

Reducing non-linear PDEs using a reduced integration proper orthogonal decomposition method

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Model order reduction methods can reduce the runtime of forward model solves by many orders of magnitude. This has important applications in creating simulation tools with real-time response rates, accelerating the solution of stochastic PDEs and inverse problems.

A popular and straightforward way of reducing linear PDEs is the method of proper orthogonal decomposition (POD) [1]. This leads to a new set of global POD basis functions that optimally represent the solution data in a set of generated solution snapshots.

In an online phase bilinear and linear operators on the original finite element space are projected onto the pre-calculated POD basis, resulting in a small dense linear system that is fast to solve [1].

However, as a natural consequence of solving non-linear PDEs using Newton's method, the bilinear and linear forms on the finite element space become functions of the current solution. Therefore at every Newton iteration we must re-assemble the finite element space operators and re-project onto the POD basis. This operation can become a dominant portion of the runtime of the solution of the reduced order model.

To overcome this issue we have developed a new method where we construct a reduced integration mesh that is adapted to the POD basis and the data contained in the snapshots. We generate this mesh via a greedy process in the offline phase using a simple error estimation and adaptive mesh refinement scheme on the sequence of POD basis functions. The reduced integration mesh has far fewer cells than the original mesh and therefore the linearised projected operators required at each Newton iteration can be assembled much faster.

We will show some results implemented in DOLFIN [2] using SLEPc [3] that show an order of magnitude speed up in forward runtime on a time-dependent two-dimensional non-linear reaction diffusion equation and a three-dimensional geometrically non-linear hyperelastic model.

References

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