# Performance Analysis of Underlay Cognitive Radio Systems: Estimation-Throughput Tradeoff

Ankit Kaushik*, *Student Member, IEEE*,
Shree Krishna Sharma†, *Member, IEEE*, Symeon Chatzinotas†, *Senior Member, IEEE*,
Björn Ottersten†, *Fellow, IEEE*, Friedrich K. Jondral* *Senior Member, IEEE*

*Abstract*—In this letter, we study the performance of cognitive Underlay Systems (USs) that employ power control mechanism at the Secondary Transmitter (ST). Existing baseline models considered for the performance analysis either assume the knowledge of involved channels at the ST or retrieve this information by means of a feedback channel, however, such situations hardly exist in practice. Motivated by this fact, we propose a novel approach that incorporates the estimation of the involved channels at the ST, in order to characterize the performance of USs under realistic scenarios. Moreover, we apply an outage constraint that captures the impact of imperfect channel knowledge, particularly on the interference power received at the primary receiver. Besides this, we employ a transmit power constraint at the ST to determine an operating regime for the US. Finally, we analyze an interesting tradeoff between the estimation time and the secondary throughput allowing an optimized performance of the US.

*Index Terms*—Cognitive radio, Underlay system, Channel estimation, Estimation-throughput tradeoff, Operating regime

## I. INTRODUCTION

Cognitive Radio (CR) communication is considered as one of the viable solutions that addresses the problem of spectrum scarcity of future wireless networks. Secondary access to the licensed spectrum can be broadly categorized into different CR paradigms, namely, interweave, underlay and overlay [1]. Among these, underlay and interweave systems are largely associated with techniques that are present at the physical layer, hence, can be considered feasible for hardware deployment. Particularly, interference tolerance capability exhibited by the Underlay Systems (USs) ensure that they do not cause harmful interference to the primary system while performing shared access to the licensed spectrum. Out of the various underlay techniques, power control is one such mechanism under which USs tend to operate below an Interference Threshold (IT) of the Primary Receiver (PR) [2].

To employ power control, the knowledge of the interference channel between the ST and the PR is of paramount importance. To this end, performance analysis subject to imperfect channel knowledge has received significant attention [3]–[6]. According to [4], [5], the ST attains the channel knowledge

over a feedback channel. Since the feedback channel and the ability to demodulate the ST's signal are non-existent in the current primary systems, the hardware feasibility of this approach becomes challenging. To overcome this issue, a novel strategy was proposed in [6], whereby the ST listens to the control-based transmission from the PR and estimates the received power to retrieve the knowledge of the interference channel. The variation due to imperfect channel knowledge, particularly, in interference power received at the PR was captured by means of a constraint on the probability of confidence [6].

However, the system model described in [6] has certain limitations. Since the USs are sensitive only to those variations that exceed the IT, it is reasonable to implement a power control mechanism subject to an outage constraint. Besides that, the transmit power at the ST should not exceed a certain value. Lastly, analyzing the performance of the secondary system in terms of achievable throughput requires the knowledge of access channel between the ST and Secondary Receiver (SR), however, this knowledge is not available at the ST. In this context, the performance analysis of the US that incorporates channel estimation at the ST subject to outage and transmit power constraints is an interesting research problem.

In this letter, we make the following contributions:

- We propose a novel model that employs a power control mechanism and incorporates channel estimation of the interacting channels, namely interference channel and access channel at the ST.
- Based on the proposed model, we capture the effect of imperfect channel knowledge by employing an outage constraint on the received power at the PR that restrains the interference encountered by a primary system. Subsequently, we investigate a tradeoff between the estimation time and the achievable secondary throughput.
- In reference to the transmit power constraint at the ST, we characterize an operating regime for the US.

