Piracy and Competition

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The effects of (private, small-scale) piracy on the pricing behavior of producers of information goods are studied within a unified model of vertical differentiation. Although information goods are assumed to be perfectly differentiated, demands are interdependent because the copying technology exhibits increasing returns to scale. We characterize the Bertrand–Nash equilibria in a duopoly. Comparing equilibrium prices to the prices set by a multiproduct monopolist, we show that competition drives prices up and may lead to price dispersion. Competition reduces total surplus in the short run but provides higher incentives to create in the long run.

1. Introduction

Over the last decade, the fast penetration of the Internet and the increased digitization of information goods like music, movies, and software have turned the issue of piracy into a topic of intense debate. Not surprisingly, economists have recently shown a renewed interest in information goods piracy.[1] Recent contributions revive the literature on the economics of copying and copyright, which was initiated some 20 years ago.[2] While early contributions focus on the effects of photocopying and examined how publishers can indirectly appropriate some revenues from illegitimate users (Novos and Waldman, 1984; Liebowitz, 1985; Johnson, 1985; Besen and Kirby, 1989), later papers concentrate on the intellectual property (IP) protection and discuss

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1. See the excellent survey by Peitz and Waelbroeck (2006) and the references therein.
2. For a recent survey (and extension) of this literature, see Watt (2000).

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the trade-off between the *ex ante* benefit of preserving the intellectual creation incentives and the *ex post* cost of the consequent restraint on the use of those goods (Landes and Posner, 1989; Besen and Raskind, 1991; and more recently Bae and Choi, 2006).[3]

Yet, the literature on the economics of copying abstracts away the strategic interaction among producers of information goods. Indeed, all the above-mentioned contributions focus on the study of monopolies, with the exception of Johnson (1985) who considers price-taking producers of information goods. However, this perspective sharply contrasts with the reality of information good industries where production decisions are concentrated in the hands of a small number of major players. The oligopolistic nature of those industries leaves economic observers unsatisfied with the monopoly or price-taking representation of production under the threat of copying. In particular, such a representation is unable to relate to some important characteristics of information good industries such as the recent consolidation phases[4] and the existence of price dispersion.[5] The present paper aims to address those issues.

The aim of the present paper is to study the strategic interactions among the producers of information goods in the presence of piracy. In particular, we want to analyze how oligopolistic producers set the prices of their information goods when users are able to purchase copying devices and to copy the goods. We also want to investigate the *ex ante* and *ex post* benefits of oligopolistic competition under this threat of copying. We finally want to relate those results to those obtained under the monopoly assumption (Bae and Choi, 2006) or under the perfect competition assumption (Johnson, 1985).

3. From an *ex ante* point of view, IP protection preserves the incentive to create information goods, which are inherently public goods. On the other hand, IP rights encompass various potential inefficiencies from an *ex post* point of view. The protection grants de facto monopoly rights, which generates the standard deadweight losses; also, by inhibiting imitation, IP rights might limit the creators' ability to borrow from, or build upon, earlier works, and thereby increase the cost of producing new ideas.

4. In the music industry, the 50–50 joint venture of BMG and Sony was cleared in 2004 both by the FTC and by the European Commission (although, in July 2006, the EU's Court of First Instance threw out the previous approval, finding that regulators did not sufficiently investigate whether the combination would create a monopoly). In the software industry, in September 2005, Oracle Corporation acquired Siebel Systems just one day after eBay acquired Skype. In the movie industry, Sony absorbed MGM and United Artists studios in 2005.

5. For instance, Brynjolfsson and Smith (2000) observe prices for a matched set of 20 books and 20 CDs sold through conventional and Internet outlets, and report average price differences ranging from 25% for books to 33% for CDs. For an updated measure of price dispersion over a wider range of products (including computer software), see the ongoing research project of Michael Baye, John Morgan, and Patrick Scholten on http://www.nash-equilibrium.com/.

Our modeling strategy is the following. As is usually done in the literature, we consider information goods with independent content.[6] All other things being equal, the demands for the (two, for simplicity) goods would be independent. However, we allow consumers to make lower-quality copies by using a technology that exhibits increasing returns to scale. The copying technology appears thus as a common substitute for the information goods. The main effect of such a common substitute is to make the demands for the goods interdependent over some range of prices. In particular, the goods become complementary when their prices are similar enough and they remain independent otherwise. More precisely, the demand function for a particular information good typically exhibits three segments and two kinks. Each segment corresponds to a different category of consumers. We call these categories "buyers," "copiers," and "switchers." For the first two categories, the demand for a good does not depend on the price of the other good: indeed, whatever the price of the other good, "buyers" buy this other good and "copiers" copy this other good. In contrast, for "switchers," the best use of one good depends on the best use of the other good: if they purchase (copy) one good, they also purchase (copy) the other one. Therefore, the demand for one good depends on both prices.

Using this framework, we contrast the behavior of a multiproduct monopolist with the behavior of two Bertrand duopolists. We also perform a welfare analysis, both from a static and from a dynamic perspective. Our main results are the following.

As far as the multiproduct monopoly is concerned, we show that the firm may set different prices for its goods. Actually, the multiproduct monopolist may follow two strategies. The first strategy is to set close prices and target the demand by switchers. The second strategy is to set a high price and target the buyers in one market while setting a low price and targetting copiers in the other market. It turns out that the former strategy dominates the latter. The firm prefers to target switchers for whom only the sum of the two prices matters. Therefore, prices are neither unique nor symmetric. The monopolist might well set different prices for the two goods although consumers value these goods exactly in the same way. Yet, the two prices cannot be too distant so as to avoid some consumers becoming buyers or copiers. This result can be interpreted as a first explanation for a form of limited price dispersion, confirming the empirical evidence.

6. Think, for instance, of software applications for games and word processing, or CD recordings of classic and pop music.

In a duopoly, the interaction between firms leads to interesting properties. A firm's best response to the price set by the competitor can depict up to four different attitudes. Because the nature of the marginal buyer suddenly changes as the competitor's price rises, the best-response function shows discontinuities and equilibria in pure strategies cannot be guaranteed. Intuitively, the inexistence of equilibria stems from the firms' free-riding behavior with respect to the threat of piracy. If all firms take this threat seriously and quote low prices to accommodate consumers, then they set too low a price and there exists an opportunity for any individual firm to raise its price while keeping a sufficiently large demand and making a larger profit. Technically, increasing returns to scale in the copying technology introduce nonconvexities in the profit functions and undermine the existence of a market equilibrium.

Consequently, we have to distinguish between two regions of parameters. The first region corresponds to a sufficiently large cost of copying. In this region, there exists a symmetric equilibrium in pure strategies. At this equilibrium, both firms target switchers and quote identical prices. The interesting feature of this equilibrium is that the duopolists set a higher price than the (average) price of a multiproduct monopoly. This is so because the two goods are perfect complements over the segment of demand corresponding to the switchers. We observe thus a manifestation of the so-called "Cournot effect" (Cournot, 1838). That is, the multiproduct monopolist has an incentive to decrease prices further than the duopolists do because it realizes that decreasing the price for one good increases demand for the other good by making copying less attractive. Although Johnson (1985) briefly mentions this effect, he does not analyze it in detail. As will be seen in the sequel, the existence of complementarities for some price ranges is important for understanding why the industry may not reach a single price equilibrium.

In the second region of parameters, where the fixed cost of copying is sufficiently low, an equilibrium in pure strategies fails to exist. Yet, we show that a symmetric equilibrium in mixed strategies exists. Each firm quotes two prices with positive probabilities. As price realizations may be different, we have an explanation for equilibrium price dispersion. Noteworthy is the fact that this explanation does not rely, as is often proposed in the literature, on asymmetric information and search frictions (see, for instance, Varian, 1980; Baye and Morgan, 2001). It is also interesting to note that the expected price in the mixed-strategy equilibrium, though smaller than the price that would prevail in the pure-strategy equilibrium, remains above the average price set by a multiproduct monopolist. The Cournot effect eases but does not disappear.

Finally, we perform a welfare analysis. Considering first *ex post* efficiency, we stress that industry concentration is welfare improving in the present context. Because of the Cournot effect, the multiproduct monopoly leads to larger consumer and producer surpluses than the duopoly. A merger is thus beneficial under the threat of copying because it eliminates the negative externality resulting from the lack of coordination about how firms should deter copying. This is a novel point that has not yet been considered by antitrust agencies. We also assess the effects of policy measures aiming at strengthening IP protection; we show that increasing the cost of copying and decreasing the quality of copies do not have the same qualitative effects. Considering next *ex ante* efficiency, we compare our framework with an economy where only a single information good is available. This exercise allows us to measure the (gross) incentives to create a new information good. Whether those incentives are larger for an entrant or for an incumbent firm is not clear *a priori*. Indeed, the entrant's incentives are reduced by the free-riding effect observed in a duopoly, whereas the incumbent's incentives are reduced by a cannibalization effect (copying becomes more attractive as the number of goods increases). Yet, we can conclude in our framework that incentives to create are always higher for an entrant, that is, if the *ex post* economy is organized as a duopoly. Therefore, although industry concentration improves welfare from a static perspective, it reduces welfare from a dynamic perspective. In other words, *ex post* competition can be seen as a necessary evil that enhances *ex ante* incentives to create. This conclusion turns on its head the traditional argument underlying IP protection, which considers *ex post* monopoly—and not competition—as the necessary evil.

To sum up, our main message is the following. The interactions between producers of information goods under the threat of piracy dramatically alter the equilibrium outcome compared to the outcome obtained under a one-good monopoly setting. Equilibrium prices in pure strategies may not exist and, if they do, they may be higher than those in the one-good monopoly case. Inferences about dynamics and welfare implications are not obvious anymore in oligopolistic industries. For instance, industry concentration may enhance static efficiency while being detrimental to dynamic efficiency.

