

# An equation-free multiscale method applied to discrete networks

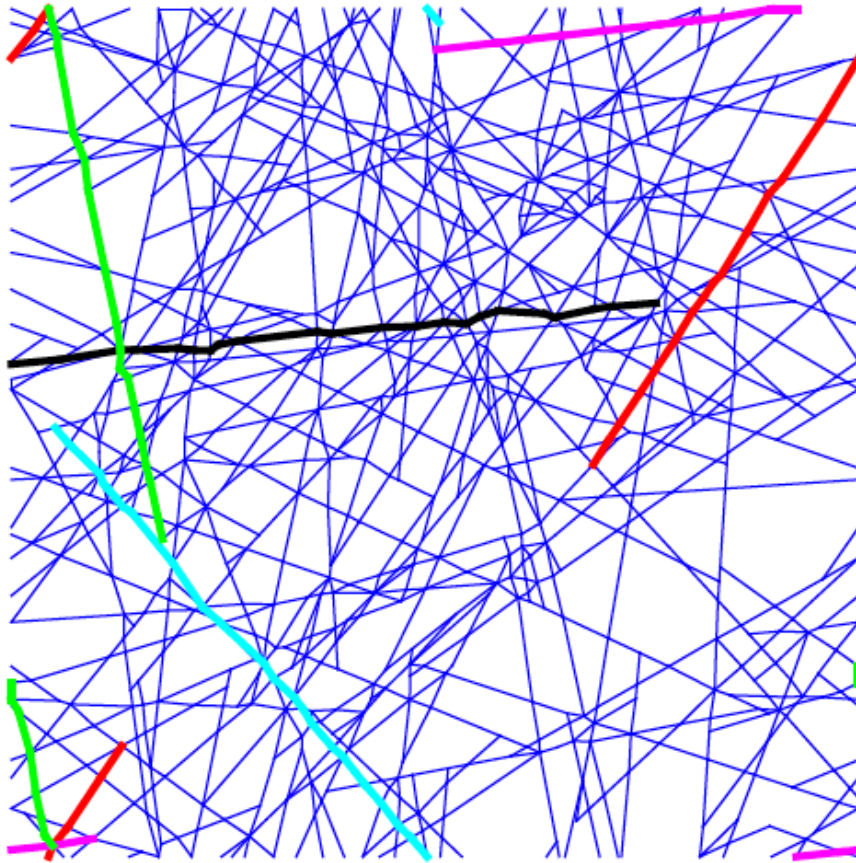
Lars Beex

University of Luxembourg

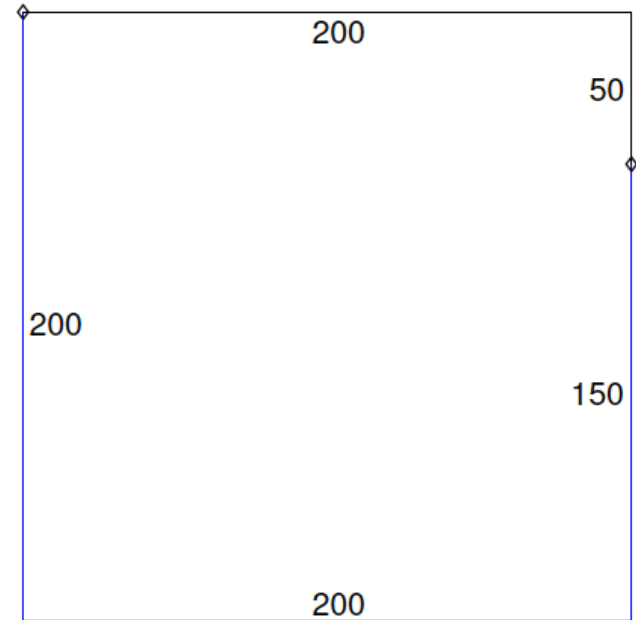
Pierre Kerfriden

Cardiff University

## Setup

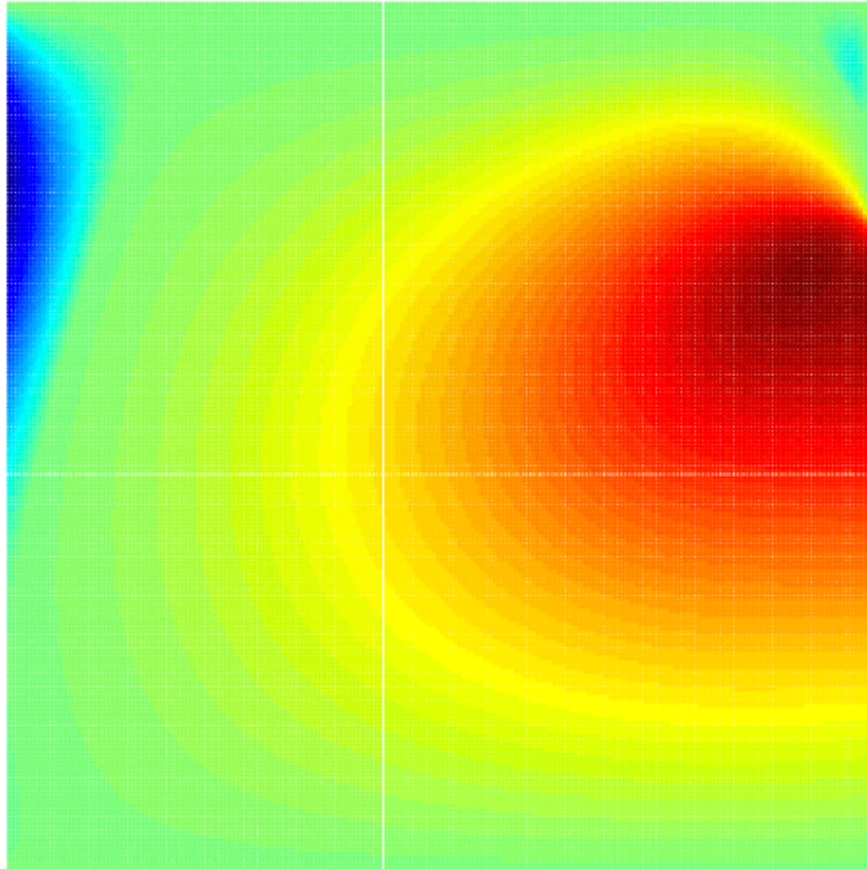


Planar unit cell of LE EB beams inc damage  
(1x1mm<sup>2</sup>)



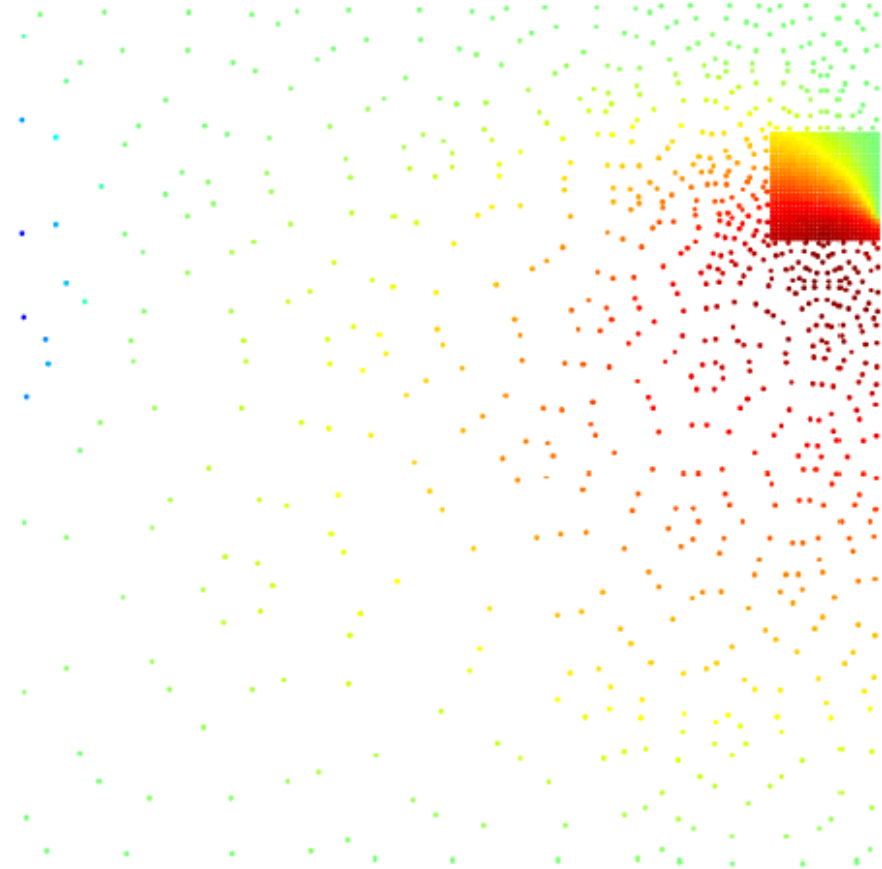
Macroscale problem

## Rotation around vertical axis



DNS:

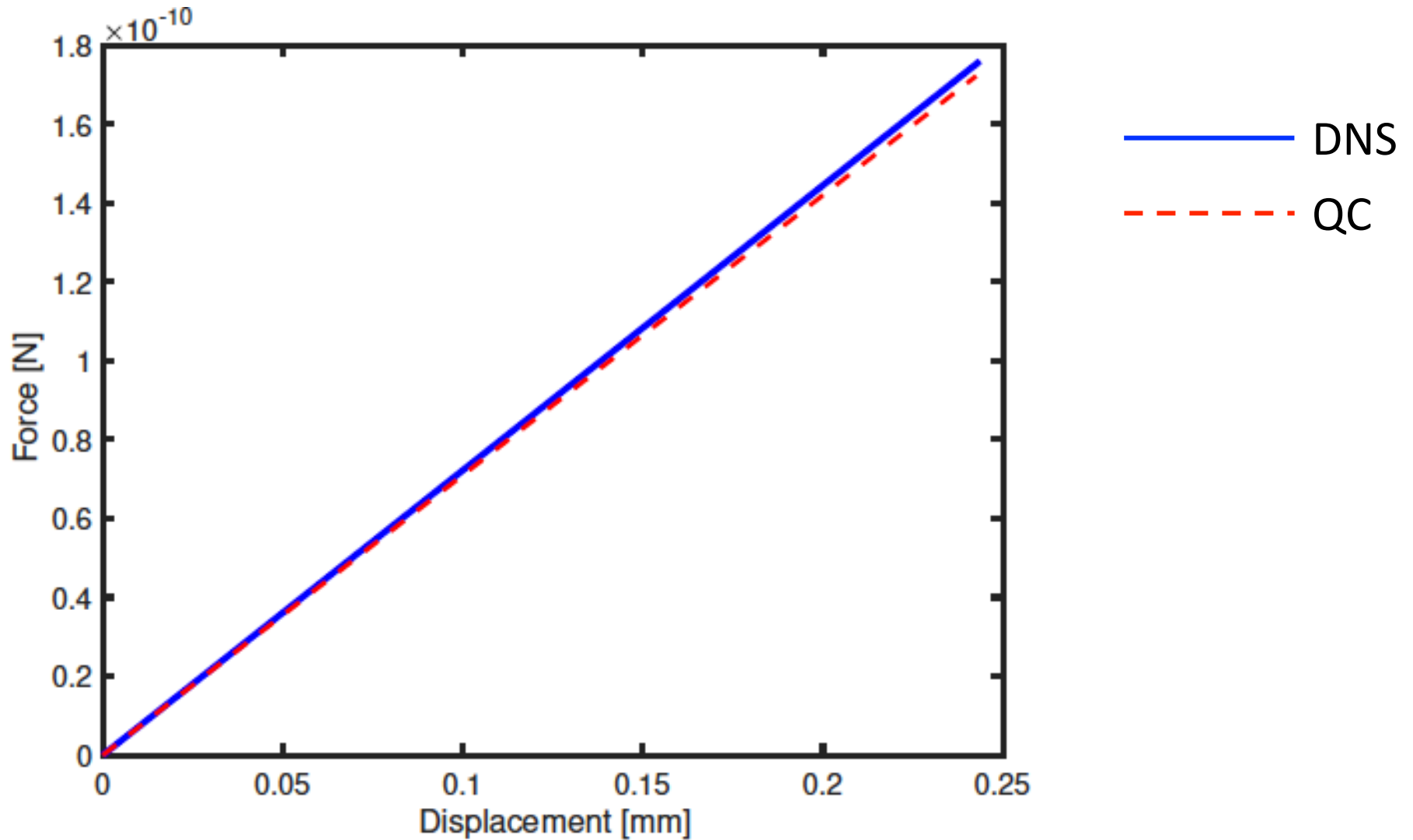
- 80M beams
- 233M DOFs



QC:

- 29x less beams
- 42x less DOFs

## Force-displacement response



## **Condition:**

Unit cell must be periodic.

