Global Energy Minimization for Multi-Crack Growth in Linear Elastic Fracture Using the Extended Finite Element Method

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MOTIVATION and FOCUS

It has been common practice to determine the onset of fracture growth and the growth direction by post-processing the solution of the linear elastostatics problem, at a particular instance in time. For mixed mode loading the available analytically derived criteria that can be used for determining the onset of crack growth rely on the assumptions of idealized geometry e.g. a single crack subjected to remote loading and that the kink angle of the infinitesimal crack increment is small. Moreover, the growth direction given by a criterion that is based on an instantaneous local crack tip field can only be valid for infinitesimally small crack growth increments. Consequently, the principal (max-hoop) stress criterion and other similar criteria disregard the changes in the solution that take place as fractures advance over a finite size propagation. Hence, due to the error committed in time-integration, fractures may no longer follow the most energetically favorable paths that theoretically could be achieved for a specific discrete problem.

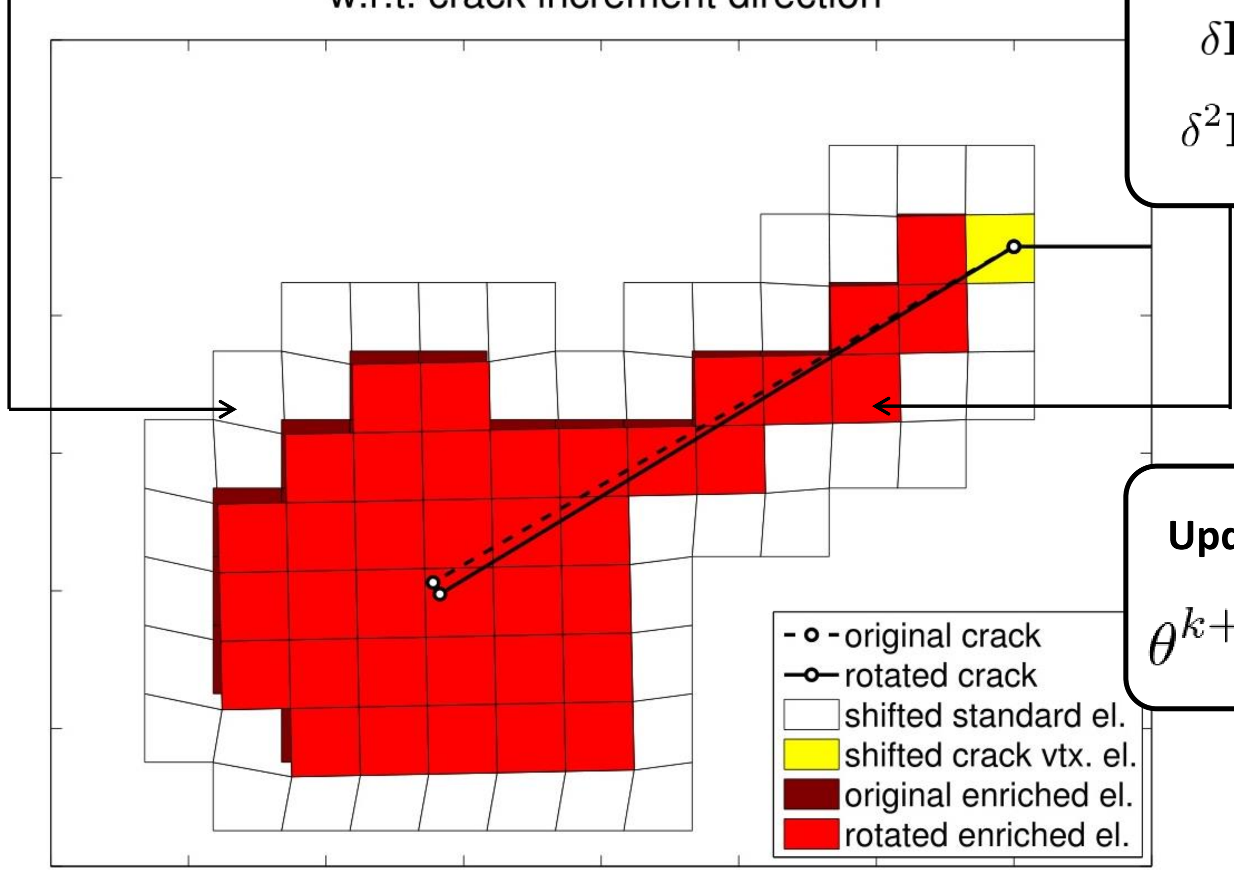
In our approach, we investigate multiple fracture evolution under quasi-static conditions based on the principle of minimum potential energy to help circumvent the aforementioned difficulties. It is found the converged fracture path lies in between the fracture paths obtained by each criterion for coarser meshes. This presents an opportunity to estimate an upper and lower bound of the true fracture path as well as an error on the crack path.