The rest of the paper is organized as follows. In Section 2, we lay out the model and we derive the demand schedule for a particular original. In Section 3, we characterize the two-good monopoly case. In Section 4, we present the two-good duopoly case. In Section 5, we perform a welfare analysis. We conclude and propose an agenda for future research in the last section.

2. Demand for Originals

There is a continuum of potential users who can consume at most two information goods. These information goods are assumed to be perfectly (horizontally) differentiated and equally valued by the users. In particular, users are characterized by their valuation, θ, for any information good. We assume that θ is uniformly distributed on the interval $[\underline{\theta}, \bar{\theta}]$, with $\underline{\theta} > 0$.

Each information good is imperfectly protected and thus "piratable." As a result, users can obtain each information good in two different ways: they can either *buy* the copyrighted product (an "original") or make a *copy* of the product. It is reasonable to assume that all users see the copy as a lower-quality alternative to the original.[7] Therefore, in the spirit of Mussa and Rosen (1978), we posit some vertical (quality) differentiation between the two variants of any information good: letting s_o and s_c denote, respectively, the quality of an original and a copy, we assume that $0 < s_c < s_o$.[8] So, a user's valuation of an original and a copy is, respectively, θs_o and θs_c. As a result, a user's valuation of the extra quality provided by an original (with respect to a copy) is $\theta(s_o - s_c)$. Thus, parameter θ also measures a user's willingness to pay for the extra quality provided by an original. As originals are perfectly horizontally differentiated, it makes sense to consider that this willingness to pay for extra quality does not depend on the very characteristics of the information good. Therefore, if a user puts a higher premium on quality than another user for good 1 (say a CD of classical music), she also puts a higher premium on quality for good 2 (say a CD of rock music).[9]

As for the relative cost of originals and copies, we let p_i denote the price of original i ($i = 1, 2$) and we assume that users have access to

7. This assumption is common (see, e.g., Gayer and Shy, 2003) and may be justified in several ways. In the case of analog reproduction, copies represent poor substitutes of originals and are rather costly to distribute. Although this is no longer true for digital reproduction, originals might still provide users with a higher level of services, insofar as they are bundled with valuable complementary products, which can hardly be obtained otherwise.

8. Similar models are used by Koboldt (1995) to consider commercial copying and by Yoon (2002) and Bae and Choi (2006) to analyze the market for a single information good.

9. This specification implies that the preferences for the two goods are positively correlated among users. This positive correlation clearly drives our results, but a negative correlation seems to square hardly with real-life situations (why would users who put a high premium on the original of one good systematically be those who put a low premium on the original of the other good?). In addition, a more general specification with independent valuations for the two goods makes the analysis untractable.

a copying technology with increasing returns to scale. To keep things simple, we assume that to be able to copy, consumers must incur a fixed cost $K > 0$.[10] Finally, for the sake of the exposition, we further assume that all users prefer copying a single original over not using any information good:

A1: $\underline{\theta} s_c - K \geq 0$.

Assumption A1 is likely to be satisfied in industries like music, software, and video because, in the digital era, (1) the quality of copies (s_c) is high, (2) the fixed cost of copying (K) is low, and (3) users have a high valuation (θ) for information goods.[11] This assumption greatly simplifies the exposition whereas it retains the main properties of the model.[12] Indeed, it follows that all users will always consume both goods, either by purchasing the original or by copying it.

The demand function for good i is therefore derived as follows. One can write that a user with type θ buys original i iff,

$$\theta s_o - p_i + \max\{\theta s_o - p_j; \theta s_c - K\}$$
$$\geq \max\{\theta s_o - p_j + \theta s_c - K; 2\theta s_c - K\}. \quad (1)$$

This inequality compares user θ's value of purchasing original i, and either purchasing or copying good j, to the best option available given that he or she does not purchase original i, namely, copying good i whereas either buying or copying good j.

In the extreme case where $K = 0$ (copies are free), expression (1) rewrites as

$$\theta s_o - p_i + \max\{\theta s_o - p_j; \theta s_c\} \quad \geq \quad \theta s_c + \max\{\theta s_o - p_j; \theta s_c\}$$
$$\iff \theta(s_o - s_c) \geq p_i.$$

In that case, the demand for good i only depends on p_i: the two goods are independent.

10. For simplicity, we assume away the possibility for a user to share the copying technology with N other users (e.g., within clubs or families). Yet, it can be shown that the effect of sharing in groups of N users is equivalent to the effect of a decrease in the cost of the copying technology from K to K/N. This equivalence holds provided that (1) users form groups of the same size N; (2) all users within a particular group have the same valuation θ; and (3) users split the cost K equally within each group.

11. Another way to justify this assumption is to say that the fraction of users we choose not to consider (i.e., those for whom $\theta s_c - K < 0$) is constantly narrowing, as copying devices become widely and cheaply available and as the "moral barrier" to illegal copying is increasingly fading. The widespread use of copied music and software in less developed countries corroborates this assumption.

12. Relaxing this assumption introduces additional complications in the analysis (i.e., additional kinks in the product demand and so, additional jumps in best response functions), without bringing any further insight.

However, for $K > 0$, increasing returns to scale in copying make the demands interdependent. Inequality (1) can take three different forms, each form corresponding to a specific category of users.

First, for high-valuation users such that $\theta(s_o - s_c) \geq p_j$, expression (1) rewrites as

$$\theta s_o - p_i + \theta s_o - p_j \geq \theta s_o - p_j + \theta s_c - K \iff \theta \geq \frac{p_i - K}{s_o - s_c}.$$

Because these users purchase the other original whether they purchase good i or copy it, we call them *buyers*. The maximum price they are willing to pay for original i is equal to

$$p_i^b(\theta) = \theta(s_o - s_c) + K.$$

That is, they are willing to pay up to the extra value that an original brings on top of a copy, augmented by the cost of the copying technology (which they save once they decide to buy i instead of copying it).

Second, for intermediate valuation users such that $p_j - K \leq \theta(s_o - s_c) \leq p_j$, expression (1) rewrites as

$$\theta s_o - p_i + \theta s_o - p_j \geq 2\theta s_c - K \iff \theta \geq \frac{p_i + p_j - K}{2(s_o - s_c)}.$$

For these users, the best use of one good depends on the best use of the other good: if they purchase good i, they also purchase good j; if they copy good i, they also copy good j. We therefore call them *switchers*. How much are switchers willing to pay for good i? Going from two copies to two originals, they earn twice the extra value of an original compared to a copy, and they trade the cost of the copying technology for the price of the other original. So, their maximum price is given by

$$p_i^s(\theta, p_j) = 2\theta(s_o - s_c) + K - p_j.$$

Finally, for low-valuation users such that $\theta(s_o - s_c) \leq p_j - K$, expression (1) rewrites as

$$\theta s_o - p_i + \theta s_c - K \geq 2\theta s_c - K \iff \theta \geq \frac{p_i}{s_o - s_c}.$$

Because these users copy good j no matter what they decide about good i, we call them *copiers*. What they are willing to pay for good i is just the extra value of an original compared to a copy (for they have already sunk the cost of the copying technology). Their maximum price is thus equal to

$$p_i^c(\theta) = \theta(s_o - s_c).$$

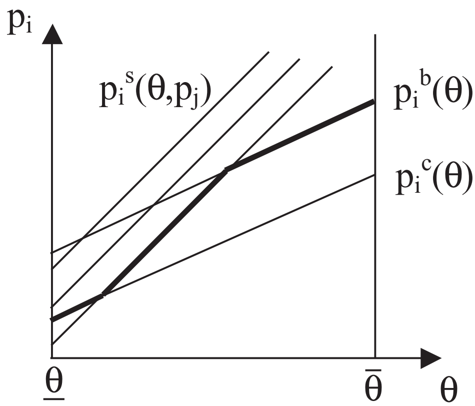

FIGURE 1. PRICES AS A FUNCTION OF θ

The three price functions are depicted in Figure 1. We observe that depending on the price of good j, the price function for good i can have up to two kinks. The price function is (increasing and) concave in θ in the neighborhood of $p_j/(s_o - s_c)$ (which separates switchers from buyers), and (increasing and) convex in θ in the neighborhood of $(p_j - K)/(s_o - s_c)$ (which separates copiers from switchers).

As a result, the demand function for good i has three segments and two kinks,

$$D_i(p_i, p_j) = \frac{1}{\bar{\theta} - \underline{\theta}} \times \begin{cases} \left(\bar{\theta} - \dfrac{p_i - K}{s}\right) & \text{if} \quad p_j + K \leq p_i \text{ (buyers)} \\ \left(\bar{\theta} - \dfrac{p_i + p_j - K}{2s}\right) & \text{if} \quad p_j - K \leq p_i < p_j \\ & \quad + K \text{ (switchers)} \\ \left(\bar{\theta} - \dfrac{p_i}{s}\right) & \text{if} \quad p_i < p_j - K \text{ (copiers)} \end{cases} \tag{2}$$

where, to ease the exposition, we denote by s the quality difference between originals and copies,

$s \equiv s_o - s_c.$

Two remarks are in order. First, as long as $K > 0$, there exists a range of prices for which goods are *complements*. Indeed, for $p_j - K \leq p_i < p_j + K$, an increase in the price of one good decreases the demand for the other. When p_j is close to p_i, the marginal users choose to copy according to the value of the bundle, $p_i + p_j$. For other prices, the goods remain independent: when p_j is relatively smaller than p_i, the marginal users are buyers; when p_j is relatively larger than p_i, the marginal users are copiers.

Second, as long as $K > 0$, the demand function has the same concave and convex kinks as the price function depicted above. These kinks result from the fact that the marginal consumer alters his or her behavior about copying at some pivot prices p_i. Similar kinks would appear for other distributions of consumers' types θ than the uniform distribution assumed here.

