Joint Compression and Feedback of CSI in Correlated Multiuser MISO Channels

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[1] ***Abstract*—The potential gains of multiple antennas in wireless systems can be limited by channel state information imperfections. In this context, this paper tackles the limited feedback in multiuser correlated multiple input single output (MU-MISO). We propose a framework to feedback the minimum number of bits with limited performance degradation. This framework is based on decor relating the channel state information by compression and then quantize the compressed (CSI) and feedback it to the base station (BS). We characterize the rate loIEEEss resulting from the proposed framework. An upper bound on the rate loss is derived in terms of the amount of feedback and the statistics of the channel. Based on this characterization, we propose an adaptive bit allocation algorithm that takes into the account the channel statistics to reduce the rate loss induced by the quantization. Moreover, in order to maintain a constant rate loss with respect to perfect CSI, it is shown that the number of feedback bits should scale linearly with the SNR (in dB) and to the rank of the user transmit correlation matrix. We validate the proposed framework by Monte-carlo simulations.**

***Index Terms*—Multiuser MISO, CSI feedback, vector quantization, Karhunen-Leóve Transform (KLT), beamforming.**

I. Introduction

The channel state information plays an important role in designing the appropriate beamforming to serve multiple users simultaneously without inducing harmful interference [1].The CSI acquisition techniques can be classified into feedback and reciprocity techniques. In the feedback systems (so-called frequency devision duplexing (FDD)), a training sequence is broadcasted by the BS, which is measured by users, and a limited feedback link is considered from the users to the base station. In [2]- [8], this mode is discussed for different scenarios. In [2], the author shows that in order to achieve full multiplexing gain in the MIMO downlink channel in the high signal to noise ratio (SNR) regime, the required feedback rate per user grows linearly with the SNR (in dB). The main result in [3] is that the extent of CSI feedback can be reduced by exploiting multi-user diversity. While in [4], it is shown that non-random vector quantizers can significantly increase the MIMO downlink throughput. Furthermore, the authors in [6] study the impact of quantization on the sum rate performance in the downlink of correlated multiple antennas single cell systems.

The CSI is usually characterized by channel direction information (CDI) and channel quality information (CQI). In the literature, CDI is usually quantized while CQI is assumed to be available at the BS [2]. In this paper, we study the performance of limited feedback scenarios assuming that the adopted CSI acquisition model is FDD. The contribution of this paper can be summarized as:

- A framework to feedback the CDI of the channel, which is based on joint compression and quantization is proposed. In this framework, a more generic characterization can be formulated and derived. We exploit the correlated channel characteristics and the capability of implementing channels in lower dimensional vectors through compression. We utilize the optimality of Karhunen-Leóve Transform (KLT) and its capability to compress the information in lower vector dimensions.
- The rate loss of the proposed feedback scheme is characterized. Depending on this characterization, we suggest a new feedback allocation strategy that exploits the benefits of compression and hence finds the number of bits that is required to feedback to information without severe degradation.

The contributions of this paper are different from the previous literature since it exploit the channel correlation to employ compression before the quantization. However the work in [2] characterized the rate loss resulted from the quantization without introducing the compression concept since it tackles the uncorrelated channels.

Notation: We use boldface upper and lower case letters for matrices and column vectors, respectively. $(\cdot)^H$, $(\cdot)^*$, and $(.)^T$ stand for Hermitian transpose, conjugate , and transpose of $(\cdot)$. $\mathbb{E}(\cdot)$ and $\|\cdot\|$ denote the statistical expectation and the Euclidean norm, we use bold upper and lower case letters for matrices and column vectors, respectively.

II. System Model

We consider a multiuser MISO channel, where a transmitter equipped with M antennas communicates with K single-antenna receivers. The received signal at k^{th} user, denoted by $y_k \in \mathbb{C}$ can be written as

$$y_k = \mathbf{h}_k\mathbf{x} + n_k, \tag{1}$$

where $\mathbf{h}_k \in \mathbb{C}^{1\times M}$ represents the channel between the base station and the k^{th} user. In addition, $\mathbf{x} \in \mathbb{C}^{M\times 1}$ stands for the transmit signal vector with $tr(\mathbf{x}\mathbf{x}^H) = P$, and n_k is the additive Gaussian noise for user k with zero mean and unit

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variance. Using $\mathbf{x} = \sum_{k=1}^{K} \mathbf{w}_k s_k$, the received signal at k^{th} can be formulated as

$$y_k = \sqrt{\frac{P}{M}} \mathbf{h}_k \mathbf{w}_k s_k + \sum_{j, j \neq k} \sqrt{\frac{P}{M}} \mathbf{h}_k \mathbf{w}_j s_j + n_k \quad (2)$$

where P is the total transmit power from the base station, and $\mathbf{w}_k \in \mathbb{C}^{M \times 1}$, s_k stand for linear beamforming vector and a data symbol for user k satisfying $\mathbb{E}[|s_k|^2] = 1$.

