# Stable distributions for alternative UCITS

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In this paper, we analyse the different methods to estimate the parameters of $\alpha$-stable distributions and choose the one which is best for our purposes.

Then, we show that hedge fund indices follow $\alpha$-stable distributions and we compute value at risk and expected shortfall for four hedge fund indices.

# Outline

1. Introduction

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# Stable distributions

A non-degenerate random variable $X$ is said to be stable, if for each pair $(X_1, X_2)$ of independent copies of $X$ and for any constants $a > 0$ and $b > 0$ the random variable $aX_1 + bX_2$ has the same distribution as $cX + d$ for some constants $c > 0$ and $d$.

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- $\gamma \in [0, \infty)$ is a scale parameter, compressing or extending the distribution.
- $\delta \in \mathbb{R}$ a location parameter, shifting the distribution to the left or right.

# Characteristic Function

There are no general closed formulas for the probability density function of stable distributions, except for particular values of the shape parameters $(\alpha, \beta)$.

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The characteristic function $\phi(t)$ of $X$ can however be written as

$$\phi(t) = \exp[it\mu - |ct|^{\alpha}(1 - i\beta \operatorname{sgn}(t)\Phi)],$$

where $\operatorname{sgn}(t)$ denotes the sign of $t$ and

$$\Phi = \tan(\pi\alpha/2), \text{ if } \alpha \neq 1$$

and

$$\Phi = -2\frac{2}{\pi}\log|t|, \text{ if } \alpha = 1.$$

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# Examples of stable distributions

A lot of famous distributions can be obtained as special cases of stable distribution.

- If $\alpha = 2$, one gets Gaussian distributions.
- If $\alpha = 1$ and $\beta = 0$, one gets Cauchy distributions.
- If $\alpha = \frac{1}{2}$ and $\beta = 1$, one gets Lévy distributions.
- If $\alpha = 1$ and $\beta = 1$, one gets Landau distributions (Landau 1944).
- If $\alpha = \frac{3}{2}$ and $\beta = 0$, one gets Holtsmark distributions (Holtsmark 1914)

# Outline

1. Introduction

2. Estimation Methods for Stable Distributions
   - Order Statistics
   - Maximum Likelihood Estimation
   - Empirical Characteristic Function

3. Risk Metrics using the Stable distribution
   - Data
   - $\alpha$-STABLE GARCH(1,1) VaR and ES

# Analysed estimation methods

**Order Statistics**

- The quantile method
- The order moment method

**Maximum Likelihood Estimation**

- Fast Fourier Transform based method
- Modified parametrization based method

**Empirical Characteristic Function**

- Sample characteristic function based parameter estimation
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We decided to use the quantile method to get good starting parameters for a modified parametrization based maximum likelihood estimation.

# The Quantile Method

Suppose we have $n$ independent drawings $x_i$ from the stable distribution $S(\alpha, \beta, \gamma, \delta)$.

Denote by $\hat{x}_p$ the p-th quantile of $X$ in the sample. The parameters of $S(\alpha, \beta, \gamma, \delta)$ can then be estimated by

$$\begin{cases} \hat{\alpha} &= \Psi_1\left(\frac{\hat{x}_{.95} - \hat{x}_{.05}}{\hat{x}_{.75} - \hat{x}_{.25}}, \frac{\hat{x}_{.95} + \hat{x}_{.05} - 2\hat{x}_{.5}}{\hat{x}_{.95} - \hat{x}_{.05}}\right). \\ \hat{\beta} &= \Psi_2\left(\frac{\hat{x}_{.95} - \hat{x}_{.05}}{\hat{x}_{.75} - \hat{x}_{.25}}, \frac{\hat{x}_{.95} + \hat{x}_{.05} - 2\hat{x}_{.5}}{\hat{x}_{.95} - \hat{x}_{.05}}\right). \\ \hat{\gamma} &= \frac{\hat{x}_{.75} - \hat{x}_{.25}}{\phi_3(\hat{\alpha}, \hat{\beta})}. \\ \hat{\delta} &= \hat{x}_{.5} + \hat{\gamma}\phi_5(\hat{\alpha}, \hat{\beta}) - \hat{\beta}\hat{\gamma}\tan\left(\frac{\pi\hat{\alpha}}{2}\right), \end{cases}$$

where $\Psi_1, \Psi_2, \phi_3$ and $\phi_5$ are tabulated functions that can be found in McCulloh (1986).

