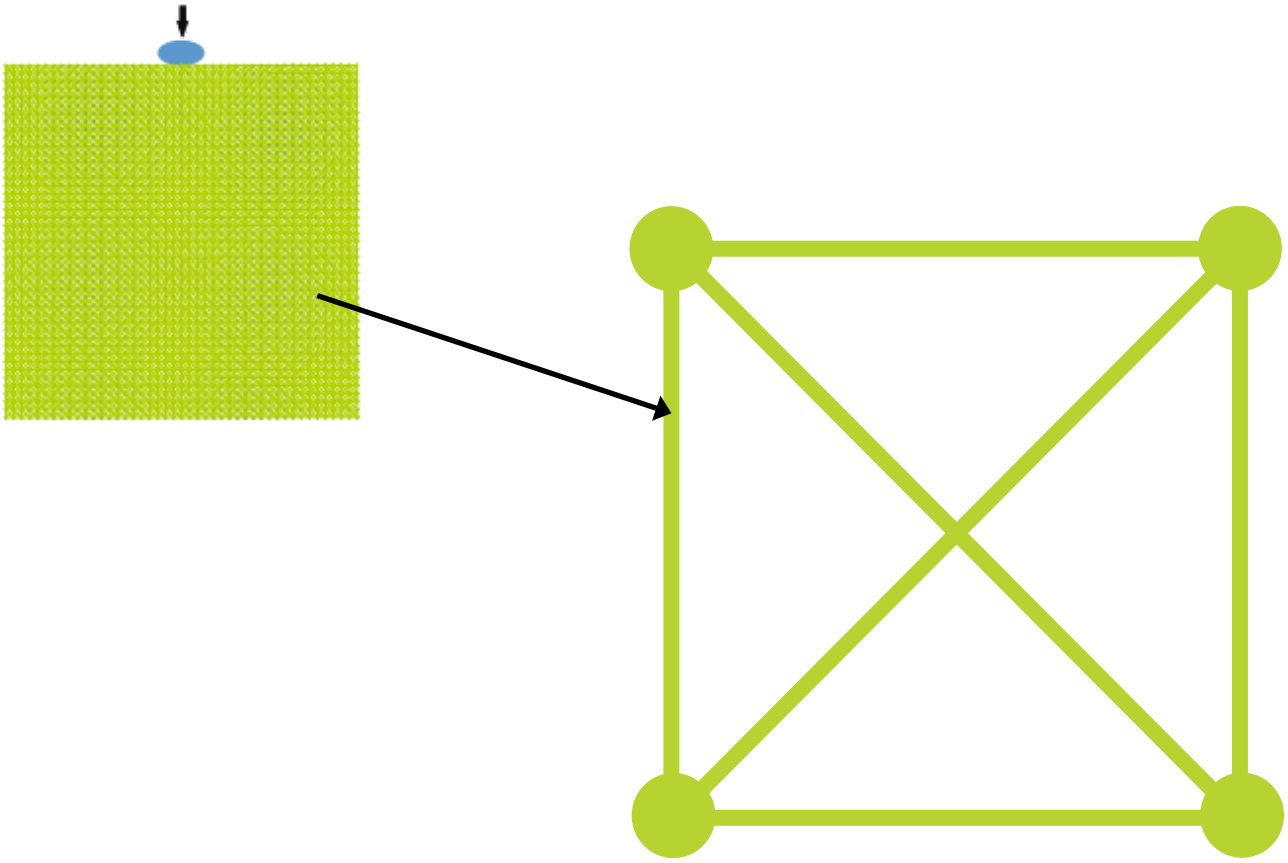
UNIVERSITÉ DU LUXEMBOURG

- The QC method in a nutshell

- From regular lattice to irregular networks
  - string of 1D LE springs
  - string of LE EB beams
  - planar unit cells of beams

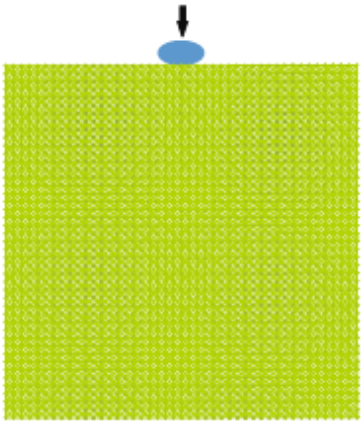
- Previous works on regular lattices
  - summation
  - plasticity in interactions
  - frictional sliding in nodes

The QC method in a nutshell



Unit cell

The QC method in a nutshell

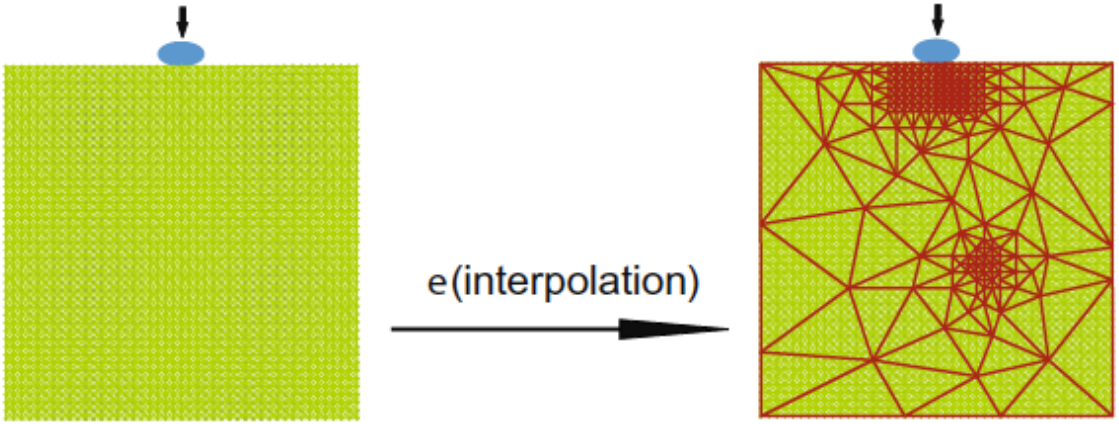


Large lattice computations are expensive due to

1. Large numbers of DOFs

2. Large numbers of interactions

The QC method in a nutshell

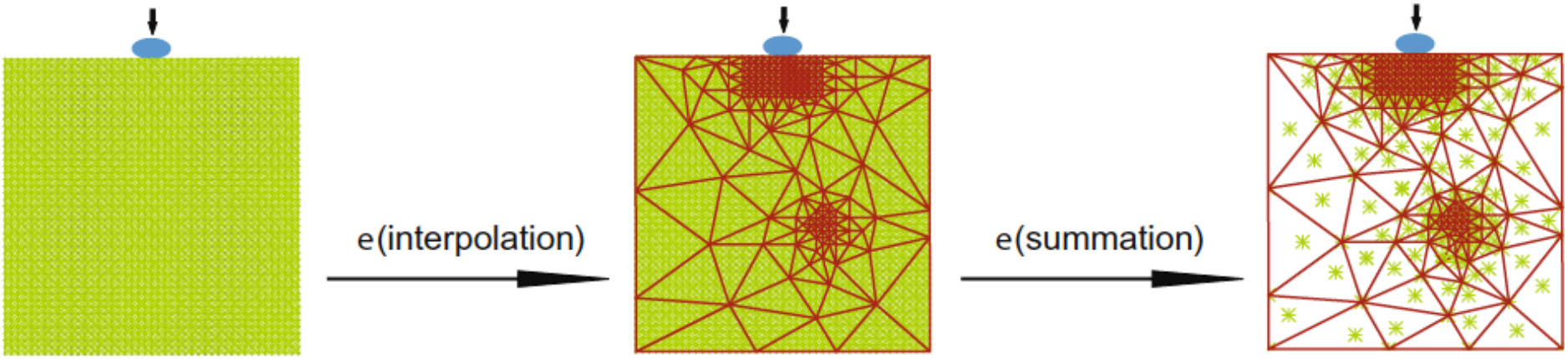


Large lattice computations are expensive due to

1. Large numbers of DOFs **interpolation**

2. Large numbers of interactions

The QC method in a nutshell

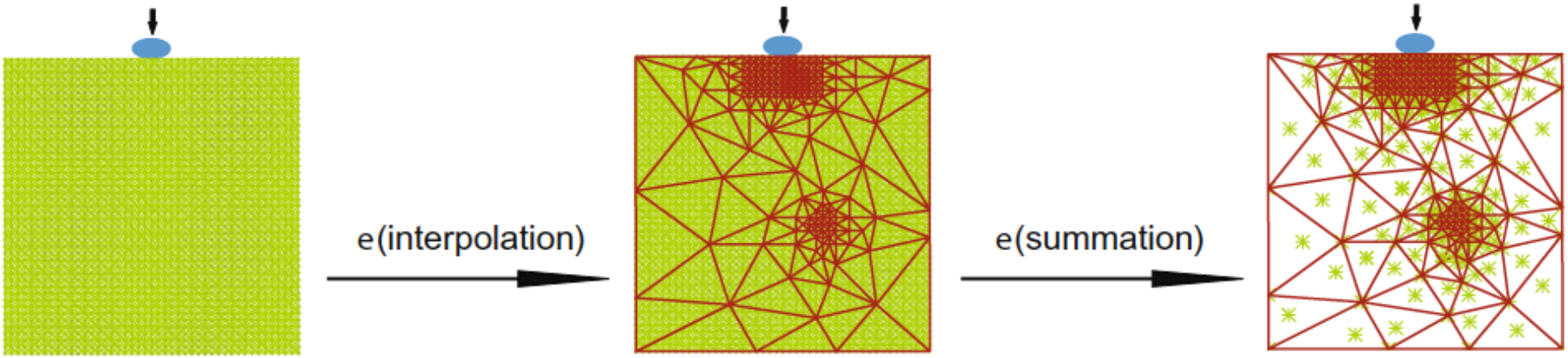


Large lattice computations are expensive due to

1. Large numbers of DOFs **interpolation**

2. Large numbers of interactions **summation/sampling**

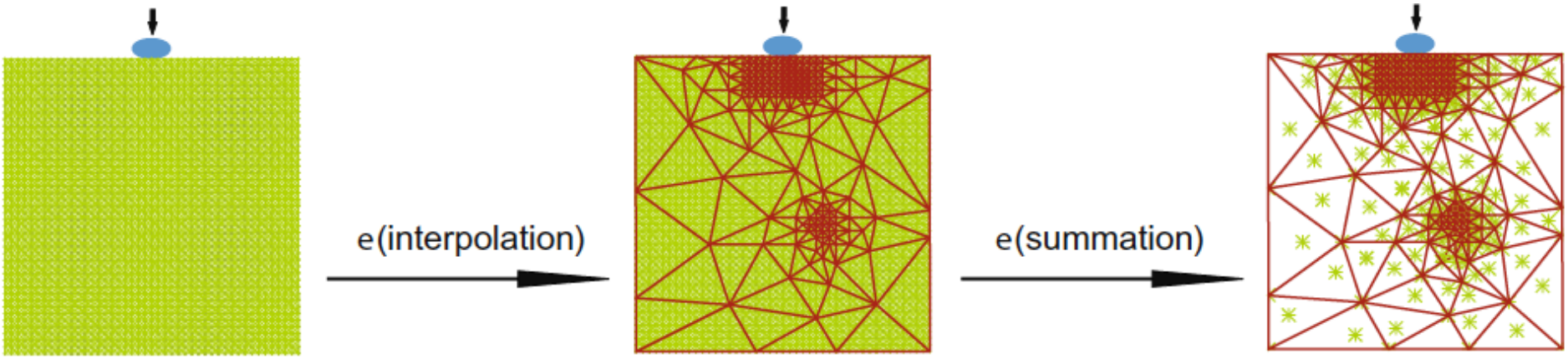
The QC method in a nutshell



The QC method is

1. a nested multiscale method

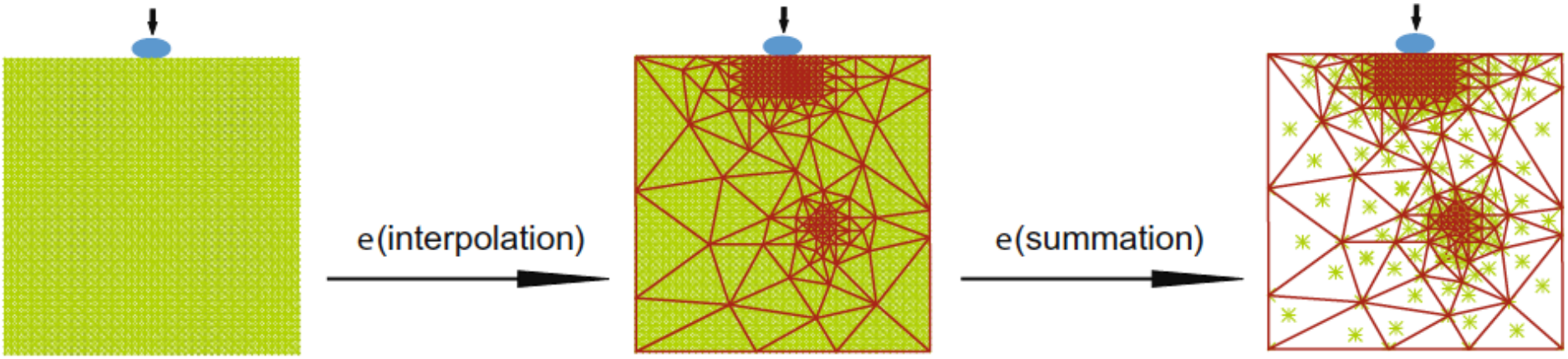
The QC method in a nutshell



The QC method is

1. a nested multiscale method
2. a nested **concurrent** multiscale method

The QC method in a nutshell



The QC method is

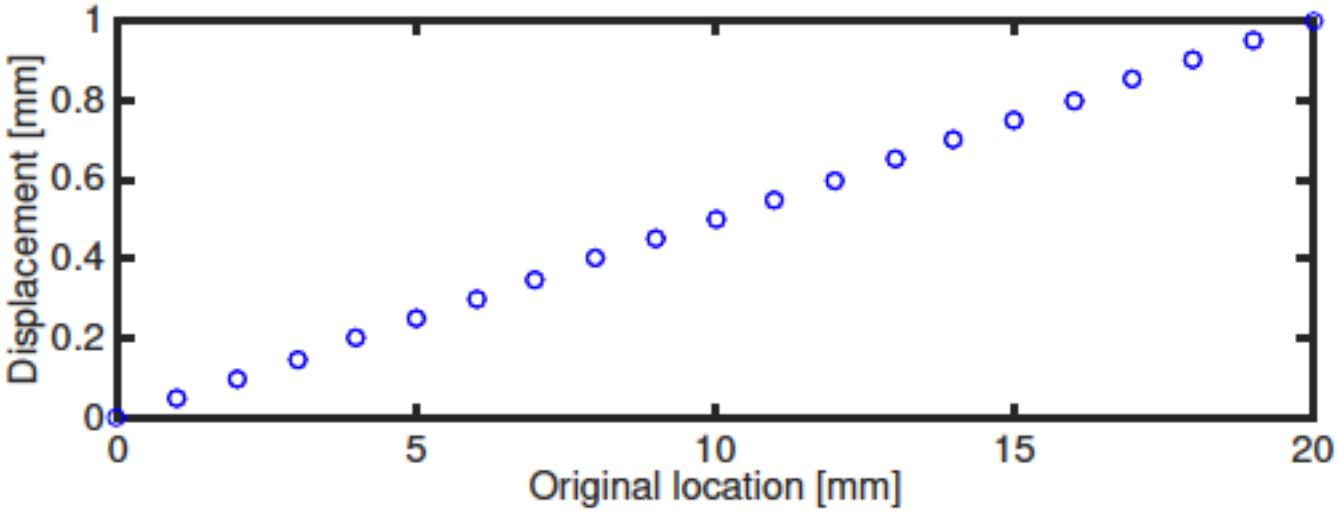
1. a nested multiscale method
2. a nested **concurrent** multiscale method
3. a reduced-order modelling approach

BUT no need for offline snapshots

relatively easy to include adaptivity (ROKOS)