## **Advantages** compared to other nested multiscale methods:

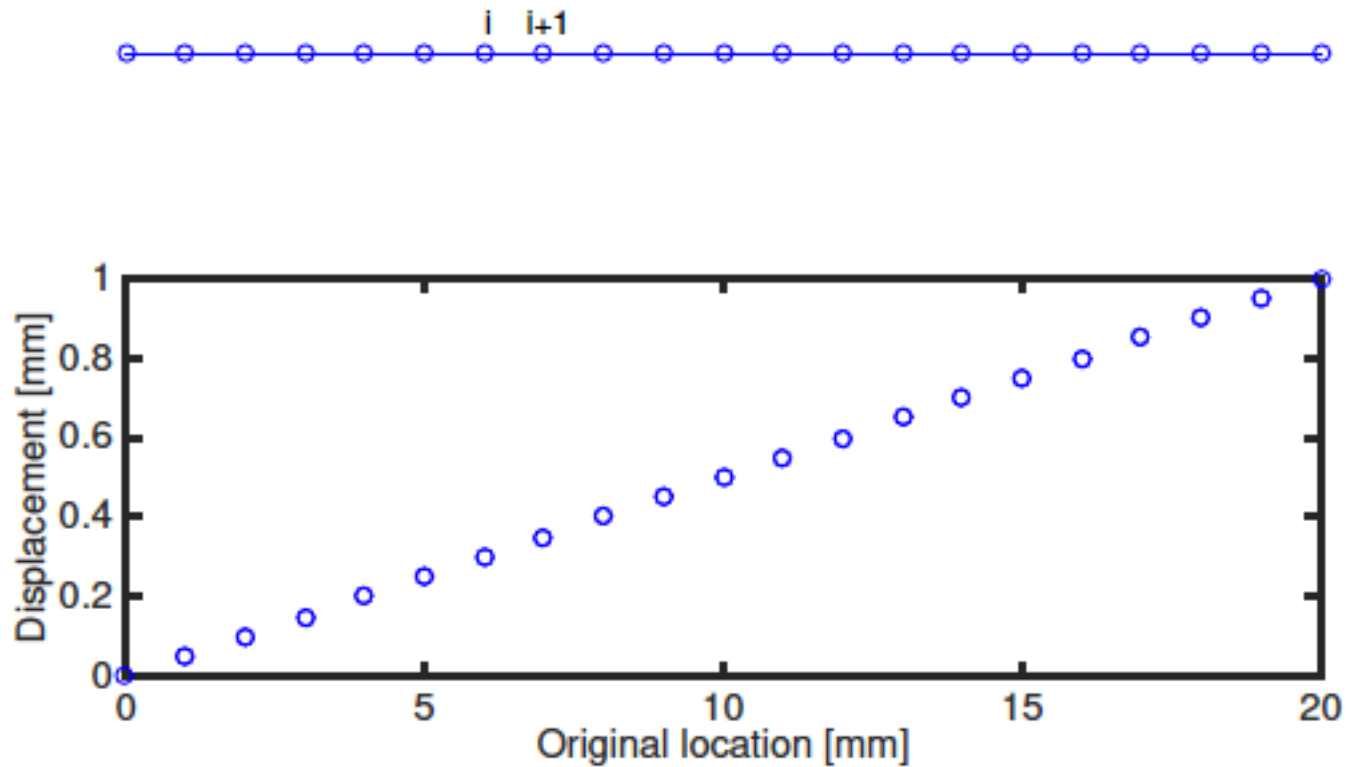
1. Higher-order macroscale interpolations are as easy to treat as linear ones
2. No scale-separation

## **Disadvantages** compared to other nested multiscale methods:

1. All DOFs in one system, instead of subdivided over the unit cells and the macroscale elements
2. More unit cells required

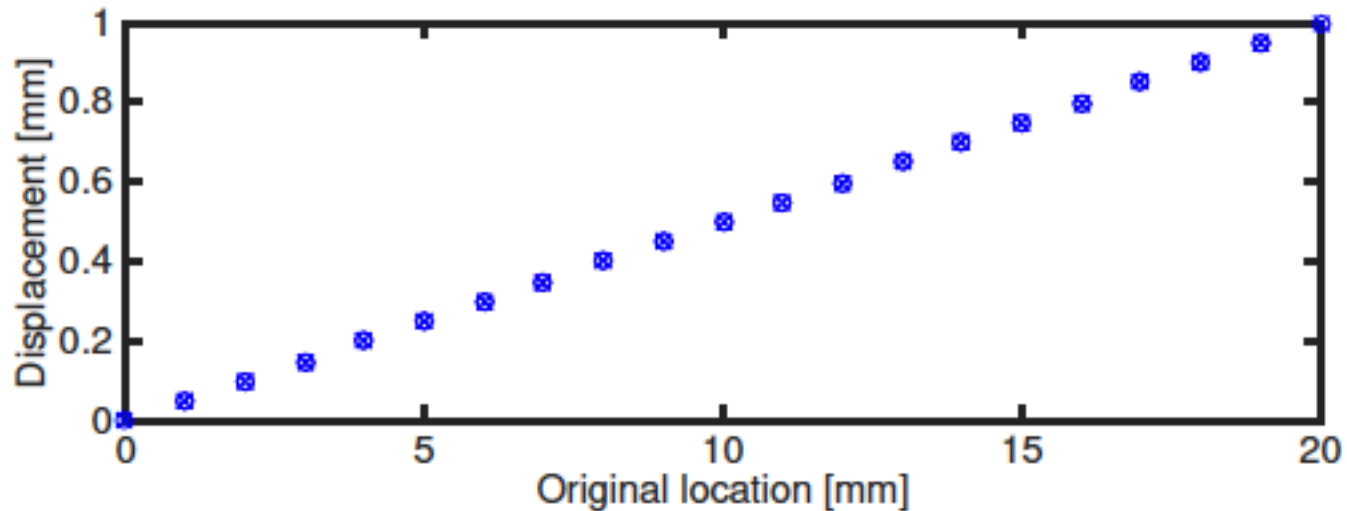
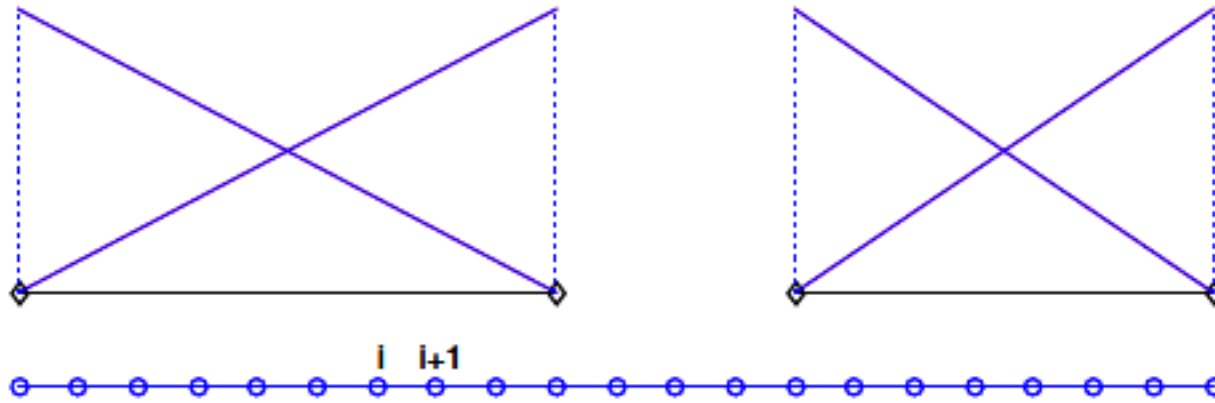
# The method's ansatz explained

1D string of LE springs: DNS



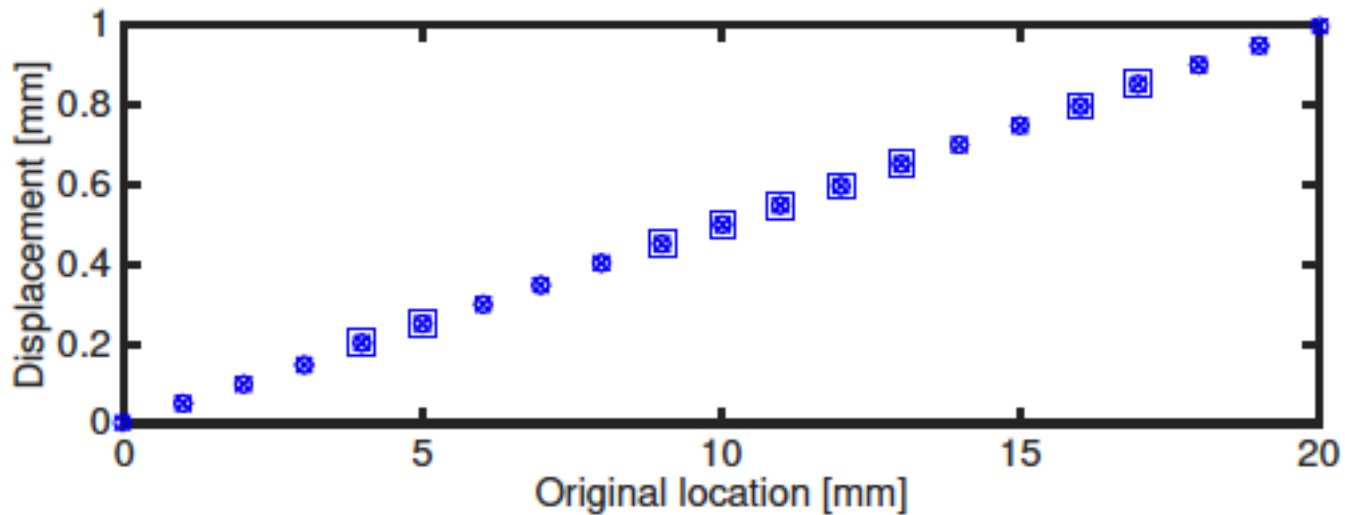
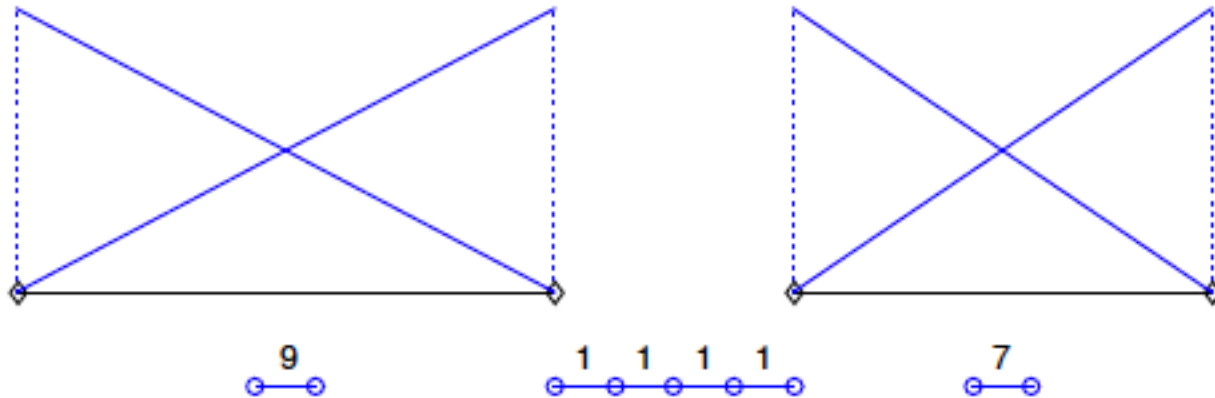
# The method's ansatz explained

1D string of LE springs: Interpolation



# The method's ansatz explained

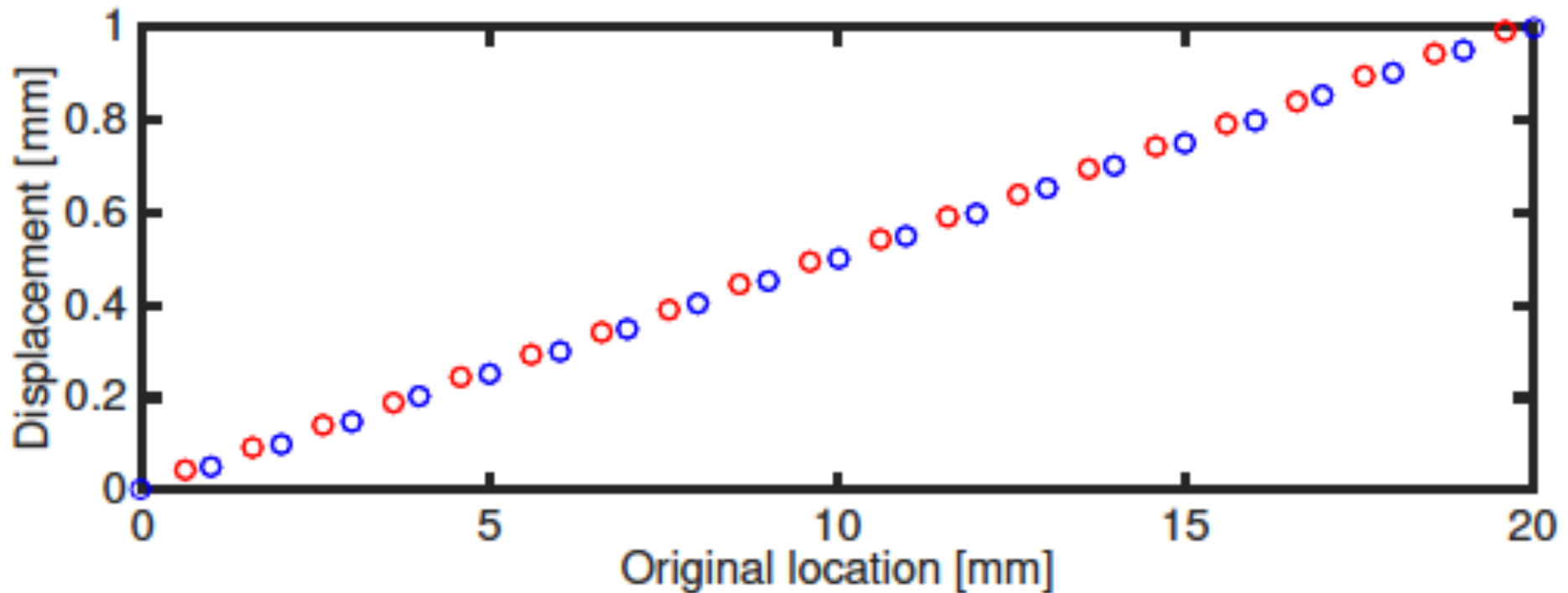
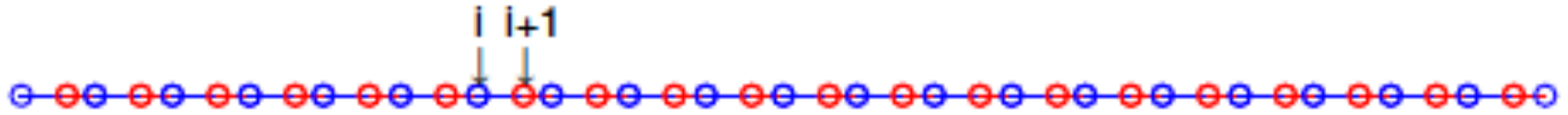
1D string of LE springs: Interpolation + Sampling