# Advantages and disadvantages of the quantile approach

The main advantages of the quantile approach:

(a) Elimination of asymptotic bias in their estimators as opposed to the similar Fama/Roll method.

(b) Stable parameters are relatively straightforward to estimate (minimal calculations, not computer-intensive).

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The main disadvantages of the quantile approach:

(a) Method does not work for $\alpha < 0.6$.

(b) to get reliable estimates, you need quite large datasets.

# Modified parametrization based method

Modified parametrization of Zolotarev (1999):

A random variable $X \sim (\alpha, \beta, \gamma, \delta)$ has the characteristic function:

$$E\exp(itX) = \begin{cases} \exp\left( -\gamma^{\alpha}\,|t^{\alpha}|\left[1 + i\beta(\tan\frac{\pi}{2}(\text{sign } t))((\gamma\,|t|)^{1-\alpha} - 1\right] + i\delta_0 t\right), & \alpha \neq 1 \\ \exp\left( -\gamma\,|t|\left[1 + i\beta\frac{2}{\pi}(\text{sign } t)(\ln|t| + \ln\gamma)\right] + i\delta_0 t\right), & \alpha = 1 \end{cases}$$

Nolan(1997) derives the following formulas to compute the probability density function values of a standardized random variable X with Zolotarev' parameterization in the case where $\gamma = 1$ and $\delta_0 = 0$:

# Modified parametrization based method

$$f(x-\zeta;\alpha,\beta) = \begin{cases} \frac{\alpha(x-\zeta)^{\frac{1}{\alpha-1}}}{\pi \ln|\alpha-1|} \int_{-\theta_0}^{\frac{\pi}{2}} V(\theta;\alpha,\beta)\exp(-(x-\zeta)^{\frac{\alpha}{\alpha-1}} V(\theta;\alpha,\beta))\mathrm{d}\theta & \alpha \neq 1 \text{ and } x > \zeta \\ \dfrac{\Gamma\left(1+\frac{1}{\alpha}\right) cos(\theta_0)}{\pi\left(1+\zeta^2\right)^{\frac{1}{2\alpha}}} & \alpha \neq 1 \text{ and } x = \zeta \\ f(-x;\alpha,-\beta) & \alpha \neq 1 \text{ and } x < \zeta \\ \frac{1}{|2\beta|} e^{-\frac{\pi x}{2\beta}} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} V(\theta;\alpha,\beta)\exp(-e^{-\frac{\pi x}{2\beta}} V(\theta;\alpha,\beta))\mathrm{d}\theta & \alpha = 1 \text{ and } \beta \neq 0 \\ \dfrac{1}{\pi(1+x^2)} & \alpha = 1 \text{ and } \beta = 0, \end{cases}$$

# Modified parametrization based method

where

$$\zeta = \zeta(\alpha, \beta) = \begin{cases} -\beta \tan \frac{\pi\alpha}{2}, & \alpha \neq 1 \\ 0, & \alpha = 1 \end{cases}$$

$$\theta_0 = \theta_0(\alpha, \beta) = \begin{cases} \frac{1}{\alpha} \arctan(\beta \tan \frac{\pi\alpha}{2}), & \alpha \neq 1 \\ \frac{\pi}{2}, & \alpha = 1 \end{cases}$$

and

$$V(\theta; \alpha, \beta) = \begin{cases} (\cos \alpha\theta_0)^{\frac{1}{\alpha-1}} \left( \frac{\cos \theta_0}{\sin \alpha(\theta_0 + \theta)} \right)^{\frac{\alpha}{\alpha-1}} \frac{\cos(\alpha\theta_0 + (\alpha-1)\theta)}{\cos \theta_0}, & \alpha \neq 1 \\ \frac{2}{\pi} \left( \frac{\frac{\pi}{2} + \beta\theta}{\cos \theta} \right) \exp\left( \exp\left( \frac{1}{\beta} \left( \frac{\pi}{2} + \beta\theta \right) \tan \theta \right) \right), & \alpha = 1, \beta \neq 0. \end{cases}$$

The log likelihood function can then be computed numerically using the quasi-Newton method.

