Investigation of Simple Algorithms for Estimation of Delay-Spread and Angle-Spread

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Abstract

In this paper we describe two simple methods for estimation of delay-spread and angle-spread, respectively. The algorithms are simple in the sense that the transmitted signal may consist of only three superimposed CW tones -and the receiver need only two antennas. The algorithms are also simple in the sense that the computational cost is very low. We verify the algorithms by applying them to wideband and multi-antenna measurement data, respectively.

I. Introduction

This technical report is a companion to the paper [1]. Thus the reader is referred to that paper for a background of the studied problem.

II. The Delay-Spread Estimation Algorithms

The propagation channel between one of the base-station antennas and a mobile-station antenna $h(\tau)$ is assumed to given be given by N discrete pulses i.e.

$$h(\tau) = \sum_{k=1}^{N} \beta_k \delta(\tau - \tau_k), , \tag{1}$$

where β_k are the instantaneous tap gains and τ_k the corresponding delays. The impulse response and the tap gains are assumed complex (i.e. a base-band representation is used). Nothing in our analysis prevents N to tend to infinity i.e. we can model also diffuse propagation scenarios. In practice the discrete components in (1) can never be observed due to smoothing due to the finite bandwidth of transmitter and receiver circuity

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and antennas. The paths β_k are assumed to varying during the observation interval. This corresponds to the three second bursts used in [1]. The average path powers during this interval are given by

$$\mathrm{E}\{|\beta_k|^2\} = \alpha_k. \tag{2}$$

The path-delays τ_k are assumed fixed during the same interval. The delay-spread is defined as

$$\sigma_{\mathrm{DS}} = \sqrt{\frac{\sum_{k=1}^{N}(\tau_k - \bar{\tau_a})^2\alpha_k}{\sum_{k=1}^{N}\alpha_k}}, \tag{3}$$

where α_k and τ_k is the power and delay of the kth multipath component, and τ_a is defined as

$$\tau_a = \frac{\sum_{k=1}^{N}\tau_k\alpha_k}{\sum_{k=1}^{N}\alpha_k}\,. \tag{4}$$

A. Power-delay Profile Estimation Method

A common method to estimate the delay-spread is through the so-called power-delay profile. The power-delay-profile (PDP), $\breve{p}(\tau)$. The PDP is defined as the expectation of the impulse response $h(t)$ squared i.e.

$$\breve{p}(\tau) = \mathrm{E}\{|h(\tau)|^2\}. \tag{5}$$

Assuming uncorrelated scattering components (i.e. the common WSSUS wide-sense uncorrelated scattering assumption) $p(t)$ is given by

$$\breve{p}(\tau) = \sum_{k=0}^{N}\alpha_k\delta(\tau - \tau_k). \tag{6}$$

In terms of $\breve{p}(\tau)$ the delay-spread is given by

$$\tau_a = \int_{\tau=0}^{\infty} p(\tau)(\tau - \tau_a)^2 d\tau, \tag{7}$$

where $p(\tau)$ is the normalized PDP defined by

$$p(\tau) = \frac{\breve{p}(\tau)}{\int_{\tau=0}^{\infty}\breve{p}(\tau)d\tau} \tag{8}$$

In practice the PDP is estimated by averaging several impulse responses during the observation interval. The measured (normalized) PDP will include the effect of the filtering caused by the hardware i.e.

$$\hat{p}(\tau) = p(\tau) \star f(\tau), \tag{9}$$

where $p(\tau)$ and $f(\tau)$ are the true normalized PDP and the normalized system response. Note that all the three functions in (9) can be interpreted as probability distributions and the the delay-spread as the variance of $p(\tau)$. Thus the variance of $\hat{p}(\tau)$ is the sum of the variance of $p(\tau)$ and $f(\tau)$ i.e.

$$\tilde{\sigma}_{\mathrm{DS}}^2 = \sigma_{\mathrm{DS}}^2 + \sigma_f^2. \tag{10}$$

B. Simple Correlation Based Method for Delay-Spread estimation

In this section we describe the RMS delay-spread estimation algorithm of [1]. We begin by assuming that the signal strength of our narrow-band signals are Rayleigh fading over our observation interval. Mathematically, the amplitude $h_n(t)$ of the narrow-band channel can be expressed in terms of the wideband channel defined in in (1) above i.e.