The method's ansatz explained

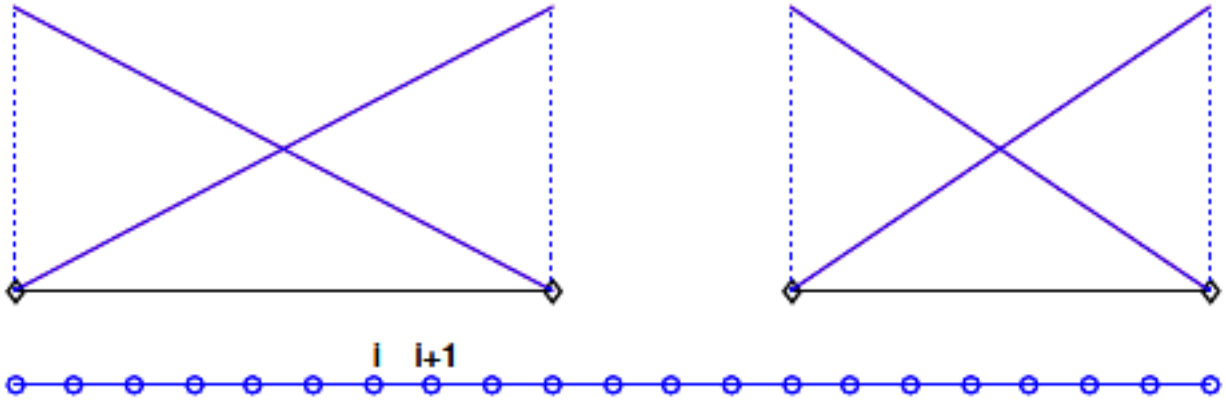
1D string of LE springs: DNS

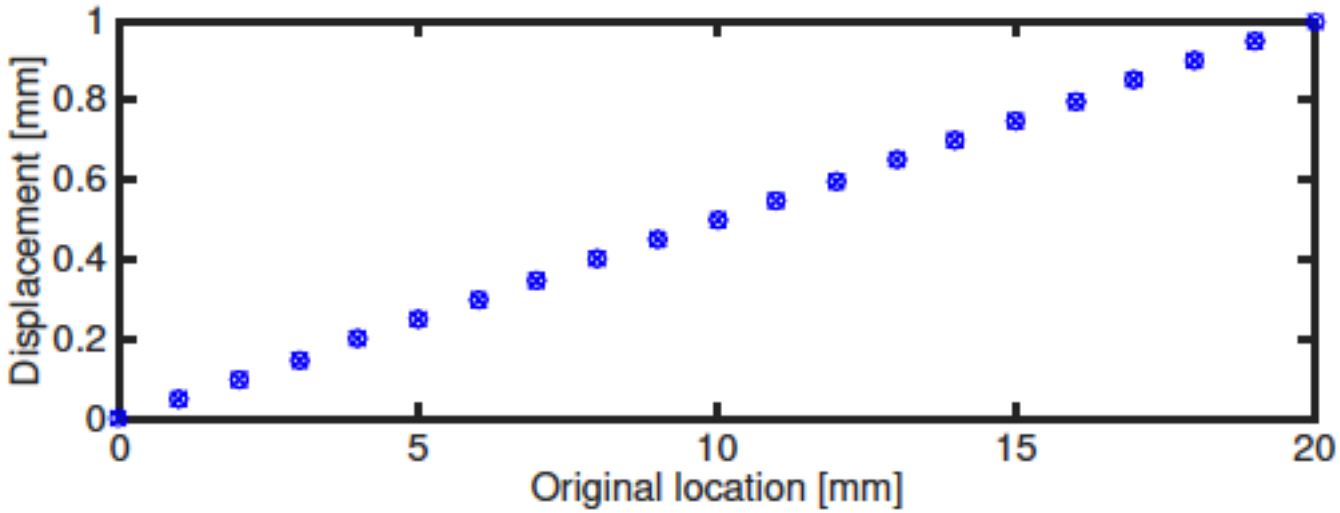


The method's ansatz explained

UNIVERSITÉ DU LUXEMBOURG

1D string of LE springs: Interpolation

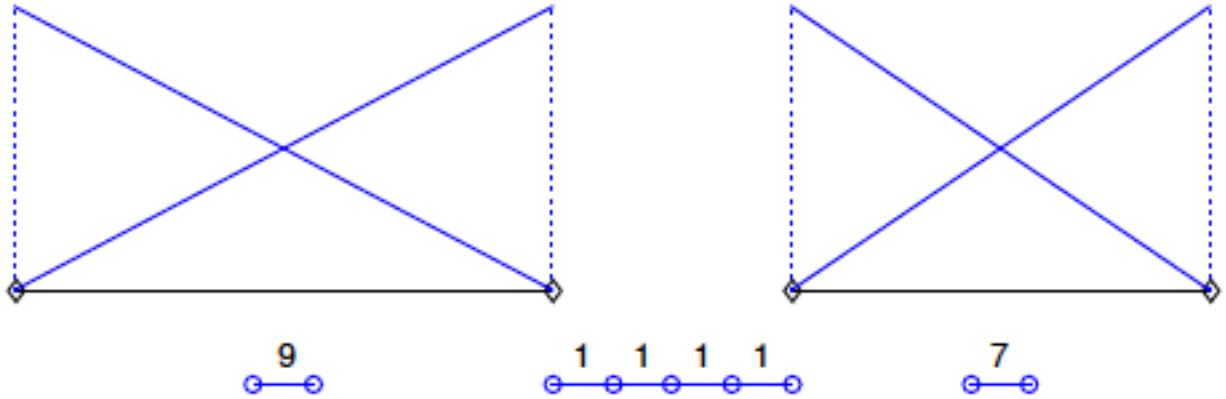


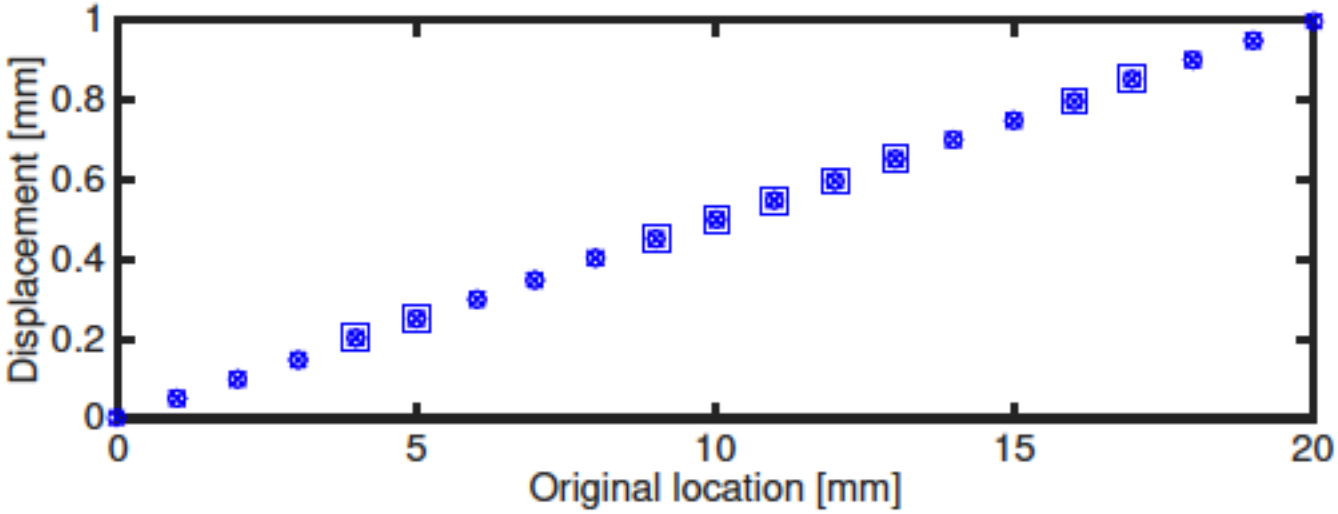


The method's ansatz explained

UNIVERSITÉ DU LUXEMBOURG

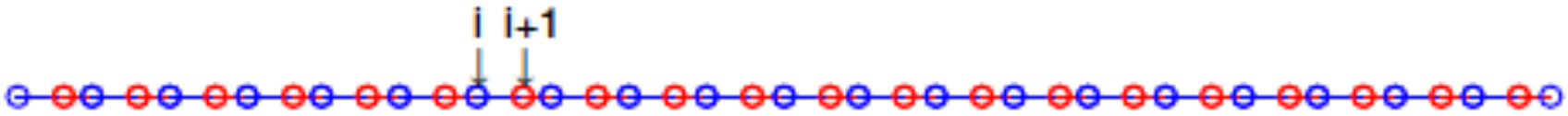
1D string of LE springs: Interpolation + Sampling

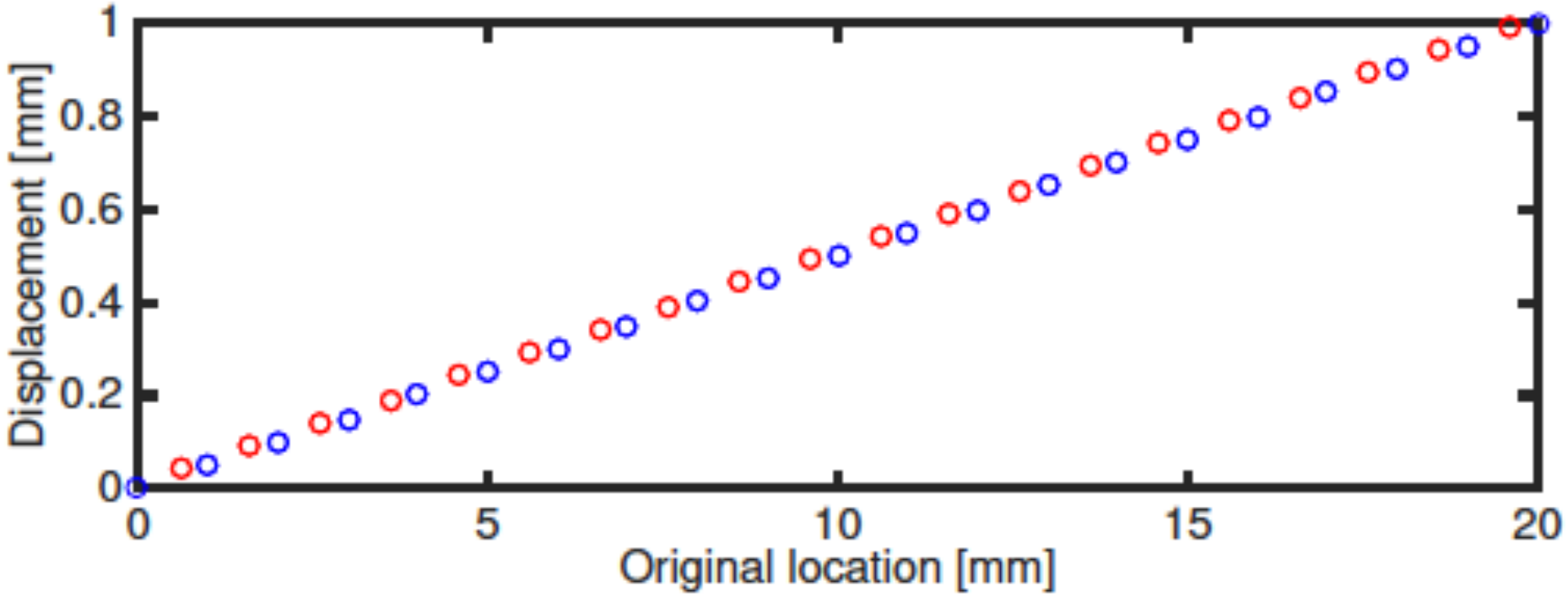




The method's ansatz explained

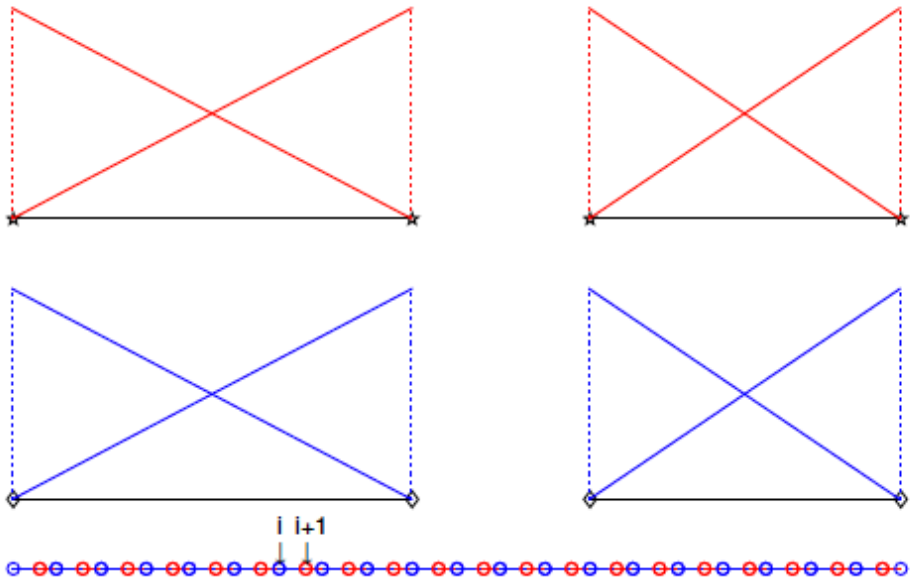
1D string of 2 types of LE springs: DNS

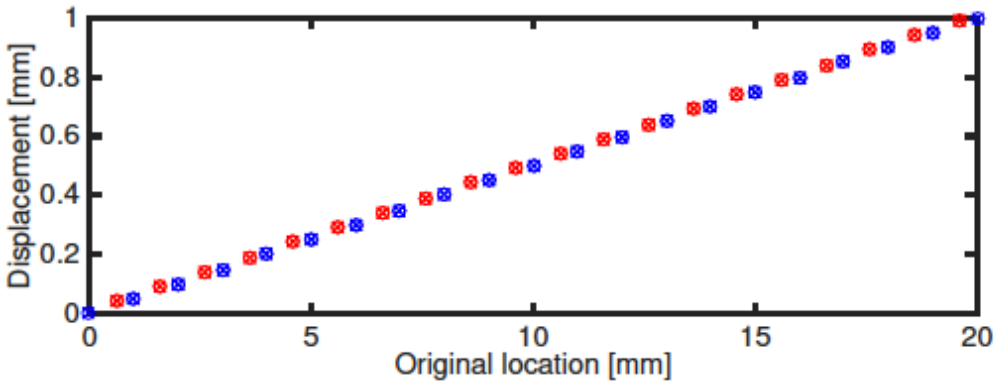




The method's ansatz explained

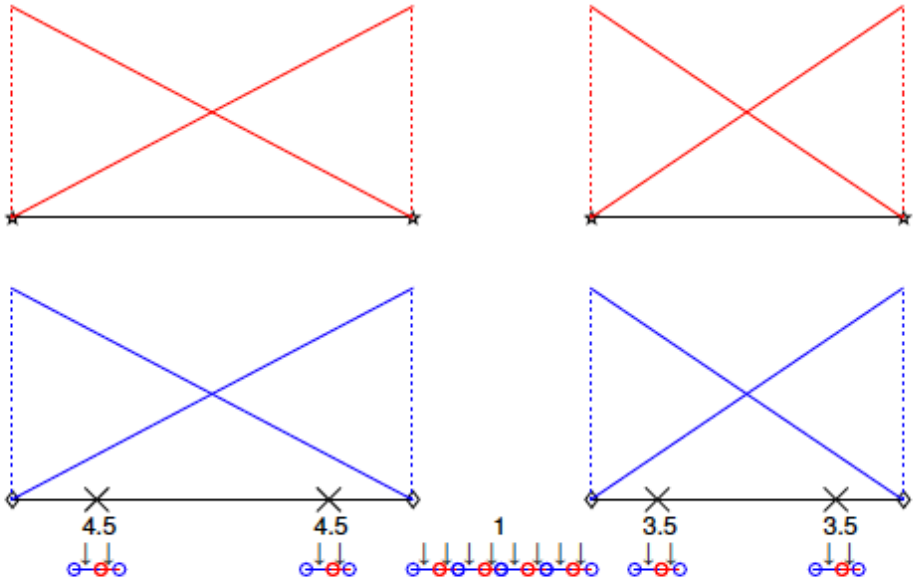
1D string of 2 types of LE springs: Interpolation

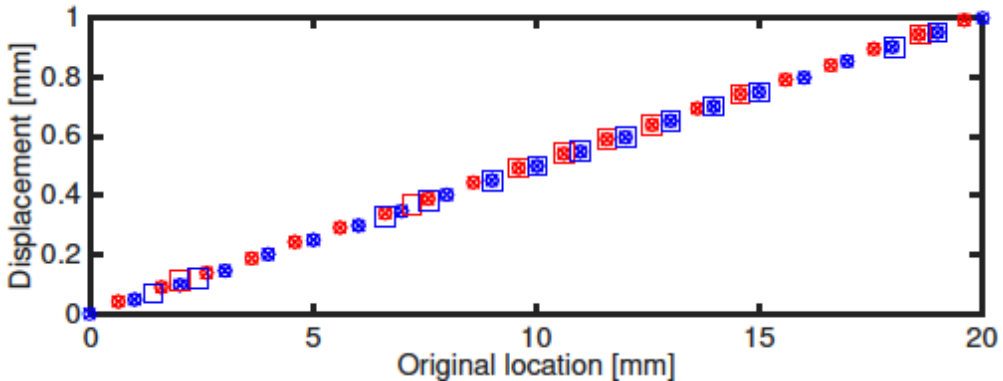




The method's ansatz explained

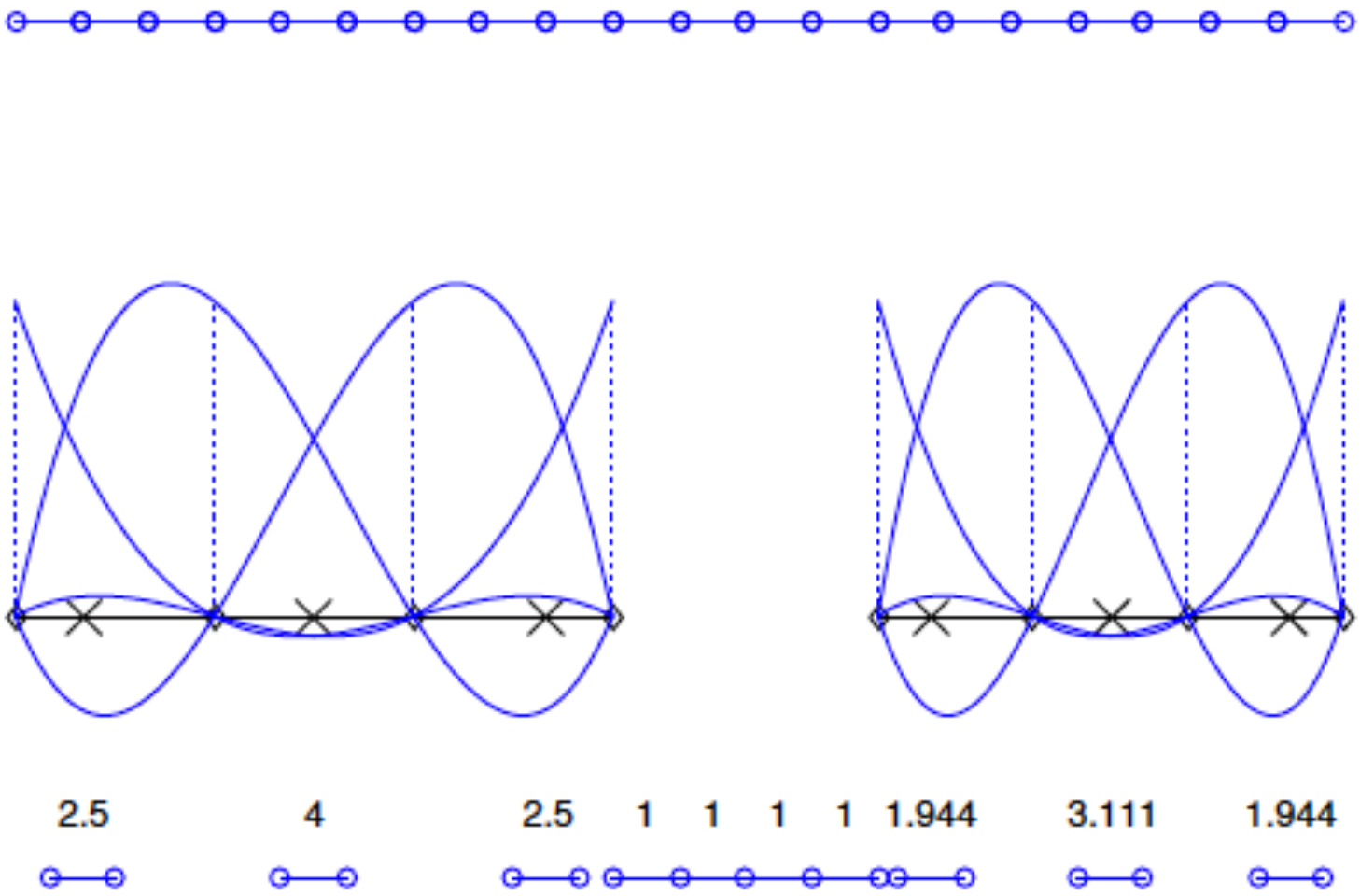
1D string of 2 types of LE springs: Interpolation + Sampling