# The method's ansatz explained

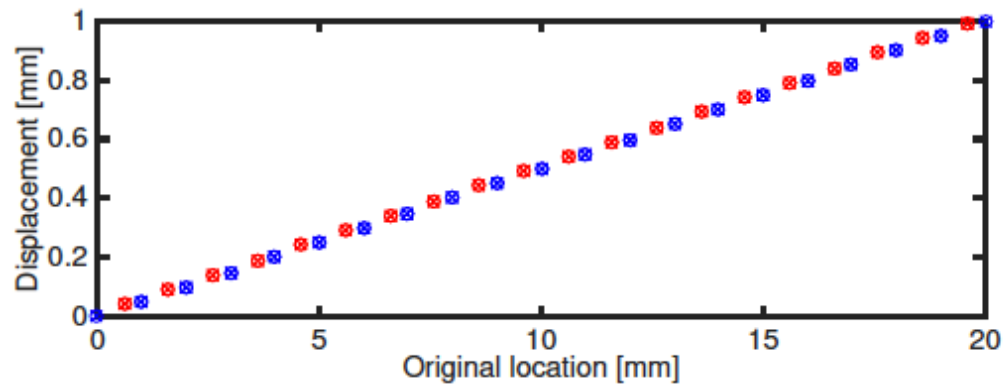
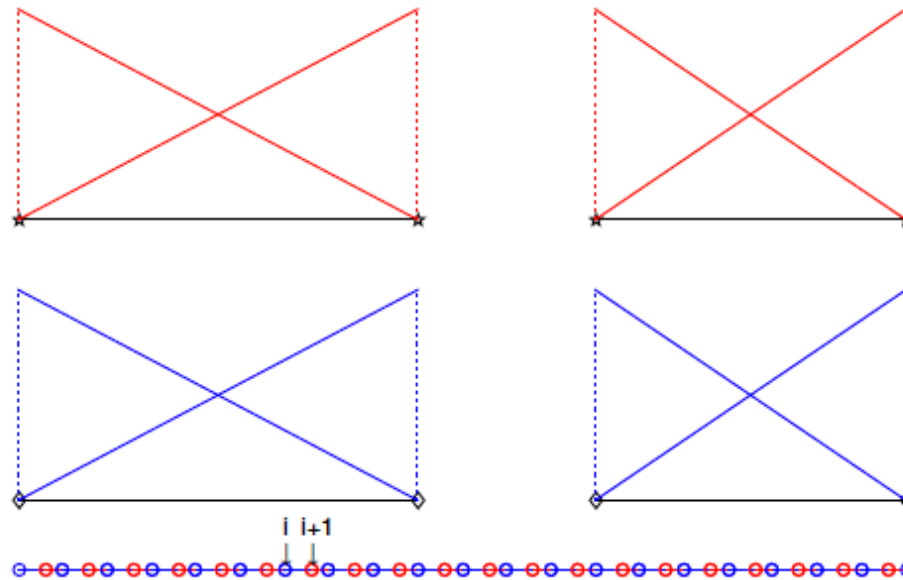
1D string of 2 types of LE springs: DNS



# The method's ansatz explained

1D string of 2 types of LE springs:

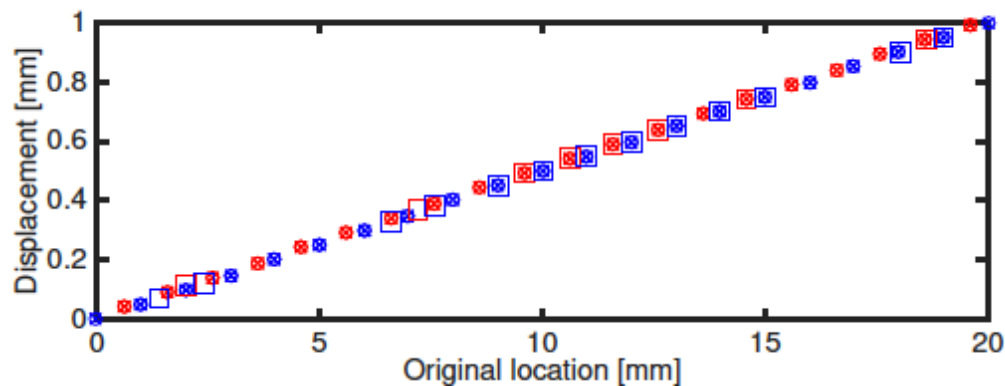
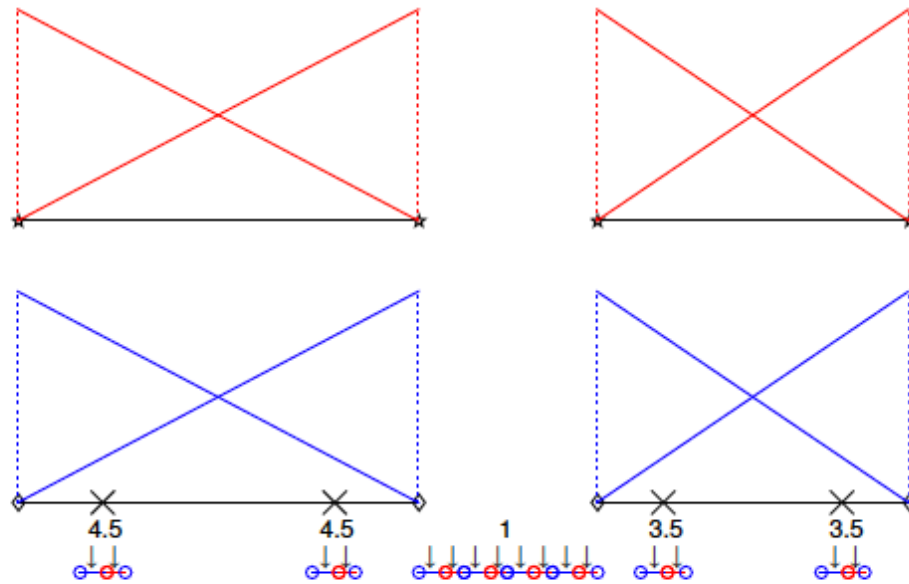
Interpolation



# The method's ansatz explained

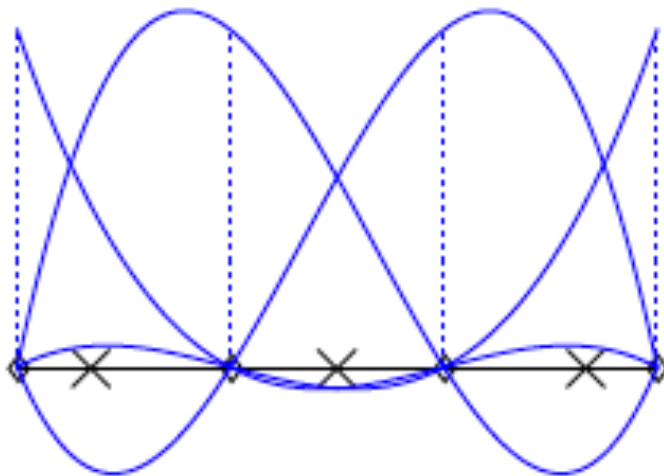
1D string of 2 types of LE springs:

Interpolation + Sampling



# The method's ansatz explained

String of LE beams: Interpolation + Sampling



2.5



4



2.5



1

1

1

1

1

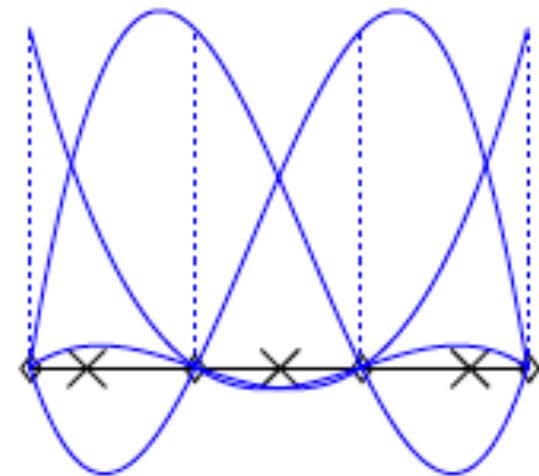
1.944



3.111

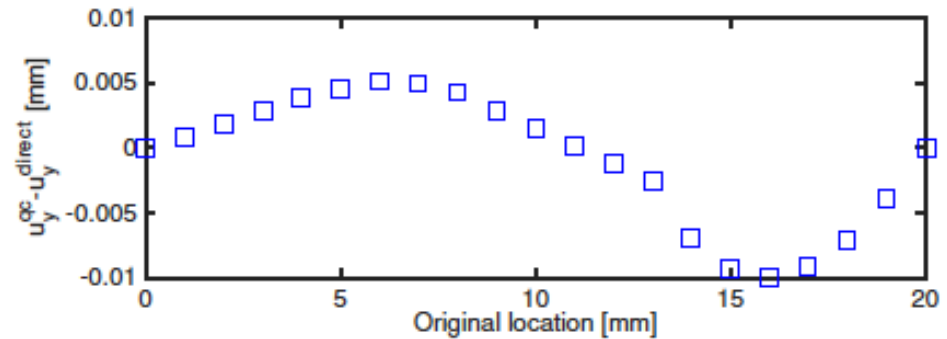
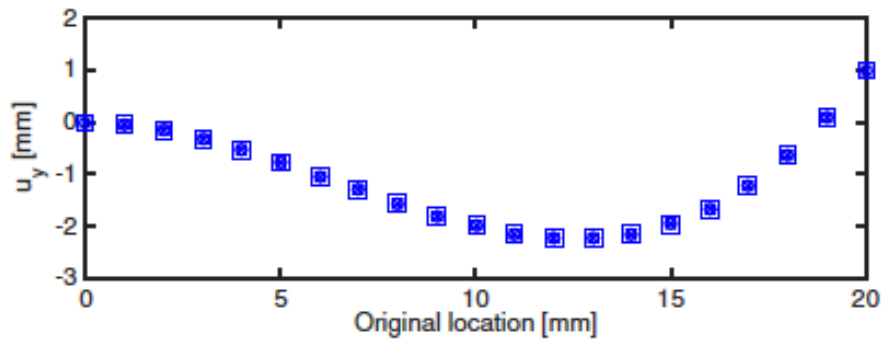
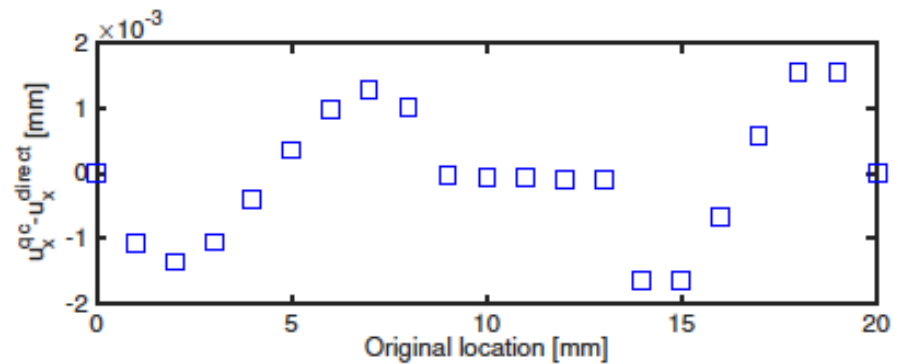
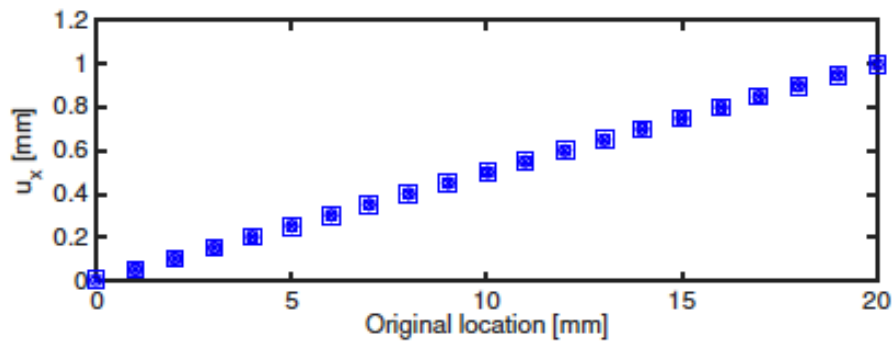