## II. SYSTEM MODEL

Cognitive Small Cell (CSC), a CR application, characterizes a small cell deployment that fulfills the spectral requirements for Mobile Stations (MSs) operating indoor, cf. Fig. 1. For the disposition of the CSC in the network, the following key elements are essential: a CSC-Base Station (CSC-BS), a Macro Cell-Base Station (MC-BS) and MS [7]. Considering the fact that the power control is employed at the CSC-BS, the CSC-BS and the MS represents ST and SR, respectively. The PR

*A. Kaushik and F. K. Jondral are with Communications Engineering Lab, Karlsruhe Institute of Technology (KIT), Germany. Email:{ankit.kaushik,friedrich.jondral}@kit.edu.
†S.K. Sharma, S. Chatzinotas and B. Ottersten are with SnT - securityandtrust.lu, University of Luxembourg, Luxembourg. Email:{shree.sharma, symeon.chatzinotas, bjorn.ottersten}@uni.lu.
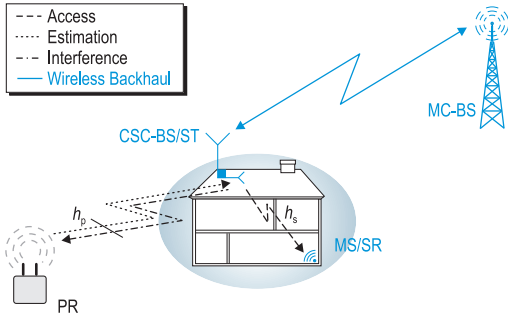
Fig. 1. A cognitive small cell scenario demonstrating: (i) the underlay paradigm, (ii) the associated network elements, which constitute Cognitive Small Cell-Base Station/Secondary Transmitter (CSC-BS/ST), Mobile Station/Secondary Receiver (MS/SR), Macro Cell-Base Station (MC-BS) and Primary Transmitter (PT), (iii) the interacting channels: interference and access channels.
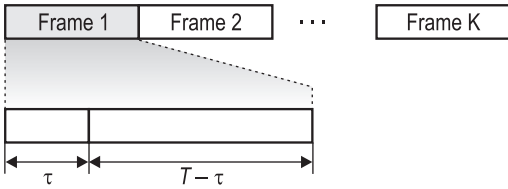


Fig. 2. Frame structure of underlay system with received power estimation for the interference channel.

performs transmission/reception of signals (interchangeably over time) to/from the Primary Transmitter (PT), cf. Fig. 1, the ST follows this alignment for transmitting signals with controlled power over the access channel.

The medium access for the US is slotted, where the time axis is segmented into frames of length $T$. The duration $T$ ($\ll$ expected ON/OFF period of the primary user) is chosen in such a way that the frames are aligned to the control-based transmissions[1] of the primary system. The frame structure is analog to periodic sensing in [6], according to which, US uses a time interval $\tau(< T)$ to estimate the received power, cf. Fig. 2. In order to consider variations due to channel fading, we assume that the interacting channels remain constant over a frame duration [6]. Hence characterized by the fading process, each frame witnesses a different received power.

In order to operate at a desired outage probability, defined later in Section III, it is reasonable to exercise estimation followed by data transmission with power control in the remaining time $T - \tau$ for each frame. Since the access channel estimation is performed by listening to the pilot symbols from the SR, no time resources are allocated for access channel estimation in the frame structure, hence $\tau$ is utilized for interference channel estimation only. In this letter, we consider this frame structure to perform short-term analysis, i.e., the performance is analyzed for a certain channel gain, without taking into account the effect of channel fading.

To simplify the analysis, we assume that during data transmission at the ST, the interference at the SR, from the PT, to be below the noise level. However, by replacing the noise

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[1]These include the beacons transmitted by the PR in the same band or a pilot channel transmitted in a separate band. With the knowledge of power transmitted by the PR and using channel reciprocity, the ST is able to determine the interference power received at the PR, thus, can control its transmit power while sustaining the IT, cf. [6] and references therein.

power with interference plus noise power in the throughput expressions, derived later in Section III, the performance of the US under the interference limited regime can be depicted.

In the estimation phase, the discrete control-based signal received from the PR at the ST is given by [6]

$$y_{\text{rcvd}}[n] = h_{\text{p}} \cdot x_{\text{tran}}[n] + w_{\text{s}}[n], \tag{1}$$

where $x_{\text{tran}}[n]$ corresponds to a discrete and complex sample transmitted by the PR with transmit power $P_{\text{tran}}$ known at the ST, $|h_{\text{p}}|^2$ represents the power gain for the interference channel and $w_{\text{s}}[n]$ is circularly symmetric complex Additive White Gaussian Noise (AWGN) at the ST with $\mathcal{CN}(0, \sigma^2)$.