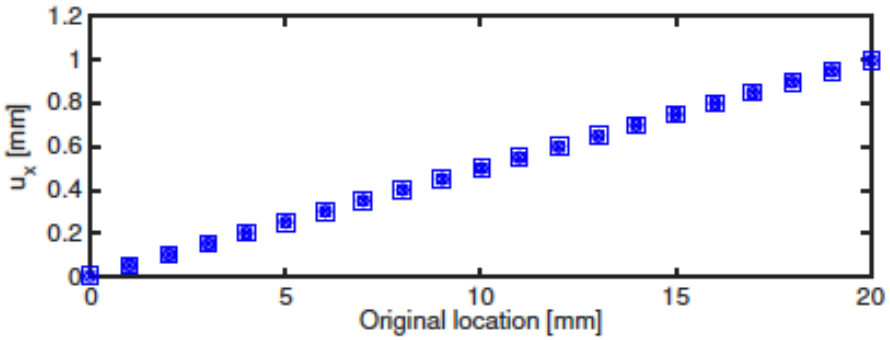
The method's ansatz explained

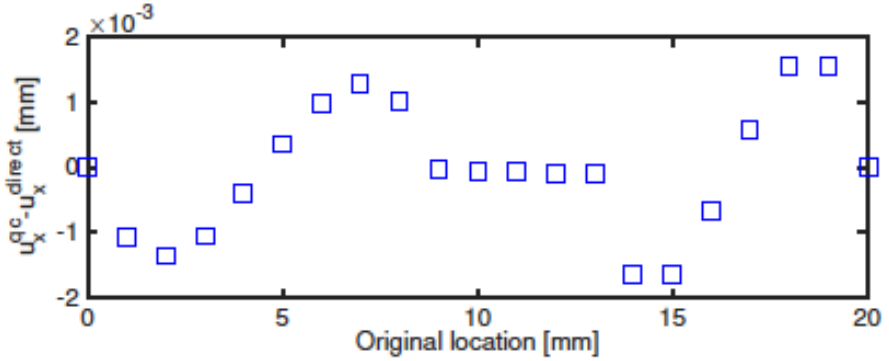
String of LE beams: Interpolation + Sampling

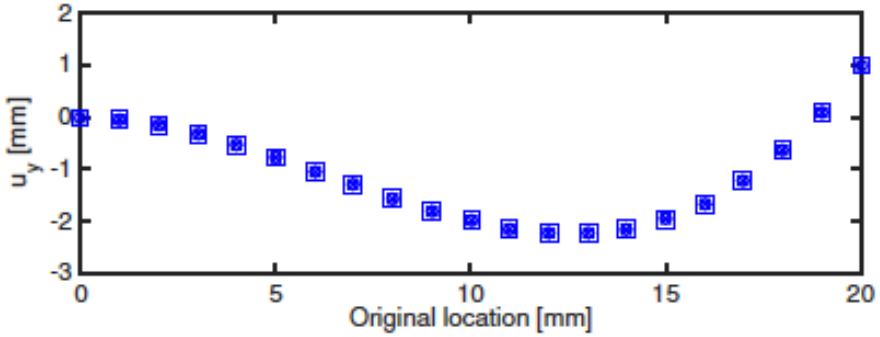


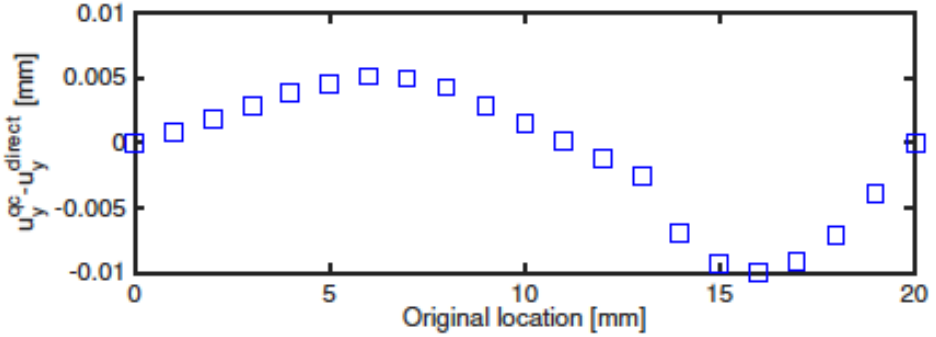
The method's ansatz explained

String of LE beams: Interpolation + Sampling



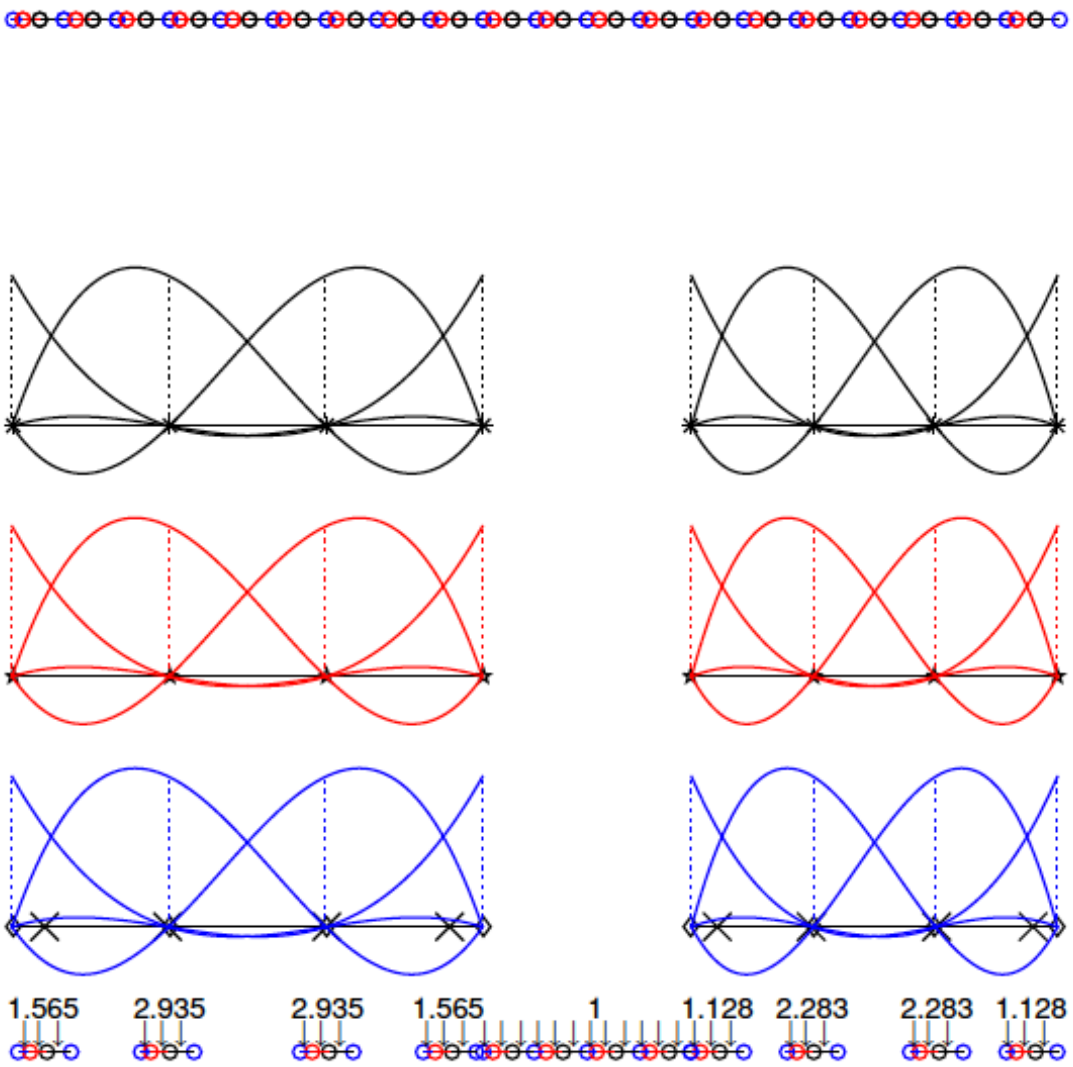






The method's ansatz explained

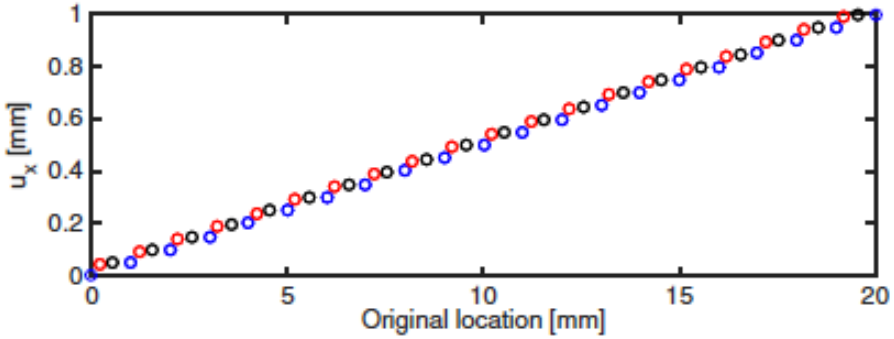
String of 3 types of LE beams: Interpolation + Sampling

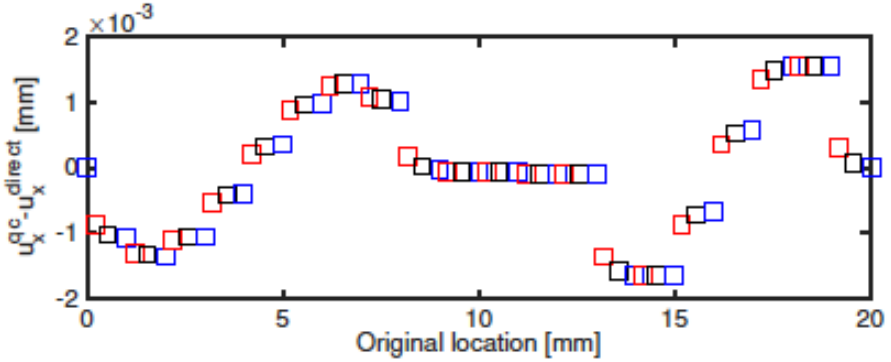


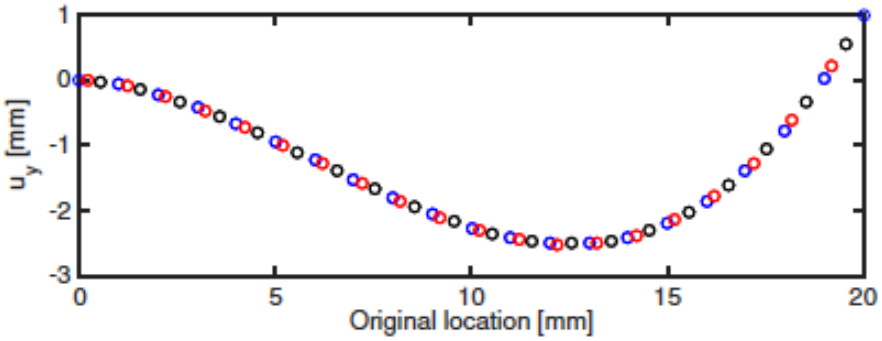
The method's ansatz explained

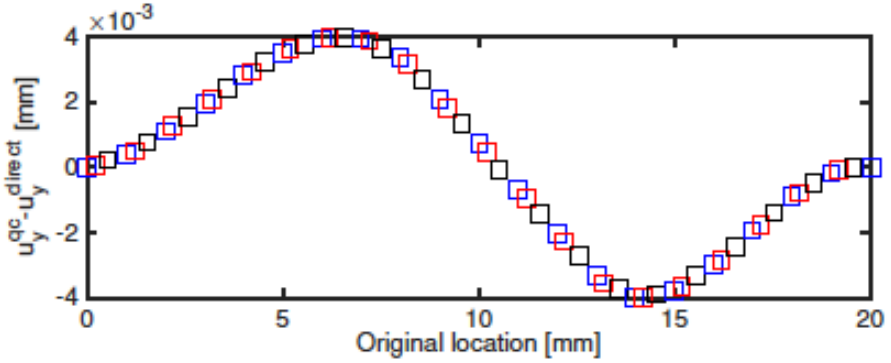
String of 3 types of LE beams:

Interpolation + Sampling

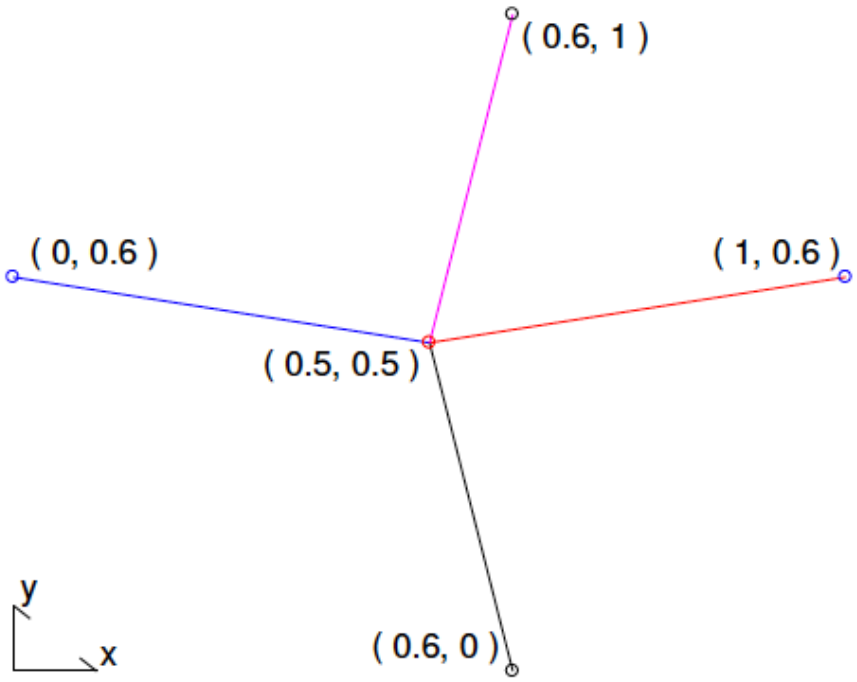






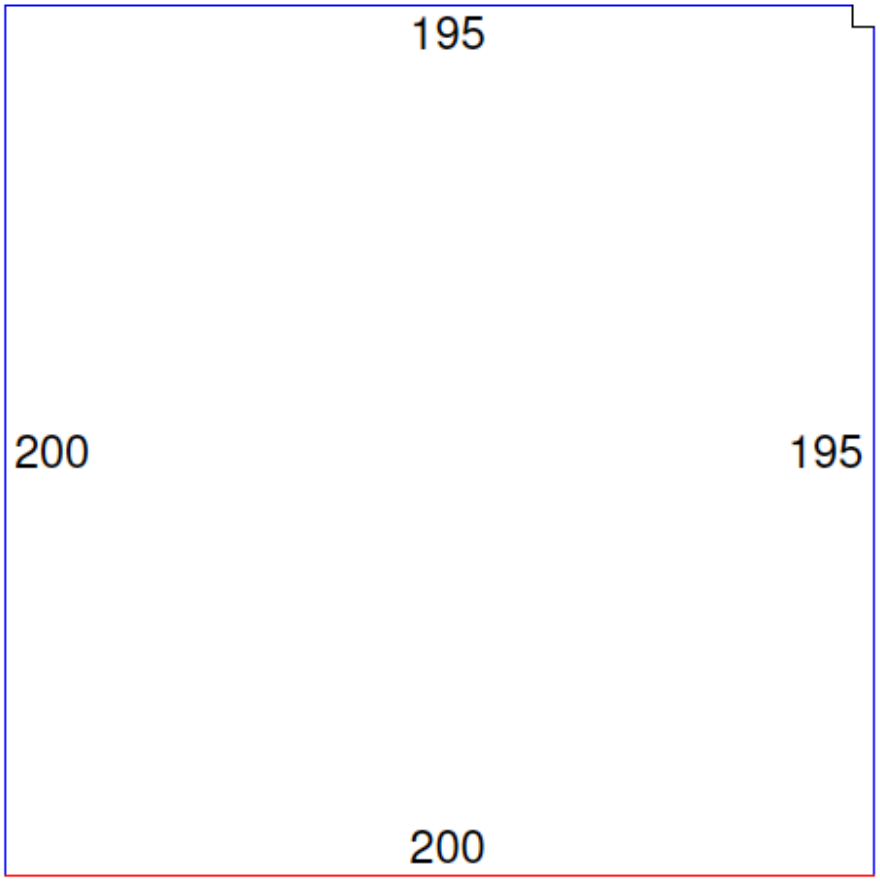


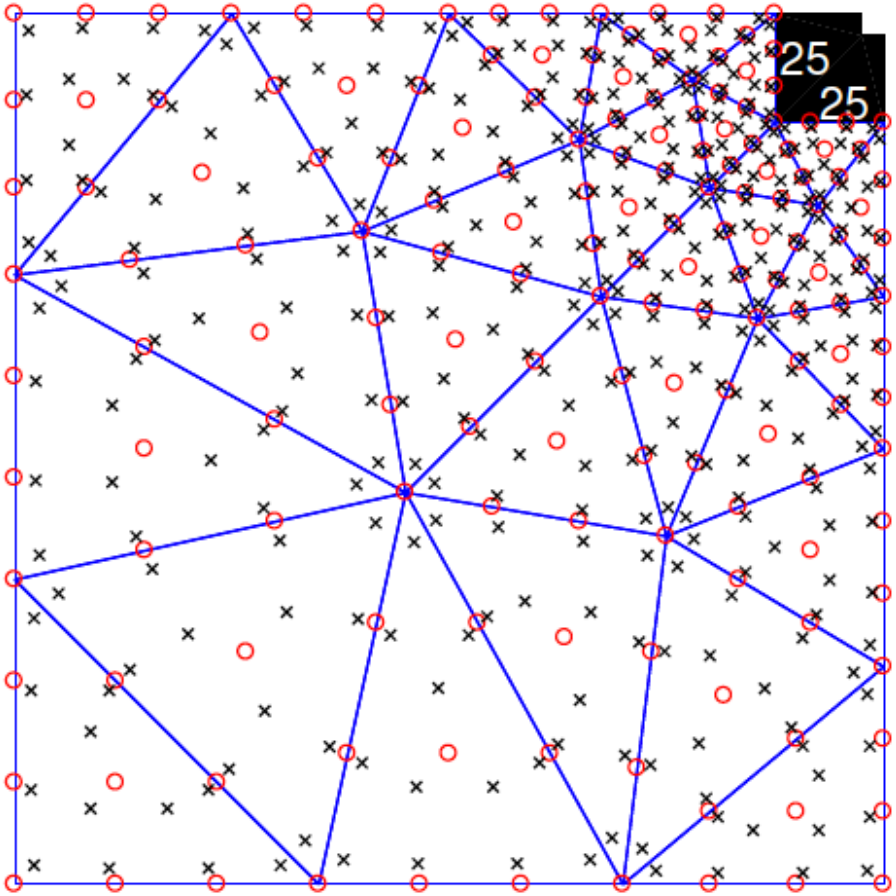
Simple planar unit cell: Setup



	Blue	Red	Black	Purple	
Area	1	3	9	27	[m^2] ($x10^{-9}$)
Y. Modulus	1	5	25	125	[MPa]
P. ratio	0.1	0.2	0.3	0.4	[-]
Failure str.	10	1	2	70	($x10^{-5}$)