The case of price competition over perfect complements is well known since Cournot (1838). Yet, the present model differs from Cournot's case as complementarity only takes place over a limited range of prices. A similar property appears in Gabszewicz et al. (2001) who consider imperfect complements (i.e., the joint consumption of two products provides an extra utility but products can still be consumed individually). In our setting, however, complementarity is not built in consumers' preferences but stems, indirectly, from the existence of a common substitute.

We now analyze the pricing decisions in the monopoly and duopoly cases. To make this analysis relevant we assume that in any possible demand regime, decision makers never find it optimal to cover the whole market. Under the present demand system, this means that high valuation consumers are willing to pay significantly more for the goods than low valuation consumers. A sufficient condition is given by the following assumption:[13]

A2: $\bar{\theta}s > \underline{\theta}(s + s_o)$.

3. Multiproduct Monopoly

Since Cournot (1838), the analysis of a multiproduct monopoly selling perfect complements is well known: the firm sets a unique price reflecting only the value of the bundle. Yet, in the present paper, the case of multiproduct monopoly deserves some attention. Indeed, as goods are complement only over a limited range of prices, it is not sure whether the firm's optimal prices will lie in that range. Indeed, when prices are close, the firm faces only switchers. By contrast, when prices differ by more than the fixed cost K, the firm faces two groups of consumers: copiers and buyers. The firm may follow two strategies. On the one

13. Together, Assumptions A1 and A2 give a fair representation of the reality as they guarantee that there are always some consumers who pirate at least one good.

hand it may set close prices and target the demand by switchers. On the other hand, the firm may set a high price and target the buyers in one market, while it sets a low price and targets the copiers in the other market, the prices being sufficiently distant to avoid some consumers to become switchers. Doing so, the firm is able to collect high revenues on the buyers who have a high willingness to pay. It nevertheless turns out that the latter strategy is not optimal.

Formally, the monopoly chooses prices p_1 and p_2 so as to maximize profits,

$$\max_{p_1, p_2} \pi_m = p_1 D_1(p_1, p_2) + p_2 D_2(p_1, p_2),$$

where demands are given by (2) and where the firm is assumed to have zero production cost.

Proposition 1: *The multiproduct monopolist sets any price* (p_1, p_2) *such that* $(p_1 + p_2)/2 = p_m \equiv \bar{\theta}s/2 + K/4$ *and* $p_2 - K < p_1 \leq p_2$.

Proof. See the Appendix.

We prove in the Appendix that the strategy where the monopolist targets buyers and copiers on separate markets is dominated because the optimal prices on each market are not distant enough (i.e., they do not satisfy $|p_i - p_j| > K$).[14] As a result, the monopolist sells the two goods at prices such that marginal buyers are switchers. The difference between the two prices is limited upward to avoid that marginal users become copiers or buyers.

It is important to note that prices are neither unique nor symmetric. The monopolist might well set different prices for the two goods although consumers value these goods exactly in the same way. It is because the copying technology offers a common substitute for the goods and because the monopolist prefers to target switchers that only the sum of the two prices matters for the consumers, as well as for the monopolist. Our setting slightly contrasts Cournot's (1838) discussion about perfect complements because products are here not genuine complements. Henceforth, the monopolist is constrained to set its prices in some price range. For instance, the firm has no possibility of setting a zero price for one good and collecting its revenue on the other as it would be possible for perfect complements. Consumers are likely to take the free good and copy the other good.

Several additional comments are in order. First, Assumption A1 implies that *the monopolist sets an average price* p_m *that is smaller than*

14. It is possible to show that this result holds for alternative specifications of the preferences for the two goods provided that types are sufficiently correlated.

the price it would set for each good if there were no threat of piracy. To compute the latter price, note that a user $\theta \in [\underline{\theta}, \bar{\theta}]$ who purchases the original good i gets a utility of $\theta s_o - p_i$. If copying is not an option, the demand for this good is simply equal to $D_i(p_i) = (\bar{\theta} - p_i/s_o)/(\bar{\theta} - \underline{\theta})$. The optimal price for the monopolist is easily found as $p_M = \bar{\theta}s_o/2$. Simple computations establish that $p_M > p_m \iff \bar{\theta}s_c > K/2$, which is clearly implied by Assumption A1. It follows that the presence of piracy also reduces profits. Second, it is easily seen that p_m *decreases as copies become a closer substitute for originals* (i.e., if K decreases and/or s_c increases). Finally, in the extreme case where $K = 0$, the demands for the two goods are independent and the monopolists sets the same price for both: $p_1 = p_2 = \bar{\theta}s/2$.

We now examine the case of the duopoly.

4. Duopoly

Under a duopoly, each information good $i \in \{1, 2\}$ is produced and sold by a separate firm. In the limiting case where $K = 0$, demands are independent and the producers act as local monopolists; they set the same prices as the multiproduct monopolist would do in this situation: $p_1 = p_2 = \bar{\theta}s/2$. However, for $K > 0$, the interdependence between the demand functions is a source of strategic interaction. To analyze this interaction, we proceed in two steps: first, we derive firm i's best reponse and then we compute the Bertrand–Nash price equilibria.

4.1 Best Response Function

Best-response functions are derived from the demand functions (2). Because the demand functions are piece-wise linear and include a convex kink, firms' best-response functions are expected to be discontinuous. In fact, the point of discontinuity will take place when marginal users shift from being switchers to copiers. We now characterize the portion of the best response of firm i below and above the discontinuity.

Targeting buyers or switchers? The optimal price on the buyers of good i is equal to

$$p_i^{b*} \equiv \arg\max_{p_i} p_i \left(\bar{\theta} - \frac{p_i - K}{s}\right) = \frac{1}{2}(\bar{\theta}s + K).$$

Firm i's best response is to set $p_i = p_i^{b*}$ as long as the competitor's price does not to entice the marginal consumer to become a switcher. Using (2), this is so as long as

$$p_j \leq p_i^{b*} - K \iff p_j \leq p^f \equiv \frac{1}{2}(\bar{\theta}s - K).$$

Note that $p^f > 0$ under Assumptions A1 and A2.

For $p_j > p^f$, some low-valuation users are enticed to switch to copying. Firm i can either accommodate these switching users by lowering its price, or it can avoid them and concentrate on higher-valuation users by increasing its price. On the one hand, when p_j is low enough, firm i sets a "limit price" to "deter" switchers. By (2), it sets a price equal to

$$p_i^D(p_j) = p_j + K$$

(or just a small amount below this price) and achieves a corresponding profit of $\pi_i^D(p_j)$. This price is an increasing function of p_j. Because more users tend to switch to the copying technology when the competitor raises its price p_j, firm i must raise its price p_i to avoid the switchers. Hence, there exists a range of prices such that prices are *strategic complements*.

When p_j gets larger, firm i has no other choice but to accommodate switchers. It sets a price equal to $p_i^{s*}(p_j)$ where

$$p_i^{s*}(p_j) \equiv \arg\max_{p_i} p_i \left(\bar{\theta} - \frac{p_i + p_j - K}{2s}\right) = \bar{\theta}s + \frac{K - p_j}{2},$$

and achieves a corresponding profit of $\pi_i^{s*}(p_j)$. The price $p_i^{s*}(p_j)$ is a decreasing function of the competitor's price; prices are then *strategic substitutes* in this range of prices.

The transition between deterrence and accommodation of switchers takes place at the price p^d such that deterence and accommodation of switchers yield the same profit and thus the same price: $p_i^D(p^d) = p_i^{s*}(p^d)$, or equivalently

$$p^d = \frac{1}{3}(2\bar{\theta}s - K) > p^f.$$

Targeting copiers? Because of the convex kink in the demand function, the shift from switchers to copiers has to be analyzed by comparing profit levels. The optimal price and profit on copiers are equal to

$$p_i^{c*} \equiv \arg\max_{p_i} p_i \left(\bar{\theta} - \frac{p_i}{s}\right) = \frac{1}{2}\bar{\theta}s \quad \text{and} \quad \pi_i^{c*} = \frac{1}{4}\bar{\theta}^2 s.$$

We readily get that

$$\pi_i^{s*}(p_j) > \pi_i^{c*} \iff p_j < p^e \equiv (2 - \sqrt{2})\bar{\theta}s + K.$$

The regime including accommodation of switchers is part of the best-response function as long as

$$p^d < p^e \iff K > \frac{3\sqrt{2}-4}{4}\bar{\theta}s. \tag{3}$$

In this case there exists a downard jump at $p_j = p^e$.

Otherwise, accommodation of switchers is not part of the best-response function and the latter has a downward jump from deterrence of switchers to accommodation of copiers for another price $p_j = p^{e\prime}$, where $\pi_i^D(p^{e\prime}) = \pi_i^{c*}$, which is equivalent to

$$p^{e\prime} = \frac{1}{2}(\bar{\theta}s - K) + \frac{1}{2}\sqrt{K(2\bar{\theta}s + K)}.$$

Summarizing our results, we have under condition (3), the best-response function is given by

$$p_i^*(p_j) = \begin{cases} p_i^{b*} & \text{if } p_j \leq p^f, \\ p_i^D(p_j) & \text{if } p^f \leq p_j \leq p^d, \\ p_i^{s*}(p_j) & \text{if } p^d \leq p_j \leq p^e, \\ p_i^{c*} & \text{if } p_j > p^e. \end{cases}$$

Otherwise it is given by

$$p_i^*(p_j) = \begin{cases} p_i^{b*} & \text{if } p_j \leq p^f, \\ p_i^D(p_j) & \text{if } p^f \leq p_j \leq p^{e\prime}, \\ p_i^{c*} & \text{if } p_j > p^{e\prime}. \end{cases}$$

Figure 2 displays these functions (in black for firms 1 and in gray for firm 2) for 'high' and 'low' fixed cost of copying (resp. in the left- and right-hand panel).