$$h_n(t,f) = \int_{\tau=0}^{\infty} h(\tau)\exp(-j2\pi f\tau)d\tau \tag{11}$$

$$= \sum_{k=1}^{N} \beta_k \exp(-j2\pi f\tau_k), \tag{12}$$

where f is the frequency offset between the carrier frequency used for the wide-band measurement of $h(\tau)$ and the frequency of the narrow-band transmission. The complex correlation coefficient between two narrow-band signals with carrier frequencies f_1 and f_2 is obtained as

$$\sqrt{\mathrm{E}\{|h_n(t,f_1)|^2\}\mathrm{E}\{|h_n(t,f_2)|^2\}}\tilde{\rho} = (\int_{\tau=0}^{\infty} p(\tau)d\tau)\tilde{\rho} \tag{13}$$

$$= \mathrm{E}\{h_n(t,f_1)h_n^*(t,f_2)\} \tag{14}$$

$$= \sum_{k=1}^{N} \alpha_k \exp(-j2\pi(f_1-f_2)\tau_k) \tag{15}$$

$$= \int_{\tau=0}^{\infty} \breve{p}(\tau)\exp(-j2\pi(f_1-f_2)\tau)d\tau. \tag{16}$$

If a certain parametrization of the power-delay-profile is assumed (e.g. exponential i.e. $p(\tau) = \frac{1}{s}\exp(-t/s)$), it is possible to express $\tilde{\rho}$ as a function of $f_1 - f_2$ and the delay-spread s. Thus one way of estimating the delay-spread is to transmit two CWs measure $\tilde{\rho}$, assume a certain delay-spread parametrization and then calculate the delay-spread. Here we will make two alterations of this basic algorithm. First we will consider a system where the phases the transmitted CWs are drifting. To cope with this situation we will consider the correlation coefficient, ρ, between signal envelopes $|h_n(t,f_1)|$, $h_n(t,f_2)$ instead of the complex channel gains. It is well known that this correlation can be approximated by the complex correlation coefficient magnitude squared. Thus instead of using this complex correlation we will use the envelope correlation coefficient. The second alteration is the use of three CWs instead of two. With three CWs we need to consider three different envelope correlation coefficients. This gives us the possibility to try different assumptions on power-delay

	Tone 1	Tone 2	Tone 3
Freq. Setting 1	0 kHz	180kHz	360kHz
Freq. Setting 2	0 kHz	480kHz	960kHz
Freq. Setting 3	0 kHz	720kHz	1440kHz

Table III: Frequency of the three tones in each measurement selection setting, in relation to the carrier frequency.

profile. We try the following possible power-delay profiles: exponential, exponential square, triangular and rectangular. For each of the four hypotheses we minimize the sum of the square errors between the measured envelope correlation coefficients with respect to the delay-spread. Finally, we select profile providing the minimum error.

C. Comparison of PDP and Correlation Based RMS Delay Spread Estimates

By courtesy of Ericsson Research AB we have access to a measurement series over a transmission bandwidth of almost 20MHz. The channel response is made available to us in the frequency domain with 162 frequency bins with a subcarrier spacing of 120kHz. The measurements were done in an urban environment with relatively uniform building height at distances up to 400 meters.

In order to estimate the PDP we need first need to convert this into a time-domain impulse response. We do this by using an inverse discrete Fourier transform. Prior this operation we multiply the frequency bins with a length 162 Hanning filter. By this operation the system impulse response will be dominated by the impulse response of the Hanning filter and sidelobes are attenuated. The variance of this filter is 29ns (i.e. $\sigma_f = 2.910^{-8}$ in the framework above). This delay-spread is negligible in comparison with the delay-spreads we expect from the channel. Thus we estimate the delay-spread directly from the estimated PDP without any compensation. However, the PDP is post-processed with a thesholding operation in order to reduce the influence of noise. Thus the PDP estimate is set to zero in positions where the level is weaker than the peak of the PDP minus 35dB.