METHOD: Global Energy Minimisation

$$\delta\mathbf{K}_e = \int_{\Omega_e} (\delta\mathbf{B}^T\mathbf{D}\mathbf{B} + \mathbf{B}^T\mathbf{D}\delta\mathbf{B})\det(\mathbf{J})\,\mathrm{d}\bar\Omega + \int_{\Omega_e} \mathbf{B}^T\mathbf{D}\mathbf{B}\,\delta\det(\mathbf{J})\,\mathrm{d}\bar\Omega$$

$$\delta^2\mathbf{K}_e = \int_{\Omega_e} (\delta^2\mathbf{B}^T\mathbf{D}\mathbf{B} + 2\delta\mathbf{B}^T\mathbf{D}\delta\mathbf{B} + \mathbf{B}^T\mathbf{D}\delta^2\mathbf{B})\det(\mathbf{J})\,\mathrm{d}\bar\Omega + \int_{\Omega_e} 2\,(\delta\mathbf{B}^T\mathbf{D}\mathbf{B} + \mathbf{B}^T\mathbf{D}\delta\mathbf{B})\,\delta\det(\mathbf{J})\,\mathrm{d}\bar\Omega + \int_{\Omega_e} \mathbf{B}^T\mathbf{D}\mathbf{B}\,\delta^2\det(\mathbf{J})\,\mathrm{d}\bar\Omega$$