During data transmission phase, the interference signal received at the PR is given by

$$y_{\text{p}}[n] = h_{\text{p}} \cdot x_{\text{cont}}[n] + w_{\text{p}}[n], \tag{2}$$

and on the other side, the received signal at the SR follows

$$y_{\text{s}}[n] = h_{\text{s}} \cdot x_{\text{cont}}[n] + w_{\text{s}}[n], \tag{3}$$

where $x_{\text{cont}}[n]$ corresponds to a discrete and complex sample transmitted by the ST with controlled power $P_{\text{cont}}$. Further, $|h_{\text{s}}|^2$ represents the power gain for the access channel and $w_{\text{p}}[n]$ is AWGN at the PR with $\mathcal{CN}(0, \sigma^2)$.

## III. PERFORMANCE ANALYSIS

### A. Ideal model

According to the ideal model, a ST as an US is required to control its transmit power in such a way that the interference power received $P_{\text{p}}$ at the PR is below IT $\theta_{\text{I}}$ [2]

$$P_{\text{p}} = |h_{\text{p}}|^2 P_{\text{cont}} \le \theta_{\text{I}}. \tag{4}$$

With controlled power at the ST determined using (4), the throughput at the SR is defined as

$$R_{\text{s}} = \log_2 \left( 1 + |h_{\text{s}}|^2 \frac{P_{\text{cont}}}{\sigma^2} \right). \tag{5}$$

### B. Proposed Model

To employ power control based on (4) and evaluate $R_{\text{s}}$ according to (5), the ideal model considers the knowledge of the involved channels $h_{\text{p}}$ and $h_{\text{s}}$ at the ST, which is not available in practice. In this regard, we incorporate channel estimation in the system model. The imperfect channel knowledge, however, translates to variations in the performance parameters, $P_{\text{p}}$ and $R_{\text{s}}$. Particularly, a variation in $P_{\text{p}}$ that exceeds the $\theta_{\text{I}}$ causes interference at the PR. Unless characterized, these variations may seriously degrade the performance of the US. In this view, we capture the variations in $P_{\text{p}}$ and $R_{\text{s}}$ by characterizing the distribution functions of the estimated channels.

*1) Estimation of interference channel:* Given $P_{\text{rcvd}} = |h_{\text{p}}|^2 P_{\text{tran}} + \sigma^2$ and the knowledge of PR's transmit power $P_{\text{tran}}$, the ST listens to the control-based transmissions from the PR and acquires the knowledge of $|\hat{h}_{\text{p}}|^2$ indirectly by estimating the received power $\hat{P}_{\text{rcvd}} = \frac{1}{\tau f_{\text{s}}} \sum_n^{\tau f_{\text{s}}} |y_{\text{rcvd}}[n]|^2$. $\hat{P}_{\text{rcvd}}$ estimated using $\tau f_{\text{s}}$ samples follows a non-central chi-squared distribution $F_{\hat{P}_{\text{rcvd}}} \sim \mathcal{X}_1^2(\lambda_{\text{p}}, \tau f_{\text{s}})$ with non-centrality parameter $\lambda_{\text{p}} = \tau f_{\text{s}} |h_{\text{p}}|^2 P_{\text{tran}}/\sigma^2 = \tau f_{\text{s}} \gamma$, where $\gamma$ is defined as the ratio

of the received control-based power (from the PR) to noise at the ST and $\tau f_s$ corresponds to the degrees of freedom.

*Approximation 1:* For all degrees of freedom, the $\mathcal{X}_1^2$ distribution can be approximated by a Gamma distribution [8]. The parameters of the Gamma distribution are obtained by matching the first two central moments to those of $\mathcal{X}_1^2$.

*Lemma 1:* The cumulative distribution function of $\hat{P}_{rcvd}$ is characterized as

$$F_{\hat{P}_{rcvd}}(x) \approx 1 - \Gamma(a_1, b_1 x), \qquad (6)$$

where $a_1 = \dfrac{\tau f_s(1+\gamma)^2}{2+4\gamma}$ and $b_1 = \dfrac{\sigma^2(2+4\gamma)}{\tau f_s(1+\gamma)}$,

and $\Gamma(\cdot, \cdot)$ represents the regularized lower-incomplete Gamma function [8].