1.944



# The method's ansatz explained

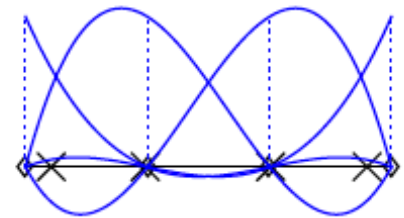
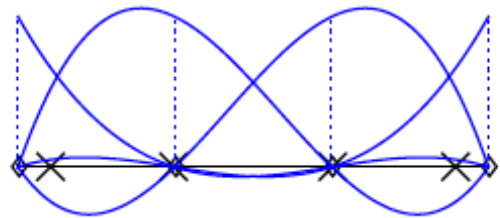
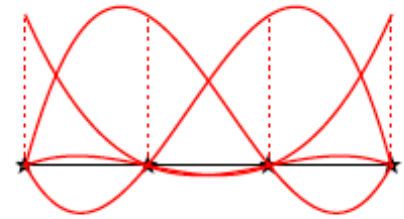
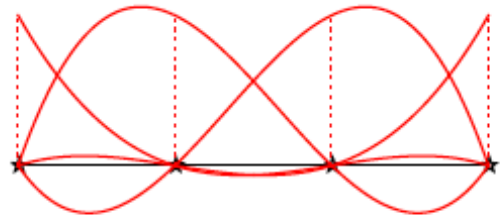
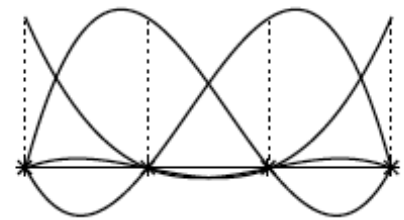
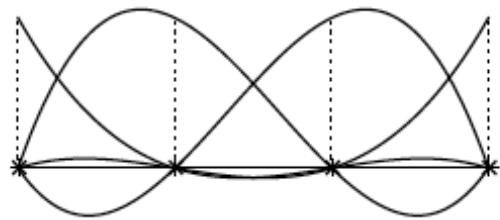
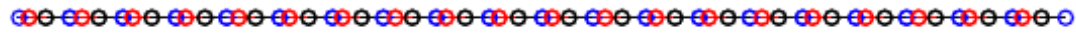
String of LE beams: Interpolation + Sampling



# The method's ansatz explained

String of 3 types of LE beams:

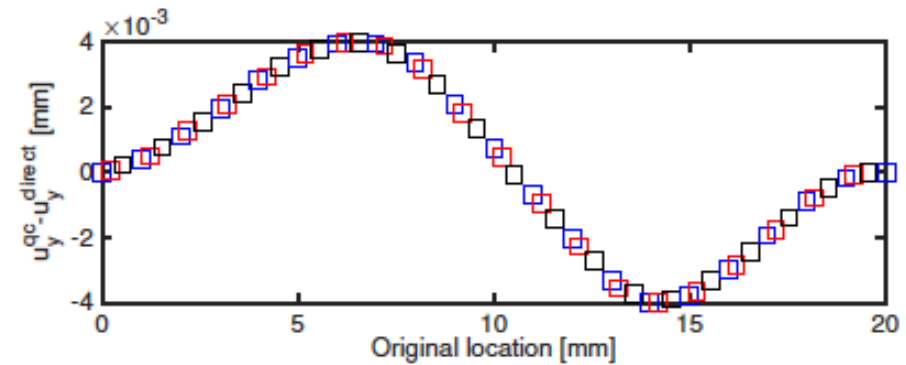
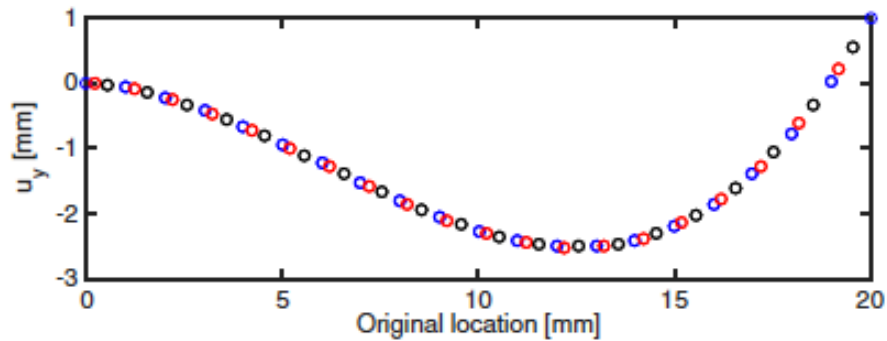
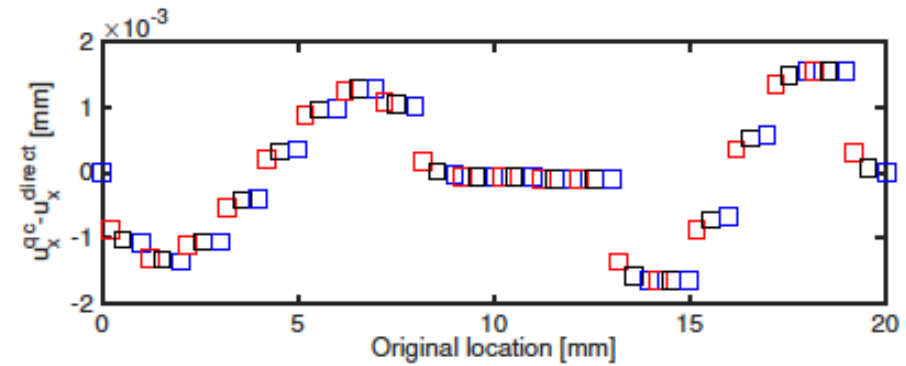
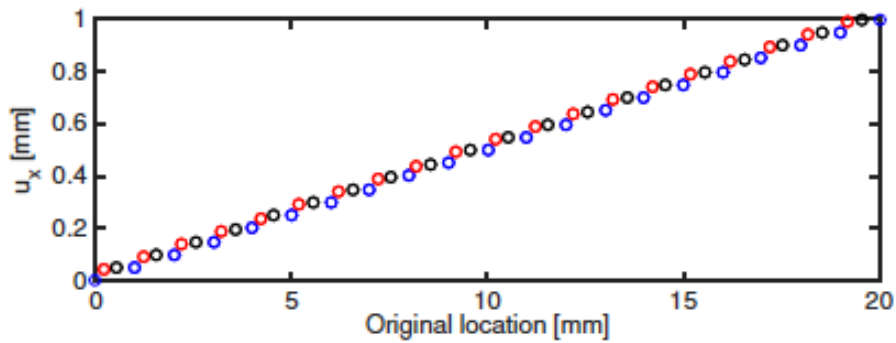
Interpolation + Sampling



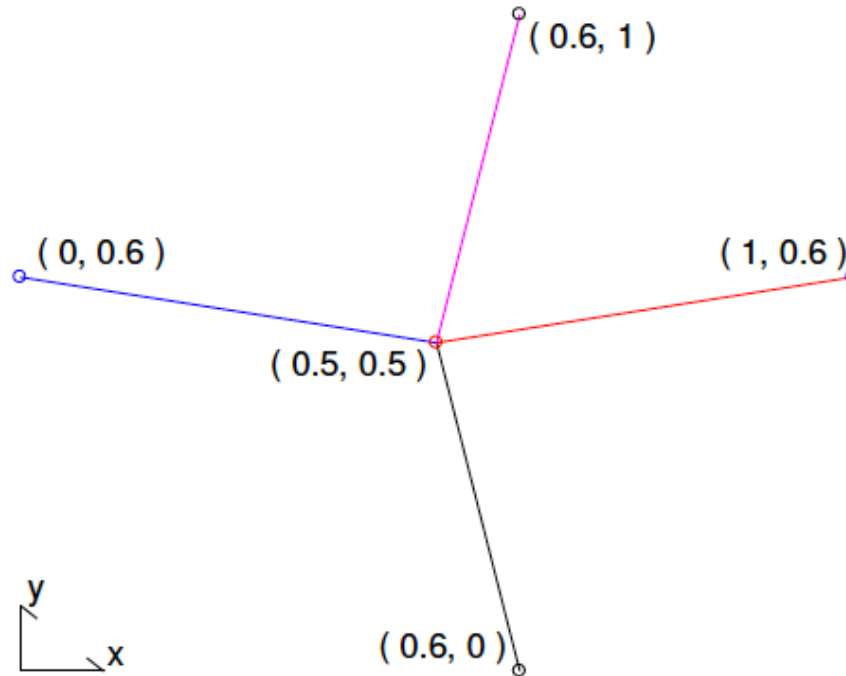
# The method's ansatz explained

String of 3 types of LE beams:

Interpolation + Sampling



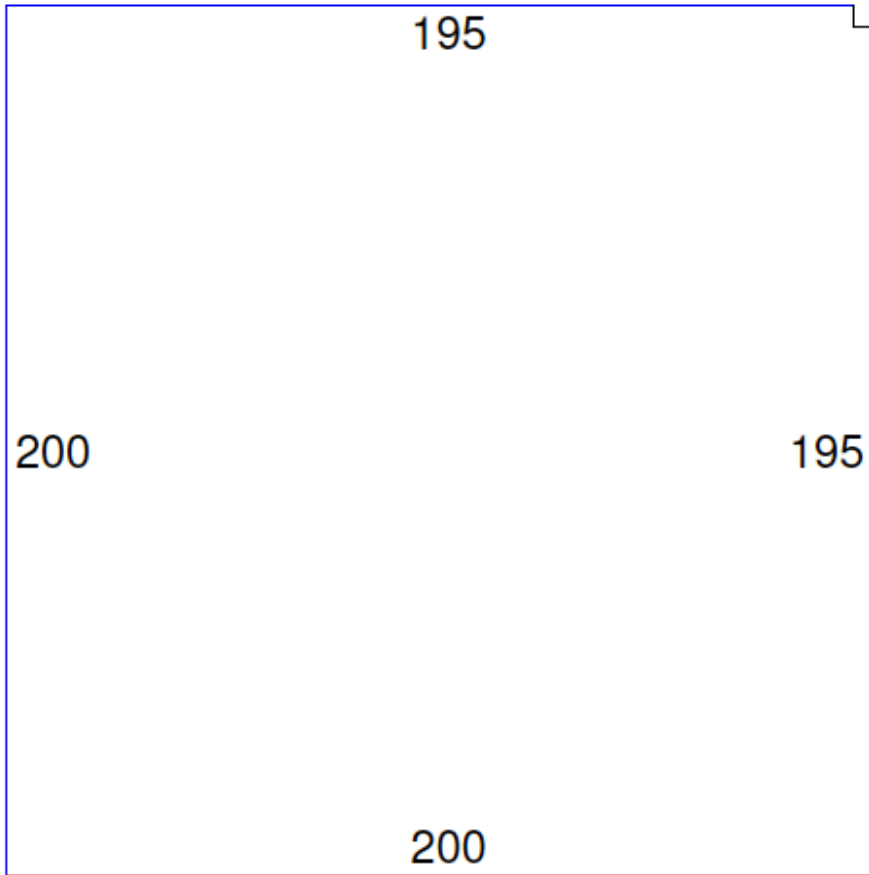
# Simple planar unit cell: Setup



	Blue	Red	Black	Purple	
Area	1	3	9	27	[m <sup>2</sup> ] (x10 <sup>-9</sup> )
Y. Modulus	1	5	25	125	[MPa]
P. ratio	0.1	0.2	0.3	0.4	[-]
Failure str.	10	1	2	70	(x10 <sup>-5</sup> )

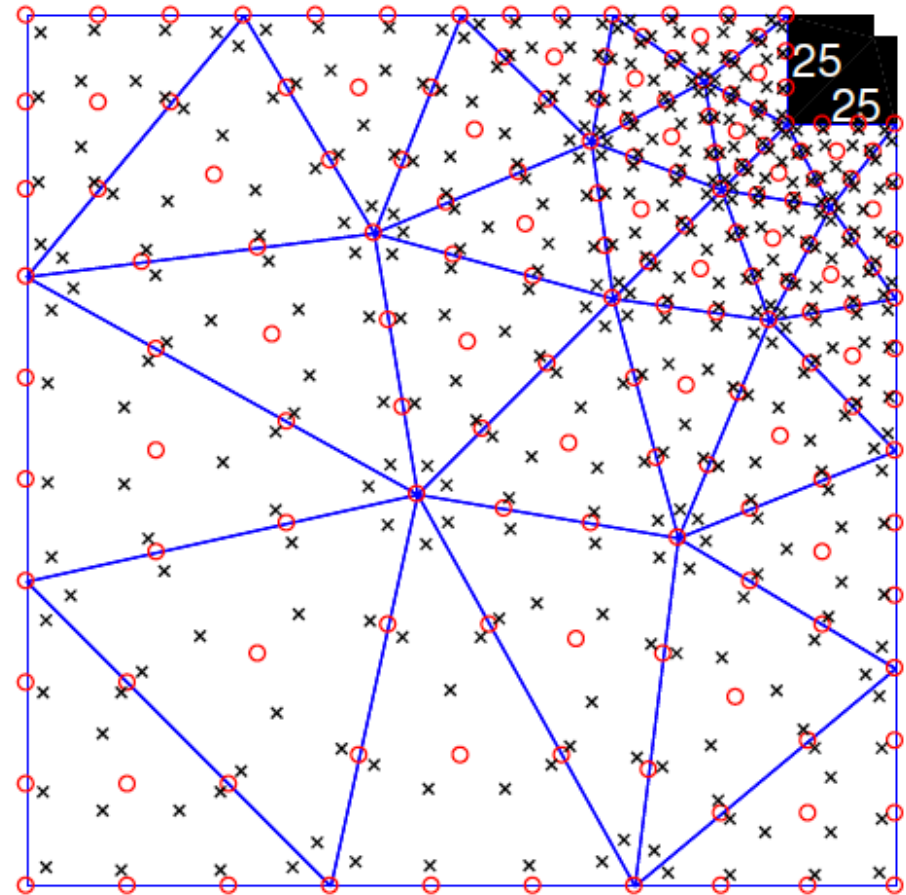


# Simple planar unit cell: Setup



DNS:

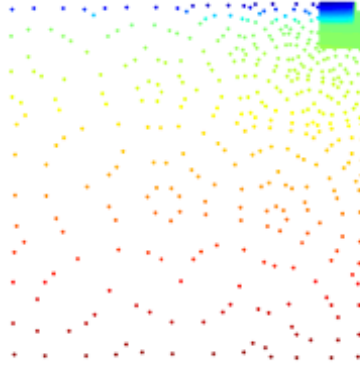
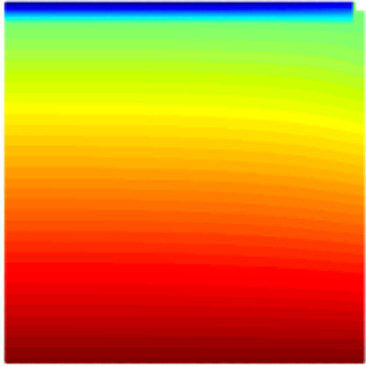
- 160k beams
- 722k DOFs



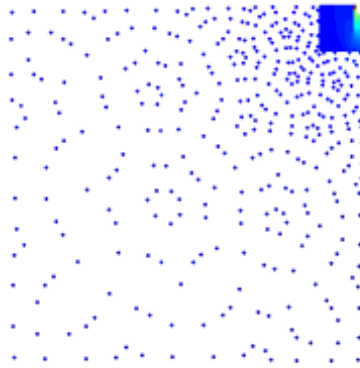
QC:

- 42x less beams
- 52x less DOFs

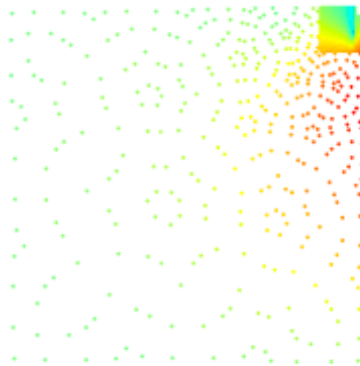
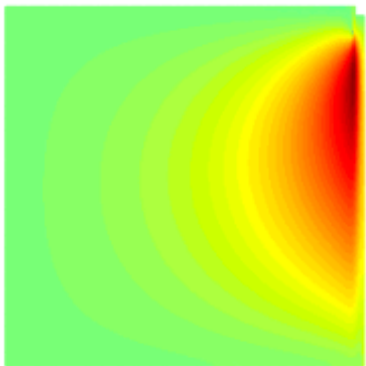
# Simple planar unit cell: Results