The data is measured using eight transmitter and eight receiver antennas. The development above basically treats a SISO channel. However here we use one of the transmitter antennas and two of the receiver antennas to match the conditions in [1]. We treat the two MIMO channel components as multiple impulse responses from one single SISO channel. We further consider twenty wavelengths of measurements within each observation interval. In all this gives typically 500-1000 channel impulse responses per estimated power-delay profile (PDP).

From the same set of data we also estimate the delay-spread according to the correlation based method described above and used in [1]. We have implemented three variants of this method using three different sets of CW settings. These variants are given in Table III. The second and third settings are readily available from the data-set used. The first requires interpolation between subcarriers. A sixth order interpolation filter

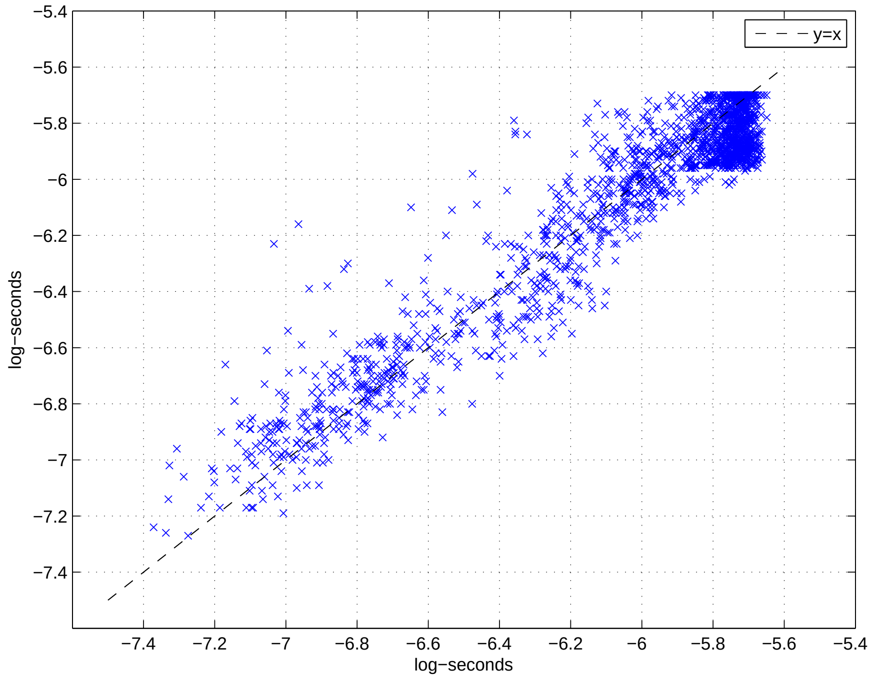

Figure 1: Performance of the correlation based method with frequency setting 1. Each "x" marks the results obtained from one observation interval. The x-axis is the result obtained using the PDP based method and the y-axis the correlation based result.

was used. The frequency spacings used are for practical purposes identical to the ones used in [1]. As in [1] we combine two receiving antennas to increase the precision of the estimates.

The estimates from the two delay-spread estimation methods are presented in Figure 1-3 using frequency setting 1-3, respectively (the estimates from the PDP based method is the same in all three plots).

The results show good results using the correlation based method and the smallest spacing, while the quality for the larger spacings are more questionable. The reason for this behavior is probably that the larger correlation values are more robust to modelling errors and noise.

III. The Angle-Spread Estimation Algorithms

As in the derivation of the delay-spread methods we assume that there are N propagation paths with instantaneous complex amplitudes β_k with average powers α_k. We assume further that the kth path arrives at azimuth angle θ_k at the base-station. We define the *power azimuth spectrum* (PAS) as

$$p_{\text{PAS}}(\theta) = \sum_{k=1}^{N} \alpha_k \delta(\theta - \theta_k).. \tag{17}$$