Simple planar unit cell: Setup





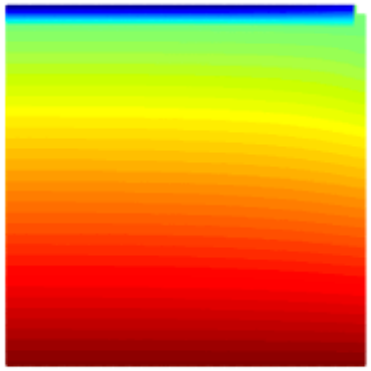
DNS:

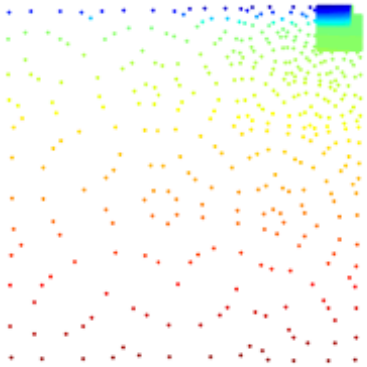
- 160k beams
- 722k DOFs

QC:

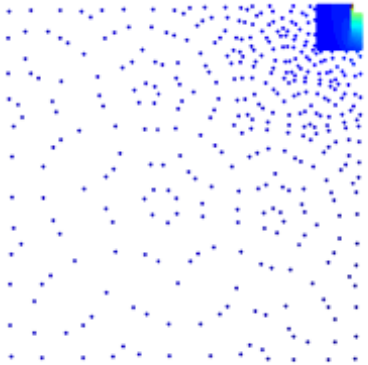
- 42x less beams
- 52x less DOFs

Simple planar unit cell: Results

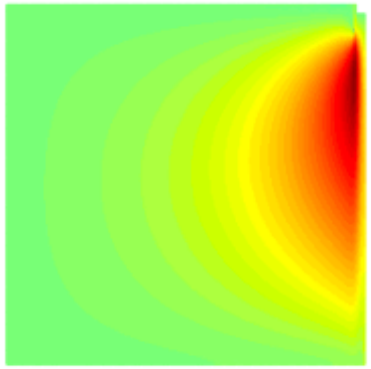


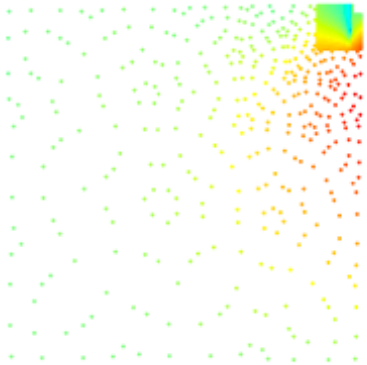


Out-of-plane displacement



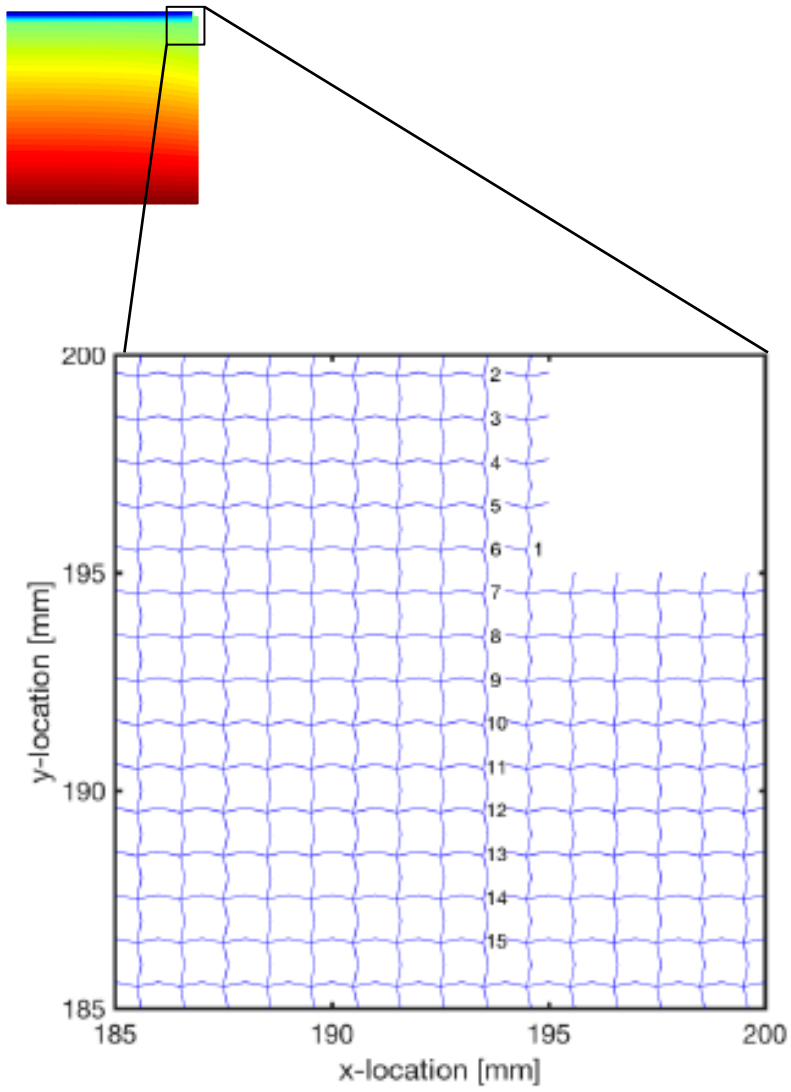
Rotation around horizontal axis



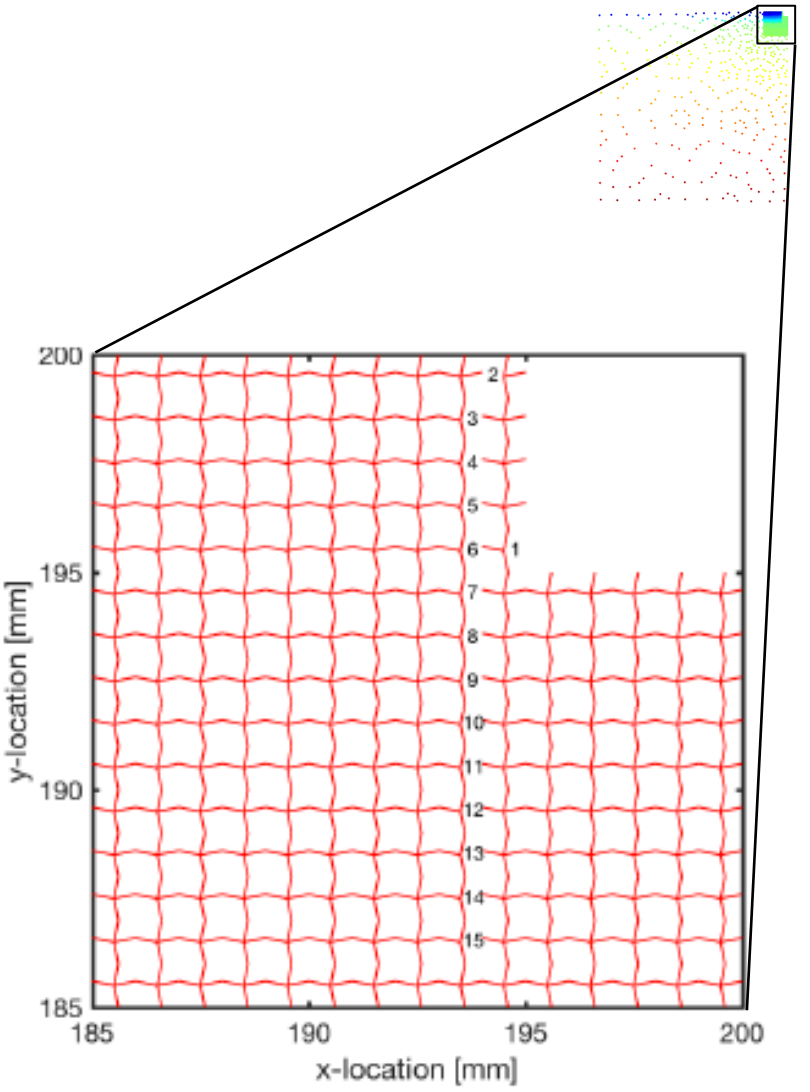


Rotation around vertical axis

Simple planar unit cell: Results



DNS

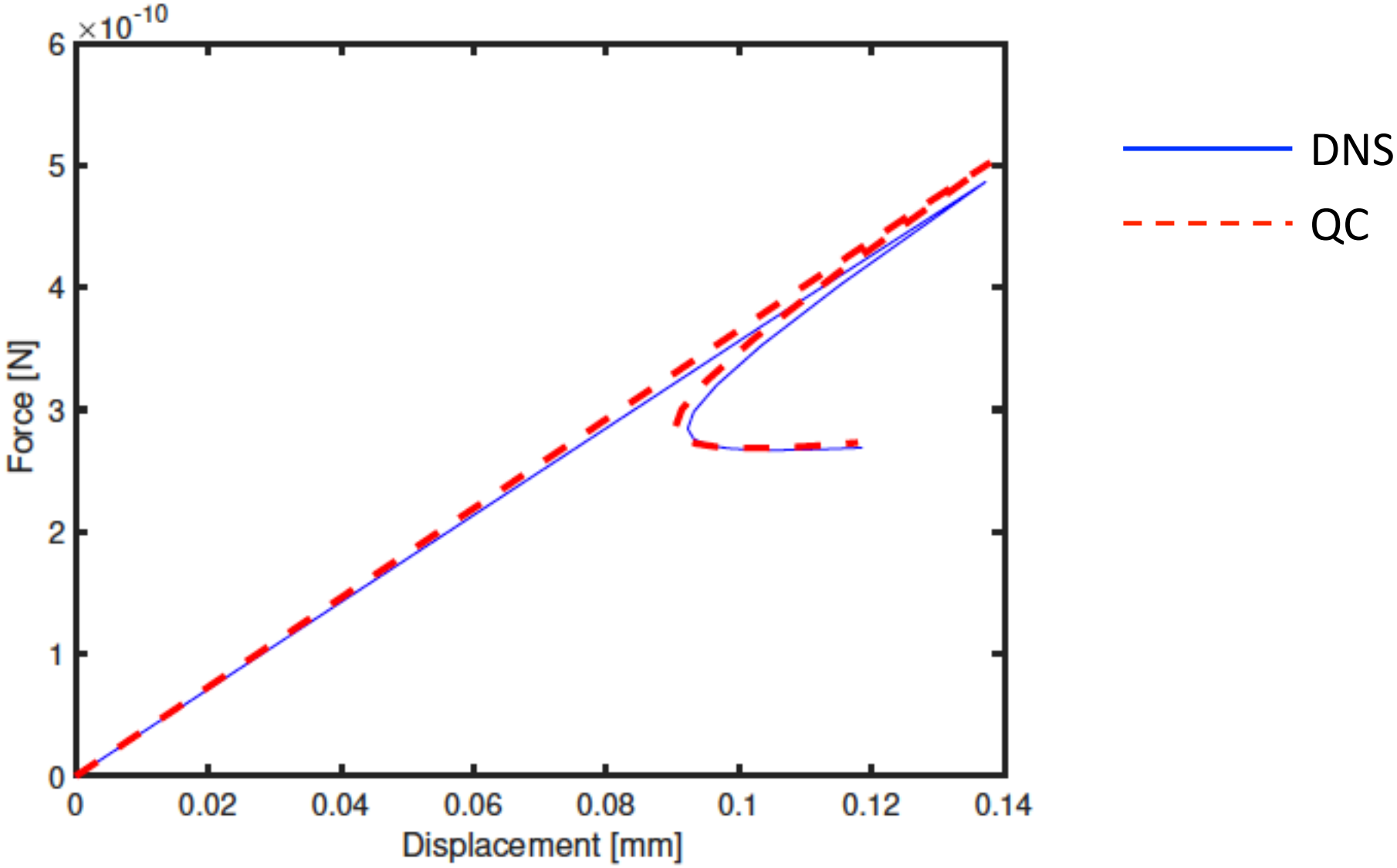


QC

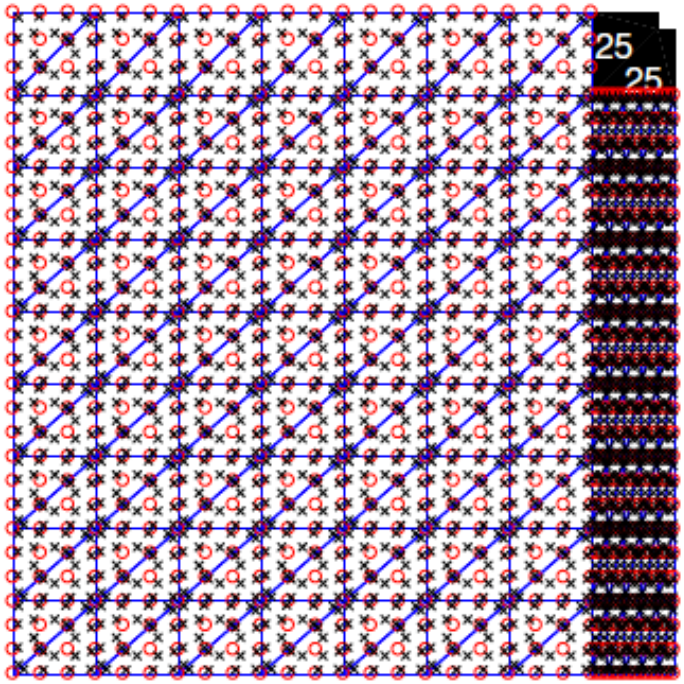
Failed beams, inc. sequence

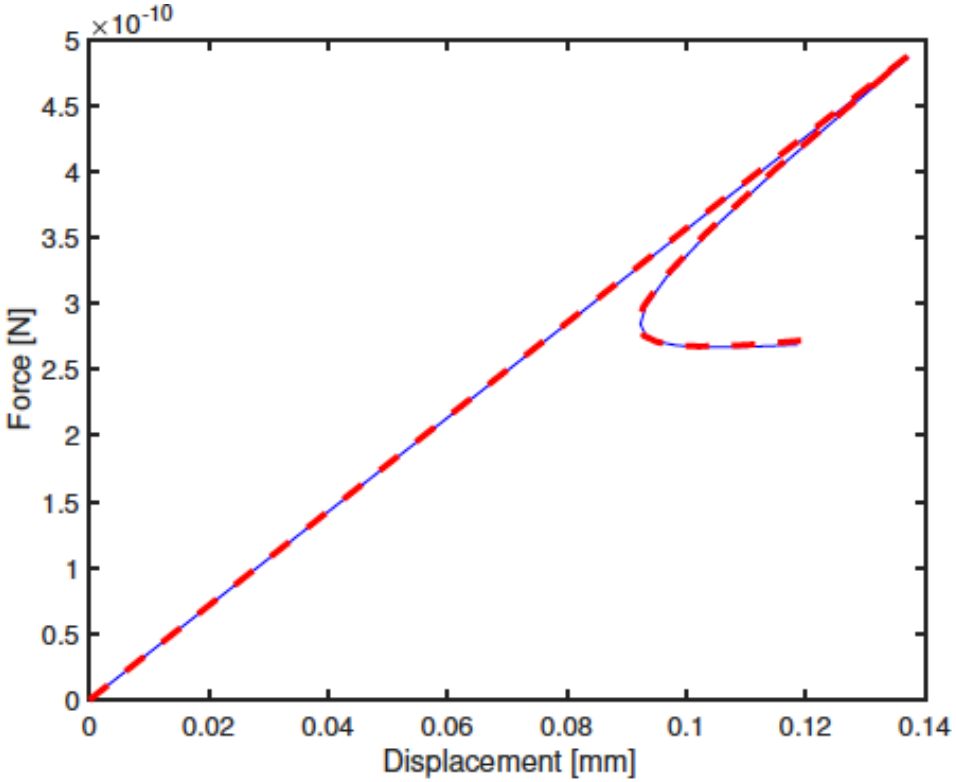
Simple planar unit cell: Results

Force-displacement response



Simple planar unit cell: New setup + Result





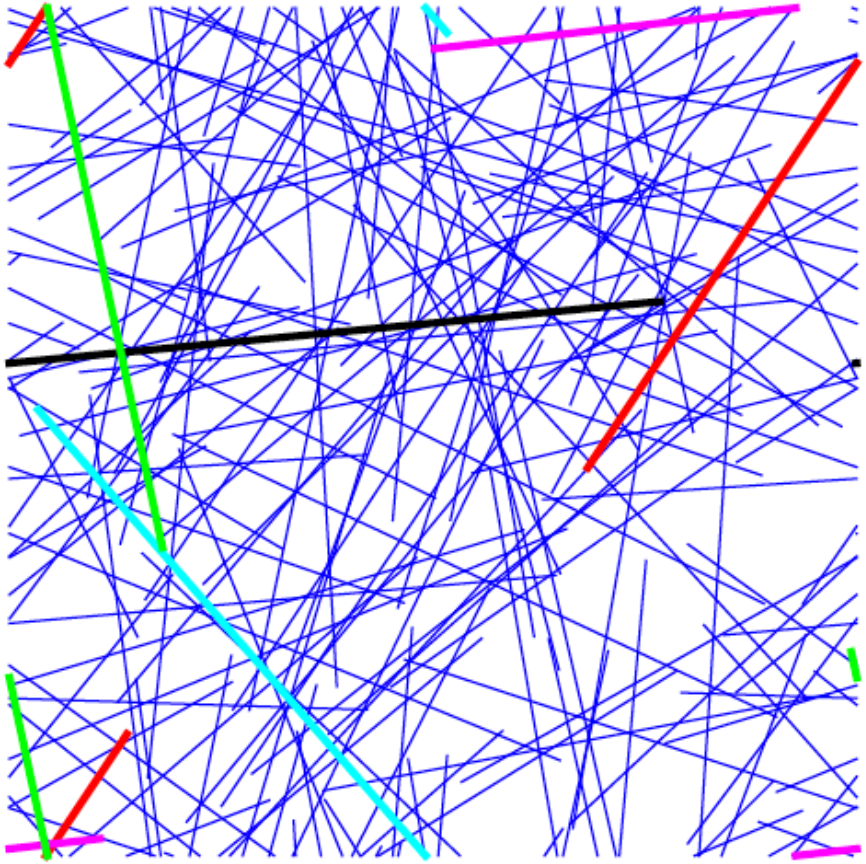
QC:

- 13x less beams
- 25x less DOFs

DNS

QC

Fibrous unit cell: Small setup

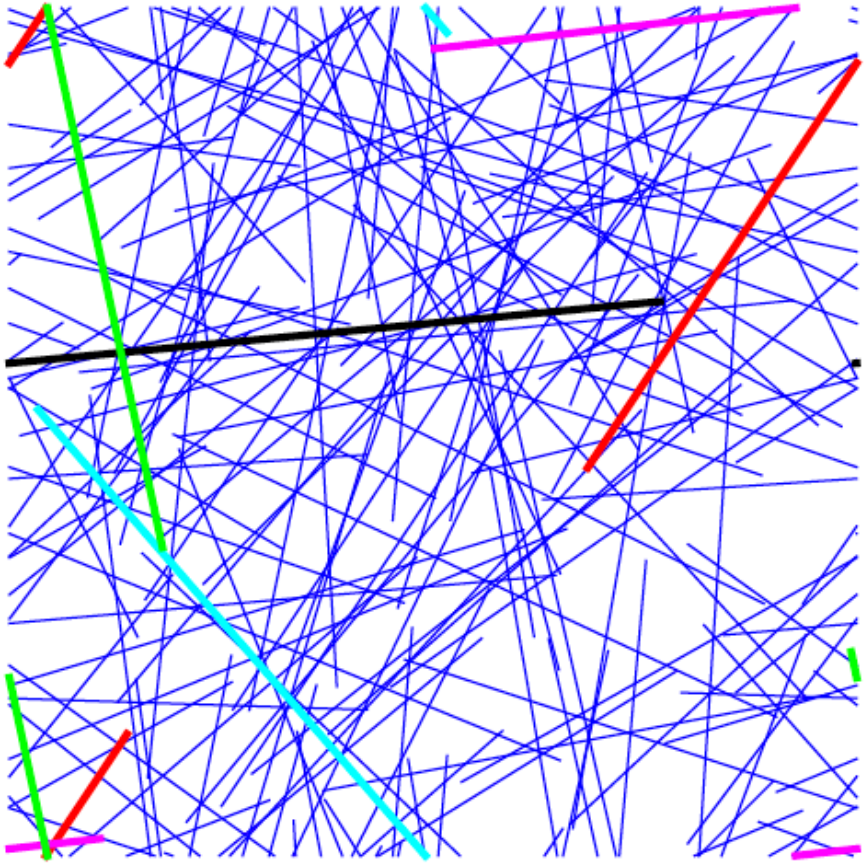


Periodic, planar unit cell of fibres (1x1mm^2)

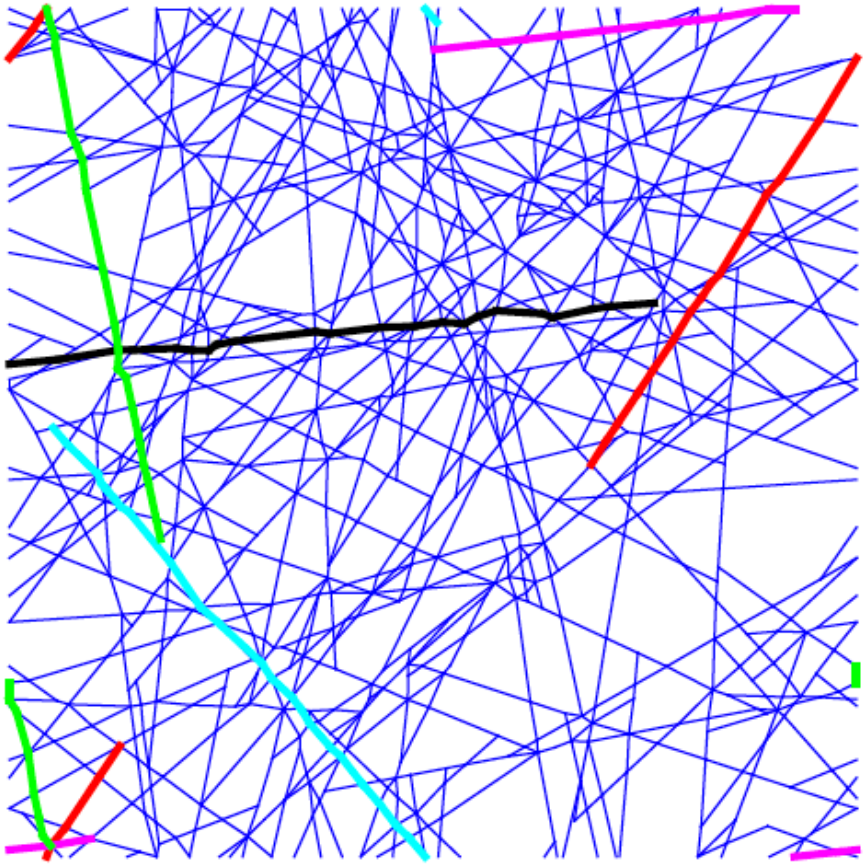
Parameters taken from U(a,b)

	a	b	
L	0.6	0.9	[mm]
A	1	2	[m^2] (x10^{-9})
E	1	2	[MPa]
v	0.2	0.4	[-]
Failure str.	1	2	(x10^{-5})

Fibrous unit cell: Small setup



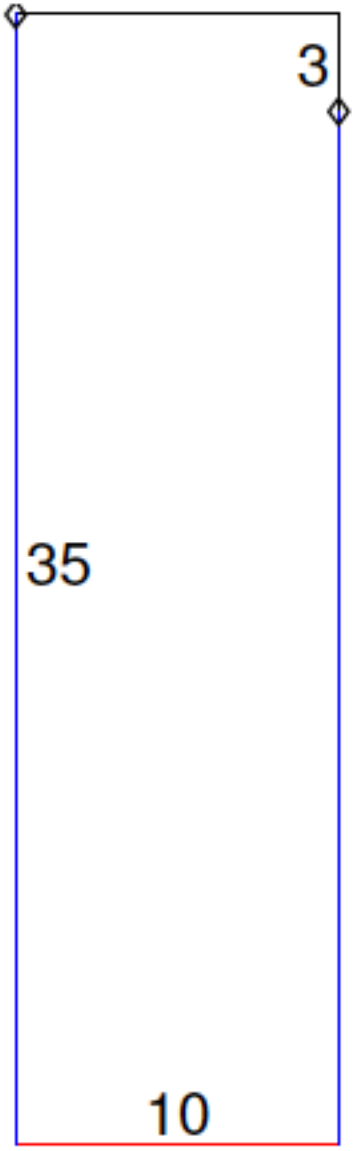
Periodic, planar unit cell of fibres (1x1mm^2)

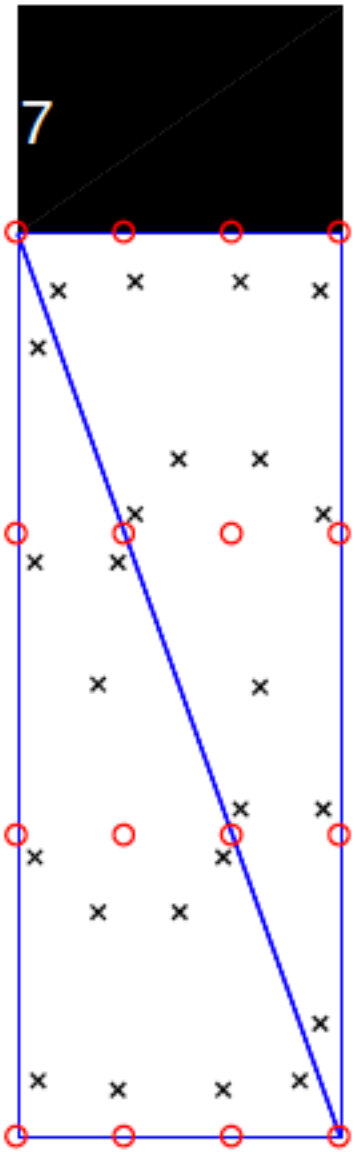


Beam discretisation
(LE EB beams with brittle damage)