Out-of-plane displacement

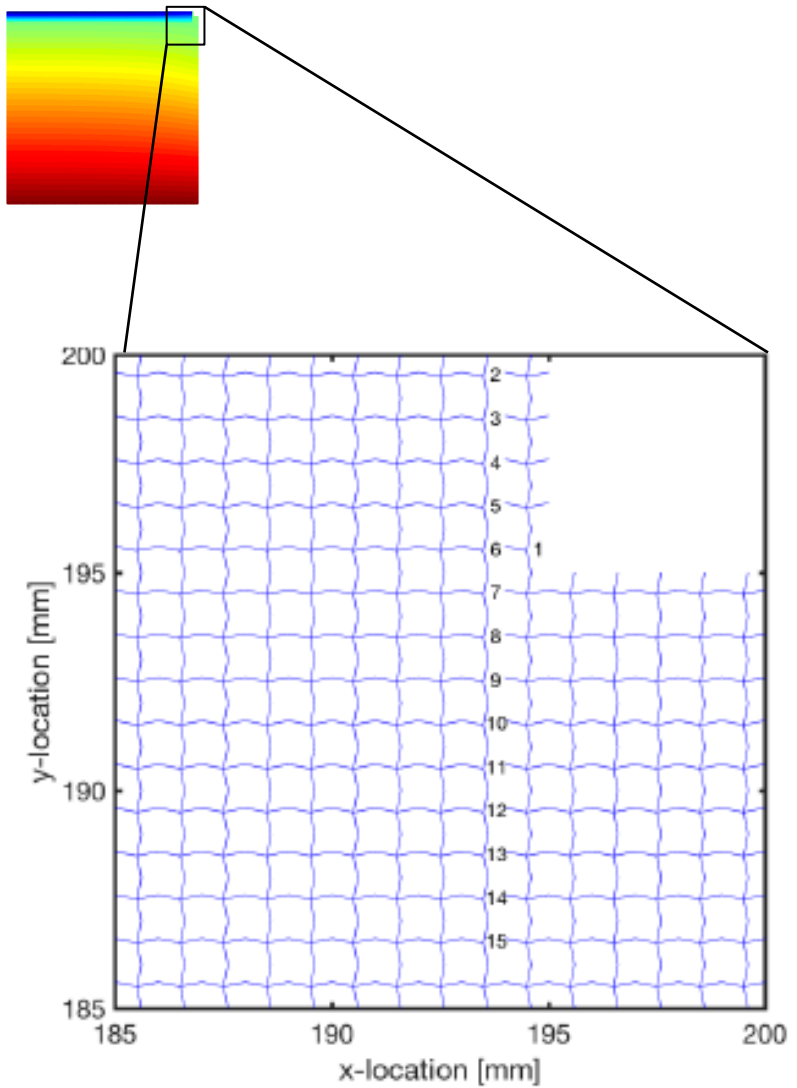


Rotation around horizontal axis

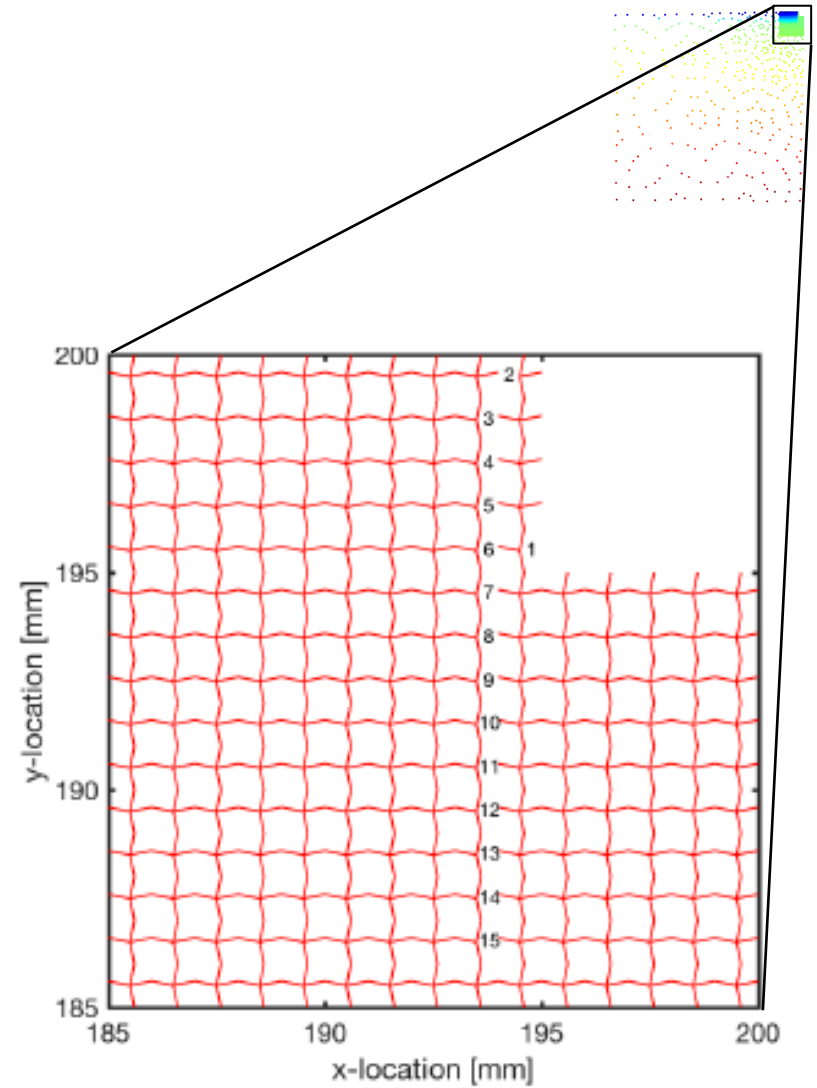


Rotation around vertical axis

# Simple planar unit cell: Results



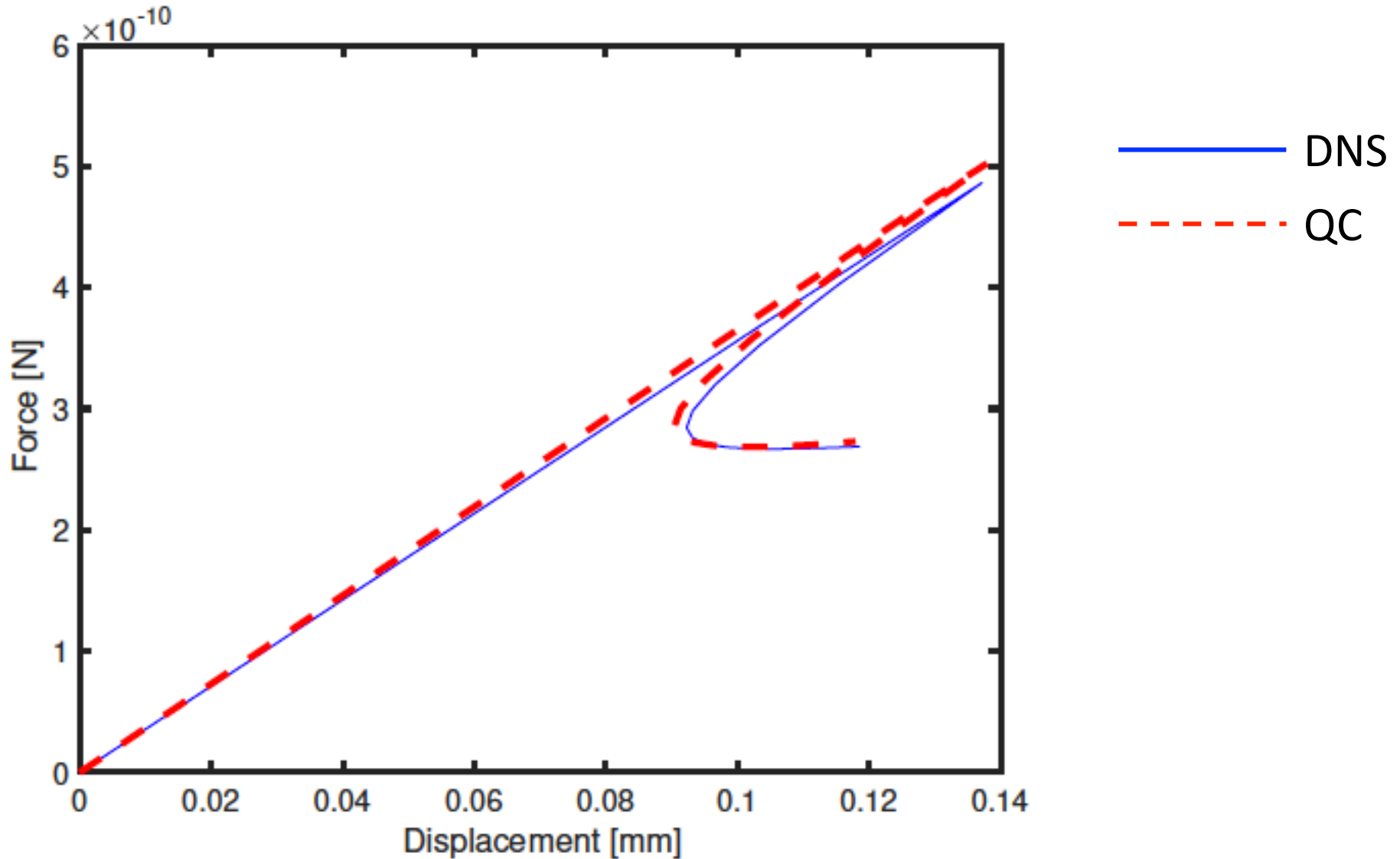
DNS



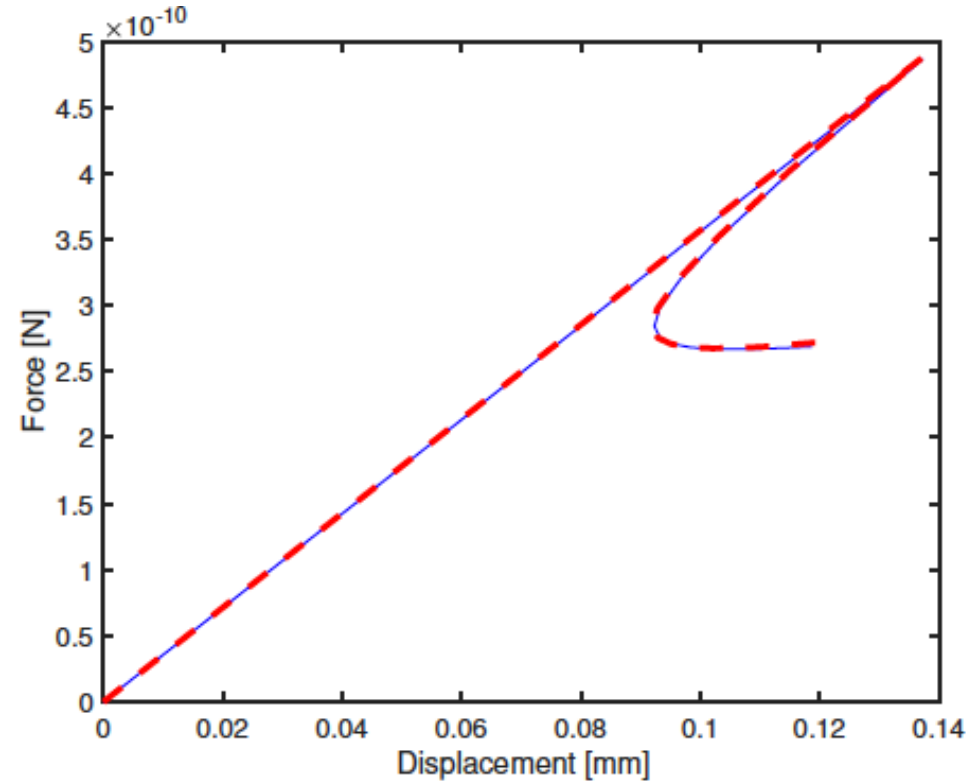
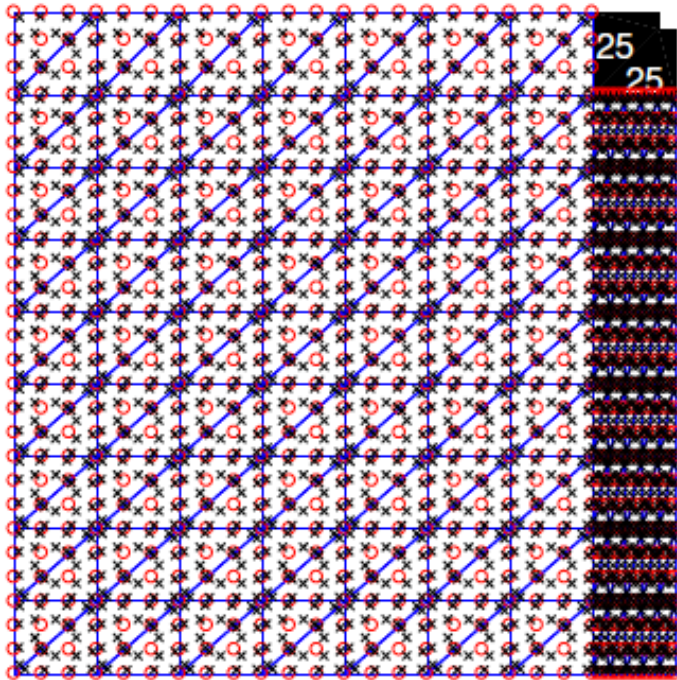
QC

Failed beams, inc. sequence

## Force-displacement response



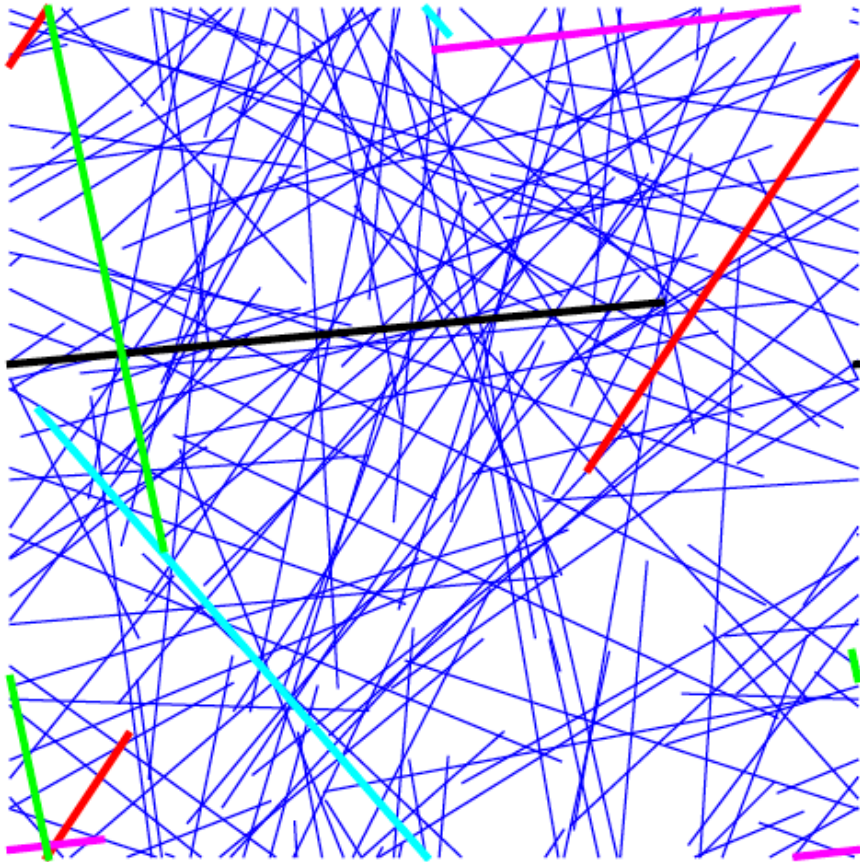
# Simple planar unit cell: New setup + Result



QC:

- 13x less beams
- 25x less DOFs

— DNS  
- - - QC

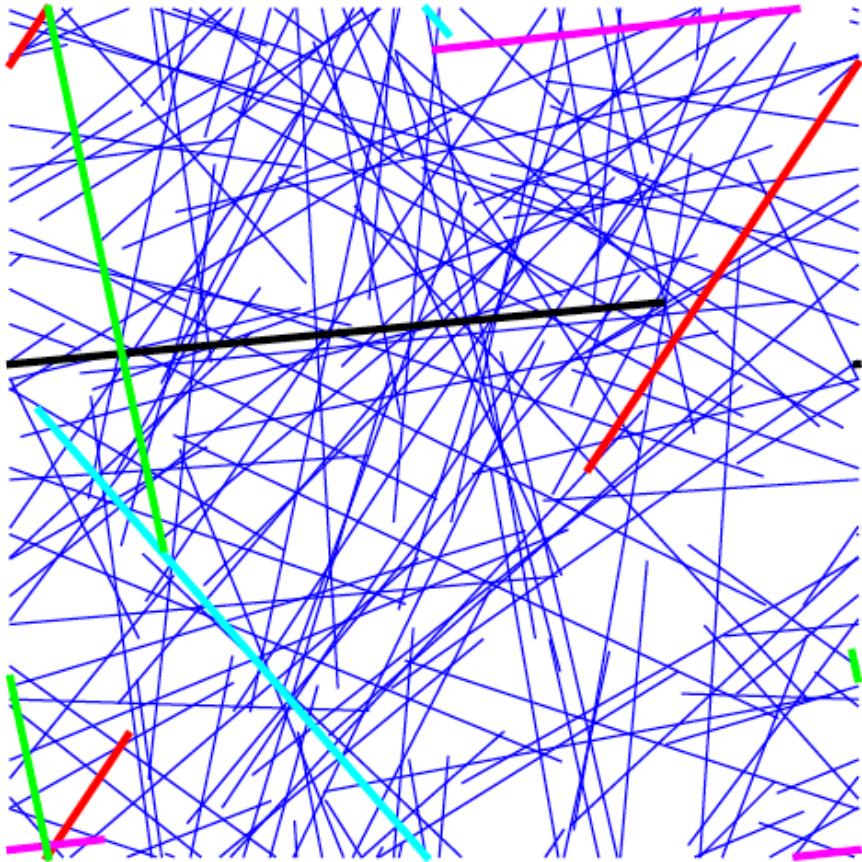


Periodic, planar unit cell of fibres (1x1mm<sup>2</sup>)

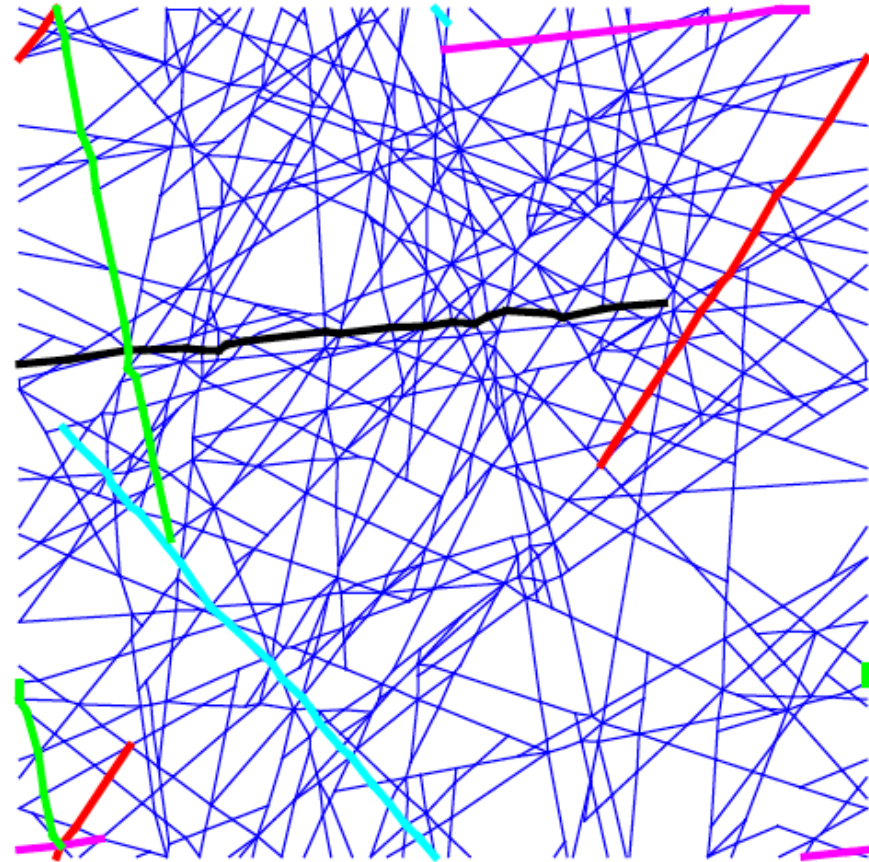
## Parameters taken from U(a,b)

	a	b	
L	0.6	0.9	[mm]
A	1	2	[m <sup>2</sup> ] (x10 <sup>-9</sup> )
E	1	2	[MPa]
$\nu$	0.2	0.4	[-]
Failure str.	1	2	(x10 <sup>-5</sup> )

# Fibrous unit cell: Small setup



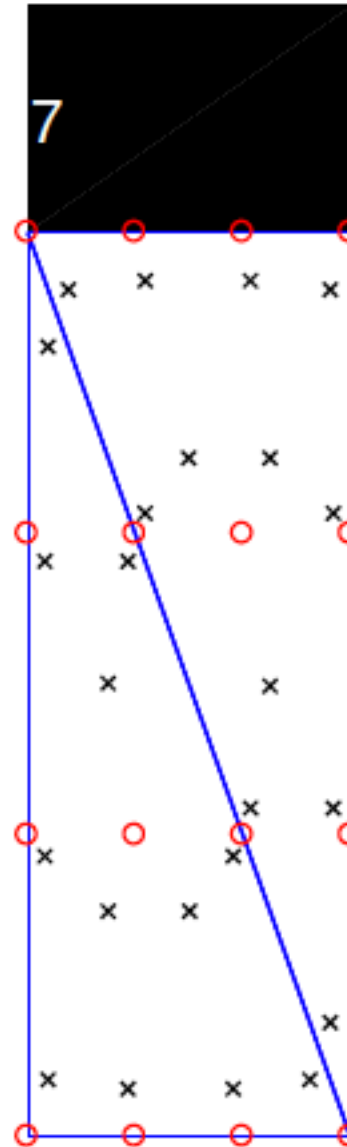
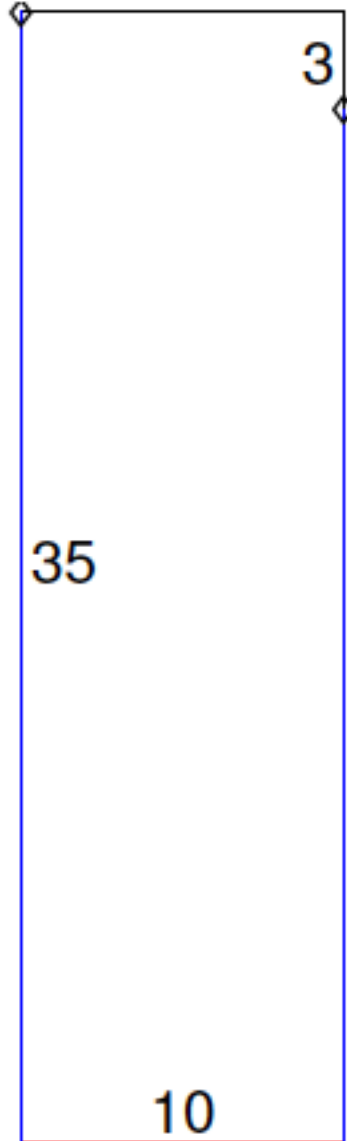
Periodic, planar unit cell of fibres (1x1mm<sup>2</sup>)