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# Data

The data used in this study consist of daily price series of 4 proprietary hedge fund indices over the period 2003 - 2015: DMXUSD, CAIXUSD, GAIXUSD, ABRXEUR.

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The diversity of the underlying consitutents (aggregation of derivatives with symmetric or asymmetric payoffs) that are part of these indices require robust management procedures, in particular within the regulated UCITS framework.

# Data

Most of the previous studies on empirical financial data were focusing on the asset level (single instrument), whereas we aim at developing risk management tools at a portfolio level, especially for UCITS Managed Futures - CTAs.

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The reason is that UCITS CTAs generally allocate their assets into cash or spreading the latter instrument into riskless fixed-income government securities, and a Total Return Swap (TRS) on a proprietary hedge fund index such as the ones used in our sample.

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As a result, the market risk of Managed Futures - UCITS CTAs is more or less identical to their corresponding indices, underlying of the TRS.

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# Descriptive Statistics

Table 1: Summary of Data Statistics

| | ABRXEUR | CAIXUSD | DMIXUSD | GAIXUSD |
|---|---|---|---|---|
| Start | 3/3/2003 | 11/28/2003 | 3/3/2003 | 3/3/2003 |
| End | 1/27/2015 | 1/27/2015 | 1/27/2015 | 1/27/2015 |
| Obs | 2945 | 2819 | 2975 | 2965 |
| Mean | 0.011% | 0.023% | 0.062% | 0.054% |
| Sigma | 0.344% | 0.226% | 0.897% | 0.537% |
| Skew | 0.366 | -0.351 | -0.016 | -0.453 |
| Kurt | 8.819 | 11.380 | 5.506 | 9.619 |

Table 2: Benchmark Data Statistics

| | CAC40 | DAX30 | STOXX50 | FTSE100 |
|---|---|---|---|---|
| Start | 3/3/2003 | 3/3/2003 | 3/3/2003 | 3/3/2003 |
| End | 1/27/2015 | 1/27/2015 | 1/27/2015 | 1/27/2015 |
| Obs | 3050 | 3040 | 3046 | 3099 |
| Mean | 0.017% | 0.047% | 0.015% | 0.020% |
| Sigma | 1.416% | 1.397% | 1.418% | 1.156% |
| Skew | 0.059 | 0.029 | 0.019 | -0.134 |
| Kurt | 6.650 | 6.155 | 6.086 | 9.035 |

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The hedge fund indices are leptokurtic just as other European stock market indices such as the French CAC 40, German DAX 30, and British Footsie 100.

Hedge fund indices exhibit larger excess kurtosis, and are significantly skewed to the left in particular for those holding derivative instruments with asymmetric payoff functions such as GAIXUSD and CAIXUSD.

An investigation of the historical profits and losses (P&L) shows that the biggest historical daily losses are respectively -2.44%, -2.52%, -5.96%, -4.67% for ABRXEUR, CAIXUSD, DMIXUSD, and GAIXUSD, whereas their corresponding maximum daily profits are 2.47%, 1.56%, 6.83%, 4.45%.