Fibrous unit cell: Small setup

DNS:
- 700k beams
- 2M DOFs

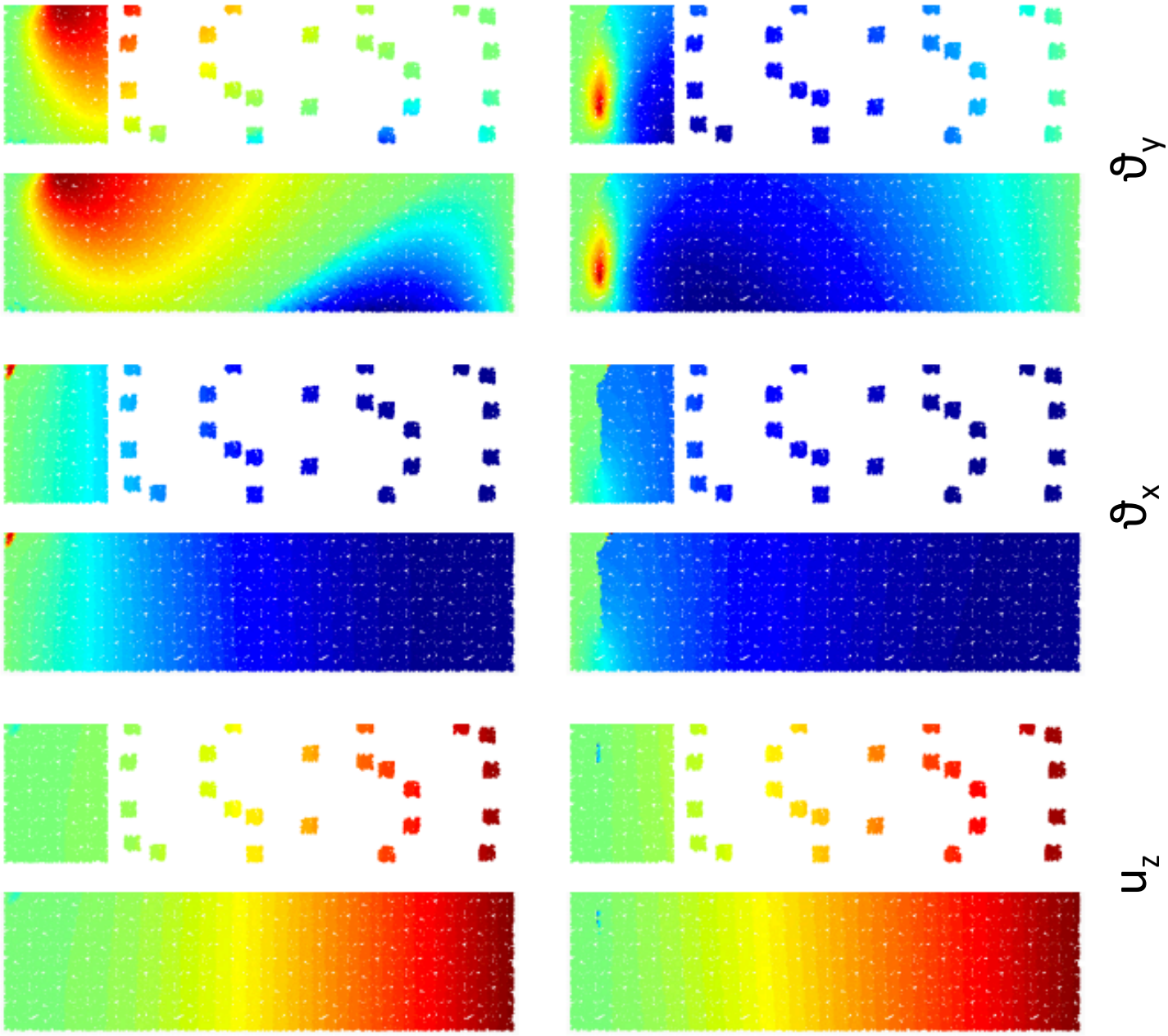


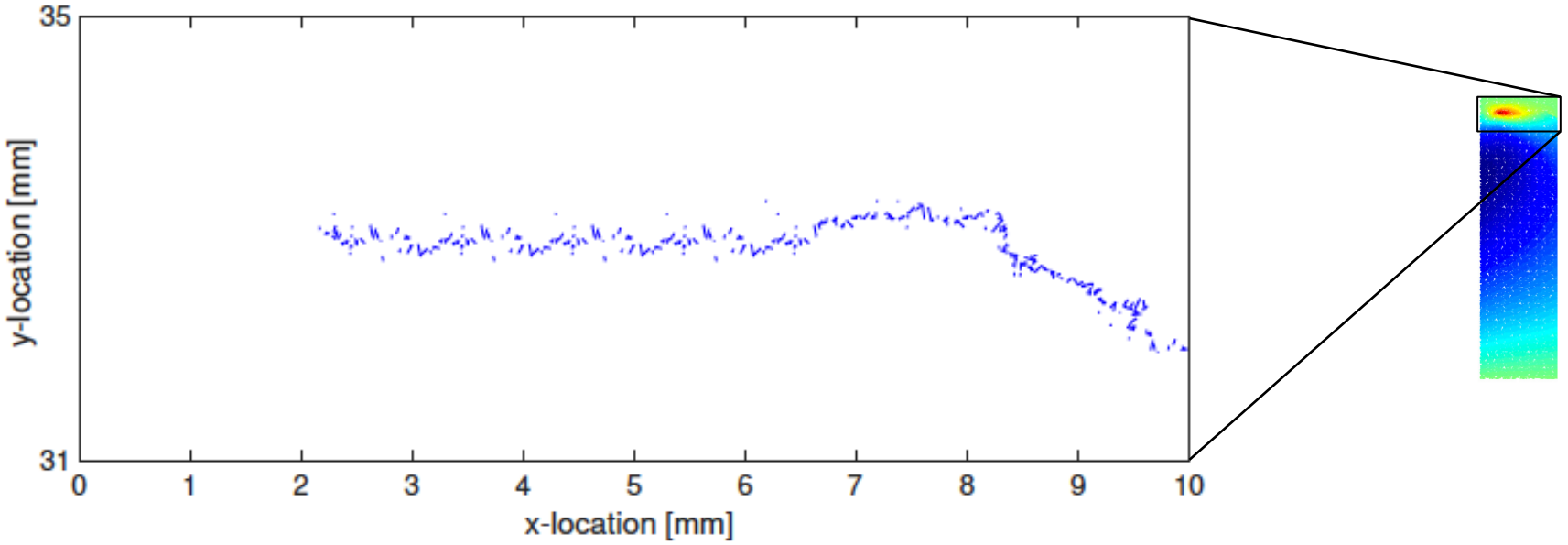


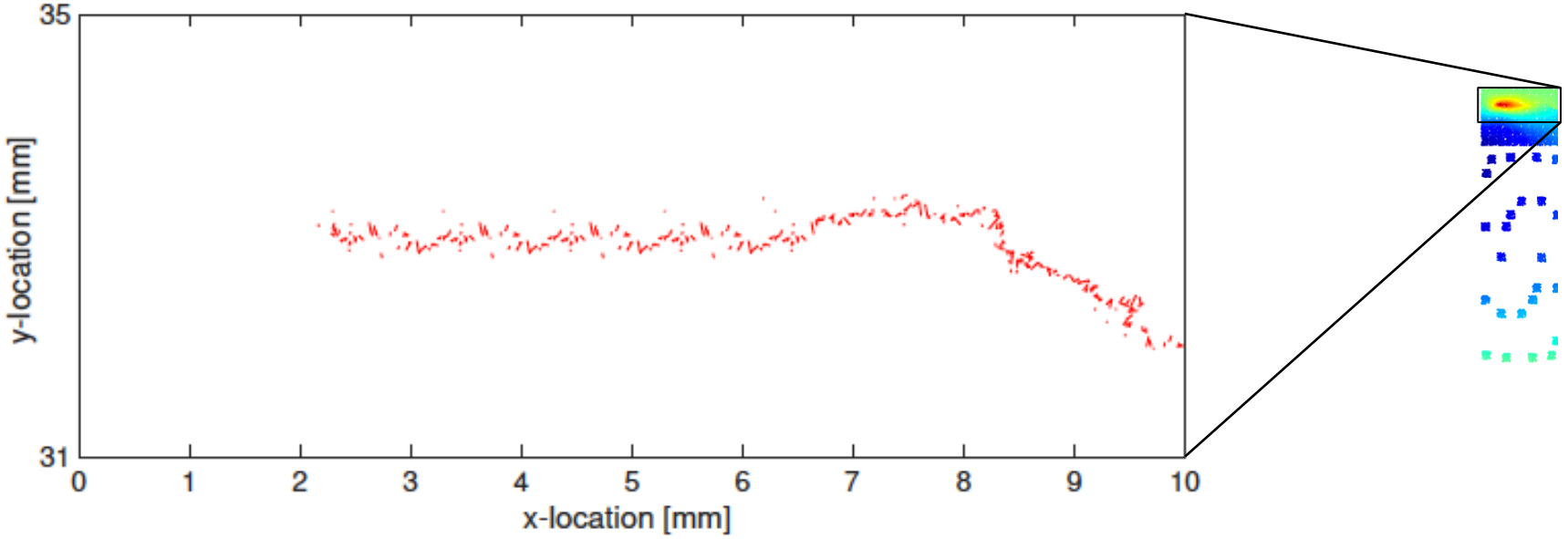
QC:
- 4 less beams
- 4 less DOFs

After 1 beam failure

After 350 beam failures



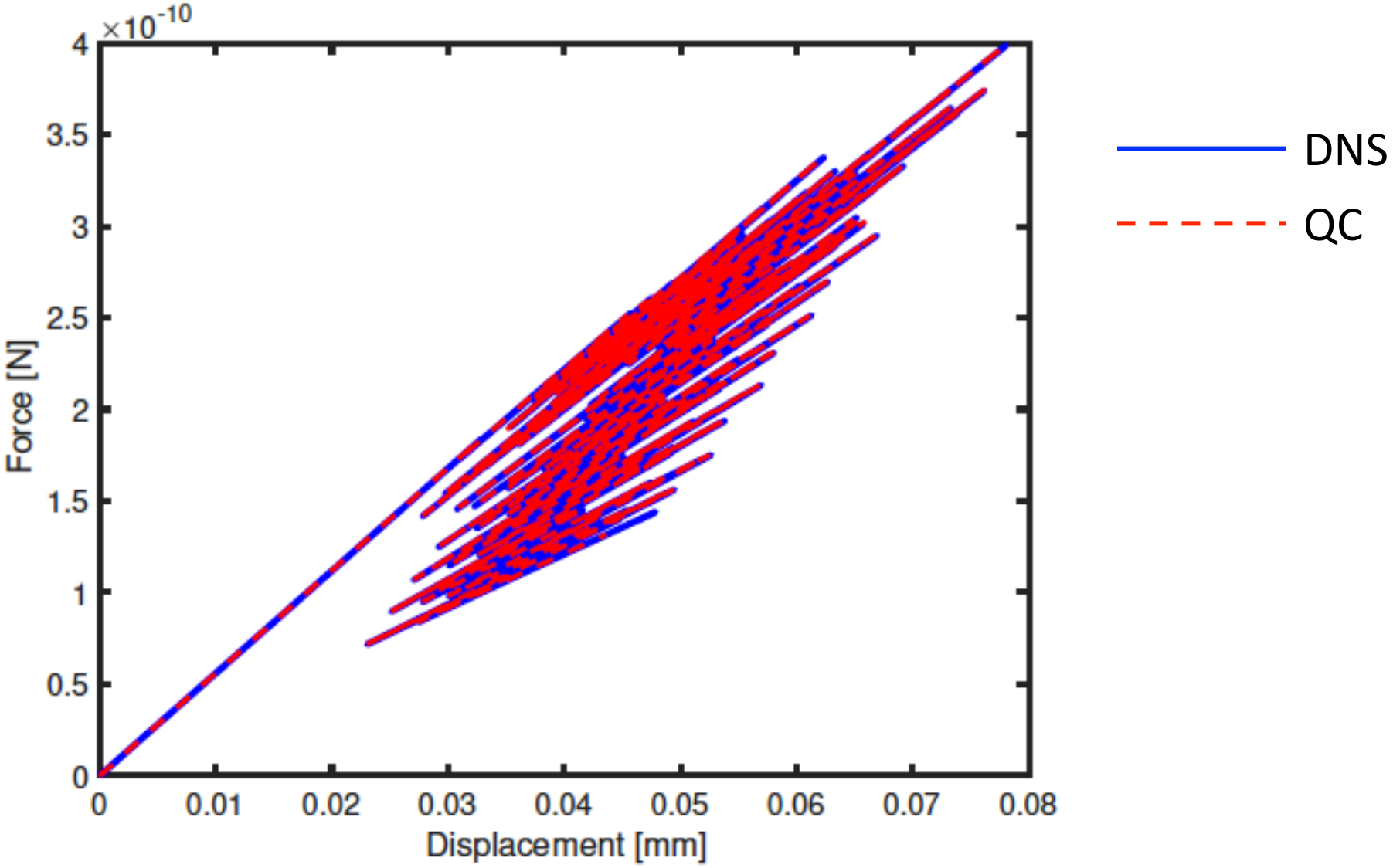




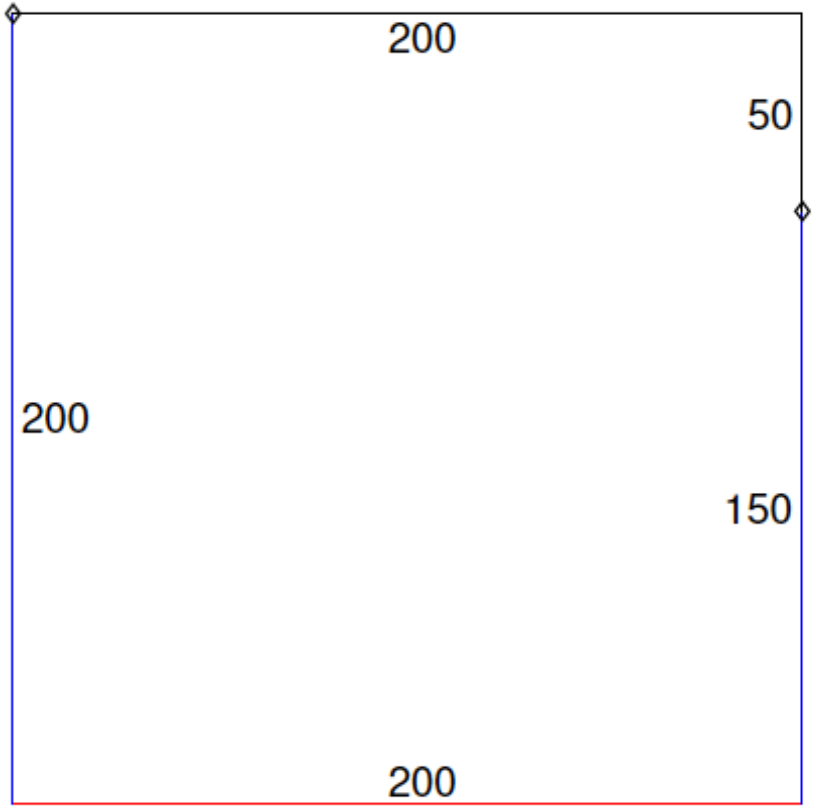
Failed beams

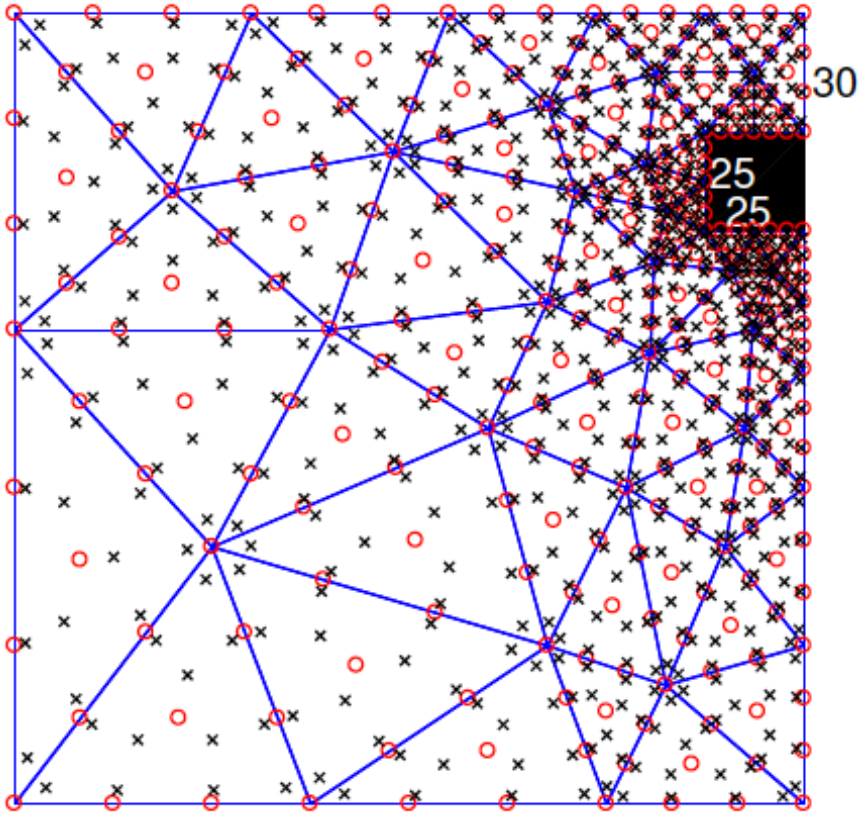
Fibrous unit cell, small setup: Results

Force-displacement response



Fibrous unit cell: Large setup

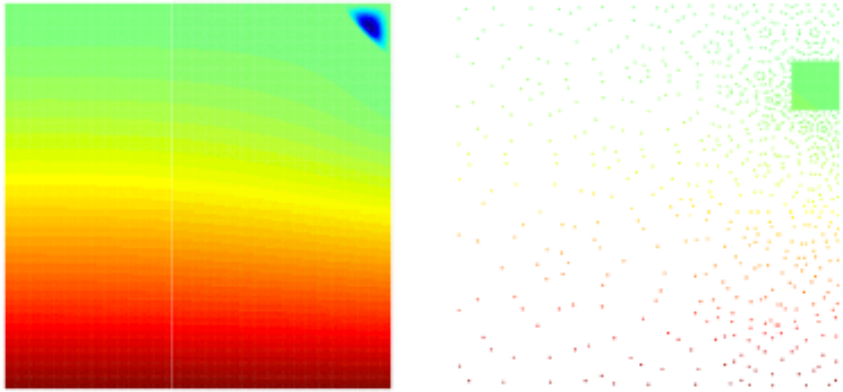




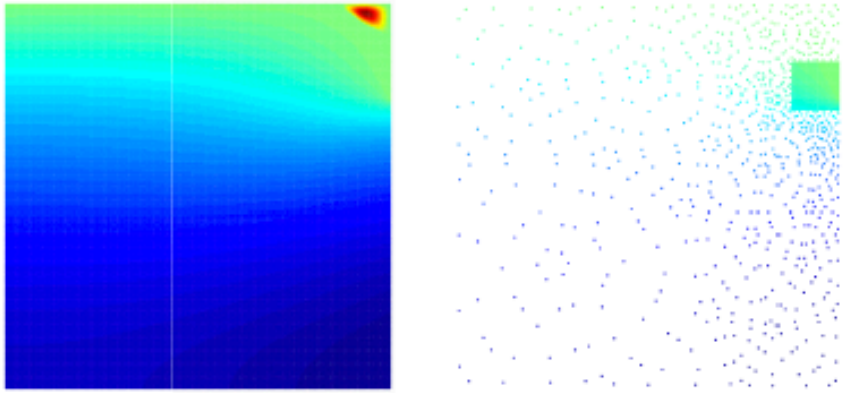
DNS:
- 80M beams
- 233M DOFs

QC:
- 29x less beams
- 42x less DOFs

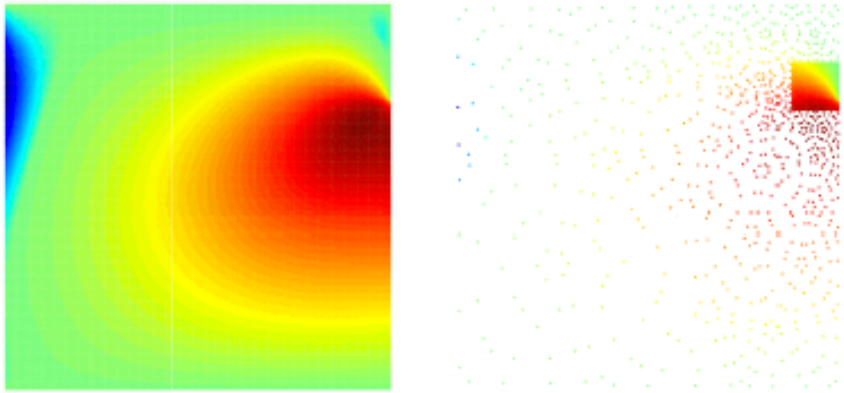
Fibrous unit cell, large domain: Results



Out-of-plane displacement



Rotation around horizontal axis

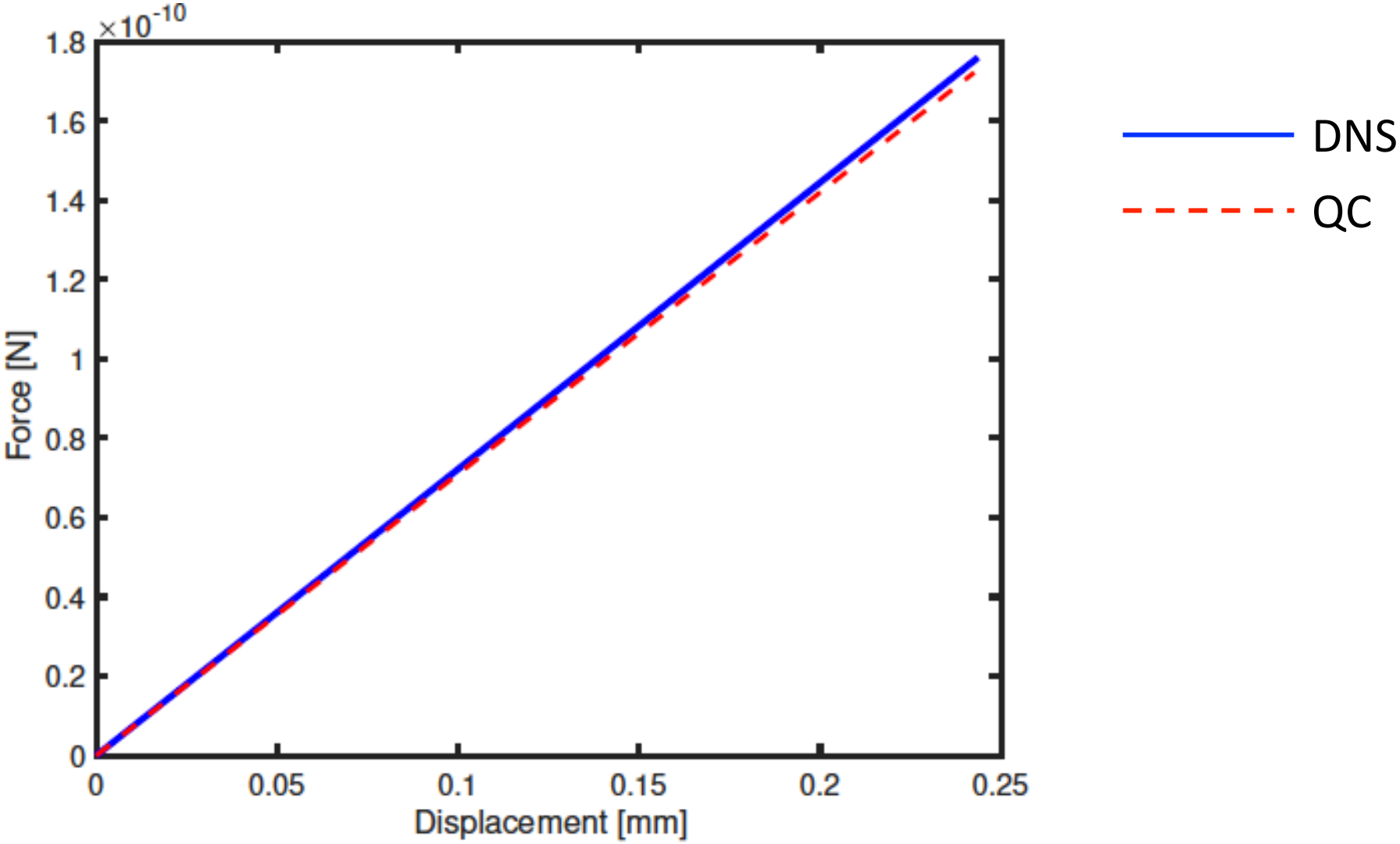


Rotation around vertical axis

Fibrous unit cell: Large setup

UNIVERSITÉ DU LUXEMBOURG

Force-displacement response



Main points of the method

UNIVERSITÉ DU LUXEMBOURG

Condition:

Unit cell must be periodic.

Main points of the method

UNIVERSITÉ DU LUXEMBOURG

Condition:

Unit cell must be periodic.

Advantages compared to other nested multiscale methods:

1. Higher-order macroscale interpolations are as easy to treat as linear ones

2. No scale-separation

Main points of the method

UNIVERSITÉ DU LUXEMBOURG

Condition:

Unit cell must be periodic.

Advantages compared to other nested multiscale methods:

1. Higher-order macroscale interpolations are as easy to treat as linear ones

2. No scale-separation

Disadvantages compared to other nested multiscale methods:

1. All DOFs in one system, instead of subdivided over the unit cells and the macroscale elements

2. More unit cells required

Future work

UNIVERSITÉ DU LUXEMBOURG

Ongoing:

Apply to matrix material + inclusions (geom. NL + mat. NL)

Future work

UNIVERSITÉ DU LUXEMBOURG

Ongoing:

Apply to matrix material + inclusions (geom. NL + mat. NL)

Future:

Apply to real materials

(Goal-oriented) adaptivity

Add randomness to structure in the fully resolved region

Chair: Do I have time left?