4.2 Existence of Equilibria in Pure Strategies

Because of discontinuities in the best-response functions, equilibria in pure strategies might fail to exist. Intuitively, the possible inexistence of equilibria stems from firms' free-riding behavior with respect to the threat of copying. If both firms take this threat seriously and quote low prices to accommodate copiers, then there exists an opportunity for either firm to raise its price while keeping a sufficiently large demand and making a larger profit. This situation is shown in the right-hand panel of Figure 2 where best-response functions do no intersect. By contrast, the left-hand panel shows the situation where firms reach

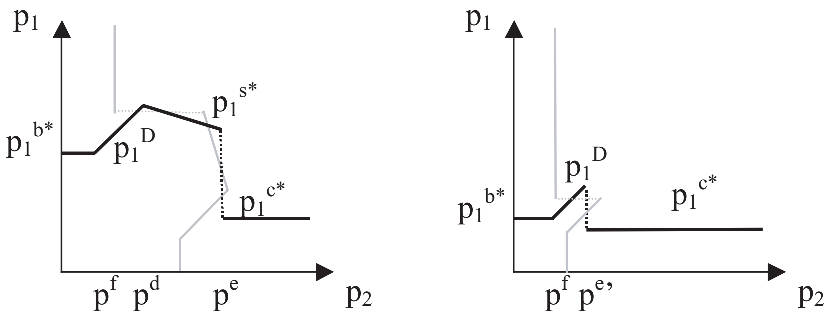

FIGURE 2. BEST RESPONSE FUNCTIONS

an equilibrium as their best-response function intersect at a symmetric equilibrium. More formally, we can state the following proposition.

Proposition 2: *There exists a unique symmetric Nash equilibrium in which both firms focus on switchers and set the price* $p_S \equiv \frac{1}{3}(2\bar{\theta}s + K)$ *if and only if*

$$K > \hat{K} \equiv \frac{3\sqrt{2} - 4}{2}\bar{\theta}s. \tag{4}$$

Otherwise, there is no Nash equilibrium in pure strategies.

Proof. See the Appendix.

Proposition 2 tells us that the only possible Nash equilibrium in pure strategies is symmetric and is such that both firms target switchers. This result could not have been guessed at the outset. Moreover, we also see that the market fails to reach an equilibrium for small fixed costs of copying because the price p_S and the profit associated to this strategy decrease with K. For a low enough value of K, profits under accommodation of copiers become more attractive and firms tend to cut their price to p_i^{c*}. As a result, the absence of duopoly equilibria for low fixed costs of the copying technology casts some doubts on the traditional analyses of the threat of copying in one-good monopoly settings.

4.3 Pure-Strategy Equilibrium: Cournot Effect

We first focus on the situation in which the market reaches a symmetric equilibrium. One observes that

$$p_S = \frac{1}{3}(2\bar{\theta}s + K) > p_m = \frac{1}{4}(2\bar{\theta}s + K).$$

Corollary 1: *The price set by duopolists at the pure-strategy equilibrium is higher than the average price set by the multiproduct monopolist.*

At the symmetric equilibrium in pure strategies, the duopolists focus on switchers. In such a case, the copying technology constitutes a common substitute for their original good. The presence of this common substitute turns goods i and j (which are a priori perfectly horizontally differentiated) into complementary goods. As a result, the so-called Cournot effect (Cournot, 1838) applies. That is, the multiproduct monopolist has an incentive to decrease prices further than the duopolists do because it realizes that decreasing the price for one good increases demand for the other good by making copying less attractive.

The externality that each firm imposes on the other can be quite important. Indeed, if the quality of copies is sufficiently low, the duopolists end up setting *prices higher than the price they would set under no threat of copying*.[15] As we noted above, in the absence of piracy, the demands for the two goods are independent. Hence, the duopolists set the same price as a monopolist, that is, $p_M = \frac{1}{2}\bar{\theta}s_o$. We observe that the price set by duopolists at the pure-strategy equilibrium (p_S) is *higher* than the monopoly price under no threat of copying (p_M) if and only if $K \geq \frac{1}{2}\bar{\theta}(4s_c - s_o)$, which is clearly satisfied if $s_c < s_o/4$.

The last two findings qualify the argument that the threat of piracy forces firms to lower their prices and that the usage of the copyrighted product increases with piracy. Instead, our model gives some evidence to the common claim of copyright holders, who assert that piracy reduces their sales. These results also cast some doubt on the social benefits of stronger competition in information good markets that are subject to potential piracy. The last two findings indeed suggest that a more concentrated industry is better equipped to provide surplus both to legal consumers and to producers. We consider the latter issue in more detail in Section 5.

4.4 Mixed-Strategy Equilibria: Price Dispersion

When $K < \hat{K}$, there exists no equilibrium in pure strategies. Nevertheless, by Glicksberg (1952), there exists a mixed-strategy Nash equilibrium because profits are continuous. Therefore, firms randomize prices

15. The possibility of the duopolists raising prices above the monopoly price has also been noted in the context of the pharmaceutical industry (we thank a coeditor for bringing this issue to our attention). As argued by Frank and Salkever (1997), when a patent expires, the former patent holder may raise price to serve the high end of the market and leave it to the firms producing generic drugs to sell to the low end of the market. This is for pure market segmentation reasons. In the present model, the price-raising effect stems from a coordination failure between the duopolists to deter piracy.

at equilibrium when the cost of the copying technology is low enough. As a result, price dispersion can be observed. In this section, we first characterize a simple and intuitive class of mixed-strategy equilibria; we then discuss the impact of the copying technology on price dispersion.

As in Boccard and Wauthy (1997, 2003), the piecewise linearity of the demand function allows us to show that firms do not use continuous densities. This allows us to focus on mixed-strategy equilibria in which firms play two prices with the same probability distributions. The following proposition shows that such equilibria exist provided that fixed costs are not too small.

Proposition 3: *When $\hat{K} > K > \tilde{K} \equiv 0.0274\bar{\theta}s$, there exists an equilibrium where firms randomize between the prices*

$$p_a = \frac{2\bar{\theta}s + xK}{4 - x} \quad \text{and} \quad p_b = \frac{2\bar{\theta}s + (x+1)K}{x+3} > p_a$$

with probabilities x and $1 - x$. The probability x is equal to zero when K is equal to $\hat{K}$, it increases when K decreases below $\hat{K}$ and it is equal to $x = 0.3603$ when K tends to $\tilde{K}$. Prices are such that $p_b > p_a + K$.

Proof. See the Appendix.

Unfortunately, the probability x has no explicit expression. Numerical simulations show that for any admissible set of parameters, the probability x monotonically increases when K falls from $\hat{K}$ to $\tilde{K}$. When $K < \tilde{K}$, symmetric mixed strategies with two prices are not equilibria; firms have to randomize over a larger number of prices. The characterization of such equilibria goes beyond the scope of this paper.

Because $p_b > p_a + K$, the mixed-strategy equilibrium yields *ex post* realizations that include the three regimes with a positive probability. Each firm faces switchers when its price realization is equal to the other firm's realization; when price realizations are different, a firm faces buyers when it quotes the highest price and copiers when it quotes the lowest price. Hence, by playing mixed strategies, firms are able to avoid the negative Cournot effect of competing on a segment of demand where goods are perfect complements. Indeed, with probability $x(1 - x)$, they set the prices (p_b, p_a) and the firm setting p_b collects revenues on high-valuation consumers (i.e., the "buyers").

One can check that prices can be ranked as in the following corollary.

Corollary 2: *(1) Prices are ranked as follows: $p_i^{c*} < p_a < p_a + K < p_b < p_S$. (2) The expected price is higher than the average monopoly price: $xp_a + (1 - x)p_b > p_m$.*

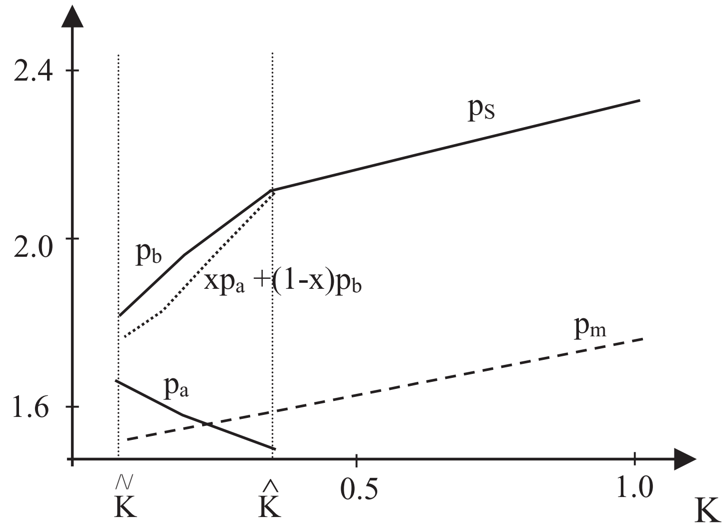

FIGURE 3. PRICES AS A FUNCTION OF K

Figure 3 illustrates these results. The intuition behind part (1) goes as follows: firm i has no incentive to set prices below p_i^{c*} because, if it does, it gets a positive marginal revenue irrespective of firm j's mixed strategy; similarly, it gets a *negative* marginal revenue whenever it sets a price *above* p_S. This result demonstrates that the Cournot effect is less acute than in the pure-strategy equilibrium. However, part (2) shows that the Cournot effect does not disappear completely: the expected price at the mixed-strategy equilibrium is still higher than the average price set by a multiproduct monopolist. In other words, firms would like to coordinate on a limit-price to eliminate piracy but are unable to commit to do so as they prefer to free-ride on the rival's effort. Therefore, they choose random prices as a way to share the burden of deterring piracy. Yet, this joint effort remains insufficient.