Beam discretisation  
(LE EB beams with brittle damage)

# Fibrous unit cell: Small setup

DNS:  
- 700k beams  
- 2M DOFs

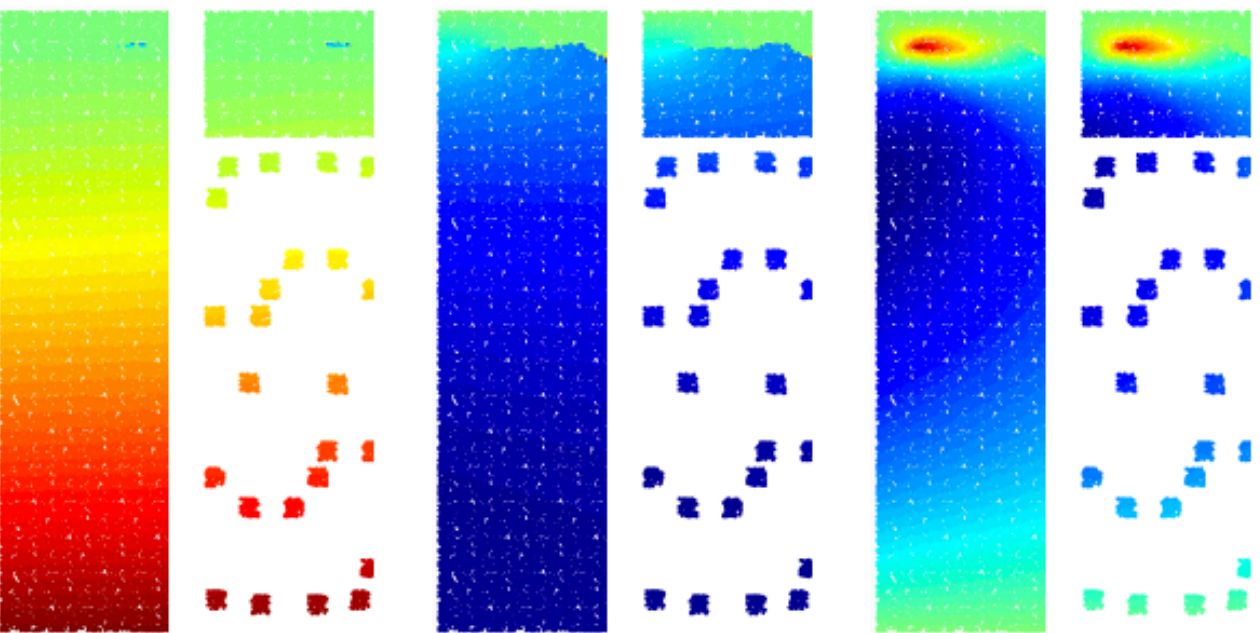
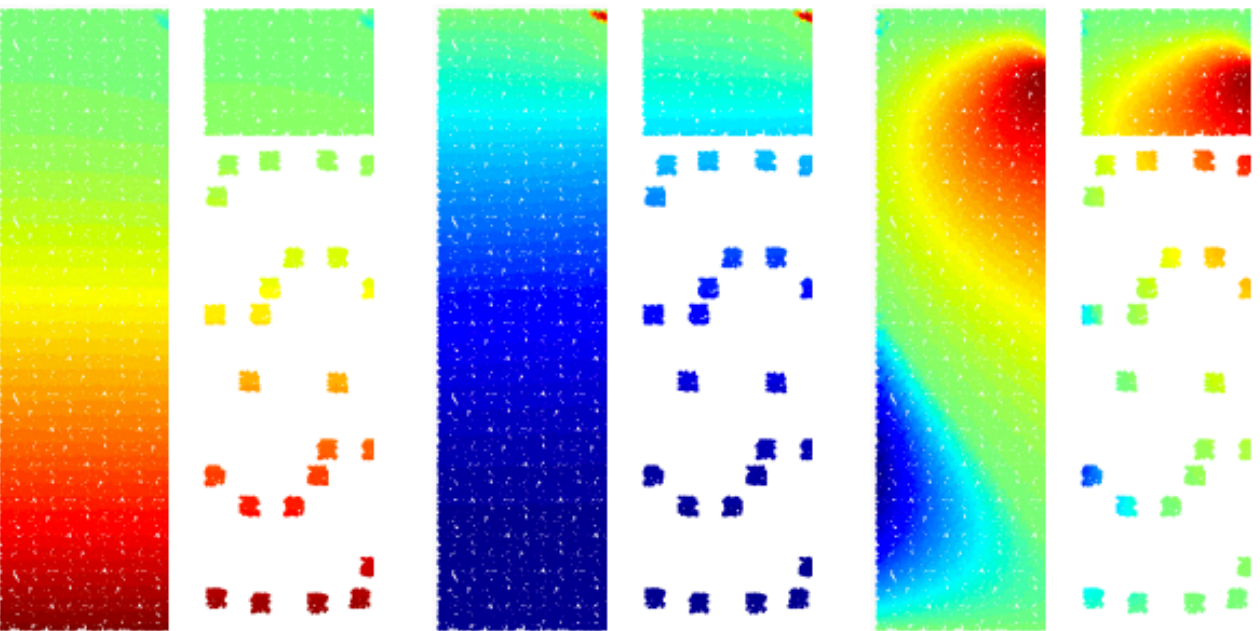


QC:  
- 4 less beams  
- 4 less DOFs



After 1 beam failure

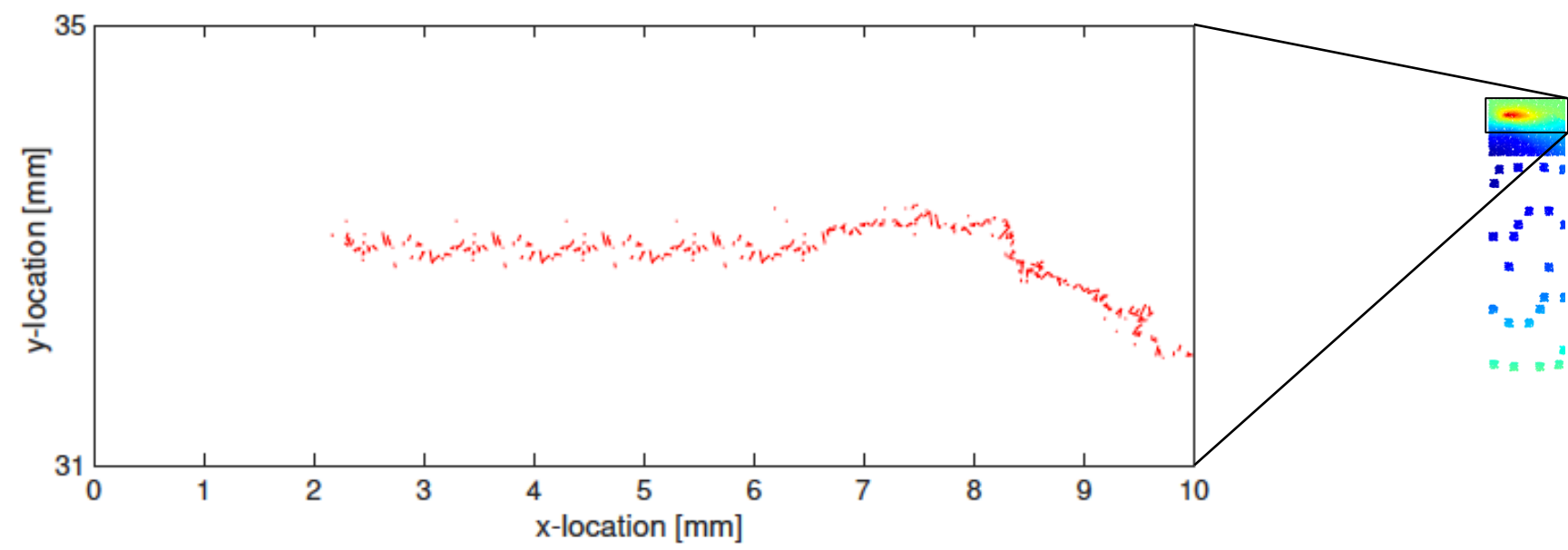
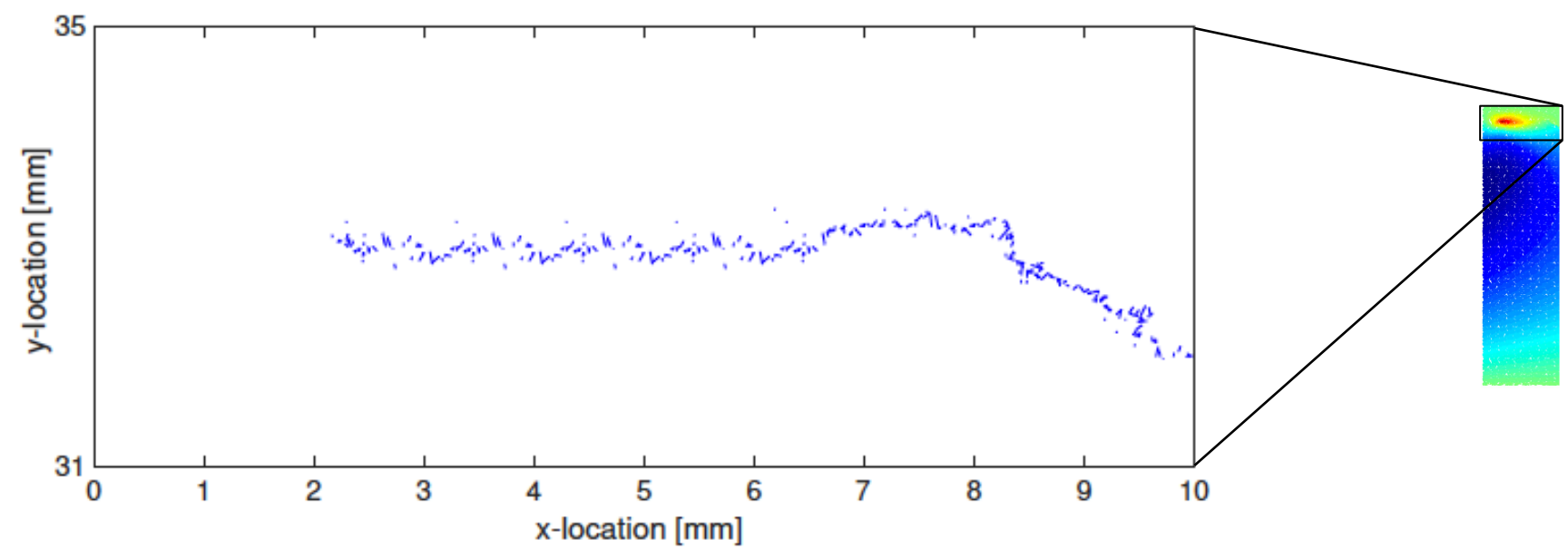
After 350 beam failures