A. Correlation model

Let us assume k^{th} user channel is modeled as $\mathbf{h}_k = \mathbf{R}_k^{1/2} \mathbf{h}_{w,k}$ where $\mathbf{R}_k^{1/2} = \mathbf{U}_k \mathbf{\Delta}_k^{1/2} \mathbf{U}_k^H$ is the square root of the transmit correlation matrix $\mathbf{R}_k$ and $\mathbf{h}_{w,k}$ is a vector whose elements are i.d.d. complex Gaussian distributed with variance equals to 1. Vectors $\mathbf{h}_{w,k}$ are assumed to be mutually independent. In this model, the eigenvalues $\mathbf{\Lambda}_k$ of the transmit correlation matrices are independent from one user to another. Additionally, let us assume that the eigenvalues of $\mathbf{\Delta}_k^{1/2}$, denoted as λ_k and ordered in decreasing order of magnitude, can be written as

$$\begin{aligned} \{\lambda_{1,k}, \ldots, \lambda_{n_t,k}\} &\overset{r_k=1}{=} \{\lambda_{1,k}, 0, \ldots, 0\} \\ &\overset{r_k=2}{=} \{\lambda_{1,k}, \lambda_{2,k}, \ldots, 0\} \\ &\vdots \\ &\overset{r_k=nt}{=} \{\lambda_{1,k}, \lambda_{2,k}, \lambda_{3,k}, \ldots, \lambda_{n_t,k}\} \end{aligned} \quad (3)$$

where r_k can be thought of as the rank of the transmit correlation matrix. The channel covariance $\mathbf{R}_k$ must be estimated. It is reasonable to assume that $\mathbf{R}_k$ changes slowly compared to the coherence time of the channel $\mathbf{h}_k$. $\mathbf{R}_k$, therefore, can be obtained at the transmitter (or BS) by the uplink in FDD systems, or by subspace tracking algorithm [12] using the downlink training. In some scenarios, the eigenvalues of $\mathbf{R}_k$ can have different values, some of them have significant impact on the response (very large in comparison to others) and the rest are insignificant but not necessary equal to zero. Therefore, the insignificant ones can be approximated to zero and truncated without influential impact on the system performance.

B. CSI Feedback Model

We assume that each user has a perfect knowledge of its channel $\mathbf{h}_k$. It is assumed that each user perfectly feeds back the CQI to the BS. Moreover, it is assumed that CQI feedback is not included in total feedback amount per user to simplify the analysis[2].

The quantization of a unit norm vector $\hat{\mathbf{h}}_k = \frac{\mathbf{h}_k}{\|\mathbf{h}_k\|}$ is chosen from distinct quantization codebook $\mathcal{C}_k = \{\mathbf{c}_{k1}, \ldots, \mathbf{c}_{kN_k}\}$ of size $N_k = 2_k^B$. By using minimum chordal distance the indices can be

$$\mathbf{c}_{k,n} = \arg \max_{1 \leq n \leq 2^{B_i}} |\mathbf{c}_{k,n}^H \hat{\mathbf{h}}_k|^2. \quad (4)$$

[2]This feedback model is exploited widely in the literature, see [2]- [9].

The codebook $\mathcal{C}_k$ is calculated offline and it is priori known at the base station and k^{th} user. Each user feedbacks only the index n to the base station using B feedback bits per user.

C. Beamforming

The utilized transmission technique is zero forcing beamforming. This aims at canceling the interference between the multiuser interference. The beamforming matrix can be formulated as $\mathbf{W} = [\mathbf{w}_1, \ldots, \mathbf{w}_K] = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H)^{-1}$. Due to imperfect CSI, the received signal to interference noise ratio can be expressed as:

$$\zeta_k = \frac{\frac{P}{K} |\mathbf{h}_k \mathbf{w}_k|^2}{\sum_{j, j \neq k} \frac{P}{K} |\mathbf{h}_k \mathbf{w}_j|^2 + \sigma_n^2}. \quad (5)$$

In ZFB, the CDI is the part of the CSI that is responsible of designing the accurate beamforming vectors while CQI accompanied by CDI is responsible for user selection and power allocation strategies at the base station.