The absence of a pure-strategy equilibrium in prices is often presented as an explanation for *equilibrium price dispersion*. In consumer search models (see, e.g., Varian, 1980; Baye and Morgan, 2001), equilibrium price dispersion arises because firms are tempted to lower price to attract informed consumers, but realize that if they do so, they forego rents from uninformed consumers; therefore, each firm intentionally randomizes prices to reduce the ability of rival firms to undercut its own price.

Our model generates similar price dispersion results and shares a similar intuition. Yet, it does not rely on search frictions and information asymmetries because our consumers are perfectly informed about prices and, therefore, do not need to search. Here, firms also face the tension between lowering price to attract low-valuation users (the copiers) and increasing price to extract rents from high-valuation consumers (the buyers). The difference with search models is that, in our setting, the existence of various classes of consumers results, endogenously, from the presence of the copying technology. In fact, price dispersion is even a prerequisite for two of these classes to exist at equilibrium; indeed, all consumers are switchers when prices do not differ by more than K.

A number of empirical studies document significant and persistent price dispersion on markets for information goods.[16] The previous argument relates such price dispersion to piracy. It is interesting to study the impact of the copying technology on price dispersion. On the one hand, supposing that firms quote two prices, it can readily be shown that the price dispersion $p_b - p_a$ decreases when K falls. On the other hand, when firms quote more than two prices, we can also show that the range $[p_i^{c*}, p_S]$, within which price atoms must lie, also shrinks as K decreases. Hence, the model predicts that price dispersion is likely to be lower for information goods that are more exposed to piracy.[17]

5. Welfare Analysis

As indicated in Section 1, the economics of IP protection discusses the trade-off between *ex ante* and *ex post* efficiency considerations: to remedy the long-run underproduction problem that might arise from insufficient incentives to create, the law grants exclusive rights to creators, which entail a short-run underutilization problem.

Our simple framework allows us to shed some new light on this policy debate. First, in a short-run perspective, we can perform comparative statics exercises to assess the effects of stronger IP protection; we can also compare the welfare performances of two market structures, namely a multiproduct monopoly versus a duopoly. Second, in a long-run perspective, we can measure incentives to create and compare again the relative merits of monopoly and duopoly. As we now explain, policy implications are not clear cut. Indeed, *from a static perspective, the multiproduct monopoly enhances welfare with respect to the duopoly, but from a dynamic perspective, the duopoly provides higher incentives to create than the monopoly.*

16. See, for example, Bailey (1998), Brynjolfsson and Smith (2000), and Iyer and Pazgal (2003).

17. Recall, nevertheless, that no dispersion is observed for $K > \hat{K}$ (as a unique pure-strategy equilibrium exists).

5.1 *Ex Post* Efficiency Considerations

In many discussions, the protection of IP rights calls for an increase in the cost of piracy (Novos and Waldman, 1984; Yoon, 2002, etc). In this model, this would call for two policy measures: first, one can increase the fixed cost of the copying technology K by, for example, applying a tax on the reproduction devices; second, one can take actions to decrease the value of a copy s_c. Therefore, we assess the effects of a change in K and in s_c for the multiproduct monopoly and for the duopoly. As for the duopoly, we restrict for now our attention to the case where K is sufficently large so that an equilibrium in pure strategies exists. We consider the mixed-strategy equilibrium case below using numerical simulations.

When a pure-strategy equilibrium exists in the duopoly, we can easily analyze the effects of a marginal strengthening of IP rights. Let $\theta_S \equiv (4\bar{\theta}s - K)/(6s)$ be the type of the switching user at the equilibrium prices p_S. This is the lowest type among the consumers who purchase an original good. Users with type $\theta \in [\theta_S, \bar{\theta}]$ purchase an original whereas, by Assumption A1, users with type $\theta \in [\underline{\theta}, \theta_S)$ make use of copies (we call them "pirates"). Similarly, in the multiproduct monopoly, let $\theta_m \equiv (2p_m - K)/(2s) = (2\bar{\theta}s - K)/(4s)$ denote the marginal user at price p_m.

We do not need to perform separate comparative statics analyses for the monopoly and for the duopoly as the equilibrium price and demand vary in the same direction in the two cases. We observe that the two policy measures have different effects. On the one hand, a rise in the copying cost ($dK > 0$) implies an upward parallel shift of the demand for original goods. It then increases equilibrium prices ($dp_S, dp_m > 0$) and increases the set of consumers buying original goods ($d\theta_S, d\theta_m < 0$). The number of pirates falls. On the other hand, a deterioration of the value of copies ($ds_c < 0$) implies a rotation of the demand, thereby reducing its elasticity. The deterioration of the value of copies then leads to an increase in price ($dp_S, dp_m > 0$) and to a reduction of the set of consumers buying original goods ($d\theta_S, d\theta_m > 0$). The number of pirates increases.

In the Appendix, we formally establish the following results, which generalize those obtained by Bae and Choi (2006) for a single-product monopoly.[18] First, *the two ways of strengthening IP protection increase (optimal or equilibrium) profits*. This result is trivial for an increase in K as both the price and the quantity demanded increase. As for a decrease in s_c, price and quantity move in opposite directions, meaning that the result is a priori ambiguous. However, under Assumptions A1

18. What Bae and Choi (2006) call the "copying regime" corresponds to the regime faced by our multiproduct monopolist and by our duopolists around the symmetric pure-strategy equilibrium.

and A2, it can be shown that the positive effect on price outweighs the negative effect on demand.

Second, *the two ways of strengthening IP protection decrease consumer surplus*. To see this, we observe that both categories of users suffer from a stronger protection: first and obviously, pirates are negatively affected by the decrease in the attractiveness of copies (either through a larger cost or through a lower quality); second, buyers of originals are charged a higher price (as $dK > 0$ and $ds_c < 0$ both imply $dp_S > 0$ and $dp_m > 0$).

Finally, we evaluate the effect on social surplus by adding the two previous effects. The change in social surplus writes as $dW = -(\theta_k - \underline{\theta})\, dK - (\theta_k - \underline{\theta})^2\, ds_c - 2p_k\, d\theta_k$ (with $k = m$ for the monopoly case and $k = S$ for the duopoly case). The overall effect is negative if the additional costs imposed on all pirates is lower than the additional revenue from having an additional legal consumer. This is so when the policy measure consists of deteriorating the copies. For $ds_c < 0$ and $dK = 0$, $dW = -(\theta_k - \underline{\theta})^2 ds_c - 2p_k\, d\theta_k < 0$: *strenghtening IP protection by making copies less valuable decreases social surplus*. On the other hand, the policy measure may consist of an increase in the fixed cost of copying ($dK > 0$, $ds_c = 0$). There are two possible cases. First, the copy technology is taxed and the proceeds of the tax are efficiently redistributed to the consumers through lump sum transfers, so that the increase of K has no direct impact on welfare. In that case, $dW = -2p_k\, d\theta_k > 0$ as θ_k decreases after the increase of K: *strenghtening IP protection by taxing the copying technology (and redistributing the tax proceeds) increases social surplus.* Second, if the tax proceeds are not (or inefficiently) redistributed, then $dW = -(\theta_k - \underline{\theta})\, dK - 2p_k\, d\theta_k$ and the sign of the welfare change is ambiguous. We show in the Appendix that *strenghtening IP protection by making copies more costly decreases social surplus if K is not too large; it increases social surplus otherwise.*

To close our static welfare analysis, we repeat the conclusion we drew in the previous section: the multiproduct monopolist always sets lower prices than duopolists do; as the monopolist also achieves higher profits, *social surplus is undoubtedly higher under a multiproduct monopoly than under a duopoly*.

Finally, we provide results from numerical simulations pointing that some of the previous results might not hold in the region of parameters where symmetric mixed strategies with two prices exist. For the sake of the exposition, we concentrate on the impact of K. Figure 4 depicts an example that shows one noticeable difference with the symmetric equilibrium regime: as K increases, expected duopoly profits ($E\pi$) *decrease* in some range. The explanation is the following: as K increases, the prices p_a and p_b move in opposite directions and therefore, although the expected price increases, expected profits decrease.

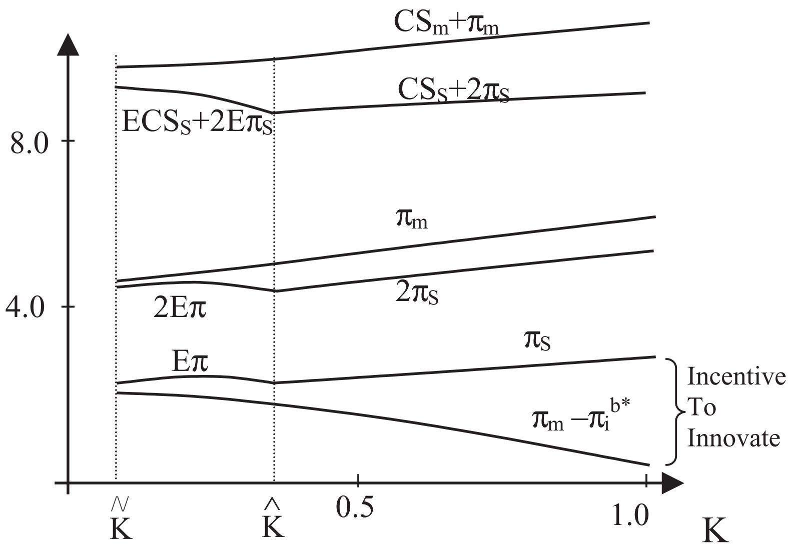

FIGURE 4. PROFITS, CONSUMER SURPLUSES AND INCENTIVE TO INNOVATE AS A FUNCTION OF K

5.2 *Ex Ante* Efficiency Considerations

Comparing our framework with an economy where only a single information good is available allows us to measure the (gross) incentive to create a new information good. In the previous model, if only one information good is available instead of two, it is easy to see that under Assumption A1, the producer of this good only faces "buyers." The demand function is thus given by $D_i(p_i) = (\bar{\theta} - (p_i - K)/s)/(\bar{\theta} - \underline{\theta})$, and the optimal price and profit are given by $p_i^{b*} = (\bar{\theta}s + K)/2$ and $\pi_i^{b*} = (\bar{\theta}s + K)^2/(4s)$.

