Ali Baba and the Thief, Convention Emergence in Games

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Game theoretical study of the emergence of conventions and social norms has been developed by both computer scientists and philosophers [1, 3–5]. In this paper we propose a model that supports the emergence of conventions or social norms which prescribe peaceful behavior via multiagent learning in social networks. In our model, individual agents repeatedly interact with their neighbors in a game called 'Ali Baba and the Thief'. Our results show that conventions prohibiting harmful behaviors, such as "don't rob", can emerge after repeated interactions among agents.

The general methodology for studying the emergence of norms in Alexander [1] is the following:

1. Identify norms with a particular strategy in a two-player game.
2. Use replicator dynamics and multiagent learning to test whether norms emerge as a result of the repeated play of the two-player game.
3. Test norm emergence with different social networks.

Two-player games studied in Alexander [1] includes prisoner's dilemma, stag hunt, cake cutting and ultimatum game. Alexander uses these games to analyze the emergence of norms of cooperation, trust, fair division and retaliation respectively. In this paper, we follow Alexander's general methodology but we study convention emergence in a game which is not explored in Alexander [1].

As a variant of the famous Hawk-Dove game, *Ali Baba and the Thief* is a 2-player game, where each agent has two strategies: Ali Baba and Thief. Each agent has initial utility x. If both agents choose Ali Baba, then their utility does not change. If they both choose Thief, then there will be a fight between them and they are both injured. The resulting utility is 0. If one chooses Ali Baba and the other chooses Thief, then Thief robs Ali Baba and the utility of the one who chooses Thief increases by d and the other one decreases by d, where $0 < d < x$. We call d the amount of robbery. The payoff matrix of this game is shown in Table 1.

	Ali Baba	Thief
Ali Baba	x,x	$x-d$,$x+d$
Thief	$x+d$,$x-d$	0,0

Table 1. Ali Baba and Thief

We identify conventions prescribing peaceful behavior with the strategy of Ali baba in this game. In our model this game is repeatedly played by a given amount of agents. Each agent adapts its strategy using a learning rule between different rounds of play. In general, we say a convention has emerged in the population if:

(1) All agents are choosing and will continue to choose the action prescribed by the convention.
(2) Every agent believes that all agents, who are relevant in its social network, will choose the action prescribed by the convention in the next round.
(3) Every agent believes that all other agents, who are relevant in its social network, believe that it is good if the agent chooses the action prescribed by the convention.

The above three criteria of convention emergence is a reformulation of Lewis' famous analysis of conventions: "Everyone conforms, everyone expects others to conform, and everyone has good reasons to conform because conforming is in each person's best interest when everyone else plans to conform" [2]. In our game, we are interested in the rounds where all agents choose Ali Baba. This can be understood as no agent is willing to be Thief, which shows conventions like "you should not rob" or "don't harm others" have emerged.

We use replicator dynamics and imitate-the-best as rules of learning. No social network is assumed when agents learn using replicator dynamics while lattice model and small world model are used when agents use imitate-the-best.

In both lattice model and small world model, our experimental results suggest that there are critical points of convention emergence which are decided by the quotient of the amount of robbery and the initial utility in the Ali baba and the Thief game. When the quotient of the amount of robbery and the initial utility is smaller than the critical point, the probability of convention emergence is high. The probability drops dramatically as long as the quotient is larger than the critical point. In the lattice model, the probability of convention emergence drops quickly as long as the amount of robbery is large than 428.57142.

Amount of robbery (d)	Probability of norm emergence
428.57142	1
428.57143	0.16

Table 2. lattice model, critical point, $x = 1000$

References

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