III. LOSSLESS COMPRESSION

In this section, we exploit the knowledge of the second order statistics at the users' terminal to employ lossless compression strategy to simplify the vector quantization. The goal of compression is to represent the data in a more compact form; i.e a representation that requires fewer dimensions for encoding the same data to simplify the quantization procedure. Therefore, closed form expressions for the feedback allocation can be formulated for any generic scenario. In [13], the authors exploited the compression capability of discrete cosine transform (DCT) to reduce the required amount of feedback bits in massive MIMO scenario.

A. Karhunen-Leóve Transform (KLT)

The Karhunen-Leóve transform is defined as the linear transformation whose basis vectors are the eigenvector of the covariance information of the related data. The idea of utilizing the eigenvector as the basis vectors ensures that the first coefficient power is maximized while keeping the orthogonality among the basis and as consequence the subsequent coefficients are maximized respectively. KLT has attractive characteristics that motivates its utilization as compression technique in our work. First, the KLT has the optimal energy compaction, reducing the number of sufficient coefficients that is required to reconstruct the data at a desired accuracy. Second, rotating the data makes all off-diagonal terms of the covariance matrix equal zero (i.e. the KLT decorrelates the data). Using $\mathbf{R}_k = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{U}_k^H$., the uncorrelated representation of the channel vector $\mathbf{h}_k$, which is denoted by $\mathbf{v}_k$ can be written as

$$\mathbf{v}_k = \mathbf{U}_k^H \mathbf{h}_k^T. \quad (6)$$

Lemma 1. *KLT transformation does not change the channel power (i.e. $\|\mathbf{v}_k\|^2 = \|\mathbf{h}_k\|^2$).*

Proof. This rises from the fact that the transformation matrix $\mathbf{U}_k^H$ is a unitary matrix. □

It should be noted that the channel covariance matrix for the KLT transformed vector $\mathbf{v}_k^T$ equals to $\mathbf{\Lambda}_k$, which is different from the original channel vector $\mathbf{R}_k$.

B. Joint Lossless Compression and Vector Quantization

This enables the alternative data to be encoded with fewer number of bits for a given distortion than the original data. The KLT is the optimal transformation in terms of minimizing the bit rate. Depending on the rank of the channel, the information content in vector $\mathbf{v}_k$ is condensed in the first few components while the trailing components have zero power, hence, they can be truncated without losing any information content. This fact can be exploited efficiently to reduce the required number of quantization bits to achieve certain distortion. Since the information contents of the $\mathbf{v}_k$ are condensed in the first r_k components, the vector $\mathbf{v}_k$ can be truncated as:

$$\hat{\mathbf{v}}_k = \mathbf{T}_k \mathbf{v}_k, \tag{7}$$

where $\mathbf{T} \in \{0,1\}^{r_k \times M} = [\mathbf{I}^{r_k \times r_k} \mathbf{0}^{r_k \times (M-r_k)}]$.

Theorem 1. *The quantization distance d for quantizing the truncated KLT compressed version of the channel vector $\hat{\mathbf{v}}_{(k,i)}$ can be bounded as:*

$$\mathbb{E}[\sin^2(\angle(\hat{\mathbf{v}}_k, \tilde{\mathbf{v}}_k))] = 2^{B_k} \beta\Big(2^{B_k}, \frac{r_k}{r_k - 1}\Big) \leq 2^{\frac{-B_k}{r_k-1}}, \tag{8}$$

where $\angle(\hat{\mathbf{v}}_k, \tilde{\mathbf{v}}_k)$ denotes the angle between the real channel direction and quantized channel direction.

Proof. Truncating the trailing zero components of the compressed vector does not affect the rest of the components as long as the zeros can be retrieved at BS. The number of zero components in each compressed channel vector equals to $M - r_k$, which leaves r_k components to be quantized. The new quantization distance can be evaluated as Theorem 1 [2]. □

C. Loss Characterization

To characterize the performance degradation, we use the rate loss metric $\Delta\mathcal{R}_k$, which can be expressed as follows:

$$\Delta\mathcal{R}_k = \mathcal{R}_{P,k} - \mathcal{R}_{L,k} \tag{9}$$

where $\mathcal{R}_{p,k}$ denotes the rate assuming perfect CSI, and $\mathcal{R}_{L,k}$ denotes the rate assuming limited feedback. The rate under full CSI assumption can be formulated as:

$$\mathcal{R}_{P,k} = \mathbb{E}_{\mathbf{h}_k}[\log_2(1 + \frac{P|\mathbf{h}_k \mathbf{w}_k|^2}{K\sigma^2})], \tag{10}$$

while the rate under limited CSI assumption can be expressed as:

$$\mathcal{R}_{P,L} = \mathbb{E}_{\mathbf{h}_k}[\log_2(1 + \frac{\frac{P}{K}|\hat{\mathbf{h}}_k \mathbf{w}_k|^2}{\sum_{j\neq k} \frac{P}{K}|\hat{\mathbf{h}}_k \mathbf{w}_j|^2 + \sigma^2})] \tag{11}$$

