Propagating uncertainty through a non-linear hyperelastic model using advanced Monte-Carlo methods

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Keywords: *Monte-Carlo methods, uncertainty propagation, sensitivity derivatives, non intrusive solvers, polynomial chaos expansion, parallel computing.*

In many soft-tissue biomechanics simulations the material parameters used in the definition of the hyperelastic energy density functional often have a significant degree of uncertainty associated with them. In a clinical environment, where safety-critical decisions must be made based on the output of simulations, being able to propagate and visualise this uncertainty is of importance.

To propagate uncertainty we recast the the geometrically non-linear Mooney-Rivlin hyperleastic model as a stochastic PDE with random coefficients. We advocate the solution of this non-linear stochastic problem with what we call *partially-intrusive* Monte-Carlo methods. These methods only use the output of the forward model and sensitivity information (tangent linear models derived from UFL expressions) [1] and polynomial chaos expansion (PCE) techniques [2, 3] to greatly improve convergence.

We implement our forward and tangent linear model solvers using DOLFIN [4] and we use chaospy [5] to generate various stochastic objects. We then use ipyparallel and mpi4py to massively parallelise individual forward model runs across a cluster.

We compare the results of our method with simple Monte-Carlo methods. By using sensitivity information we demonstrate that computational workload can be reduced by one order of magnitude over commonly used schemes.

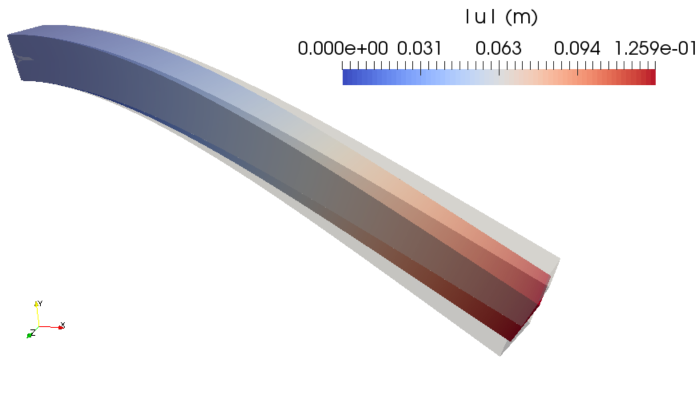

Figure 1: Deformation of the beam: mean +/- standard deviation.

References

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