UNIVERSITÉ DU LUXEMBOURG

If NO

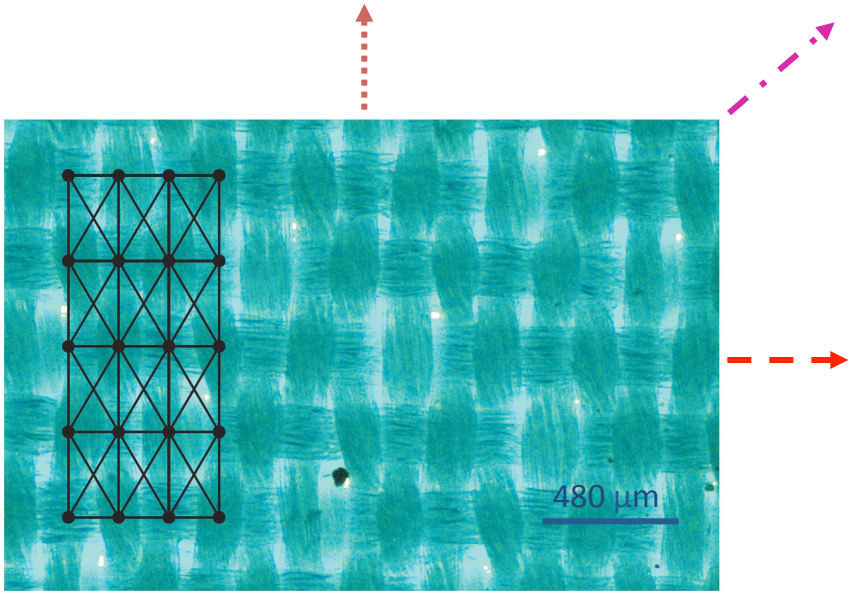
→ Thank you for your attention

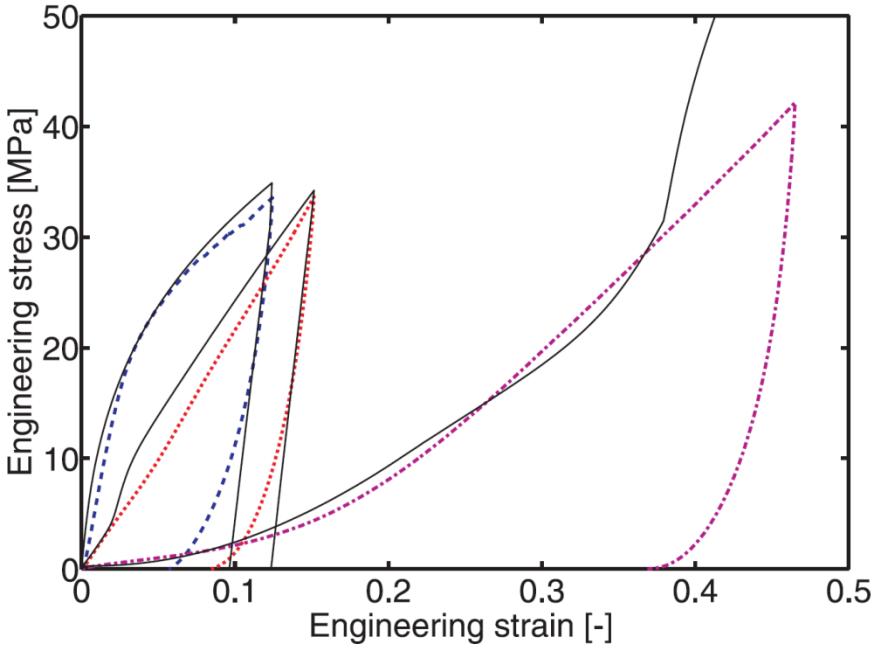
Else

→ Previous work on regular structures

Lattices instead of irregular networks

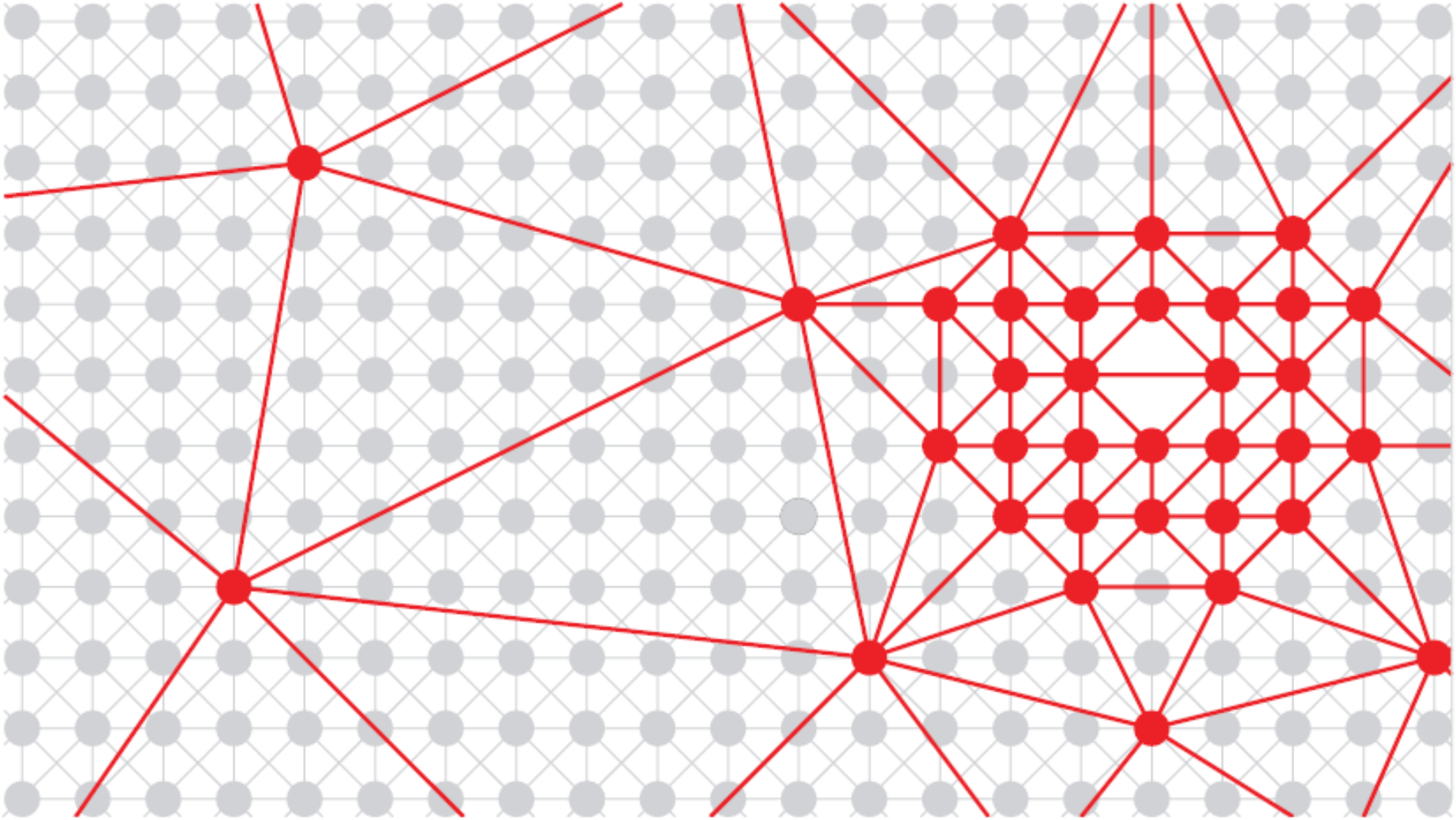
A mesoscale spring lattice for textiles





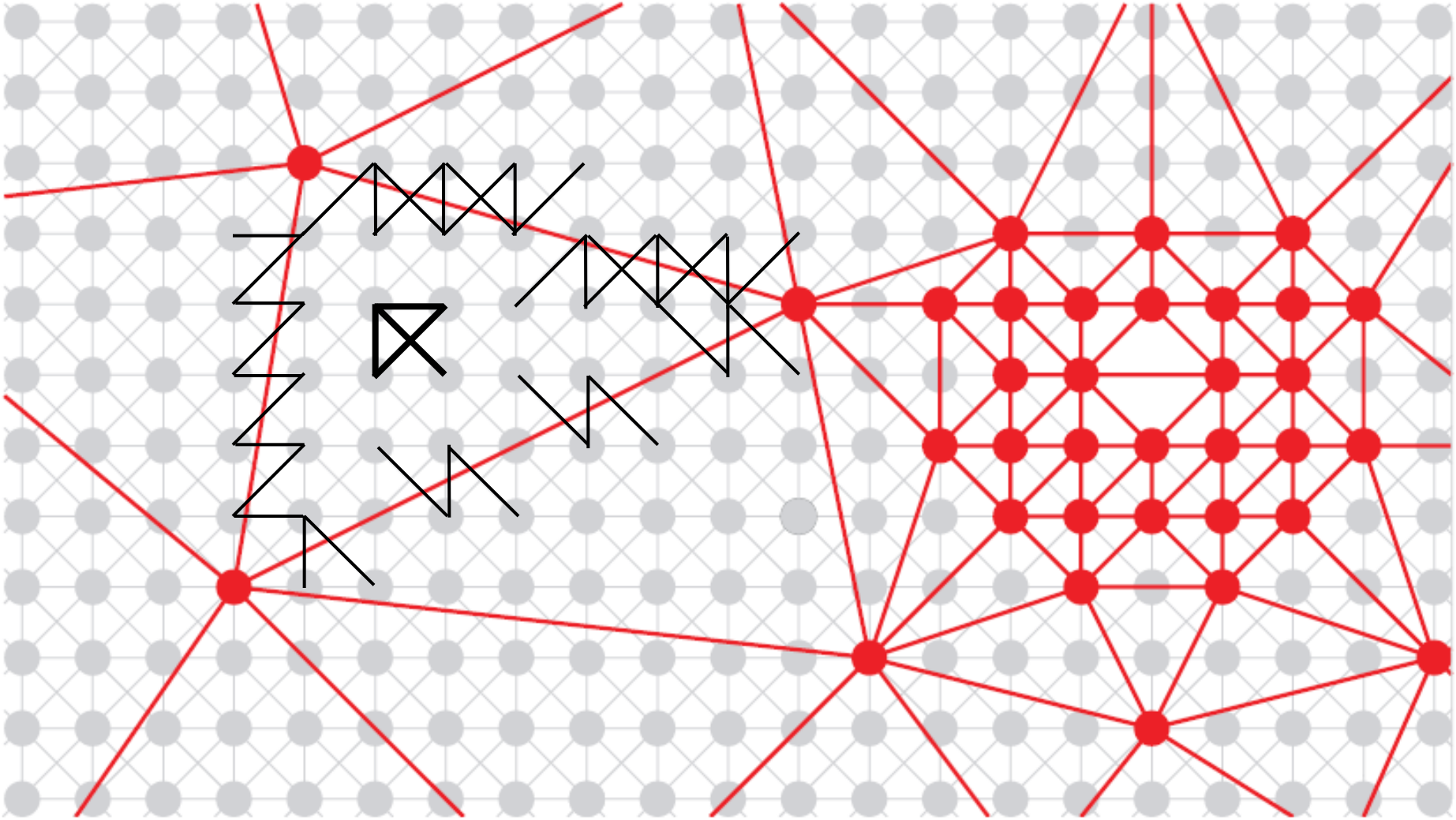
Beex, Verberne, Peerlings, 2013

Summation/sampling: approach 1



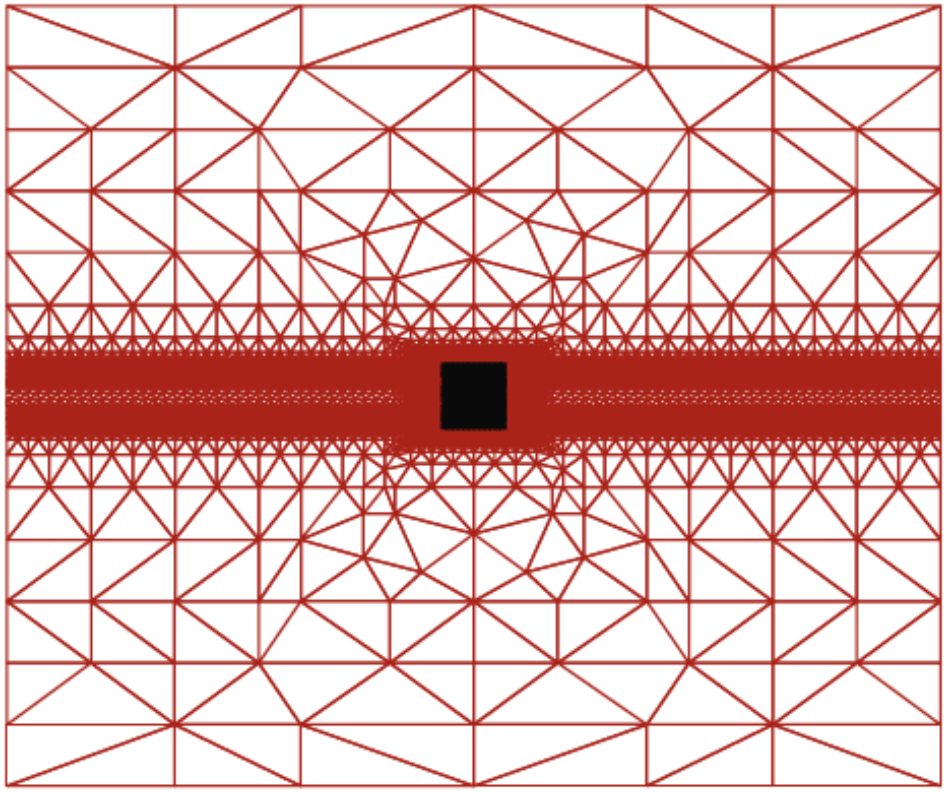
Beex, Peerlings, Geers, 2011

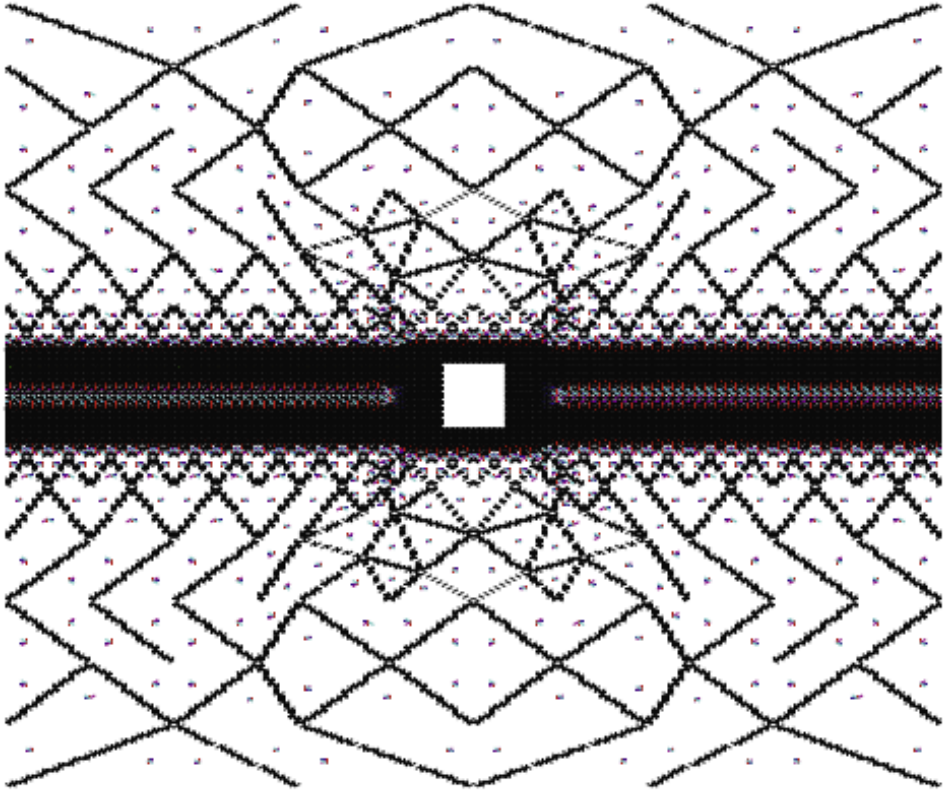
Summation/sampling: approach 1



Beex, Peerlings, Geers, 2011

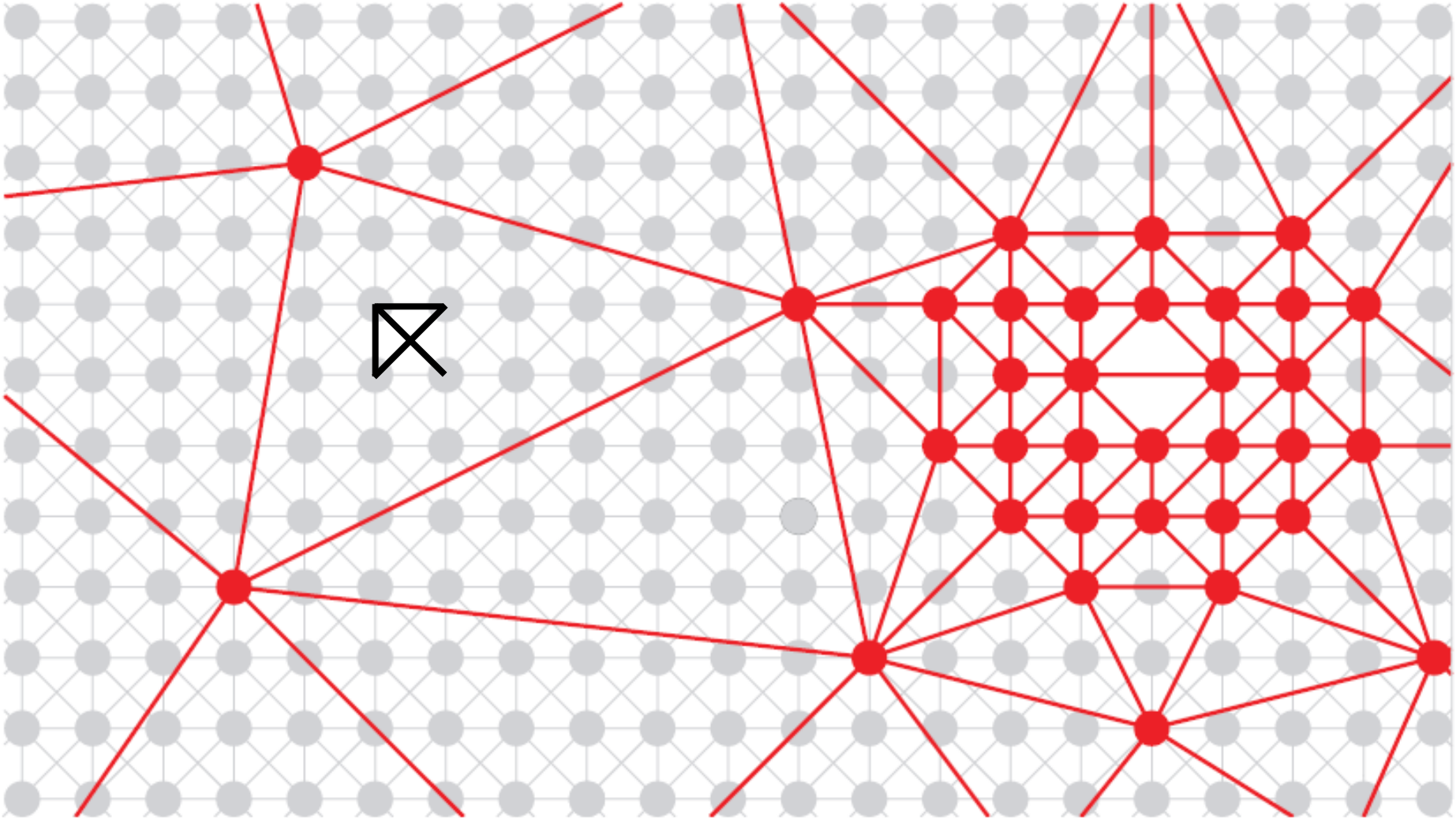
Sampling approach 1 applied to textile lattice