Using the compress and quantize feedback strategy, the upper bound on the rate can be formulated as the following theorem

Theorem 2. *If compress and quantize-finite scheme is utilized, the rate loss per user due to rate feedback is function of the expected quantization error, δ, which is*

$$\Delta\mathcal{R}_k(P, K, \mathbf{h}_k, r_k, M) \leq \log(1 + \phi_{CQ})$$

where $\phi_{CQ} = \frac{(M-1)P}{M} 2^{B_i} \beta\Big(2^{B_i}, \frac{r_i}{r_i-1}\Big) \leq \frac{(M-1)P}{M} 2^{-\frac{B_i}{r_i-1}}$.

Proof. The rate loss can be formulated as:

$$\begin{aligned} \Delta\mathcal{R}_k &\approx \mathbb{E}_{\mathbf{h}_k}[\log_2(1 + \sum_{i,i\neq k} \frac{P}{M}|\hat{\mathbf{h}}_k \mathbf{w}_i|^2)] && (12) \\ &\leq \log_2(1 + \sum_{i,i\neq k} \mathbb{E}_{\mathbf{h}_k}[\frac{P}{M}|\hat{\mathbf{h}}_k \mathbf{w}_i|^2]) && (13) \\ &\leq \log_2(1 + \sum_{i,i\neq k} \mathbb{E}_{\mathbf{v}_k}[\frac{P}{M}|\tilde{\mathbf{v}}_k^T \mathbf{U}_k^T \mathbf{T}_k^T \mathbf{w}_i|^2]), && (14) \end{aligned}$$

where $\tilde{\mathbf{v}}_k$ is the quantized version of $\hat{\mathbf{v}}_k$, and it can be formulated as:

$$\tilde{\mathbf{v}}_k = \hat{\mathbf{v}}_k \cos(\angle(\hat{\mathbf{v}}_k, \tilde{\mathbf{v}}_k)) + \mathbf{q}_k \sin(\angle(\hat{\mathbf{v}}_k, \tilde{\mathbf{v}}_k)). \tag{15}$$

and $\mathbf{q}_k$ stands for error vectors due to channel quantization. This makes the term $\mathbb{E}_{\mathbf{v}_k}[\tilde{\mathbf{v}}_k^T \mathbf{U}_k^T \mathbf{T}_k^T \mathbf{w}_i|^2]$ equal to $\mathbb{E}_{\mathbf{v}_k}[\big(\hat{\mathbf{v}}_k \cos(\angle(\hat{\mathbf{v}}_k, \tilde{\mathbf{v}}_k)) + \mathbf{q}_k \sin(\angle(\hat{\mathbf{v}}_k, \tilde{\mathbf{v}}_k))\big)^T \mathbf{U}_k^T \mathbf{T}_k^T \mathbf{w}_i|^2]$, and finally it can be simplified to $\mathbb{E}_{\mathbf{v}_k}[\sin^2(\angle(\hat{\mathbf{v}}_k, \tilde{\mathbf{v}}_k))]$. □

D. Joint Lossy Compression and Vector Quantization

In practice, the channel can be rank deficient which means that some eigen directions have dominant contribution to the signal power. Therefore, the eigen directions with less power can be neglected without influential impact on the acquired channel information. If the eigenvalues $\lambda_{k,i}$ for certain eigen directions below a certain threshold , the corresponding components in the KLT domain can be approximated to zero. The vector of discarded components from the response $\mathbf{z}_k$ can be expressed as:

$$\mathbf{z}_k = \mathbf{v}_k - \bar{\mathbf{v}}_k. \tag{16}$$

The power of approximated vector $\bar{\mathbf{v}}_k$ can be bounded by [16]:

$$\|\bar{\mathbf{v}}_k\|^2 \leq \|\mathbf{v}_k\|^2 = \sum_{i=1}^{\hat{r}_k} \lambda_{k,i}. \tag{17}$$