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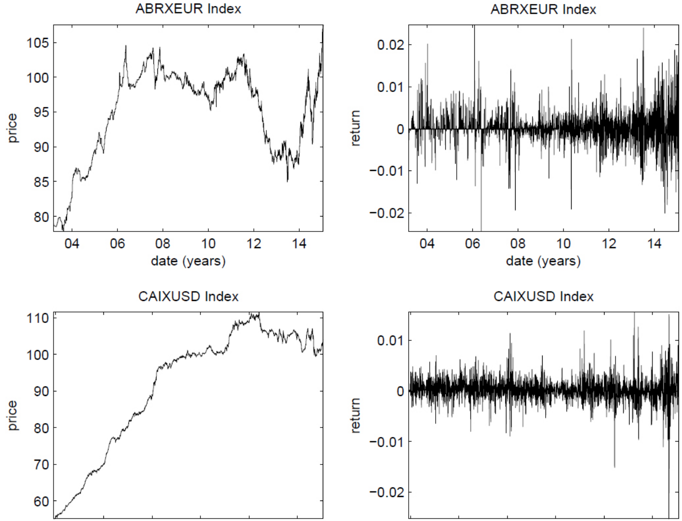


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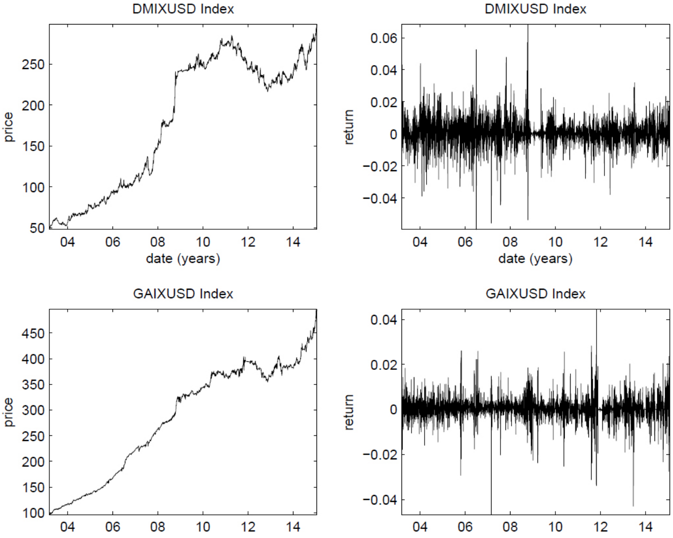
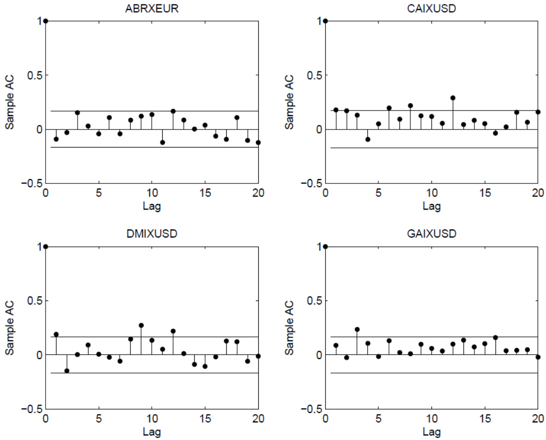

DMIXUSD Index
price
250
200
150
100
50
04
06
08
10
12
14
date (years)
DMIXUSD Index
return
0.06
0.04
0.02
0
−0.02
−0.04
04
06
08
10
12
14
date (years)
GAIXUSD Index
price
450
400
350
300
250
200
150
100
04
06
08
10
12
14
GAIXUSD Index
return
0.04
0.02
0
−0.02
−0.04
04
06
08
10
12
14

Table 4: KPSS Test for Stationarity

| lags | ABRXEUR | CAIXUSD | DMIXUSD | GAIXUSD |
|---|---|---|---|---|
| 0 | 0.0189* | 0.1* | 0.0705* | 0.0396* |
| | *0.1922*** | *0.0399*** | *0.1349*** | *0.1585*** |
| 1 | 0.0113* | 0.1* | 0.1* | 0.0448* |
| | *0.2124*** | *0.0384*** | *0.1152*** | *0.1522*** |
| 2 | 0.01* | 0.1* | 0.0962* | 0.0404* |
| | *0.2258*** | *0.0372*** | *0.1211*** | *0.1575*** |
| 3 | 0.0108* | 0.1* | 0.0886* | 0.0499* |
| | *0.2139*** | *0.037*** | *0.1252*** | *0.1461*** |
| 4 | 0.0141* | 0.1* | 0.0897* | 0.0661* |
| | *0.2051*** | *0.0412*** | *0.1246*** | *0.1373*** |
| 5 | 0.0149* | 0.1* | 0.0887* | 0.0713* |
| | *0.203*** | *0.0461*** | *0.1251*** | *0.1345*** |

*denotes the test statistics .** p-value at 5% significance level.

The Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test fails to reject at a 5% significance level lags the null hypothesis that CAIXUSD, DMIXUSD, and GAIXUSD (up to 3 lags) are trend stationary whereas ABRXEUR return series follow a nonstationary unit root process.




The alternative indices exhibit significant autocorrelation.

The Global Alpha Strategy displays a significant third-order autocorrelation of at least 0.2, results for the remaining Single-Manager strategies suggest AR(1) processes.

# $\alpha$-STABLE GARCH(1,1)

Given the simultanous presence of leptokurtosis and volatility clustering phenomena exhibited by the data, we use a variation of Liu and Brorsen's model (1995) in which a random variable $Y$ follows a univariate Stable - Garch process if $\epsilon$ is *i.i.d* and modeled with an $\alpha$-Stable distribution :

$$y_t = \delta + \varepsilon_t \gamma_t \tag{1}$$

$$\gamma_t^\lambda = \omega + \phi_1 \gamma_{t-1}^\lambda + \tau_1 \left| y_{t-1} - \delta \right|^\lambda \tag{2}$$

where $\gamma, \delta, \varepsilon$, and $\alpha$ represent respectively the scale, location, real-valued discrete-time stochastic process, and characteristic exponent, $\lambda = 1$, whereas $\omega, \phi, \tau$ are estimated from the following regression:

$$\left|\varepsilon_t\right|^\alpha = \omega + (\phi_1 + \tau_1) \left|\varepsilon_{t-1}\right|^\alpha - \phi_1 \upsilon_{t-1} + \upsilon_t \tag{3}$$

where $\upsilon$ represents the residual term.

As demonstrated by Mittnik et al. (2002) stationarity criteria involves $\lambda(\phi_1 + \tau_1) \leq 1$, and as opposed to Liu and Brorsen(1995) the parameter constraints $\omega \geq 0$, $\phi \geq 0$, $\tau \geq 0$ have been relaxed.

Finally, the risk metrics are derived from the usual definitions:

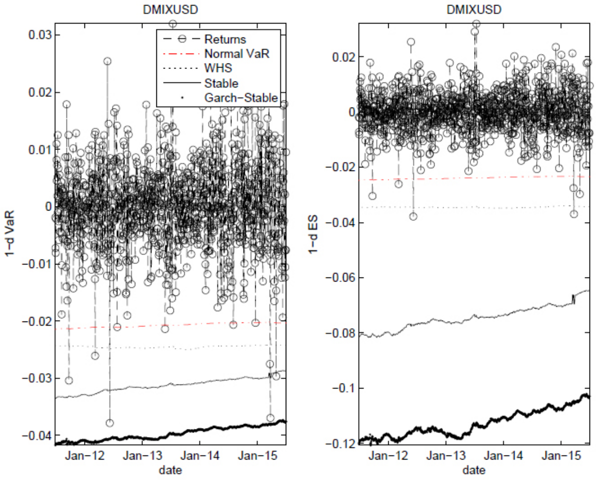
$$\text{VaR} = q_k = F_k^{-1}(\hat{\alpha}, \hat{\beta}, \hat{\gamma}, \hat{\delta}),$$

where $F$ denotes the cumulative distribution function, and

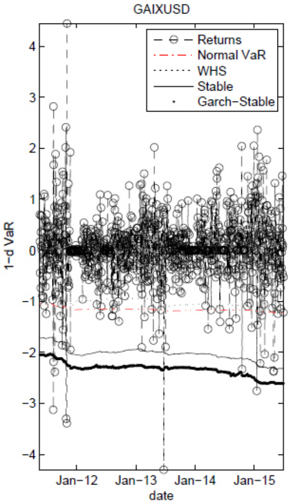
$$\text{ES} = \frac{1}{1 - cl} \int_{cl}^{1} q_k dk,$$

where $cl$ represents the confidence level.