The angle-spread is defined as

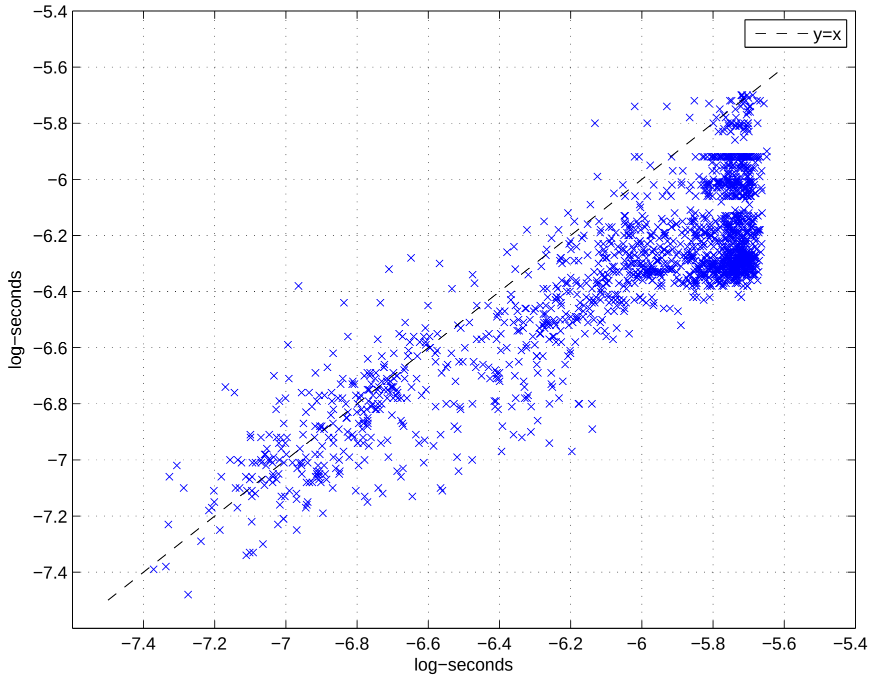

Figure 2: Performance of the correlation based method with frequency setting 1. Each "x" marks the results obtained from one observation interval. The x-axis is the result obtained using the PDP based method and the y-axis the correlation based result.

$$\sigma_{\mathrm{AS}}^2 = \sum_{k=1}^{N} \alpha_k (\theta_k - \bar{\theta})^2, \tag{18}$$

where

$$\bar{\theta} = \frac{1}{\sum \alpha_k} \sum \alpha_k \theta_k. \tag{19}$$

This definition is slightly different from the circular definition in [1] but for limited spreads the two definitions are practically identical.

A. Power Azimuth Spread Based Estimate

Here we derive an azimuth spread estimation algorithm which is similar to the one described in [2]. If we point a narrow-beam antenna with antenna pattern $p_{\mathrm{a}}(\theta)$ towards direction θ_0 the power of the received signal becomes

$$\breve{p}_{\mathrm{PAS}}(\theta_0) = \sum_{k=1}^{N} \alpha_k p_{\mathrm{a}}(\theta_k - \theta_0) = p_{\mathrm{PAS}}(\theta) \star p_{\mathrm{a}}(-\theta) \tag{20}$$