*Proof:* Applying Approximation 1 to $\mathcal{X}_1^2(\lambda_p, \tau f_s)$ yields (6). ∎

*2) Estimation of access channel:* The pilot signal received from the SR undergoes matched filtering and demodulation at the ST, hence, we employ a pilot-based estimation at the ST to acquire the knowledge of the access channel. According to [9], the maximum-likelihood estimate with $N_s$ pilot symbols is given by

$$h_s = \hat{h}_s + \frac{\sum_n^{N_s} p[n]}{2N_s}, \qquad (7)$$

where $p[n]$ denotes the discrete pilot symbol and $\frac{\sum_n^{N_s} p[n]}{2N_s}$ represents the estimation error. As a result, the estimate $\hat{h}_s$ is unbiased, efficient, i.e., achieves the Cramér-Rao bound with equality, with asymptotic variance $\mathbb{E}\left[|h_s - \hat{h}_s|^2\right] = \frac{\sigma^2}{2N_s}$ [9]. Hence, $\hat{h}_s$ conditioned on $h_s$ follows a Gaussian distribution

$$\hat{h}_s | h_s \sim \mathcal{N}\left(h_s, \frac{\sigma^2}{2N_s}\right). \qquad (8)$$

Consequently, the estimated power gain $|\hat{h}_s|^2$ follows a non-central chi-squared $\mathcal{X}_1^2(\lambda_s, 1)$ distribution with 1 degree of freedom and non-centrality parameter $\lambda_s = \frac{2N_s|h_s|^2}{\sigma^2}$.

*Lemma 2:* The cumulative distribution function of $|\hat{h}_s|^2$ is characterized as

$$F_{|\hat{h}_s|^2}(x) \approx 1 - \Gamma(a_2, b_2 x), \qquad (9)$$

where $a_2 = \dfrac{(1+\lambda_s)^2}{2+4\lambda_s}$ and $b_2 = \dfrac{\sigma^2(2+4\lambda_s)}{(1+\lambda_s)}$.

*Proof:* Applying Approximation 1 to $\mathcal{X}_1^2(\lambda_s, 1)$ yields (9). ∎

Next, we employ an outage probability constraint at the ST to capture the variation in the $P_p$ incurred due to channel estimation, defined as

$$\mathbb{P}\left(\underbrace{\left(\frac{\hat{P}_{rcvd} - \sigma^2}{P_{tran}}\right)}_{|\hat{h}_p|^2} P_{cont} \geq \theta_I\right) \leq \rho_{out}, \qquad (10)$$

where $\rho_{out}$ corresponds to an outage constraint. Besides the outage constraint, $P_{cont}$ is limited by a predefined transmit power $\rho_{cont}$. To capture this aspect, the transmit power constraint at the ST is defined as
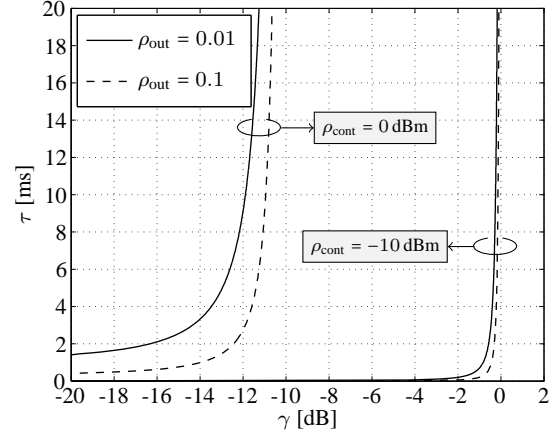
$$P_{cont} \leq \rho_{cont}. \qquad (11)$$



Fig. 3. An illustration of operating regime ($\gamma^*$) for the US depicted in terms of estimation time ($\tau$) and the ratio of the received control-based power (from the PR) to noise ($\gamma$) at the ST.

Based on the aforementioned constraints, we determine the expression of controlled power for the proposed model.