The mean square error (MSE) of approximating $\mathbf{v}_k$ by $\bar{\mathbf{v}}_k$ can be formulated as:

$$\mathbb{E}[\|\bar{\mathbf{v}}_k - \mathbf{v}_k\|^2] = \sum_{i=\hat{r}_k+1}^{n_t} \lambda_{k,i}, \tag{18}$$

where $\hat{r}_k$ is the number of the components that their corresponding eigenvalues are higher than the predefined threshold. The quantization distance d for the truncated lossy compression can be expressed as:

$$\mathbb{E}[\sin^2(\angle\hat{\mathbf{v}}_k, \hat{\tilde{\mathbf{v}}}_k)] = 2^{B_k} \beta\Big(2^{B_k}, \frac{\hat{r}_k}{\hat{r}_k - 1}\Big) \leq 2^{\frac{-B_k}{\hat{r}_k-1}}. \tag{19}$$

where $\hat{r}_k$ denotes the number of the significant components (i.e. cannot be approximated to zero). The acquired $\mathbf{h}_k$ after the lossy compression and the vector quantization can be formulated as:

$$\hat{\hat{\mathbf{h}}}_k = \hat{\mathbf{v}}_k^T \mathbf{T}_k^T \mathbf{U}_k^T \quad (20)$$

$$\approx \left(\sqrt{\frac{(1-2^{-\frac{B_k}{\hat{r}_k-1}})}{\sum_{i=1}^{\hat{r}_k} \lambda_{k,i}}} (\mathbf{h}_k - \mathbf{U}_k \mathbf{z}_k)^T \right) \mathbf{T}_k^T \mathbf{U}_k^T + \mathbf{e}(\mathbf{21}) \quad (21)$$

The means square error resulted the joint lossy KLT and vector quantization can be expressed as:

$$\mathbb{E}[\|\hat{\hat{\mathbf{h}}}_k - \mathbf{h}_k\|^2] \approx (\frac{1}{\sum_{i=1}^{\hat{r}_k} \lambda_{k,i}} - 1)(1 - 2^{-\frac{B_k}{\hat{r}_k-1}}) + 2^{-\frac{B_k}{\hat{r}_k-1}}. \quad (22)$$

If we take $\lim_{B_k \to \infty} \mathbb{E}[\|\hat{\hat{\mathbf{h}}}_k - \mathbf{h}_k\|^2] = \frac{1}{\sum_{i=1}^{\hat{r}_k} \lambda_{k,i}} - 1$ and $\lim_{\sum_{i=1}^{\hat{r}_k} \lambda_{k,i} \to 1} \mathbb{E}[\|\hat{\hat{\mathbf{h}}}_k - \mathbf{h}_k\|^2] = 2^{-\frac{B_k}{\hat{r}_k-1}}$, we can see the effect of vector quantization and KLT lossy compression on the performance of the system. Moreover, it can be concluded that in the scenario $\sum_{i=1}^{\hat{r}_k} \lambda_{k,i} \geq 0.5$, the vector quantization effect is dominant over the compression effect.

IV. Feedback Bits Scaling and Allocation

A. Optimal Bit Allocation

In many scenarios, we have limited number of total feedback bits that should be allocated to all users. An adaptive feedback that exploits KLT with random vector quantization is proposed to reduce the rate loss resulted from the limited feedback. For $\sum_{i=1}^{\hat{r}_k} \lambda_{k,i} > 0.5$, the optimization problem to find the optimal bit allocation can be expressed as:

$$\min_{B_i \in \{0, \mathcal{Z}^+\}} \sum_i \frac{P}{M} 2^{B_k} \beta\Big(2^{B_k}, \frac{\hat{r}_k}{\hat{r}_k - 1}\Big) + u_k(1 - 2^{B_k} \beta\Big(2^{B_k}, \frac{\hat{r}_k}{\hat{r}_k-1}\Big))$$
$$s.t. \quad \sum_{k=1}^K B_k \leq B_t, \quad (23)$$

where $u_k = \frac{1}{\sum_{i=1}^{\hat{r}_k} \lambda_{k,i}} - 1$. The optimization problem in (23) is non-linear integer programing (NIP) one. Therefore, the optimal solution of the NIP problems is obtained by an exhaustive search method using combinatorial optimization.

B. Suboptimal Bit Allocation

Due to the high computational complexity of the problem (23), we propose a sub-optimal feedback bits allocation scheme with explicit solution. By changing the problem using the upper bound of the quantization error and applying a continuous relaxation technique to the integer constraint, we can relax the optimization problem and formulate it as:

$$\min_{B_k \in \{0, \mathbb{R}^+\}} \quad \sum_i \frac{Pc_k}{K} 2^{-\frac{B_k}{\hat{r}_k-1}}$$
$$s.t. \quad \sum_{k=1}^K B_k \leq B_t, \quad (24)$$

where $c_k = 1 - u_k = (2 - \frac{1}{\sum_{i=1}^{\hat{r}_k} \lambda_{k,i}})$. Using the fact that the objective function is logarithmically convex, we apply a convex optimization technique to solve the problem in (23). The related Lagrange function can be expressed as follows:

$$\mathcal{L}(B_k) = \sum_i \frac{Pc_k}{K} 2^{-\frac{B_k}{r_k-1}} + \lambda(\sum_{k=1}^K B_k - B_t). \quad (25)$$