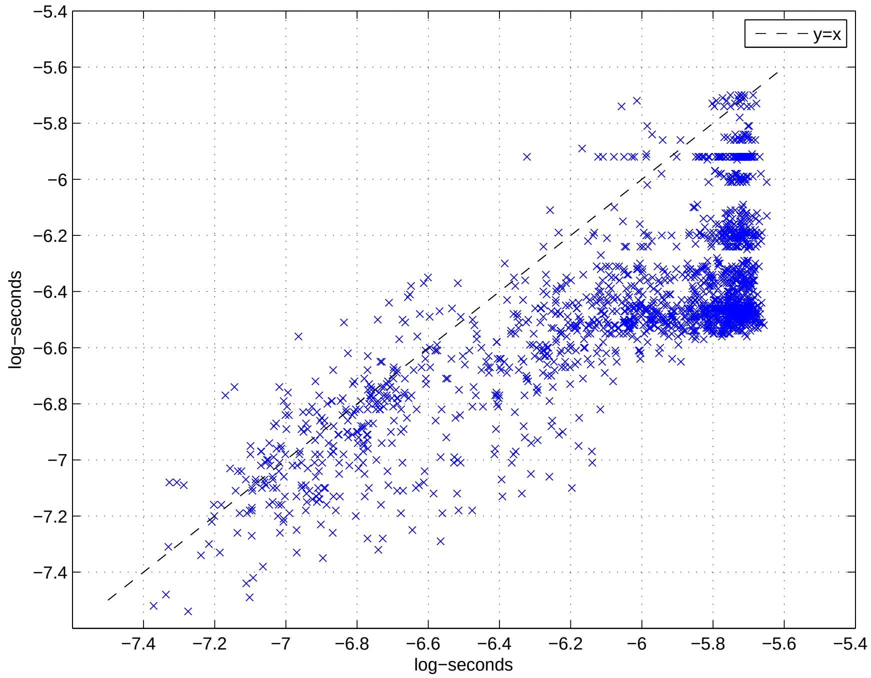

Figure 3: Performance of the correlation based method with frequency setting 1. Each "x" marks the results obtained from one observation interval. The x-axis is the result obtained using the PDP based method and the y-axis the correlation based result.

Thus if we could sweep a narrow-beam over the azimuth we could get an estimate of the PAS just as described for the PDP above. Can we do that? Here we will use the data-set described in [3] (2004 measurements, Vanadis sector B). By applying weights

$$\mathbf{w}(\theta_0) = \frac{1}{|\mathbf{a}(\theta_0)|}\mathbf{a}(\theta_0), \tag{21}$$

we can realize a beam pointing in direction θ_0 with pattern

$$p_{\mathrm{a}}(\theta, \theta_0) = |\mathbf{a}^*(\theta_0)\mathbf{a}(\theta)|^2. \tag{22}$$

where $\mathbf{a}(\theta)$ is the array response vector given by

$$\mathbf{a}(\theta) = e(\theta)[1, \exp(-j2\pi\Delta\sin(\theta)), \exp(-j4\pi\Delta\sin(\theta)), \exp(-j6\pi\Delta\sin(\theta))]^T, \tag{23}$$

Δ is the spacing between the antenna elements in wavelengths, and $e(\theta)$ is the element amplitude pattern. With this beam our PAS estimate becomes

$$\hat{p}_{\text{PAS}}(\theta_0) = \int_\theta p_{\text{PAS}}(\theta) \star p_{\text{a}}(\theta - \theta_0, \theta_0) d\theta \tag{24}$$

$$= \int_\theta p_{\text{PAS}}(\theta) |\mathbf{a}^*(\theta_0)\mathbf{a}(\theta - \theta_0)|^2 d\theta \tag{25}$$

One difference between this expression and (9) is that the system response is not just a function of $\theta - \theta_0$ i.e. it is not angular invariant (compare time invariant). Therefore we can not directly apply an analogy of (10). Instead we define a local beam-variance as

$$\sigma_a^2(\theta_0) = k(\theta_0) \int_\theta (\theta - \bar{\theta})^2 p_{\text{a}}(\theta, \theta_0) d\theta, \tag{26}$$

where

$$\bar{\theta} = k(\theta_0) \int_\theta p_a(\theta, \theta_0) d\theta \tag{27}$$

and

$$k(\theta_0) = \frac{1}{\int_\theta p_a(\theta, \theta_0) d\theta}. \tag{28}$$

Figure 4 illustrates $\sigma_a(\theta_0)$ for a four-element antenna array with an antenna spacing of 0.56 wavelengths and an element pattern $e(\theta) = \max(\cos(\theta), 0.1)$, which models the antenna used herein and the one used in [1]. If the maxima of the PAS is located at $\tilde{\theta}_0$, we may assume that most of the energy is concentrated at the vicinity of $\tilde{\theta}_0$. Therefore the "spread" of the beam should be given by $\sigma_a(\tilde{\theta}_0)$. Thus we may obtain the angle-spread from the measurements from the following approximation

$$\sigma_{\text{AS}}^2 \approx \tilde{\sigma}_{\text{AS}}^2 - \sigma_a^2(\tilde{\theta}_0), \tag{29}$$

where $\tilde{\sigma}_{\text{AS}}$ is the uncompensated angle-spread obtained directly from $\hat{p}_{\text{PAS}}(\theta)$.