$u_z$

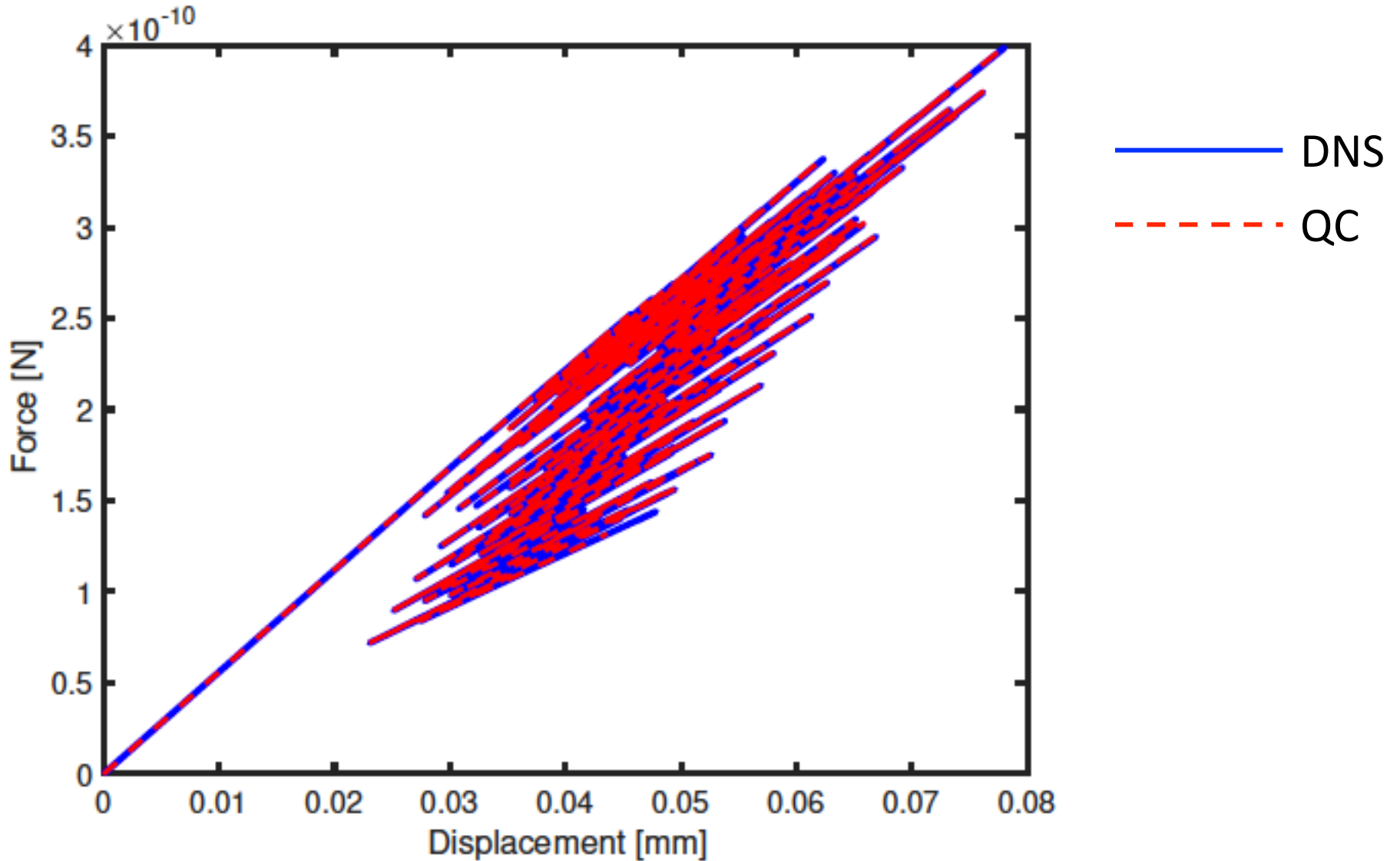
$v_x$

$v_y$

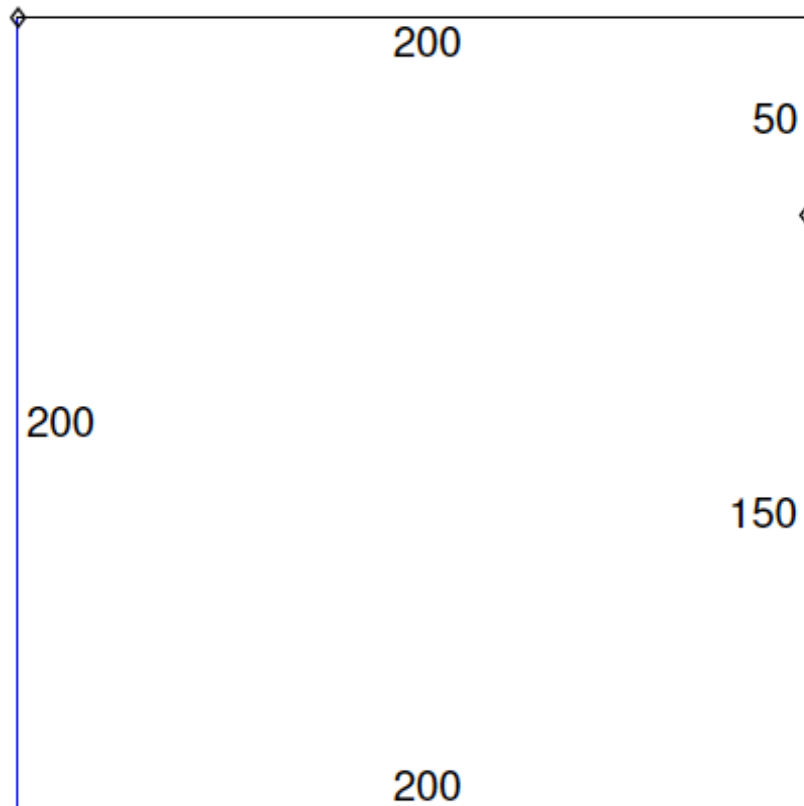


Failed beams

## Force-displacement response

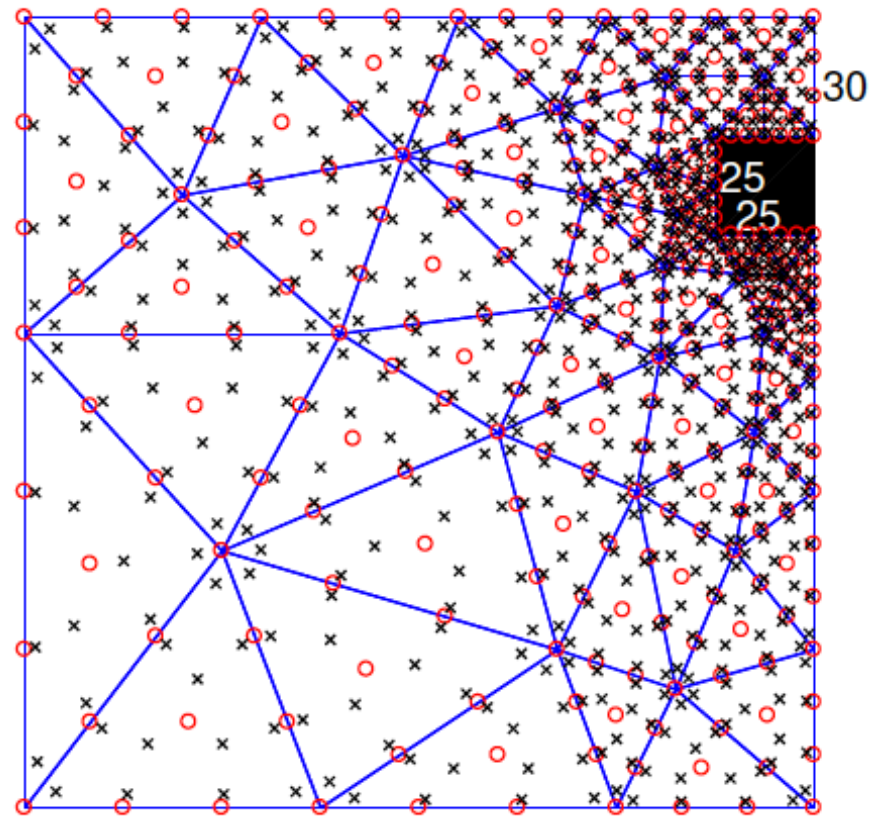


# Fibrous unit cell: Large setup



DNS:

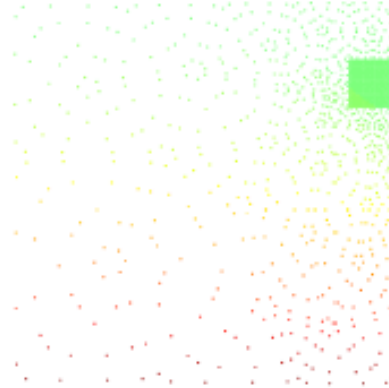
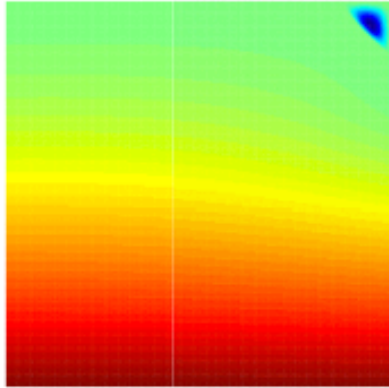
- 80M beams
- 233M DOFs



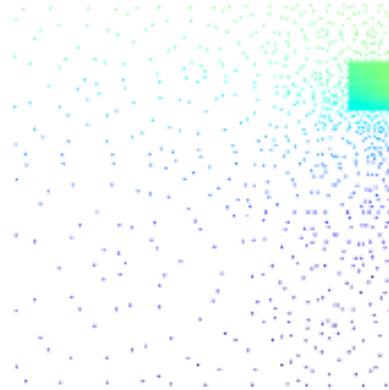
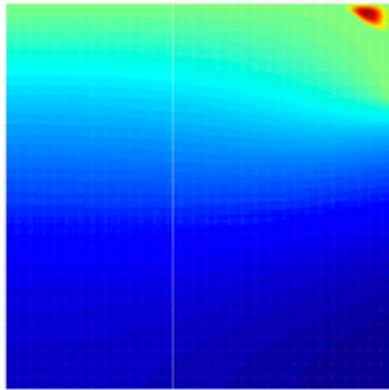
QC:

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- 42x less DOFs

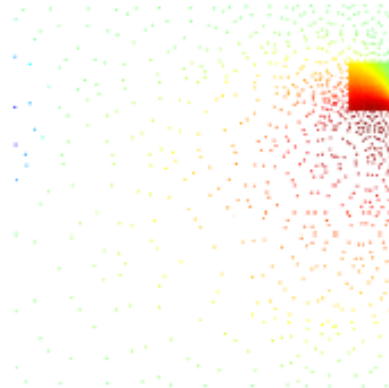
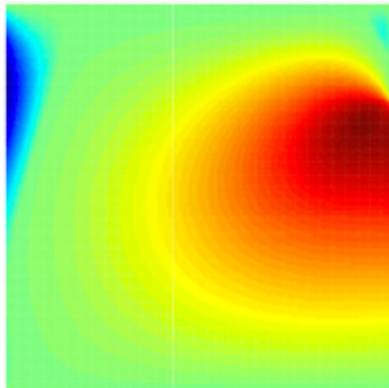
# Fibrous unit cell, large domain: Results



Out-of-plane displacement

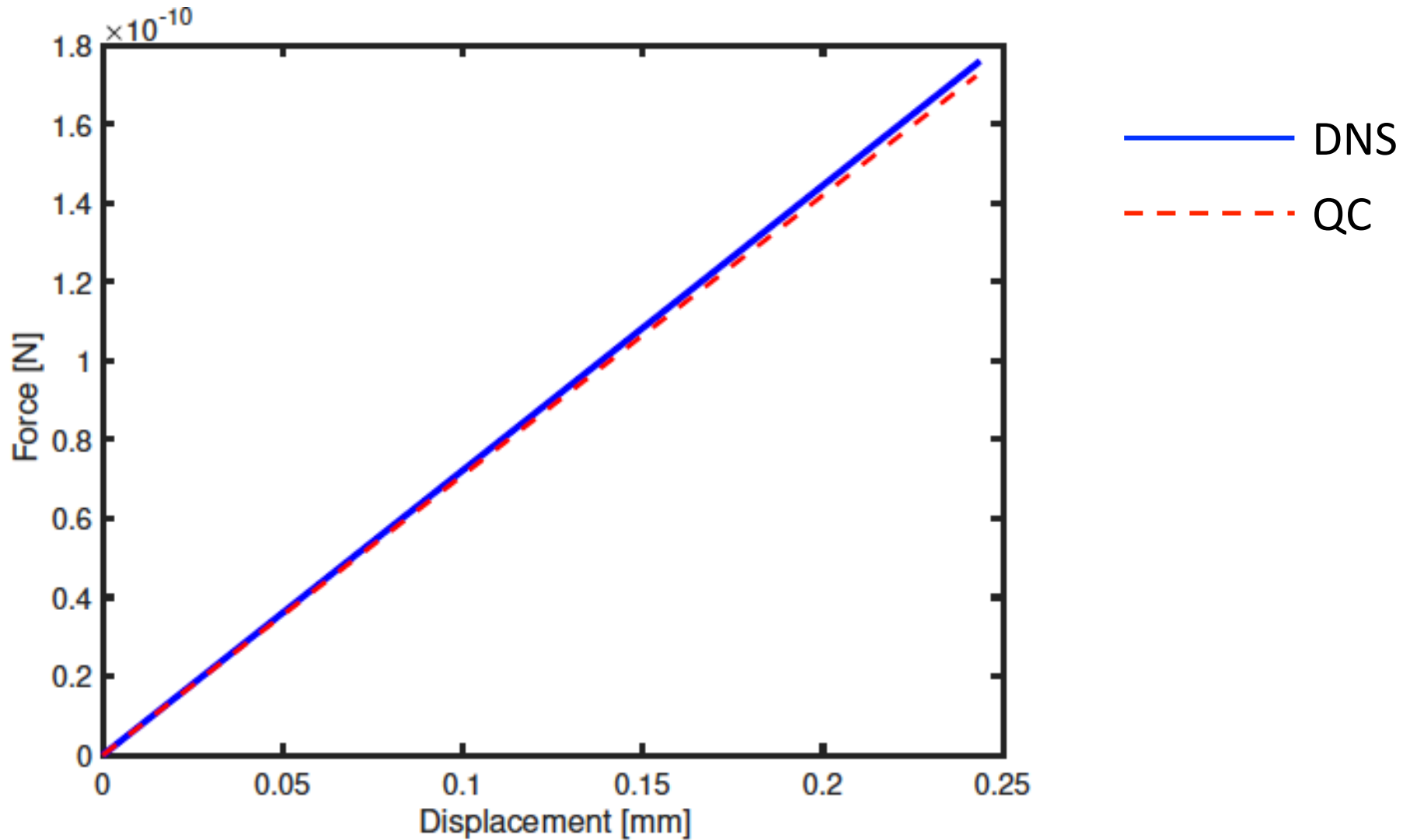


Rotation around horizontal axis



Rotation around vertical axis

## Force-displacement response



## **Condition:**

Unit cell must be periodic.

## **Advantages** compared to other nested multiscale methods:

1. Higher-order macroscale interpolations are as easy to treat as linear ones
2. No scale-separation

## **Disadvantages** compared to other nested multiscale methods:

1. All DOFs in one system, instead of subdivided over the unit cells and the macroscale elements
2. More unit cells required

## Ongoing:

Apply to matrix material + inclusions (geom. NL + mat. NL)

## Future:

Apply to real materials (e.g. PAPER/CARDBOARD??)

(Goal-oriented) adaptivity

Add randomness to structure in the fully resolved region