To solve the optimization problem, we need to find the derivative of the Lagrange function with respect to all variables B_i, λ, i.e. Karush-Kuhun Tucker (KKT) conditions as:

$$\frac{\partial \mathcal{L}(B_k)}{\partial B_k} = -\frac{Pc_k}{K \log_e 2(\hat{r}_k - 1)} 2^{-\frac{B_i}{\hat{r}_k-1}} + \lambda \quad (26)$$

$$\frac{\partial \mathcal{L}(B_k)}{\partial \lambda} = \sum_{k=1}^K B_k - B_t. \quad (27)$$

Using (26)-(27), the final bit allocation can be expressed as:

$$B_k = \min\left\{B_t, \Big[\frac{B_t(\hat{r}_k - 1)}{\sum_{k=1}^K \hat{r}_k - K} + (\hat{r}_k - 1)\log_2\Big(\frac{\prod_k \hat{c}_k^{\frac{\hat{r}_k-1}{\sum \hat{r}_k - K}}}{\hat{c}_k}\Big)\Big]^+\right\}, \quad (28)$$

where $\hat{c}_k = \frac{Pc_k}{K \log_e 2(r_k-1)}$ and $[x]^+ = \max(x, 0)$. It can be noted that the bit allocation depends on the number of the significant components in the KLT response and their total power. Assuming no lossy compression $c_k = 1$, the final bit allocation in the scenario of lossless compression can be formulated as:

$$B_k = \min\left\{B_t, \Big[\frac{B_t(r_k - 1)}{\sum_{k=1}^K r_k - K} + (r_k - 1)\log_2\Big(\frac{\prod_k (r_k-1)^{\frac{r_k-1}{\sum r_k - K}}}{(r_k - 1)}\Big)\Big]^+\right\}.$$

It can be noted that the bit allocation depends on the rank of each user as well as the sum of all users' channel rank.

C. Proposed Compress and Quantize Feedback Algorithm

We exploit the lossless compression capability of KLT in FDD setup in multiuser single cell MISO system. The compress and quantize (CQ) feedback algorithm can be summarized as follows:

- Initialization (at each receiver); find the correlation matrix rank r_k, or the number of significant components $\hat{r}_k$.
- Data compression (at each receiver). Acquire the compressed version in KLT domain $\mathbf{v}_k = \mathbf{U}_k^H \mathbf{h}_k$.
- Data truncation (at each receiver). The KLT reduces the information representation in fewer useful elements and the rest are zeros. So the zeros can be removed from the data representation $\hat{\mathbf{v}}_k \in \mathbb{C}^{r_k} = \mathbf{T}_k \mathbf{v}_k$, where $\mathbf{T}_k \in \{0,1\}^{r_k \times M} = [\mathbf{I}^{r_k \times r_k} \mathbf{0}^{r_k \times (M-r_k)}]$
- Data quantization (at each receiver). Find the relevant quantization codebook size based on solving the optimization (28). Find the closest qunatization codeword $\tilde{\mathbf{v}}_k = \mathbf{c}_{D,n_i}, n_i = \arg \max_{1 \leq j \leq 2^{B_i}} |\mathbf{c}_{D,n_i}^H \hat{\mathbf{v}}_k|^2$
- Data feedback (at each receiver). The receiver feedbacks the quantized channels to transmitter.
- Data recovery (at the transmitter). The transmitter regenerate the $M \times 1$ channel vector by appending the vector $\tilde{\mathbf{v}}_k$ by $M - r_k$ zeros to get the vector $\hat{\mathbf{v}}_k = \mathbf{T}^T \tilde{\mathbf{v}}_k$. A further step is required to get the CSI data on their domain $\tilde{\mathbf{h}}_k = \mathbf{U}_k \tilde{\mathbf{v}}_k$.