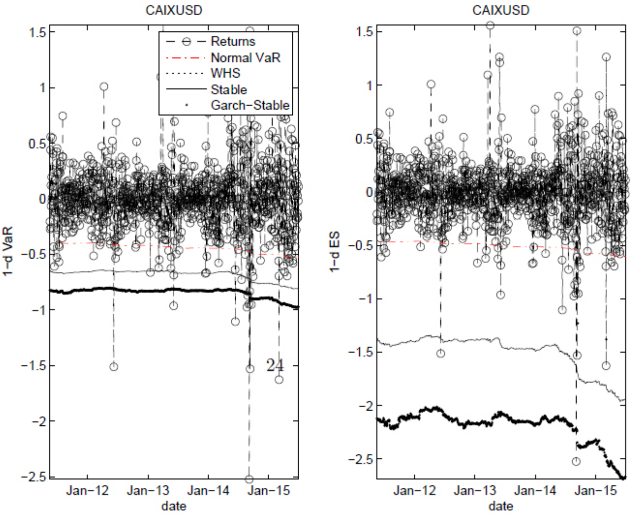
# Backtests


DMIXUSD
Returns
Normal VaR
WHS
Stable
Garch−Stable
1−d VaR
0.03
0.02
0.01
0
−0.01
−0.02
−0.03
−0.04
Jan−12
Jan−13
Jan−14
Jan−15
date
DMIXUSD
1−d ES
0.02
0
−0.02
−0.04
−0.06
−0.08
−0.1
−0.12
Jan−12
Jan−13
Jan−14
Jan−15
date

# Backtests


GAIXUSD
Returns
Normal VaR
WHS
Stable
Garch−Stable
1−d VaR
4
3
2
1
0
−1
−2
−3
−4
Jan−12
Jan−13
Jan−14
Jan−15
date
GAIXUSD
1−d ES
4
2
0
−2
−4
−6
−8
Jan−12
Jan−13
Jan−14
Jan−15
date

# Backtests


CAIXUSD
Returns
Normal VaR
WHS
Stable
Garch−Stable
1−d VaR
date
Jan−12
Jan−13
Jan−14
Jan−15
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5
24
CAIXUSD
1−d ES
date
Jan−12
Jan−13
Jan−14
Jan−15
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−2.5

# Regulatory breaches

Table 5: Summary of regulatory breaches

| DMIXUSD | Normal | | WHS | | Stable | | Garch-Stable | |
|---|---|---|---|---|---|---|---|---|
| Period | VaR | ES | VaR | ES | VaR | ES | VaR | ES |
| 6/23/2014 - 6/30/2015 | 5 | 3 | 3 | 1 | 2 | 0 | 0 | 0 |
| 6/29/2011 - 6/30/2015 | 9 | 6 | 6 | 2 | 3 | 0 | 0 | 0 |

| GAIXUSD | Normal | | WHS | | Stable | | Garch-Stable | |
|---|---|---|---|---|---|---|---|---|
| Period | VaR | ES | VaR | ES | VaR | ES | VaR | ES |
| 6/24/2014 - 6/30/2015 | 14 | 11 | 17 | 7 | 4 | 0 | 1 | 0 |
| 6/29/2011 - 6/30/2015 | 38 | 30 | 52 | 21 | 12 | 0 | 7 | 0 |

| CAIXUSD | Normal | | WHS | | Stable | | Garch-Stable | |
|---|---|---|---|---|---|---|---|---|
| Period | VaR | ES | VaR | ES | VaR | ES | VaR | ES |
| 6/24/2014 - 6/30/2015 | 24 | 16 | 76 | 57 | 9 | 1 | 7 | 1 |
| 6/29/2011 - 6/30/2015 | 53 | 34 | 205 | 141 | 15 | 2 | 10 | 1 |

# Conclusion

Risk management tool using stable distributions are perfectly suited to significantly skewed and leptokurtic data.