*Lemma 3:* Subject to the outage constraint and transmit power constraint, the controlled power at the ST is given by

$$P_{cont} = \begin{cases} \frac{\theta_I P_{tran}}{(b_1 \Gamma^{-1}(1-\rho_{out}, a_1) - \sigma^2)} & \text{if } P_{cont} < \rho_{cont} \\ \rho_{cont} & \text{if } P_{cont} \geq \rho_{cont} \end{cases}, \qquad (12)$$

where $a_1$ and $b_1$ are defined in (6) and $\Gamma^{-1}(\cdot, \cdot)$ is the inverse function of regularized lower-incomplete Gamma function [8].

*Proof:* Substituting the distribution function for $\hat{P}_{rcvd}$, defined in (6), in (10) and combining with (11) yields (12). ∎

Clearly, $P_{cont}$ increases with increase in $|h_s|^2$, which depicts low $\gamma$, consequently a better performance in terms of secondary throughput is achieved by the US for low $\gamma$, however with the presence to $\rho_{cont}$ an upper limit is imposed on the achievable performance. We define this performance limit in terms of $\gamma$ as an operating regime $\gamma^*$ for the US.

*Corollary 1:* Subject to a transmit power constraint, an operating regime at the ST is defined as[2]

$$\rho_{out} \leq 1 - \Gamma\left(a_1, \frac{1}{b_1}\left(\frac{\theta_I P_{tran}}{\rho_{cont}} + \sigma^2\right)\right). \qquad (13)$$

*Proof:* Substituting $P_{cont}$, cf. (12), in (11) results in (13). Replacing (13) with equality yields $\gamma^*$. ∎

In other words, below a certain $\gamma(\leq \gamma^*)$ no performance gain is witnessed by the CR system, cf. Fig. 3. As a result, by replacing $\gamma^*$ in the following expression of secondary throughput, we determine the performance limits of operation for the US.

Besides that, for the estimation model, the expected throughput for the access link at the SR is defined as

$$R_s(\tau) = \mathbb{E}_{|\hat{h}_s|^2}\left[\frac{T-\tau}{T} \log_2\left(1 + \frac{|\hat{h}_s|^2 P_{cont}}{\sigma^2}\right)\right], \qquad (14)$$

where $\mathbb{E}_{|\hat{h}_s|^2}[\cdot]$ corresponds to an expectation over $|\hat{h}_s|^2$, whose distribution function is characterized in Lemma 2.

At this stage, it is worthy to note that $P_{cont}$ and $R_s$ depend on $\tau$, cf. (12) and (14), respectively. Hence, the proposed model

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[2]Please note that $\tau f_s$ and $\gamma$ are included in the parameters $a_1$ and $b_1$, cf. (6).
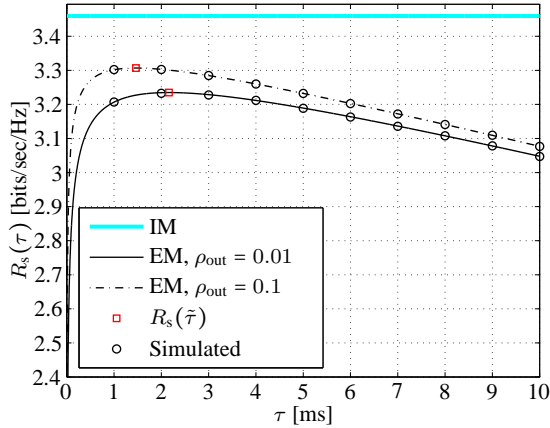
Fig. 4. Estimation-throughput tradeoff with $\gamma = 0\,\text{dB}$, $\rho_{\text{out}} \in \{0.01, 0.1\}$ and $\rho_{\text{cont}} = 0\,\text{dBm}$.

exhibits a fundamental tradeoff between the estimation time and the achievable secondary throughput.

*Theorem 1:* The expected achievable secondary throughput subject to the outage constraint on the received power at the PR and transmit power constraint at the ST is defined as

$$R_s(\tilde{\tau}) = \max_{\tau} R_s(\tau), \tag{15}$$
$$\text{s.t. (10), (11),}$$

where $R_s(\tilde{\tau})$ corresponds to optimum throughput at $\tilde{\tau}$.