There are two cases to consider when going from one to two goods. A first possibility is that the new good is created by an incumbent firm that already produces the extant good; the *ex post* economy is then organized as a multiproduct monopoly. From Proposition 1, we know that the multiproduct monopolist's optimal (average) price and profit are given by $p_m = (2\bar{\theta}s + K)/4$ and $\pi_m = (2\bar{\theta}s + K)^2/(8s)$. One readily observes that, under Assumptions A1 and A2, we have that $\pi_m > \pi_i^{b*}$, meaning that an incumbent firm has a gross incentive to introduce a second good. Still, although goods are genuinely independent, the profit per good decreases when the number of goods rises: $\pi_m < 2\pi_i^{b*}$. Indeed, *the monopolist jeopardizes the sales of the first good when it introduces the second good.* Copying becomes more attractive when the number of goods is larger and the firm is compelled to reduce the average price of originals.

Alternatively, the new good could be created by an entrant firm, turning the *ex post* economy into a duopoly. Supposing for now that the condition of Proposition 2 is met, prices and profits at the pure-strategy equilibrium are given by $p_S \equiv (2\bar{\theta}s + K)/3$ and $\pi_S = (2\bar{\theta}s + K)^2/(18s)$.

Let us now compare the two scenarios. Comparing prices, it is easily checked that Assumptions A1 and A2 imply the following ranking: $p_m < p_i^{b*} < p_S$. Therefore, *the average price decreases when the new good is introduced by an incumbent firm, whereas it increases when the new good is introduced by an entrant.* This is another illustration of the negative externality that independent producers impose on each other, and on consumers, in the presence of copying.

Next, comparing profits, we can gauge the (gross) incentive to create in the two settings. We say that an entrant has higher incentives to introduce a second good than an incumbent if the following condition is met: $\pi_S > \pi_m - \pi_i^{b*}$. Under no threat of piracy, goods are independent and incentives for the incumbent and the entrant are exactly equal. However, under piracy, the question is whether the free-riding problem between duopolists harms the entrant to a greater or to a lesser extent than the cannibalization effect hurts the incumbent. In the Appendix, we show that *when a pure-strategy equilibrium exists in the duopoly game, the duopoly always yields larger incentives to create than the multiproduct monopoly.* Formally, we show that $K > \hat{K}$ implies that $\pi_S > \pi_m - \pi_i^{b*}$: the cannibalization effect is stronger than the free-riding effect.

As illustrated in Figure 4, the previous conclusion may still apply in the region of parameters where symmetric mixed-strategy equilibria with two prices exist.

6. Conclusion

In this paper, we qualify the traditional results and insights about the impact of piracy obtained in a one-good monopoly setting. When there exist more than one information good, increasing returns to scale in the copying technology create an interdependence between the demands for information goods, which would be independent otherwise. We first show that a multiproduct monopoly may set different prices for its goods. We then show that two-product duopolies are subject to free-riding behaviors with respect to the threat of piracy. If the two firms take this threat seriously by quoting low prices, then there exists an opportunity for a firm to take advantage of this situation and to raise its price. This can lead to the absence of an equilibrium in pure strategies if the fixed cost of copying is low enough. In this case, firms may randomize between several prices. To the best of our knowledge, this is the first contribution showing that price dispersion in the information good industries can be generated by the presence of piracy. When the fixed cost of copying is not too small, the market can yield a symmetric equilibrium with prices that are larger than the (average) price of the multiproduct monopoly. Furthermore, those prices can even become larger than the price of a monopoly that faces no threat of piracy. The externality that firms impose on each other can therefore be quite important and it can drastically reduce the demand for legal copies.

In sum, the interactions between producers of information goods under the threat of piracy dramatically alter the equilibrium outcome compared to the outcome obtained under a one-good monopoly setting. Taking those interactions into account also yields surprising welfare implications: concentration appears as welfare-enhancing from a static perspective but welfare-detrimental from a dynamic perspective.

The present model suggests several avenues of future research. First, the current study is limited to the production of two perfectly differentiated information goods. It would be worthwhile to explore the pricing decisions and welfare aspects under piracy threat in a setting with more numerous and less differentiated varieties. Second, by assuming exogenous production and pricing of the copying technology, the current model sets aside the strategic issue of integration between the creators (or distributors) of information goods and the sellers of copying devices. It seems natural to investigate about the competition and welfare implications of such integration processes.

Appendix

A.1 Proof of Proposition 1

Suppose w.l.o.g. that $p_1 \leq p_2$. Then, the monopolist gets the following profits according to whether its two prices significantly differ or not:[19]

$$\text{either } \max_{p_1, p_2} \pi_m^{(1)} = (p_1 + p_2)\left(\bar{\theta} - \frac{p_1 + p_2 - K}{2s}\right) \text{ s.t. } p_1 \geq p_2 - K,$$

$$\text{or } \max_{p_1, p_2} \pi_m^{(2)} = p_1\left(\bar{\theta} - \frac{p_1}{s}\right) + p_2\left(\bar{\theta} - \frac{p_2 - K}{s}\right) \text{ s.t. } p_1 \leq p_2 - K.$$

19. Profits are actually multiplied by the constant $(\bar{\theta} - \underline{\theta})$, which we forget from now on as it does not affect optimal decisions.

The first problem is equivalent to

$$\max_p \pi_m = 2p\left(\bar{\theta} - \frac{2p-K}{2s}\right)$$

where $p \equiv (p_1 + p_2)/2$. Optimal price and profit are easily found as

$$p_m = \frac{\bar{\theta}}{2}s + \frac{K}{4} \text{ and } \pi_m^{(1)*} = \frac{(2\bar{\theta}s + K)^2}{8s}.$$

This problem includes an infinity of prices (p_1, p_2) such that $(p_1 + p_2)/2 = p_m$ subject to the contraint set in this first problem: $p_1 \geq p_2 - K$.

The unconstrained solution to the second problem is $p_1 = \bar{\theta}s/2$ and $p_2 = \bar{\theta}s/2 + K/2$. This solution does not meet the constraint because $p_1 > p_2 - K$. To find the corner solution, we set $p_1 = p_2 - K$ and rewrite the problem as

$$\max_{p_2} \pi_m^{(2)} = (p_2 - K)\left(\bar{\theta} - \frac{p_2 - K}{s}\right) + p_2\left(\bar{\theta} - \frac{p_2 - K}{s}\right).$$

The solution is found as $p_2 = (2\bar{\theta}s + 3K)/4$, which yields $p_1 = (2\bar{\theta}s - K)/4$ and

$$\pi_m^{(2)*} = \frac{(2\bar{\theta}s + K)^2}{8s} = \pi_m^{(1)*}.$$

As $p_1 = p_2 - K$, we have that the corner solution to the second problem belongs to the set of combinations (p_1, p_2) that solve the first problem, which completes the proof. □

A.2 Proof of Proposition 2

We first show that the optimal prices p_i^{b*}, $p_i^{s*}(p_j)$, and p_i^{c*} are interior solutions and do not lead to full-market coverage under Assumptions A1 and A2. Indeed, the price p_i^{b*} is an interior solution iff $p_i^{b*} > p_i^b(\underline{\theta}) \iff K < (\bar{\theta} - 2\underline{\theta})s$, which is true under A1 and A2. The price $p_i^{s*}(p_j)$ with $p_j \geq 0$ is an interior solution iff $p_i^s(\underline{\theta}, p_j) < p_i^{s*}(p_j) \iff K < p_j + 2(\bar{\theta} - 2\underline{\theta})(s_o - s_c)$, which is less stringent than the previous condition. Finally, the price p_i^{c*} is an interior solution iff $p_i^{c*} > p_i^c(\underline{\theta}) \iff \bar{\theta} - 2\underline{\theta} > 0$, which follows from A2.

Each best-response function $p_i^*(\cdot)$ and $p_j^*(\cdot)$ can have four segments. Removing symmetric configurations, we need to check the existence of a pure strategy equilibria for the 10 following configurations. For some configurations we will need to distinguish equilibrium conditions in which the switchers' branch $p_j^{s*}(\cdot)$ exists (i.e., $p^d \leq p^e$ or condition (3)) or in which it does not (i.e., $p^d > p^e$ or the reverse of condition (3)).