B. Comparison of PAS and Correlation Based RMS Angle Spread Estimates

The estimation algorithm described above is on the data-set described in [3] (2004 measurements, Vanadis sector B). Only mobile-positions within ±20 degrees is used. Moreover, the PAS is estimated only within a sector of ±50 degrees. These two measures are used to avoid the problem with "spatial aliasing" occurring for the PAS based method when a signal enters the array at angles around 50 degrees or greater (the array used has an inter-element spacing of 0.56 wavelengths). The correlation based method described in [1] is applied on the same set of data in order to compare the estimates. The results are shown in Figure 5 below. The correlation based method gives only integer values on the delay-spread as it uses a grid based search with the RMS angle-spread quantized to integer degrees. The difference between the two methods is typically 2-3 degrees. Note that the two algorithms use a different set of antennas elements. If the correlations between

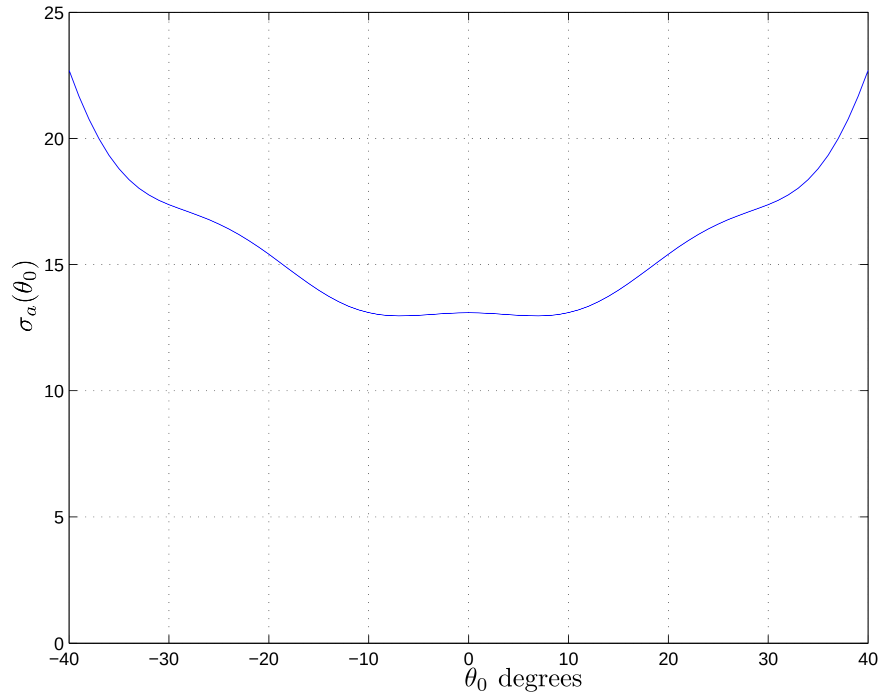

Figure 4: Illustration of the variance due to the array beamwidth.

the antenna pairs #1-#2, #2-#3, and #3-#4 are averaged, the estimates from the two algorithms become more similar. The results are as shown in Figure 6 below. The results from the two algorithms are similar. The angle-spreads from the PAS based method is generally larger. The reason for this is probably that it is more susceptible to calibration errors and antenna modeling errors. Such errors will generally widen it's beam which increases the angle-spread estimate.

IV. Conclusion

We conclude that the simple correlation based delay-spread method works well with the smallest frequency spacing, i.e. the three tones are separated some 180kHz, when the RMS angle-spread is in the region 40ns to 1μs. The simple correlation based angle-spread estimation method gives similar results as a power azimuth spectrum PAS based method - thereby showing the reliability of the results.