Differentiation of the stiffness matrix w.r.t. crack increment direction

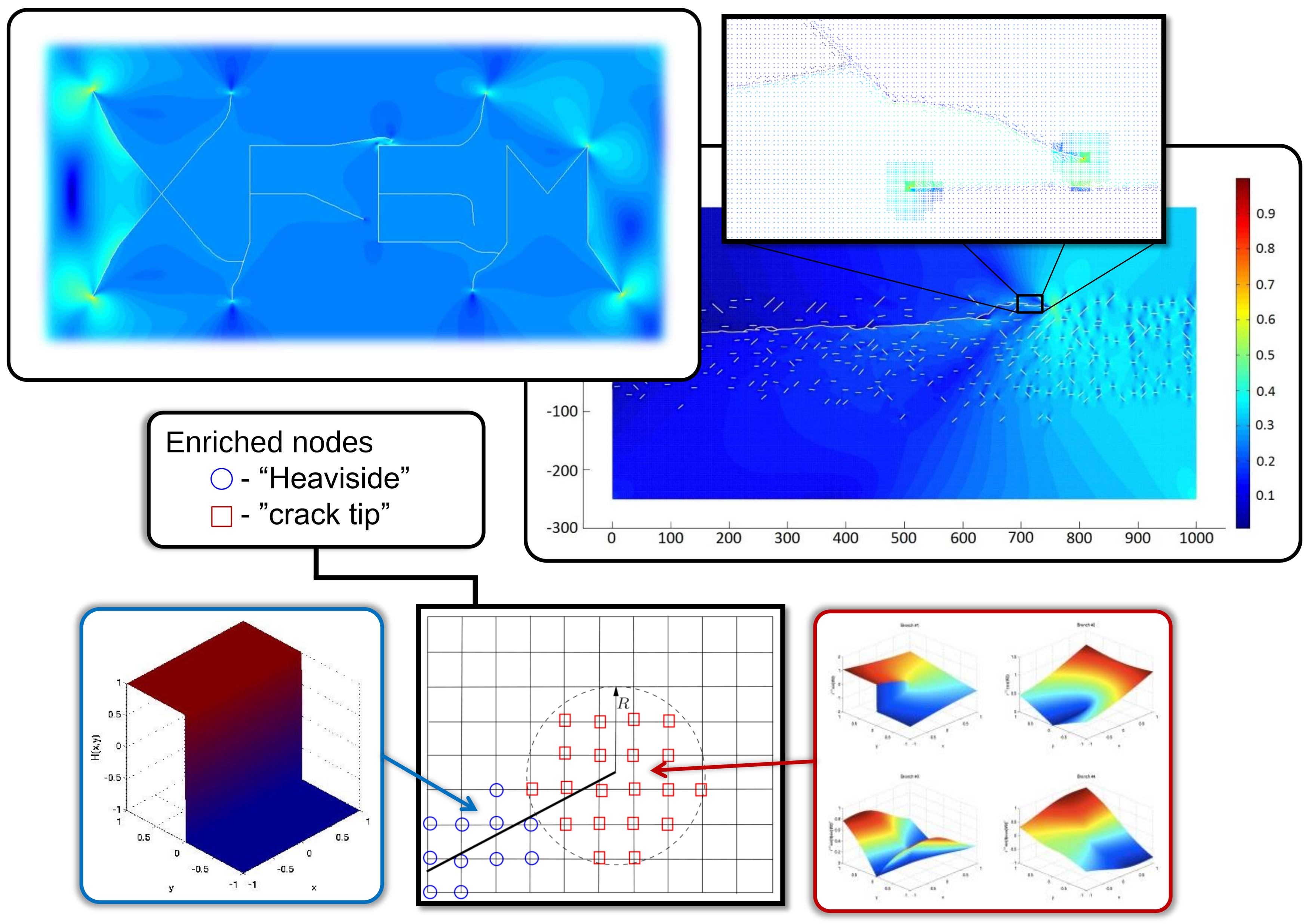


$$\delta\mathbf{K_e} = \mathbf{T}^T\mathbf{K_e} + \mathbf{K_e}\mathbf{T}$$
$$\delta^2\mathbf{K_e} = 2(\mathbf{T}^T\mathbf{K_e}\mathbf{T} - \mathbf{K_e})$$

Updated directions:

$$\theta^{k+1} = \theta^k - \mathbf{Hs}^{-1}\mathbf{Gs}$$

METHOD: Discretisation Using XFEM



$$\mathbf{u}^h(\mathbf{x}) = \underbrace{\sum_{I\in\mathcal{N}_I} N_I(\mathbf{x})\mathbf{u}^I}_{\text{standard part}} + \underbrace{\sum_{J\in\mathcal{N}_J} N_J(\mathbf{x})H(\mathbf{x})\mathbf{a}^J}_{\text{discontinuous enrichment}} + \underbrace{\sum_{K\in\mathcal{N}_K} N_K(\mathbf{x})\sum_{\alpha=1}^{4} f_\alpha(\mathbf{x})\mathbf{b}^{K\alpha}}_{\text{singular tip enrichment}}$$

$$H(\mathbf{x}) = \begin{cases} +1 & \text{if } \mathbf{x} \text{ above crack} \\ -1 & \text{if } \mathbf{x} \text{ below crack} \end{cases}$$

$$\{f_\alpha(r,\theta), \alpha = 1,4\} = \left\{\sqrt{r}\sin\frac{\theta}{2}, \sqrt{r}\cos\frac{\theta}{2}, \sqrt{r}\sin\frac{\theta}{2}\sin\theta, \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right\}$$

RESULTS: fracture path comparison by criteria (max-hoop VS. energy min. VS. mixed criterion)