D. Feedback scaling

In this section, we derive the scaling law of feedback bits per user for maintaining a constant rate loss. By using the proposed CQ feedback bit allocation strategy, we can propose the following lemma as:

Lemma 2. *For ZFB and lossless compression, if user's channel has the rank r_k, the bit scaling that can achieve a constant rate loss δ_k can be written as*

$$B_k \leq (r_k - 1)\log_2\left(\frac{P(K-1)}{K(2^{\delta_k}-1)}\right). \tag{29}$$

To have a total rate loss $\sum_k \delta_k$, the total numbers of feedback bits should scale as:

$$B_t \leq \left(\left(\sum_k r_k - K\right)\log_2\left(\frac{P(K-1)}{K}\right) - \sum_k \left(r_k - 1\right)\log_2\left(2^{\delta_k}-1\right)\right) \tag{30}$$

For the full rank transmit correlation, the number of zero components equals to zero. The advantage of employing lossless compression lies in decorrelating the channel vector $\mathbf{h}_k$, which simplifies the bit allocation in the previous sections. In the scenario of rank deficient correlation matrices, the gains of proposed algorithms are anticipated to be higher due the compression capability of KLT. Moreover, the proposed techniques can be used for CSI acquisition in Massive MIMO scenarios [13]- [17], where the channel has high probability to be spatially correlated or rank deficient.

V. Numerical results

In this section, numerical results are provided to demonstrate and get more insights of results derived in the previous sections. The assumed secenario for the first two figures: $\mathbb{E}[\mathbf{h}_k] = 1 \forall k \in K$, the users' channel ranks equal to $[4\ 2\ 4\ 3]$, $\mathbf{\Lambda}_1 = \text{diag}[0.45, 0.25, 0.25, 0.05]$, $\mathbf{\Lambda}_2 = \text{diag}[0.71, 0.29, 0, 0]$, $\mathbf{\Lambda}_3 = \text{diag}[0.42, 0.28, 0.15, 0.15]$, $\mathbf{\Lambda}_2 = \text{diag}[0.44, 0.39, 0.17, 0]$.

Fig. 1 depicts the comparison of the sum rate under limited and full CSI scenarios. For limited CSI scenarios, we study the performance of our proposed algorithm (CQ) and compare it to the uniform bit allocation, in which all the users are allocated the same amount of bits. It can be concluded that at low SNR regime the sum rate performance is not influenced by the limited feedback whether it is done uniformly or by using CQ. This is due to the fact that the dominant factor in this regime is the noise, which makes the interference terms resulted from the quantization insignificant. However, this trend changes at high SNR regime, where the interference is the dominant factor. It can be deduced that CQ achieves higher sum rates than the uniform quantization at $B_t = 10$, $B_t = 20$. This can be explained by the higher resolution provided by the quantization for the compressed channel components. This means that if a certain user has a higher rank, it is better to allocate it more bits than the other users. Finally, it can be noted the gain of CQ over uniform quantization reaches to 4,5 dB at high SNR regime.

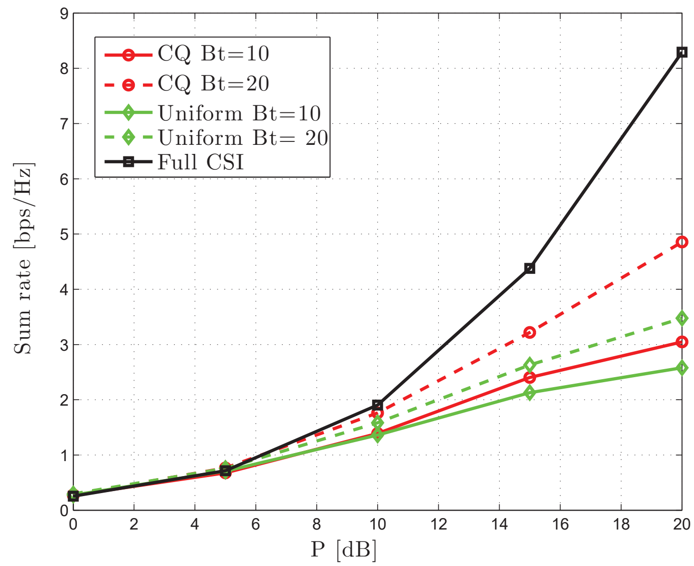

Fig. 1. Sum rate vs transmit power. $M = 4$, $K = 4$.

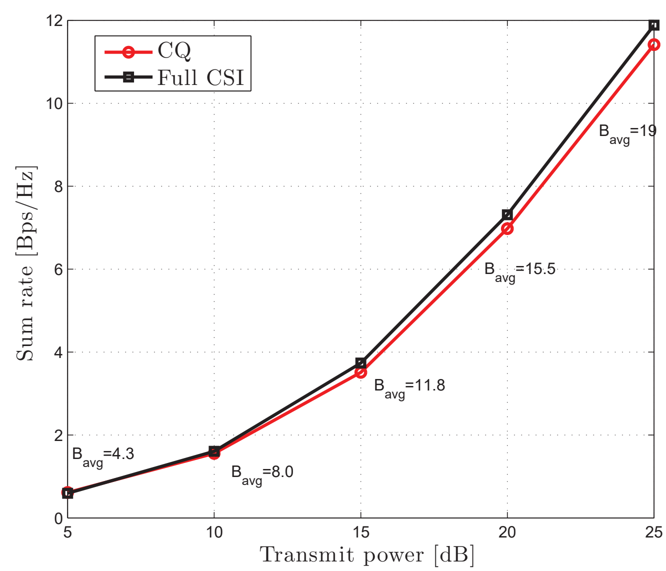

Fig. 2. Sum rate vs transmit power, $M = 4$, $K = 4$.