References

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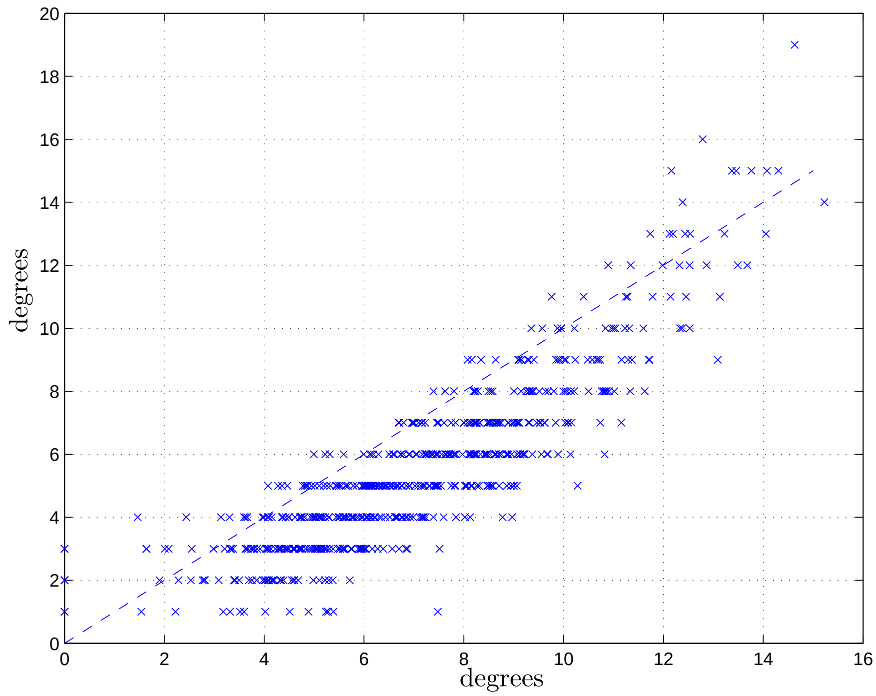

Figure 5: Comparison of PAS and Correlation Based Angle-Spread Estimation. Each measurement is marked with an 'x'. The x-axis of the 'x' correspond to the PAS method and the y-axis the correlation based estimate.

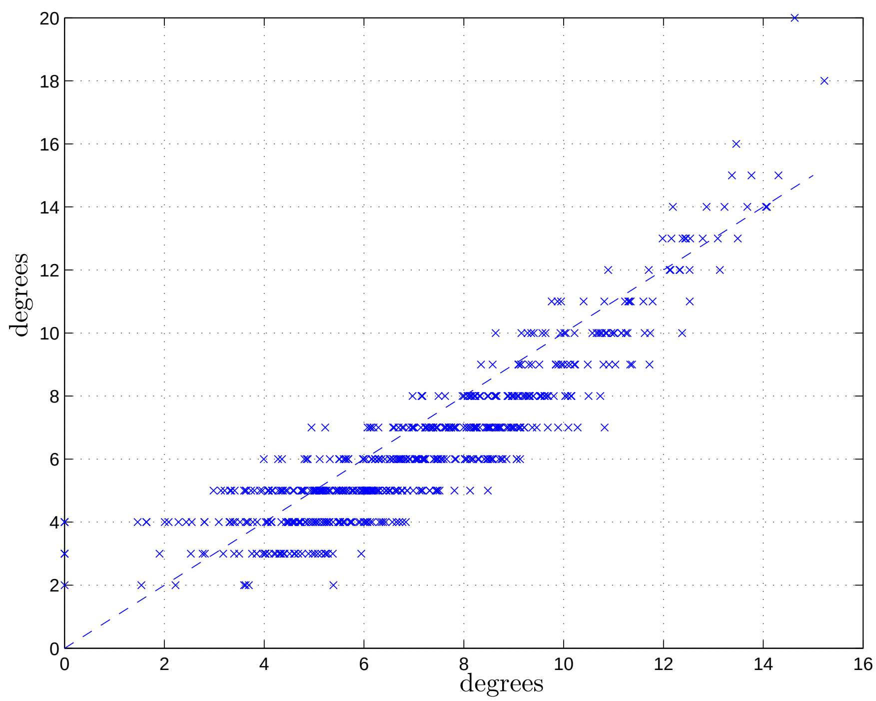

Figure 6: Comparison of PAS and Correlation Based Angle-Spread Estimation. Each measurement is marked with an 'x'. The x-axis of the 'x' correspond to the PAS method and the y-axis the correlation based estimate.