Double cantilever problem

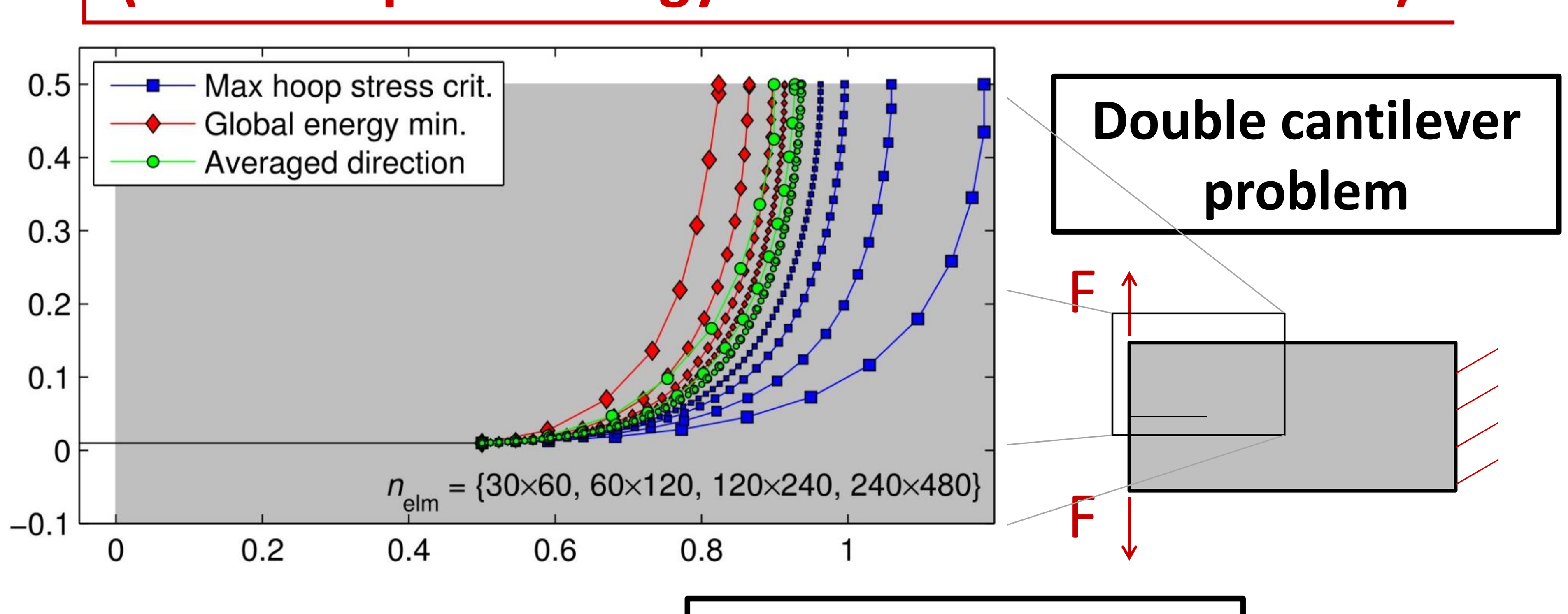
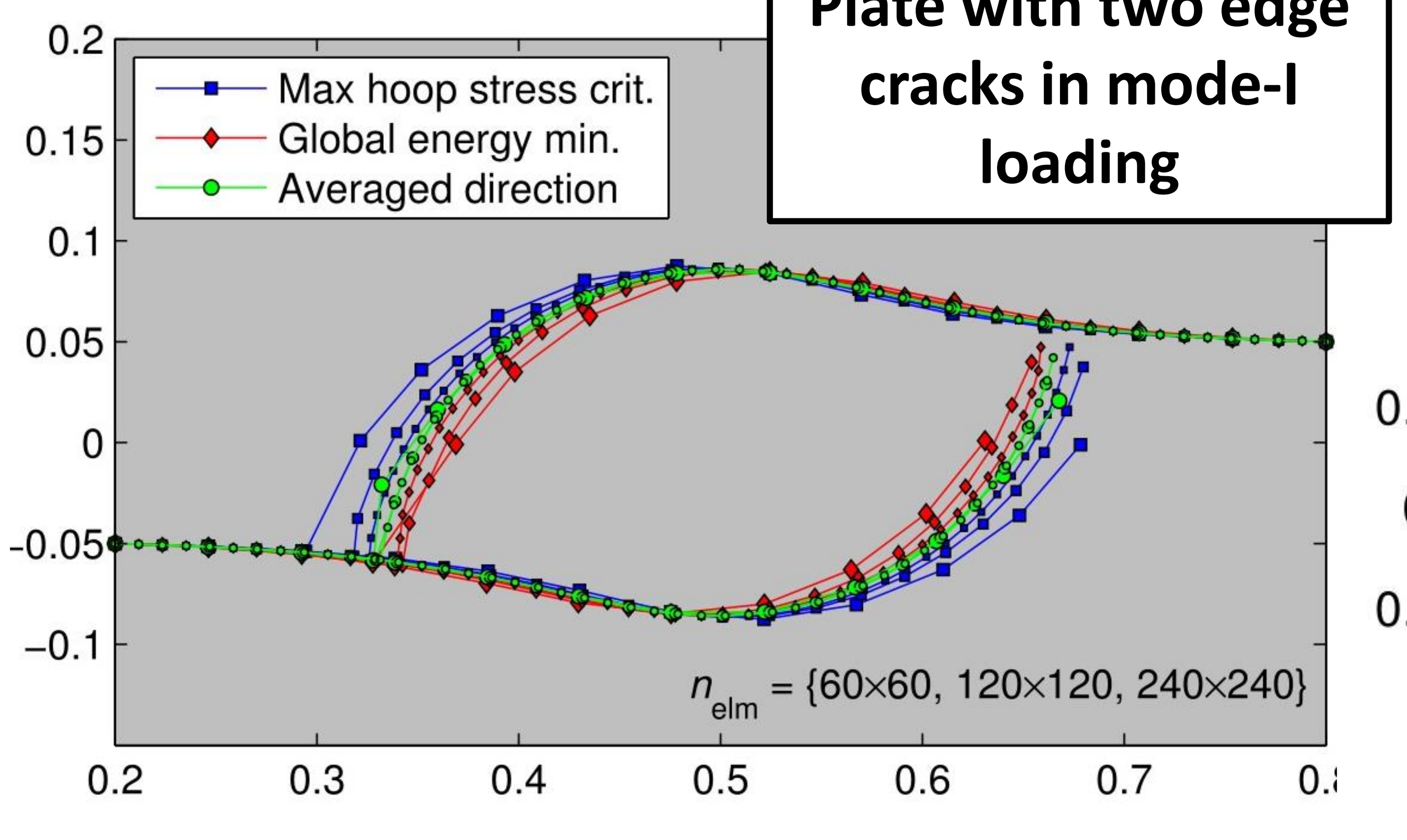