1. The configuration (p_i^{b*}, p_j^{b*}) cannot be an equilibrium because $p_j^{b*} > p^f$ and thus the best response of i cannot be equal to $p_i^{b*} : p_i^*(p_j^{b*}) \neq p_i^{b*}$.
2. The configuration $(p_i^D(\cdot), p_j^D(\cdot))$ cannot be an equilibrium because the system $p_i = p_i^D(p_j)$ and $p_j = p_j^D(p_i)$ has no solution.
3. The configuration $(p_i^{s*}(\cdot), p_j^{s*}(\cdot))$ is an equilibrium if and only if $K > \hat{K} \equiv \frac{3\sqrt{2}-4}{2}\bar{\theta}s$. Indeed, solving the system $p_i = p_i^{s*}(p_j)$ and $p_j = p_j^{s*}(p_i)$, we find $p_i = p_j = p_S \equiv \frac{1}{3}(2\bar{\theta}s + K)$. It is a best response for both firms to set $p_i = p_i^{s*}(p_j)$ if and only if $p^d \leq p_S \leq p^e$. The first inequality is clearly met, whereas the second is met provided that $p_S < p^e \iff K > \frac{3\sqrt{2}-4}{2}\bar{\theta}s$. This last condition is compatible with $p^d \leq p^e$.
4. The configuration (p_i^{c*}, p_j^{c*}) cannot be an equilibrium because one can check that $p_j^{c*} < p^e$ and $p_j^{c*} < p^{e\prime}$. Hence, $p_i^*(p_j^{c*}) \neq p_i^{c*}$.
5. The configuration $(p_i^{b*}, p_j^D(\cdot))$ cannot be an equilibrium because $p_j^D(p_i^{b*}) = p_i^{b*} + K > p^f$ and thus $p_i^*[p_j^D(p_i^{b*})] \neq p_i^{b*}$.
6. Similarly, the configuration $(p_i^{b*}, p_j^{s*}(\cdot))$ cannot be an equilibrium because $p_j^{s*}(p_i^{b*}) = \frac{3}{4}\bar{\theta}s + \frac{K}{4} > p^f$, and hence $p_i^*[p_j^{s*}(p_i^{b*})] \neq p_i^{b*}$.
7. The configuration (p_i^{b*}, p_j^{c*}) cannot be an equilibrium because when $p^d \leq p^e$, one can easily check that $p_i^{b*} < p^e$ so that $p_j^*(p_i^{b*}) \neq p_j^{c*}$. Also, when $p^d > p^e$, we get $p_i^{b*} < p^{e\prime}$ iff $K < \frac{2}{3}\bar{\theta}s$ which follows from Assumptions A1 and A2. Therefore, $p_j^*(p_i^{b*}) \neq p_j^{c*}$ when $p^e < p^d$.
8. The configuration $(p_i^D(\cdot), p_j^{s*}(\cdot))$ cannot be an equilibrium because solving for $p_i = p_i^D(p_j)$ and $p_j = p_j^{s*}(p_i)$, we get $p_j = \tilde{p}_j \equiv \frac{2}{3}\bar{\theta}s > p^d$, meaning that $p_i^*(\tilde{p}_j) \neq p_i^D(\tilde{p}_j)$.
9. The configuration $(p_i^D(\cdot), p_j^{c*})$ cannot be an equilibrium because when $p^d \leq p^e$, we have $p_i^D(p_j^{c*}) = p_j^{c*} + K = \frac{1}{2}\bar{\theta}s + K < p^e$. When $p^d > p^e$, we have $p_i^D(p_j^{c*}) = \frac{1}{2}\bar{\theta}s + K < p^{e\prime}$ iff $K < \frac{1}{4}\bar{\theta}s$, which is always true. Therefore, $p_j^*[p_i^D(p_j^{c*})] \neq p_j^{c*}$.
10. The configuration $(p_i^{c*}, p_j^{s*}(\cdot))$ cannot be an equilibrium because, for this to be an equilibrium, we should have (1) $p_j^{s*}(p_i^{c*}) = \frac{3}{4}\bar{\theta}s + \frac{K}{2} \geq p^e \iff K \leq 0.328\bar{\theta}s$, and (2) $p_i^{c*} \geq p^d \iff K \geq 0.5\bar{\theta}s$, which is incompatible with (1). □

A.3 Proof of Proposition 3

Firm i's profit is equal to $\pi(p_i, p_j)$ where

$$\pi(p_i, p_j) = \begin{cases} \pi^b(p_i) = p_i\left(\bar{\theta} - \dfrac{p_i - K}{s}\right) & \text{if } \ p_j \in [0, p_i - K) \\ \pi^s(p_i, p_j) = p_i\left(\bar{\theta} - \dfrac{p_i + p_j - K}{2s}\right) & \text{if } p_j \in [p_i - K, p_i + K), \\ \pi^c(p_i) = p_i\left(\bar{\theta} - \dfrac{p_i}{s}\right) & \text{if } p_j \in [p_i + K, \infty) \end{cases}$$

Each section of the profit function is concave in p_i.

We consider mixed-strategy equilibria that include two price atoms p_{ai} and p_{bi} played by player i with probabilities x_i and $1 - x_i$. Firm i's expected profit is equal to

$$\Pi_i = x_i x_j \pi(p_{ai}, p_{aj}) + x_i(1 - x_j)\pi(p_{ai}, p_{bj}) + (1 - x_i)x_j\pi(p_{bi}, p_{aj}) + (1 - x_i)(1 - x_j)\pi(p_{bi}, p_{bj}).$$

We look for a symmetric mixed-strategy equilibrium. This exists if we can find probabilities and prices such that $(x, p_a, p_b) = (x_i, p_{ai}, p_{bi})$, $i = 1, 2$ and

$$(x_i, p_{ai}, p_{bi}) = \arg\max_{(x_i, p_{ai}, p_{bi})} \Pi_i \text{ s.t. } (x_j, p_{aj}, p_{bj}) = (x, p_a, p_b), \quad i = 1, 2.$$

First, suppose that the symmetric equilibrium is such that $p_b \leq p_a + K$. For (x_i, p_{ai}, p_{bi}), $i \in \{1, 2\}$ close enough to (x, p_a, p_b), firm i's payoff is given by the function

$$\Pi_i = x_i x_j \pi^s(p_{ai}, p_{aj}) + x_i(1 - x_j)\pi^s(p_{ai}, p_{bj}) + (1 - x_i)x_j\pi^s(p_{bi}, p_{aj}) + (1 - x_i)(1 - x_j)\pi^s(p_{bi}, p_{bj}).$$

Because π^s is strictly concave, it is easy to show that there is a unique symmetric equilibrium with $p_S = p_{ai} = p_{bi}$, $i \in \{1, 2\}$. This is the equilibrium with pure strategies found in Proposition 2.

Second, suppose that the symmetric equilibrium is such that $p_b > p_a + K$. For (x_i, p_{ai}, p_{bi}), $i \in \{1, 2\}$ close enough to (x, p_a, p_b), firm i's expected payoff writes as

$$\Pi_i = x_i x_j \pi^s(p_{ai}, p_{aj}) + x_i(1 - x_j)\pi^c(p_{ai}) + (1 - x_i)x_j\pi^b(p_{bi}) + (1 - x_i)(1 - x_j)\pi^s(p_{bi}, p_{bj}).$$

There are three first-order conditions for equilibrium,

$$
\begin{aligned}
\frac{\partial \Pi_i}{\partial p_{ai}} = 0 &\iff x_i = 0 \quad \text{or} \quad x_j \pi^s_{p_i}(p_{ai}, p_{aj}) + (1 - x_j)\pi^c_{p_i}(p_{ai}) = 0 \\
\frac{\partial \Pi_i}{\partial p_{bi}} = 0 &\iff x_i = 1 \quad \text{or} \quad x_j \pi^b_{p_i}(p_{bi}) + (1 - x_j)\pi^s_{p_i}(p_{bi}, p_{bj}) = 0 \\
\frac{\partial \Pi_i}{\partial x_i} = 0 &\iff \left\{ \begin{array}{c} x_j \pi^s(p_{ai}, p_{aj}) + (1 - x_j)\pi^c(p_{ai}) \\ -x_j \pi^b(p_{bi}) - (1 - x_j)\pi^s(p_{bi}, p_{bj}) \end{array} \right\} = 0, \qquad \text{(A1)}
\end{aligned}
$$

where the subscript p_i denotes a partial differentiation w.r.t. to p_i. These three conditions guarantee a maximum because $\frac{\partial^2 \Pi_1}{\partial p_{a1}^2} < 0$, $\frac{\partial^2 \Pi_1}{\partial p_{b1}^2} < 0$ and the Hessian determinant is zero,

$$
|H| = \begin{vmatrix} \dfrac{\partial^2 \Pi_i}{\partial p_{ai}^2} & 0 & 0 \\ 0 & \dfrac{\partial^2 \Pi_i}{\partial p_{bi}^2} & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0.
$$

We now determine the symmetric equilibrium by setting $(x, p_a, p_b) = (x_i, p_{ai}, p_{bi})$, $i = 1, 2$ and $x_i = x \in (0, 1)$. We successively get

$$
p_a = \frac{2\bar{\theta}s + xK}{4 - x}, \quad p_b = \frac{2\bar{\theta}s + (x + 1)K}{x + 3}
$$

and

$$
x = \frac{\pi^s(p_b, p_b) - \pi^c(p_a)}{\pi^s(p_a, p_a) + \pi^s(p_b, p_b) - \pi^c(p_a) - \pi^b(p_b)}.
$$

One can check that $p_b > p_a + K$, which is consistent with the condition $p_{bi} > p_{ai} + K$, $i \in \{1, 2\}$.

Equilibrium profits can be computed as the following functions of x:

$$\pi^c(p_a) = ((2-x)\bar{\theta}s - xK)\frac{(xK + 2\bar{\theta}s)}{(x-4)^2 s},$$

$$\pi^s(p_a, p_a) = \frac{1}{2}(2(2-x)\bar{\theta}s + (4-3x)K)\frac{(xK + 2\bar{\theta}s)}{(x-4)^2 s},$$

$$\pi^s(p_b, p_b) = \frac{1}{2}((x+1)2\bar{\theta}s + (1-x)K)\frac{(2\bar{\theta}s + (x+1)K)}{(x+3)^2 s},$$

$$\pi^b(p_b) = (\bar{\theta}s(x+1) + 2K)\frac{(2\bar{\theta}s + (x+1)K)}{(x+3)^2 s}.$$

After some substitutions, x solves

$$x = \frac{\left[\begin{array}{c} (K^2 + 4\bar{\theta}sK)x^4 + (20K^2 - 2\bar{\theta}sK + 8\bar{\theta}^2 s^2)x^3 \\ + (3K^2 + 38\bar{\theta}sK - 12\bar{\theta}^2 s^2)x^2 + (-8K^2 + 20\bar{\theta}^2 s^2)x \\ + 16K^2 - 8\bar{\theta}^2 s^2 + 64\bar{\theta}sK \end{array}\right]}{2K(x+3)(4-x)(x^2 K + 2\bar{\theta}sx - \bar{\theta}s - 2K)}. \tag{A2}$$

Hence x is the solution of a polynomial with degree 5. There is at least one real solution. We have found no analytical solution.