*Proof:* The constrained optimization problem is solved by substituting $P_{\text{cont}}$ from Lemma 3, determined by applying outage and transmit power constraints defined in (10) and (11), in (14).

Using the distribution function of $|\hat{h}_s|^2$ in (9) to determine an expression of expected throughput as a function of $\tau$. Solving numerically this expression yields $\tilde{\tau}$ and $R_s(\tilde{\tau})$. ∎

## IV. NUMERICAL ANALYSIS

Here, we investigate the performance of the US based on the proposed model. To accomplish this: (i) we perform simulations to validate the expressions obtained, (ii) we analyze the performance loss incurred due to the estimation. In this regard, we consider the ideal model for benchmarking and evaluating the performance loss. Unless stated explicitly, the following choice of the parameters is considered for the analysis, $f_s = 1\,\text{MHz}$, $h_p = -100\,\text{dB}$, $h_s = -80\,\text{dB}$, $\theta_I = -110\,\text{dBm}$, $T = 100\,\text{ms}$, $\rho_{\text{out}} \in \{0.01, 0.10\}$, $\rho_{\text{cont}} \in \{-10, 0\}\text{dBm}$, $\sigma^2 = -100\,\text{dBm}$, $\gamma = 0\,\text{dB}$, $P_{\text{tran}} = 0\,\text{dBm}$, $N_s = 10$.

Fig. 4 analyzes performance of US in terms of estimation-throughput tradeoff, cf. Theorem 1, corresponding to the Ideal Model (IM) and the Estimation Model (EM). It is indicated that the estimation-throughput tradeoff yields a suitable estimation time $\tilde{\tau}$ that results in an optimum throughput $R_s(\tilde{\tau})$. Hereafter, for the analysis, we consider the theoretical expressions and choose to operate at suitable estimation time. To procure further insights, the variation of $R_s(\tilde{\tau})$ with $\gamma$ for different choices of $\rho_{\text{cont}}$ and $\rho_{\text{out}}$ are considered in Fig. 5. It is observed that $R_s(\tilde{\tau})$ gets saturated below a certain $\gamma$, thereby limiting the performance of the US. Particularly for $\rho_{\text{cont}} = -10\,\text{dBm}$, a severe performance loss indicated by the margin between the IM and the EM is witnessed by the US for $\gamma \le -2\,\text{dB}$.



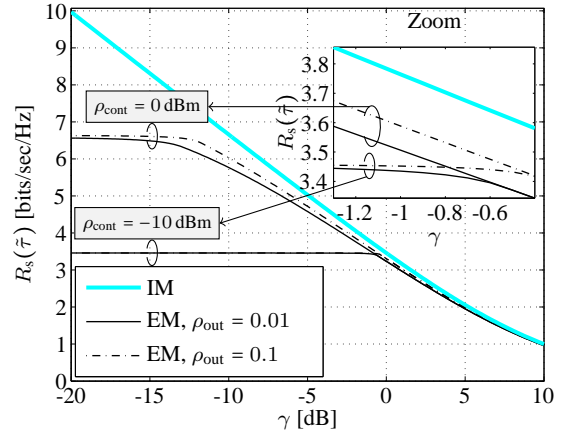Fig. 5. Optimum throughput ($R_s(\tilde{\tau})$) versus the ratio of received control-based power to noise ($\gamma$) with $\rho_{\text{out}} \in \{0.01, 0.1\}$ and $\rho_{\text{cont}} \in \{-10, 0\}$ dBm.

## V. CONCLUSION

In this letter, we studied the performance of the USs from a deployment perspective. In this view, a novel model that incorporates channel estimation has been proposed. To capture the impact of imperfect channel knowledge, an outage constraint that forbids performance degradation in terms of interference power received at the primary receiver has been employed. With the inclusion of a transmit power constraint, an operating regime that determines performance limits for the US has been established. Further, a power control mechanism subject to the outage and transmit power constraints has been proposed. Finally, the estimation-throughput tradeoff has been investigated to determine the achievable secondary throughput for the US. In future work, we plan to extend the proposed analysis to include the effect of channel fading in order to characterize the long-term performance of the USs.

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