Plate with two edge cracks in mode-I loading



APPLICATION: Si-wafer post-split roughness

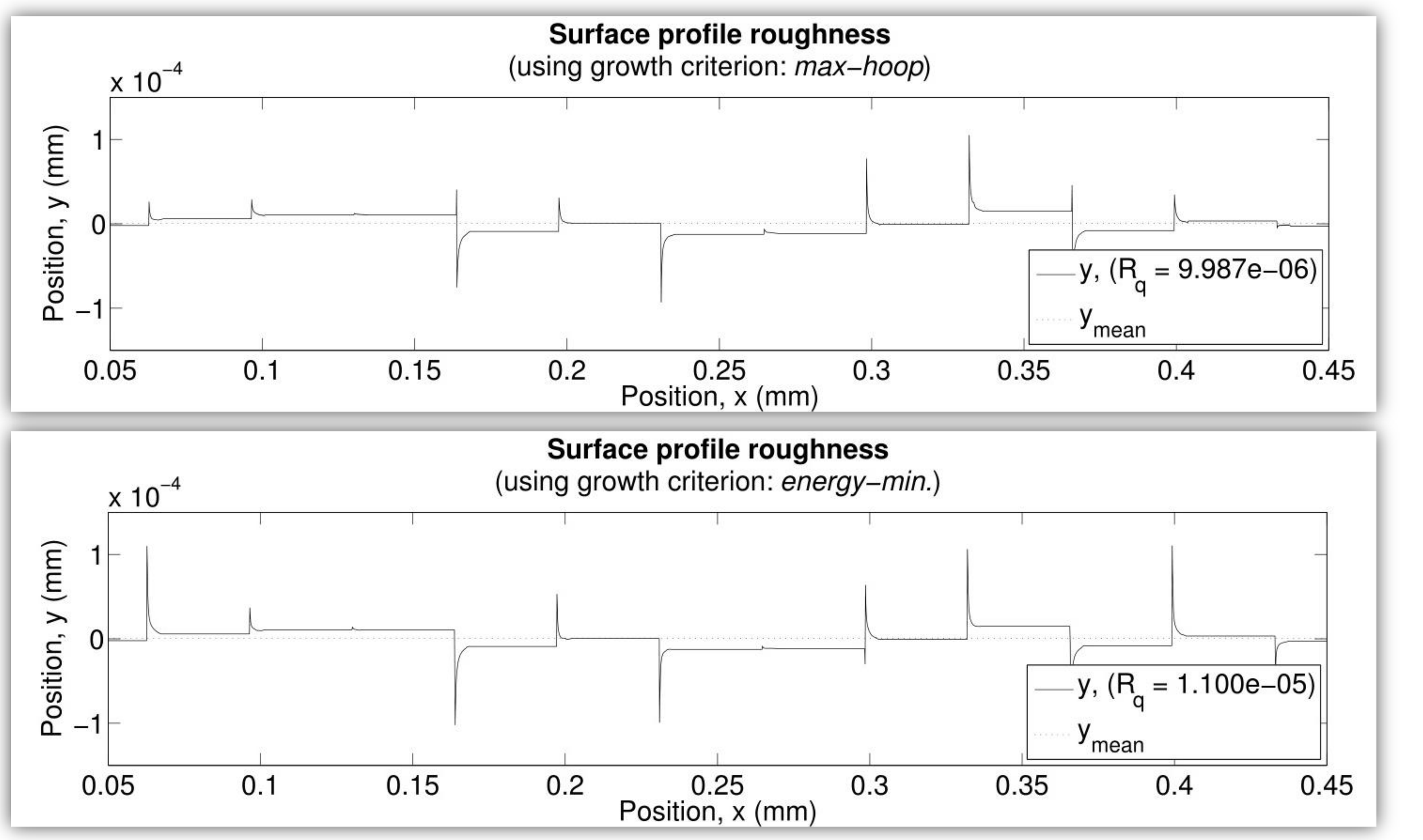


Plate with two holes in mode-I loading