Fig. 2 depicts the number of the required average feedback bits per users B_{avg} (the total number of feedback bits KB_{avg}) using CQ to achieve the sum rate under full CSI scenario. It is clear that the CQ curve keeps following the one obtained with perfect CSIT, suggesting that the trend predicted by Lemma 2 is correct. It should be noted that Lemma 2 overestimates the required number of bit since the calculation tackles the upper bound of the rate loss.

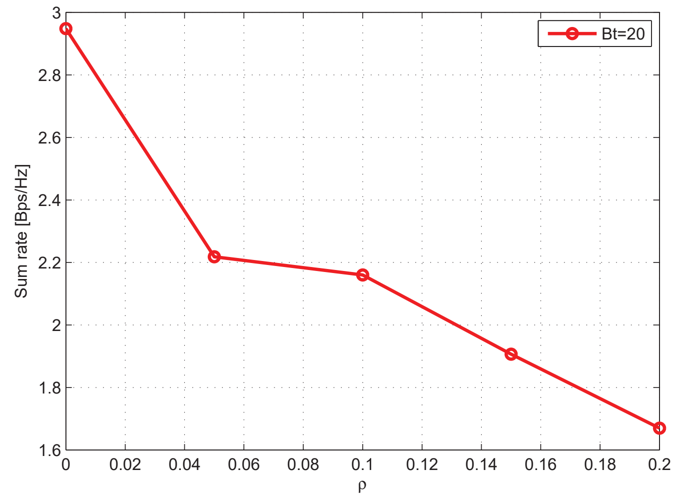

Fig. 3. Sum rate vs compression value $\rho = \sum_{i=1}^{\hat{r}_k} \lambda_{k,i}$. $M = 6$, $K = 6$.

Fig. (3) depicts the sum rate performance with respect the compression value ρ, i.e. $\hat{r}_k$ is selected to satisfy $\sum_{i=1}^{\hat{r}_k} \lambda_{k,i} \leq \rho$ for each user. The eigenvalues for the assumed scenario in Fig. (3) can be illustrated as

$$\begin{aligned}
\mathbf{\Lambda}_1 &= \text{diag}[0.2439, 0.2439, 0.2439, 0.1463, 0.0732, 0.0488] \\
\mathbf{\Lambda}_2 &= \text{diag}[0.2439, 0.2439, 0.2195, 0.1220, 0.1220, 0.0488] \\
\mathbf{\Lambda}_3 &= \text{diag}[0.55, 0.45, 0, 0, 0, 0] \\
\mathbf{\Lambda}_4 &= \text{diag}[0.2381, 0.2143, 0.1905, 0.1667, 0.1667, 0.0238] \\
\mathbf{\Lambda}_5 &= \text{diag}[0.2759, 0.2759, 0.2414, 0.1379, 0.0690, 0] \\
\mathbf{\Lambda}_6 &= \text{diag}[0.75, 0.25, 0, 0, 0, 0].
\end{aligned}$$

For lossless compression $\rho = 0$, the number of truncated components equal to $0, 0, 4, 0, 1, 4$ respectively It can be noted the increasing compression factor ρ decreases the total sum rate due to the fact that lossy compression truncates the information that cannot be retrieved at the base station and the increased interference resulted from the increased inaccuracy of the acquired CSI.

VI. Conclusion

In this work, we proposed a new feedback algorithm based on jointly using KLT and random vector quantization of correlated multiuser MISO channel. Users have the capability of adapting their codebook as a function of their spatial channel statistics to assign the suitable number of bits to quantize the channel direction information under a total number of bits constraints. We exploit the KLT capability of compression to represent the CDI in lower dimension to enable more efficient vector quantization. An analytical upper bound of the rate loss induced by quantization is derived. Based on this derivation, we proposed a closed form feedback bits allocation scheme which minimizes the expected quantization error by adaptively distributing given feedback bits per user according to the users' channel rank. It can be concluded that the proposed scheme can minimize the rate loss induced by the quantization and it can achieve better performance in comparison with uniform bit allocation.

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