Two cases can readily be studied. On the one hand, when $K = \hat{K} \equiv \frac{3\sqrt{2}-4}{2}s\bar{\theta}$, expression (A2) implies that $x = 0$. Furthermore, one can check that $[\frac{\partial^2 \Pi_i}{\partial x_i \partial x_j}]_{x_j=0} < 0$ and that $[\frac{\partial^2 \Pi_i}{\partial x_i \partial K}]_{x_j=0} < 0$ so that $[\frac{\partial x_j}{\partial K}]_{x_j=0} = [\frac{\partial x}{\partial K}]_{x=0} < 0$. A smaller K increases the probability x above zero. Hence, mixed strategy equilibria occur for $K < \hat{K}$ and pure strategy equilibria occur otherwise. Furthermore, when $x = 0$ and $K = \hat{K}$, we have that $\Pi_i = \pi_i^{c*}$ and that $d\Pi_i/dK = \partial\Pi_i/\partial K + (dx/dK)(\partial\Pi_i/\partial x) = (-10 + \sqrt{2})\bar{\theta} < 0$. Therefore, expected profits under symmetric mixed strategy increase above π_i^{c*} as K decreases below $\hat{K}$. For K smaller and close enough to $\hat{K}$, symmetric mixed strategy dominates the price strategy p_i^{c*}.

On the other hand, when $K \to 0$, we have that $\pi_1^s(p_a, p_a) = \pi_1^s(p_b, p_b) = \pi_1^c(p_a) = \pi_1^b(p_b)$. So the right-hand side of expression (6) is indefinite. To solve this problem, we approximate expression (6) by dropping terms in K of order larger than one and we get

$$K(x^4 - x^3 - x^2 + 3x + 8) + \frac{1}{2}\bar{\theta}s(2x-1)(x^2 - x + 2) = 0,$$

which yields the unique solution $x = 1/2$ when $K \to 0$. Applying this result, we get that the expected profits are equal to $\pi_i^{c*} - \bar{\theta}s/196$. Therefore, the symmetric strategy is dominated by the strategy p_i^{c*} when $K \to 0$.

The previous argument suggests that the two-price symmetric mixed strategy is a maximum as long as the firms set prices such that $p_{ai} + K < p_{bi}$, but that it can be dominated by the one-price strategy p_i^{c*} for small enough K. To check when the symmetric mixed strategy is a global maximum, we fix firm j's strategy as $(x_j, p_{aj}, p_{bj}) = (x, p_a, p_b)$ where $x > 0$, and we verify whether firm i can profitably deviate by fixing any other pair of prices. To this purpose, we sketch firm i's expected profit as a function of price p_i,

$$\Pi_i(p_i) = x\pi(p_i, p_a) + (1 - x)\pi(p_i, p_b).$$

Recall that $\pi(p_i, p_a)$ and $\pi(p_i, p_b)$ are combinations of three quadratic and concave functions. Then, it is readily shown that $\Pi_i(p_i)$ is a piece-wise quadratic and concave function. Consider p_i increasing from zero. One can check that the first section of $\Pi_i(p_i)$ is either increasing if $p_i^{c*} \geq p_a - K$, or bell-shaped with the maximum at p_i^{c*} if $p_i^{c*} < p_a - K$; the second section is bell-shaped with the maximum at $p_i = p_a$; the third section is monotonically increasing; the fourth section is bell-shaped with maximum at $p_i = p_b$, and the last section is monotonically decreasing. If $p_i^{c*} \geq p_a - K$, $p_i = p_a, p_b$ are the only candidates for a global maximum and the above solution is the unique mixed-strategy equilibrium with two price atoms.

On the other hand, if $p_i^{c*} < p_a - K$, there are three candidates for a global maximum: $p_i = p_a, p_b$ *and* $p_i = p_i^{c*}$. We need thus to check if $p_i = p_i^{c*}$ can yield a higher profit. If firm i plays $p_i = p_a$ or $p_i = p_b$, it achieves an expected profit equal to

$$\Pi_i^* \equiv x\pi^s(p_a, p_a) + (1 - x)\pi^c(p_a) = (2 - x)\frac{(2s\bar{\theta} + Kx)^2}{2s(4 - x)^2}.$$

Otherwise, if firm i plays $p_i = p_i^{c*}$, it achieves a profit equal to π_i^{c*}. Some computations show that $\Pi_i^* \geq \pi_i^{c*}$ is equivalent to

$$2x(2 - x)K^2 + 8\bar{\theta}s(2 - x)K - xs^2\bar{\theta}^2 \geq 0.$$

Hence, $K \geq K_1(x) \equiv \bar{\theta}s(2(4 - 2x) + (4 - x)\sqrt{4 - 2x})/[2x(x - 2)]$. Note that the probability x depends on K. To get the probability x that makes this inequality binding and that is simultaneously compatible with a mixed-strategy equilibrium, we insert $K_1(x)$ in expression (5), we evaluate at the symmetric mixed-strategy equilibrium to get

$$32 - 4(4 - x)\sqrt{4 - 2x}(1 + x^2)$$
$$+ x(48 + x(-1 + x(-8 + x(-1 + 2x)))) = 0.$$

This equation has a unique solution in the interval $x \in (0, 1)$, which is equal to $\tilde{x} = 0.3603$. The associated level of fixed cost of copying is equal to $\tilde{K} \equiv 0.02739\bar{\theta}s < \hat{K}$.

Hence, the above solution is not an equilibrium if $K < \tilde{K}$ and if $p_i^{c*} < p_a - K$. The latter condition rewrites as

$$K < \frac{x}{4(2-x)}\bar{\theta}s.$$

Using equation (A2), this is equivalent to $K < \tilde{K}' \equiv 0.04233\bar{\theta}s$, which is implied by $K < \tilde{K}$.

We conclude that the solution $(x_i, p_{ai}, p_{bi}) = (x, p_a, p_b)$, $i = 1, 2$, is a mixed-strategy equilibrium with two atoms provided that $K > \tilde{K}$. $\square$

A.4 Welfare Properties

As indicated in Section 5, we can apply the same analysis to the multiproduct monopoly and to the duopoly with a symmetric pure-strategy equilibrium. We note p_k and θ_k, respectively, for the optimal or equilibrium price and for the marginal user, with $k = m, S$.

Effects on profit. An increase in K induces variations in profits through effects on price and demand. Changes in the surplus of the two producers can be written as $dPS = 2(\bar{\theta} - \theta_k)\, dp_k - 2p_k\, d\theta_k$. This expression is clearly positive for $dK > 0$ as it implies $dp_k > 0$ and $d\theta_k < 0$. The sign for $ds_c < 0$ is a priori ambiguous. It is readily verified that $dPS/ds_c < 0$ iff $K < 2\bar{\theta}s$, which follows from the combination of Assumptions A1 and A2. Therefore, profits increase when copies are damaged or made more expensive.

Effects on consumer surplus. The consumers' surplus obtained from the use of both information goods includes four effects: the negative effect on illegal copiers because of the increase in the copying cost K; the negative effect of the deterioration of copies on the copying users; the negative effect of larger prices on legal consumers; and finally the effect on the switching users who move from copying to purchasing an original. At the price p_k, the latter effect on marginal users is nil because they are indifferent between copying and purchasing the orginals. Hence, $dCS = -(\theta_k - \underline{\theta})\, dK + (\theta_k - \underline{\theta})^2\, ds_c - 2(\bar{\theta} - \theta_k)\, dp_k + 0 * d\theta_k$, which is negative because all terms are nonpositive ($dK > 0$, $ds_c < 0$, $dp_k > 0$). Consumers are negatively affected by both policy measures.

Effects on social surplus. The change in social surplus writes as $dW = dPS + dCS = -(\theta_k - \underline{\theta})\, dK - (\theta_k - \underline{\theta})^2\, ds_c - 2p_k\, d\theta_k$. This expression is negative if the additional costs imposed on all copiers is smaller than the additional revenue from having an additional legal consumer. This is the case for $ds_c < 0$. For $dK > 0$ (and assuming no or inefficient redistribution of tax proceeds), the sign of the welfare change is ambiguous.

- In the monopoly case, one can show that $dW < 0$ iff $K < K_m \equiv \frac{2}{3}s(\bar{\theta} - 4\underline{\theta})$. The following numerical example shows that both cases can occur under our assumptions. Take $s_o = 7$, $s_c = 4$, $\bar{\theta} = 500$ and $\underline{\theta} = 100$. One checks that these values satisfy Assumption A2. To meet Assumption A1, we need $K < \underline{\theta}s_c = 400$. We check that $K_m = 200 < \underline{\theta}s_c$.
- In the duopoly case, one can show that $dW < 0$ iff $K < K_S \equiv \frac{2}{5}s(4\bar{\theta} - 9\underline{\theta})$. The following numerical example shows that both cases can occur under our assumptions. Take $s_o = 4$, $s_c = 1$, $\bar{\theta} = 244$ and $\underline{\theta} = 100$. One checks that these values satisfy Assumption A2. To meet the condition for a pure-strategy equilibrium and Assumption A1, we need $K \in (\hat{K}, \underline{\theta}s_c) = (88.8, 100)$. We check that $K_S = 91.2$ falls in the latter interval.

Higher incentives to create in duopoly. It is easy to check that $\pi_S > \pi_m - \pi_i^{b*}$ is equivalent to $K > (3\sqrt{10} - 8)\bar{\theta}s/13 \simeq 0.114\bar{\theta}s$, which is implied by $K > \hat{K} \simeq 0.121\bar{\theta}s$.

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