Beex, Peerlings, Van Os, Geers, 2013

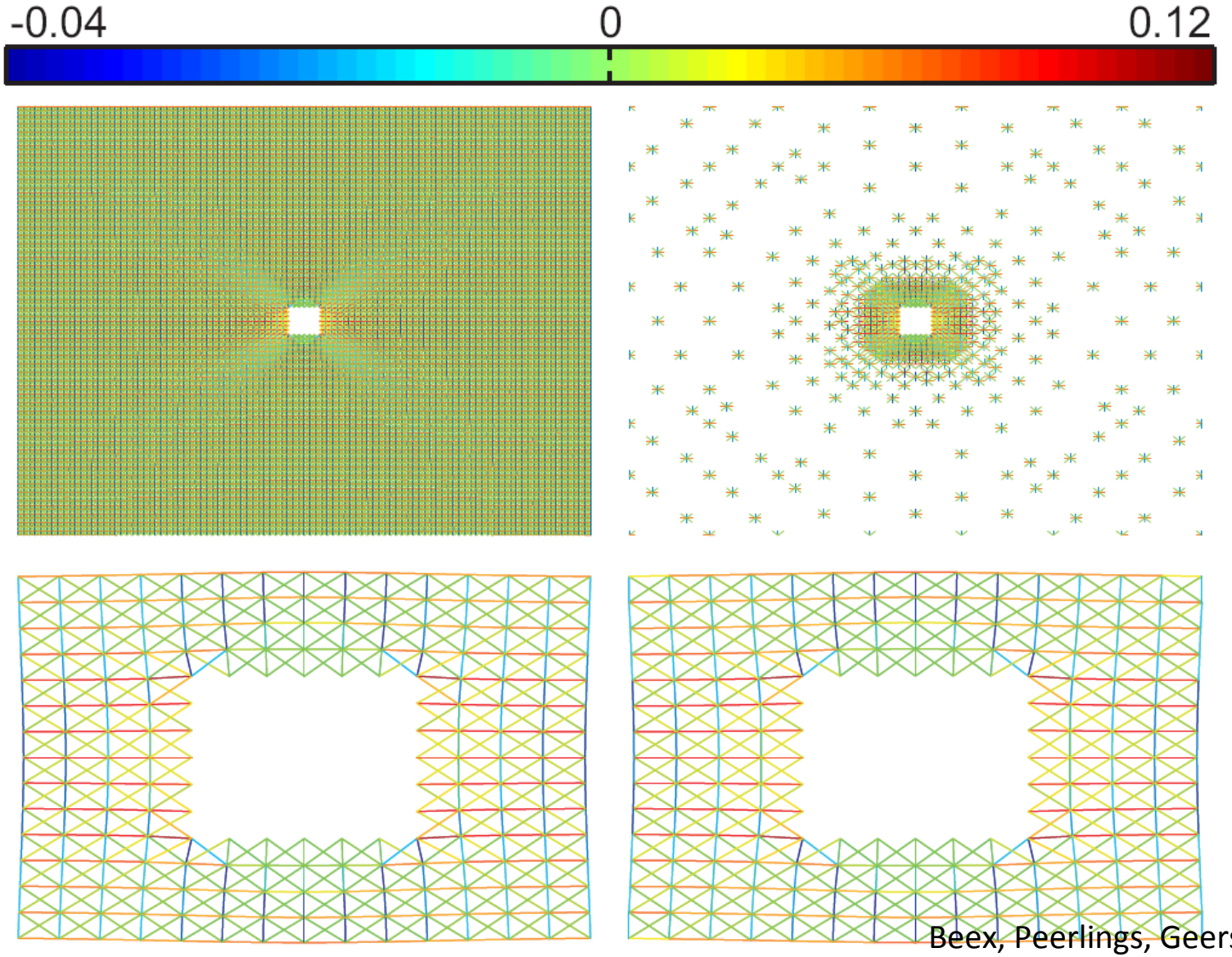
Summation/sampling: approach 2



Beex, Peerlings, Geers, 2014

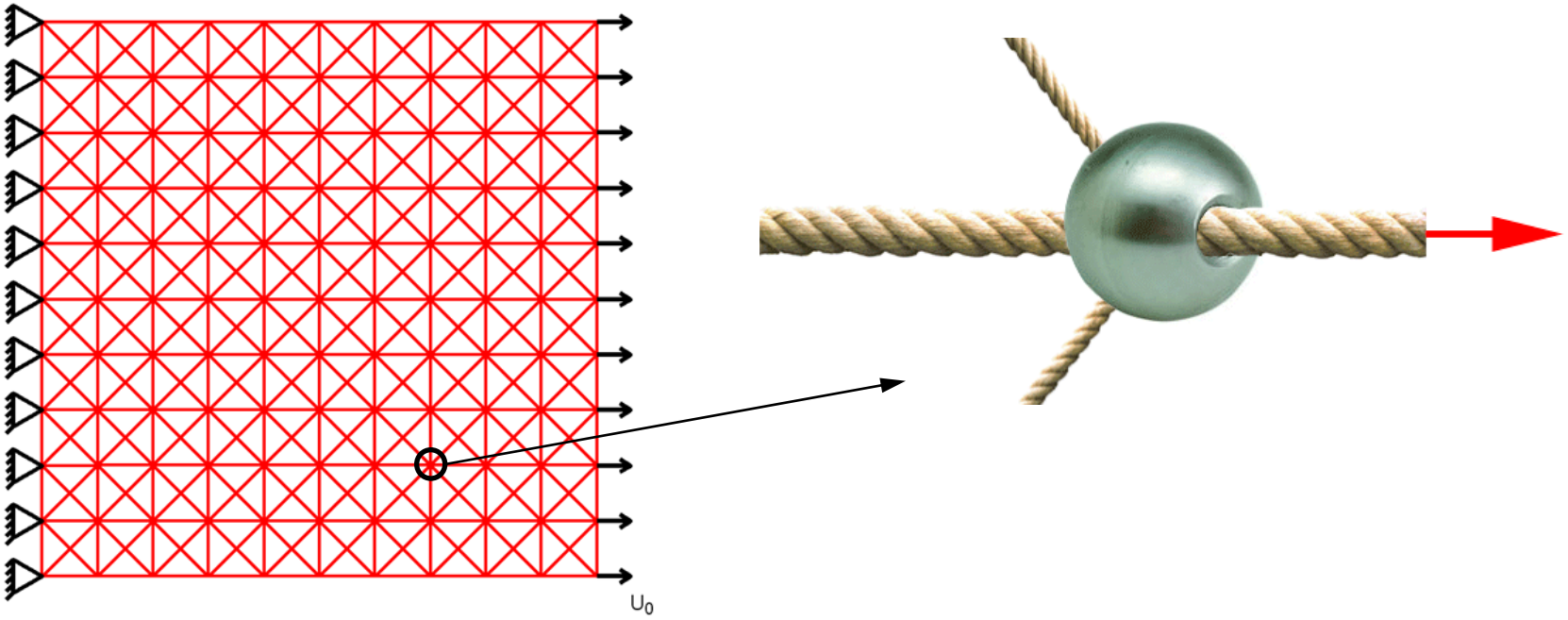
Summation/sampling: approach 2

Plastic strains at 10% horizontal stretch



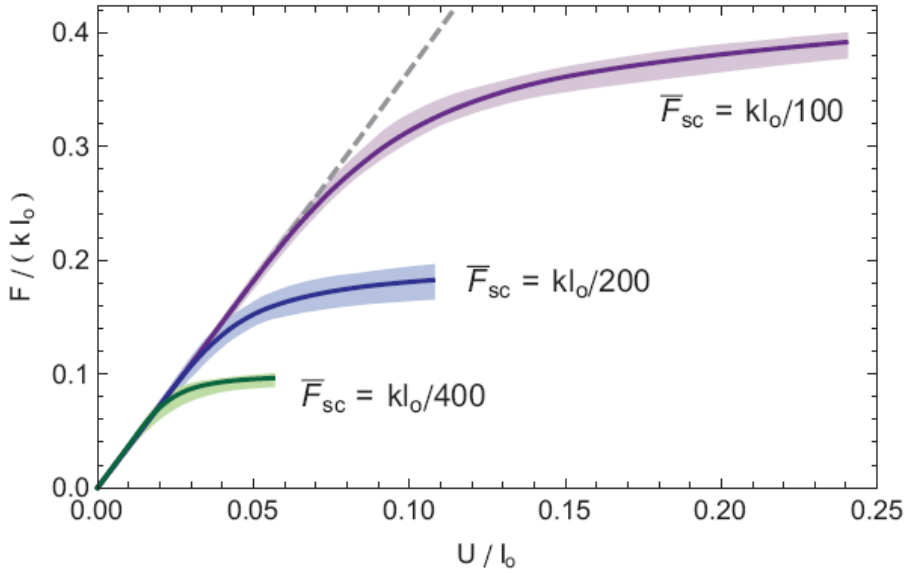
Beex, Peerlings, Geers, 2014

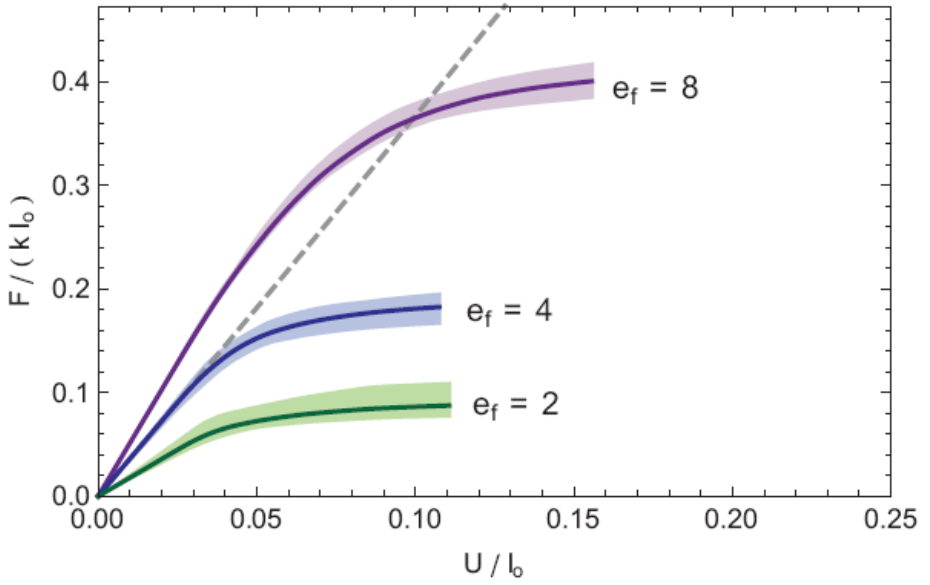
A spring lattice with frictional fibre sliding



Wilbrink, Beex, Peerlings, 2013

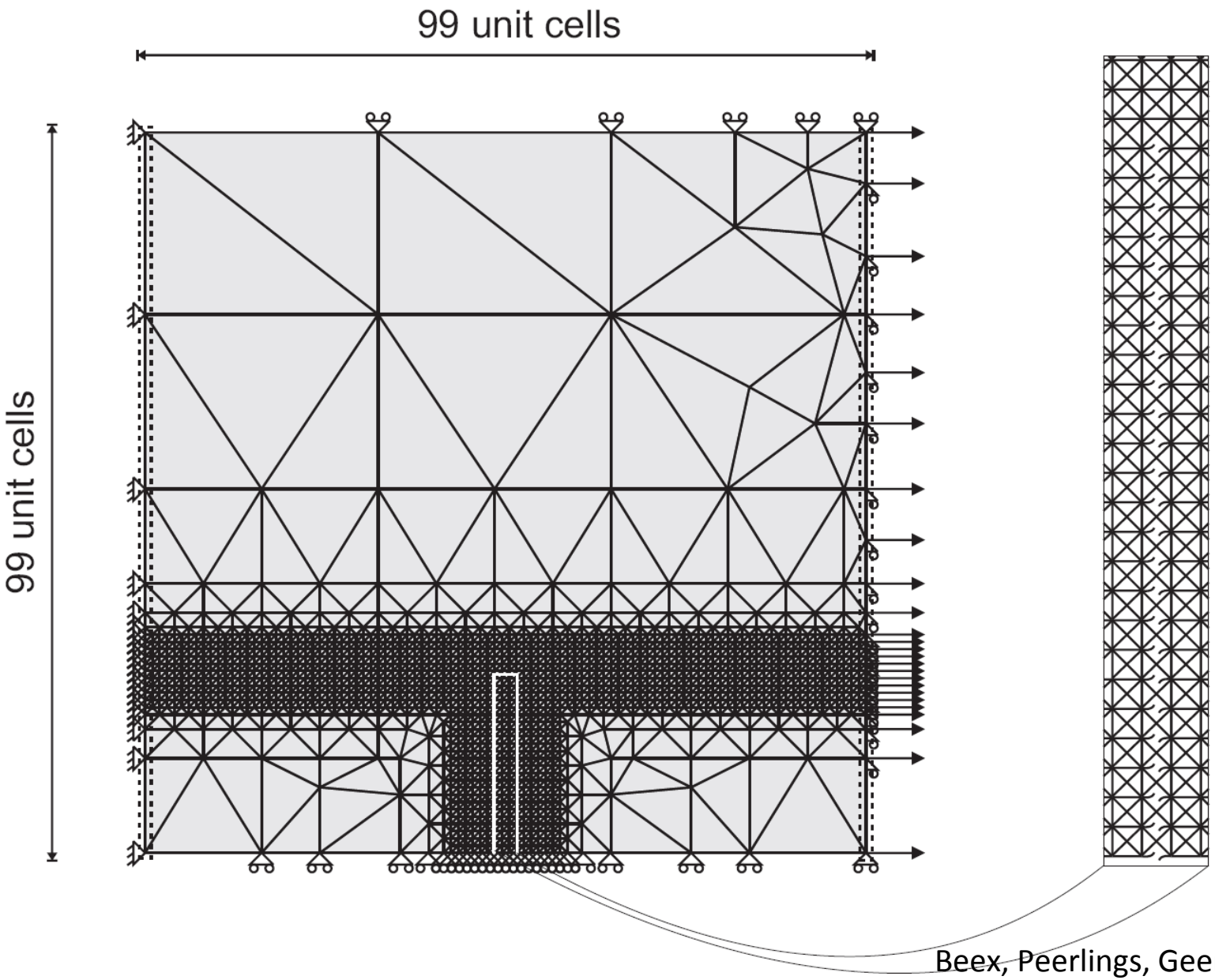
A spring lattice with frictional fibre sliding





Wilbrink, Beex, Peerlings, 2013

A spring lattice with frictional fibre sliding

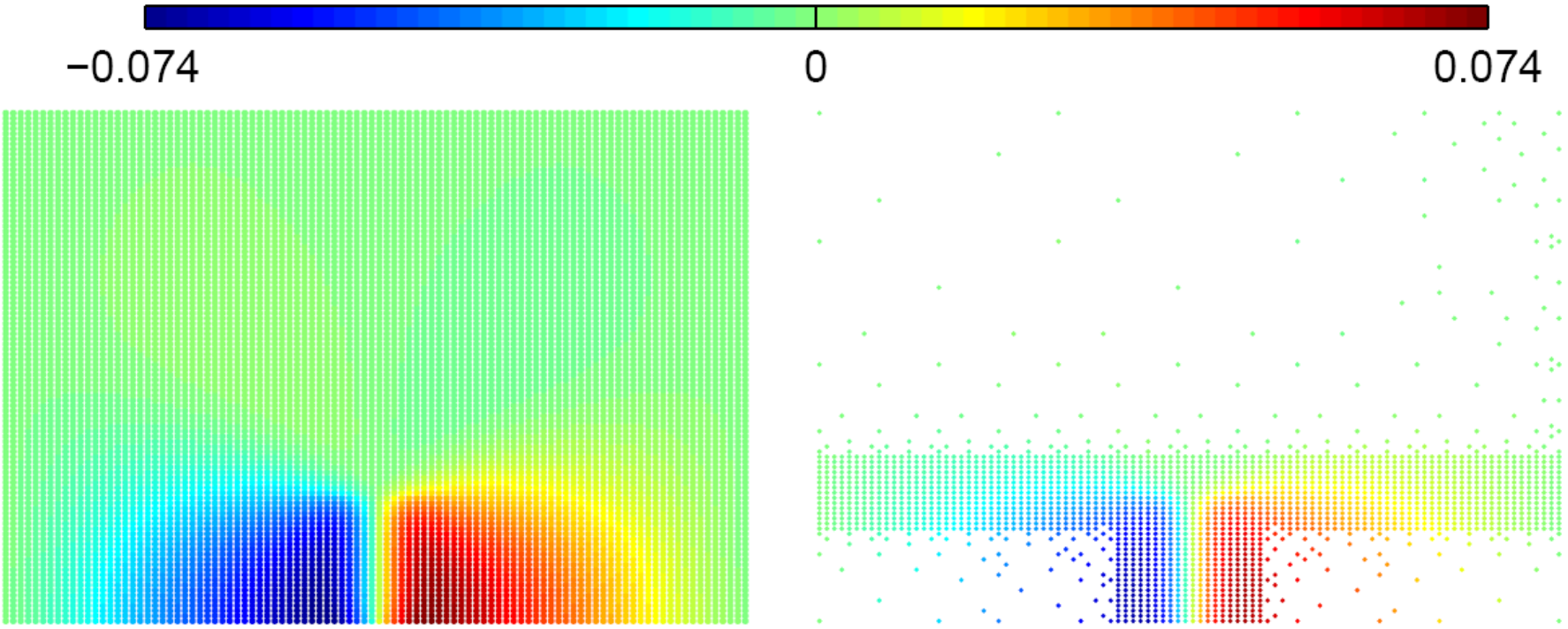


Beex, Peerlings, Geers, 2014

A spring lattice with frictional fibre sliding

UNIVERSITÉ DU LUXEMBOURG

Horizontal displacement, relative to uniform horizontal displacement



Beex, Peerlings, Geers, 2014